Mott dissociation of pions and kaons in quark matter

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Hadronic matter under extreme conditions,
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Plan

1. Motivation
2. PNJL N_f=2+1
3. Phase shifts
4. Kaon to pion ratio
5. Conclusions

Motivation. I

- The well-developed methods of perturbative QCD are applicable for description of the hard processes, with small distances between quarks and gluons.
- The most interesting region the QCD phase diagram should be the subject of nonperturbative study.
- Effective theories based on symmetries in nonperturbative region are legitimate tools.

Motivation. II

- One of the successful effective model is the local Nambu–Jona-Lasinio model.
  NJL is one of the simplest models for description of "nonperturbative" dynamics. It provides for spontaneous chiral symmetry breaking and the formation of a quark condensate as well as chiral restoration transition in a hot and dense medium.
- Polyakov loop extension solve problem of "free" quarks in hadronic phase
- For description of hadron phase where the fluctuations plays major role one need to go beyond mean field approximation

PNJL N_f=2+1

$$\mathcal{L} = \bar{q}\left(i\gamma^{\mu}D_{\mu} + \hat{m}_0\right)q + G_S\sum_{a=0}^{8}\left[\left(\bar{q}\lambda^a q\right)^2 + \left(\bar{q}i\gamma_5\lambda^a q\right)^2\right] - \mathcal{U}\left(\Phi[A], \bar{\Phi}[A]; T\right)$$

where q quark field with three flavors, $f = u, d, s$, and three colors, $N_c = 3$; λ^a are the flavor $\text{SU}_f(3)$ Gell-Mann matrices, G_S is four-quark coupling constant. Current quark masses induces an explicit breaking of chiral symmetry. Constant temporal background gauge field is minimally coupled to the quarks. Φ denotes the Polyakov loop expectation value and $\bar{\Phi}$ its conjugate.

Equations of motion

Equations of motion of mean fields are

$$\frac{\partial \Omega}{\partial m} = 0, \quad \frac{\partial \Omega}{\partial \Phi} = 0, \quad \frac{\partial \Omega}{\partial \bar{\Phi}} = 0$$

Light current quarks transform to massive constituent quarks as a result of spontaneous chiral symmetry breaking ($< \bar{q}q > \neq 0$).

Gap equation for quark masses m_f

$$m_f = m_{0,f} + 16\, m_f G_S I_1^f(T, \mu),$$

Generalized fermion distribution

$$I_1^f(T,\mu_f) = \frac{N_c}{4\pi^2}\int_0^\Lambda \frac{dp\,p^2}{E_f}\left(n_f^- - n_f^+\right),$$

$n_f^\pm = f_\Phi^+(\pm E_f)$ are generalized fermion distribution functions in the presence of the Polyakov loop

$$f_\Phi^+(E_f) = \frac{(\bar\Phi + 2\Phi Y)Y + Y^3}{1 + 3(\bar\Phi + \Phi Y)Y + Y^3},$$

$$f_\Phi^-(E_f) = \frac{(\Phi + 2\bar\Phi\bar Y)\bar Y + \bar Y^3}{1 + 3(\Phi + \bar\Phi\bar Y)\bar Y + \bar Y^3},$$

where $Y = \mathrm{e}^{-(E_f-\mu_f)/T}$ and $\bar Y = \mathrm{e}^{-(E_f+\mu_f)/T}$.

$$f_\Phi^+(-E_f) = 1 - f_\Phi^-(E_f)$$

Fermion distribution. Limiting cases

$$
\begin{aligned}
f_\Phi^+(E_f) &= \frac{(\bar\Phi + 2\Phi Y)Y + Y^3}{1 + 3(\bar\Phi + \Phi Y)Y + Y^3}\,, \\
f_\Phi^-(E_f) &= \frac{(\Phi + 2\bar\Phi \bar Y)\bar Y + \bar Y^3}{1 + 3(\Phi + \bar\Phi \bar Y)\bar Y + \bar Y^3}\,,
\end{aligned}
$$

Limiting cases of confined phase ($\Phi = \bar\Phi = 0$) and deconfined phase ($\Phi = \bar\Phi = 1$)

$$
f_0^\pm(E_f) = (1 + \mathrm{e}^{3(E_f \mp \mu_f)/T})^{-1}
$$
$$
f_1^\pm(E_f) = (1 + \mathrm{e}^{(E_f \mp \mu_f)/T})^{-1}
$$

Polarization loops

$$\Pi^{M^a}_{ff'}(q_0,\mathbf{q}) = 2N_cT\sum_n\int\frac{d^3p}{(2\pi)^3}\times$$

$$\times\ \mathrm{tr}_D\left[S_f(p_n,\mathbf{p})\Gamma^{M^a}_{ff'}S_{f'}(p_n+q_0,\mathbf{p}+\mathbf{q})\Gamma^{M^a}_{ff'}\right]$$

where $S^{-1}_f(p_n,\mathbf{p}) = \gamma_0(i\omega_n+\mu_f+A^0)-\gamma\mathbf{p}-m_f$,
$\omega_n = (2n+1)\pi T$, $\Gamma^{P^a}_{ff'} = i\gamma_5\ T^a_{ff'}$, $\Gamma^{S^a}_{ff'} = T^a_{ff'}$

$$T^a_{ff'} = \begin{cases}(\lambda_3)_{ff'},\\ (\lambda_1\pm i\lambda_2)_{ff'}/\sqrt{2},\\ (\lambda_4\pm i\lambda_5)_{ff'}/\sqrt{2},\\ (\lambda_6\pm i\lambda_7)_{ff'}/\sqrt{2},\end{cases}$$

$P^a = \pi^0,\pi^\pm,K^\pm,K^0,\bar{K}^0$, $S^a = a_0^0,a_0^\pm,\kappa^\pm,\kappa^0,\bar{\kappa}^0$.

Polarization loops

For mesons at rest in the medium ($\mathbf{q} = \mathbf{0}$)

$$\Pi_{ff'}^{P^a,S^a}(q_0 + i\eta, \mathbf{0}) = 4\Big\{I_1^f(T,\mu_f) + I_1^{f'}(T,\mu_{f'})$$
$$\mp \left[(q_0 + \mu_{ff'})^2 - (m_f \mp m_{f'})^2\right] I_2^{ff'}(z,T,\mu_{ff'})\Big\}\,,$$

where $\mu_{ff'} = \mu_f - \mu_{f'}$.

$$I_2^{ff'}(z,T,\mu_{ff'}) = \frac{N_c}{4\pi^2}\int_0^\Lambda \frac{dp\,p^2}{E_f E_{f'}} \times$$
$$\times \Bigg[\frac{E_{f'} n_f^-}{(z - E_f - \mu_{ff'})^2 - E_{f'}^2} - \frac{E_{f'} n_f^+}{(z + E_f - \mu_{ff'})^2 - E_{f'}^2}$$
$$+ \frac{E_f n_{f'}^-}{(z + E_{f'} - \mu_{ff'})^2 - E_f^2} - \frac{E_f n_{f'}^+}{(z - E_{f'} - \mu_{ff'})^2 - E_f^2}\Bigg]$$

Polarization loops

For mesons at rest in the medium ($\mathbf{q} = \mathbf{0}$)

$$\Pi_{ff'}^{P^a,S^a}(q_0 + i\eta, \mathbf{0}) = 4\Big\{ I_1^f(T,\mu_f) + I_1^{f'}(T,\mu_{f'}) \mp \left[(q_0 + \mu_{ff'})^2 - (m_f \mp m_{f'})^2 \right] I_2^{ff'}(z, T, \mu_{ff'}) \Big\} ,$$

where $\mu_{ff'} = \mu_f - \mu_{f'}$. In vacuum

$$I_1^f(0,0) = \frac{N_c}{4\pi^2} \int_0^\Lambda \frac{dp\, p^2}{E_f},$$

$$I_2^{ff'}(z,0,0) = \frac{N_c}{4\pi^2} \int_0^\Lambda \frac{dp\, p^2}{E_f E_{f'}} \frac{E_f + E_{f'}}{z^2 - (E_f + E_{f'})^2}.$$

Meson propagator

Meson mass spectrum is obtained from pole condition for meson propagator

$$[\mathcal{S}^{M^a}_{ff'}(M_{M^a},\mathbf{0})]^{-1} = (2G_S)^{-1} - \Pi^{M^a}_{ff'}(M_{M^a}+i\eta,\mathbf{0}) = 0.$$

'Polar' representation for propagator of meson

$$\mathcal{S}^{M^a}_{ff'}(\omega,\mathbf{q}) = |\mathcal{S}^{M^a}_{ff'}(\omega,\mathbf{q})| e^{\delta_{\mathrm{M}}(\omega,\mathbf{q})}$$

Mesonic phase shift has the form

$$\delta_{\mathrm{M}}(\omega,\mathbf{q}) = -\arctan\left\{\frac{\mathrm{Im}\left([\mathcal{S}^{M}_{ff'}(\omega-i\eta,\mathbf{q})]^{-1}\right)}{\mathrm{Re}\left([\mathcal{S}^{M}_{ff'}(\omega+i\eta,\mathbf{q})]^{-1}\right)}\right\}.$$

Pressure

$$P = \sum_{f=u,d,s} P_f - U(\Phi, \bar{\Phi}) + \sum_M P_M \ ,$$

The quark pressure of flavor f

$$P_f \ = \ -\frac{(m_f - m_{0,f})^2}{8G} + \frac{N_c}{\pi^2}\int_0^\Lambda dp\, p^2\, E_f$$

$$+ \ \frac{N_c}{3\pi^2}\int_0^\infty \frac{dp\, p^4}{E_f}\left[f_\Phi^+(E_f) + f_\Phi^-(E_f)\right] \ ,$$

Contribution to pressure from mesonic fluctuations (ring sum)

$$P_M \ = \ -\frac{d_M}{2} T \sum_n \int \frac{d^3q}{(2\pi)^3} \ln\left[\mathcal{S}_{ff'}^{M^a}(iz_n, \mathbf{q})\right]^{-1}\right]$$

Pressure

Mesonic pressure can be rewritten in the Beth-Uhlenbeck form with the phase shifts

$$P_M \;=\; d_{\mathrm{M}} \int \frac{\mathrm{d}^3 q}{(2\pi)^3} \int_0^\infty \frac{\mathrm{d}\omega}{2\pi}\, g(\omega \pm \mu_{\mathrm{M}}) \delta_{\mathrm{M}}(\omega, \mathbf{q}) \;,$$

where $g(E) = (\mathrm{e}^{E/T} - 1)^{-1}$ is the Bose function. One can simplify expression under assumption that, even in the medium, the phase shifts are Lorentz invariant and depending on ω and $\mathbf{q}$ only via the Mandelstam variable $s = \omega^2 - \mathbf{q}^2$ in the form $\delta_{\mathrm{M}}(\omega, \mathbf{q}) = \delta_{\mathrm{M}}(\sqrt{s}, \mathbf{q} = 0) \equiv \delta_{\mathrm{M}}(\sqrt{s}; T, \mu_{\mathrm{M}})$ for given temperature and chemical potential of the medium.

Partial number densities

Then the BU formula for the mesonic pressure can be given the following form

$$P_{\rm M} = d_{\rm M}\int_0^\infty \frac{{\rm d}\mathcal{M}}{2\pi}\delta_{\rm M}(\mathcal{M})\int \frac{{\rm d}^3q}{(2\pi)^3}\frac{\mathcal{M}}{E}g(E\pm\mu_{\rm M})$$
$$E = \sqrt{q^2+\mathcal{M}^2}$$

Analogously, the partial number densities is

$$n_{\rm M} = d_{\rm M}\int_0^\infty \frac{{\rm d}\mathcal{M}}{\pi}\frac{\delta_{\rm M}(\mathcal{M})}{d\mathcal{M}}\int \frac{{\rm d}^3q}{(2\pi)^3}\frac{\mathcal{M}}{E}g(E\pm\mu_{\rm M}) =$$
$$= \frac{d_{\rm M}}{T}\int_0^\infty \frac{{\rm d}\mathcal{M}}{\pi}\delta_{\rm M}(\mathcal{M}) \times$$
$$\times \int \frac{{\rm d}^3q}{(2\pi)^3}\frac{\mathcal{M}}{E}g(E\pm\mu_{\rm M})\left(1+g(E\pm\mu_{\rm M})\right)$$

Model parameters

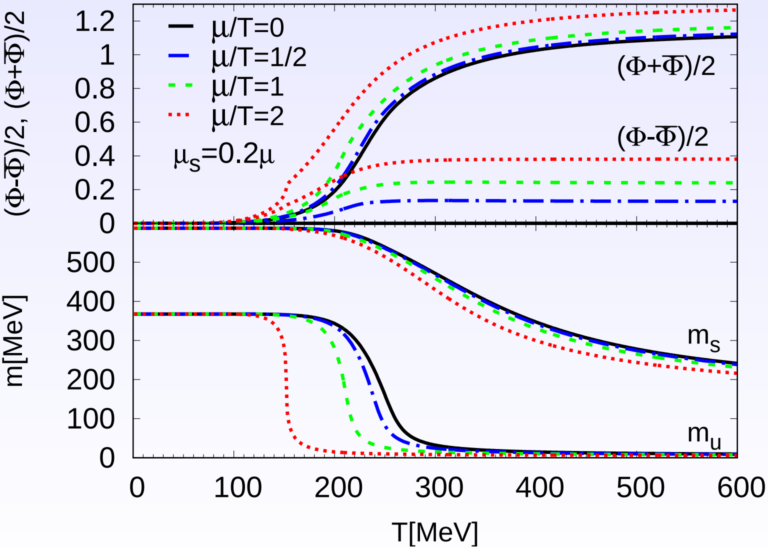
Parameters used for the numerical studies are the current quark masses $m_{0(u,d)} = 5.5$ MeV and $m_{0s} = 138.6$ MeV, the three-momentum cut-off $\Lambda = 602$ MeV and the scalar coupling constant $G_S\Lambda^2 = 2.317$. (Fixed to pion and kaon masses and f_π.) In vacuum constituent quark mass for light quarks 367 MeV and 587 MeV for strange. For PL potential the polynomial form is used[1]

$$\frac{\mathcal{U}(\Phi, T)}{T^4} = -\frac{b_2(T)}{2}\bar{\Phi}\Phi - \frac{b_3}{6}(\Phi^3 + \bar{\Phi}^3) + \frac{b_4}{4}(\Phi\bar{\Phi})^2,$$

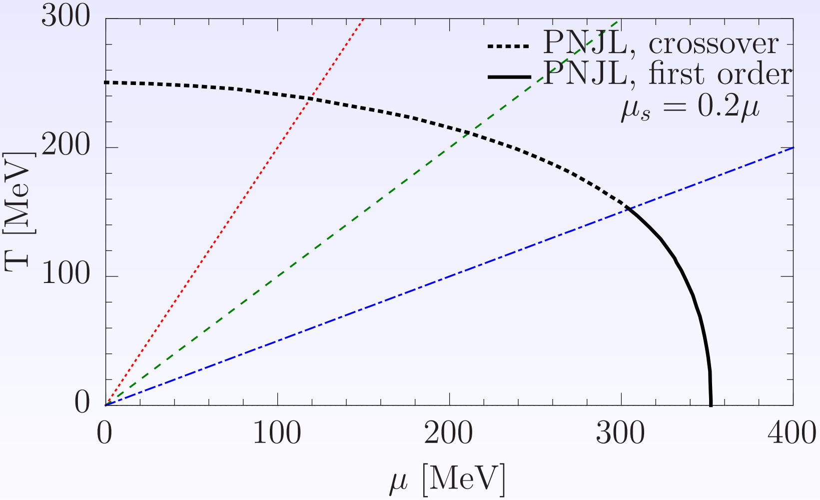
$$b_2(T) = a_0 + a_1 \left[\frac{T_0}{T}\right] + a_2 \left[\frac{T_0}{T}\right]^2 + a_3 \left[\frac{T_0}{T}\right]^3$$

[1] C. Ratti, M. A. Thaler and W. Weise, PRD **73** (2006) 014019

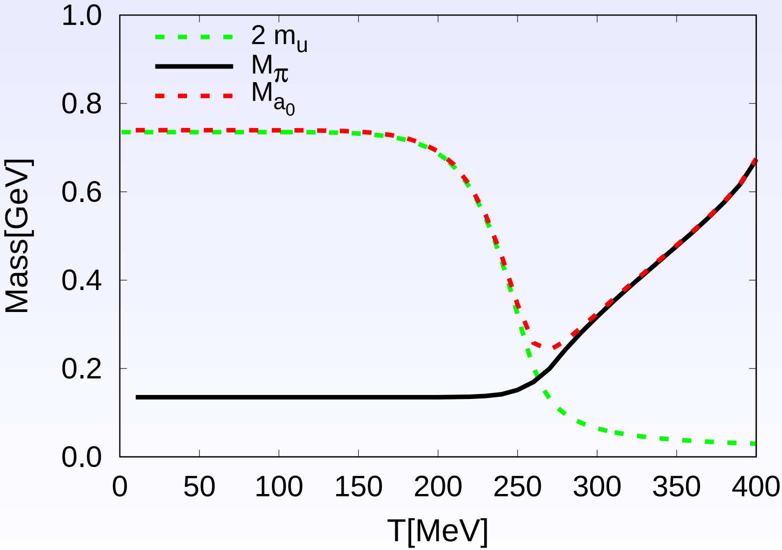
T dependence of quark masses and Φ, $\bar{\Phi}$



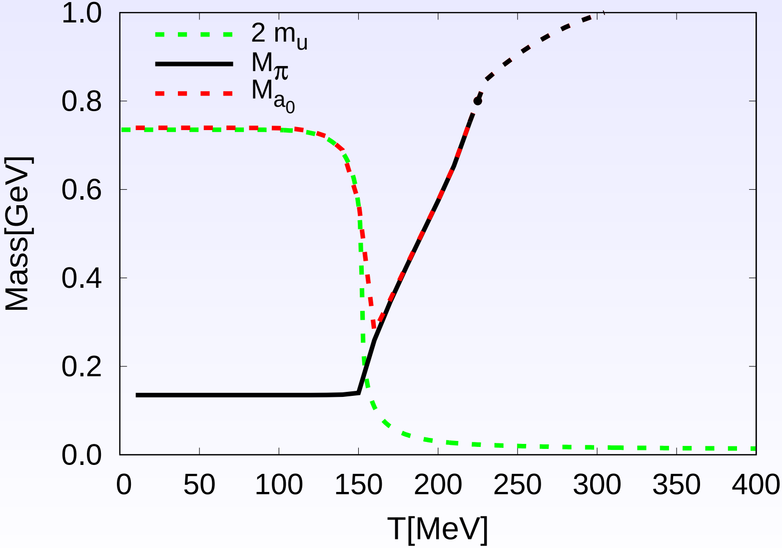
Phase diagram



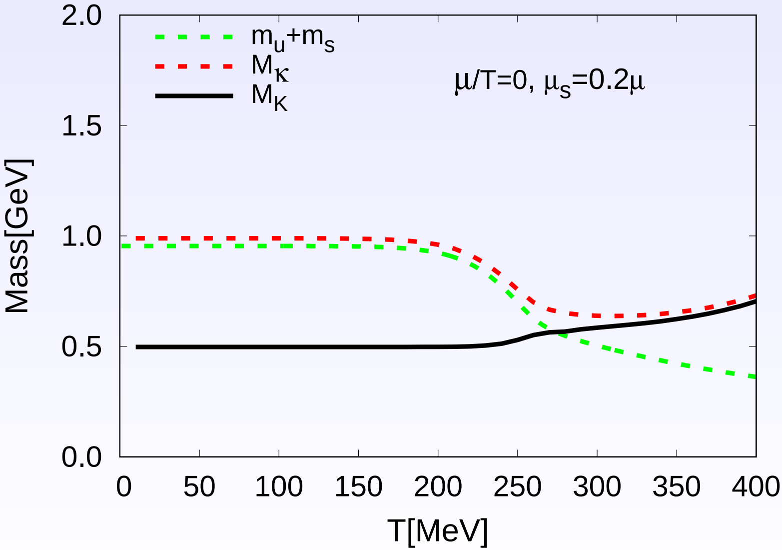
π and a_0 masses. $\mu/T = 0$



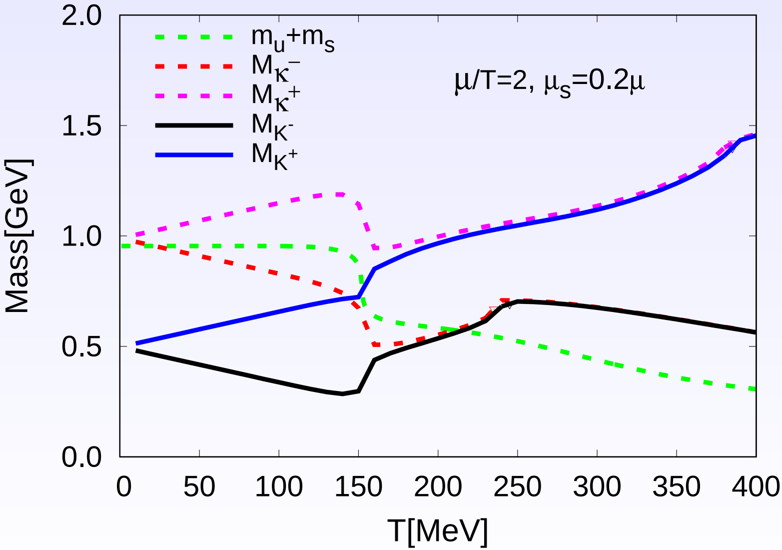
π and a_0 masses. $\mu/T = 2$



K and κ masses. $\mu/T = 0$



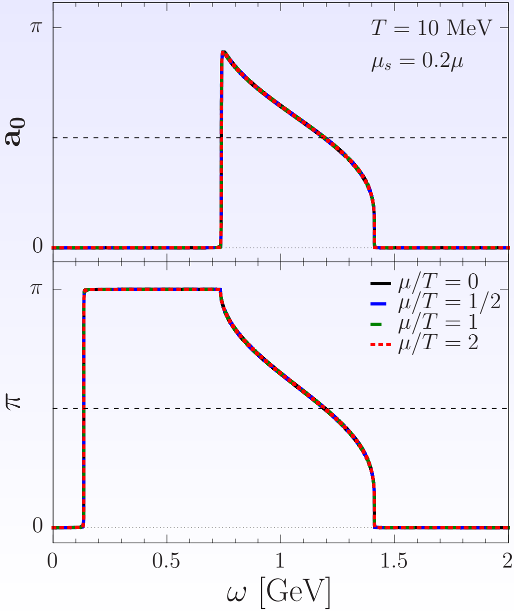
K and κ masses. $\mu/T = 2$



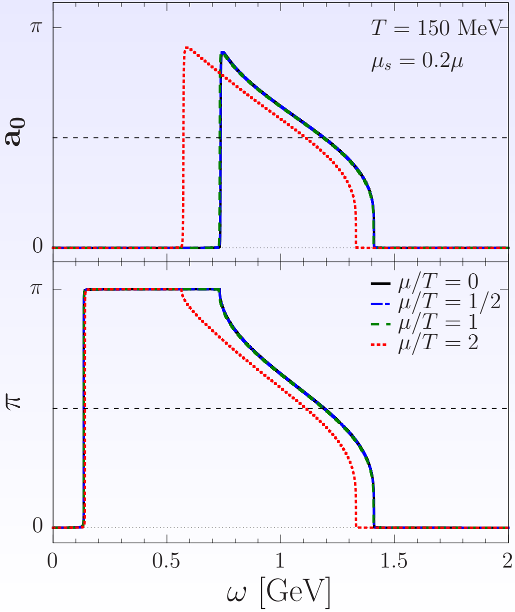
Phase shift

In a hot and dense medium the dynamically generated quark masses drop as a function of temperature and chemical potentials.
At the same time continuum thresholds for $\bar{q}q$ scattering channels drop, which results in a lowering of the binding energy for bound states and finally in their dissociation (Mott effect).
The behaviour of the phase shift at the threshold can be used as an indicator for the Mott transition of a bound state to the scattering state continuum.

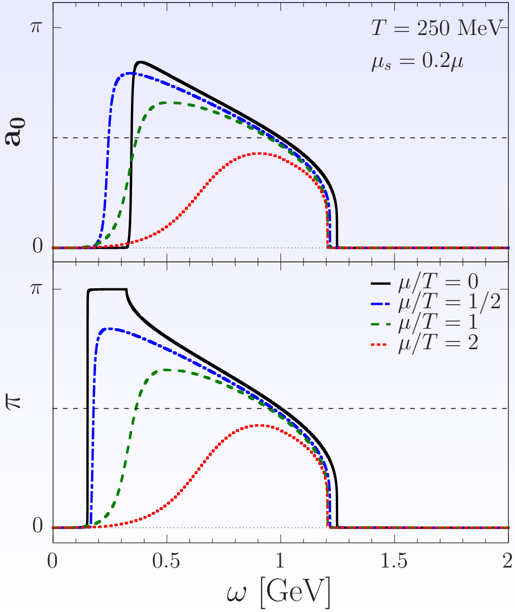
Phase shift for π and a_0. T=10 MeV



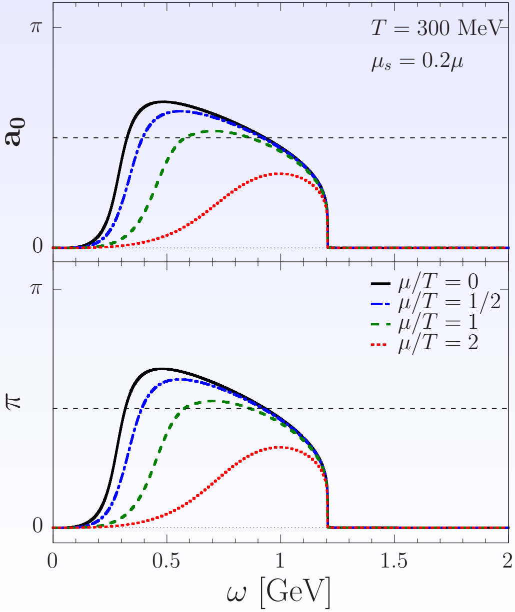
Phase shift for π and a_0. T=150 MeV



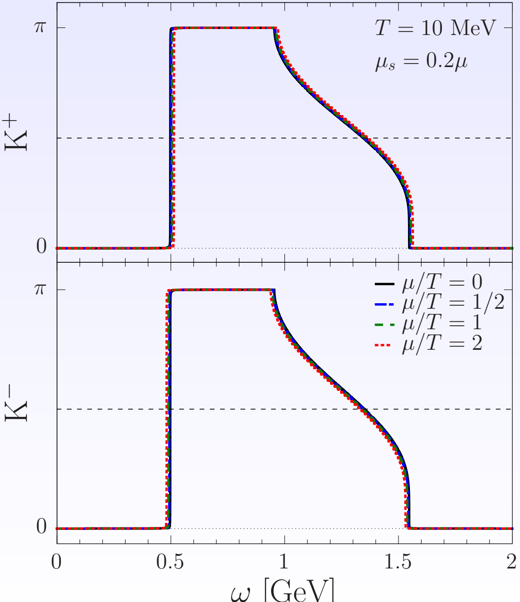
Phase shift for π and a_0. T=250 MeV



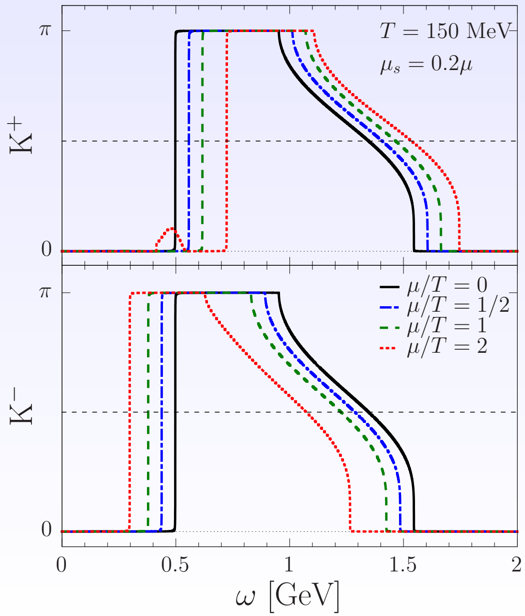
Phase shift for π and a_0. T=300 MeV



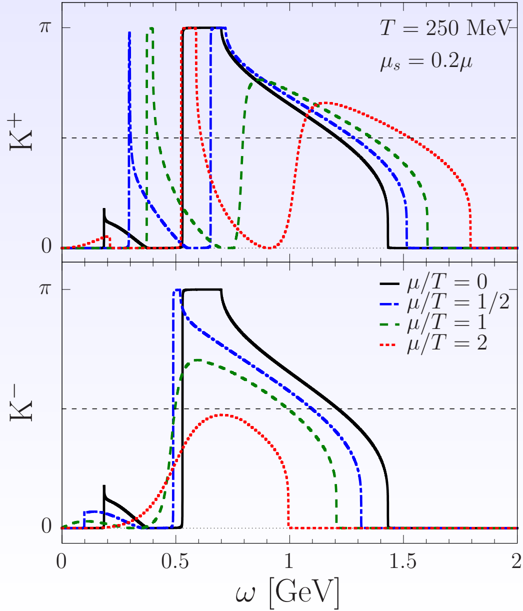
Phase shift for K and κ. T=10 MeV



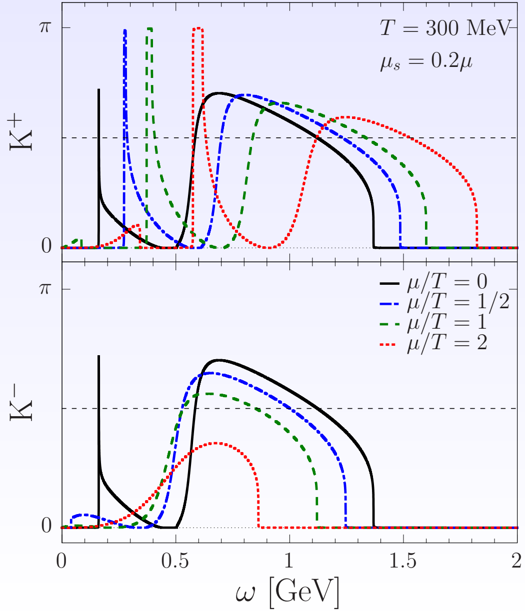
Phase shift for K and κ. T=150 MeV



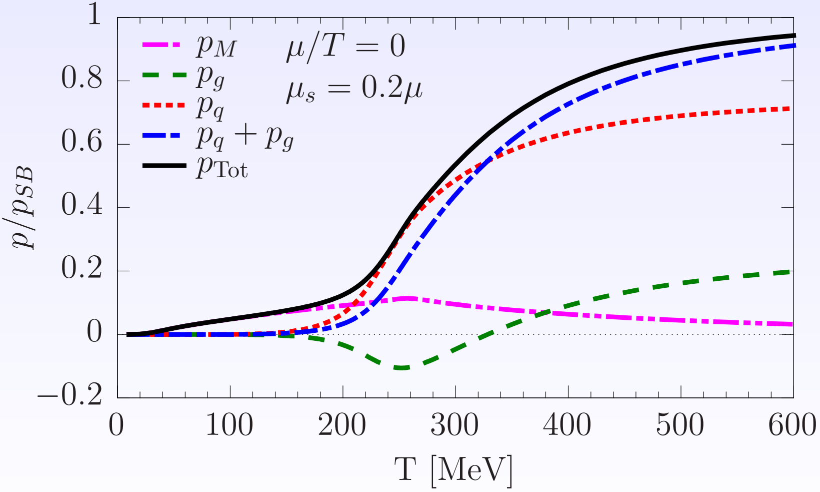
Phase shift for K and κ. T=250 MeV



Phase shift for K and κ. T=300 MeV

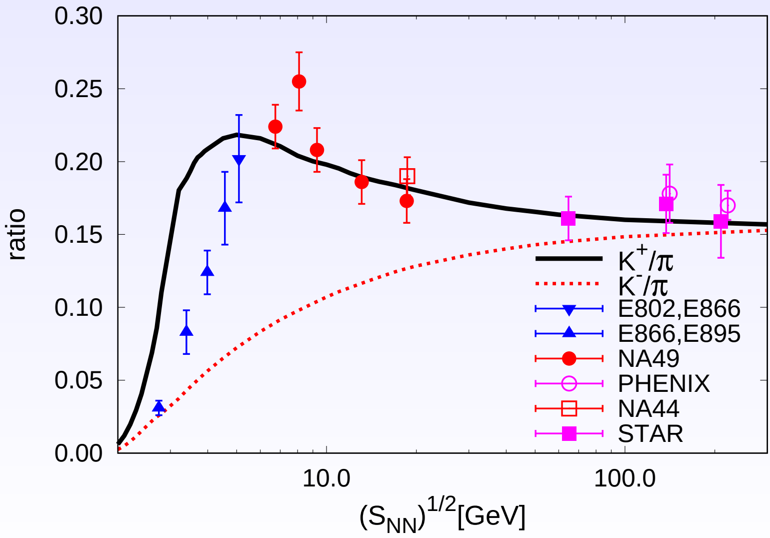


Pressure



p_M
p_g
p_q
$p_q + p_g$
p_{Tot}
$\mu/T = 0$
$\mu_s = 0.2\mu$
p/p_{SB}
1
0.8
0.6
0.4
0.2
0
−0.2
0
100
200
300
400
500
600
T [MeV]

K/π ratio

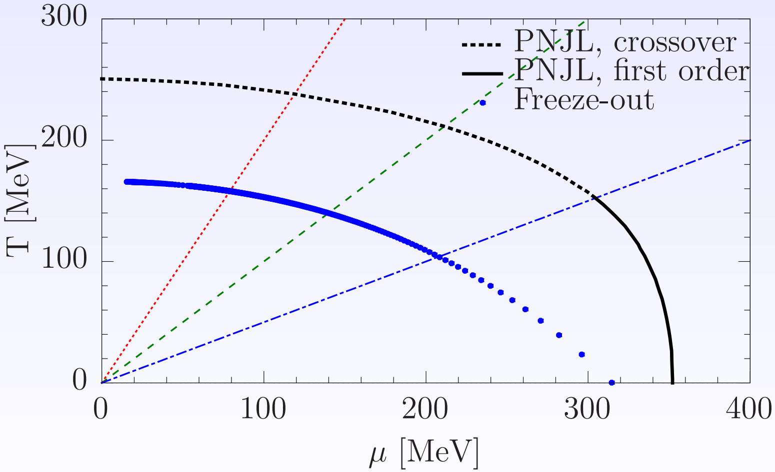


0.30
0.25
0.20
0.15
0.10
0.05
0.00
ratio
10.0
100.0
(S_NN)^1/2[GeV]
K+/π
K-/π
E802,E866
E866,E895
NA49
PHENIX
NA44
STAR

Conclusions

- We considered the thermodynamics of $N_f = 2 + 1$ meson-quark-gluon matter at finite temperature and chemical potential in the framework of the Beth-Uhlenbeck approach.
- The $\bar{q}q$ correlations in scalar and pseudoscalar channels are accounted for by the corresponding phase shifts as solutions of the BSE equations for the meson propagators.
- The mesonic pressure expressed in the BU form reflects the Mott dissociation effect.
- In the behaviour of the phase shifts for $K^{\pm}$ and $\kappa^{\pm}$ mesons we obtain an anomalous low-energy mode (possible relation to "horn").

Phase diagram + freeze-out curve[2]



[2] J. Cleymans, et.al. PRC 73 (2006) 034905