Lattice QCD simulations with external backgrounds

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Mini-Workshop on Lattice and Functional Techniques for Exploration of Phase Structure and Trasport Properties in Quantum Chromodynamics - Dubna, July 10 - 14, 2017

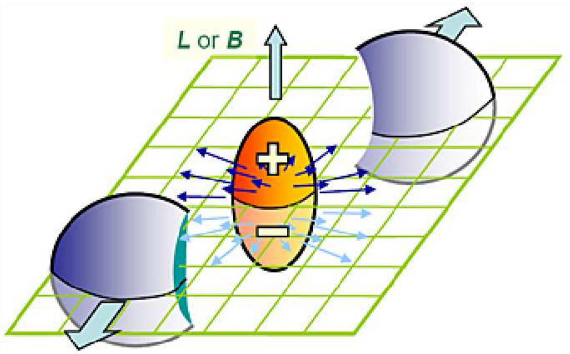
OUTLINE (I + II)

- **Lattice QCD in magnetic background fields:**
  - General properties
  - $Q\bar{Q}$ interactions in a strong magnetic background
  - Effects on color flux tubes
  - New phases for compactified $SU(N)$ gauge theories in a magnetic background

- **Lattice QCD with imaginary chemical potentials:**
  - General properties
  - Generalized susceptibities from imaginary chemical potentials
  - Discussion on the possible location of the critical point

QCD IN EXTERNAL MAGNETIC BACKGROUNDS

Quarks are subject to electroweak interactions, which in general induce small corrections to strong interaction dynamics. Exceptions are expected in the presence of strong e.m. backgrounds, a situation relevant to many contexts:

- **Large magnetic fields are expected in a class of neutron stars known as magnetars ($B \sim 10^{10}$ Tesla on the surface) (Duncan-Thompson, 1992).**
- **Large magnetic fields ($B \sim 10^{16}$ Tesla, $\sqrt{|e|B} \sim 1.5$ GeV), may have been produced at the cosmological electroweak phase transition (Vachaspati, 1991).**
- 

in non-central heavy ion collisions, largest magnetic fields ever created in a laboratory (B up to 10^{15} Tesla at LHC) with a possible rich associated phenomenology (e.g., chiral magnetic effect)

Numerical QCD+QED studies go back to the early days of LQCD

- G. Martinelli, G. Parisi, R. Petronzio and F. Rapuano, Phys. Lett. B 116, 434 (1982).
- C. Bernard, T. Draper, K. Olynyk and M. Rushton, Phys. Rev. Lett. 49, 1076 (1982).

An e.m. background field a_μ modifies the covariant derivative as follows:

$$D_\mu = \partial_\mu + i\, g A^a_\mu T^a \quad \to \quad \partial_\mu + i\, g A^a_\mu T^a + i\, q a_\mu$$

in the lattice formulation:

$$D_\mu \psi \to \frac{1}{2a}\left(U_\mu(n) u_\mu(n) \psi(n+\hat{\mu}) - U^\dagger_\mu(n-\hat{\mu}) u^*_\mu(n-\hat{\mu}) \psi(n-\hat{\mu})\right)$$

$$U_\mu \in SU(3) \qquad \mathbf{u}_\mu \simeq \exp(\mathbf{i}\,\mathbf{q}\,\mathbf{a}_\mu(\mathbf{n})) \in \mathbf{U(1)}$$

- $F^{(em)}_{ij} \neq 0 \implies$ **non-zero magnetic field (no sign problem)**
- $F^{(em)}_{0i} \neq 0 \implies$ **non-zero imaginary electric field (sign problem for real e. f.)**
- **Uniform background field are quantized in the presence of periodic boundary conditions**

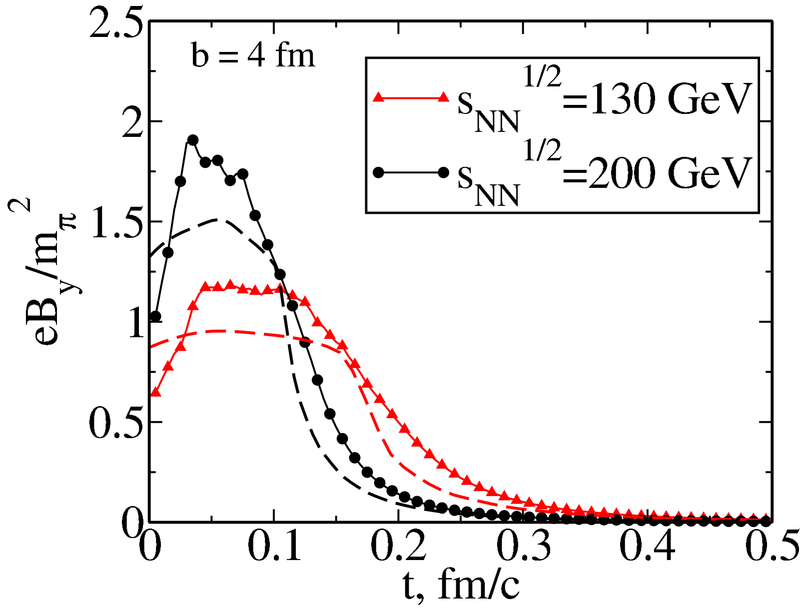
We can only simulate constant magnetic fields and compute equilibrium properties

That may be different from experimental conditions and probes

Estimate of eB time evolution @ RHIC for $Au - Au$ collisions for two values of $\sqrt{s_{NN}}$.

As the collision energy increases the magnetic field increases, but it gets more shrinked in time.

[Skokov, Illarionov and Toneev, '09]

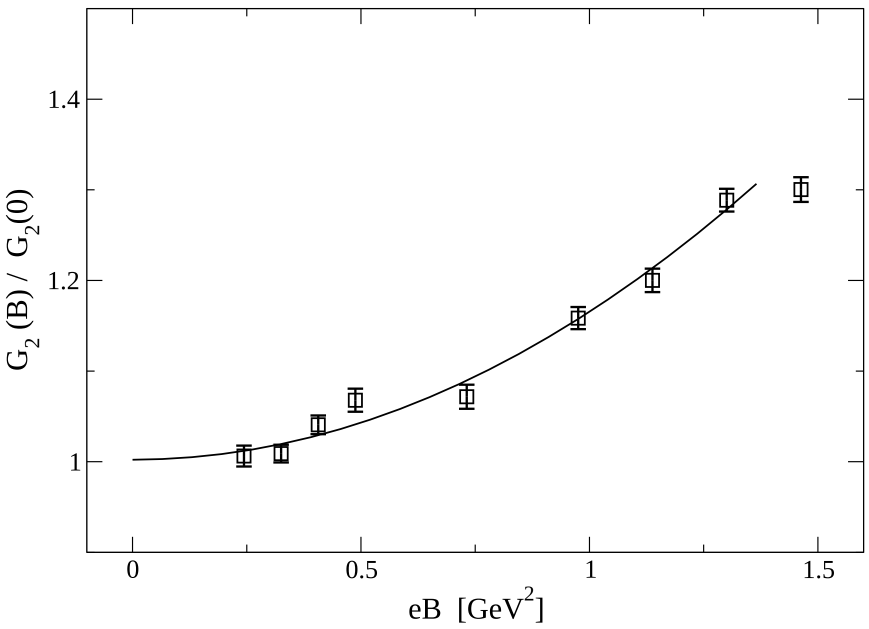


Accurate predictions about the magnetic field evolution requires knowledge of the medium conductivity

Recent years have seen an increasing activity in the lattice study of QCD in magnetic backgrounds. An incomplete summary of results:

Magnetic catalysis (increase of chiral symmetry breaking) of the QCD vacuum has been extensively verified (P. V. Buividovich et al. 2010; MD, F. Negro, 2011; G. S. Bali et al. 2012; E.-M. Ilgenfritz et al., 2012, 2014)

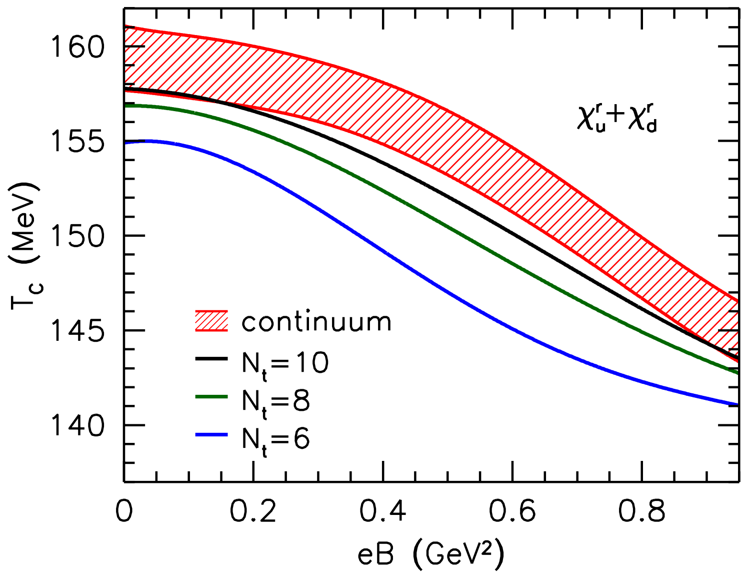
A large effect on gluon fields manifests in anisotropies of gauge observable and in an increase of the gluon condensate as a function of B (gluon magnetic catalysis) (M. Ilgenfritz et al, arXiv:1203.3360; G. Bali et al., arXiv:1303.1328; MD, M. Mesiti, E. Meggiolaro and F. Negro, arXiv:1510.07012)



from arXiv:1510.07012

The magnetic field has strong effects also on QCD thermodynamics and leads to a decrease of the pseudo-critical temperature (inverse magnetic catalysis)

G. S. Bali et al., arXiv:1111.4956

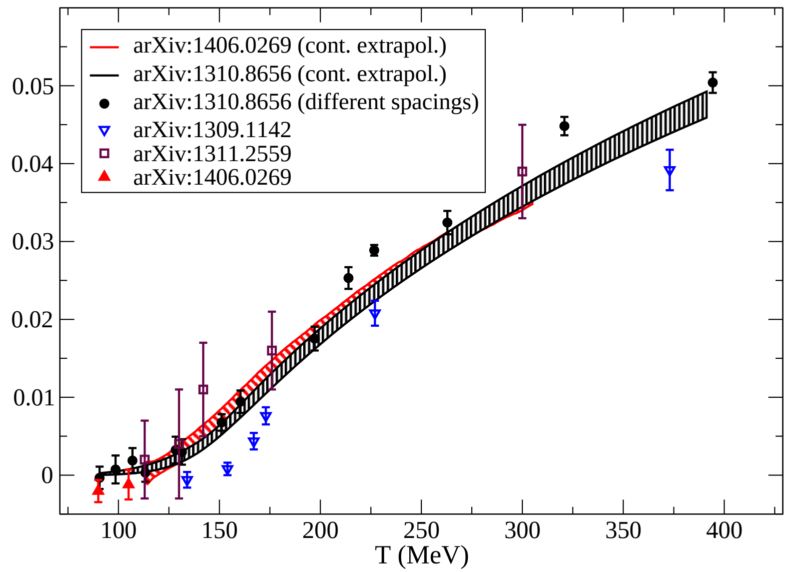


The thermal QCD medium becomes strongly paramagnetic right above T_c

C. Bonati et al., arXiv:1307.8063, arXiv:1310.8656;

L. Levkova and C. DeTar, arXiv:1309.1142;

G. S. Bali et al., arXiv:1406.0269



magnetic susceptibility

The focus here will be on:

Effects of the magnetic field on the static quark potential

- **Is confinement affected by the magnetic background field?**

  A $T = 0$ study has shown that the quark-antiquark potential becomes anisotropic, with a string tension smaller (larger) in the direction parallel to $\vec{B}$ (C. Bonati et al., arXiv:1403.6094)

- The issue is interesting both by itself and for possible phenomenological consequences, e.g. for heavy quark bound states.

I discuss some recent results reported in arXiv:1607.08160 and arXiv:1506.07890 where we try to achieve the following goals:

- A complete determination of the angular dependence of the potential
- An extrapolation to the continuum limit
- An extension to finite temperature, both below and above the pseudocritical temperature

LATTICE SETUP

$$Z(B) = \int \mathcal{D}U \, e^{-\mathcal{S}_{YM}} \prod_{f=u,\,d,\,s} \det \left(D^{f}_{\textrm{st}}[B] \right)^{1/4} .$$

- pure gauge: Symanzik tree level improved gauge action

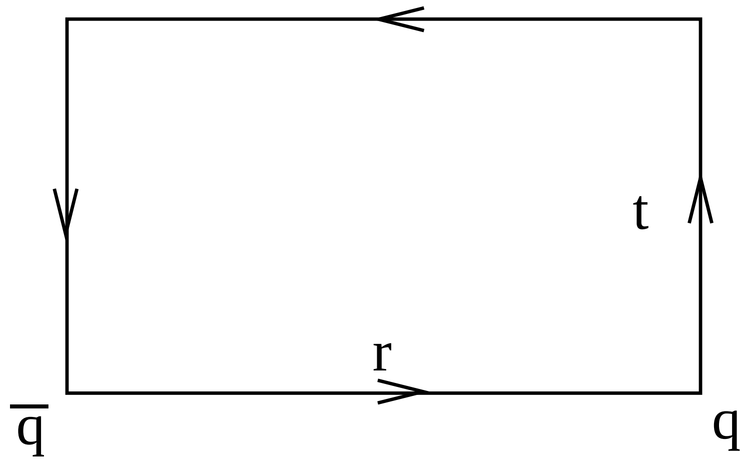
- fermion sector: 2-level stout improved rooted staggered fermions

- physical quark masses

- explored lattice spacings and sizes:
$a = 0.2173, \; 0.1535, \; 0.1249, \; 0.0989$ fm $\quad L_s a \sim 5$ fm in all cases

- numerical simulations on FERMI (BG/Q at CINECA) thanks to PRACE allocation

At $T = 0$, the potential is determined through Wilson loop expectation values

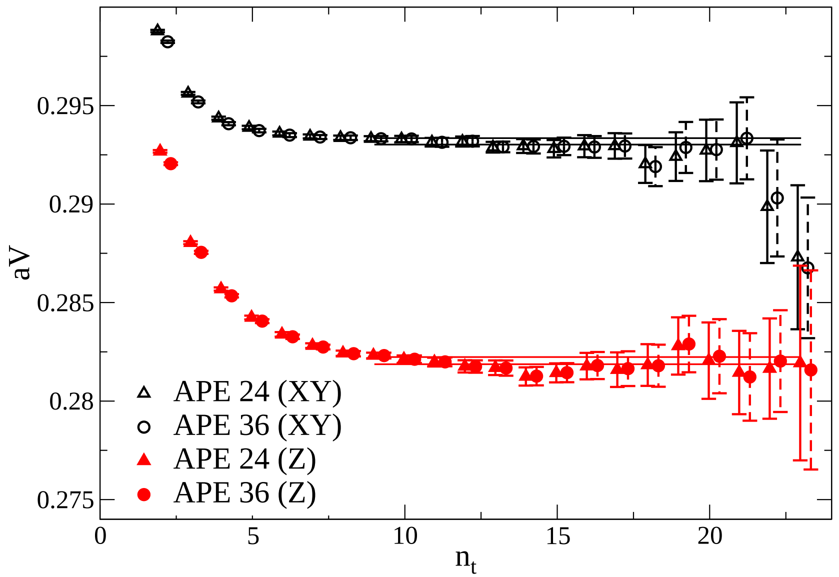


1 HYP smearing for temporal links and various APE smearings for spatial links to reduce UV fluctuations

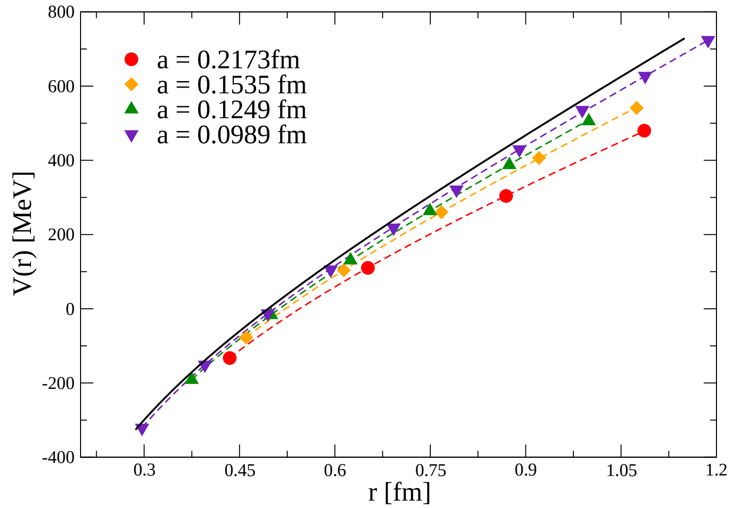
As usual

$$aV(a\vec{n}) = \lim_{n_t \to \infty} \log \left(\frac{\langle W(\vec{n}, n_t) \rangle}{\langle W(\vec{n}, n_t + 1) \rangle} \right)$$

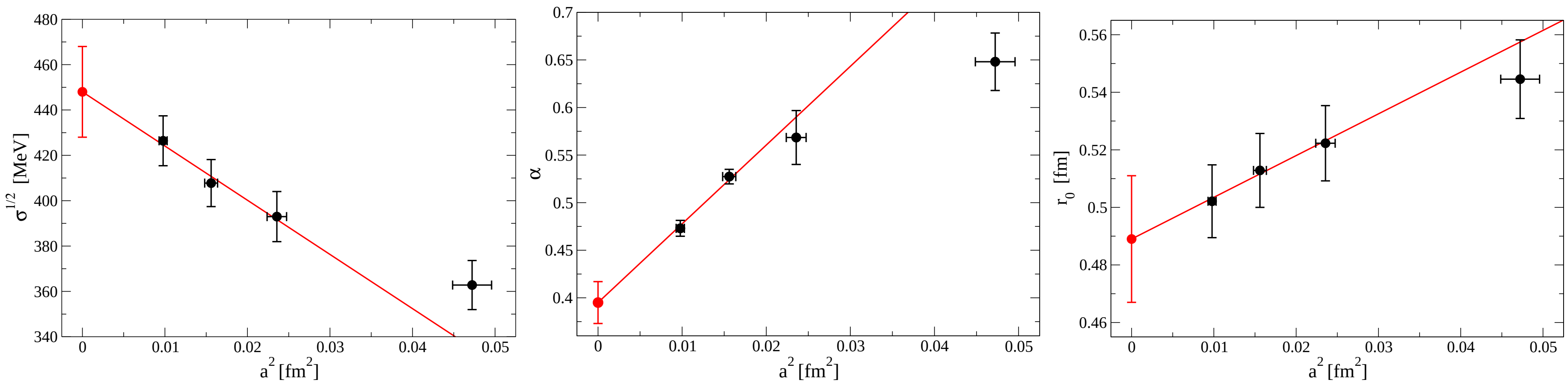
results in the figure refer to two different orientations with respect to $\vec{B} = B\hat{z}$, and for simulations performed at $a \simeq 0.0989$ fm with $|e|B \simeq 1$ GeV^2.



we have first studied the potential at $B = 0$, adopting the Cornell potential as an ansatz for all lattice spacings



$$V(r) = -\frac{\alpha}{r} + \sigma r + V_0 \ ,$$



In this way we obtain continuum extrapolated results for σ, α and for the Sommer parameter r_0

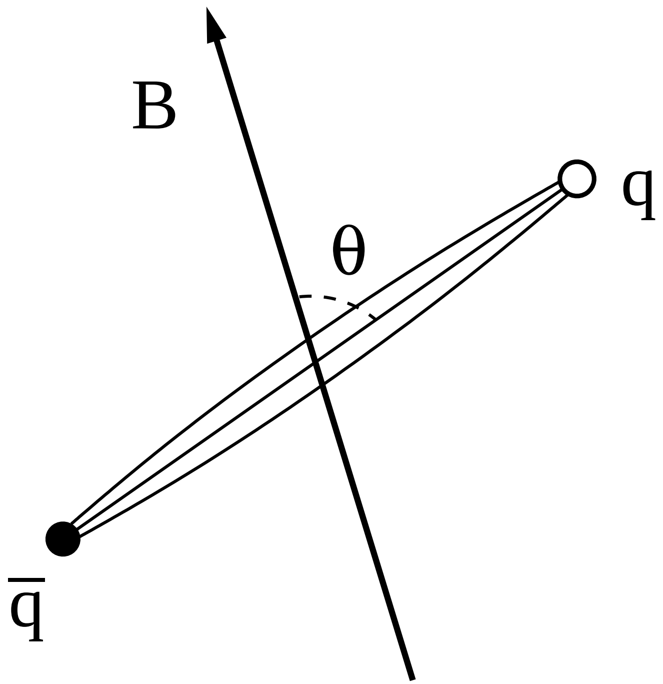
$$r_0^2 \left. \frac{dV}{dr} \right|_{r_0} = 1.65$$

α	**0.395(22)**
$\sqrt{\sigma}$	**448(20) MeV**
r_0	**0.489(20) fm**

For non-zero background field $\vec{B}$, we want to study the potential not just for parallel or orthogonal directions, but for generic orientations.

In principle, one can either rotate the spatial side of the Wilson loop, or rotate $\vec{B}$ and perform new simulations.

Rotating the loop on the lattice introduces new cusps and renormalization effects, so we chose the second solution



Each component of the field gets quantized in the presence of spatial periodic b.c.

$$eB_x = 6\pi b_x/(a^2 N_z N_y)\,; \quad b_x \in \mathbb{Z}$$

$$eB_y = 6\pi b_y/(a^2 N_x N_z)\,; \quad b_y \in \mathbb{Z}$$

$$eB_z = 6\pi b_z/(a^2 N_x N_y)\,; \quad b_z \in \mathbb{Z}$$

we performed different simulations at fixed $B_x^2 + B_y^2 + B_z^2$ and different $\vec{B}$ orientations

Expected symmetries and ansatz for $V(r,\theta,\phi)$

- **by residual rotational symmetry around $\vec{B}$:** $V(r,\theta,\phi) = V(r,\theta)$
- **by symmetry under $\vec{B} \to -\vec{B}$:** $V(r,\pi-\theta) = V(r,\theta)$
- **We make the assumption the potential is Cornell like along each direction**

$$V(r,\theta) = -\frac{\alpha(\theta,B)}{r} + \sigma(\theta,B)r + V_0(\theta,B)$$

and write a Fourier expansion in θ for each term:

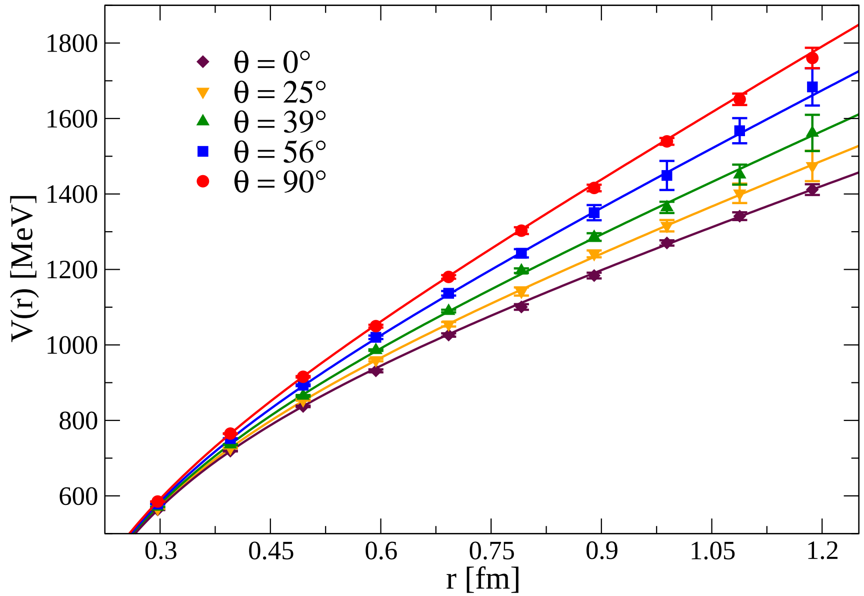
$$\begin{aligned} V(r,\theta) = &-\frac{\bar{\alpha}(B)}{r}\Big(1 - \sum_{n=1} c_{2n}^{\alpha}(B)\cos(2n\theta)\Big) \\ &+ \bar{\sigma}(B) r \Big(1 - \sum_{n=1} c_{2n}^{\sigma}(B)\cos(2n\theta)\Big) \\ &+ \bar{V}_0(B)\Big(1 - \sum_{n=1} c_{2n}^{V_0}(B)\cos(2n\theta)\Big) \,. \end{aligned}$$

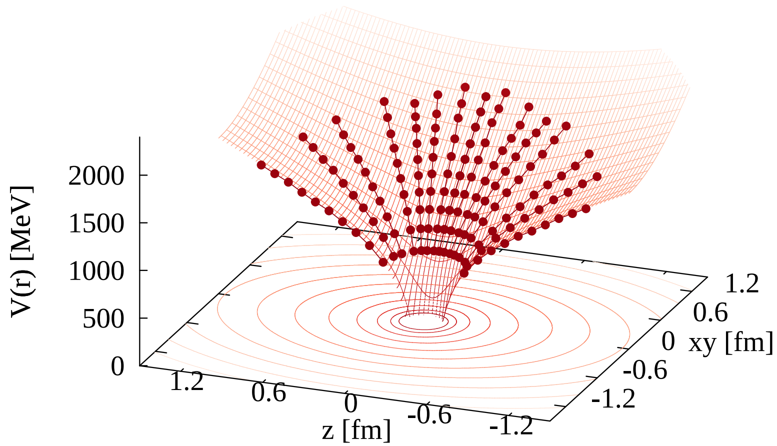
RESULTS

- **full angular dependence studied at just two lattice spacings, $a \simeq 0.1, 0.15$ fm and for $eB \sim 1$ GeV^2. Results shown for $a \sim 0.1$ fm**

- **At fixed r, the potential is an increasing function of the angle and reaches a maximum for orthogonal directions**

- **Our ansatz works well ($\chi^2/d.o.f. \sim 1$) with only the first term in the expansion $c_2 \neq 0$ (quadrupole-like deformation)**





Strategy followed for other B and a: The simplified angular dependence (only $c_2 \neq 0$), permits to reconstruct V from data at $\theta = 0, \pi$ only.

Let $\mathcal{O}$ be α, σ, V and define

$$\delta^{\mathcal{O}}(|e|B) = \frac{\mathcal{O}_{XY}(|e|B) - \mathcal{O}_Z(|e|B)}{\mathcal{O}_{XY}(|e|B) + \mathcal{O}_Z(|e|B)} \quad ; \qquad R^{\mathcal{O}}(|e|B) = \frac{\mathcal{O}_{XY}(|e|B) + \mathcal{O}_Z(|e|B)}{2\mathcal{O}(|e|B = 0)}$$

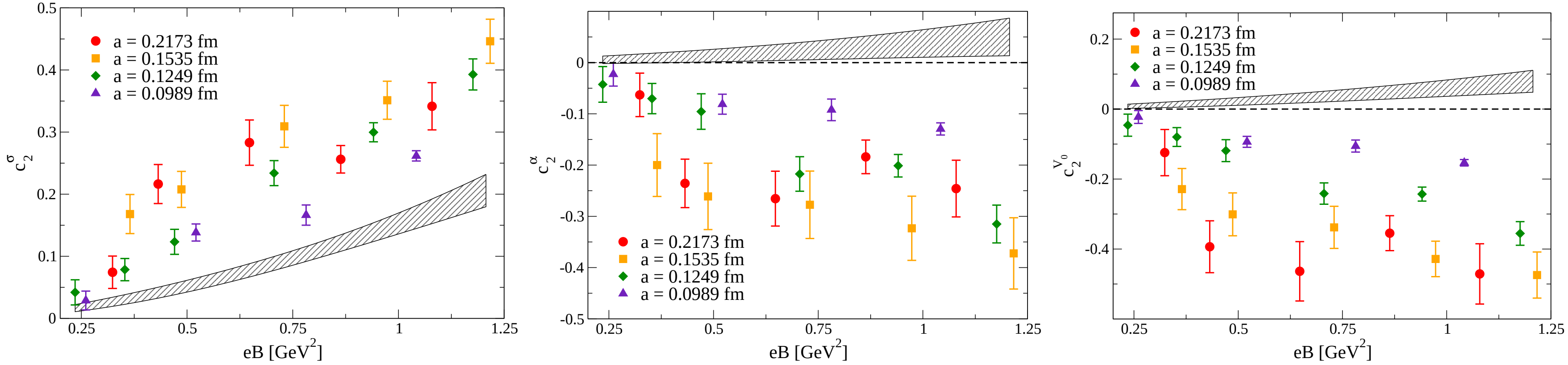
then

$$\delta^{\mathcal{O}} = c_2^{\mathcal{O}} + c_4^{\mathcal{O}} + \cdots = \sum_n c_{2n}^{\mathcal{O}} \simeq c_2^{\mathcal{O}}$$

$$R^{\mathcal{O}}(|e|B) = \frac{\bar{\mathcal{O}}(|e|B)}{\mathcal{O}(|e|B = 0)} \left(1 - \sum_{n \text{ even}} c_{2n}^{\mathcal{O}}\right) \simeq \frac{\bar{\mathcal{O}}(|e|B)}{\mathcal{O}(|e|B = 0)}$$

i.e. such quantities are enough to fix all the coefficients giving a non-trivial contribution to $V(r, \theta, B)$.

We perform the continuum extrapolation from data along longitudinal and orthogonal directions only

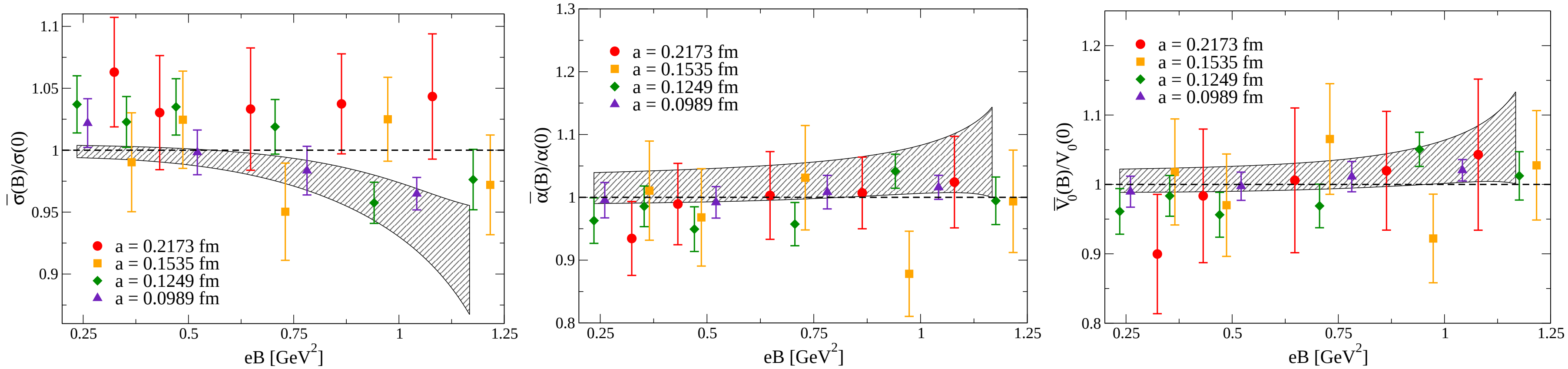


Continuum limit extrapolation of our data according to the following power-law ansatz

$$c_2^{\mathcal{O}} = A^{\mathcal{O}}(1 + C^{\mathcal{O}}a^2)(|e|B)^{D^{\mathcal{O}}(1+E^{\mathcal{O}}a^2)}$$

$$R^{\mathcal{O}} = 1 + \bar{A}^{\mathcal{O}}(1 + \bar{C}^{\mathcal{O}}a^2)(|e|B)^{\bar{D}^{\mathcal{O}}}$$

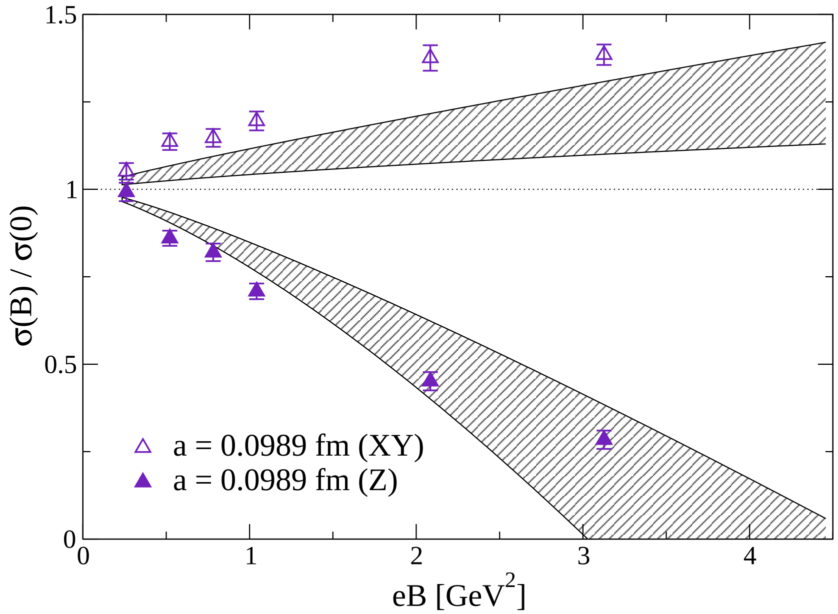
$\rightarrow$ The only quantities with non-trivial B dependence in the continuum are c_2^σ and $\bar{\sigma}$



The continuum extrapolated results for σ predict a vanishing longitudinal string tension for $eB \sim 4$ GeV^2

This is outside the range explored for the continuum extrapolation, $eB \lesssim 1$ GeV^2.

Can we trust the prediction?



Cut-off effects are large for $eB \gtrsim 1/a^2$. We could extend to larger B just on the finest lattice spacing.

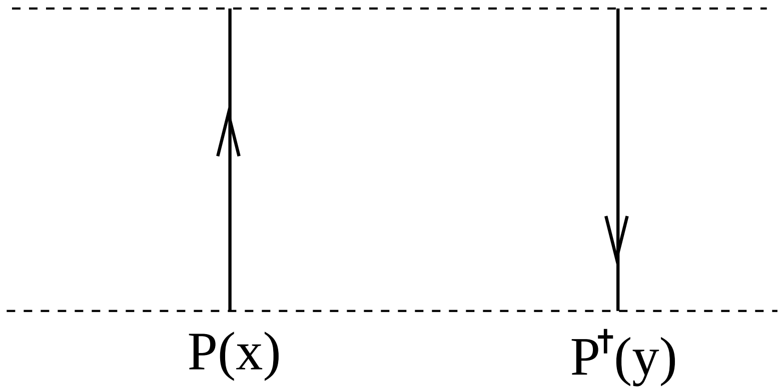
The decrease of $\sigma_{\parallel}$ is steady, even if it somewhat undershoots the continuum band extrapolated to large B.

Simulations at finer lattice spacings should clarify the issue in the future.

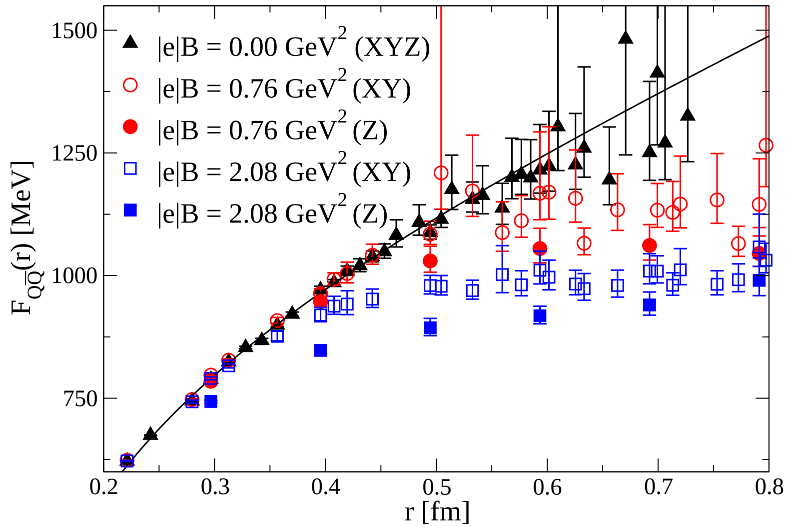
Finite T results

At finite T, the quark-antiquark potential is measured from Polyakov loop correlators

$$\langle \mathrm{Tr} P(\vec{x})\, \mathrm{Tr} P^\dagger(\vec{y}) \rangle \sim \exp\left(-\frac{F_{\bar{q}q}(r,T)}{T} \right)$$



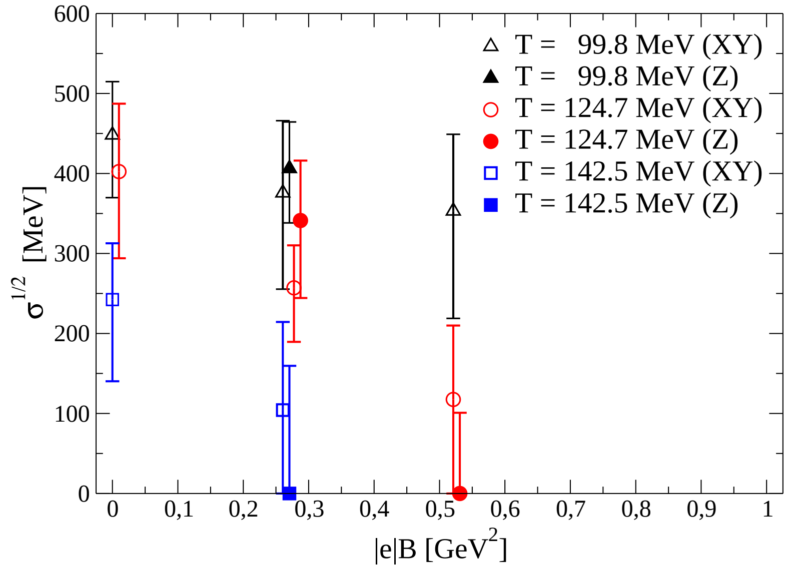
Results at $T \sim 100$ MeV on a $N_t = 20$ lattice



Although a small anisotropy is still visible, the main effect of B seems to suppress the potential in all directions

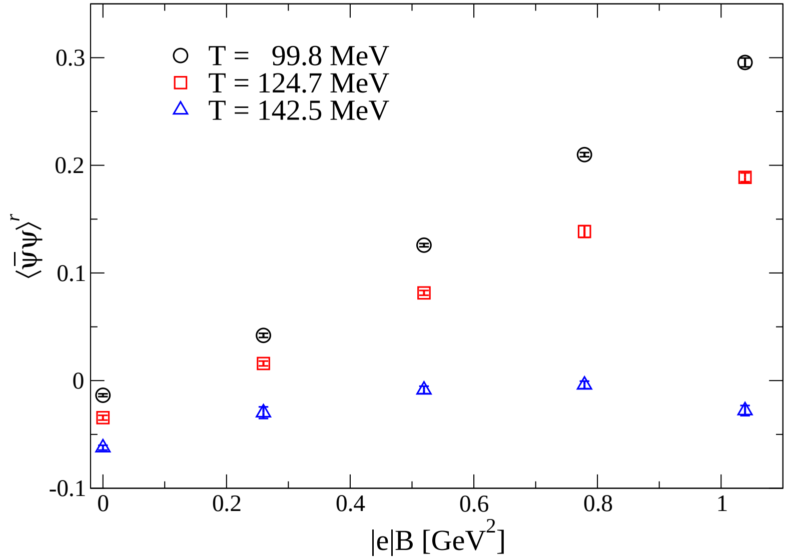
The string tension tends to disappear

A fit to the Cornell potential works in a limited range of distances and permits to obtain a determination of σ, which shows a steady decrease in all directions.

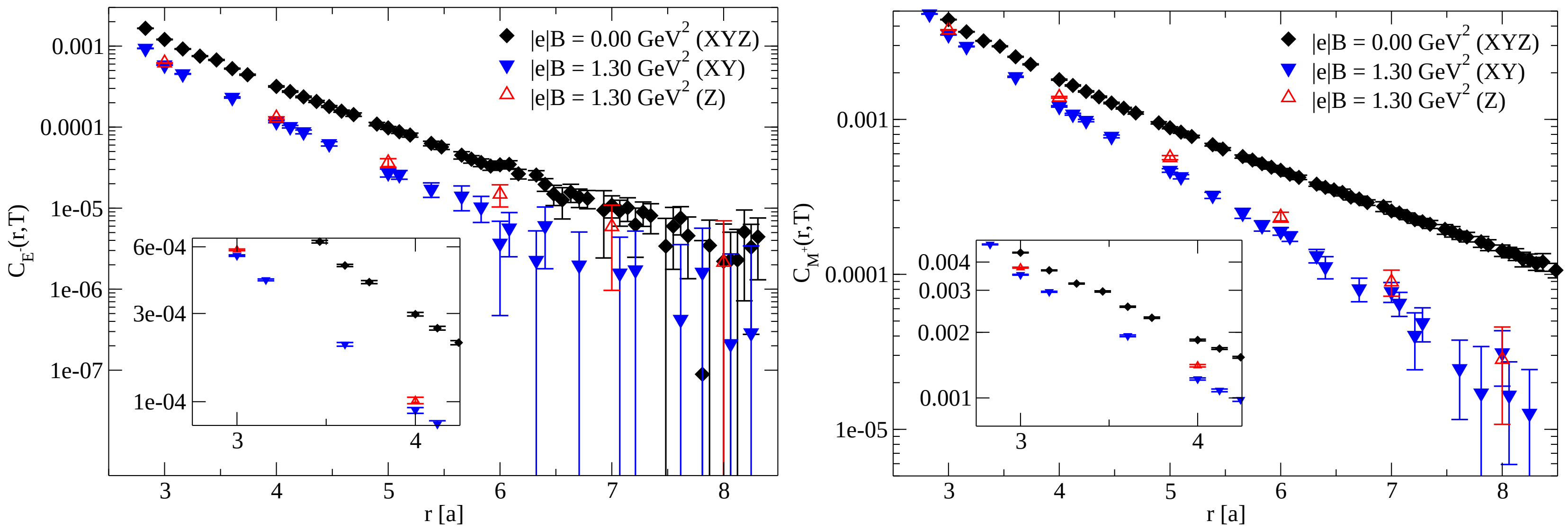


We can call this effect deconfinement catalysis

It is interesting to notice that this happens before (in temperature) inverse magnetic catalysis is visible in the chiral condensate



Is the decrease of T_c as a function of B related to a change in the confining properties?



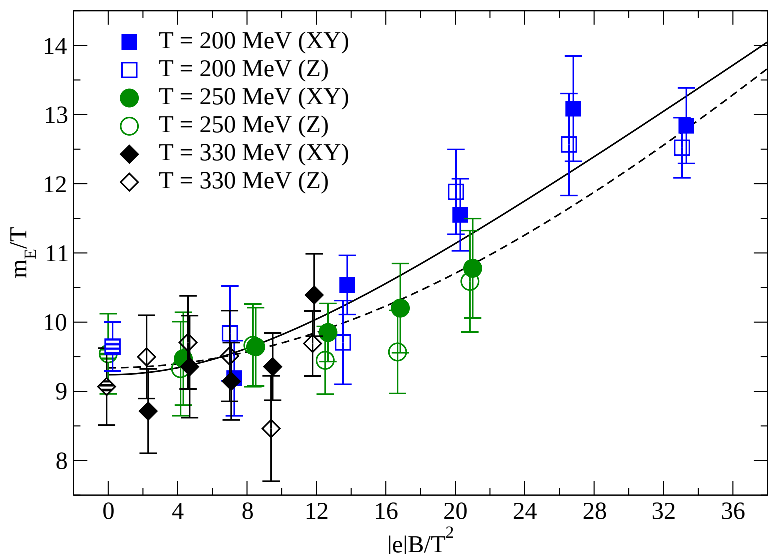
In the deconfined phase, Polyakov loop correlators give access to electric and magnetic screening masses

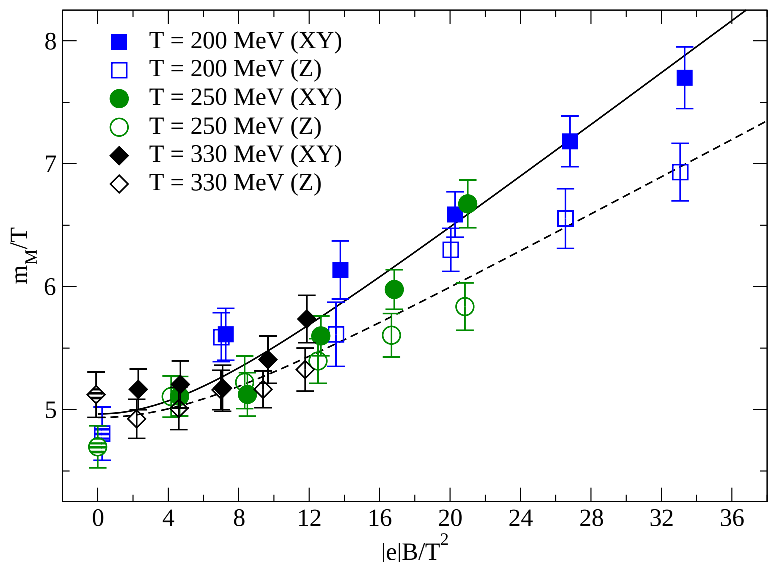
$$C_{M^+} = +\frac{1}{2}\mathrm{Re}\big[C_{LL} + C_{LL^\dagger}\big] - |\langle \mathrm{Tr}L\rangle|^2$$

$$C_{E^-} = -\frac{1}{2}\mathrm{Re}\big[C_{LL} - C_{LL^\dagger}\big] \; .$$

$$C_{E^-}(\mathbf{r},T)\big|_{r\to\infty} \simeq \frac{e^{-m_E(T)r}}{r}$$

$$C_{M^+}(\mathbf{r},T)\big|_{r\to\infty} \simeq \frac{e^{-m_M(T)r}}{r}$$





Such masses show a clear (increasing) dependence on B: the magnetic background field enhances the color screening properties of the QGP

$$\frac{m^d_{E/M}}{T} = a^d_{E/M}\left[1 + c^d_{1;E/M}\frac{|e|B}{T^2}\,\mathrm{atan}\left(\frac{c^d_{2;E/M}}{c^d_{1;E/M}}\frac{|e|B}{T^2}\right)\right] ,$$

Does B have any influence on heavy quark bound state suppression? (Satz, Matsui)

Actually, one should measure also quarkonia radii as a function of B and compare the two lengths ...

as an alternative a direct determination of quarkonia spectral functions in the presence of B would be the most direct way (possible project for the future? ...)

Color flux tubes in strong magnetic fields

We now go back to $T = 0$. Is the deformation of the static quark-antiquark potential associated with a corresponding deformation of the color flux tube?

In principle, two different phenomena may happen:

- The flux tube for longitudinal separation is less intense than that for transverse separation;
- The flux tube for transverse separation loses cilindrical symmetry and becomes anisotropic

Lattice determinations of color flux tubes make use of correlation between Wilson loops and plaquette operators.

Connected correlators allow the determination of the field strength itself

[Di Giacomo, Maggiore, Olejnik, 1990] [Cea, Cosmai, Cuteri, Papa, 2017]

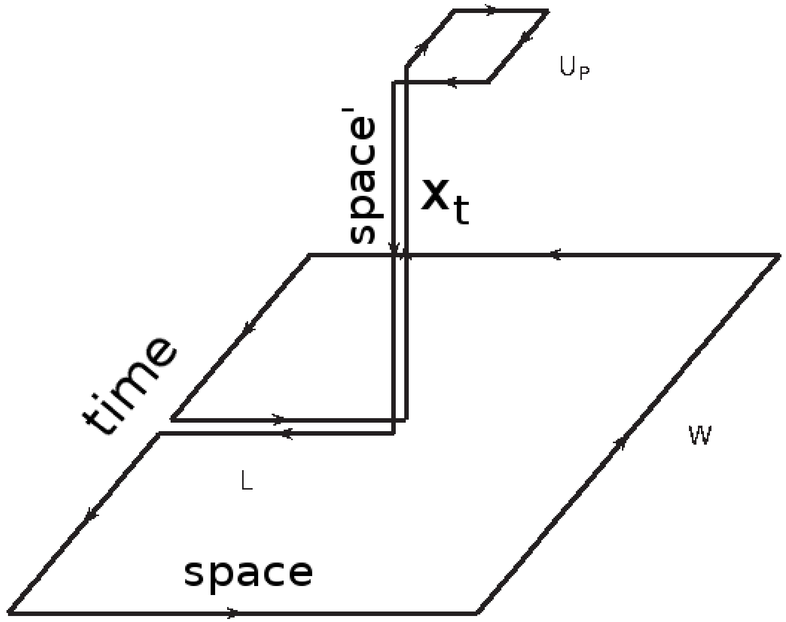
$$E_l^{chromo} = \lim_{a \to 0} \frac{1}{a^2 g} \left[\frac{\langle Tr(W L U_P L^\dagger) \rangle}{\langle Tr(W) \rangle} - \frac{\langle Tr(W) Tr(U_P) \rangle}{\langle Tr(W) \rangle} \right]$$

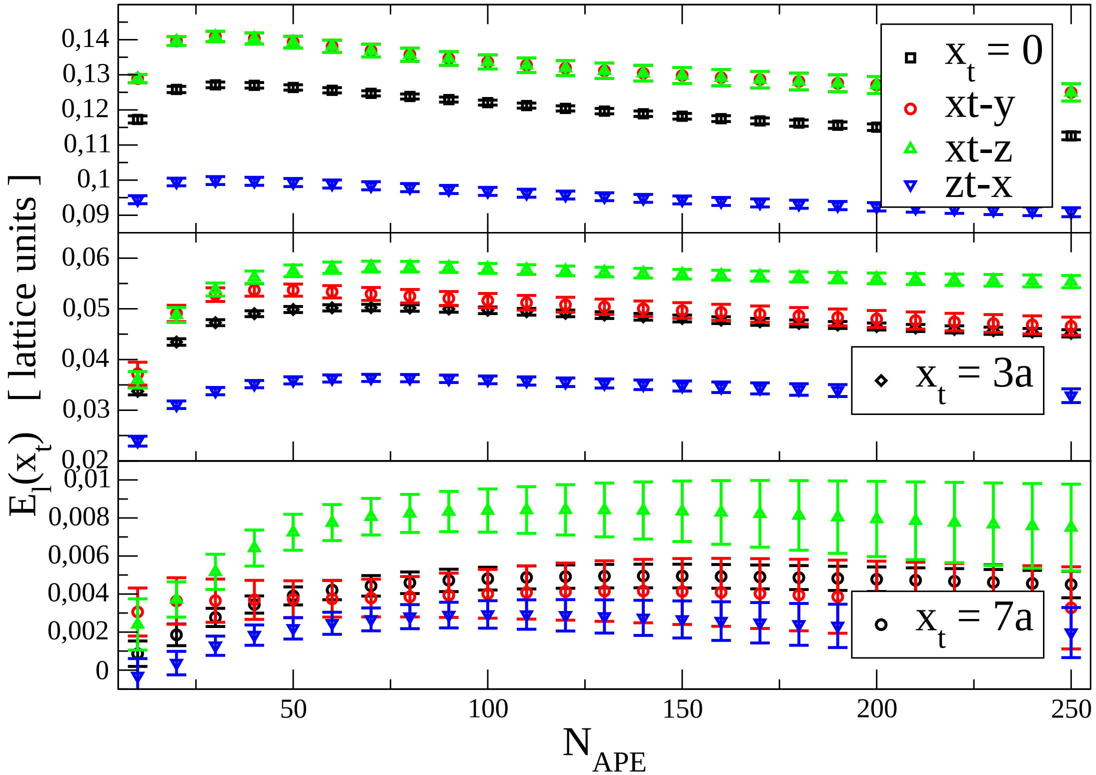
W is the open Wilson loop operator

U_P is the open plaquette operator

L is the adjoint parallel transport

A smearing procedure is adopted (1 HYP for temporal links, several APE for spatial links) as a noise reduction technique





Results at $eB = 0$ vs $eB \sim 2$ GeV^2 for $Q\bar{Q}$ separation 0.7 fm ($a \simeq 0.1$ fm), different transverse distances x_t, as a function of the APE smearing steps $\alpha_{APE=1/6}$

BLACK: $B0$

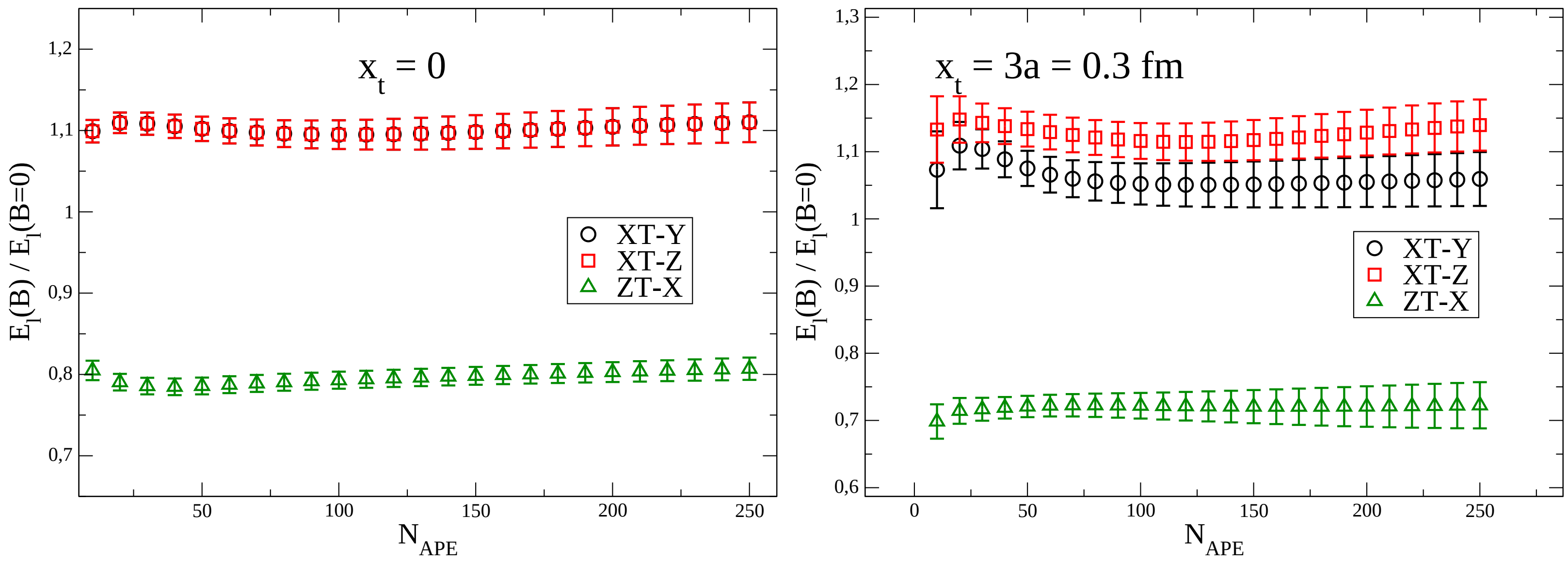
ZT-X: $Q\bar{Q}$ **separation parallel to** B

XT-Y: $Q\bar{Q}$ **separation orthogonal to** B**; transverse direction orthogonal to** B

XT-Z: $Q\bar{Q}$ **separation orthogonal to** B**; transverse direction parallel to** B

Dependence on the smearing step is non-trivial, however ...

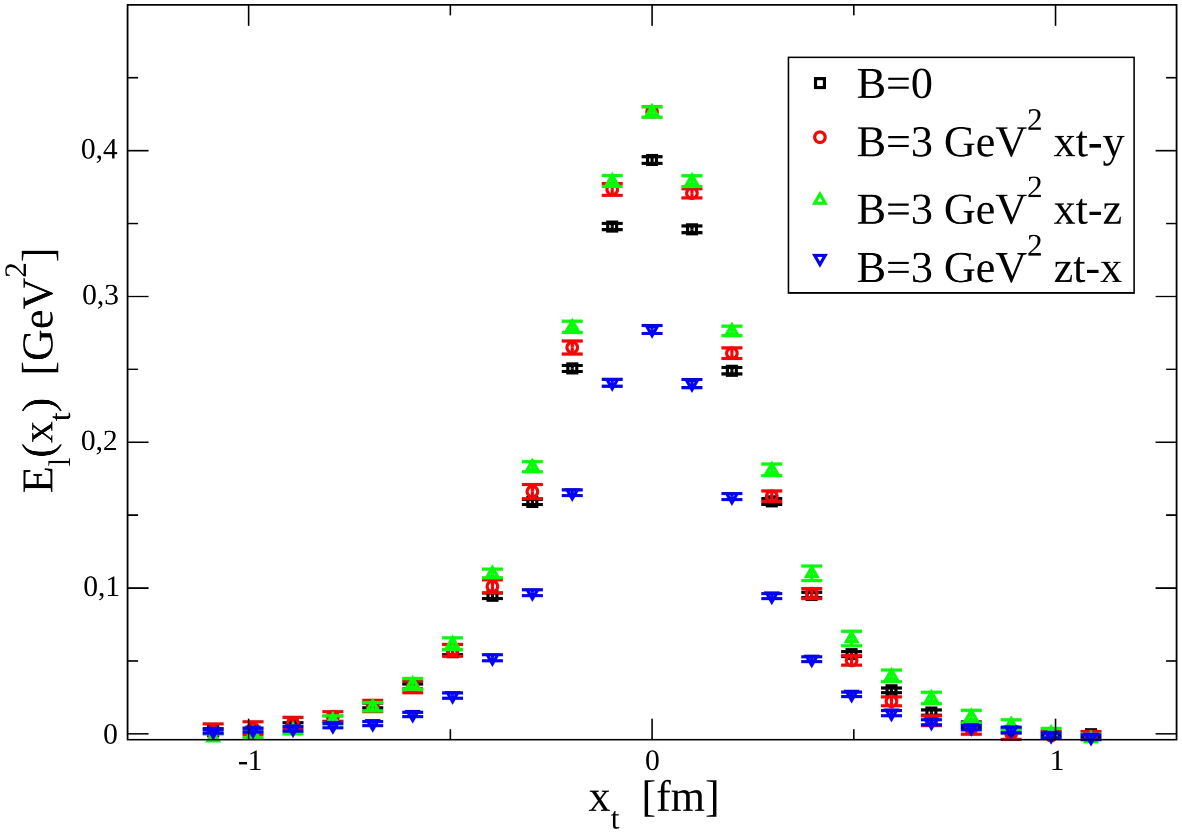
... the dependence almost completely disappears as we consider the ratio of $B \neq 0$ to $B = 0$ quantities



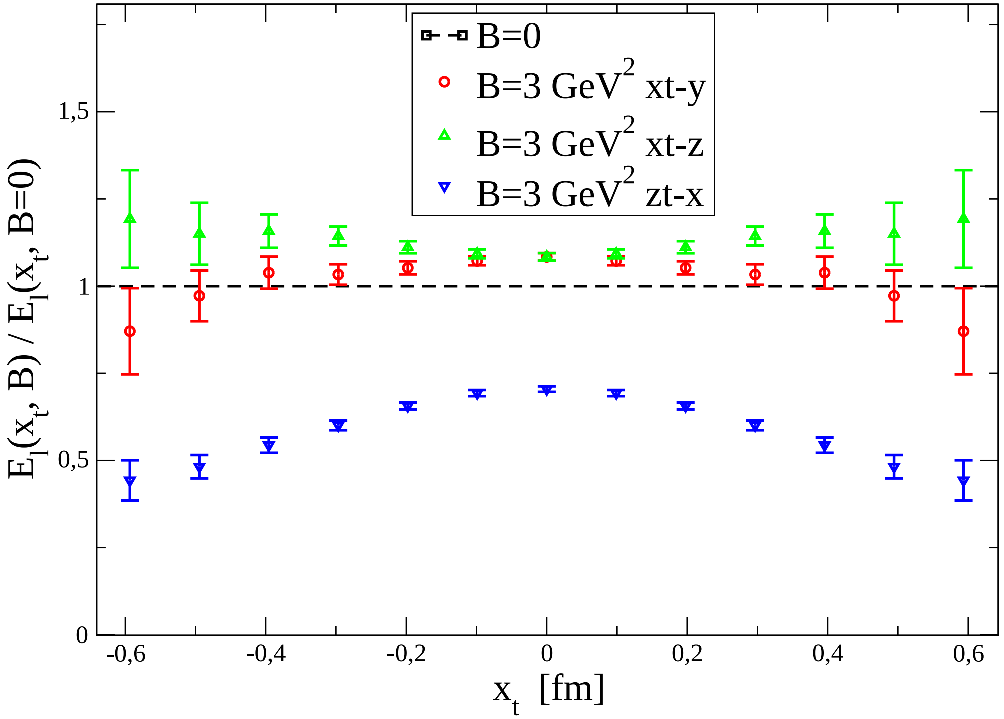
following analysis (PRELIMINARY) mostly based on such ratios

Signals of both kinds of anisotropy already visible:

- field strength for $Q\bar{Q}$ separation parallel/orthogonal to B is suppresses/enhanced
- for separation orthogonal to B, field strength keeps larger when moving along B



These are the flux tube profiles for $eB \sim 3$ GeV^2 compared to $B = 0$ at a fixed number of smearing steps $N_{APE} = 80$



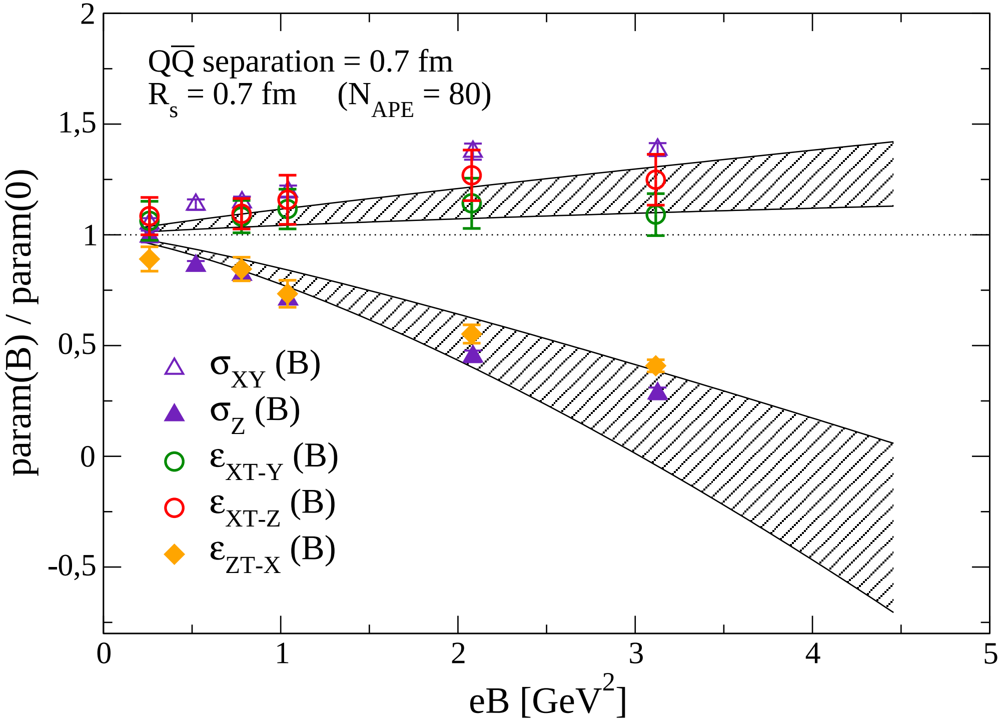
These are the same data (for $eB = 3$ GeV^2) normalized to those at $B = 0$. It is not just the overall normalization of the flux tube which changes, but also the flux tube profile:

- it clearly shrinks for $Q\bar{Q}$ separation parallel to B

- it is more or less stable for orthogonal separation, with a tendency to shrink/widen in directions orthogonal/parallel to B.

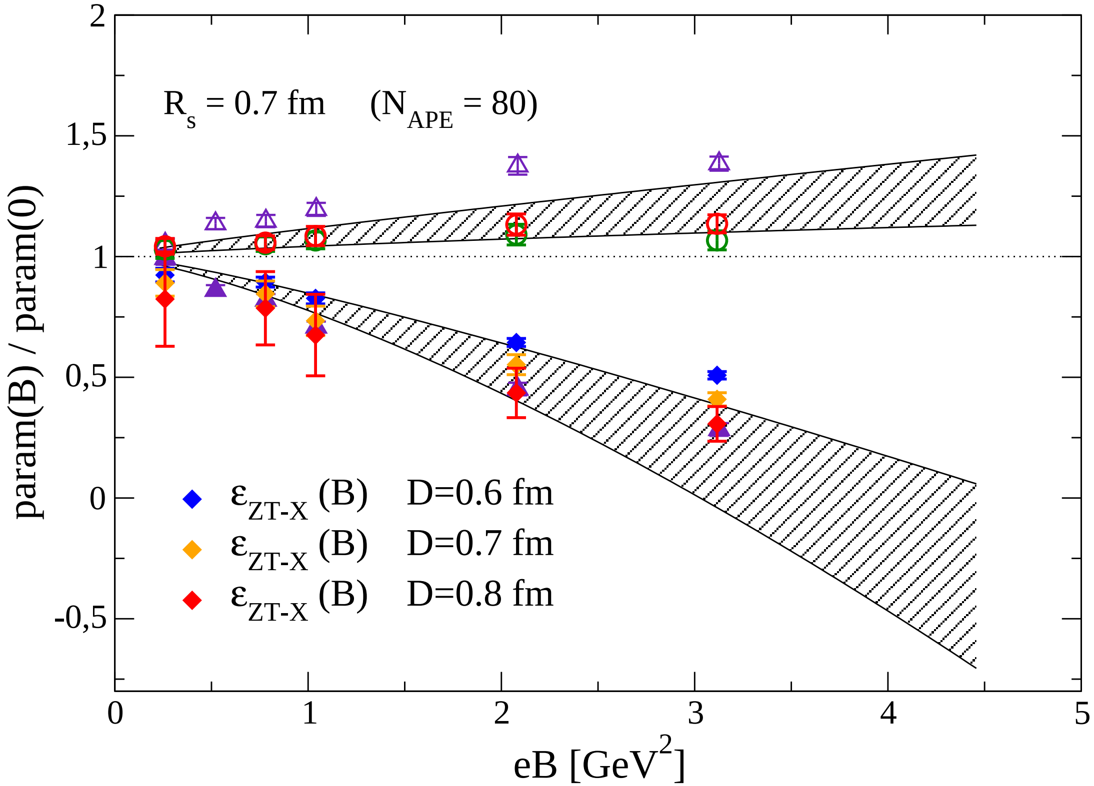
What if we consider the field strength "energy flux" across the tube? **(energy per unit length)**

$$\varepsilon = \int d^2x_t \, \frac{E_l^2(x_t)}{2}$$



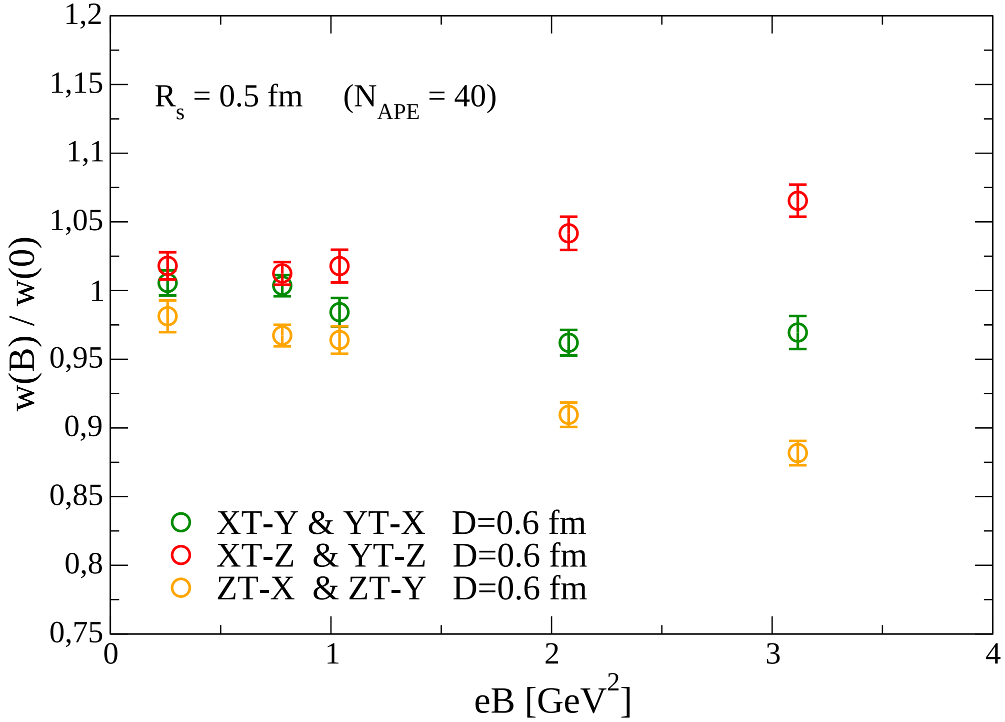
As we consider flux ratios (to $B = 0$), we can reproduce "semi-quantitatively" the string tension suppression/enhancement.

"semi-quantitatively" is perfectly satisfactory, since ε contains also other contributions apart from pure string (Coulomb term)



Indeed, the "semi-quantitative" agreement improves as the $Q\bar{Q}$ separation increases:

we are measuring the energy per unit length in the middle of the tube, and the pure string term dominates as the separation increases



A direct measurement of the flux tube width

$$w \equiv \sqrt{\langle x_t^2 \rangle} \quad ; \qquad \langle x_t^2 \rangle \equiv \frac{\int d^2x_t \, E_l(x_t) x_t^2}{\int d^2x_t \, E_l(x_t)}$$

confirms the shrinking/widening of the flux tube, which:

- clearly shrinks for $Q\bar{Q}$ separation parallel to B

- for orthogonal separation has a tendency to shrink/widen in directions orthogonal/parallel to B

CONCLUSIONS (part I)

- **Properties of strong interactions in magnetic backgrounds are highly non-trivial**

- **Confining properties, in particular, are showing a very rich phenomenology which is likely still largely uncovered**

- **Are there new exotic phases in the presence of extremely large fields? Maybe not interesting for phenomenology (too large magnetic fields) but theoretically appealing**