Chiral symmetry breaking, instantons, and monopoles in Lattice QCD

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Research Collaboration with
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Hadronic Matter under Extreme Conditions,
BLTP, JINR, Dubna, Russia, 1st, November, 2016

My proposal for the NICA

(1) Observation of the magnetic (Dirac) monopole in Bose-Einstein condensates.

A research group insists that they create isolated monopoles (topological defects) and detect in the condensed matter [**NATURE**, 505 (2014) 657 and **SCIENCE**, 348, 6234 (2015) 544]**.**

(2) Search for the magnetic monopole or the dyon in the Monopole and Exotics Detector at the LHC (MoEDAL) experiment.

This experiment is to search for the magnetic monopole or the dyon in the LHC. This experiment is approved in 2010 by the CERN. Preparations for the experiment is underway.

Can we observe QCD monopoles in the NICA?

My proposal for the NICA

- If we can observe monopoles in QCD in the NICA, the research would be really interesting.
- If monopoles really exist in the QCD vacuum, the observation of monopoles would be a direct evidence that the quarks are confined by the monopole condensation.
- If we can give some indications, for example, which signals are enhanced by the monopole condensation, the research would be also really useful for experiments.

So, I would like to suggest that we study the QCD monopoles in the NICA.

Contents

1. Monopole mass

- The scalar masses of the **gauge-independent monopole** and the glueball in quenched SU(2).

Previous research collaboration (2013 - 2014) with
K. Ishiguro (Univ. Kochi) and T. Suzuki (Univ. Kanazawa)

2. Effects of monopoles

- We estimate effects of monopoles on the chiral condensate, f_π, quark masses, and **PDF** (future).

Research collaboration with A. Di Giacomo (Univ. Pisa)

We further extend our studies, we would like to give ideas to detect monopoles in experiments.

[**Article,** M. H., *Chiral symmetry breaking, instantons, and monopoles in lattice QCD by Supercomputer SX*, Annual Report 2015, Topics 2015 of Research Group, RCNP, Osaka University, Japan (2016).]

Monopoles

What are monopoles in our study?

- **We focus on the Abelian and the Abelian-like monopole which are defined in QCD.**

(1) Abelian-like monopole in SU(2);

The gauge-invariant quantity is defined from the violation of the non-Abelian Bianchi identities [T. Suzuki, arXiv: 1402.1294].

The Abelian monopole is defined by ‘t Hooft [Proc, EPS, P. 1225, (1976)] and Mandelstam [Phys. Rept. 23 (1976) 245].

(2) Abelian monopole in SU(3);

The Wu-Yang form, (Abelian ‘t Hooft – Polyakov monopole in the non-Abelian gauge theory)

Glueball and monopole masses

- **Operator of the glueball [**Textbook, Rothe, P. 331**]**

$$\mathcal{O}_g(t) = \sum_{\mathbf{x}} \mathrm{Tr}\left(\tilde{U}_{12}(\mathbf{x},t) + \tilde{U}_{23}(\mathbf{x},t) + \tilde{U}_{31}(\mathbf{x},t)\right)$$

- **Operator of the gauge invariant monopole**
  [T. Suzuki, arXiv:1402.1294]

$$J_\mu = K_\mu,\ \left(K^a_\mu(s) = \frac{1}{2}\epsilon_{\mu\alpha\beta\gamma}\partial_\alpha n^a_{\beta\gamma}(s+\hat{\mu}\right)$$

- **The correlation function**

$$Z \cdot \exp(-M \cdot \Delta t) \sim \left\langle \sum_{\vec{x},t} \sum_{\vec{x}'} \mathcal{O}(\vec{x},t)\mathcal{O}(\vec{x}',t') \right\rangle - \left\langle \sum_{\vec{x},t} \mathcal{O}(\vec{x},t) \right\rangle^2,\ \Delta t = |t-t'|$$

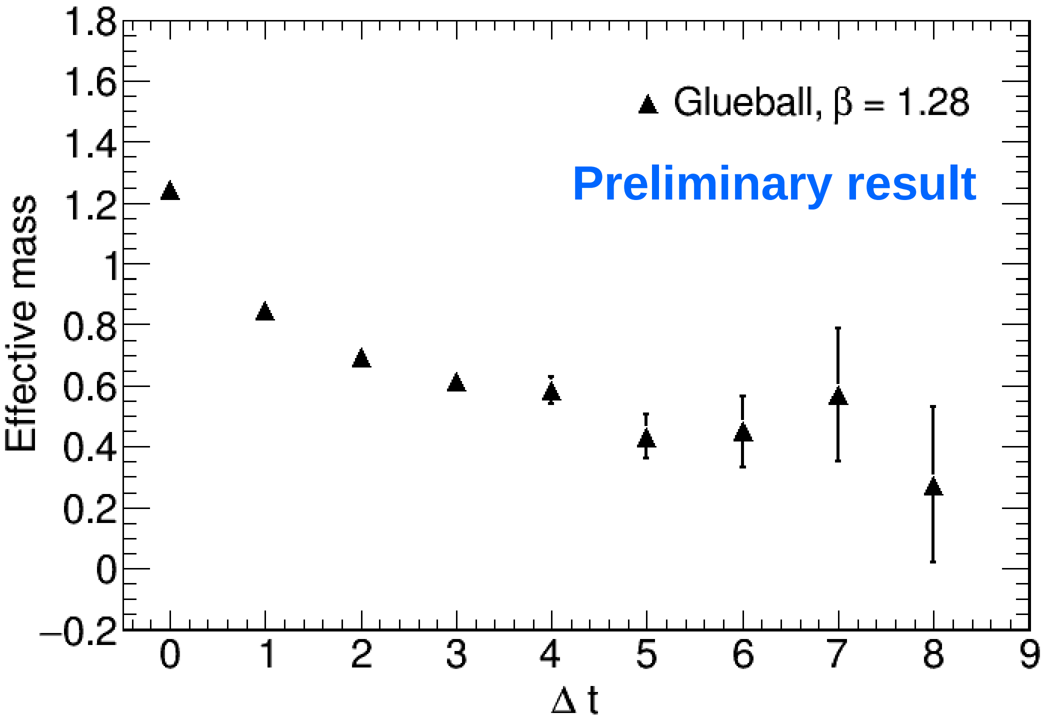
Simulation parameter

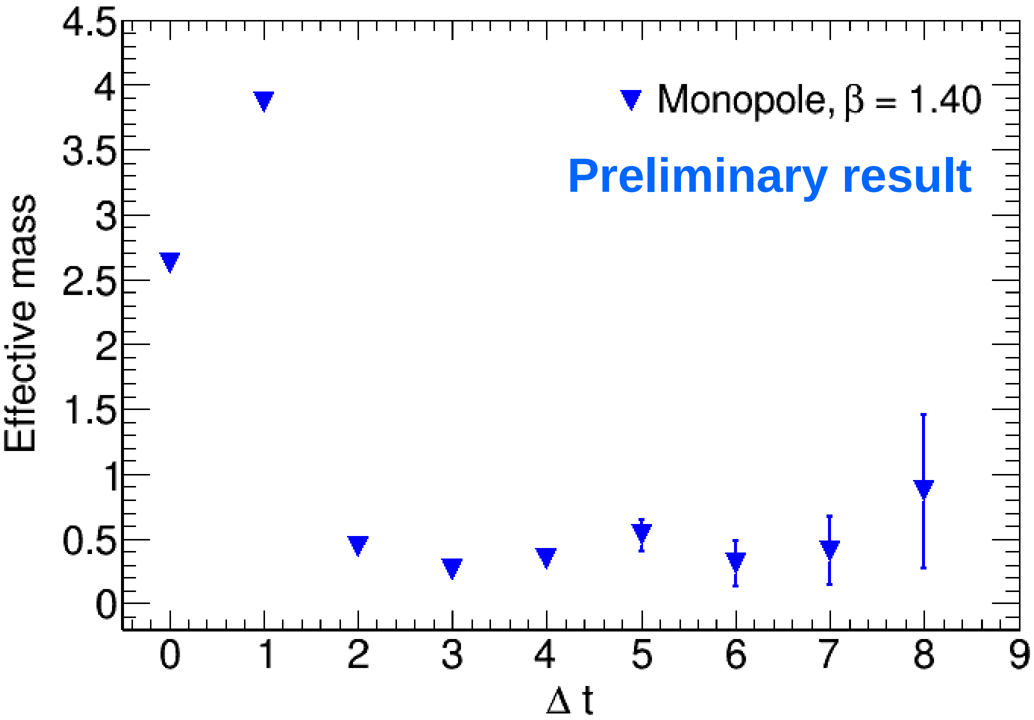
β	Volume	lattice spacing [fm]	N_{conf}
1.10	$16^3 48$	0.1068(8)	40200
1.20	$24^3 72$	$7.92(2) \times 10^{-2}$	4396
1.28	$24^3 72$	$6.35(5) \times 10^{-2}$	15650
1.40	$24^3 72$	$4.65(2) \times 10^{-2}$	12078

The effective mass of monopoles

- **The effective mass plots of the Abelian-like monopole and the glueball in quenched SU(2).**

$$Eff(\Delta t) \equiv -\log\left\{\frac{\exp(-M\cdot(\Delta t+1))}{\exp(-M\cdot\Delta t)}\right\}$$





[**NOTE:** The idea of the scalar mass of the monopole is given by T. Suzuki and K. Ishiguro. The computational results are produced by M. H.]

The scalar mass of monopoles

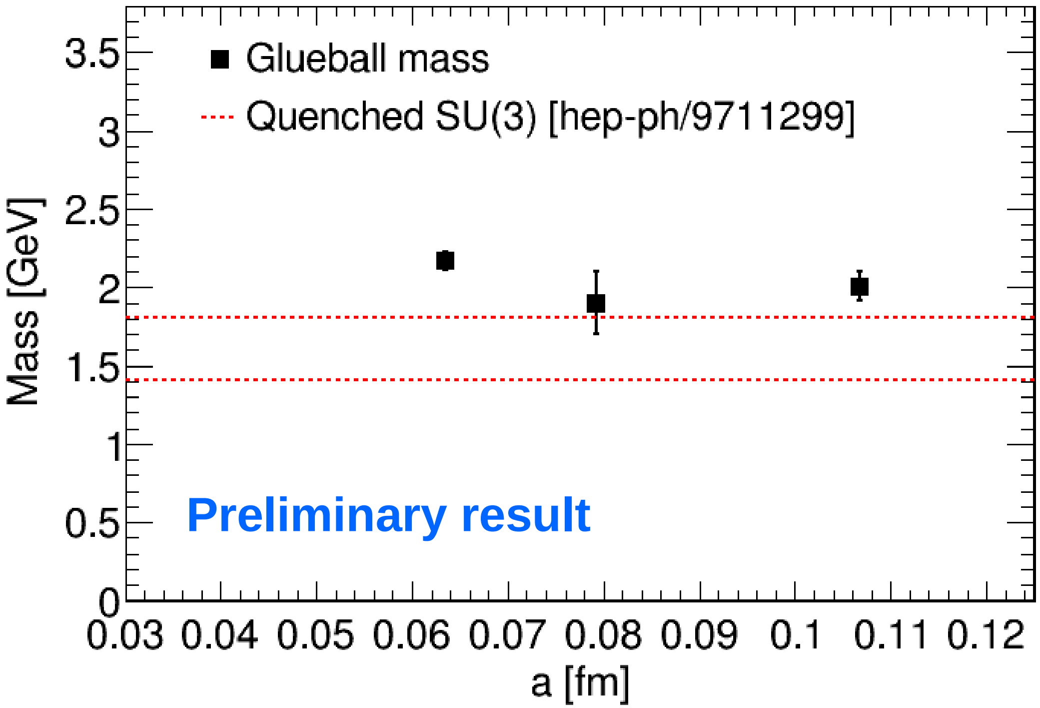
- **The correlation function**

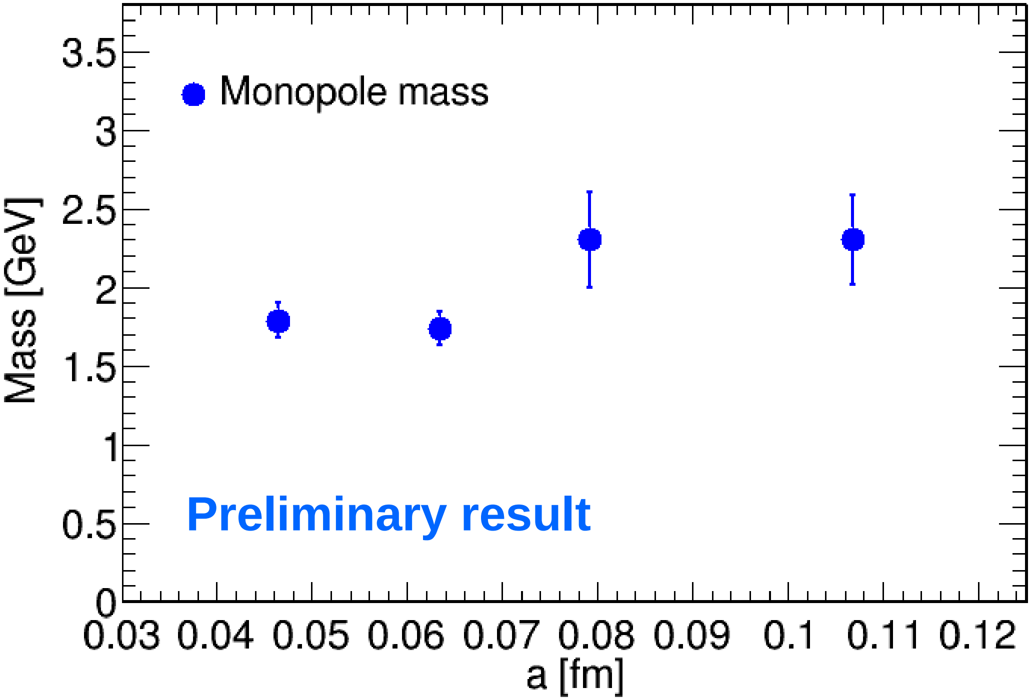
$$Z \cdot \exp(-M \cdot \Delta t) \sim \left\langle \sum_{\vec{x},t} \sum_{\vec{x}'} \mathcal{O}(\vec{x},t)\mathcal{O}(\vec{x}',t') \right\rangle - \left\langle \sum_{\vec{x},t} \mathcal{O}(\vec{x},t) \right\rangle^2$$

Theoretical prediction in SU(3) **MA gauge**, **3.93 ~ 6.29** [GeV], [Prog. Theo. Phys. 81 (1989) 229]

- **The fitting function:** $F(\Delta t) = A \exp(-M \cdot \Delta t) + B$

$$\chi^2/d.o.f. \approx 1$$





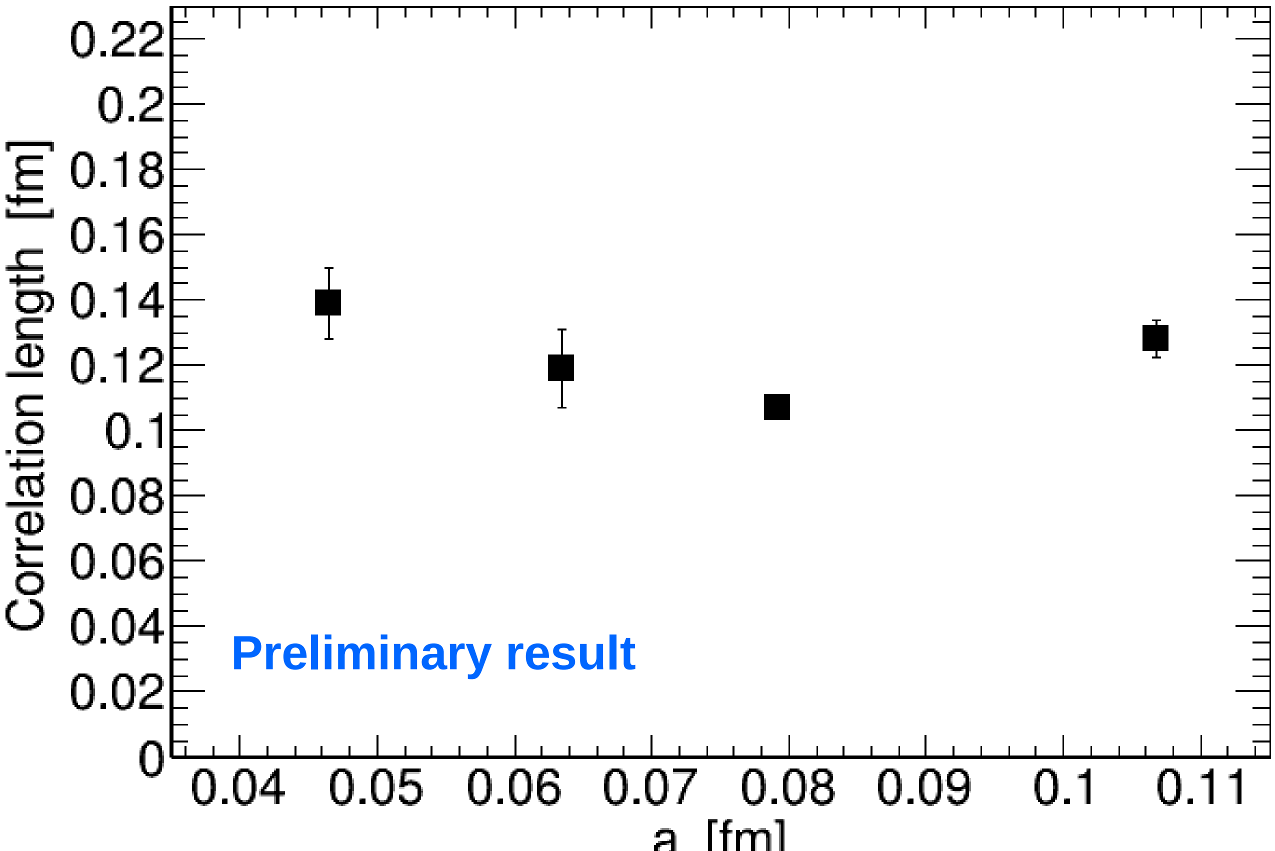
The masses are about 2 [GeV] in quenched SU(2).

The correlation length

- **The correlation between the scalar Abelian-like monopole and the scalar glueball.**

$$\mathcal{C}_{gm}(\Delta t) = \sum_t \langle \mathcal{O}_g(t)\mathcal{O}_m(t')\rangle - \langle \mathcal{O}_g\rangle\langle \mathcal{O}_m\rangle$$

$$F(\Delta t) = A\exp(-1/\xi\cdot\Delta t) + B \qquad \chi^2/d.o.f. \approx 1$$



The scalar mass of monopoles

- The mass of the gauge independent (Abelian-like) monopole is the approximately same mass as the scalar glueball, 2 [GeV] in quenched SU(2).
- The Abelian-like monopole correlates with the glueball.
- **If the glueball will be observed in experiments, we may observe QCD monopoles.**
- **However, those are similar. To distinguish signals of monopoles and glueball, we estimate effects of monopoles on the physical quantities.**

Effects of monopoles

- **We want to show that monopoles condensing in the QCD vacuum are closely related to instantons, and chiral symmetry breaking.**
- **We add monopoles by a monopole creation operator in SU(3) quenched configurations** [C. Bonati, et al., PRD 85 (2012) 065001]**.**
- **We use the Overlap fermion, which preserve the chiral symmetry in the lattice gauge theory, as an analytical tool** [R. G. Edwards, et al., PRD 61 (2000) 074504; L. Giusti, et al., JHEP 11 (2003) 023; L. Del Debbio, et al., PRL 94 (2005) 032003; L. Del Debbio, et al., JHEP 02 (2004) 003]**.**

Effects of monopoles

We have demonstrated the results as follows:

(1) Monopoles make instantons.
[A. Di Giacomo and M. H. PRD 91 (2015) 054512]

(2) In random matrix theory, the low-lying eigenvalues of the Overlap Dirac operator are not affected by the monopoles.

The chiral condensate decrease by increasing the values of monopole charges.
[A. Di Giacomo, M. H., and F. Pucci, Proc. Sci., CD15 (2015) 127]

(3) Quark masses become slightly heavy by increasing the values of monopoles charges.
[A. Di Giacomo and M. H., Proc. Sci., Lat2015 (2015) 313]

Chiral symmetry in the lattice gauge theory

- If the operator ***D*** satisfies the Ginsparg-Wilson relation, the operator preserves the chiral symmetry in the lattice gauge theory.

$$\gamma_5 \mathbf{D} + \mathbf{D}\gamma_5 = \mathbf{aDR}\gamma_5\mathbf{D} \qquad (\mathcal{L} = \bar{\psi} D \psi)$$

- The exact form of the operator ***D*** is defined as follows:

The massless Overlap Dirac operator

$$D(\rho) = \frac{\rho}{a}\left[1 + \frac{D_W(\rho)}{\sqrt{D_W^\dagger(\rho) D_W(\rho)}}\right] = \frac{\rho}{a}\{1 + \gamma_5 \epsilon(H_W(\rho))\}$$

D: Massless Overlap Dirac operater.
Є: Sign function.
D_w: Massless Wilson Dirac operator.
ρ: Parameter, **ρ = 1.4.**
H_W: Hermitian Wilson Dirac operator.

There are the exact zero modes in the spectra of the Overlap Dirac operator.

Computations of the Overlap Dirac operator

- We approximate the sign function $\epsilon(H_W(\rho))$ by the Chebyshev polynomials [L. Giusti, et al., Com. Phys. Comm. 153 (2003) 31, etc].
- We then solve eigenvalue problems using by ARPACK.

$$\mathbf{D}(\rho)|\psi_{\mathbf{i}}\rangle = \lambda_{\mathbf{i}}|\psi_{\mathbf{i}}\rangle$$

- **Almost all computational time are spent for the computations of the eigenvalue problems.**
- We compute ***O***(60-100) pairs of low-lying eigenvalues and eigenvectors, and save them in storage elements.
- The lattice spacing is computed from the analytic interpolation [S. Necco, et al., NPB 622 (2002) 328]. $r_0 = 0.5\ [\mathrm{fm}]$.

Super computers

NEC, SX-ACE
(CMC and RCNP, Osaka Univ.)

- **Vector Processors**
- 1CPU / 1-node
- 4-core / 1-node
- Total node: 1536
- Memory: 64GB / 1-node
- Performance:
  276 Gflops / 1-node

Photo from NEC web site.

Cray XC40, (2016)
(YITP, Kyoto Univ.)

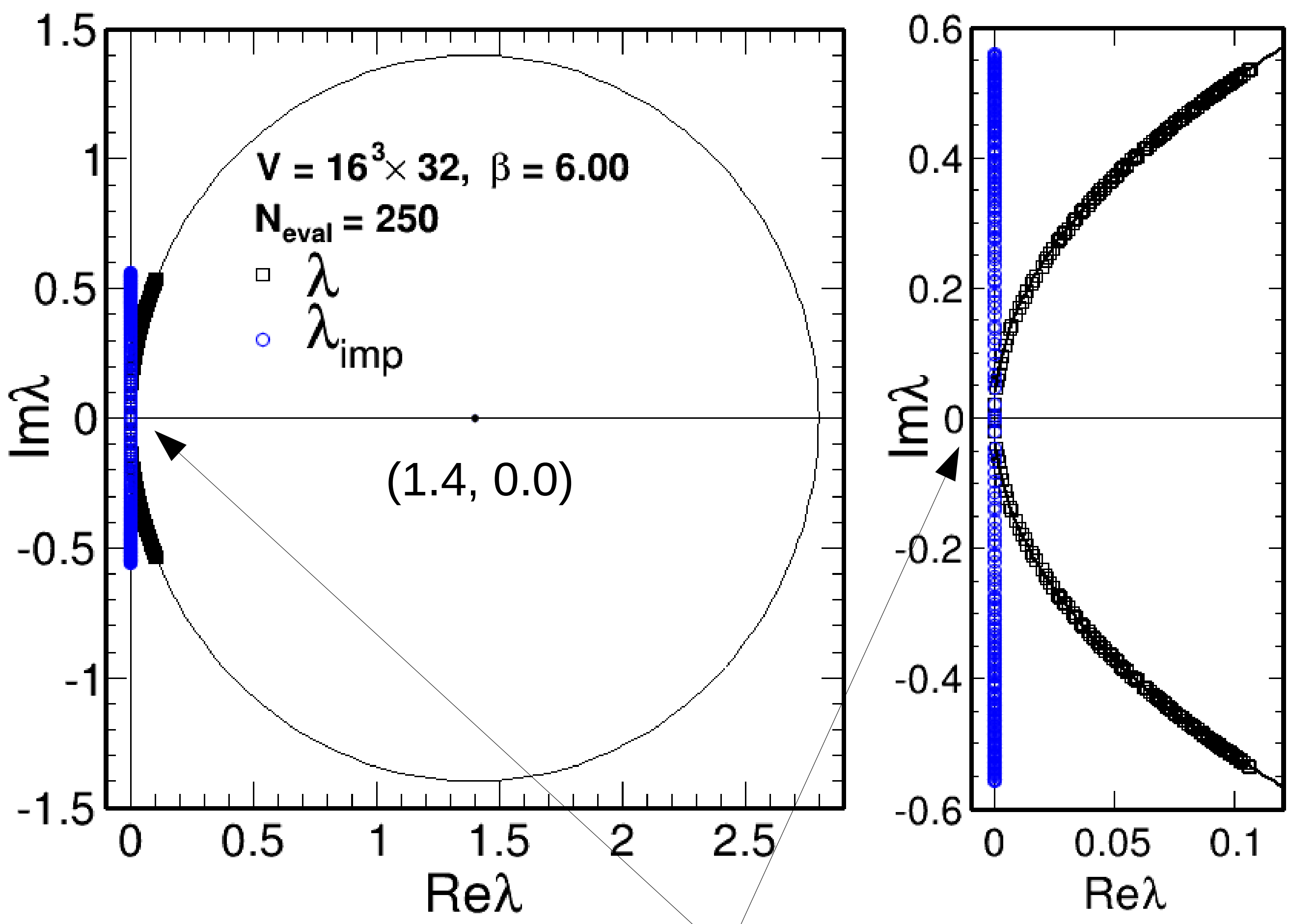
- **Parallel processing**
- 32-core / 1-node
- Total node: 292
- Memory: 128 GB / 1-node
- Total core: 9344
- Performance:
  588 Gflops / 1-cup

Photo from Cray web site.

The spectral of low-lying eigenvalues

- The eigenvalues **λ** on the circle, and the improved eigenvalues $\boldsymbol{\lambda}_{\mathbf{imp}}$ are on the imaginary axis.
- $\boldsymbol{\lambda}^{imp}$ is computed from the improved massless Overlap Dirac operator $\boldsymbol{D}^{imp}$ [S. Capitani, et al., PLB 468 (1999) 150].

$$D^{\mathrm{imp}} = \left(1 - \frac{a}{2\rho} D\right)^{-1} D$$



We find zero modes in the spectra.

Fermion zero modes

- **The Overlap fermion has the exact zero modes.**

$\mathbf{n_+}$: The number of zero modes of the ***positive chirality***.

$\mathbf{n_-}$: The number of zero modes of the ***negative chirality***.

How to find the zero modes in our computations?

- The machine precision is $\mathbf{O(10^{-14})} \sim \mathbf{O(10^{-16})}$.
- We set the tolerance of the the Chebyshev polynomial approximation to $\mathbf{O(10^{-8})}$.
- We suppose that the exact zero modes are $|\lambda_{\mathbf{zero}}| \leq \mathbf{O(10^{-8})}$.

Ex. $\lambda_{zero} = (O(10^{-10}), O(10^{-16})), \lambda_{sec} = (O(10^{-4}), O(10^{-2}))$

- The chirality of the zero modes is computed as follows:

$$\sum_x \psi^*_{zero}(x)\gamma_5\psi_{zero}(x) = \pm 1$$

Zero modes, topological charge, and topological susceptibility

- **However,** we have **never** observed zero modes of the positive chirality and negative chirality in the same configuration at the same time.
- **The observed zero modes are only the positive chirality or the negative chirality.**

  Suppose that observed zero modes in our system are the topological charges *Q*.
- Topological charge ***Q*** is $\mathbf{Q = n_+ - n_-}$.
- Topological susceptibility is defined as follows:

$$\chi \mathbf{r_0^4} = \langle \mathbf{Q^2} \rangle \mathbf{r_0^4} / \mathbf{V}.$$

- The theoretical expectation from Refs. by Witten [NPB 156 (1979) 269] and Veneziano [NPB 159, (1979) 213] is

$$\chi = (\mathbf{180}\ [\mathrm{MeV}])^{\mathbf{4}}.$$

Topological susceptibility in the continuum limit

Our results

- $V/r_0^4 = 49.96$
  $\chi = (\mathbf{194(3)}[\mathrm{MeV}])^{\mathbf{4}}$
- $V/r_0^4 = 126.5$
  $\chi = (\mathbf{201(4)}[\mathrm{MeV}])^{\mathbf{4}}$
- $V/r_0^4 = 157.9$
  $\chi = (\mathbf{193(3)}[\mathrm{MeV}])^{\mathbf{4}}$
  $(r_0 = 0.5[\mathrm{fm}])$

Another group result

- PRL 94 (2005) 032003
  $\chi = (\mathbf{191(5)}\ [\mathrm{MeV}])^{\mathbf{4}}$
  $(F_K = 160(2)[\mathrm{MeV}])$

Theoretical expectation

- Witten, and Veneziano
  [NPB 156 (1979) 269,
  NPB 159 (1979) 213]
  $\chi = (\mathbf{180}\ [\mathrm{MeV}])^{\mathbf{4}}$

We can properly evaluate the topological susceptibility in the continuum limit.

The observed zero mode and topological charge

- We can properly compute the topological charges and evaluate the topological susceptibility in the continuum limit.
- **We find that the observed zero mode in our system is the topological charge *Q*.**
- Topological charge ***Q*** is $\mathbf{Q = n_+ - n_-}$.

How to compute the number of zero modes?

- Because there is the Atiyah–Singer index theorem, we want to count the number of instantons from the number of zero modes.

The number of instantons

- We suppose that the Atiyah–Singer index theorem,

$\mathbf{n}_{+}$ **:** The number of instantons of the ***positive charge*****.**

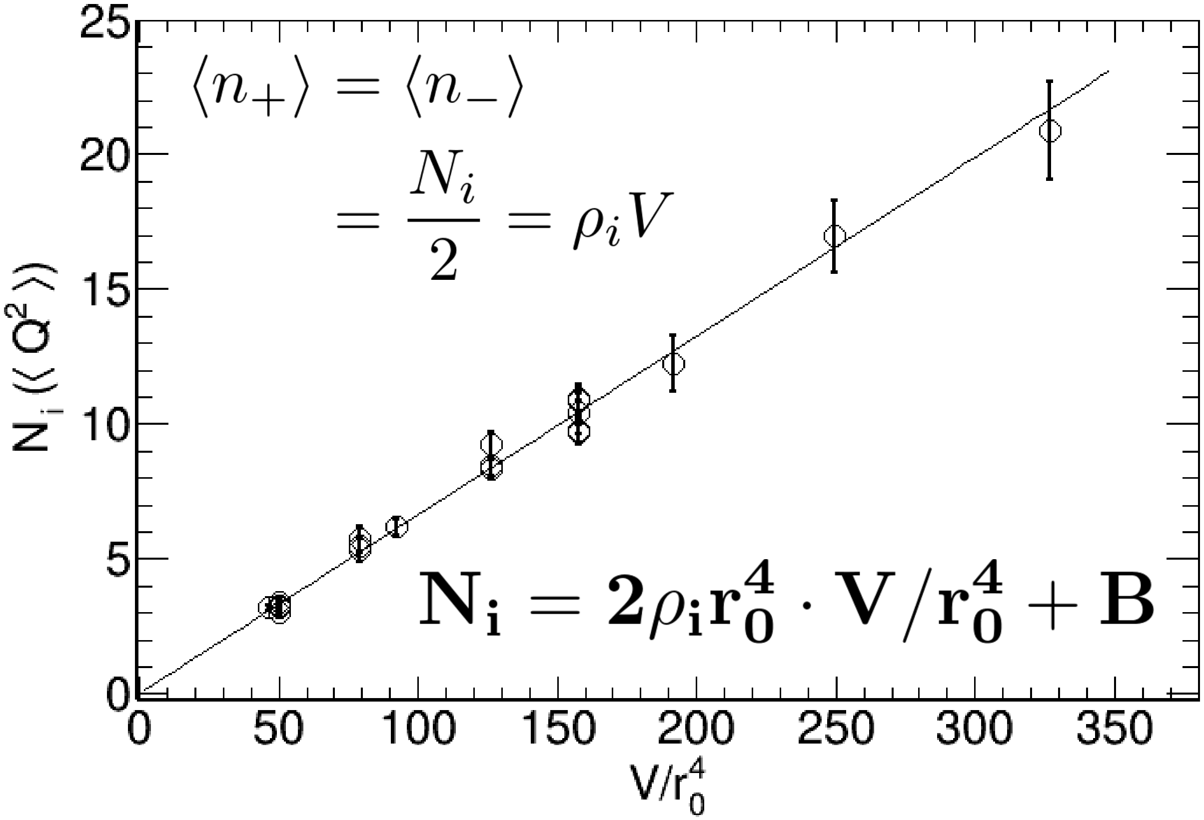
$\mathbf{n}_{-}$ **:** The number of instantons of the ***negative charge*****.**

- **However**, we **never observed** the numbers of zero modes of the positive chirality and the negative chirality in the same configuration at the same time.
- Therefore, we make a hypothesis and check the consistency.
- **We have shown that the number of instantons *N_i* can be counted from the average square of the topological charges** [A. Di Giacomo and M. H. PRD 91 (2015) 054512]**:**

$$\mathbf{N_i = \langle Q^2 \rangle}$$

The instanton density

- To confirm our supposition, we evaluate the instanton density, by fitting a function.



$$2\rho_i r_0^4 = 6.673(14) \times 10^{-2}$$

$$\mathbf{B = -0.02(0.1) = 0,\ \chi^2/d.o.f = 20.4/21.0}$$

- **The instanton density ρ_i is**

$$\rho_\mathbf{i} = \mathbf{8.09(17)} \times \mathbf{10^{-4}}\ [\mathrm{GeV}^\mathbf{4}].$$

- **The instanton liquid mode** [E. V. Shuryak, NPB 203 (1982) 93]**:**

$$\mathbf{n_c = 8 \times 10^{-4}}\ [\mathrm{GeV}^\mathbf{4}]$$

Consistent!

- **We properly calculate the number of instantons.**

Instantons and monopoles

- So, we have confirmed that we can compute the number of instantons correctly from the topological charges.
- **We add one pair of a monopole and an anti-monopole by the monopole creation operator in quenched SU(3) configurations** [Eq. (38), C. Bonati, et al., PRD 85 (2012) 065001].
- The monopole creation operator is defined as follows:

$$\bar{\mu} \equiv \exp\left(\beta \overline{\Delta S}\right)$$

- The vacuum expectation values of the creation operator becomes the disorder parameter [L. Del Debbio, et al., PLB 349 (1995) 513, A. Di Giacomo, et al., PRD 56 (1997) 6816, etc.].
- **We then compute the number of instantons, and show the relation between the monopoles and instantons.**

The monopole creation operator

- **The Plaquette gauge action is shifted as follows:**

$$S + \overline{\Delta S} \equiv \sum_{n,\mu<\nu} \mathrm{Re}(1 - \bar{\Pi}_{\mu\nu}(n))$$

$$\bar{\Pi}_{i0}(t,\vec{n}) = \frac{1}{\mathrm{Tr}[I]} \mathrm{Tr}[U_i(t,\vec{n}) M_i^\dagger(\vec{n}+\hat{i}) U_0(t,\vec{n}+\hat{i}) \times M_i(\vec{n}+\hat{i}) U_i^\dagger(t+1,\vec{n}) U_0^\dagger(t,\vec{n})]$$

$$M_i(\vec{n}) = \exp(+\mathbf{m_c} i A_i^0(\vec{n}-\vec{x}_1))$$

$$M_i^\dagger(\vec{n}) = \exp(-\mathbf{m_c} i A_i^0(\vec{n}-\vec{x}_2))$$

$$g \equiv \sqrt{\frac{6}{\beta}} : \text{Electric charge}$$

A_i^0 : Abelian monopole (Wu - Yang form) in SU(3)

$+\mathbf{m_c}$: Magnetic charges of the monopole

$-\mathbf{m_c}$: Magnetic charges of the anti-monople

$$\mathbf{m_c = 0, 1, 2, 3, 4, 5, 6}$$

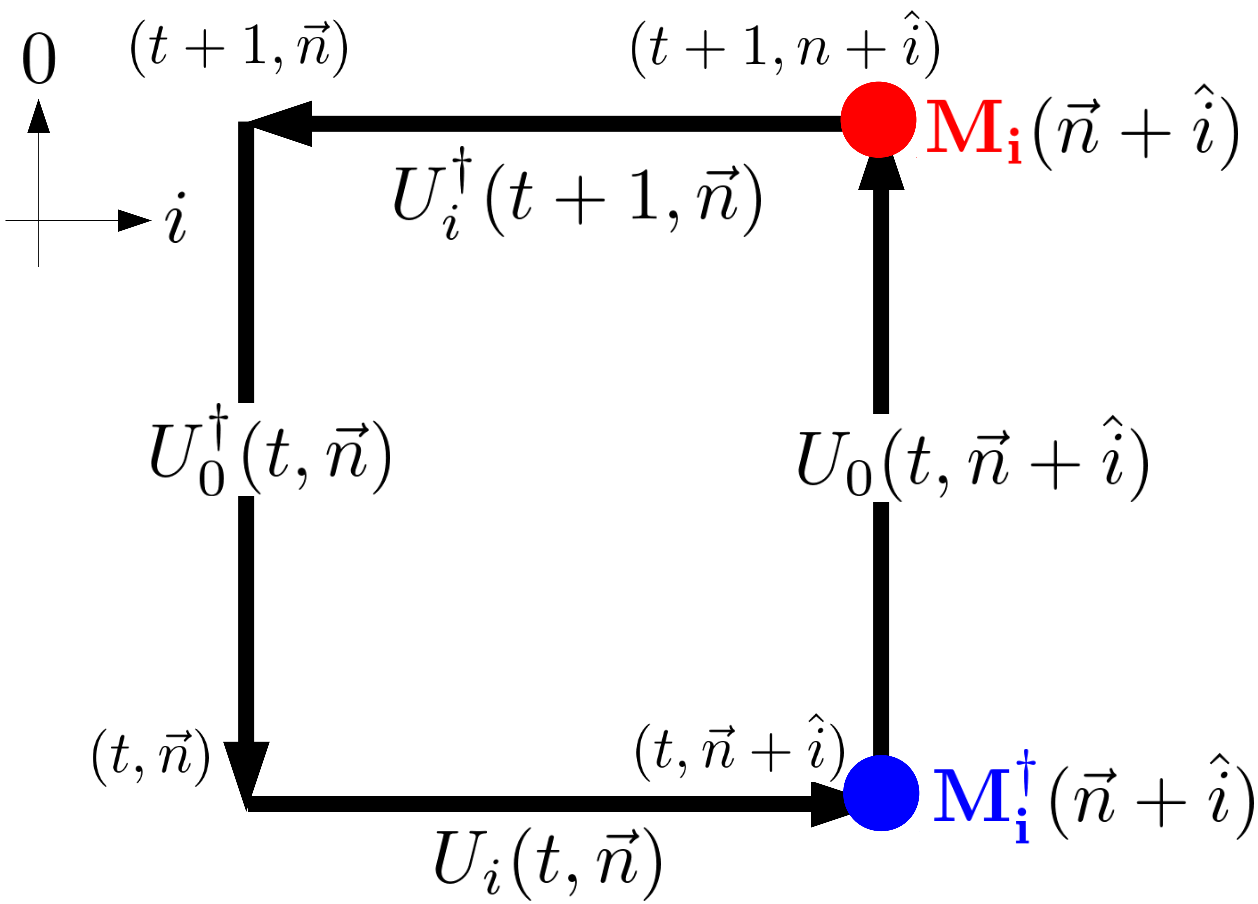
The locations of monopoles

- The Plaquette action is slightly sifted [C. Bonati, et al., PRD 85 (2012) 065001].

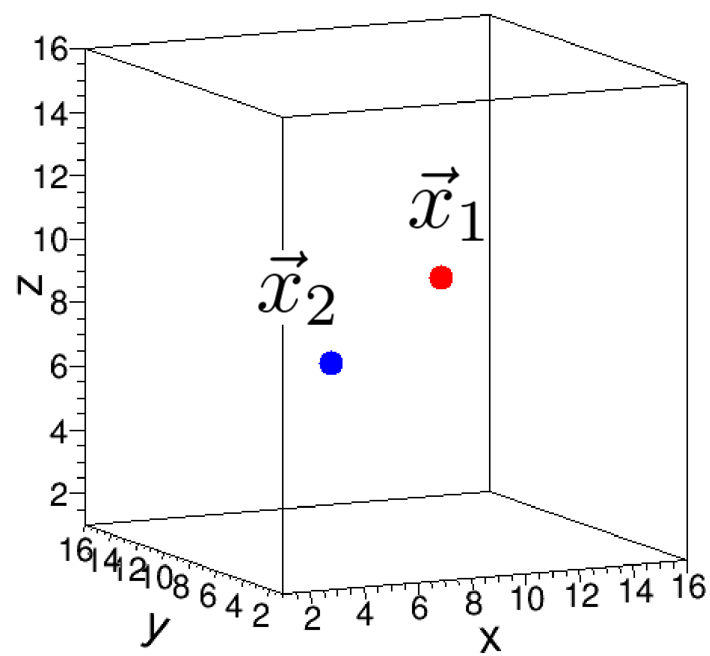
- The locations of the monopole and the anti-monopole.

$$\mathbf{M_i}(\vec{n}) = \exp(m_c i A_i^0(\vec{n} - \vec{x}_1))$$

$$\mathbf{M_i^\dagger}(\vec{n}) = \exp(-m_c i A_i^0(\vec{n} - \vec{x}_2))$$

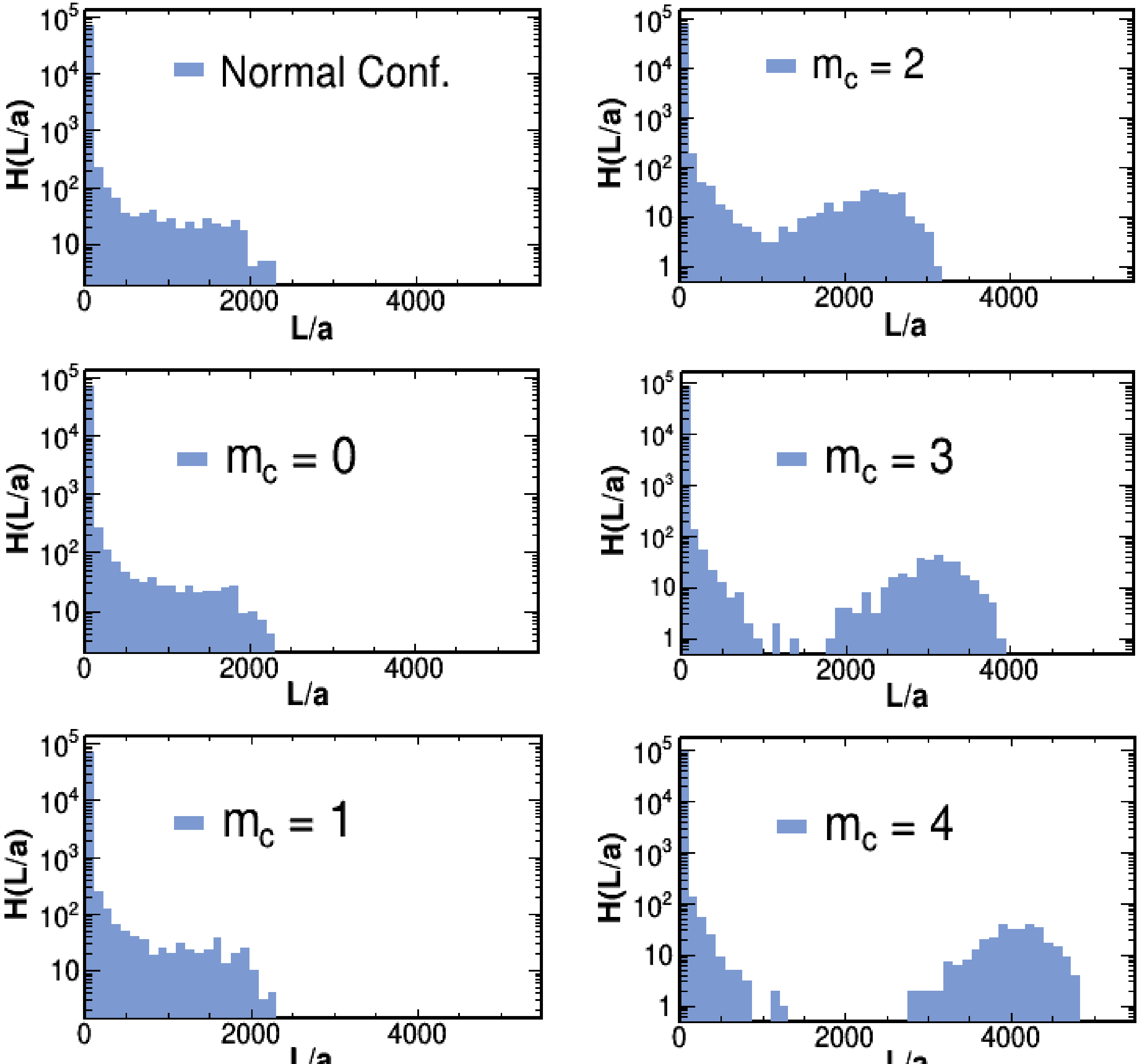


$$\mathbf{V = 16^3 \times 32}$$



$$D = |\vec{x}_1 - \vec{x}_2| \approx 1.1 \ [\mathrm{fm}]$$

Monopole loops



$V = 16^3 \mathrm{x} 32$, $\beta = 6.00$.

- The length of monopole loops becomes long by the increasing of the values of monopole charges.
- The monopole creation operator makes only the long monopole loops

[A. Di Giacomo and M. H., PRD 91 (2015) 054512].

Instantons and monopoles

- We have confirmed that the monopole creation operator creates the monopoles in the QCD vacuum.
- Next, we add one pair of monopoles varying the magnetic charges; $\mathbf{m_c = 0, 1, 2, 3, 4, 5, 6}$
- We compute the number of instantons in configurations, and show quantitative relation between the monopoles and instantons.
- **We make an assumption; one monopole of $\mathrm{m_c} = 1$ and one anti-monopole of $\mathrm{m_c} = -1$ make one instanton of a positive charge or a negative charge.**

Instantons and monopoles

- **Simulation parameters**

The finite lattice volume effect

β					
6.00	14^4	46.28	-	Normal conf	O(2000)
			1.06	$m_c = 0 - 4$	O(2000)
	$14^3 \times 28$	92.55	-	Normal conf	O(1000)
			1.06	$m_c = 0 - 4$	O(1000)
	$16^3 \times 32$	157.8	-	Normal conf	O (800)
			1.06	$m_c = 0 - 5$	O (700$\sim$ 800)
5.85	$12^3 \times 24$	157.8	-	Normal conf	O(1000)
			1.06	$m_c = 0 - 4$	O(1000)
5.93	$14^3 \times 28$	157.8	-	Normal conf	O(800)
			1.06	$m_c = 0 - 5$	O(700$\sim$ 800)
6.05	$18^3 \times 32$	157.8	-	Normal conf	O(800)
			1.09	$m_c = 0 - 6$	O(700)

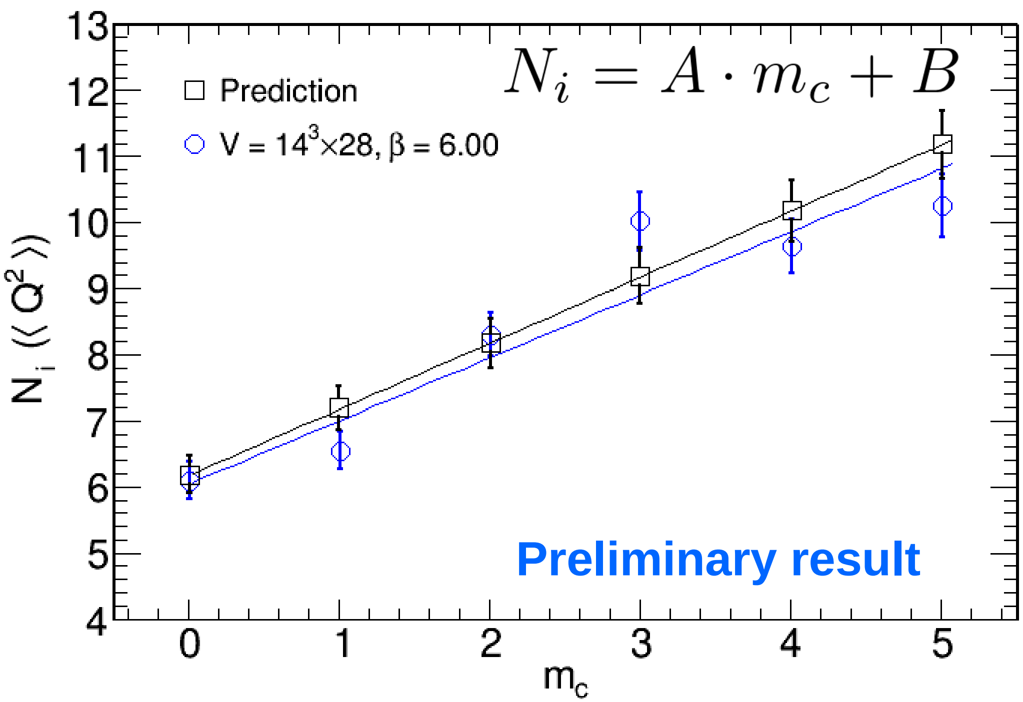
Instantons and monopoles

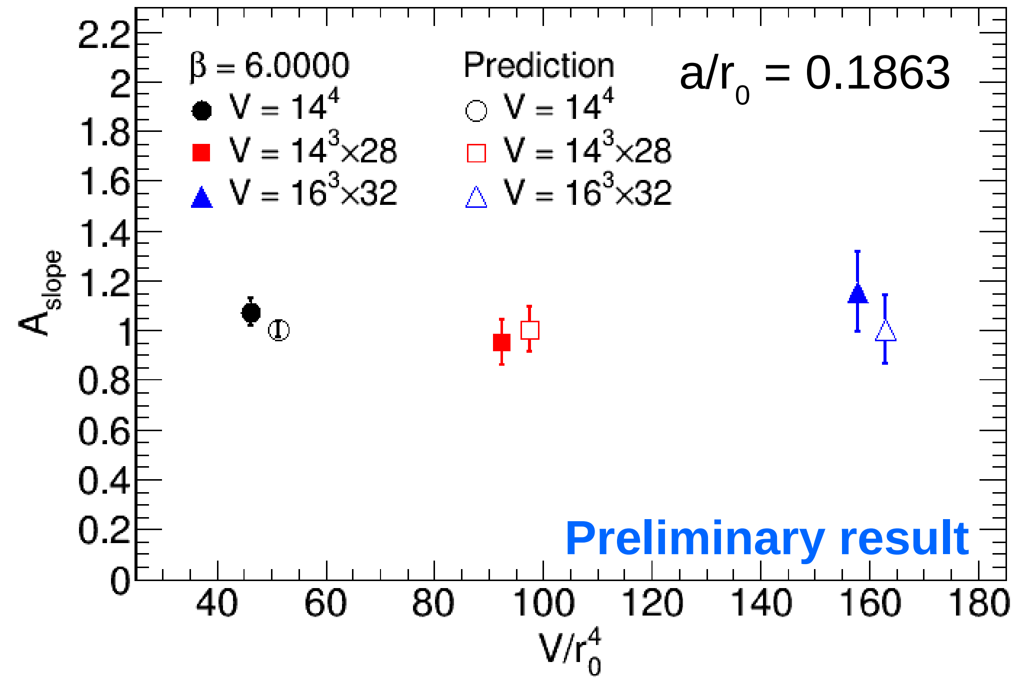
- **Simulation parameters**

β	V	V/r_0^4	D [fm]	Kinds of conf	N_{conf}
6.00	14^4	46.28	-	Normal conf	O(2000)
			1.06	$m_c = 0 - 4$	O(2000)
	$14^3 \times 28$	[illegible]	[illegible]	[illegible]	O(1000)
			[illegible]	[illegible]	O(1000)
	$16^3 \times 32$	**157.8**	-	Normal conf	O (800)
			1.06	$m_c = 0 - 5$	O (700~ 800)
5.85	$12^3 \times 24$	**157.8**	-	Normal conf	O(1000)
			1.06	$m_c = 0 - 4$	O(1000)
5.93	$14^3 \times 28$	**157.8**	-	Normal conf	O(800)
			1.06	$m_c = 0 - 5$	O(700~ 800)
6.05	$18^3 \times 32$	**157.8**	-	Normal conf	O(800)
			1.09	$m_c = 0 - 6$	O(700)

The continuum limit

Instantons and monopoles





- **Analytic Prediction**

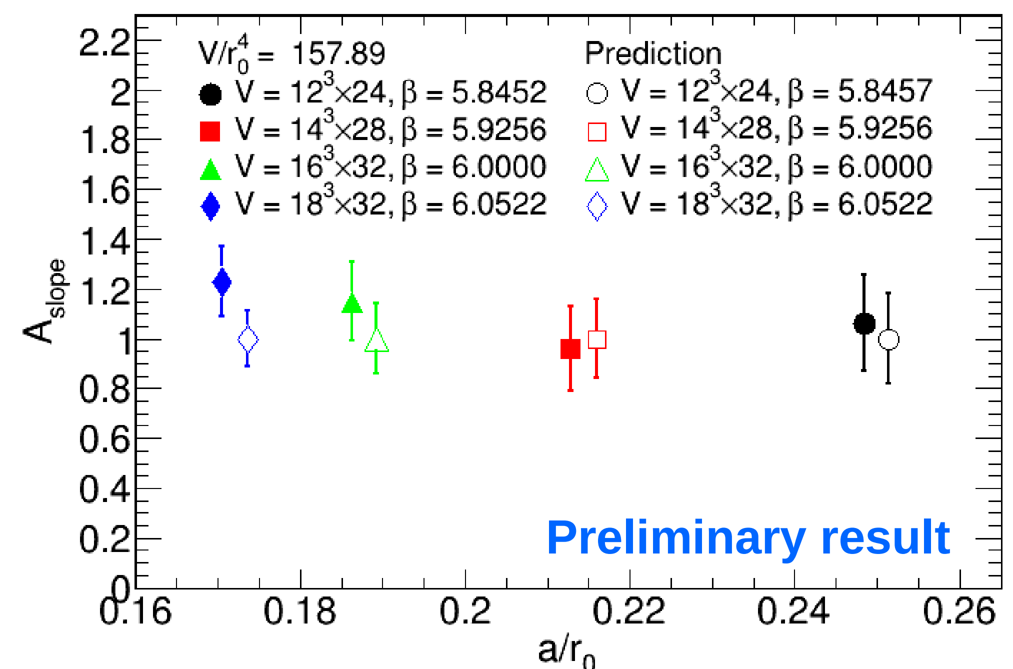
$\mathbf{A_{pre} = 1.00(9)}$, $B_{pre} = 6.2(2)$,

$\chi^2/d.o.f. = 0.0/4.0$

- **Lattice**

$\mathbf{A = 0.95(9)}$, $B = 6.0(2)$,

$\chi^2/d.o.f. = 11.6/4.0$



Instantons and monopoles

- **The quantitative relation between the number of monopoles and the number of instantons.**

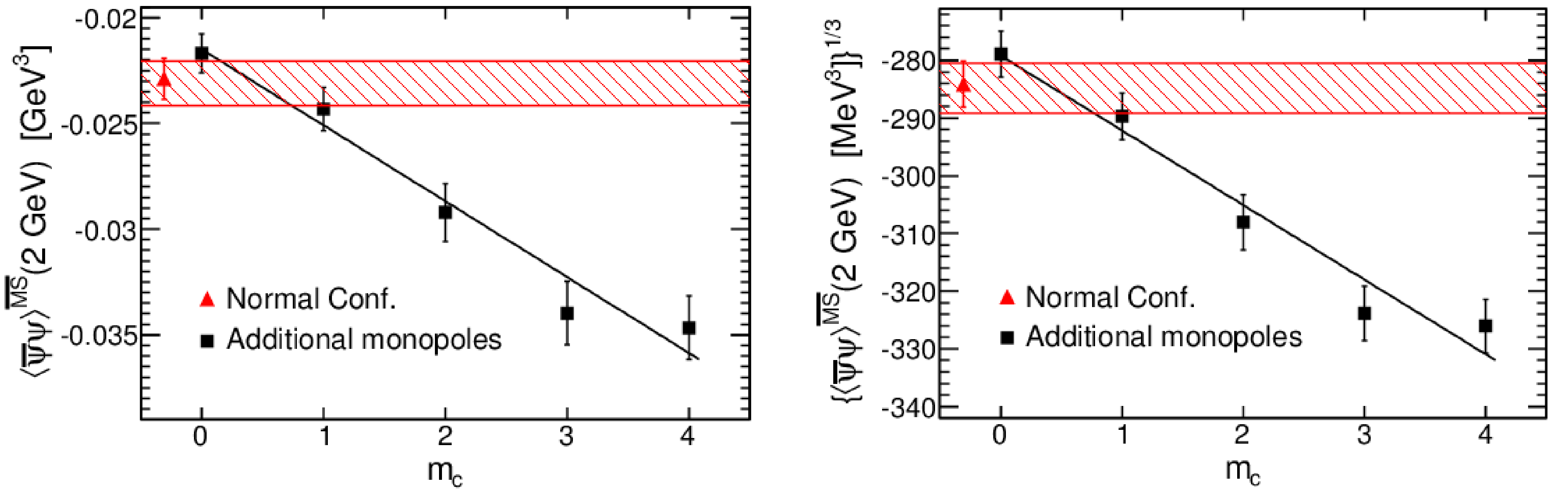
Preliminary result

β	V	$A_{\mathbf{Pre}}$	A	$B_{\mathbf{Pre}}$	B	$\chi^2/d.o.f.$
5.85	$12^3 24$	1.00 (18)	1.02 (19)	10.9 (4)	11.0 (4)	3/3
6.00	14^4	1.00 (3)	1.07 (5)	3.19 (7)	3.07 (10)	43/3
	$14^3 28$	1.00(14)	0.95(9)	6.2 (2)	6.0 (2)	12/4
	$16^3 32$	1.00(14)	1.15(16)	9.8(4)	10.1(4)	9/4
5.93	$14^3 28$	1.00 (16)	0.96 (17)	10.9 (4)	11.7 (5)	1/3
6.05	$18^3 32$	1.00 (11)	1.23(14)	9.7(3)	10.5(4)	5/5

One monopole of $\mathbf{m_c = 1}$ and one anti-monopole of $\mathbf{m_c = -1}$ make one instanton of a positive charge or a negative charge.

Chiral condensate and monopoles

- We evaluate the chiral condensate from the scale parameter Σ in RMT, considering the renormalization constant Zs.

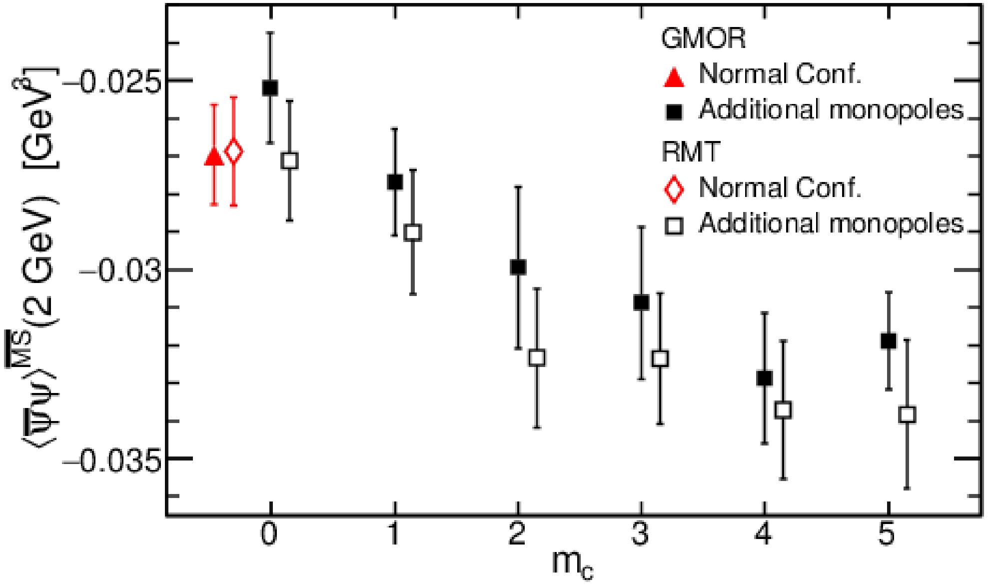


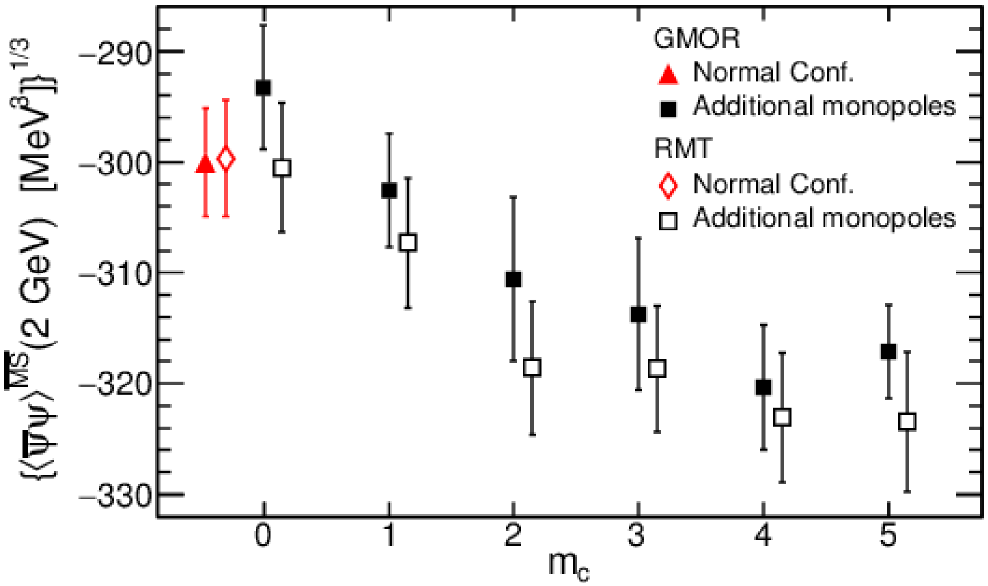
- **The chiral condensate linearly decreases by the increasing the values of monopole charges.**

The chiral condensate and monopoles (GMOR)

- The chiral condensate is computed from the Gell-Mann-Oakes-Renner mass formula [M. Gell-Mann, et al., PR 175 (1968) 2195].

$$a^3\langle\bar{\psi}\psi\rangle = -\lim_{am_q\to 0}\frac{(af_P)^2(am_{PS})^2}{2(am_q)}, \quad af_P = 2(am_q)\frac{\sqrt{G_{PP-SS}}}{(am_{PS})^2}.$$

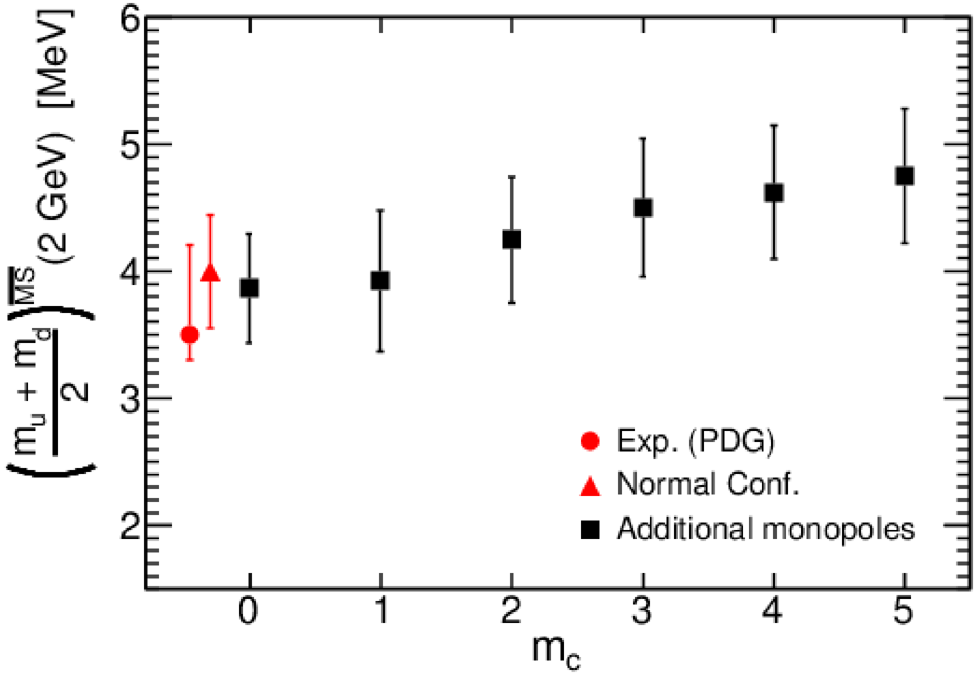


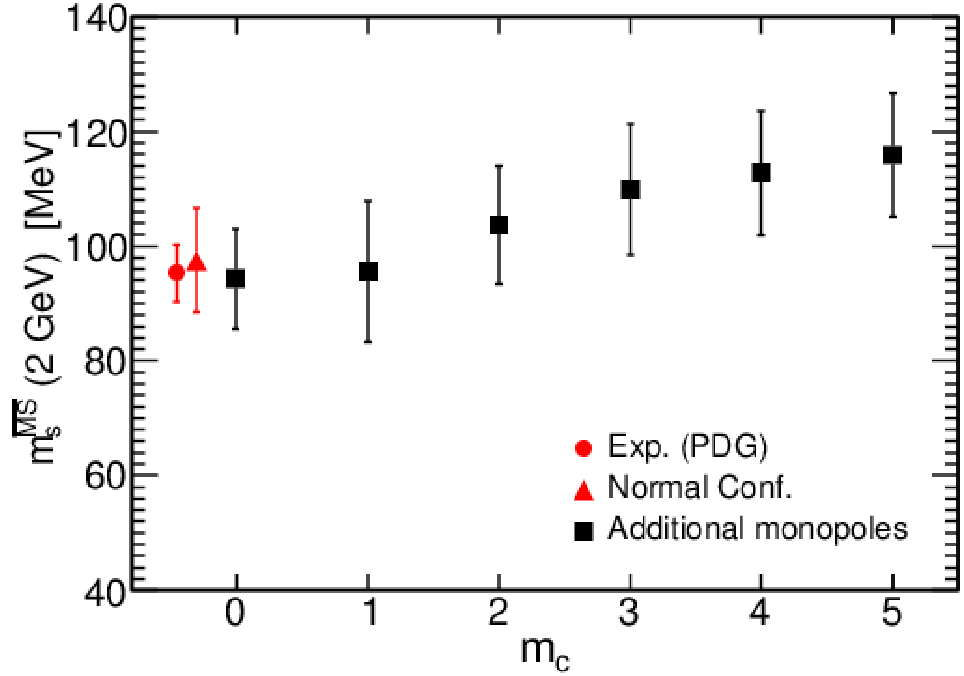


- **The chiral condensate decrease with increasing the number of monopoles charges.**

Quark masses

- Lastly, we estimate the quark masses, $\bar{m} = \frac{m_u + m_d}{2}$ and m_s, based on Ref. [L. Giusti, et al., PRD 64 (2001) 114508].
- The lattice scale is determined from Kaon decay constant and mass $a^{-1} = 2.00(8)[\mathrm{GeV}]$.





- **The masses slightly become heavy with increasing the monopole charges**

Conclusions

- Monopoles and anti-monopoles are successfully added to configurations by the monopole creation operator.
- One pair of monopoles with magnetic charges makes instantons.
- The chiral condensate decreases by increasing the monopole charges (RMT and GMOR).
- The quark masses increase by increasing the monopole charges.

Acknowledgment

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Thank you for attention!

Renormalization constant Zs