Finite-volume effects on the QCD phase structure

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in collaboration with **Simon Resch**

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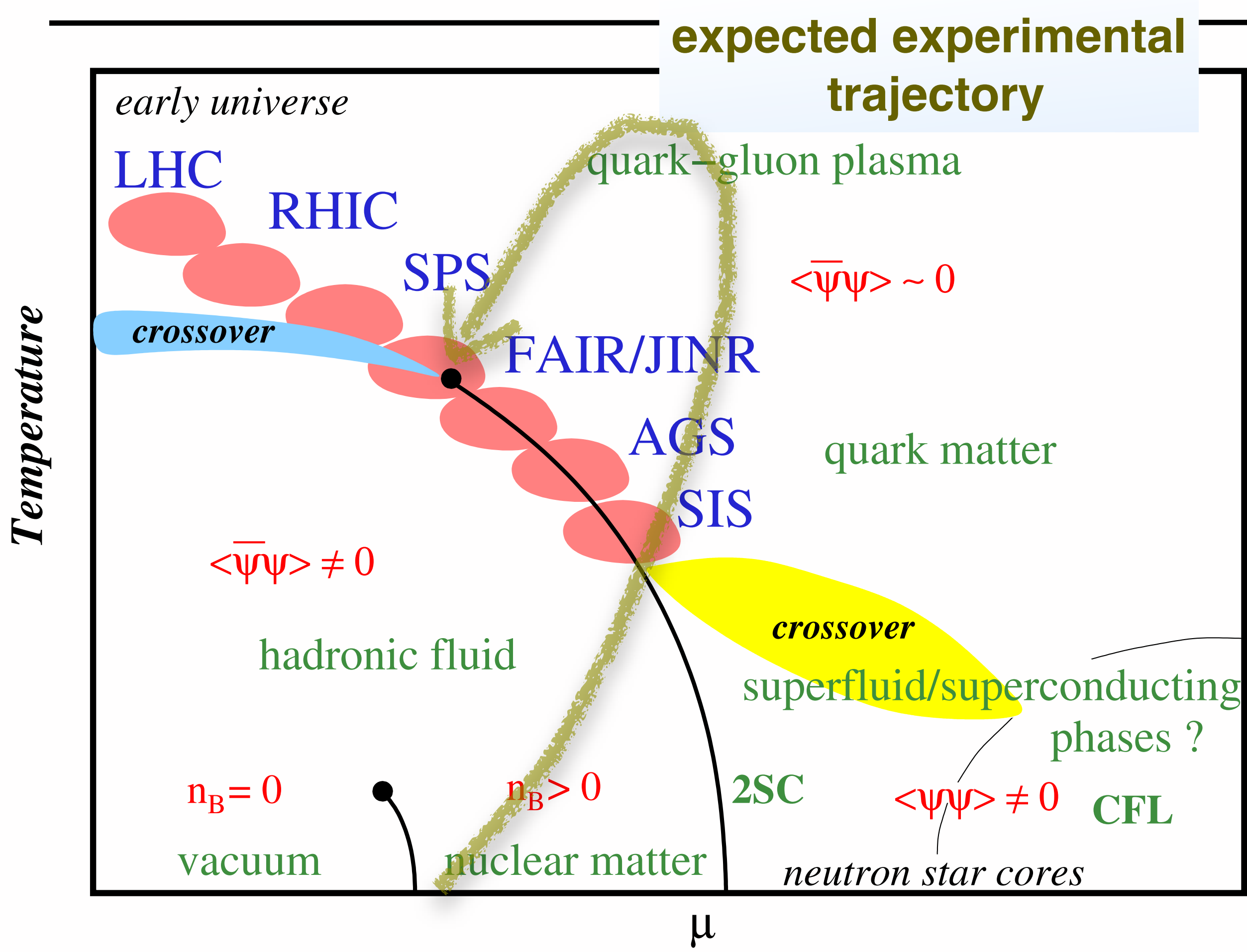
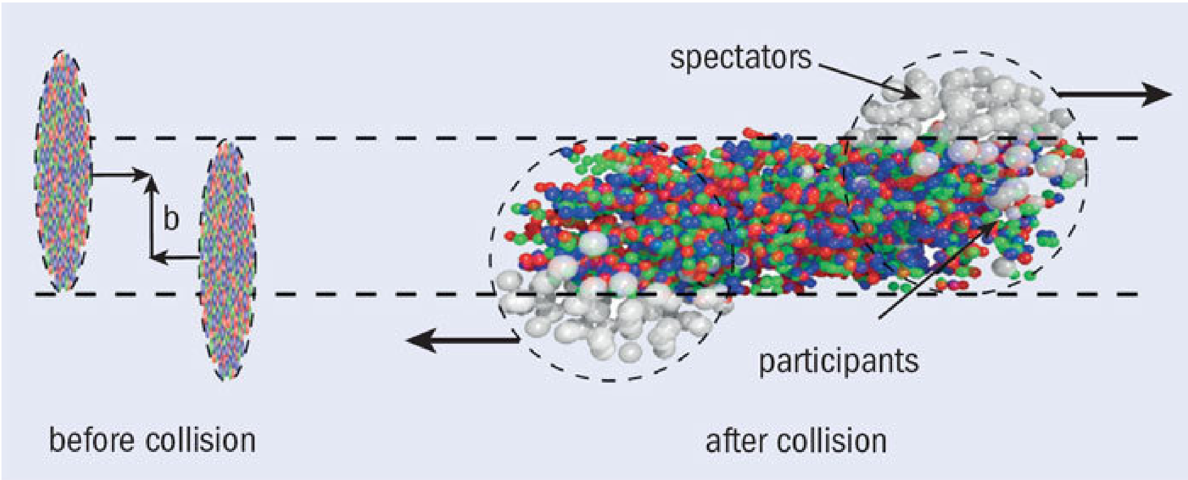
Mini-Workshop on "Lattice and Functional Techniques for Exploration of Phase Structure and Transport Properties in Quantum Chromodynamics", Dubna, July 10 - 14, 2017

Conjectured QC₃D phase diagram



Experiment:



- **CEP:** existence/location/number
- relation between chiral & deconfinement?

  chiral ⇔ deconfinement CEP? [Braun, Janot, Herbst 12/14]
- **Quarkyonic phase:** coincidence of both transitions at μ=0 & μ>0?
- **inhomogeneous phases?** ➜ more favored? [Carignano, BJS, Buballa 14]
- **axial anomaly restoration** around chiral transition? [Mitter, BJS 14]
- **finite volume effects?** ➜ lattice comparison/ influence boundary conditions
- **role of fluctuations?** so far mostly Mean-Field results

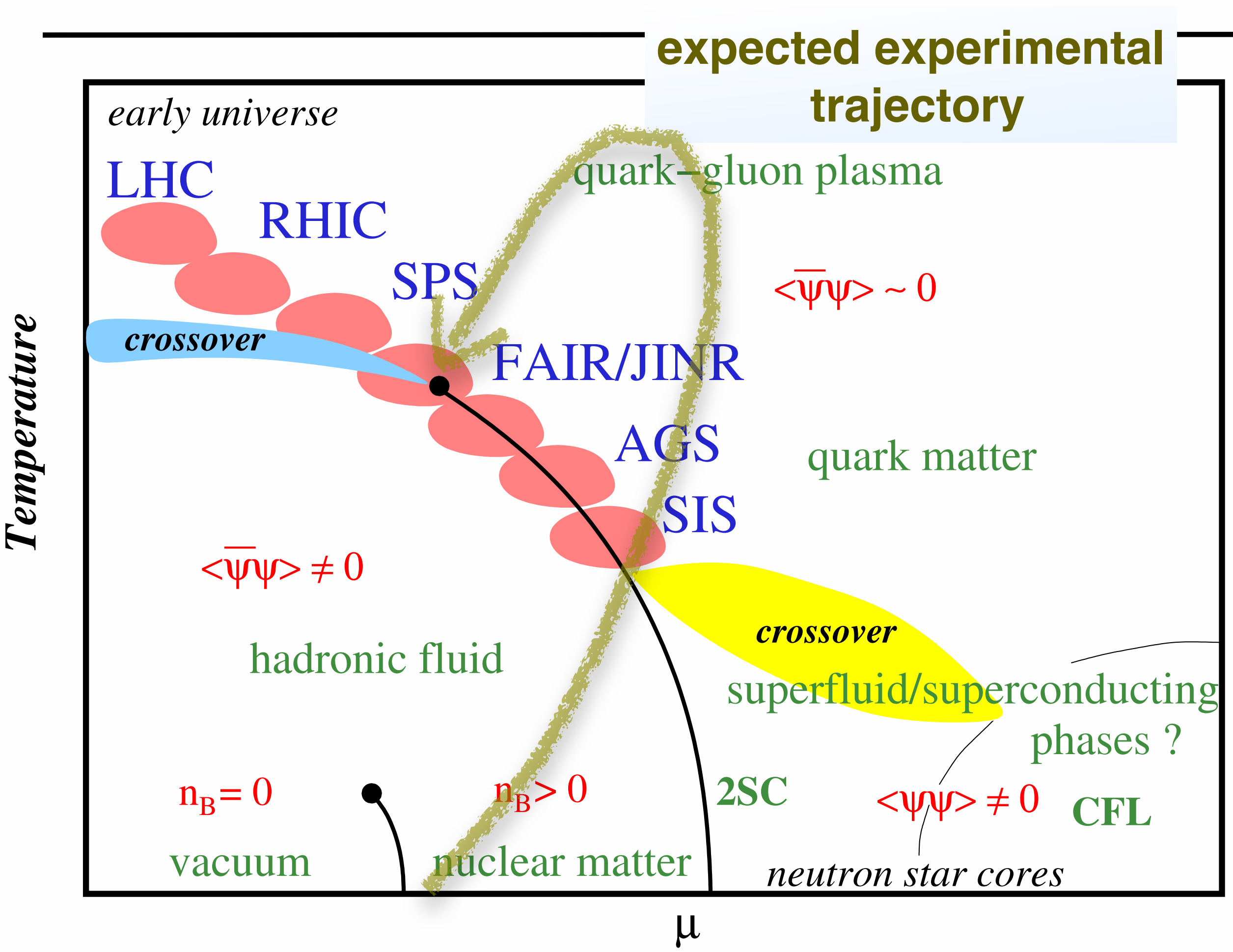
  ➜ **effects of fluctuations important** [Rennecke, BJS 16]

  examples: size of crit reg. around CEP
- **What are good experimental signatures?**

  ➜ higher moments more sensitive to criticality

  deviation from HRG model

Conjectured QC₃D phase diagram



Theory:

➜ Lattice: but simulations restricted to small μ

➜ Functional QFT methods: FRG,DSE, nPI

➜ Models: effective theories parameter dependency

Experiment: (non-equilibrium? ➜ most likely thermal equilibrium)

➜ in a finite box (HBT radii: freeze-out vol. ~ 2000-3000 fm³)

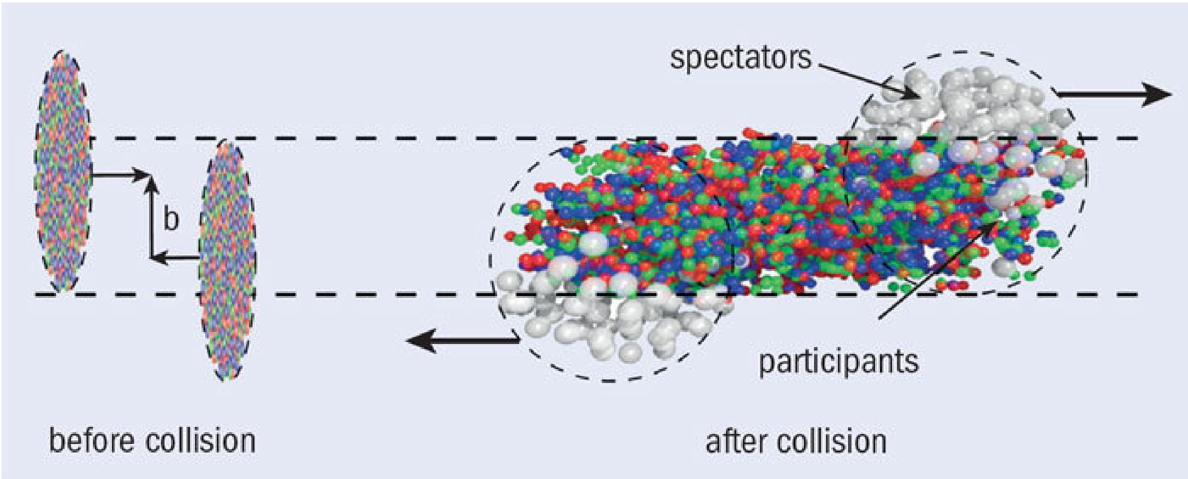
(UrQMD ($\sqrt{s}$): system vol. ~ 50 - 250 fm³)

Theoretical aim:

deeper understanding & more realistic HIC description

➜ existence of critical end point(s)?

Experiment:



Agenda

complementary to NICA

- **Motivation: physics in a finite volume**

many open theoretical issues

➜ long term project

- **Generalized susceptibilities**

  ➜ towards chiral phase transition

- **Role of Fluctuations: from mean-field approximations to RG**

- **Comparison: Finite/infinite volume effects**

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complementary to NICA

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FAIR
Facility for Antiproton and Ion Research in Europe GmbH

many open theoretical issues

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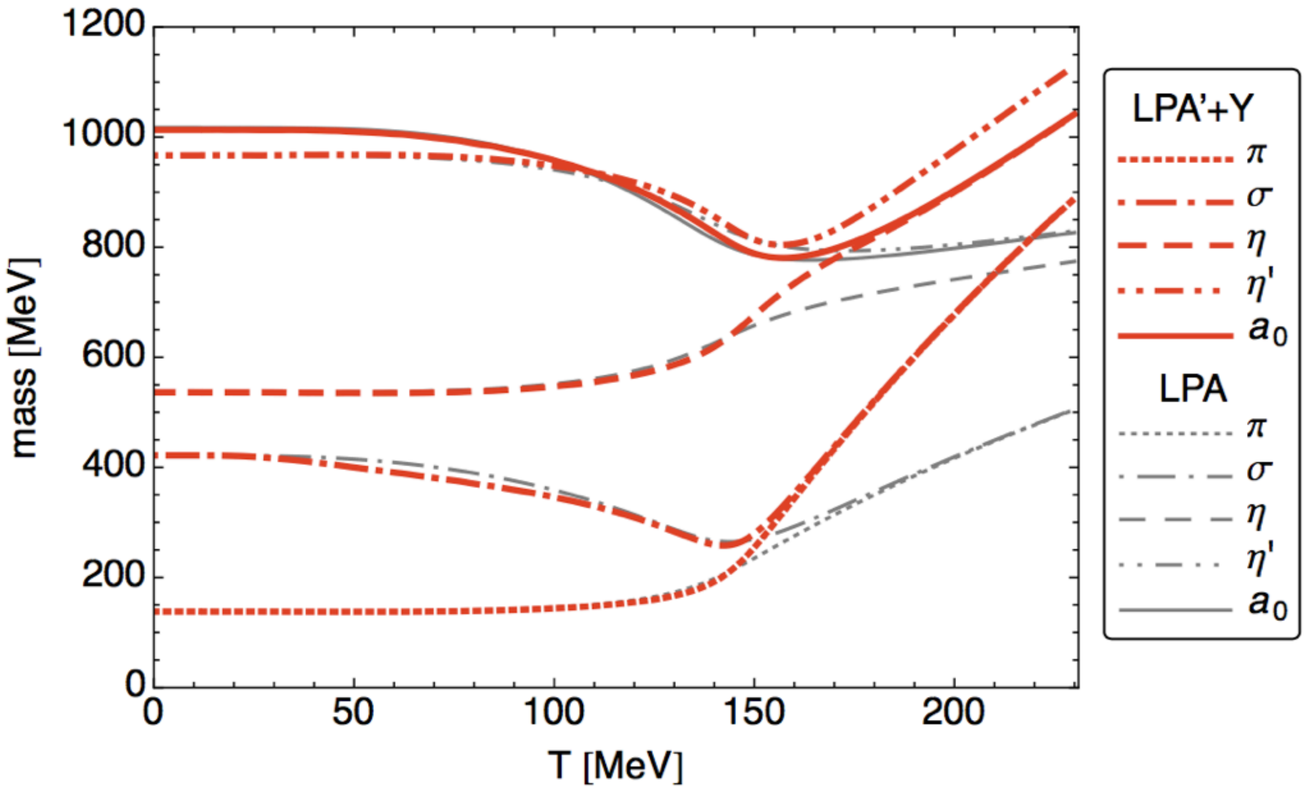
  Finite/infinite volume effects

Example:

Fluctuations are important

FRG Nf=2+1 beyond “usual” LPA truncation

[F. Rennecke, BJS 1610.08748 PRD 2017]



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Example:

mean-field/LPA	LPA′ + Y
(η, f_0)	(η, a_0)
(η', a_0)	(η', f_0)

Fluctuations are important

FRG Nf=2+1 beyond “usual” LPA truncation

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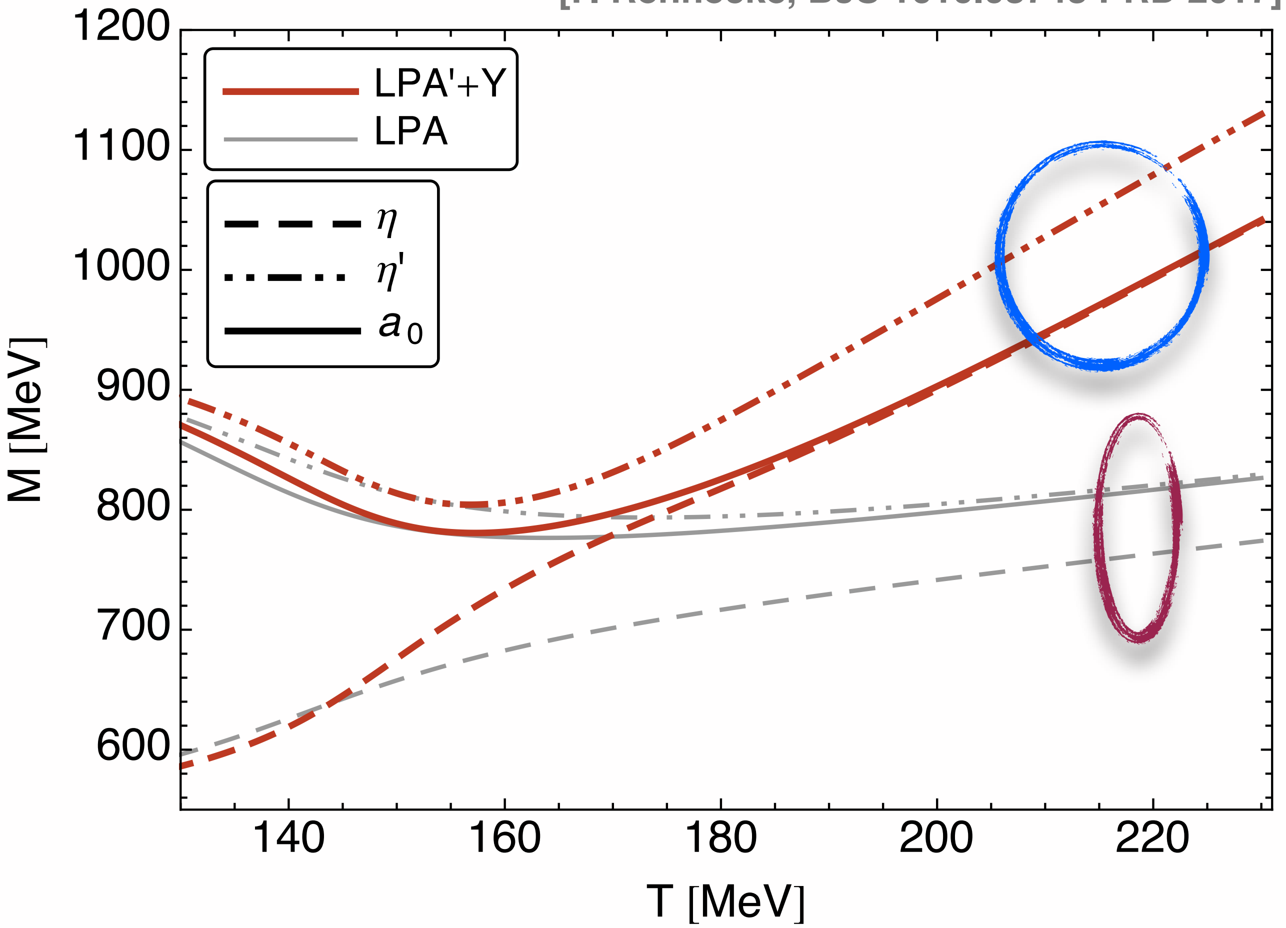


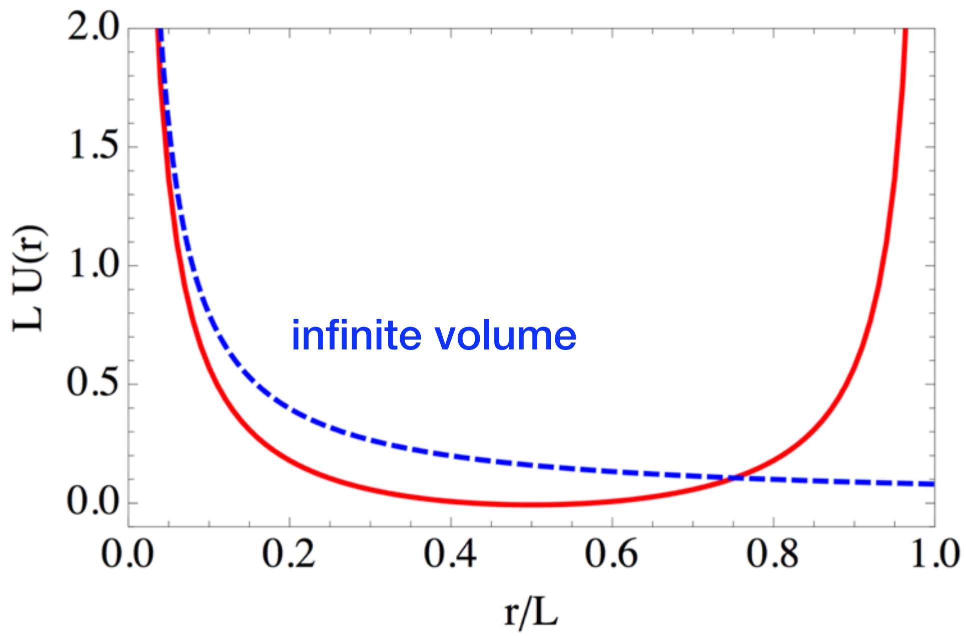
Motivation:Physics in a finite Volume

Lattice simulations:

QCD (short-ranged) with QED (long-ranged ➜ truncated) corrections

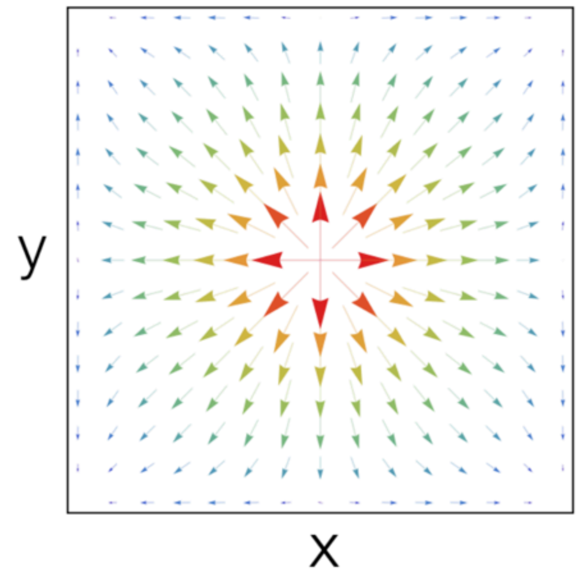
➜ violation Gauss's & Ampere's law

if EM gauge field subject to periodic boundary condition



finite volume Coulomb potential between two charges

[Davoudi, Savage 2014]



point charge at the center

Motivation:Physics in a finite Volume

Quantum Field Theory in a finite volume:

➜ no spontaneous symmetry breaking

if only finite number of degrees of freedom

QCD: [Gasser, Leutwyler 1988]

chiral condensate: non-perturbative phenomenon

example:

chiral symmetry

$$N_f = 2 : SU(2) \times SU(2) \cong O(4)$$

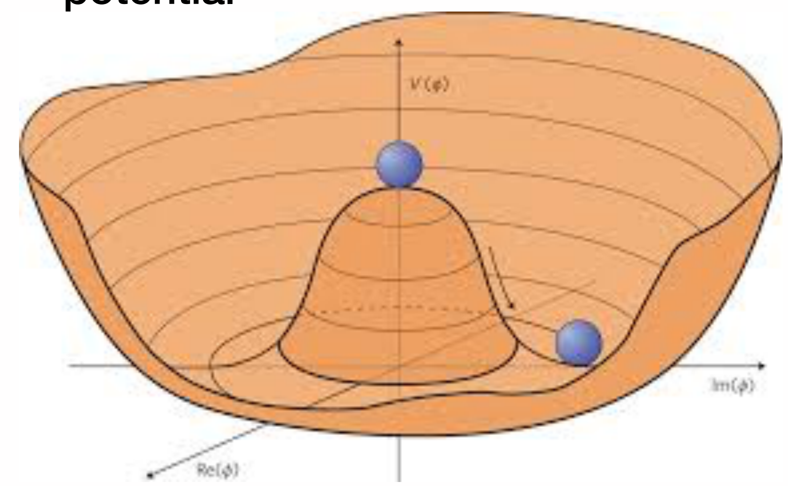
$$O(4) \to O(3)$$ infinite volume

massless Goldstone Bosons

finite volume:

fluctuations of Goldstone bosons always restore symmetry

potential



minimum: zero-momentum mode of the field

$$Z_2 : \varphi \to -\varphi$$

probability of tunneling: $P_{\text{tunnel}} \sim e^{-L}$

exponentially suppressed with volume

$O(N)$ - case: rotation ➜ averaging to zero (no breaking)

infinite volume ➜ no tunneling ➜ symmetry broken

Motivation:Physics in a finite Volume

long-range correlations are necessary to obtain spontaneous SB (for a continuous symmetry)

chiral limit: massless Goldstone boson fluctuations in a finite box avoid symmetry breaking

but

symmetry breaking in mean-field approximations are possible:

Goldstone-fluctuations are absent

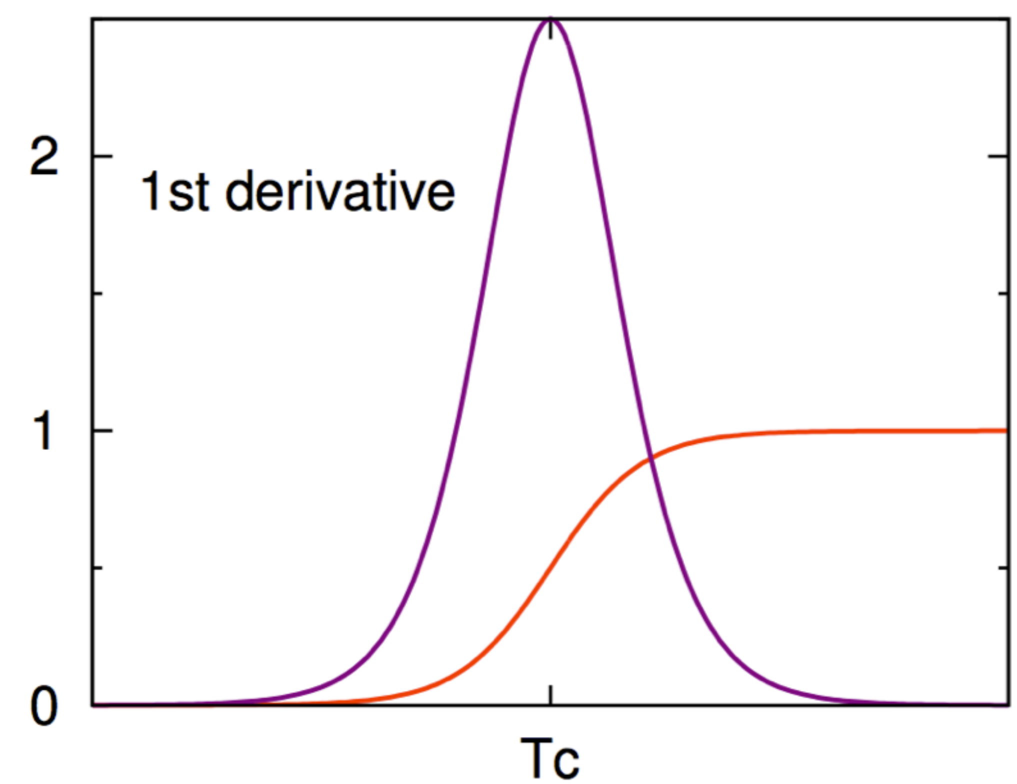
<u>Thermodynamics on a torrus:</u>

correlation length always finite ➜ no real 2nd order phase transition

criterion for phase transition: (generalized) susceptibilities

➜ derivatives of order parameter reveal more details

derivatives of thermodynamic quantities ↔ fluctuations



Fluctuation observables

- generalized susceptibilities:

$$\chi_n = \frac{\partial^n p(T,\mu)/T^4}{\partial(\mu/T)^n}\bigg|_T$$

- Fluctuations of conserved charges

$$\delta Q_X = Q_X - \langle Q_X \rangle \qquad X = Q, B, S, \ldots$$

mean value: $\chi_1 \sim \langle Q \rangle$

$\chi_2 \sim \langle (\delta Q)^2 \rangle$

$\chi_3 \sim \langle (\delta Q)^3 \rangle$

- Measured in event-by-event multiplicity distributions

variance: $\sigma^2 \sim \frac{\chi_2}{\chi_1}$

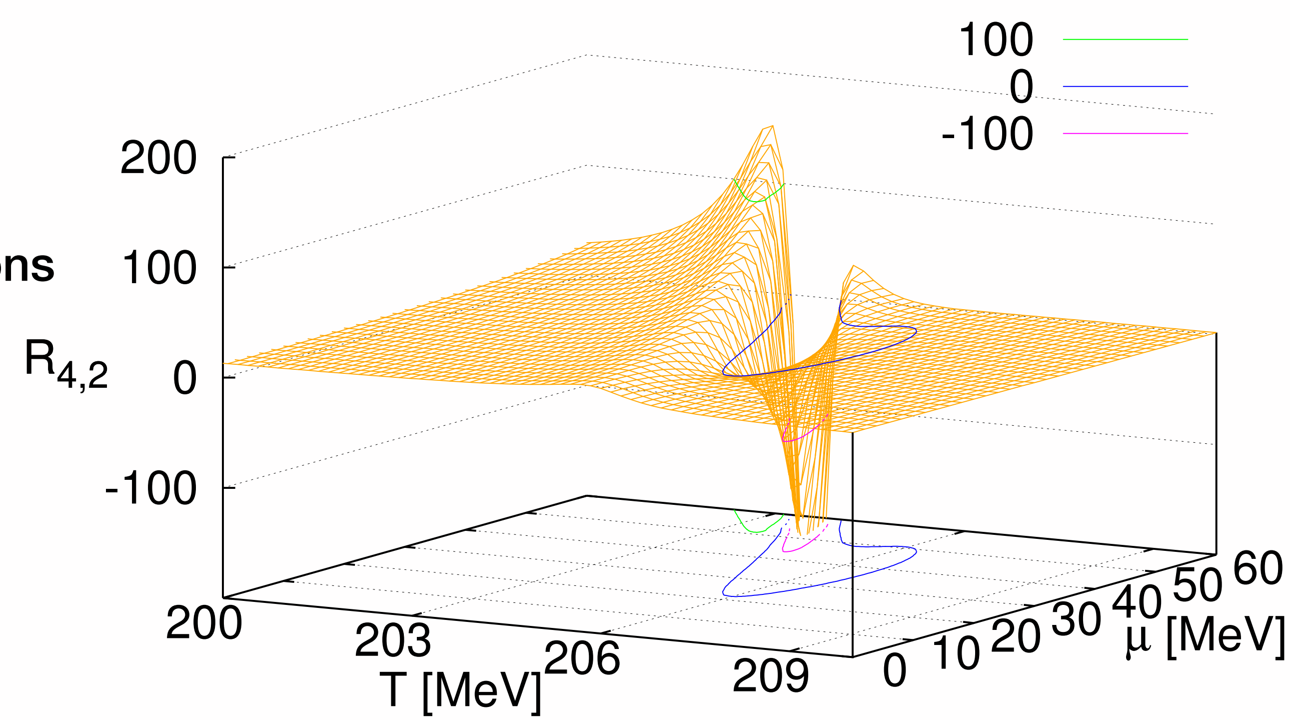
skewness: $S\sigma = \frac{\chi_3}{\chi_2}$

kurtosis: $\kappa\sigma^2 = \frac{\chi_4}{\chi_2}$

[STAR Coll. 2014; PHENIX Coll 1506.07834]

strong temperature & density dependence of ratios

[BJS, M. Wagner 2012]

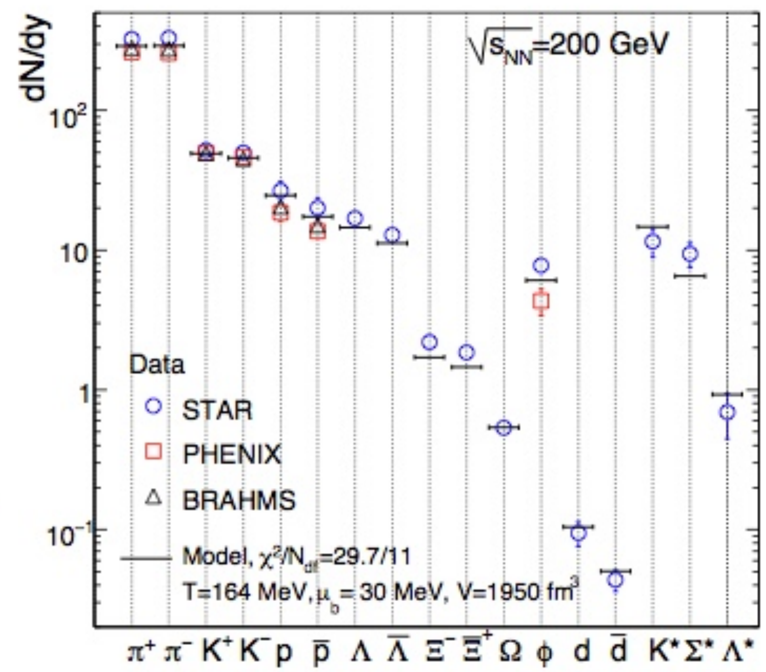


Hadron Resonance Gas Model

HRG model: **good lattice data description**

HRG model versus experiment

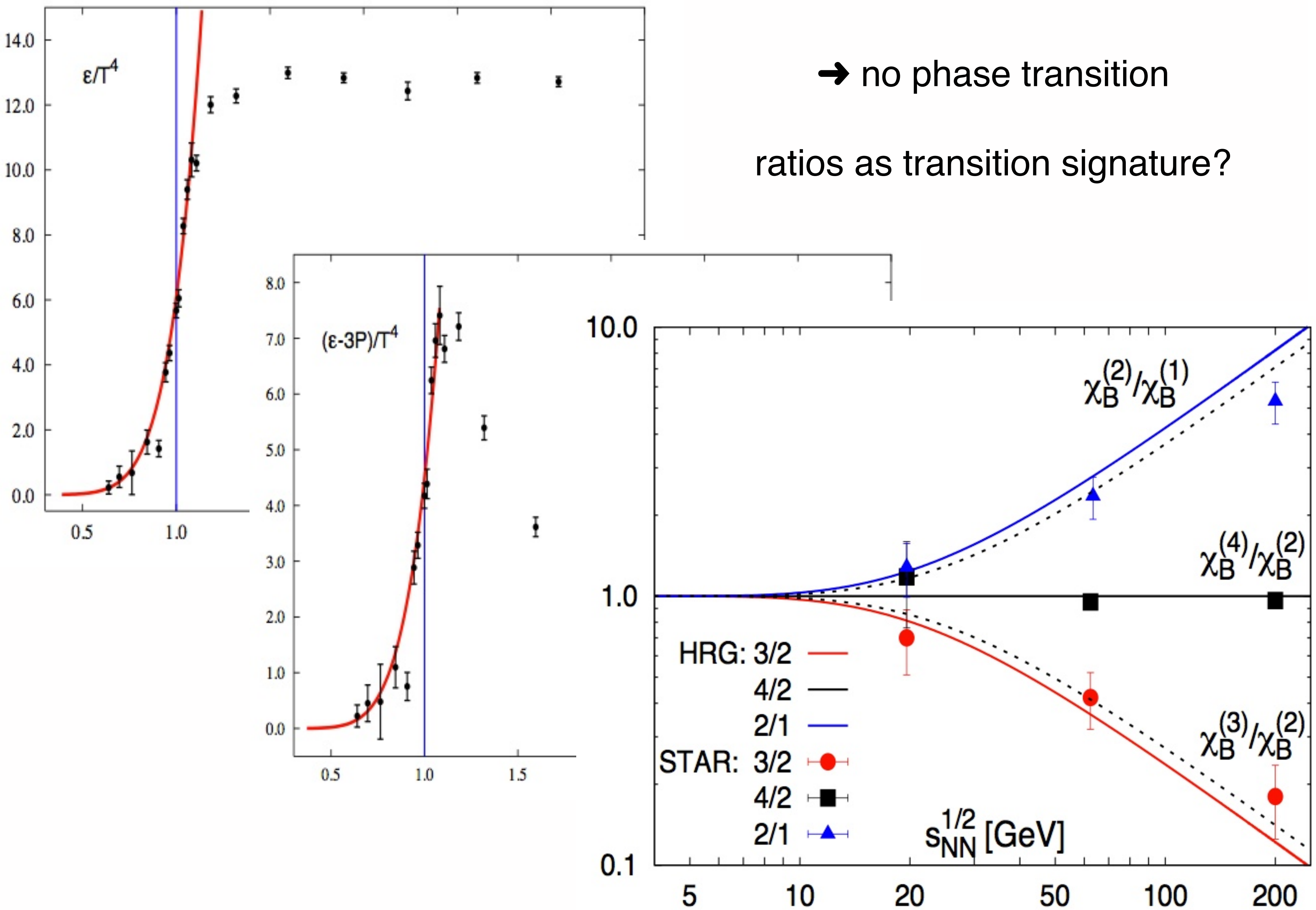
[Andronic et al. 2011]



HRG model: **no critical fluctuations**

➜ no phase transition

ratios as transition signature?



Fluctuations of higher cumulants

exhibit strong variation

from HRG model

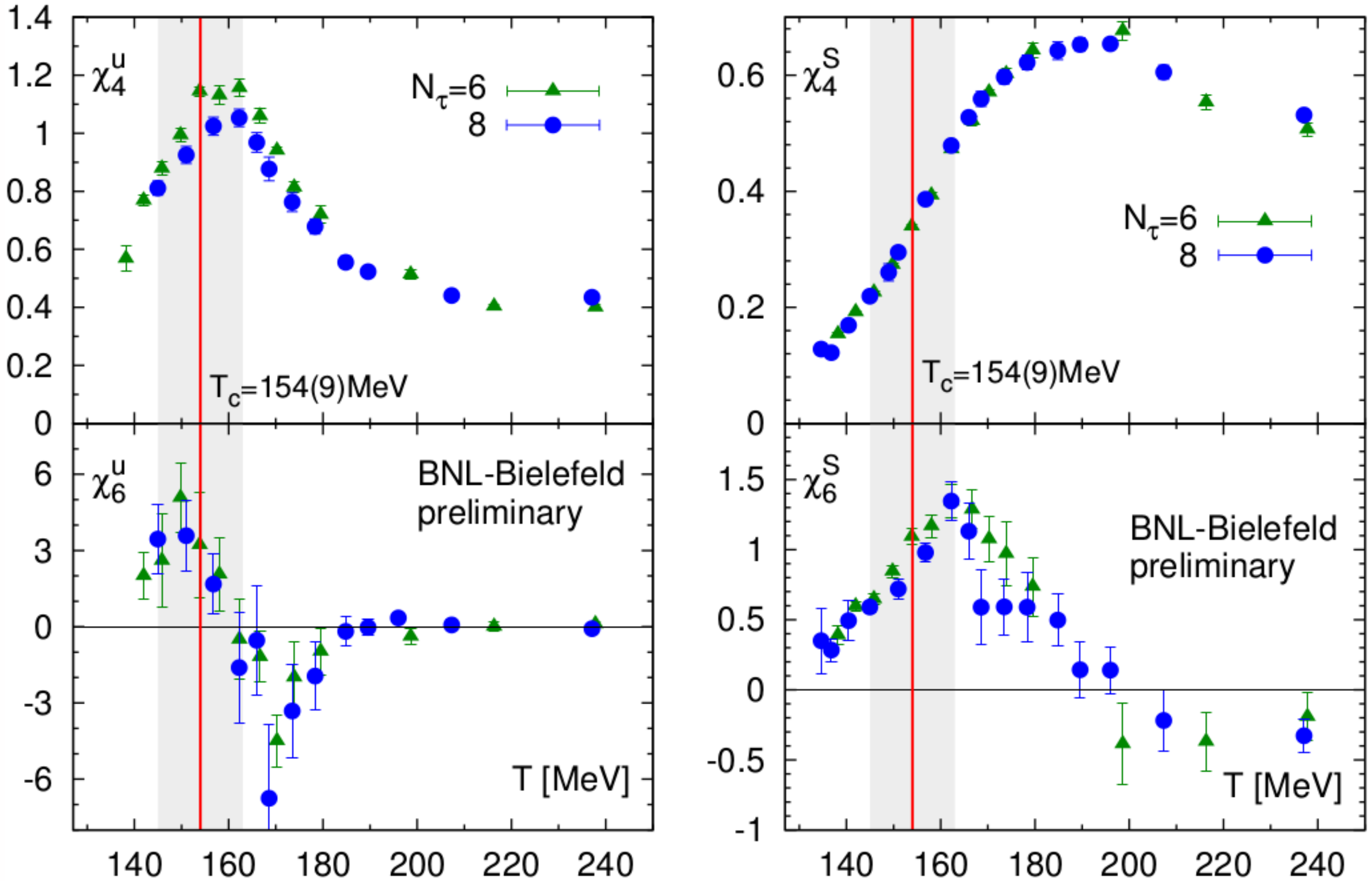
[Karsch, Redlich 2010]

Fluctuation observables

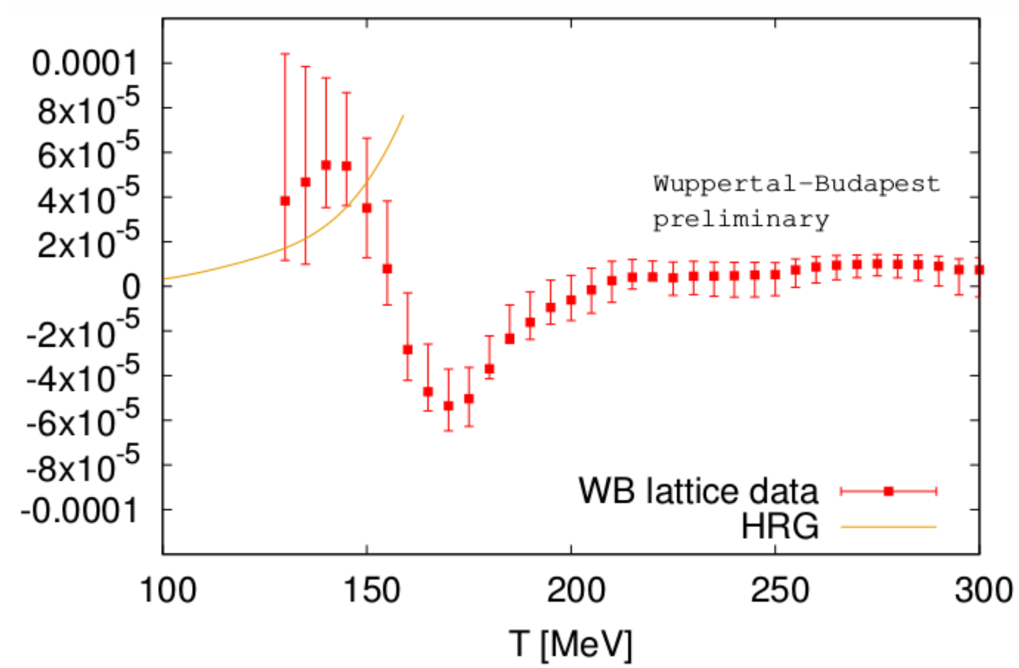
$$\chi_n = \left. \frac{\partial^n p(T,\mu)/T^4}{\partial(\mu/T)^n} \right|_T$$

change of sign ➜ deviations from HRG ➜ criticality

[Schmidt et al. 2015]



[Bellwied et al. 2016]



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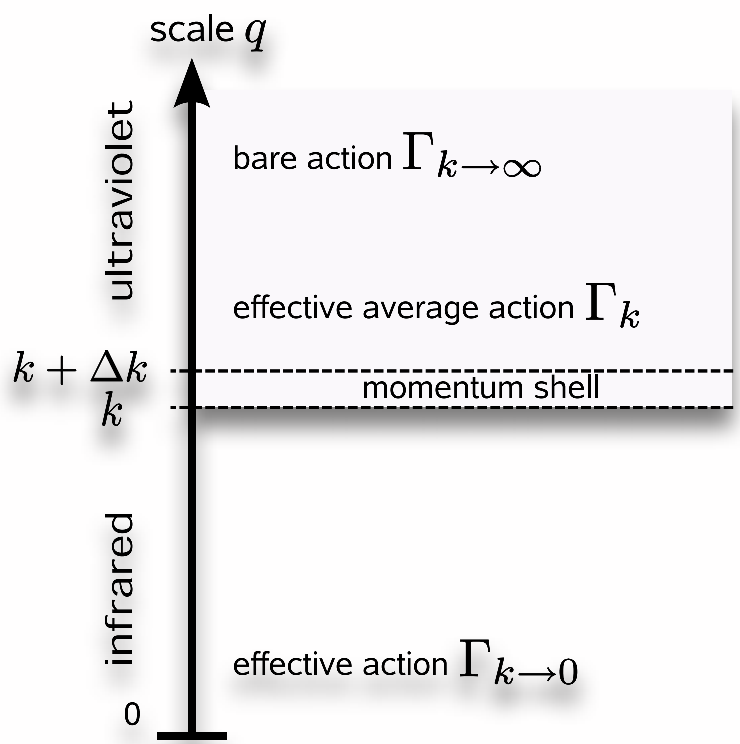
Functional Renormalization Group

- $\Gamma_k[\phi]$ scale dependent effective action $\quad t = \ln(k/\Lambda) \quad R_k$ regulators $\quad \Gamma_k^{(2)} = \frac{\delta^2 \Gamma_k}{\delta\phi\delta\phi}$



FRG (average effective action)

$$\partial_t \Gamma_k[\phi] = \frac{1}{2} \mathrm{Tr}\, \partial_t R_k \left(\frac{1}{\Gamma_k^{(2)} + R_k} \right)$$

$$k\partial_k \Gamma_k[\phi] \sim \frac{1}{2}$$

[Wetterich 1993]

- Ansatz for Γ_k: **Example: Leading order derivative expansion** $\quad$ arbitrary potential

$$\Gamma_k = \int d^4x \bar{q}[i\gamma_\mu\partial^\mu - g(\sigma + i\vec{\tau}\vec{\pi}\gamma_5)]q + \frac{1}{2}(\partial_\mu\sigma)^2 + \frac{1}{2}(\partial_\mu\vec{\pi})^2 + V_k(\phi^2)$$

$$V_{k=\Lambda}(\phi^2) = \frac{\lambda}{4}(\sigma^2 + \vec{\pi}^2 - v^2)^2 - c\sigma$$

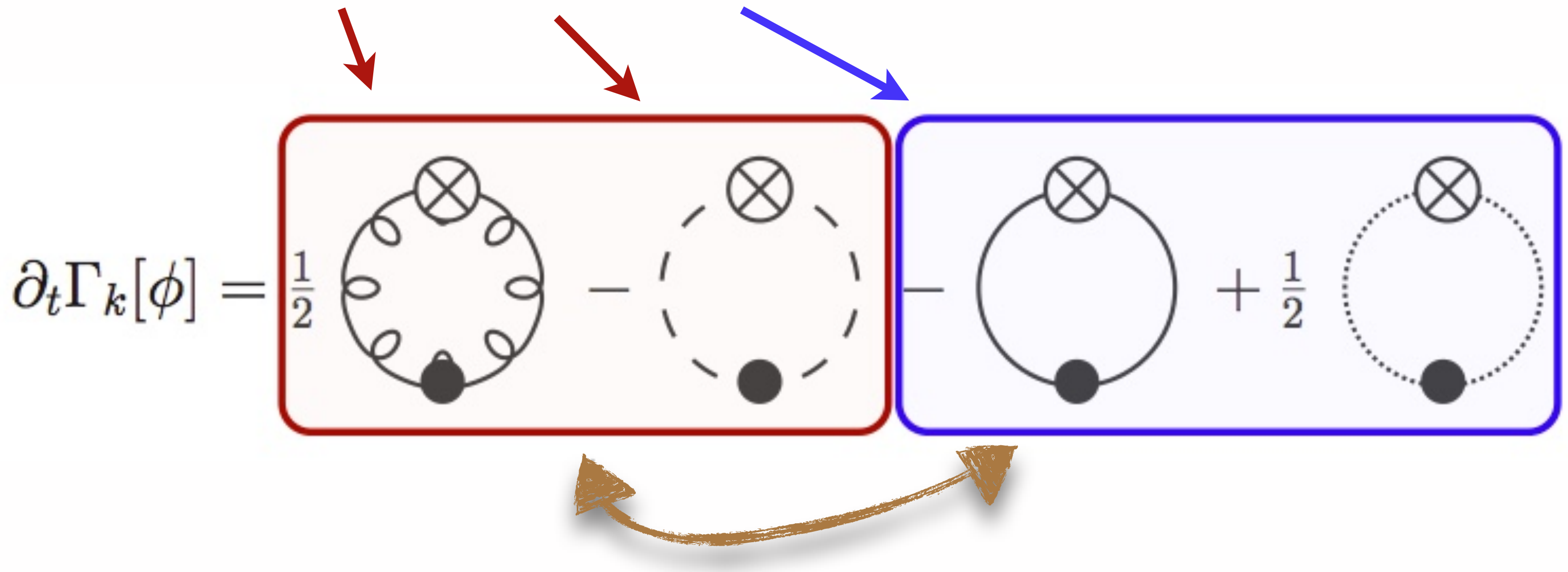
solution techniques:
grid, polynomial, bilocal; pseudo-spectral

FRG and QCD

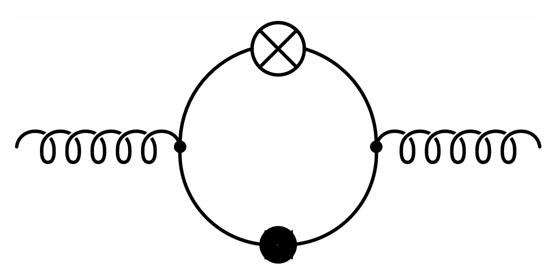
- **full dynamical QCD FRG flow:**

  see talk by J.M. Pawlowski

  fluctuations of **gluon**, **ghost**, **quark** and (via hadronization) **meson**



in presence of **dynamical quarks**:
gluon propagator is modified

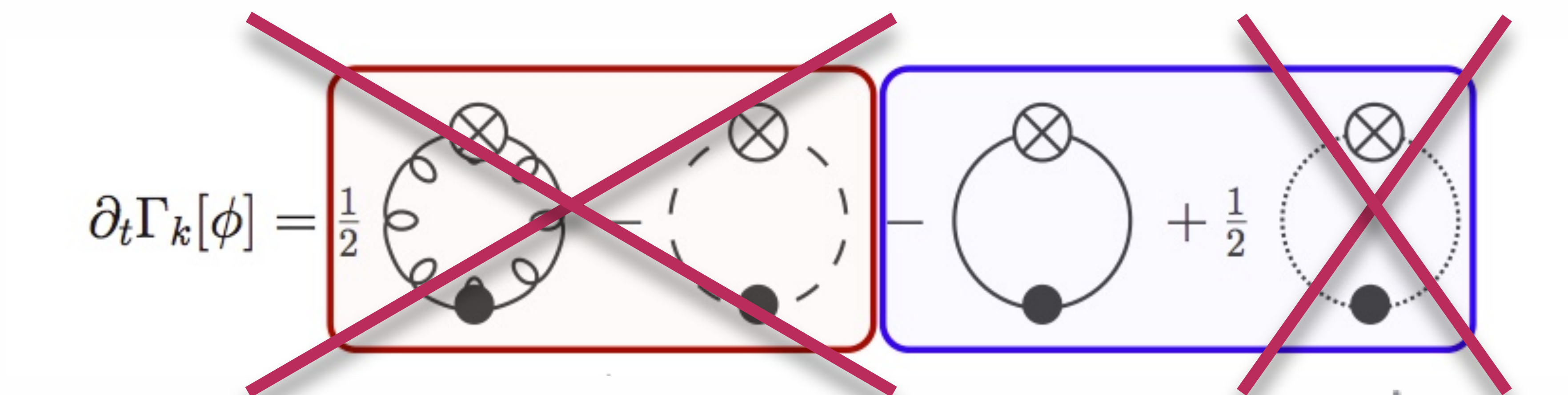


pure Yang Mills flow + matter back-coupling

quark-meson truncation

- **Quark-meson (QM) truncation and mean-field approximation**

flow for **quark-meson** model truncation:
neglect
YM contributions and bosonic fluctuations

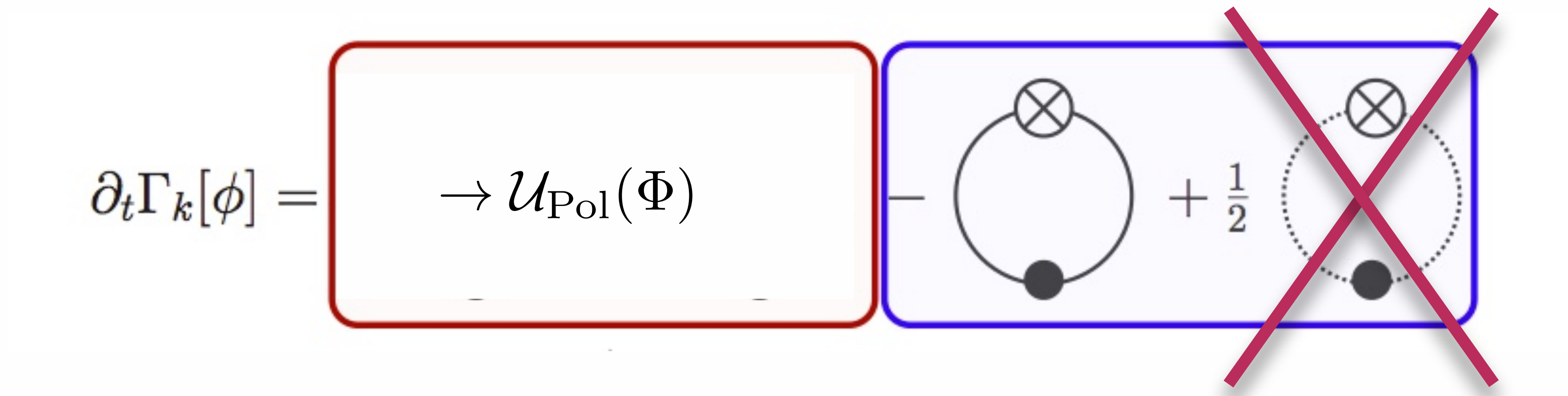


without bosonic fluctuations: MFA

Polyakov-quark-meson truncation

- **Polyakov-loop improved quark-meson truncation (PQM):** [Herbst, Pawlowski, BJS 2007 2013]

  fluctuations of **Polyakov-loop**, **quark** and **meson**

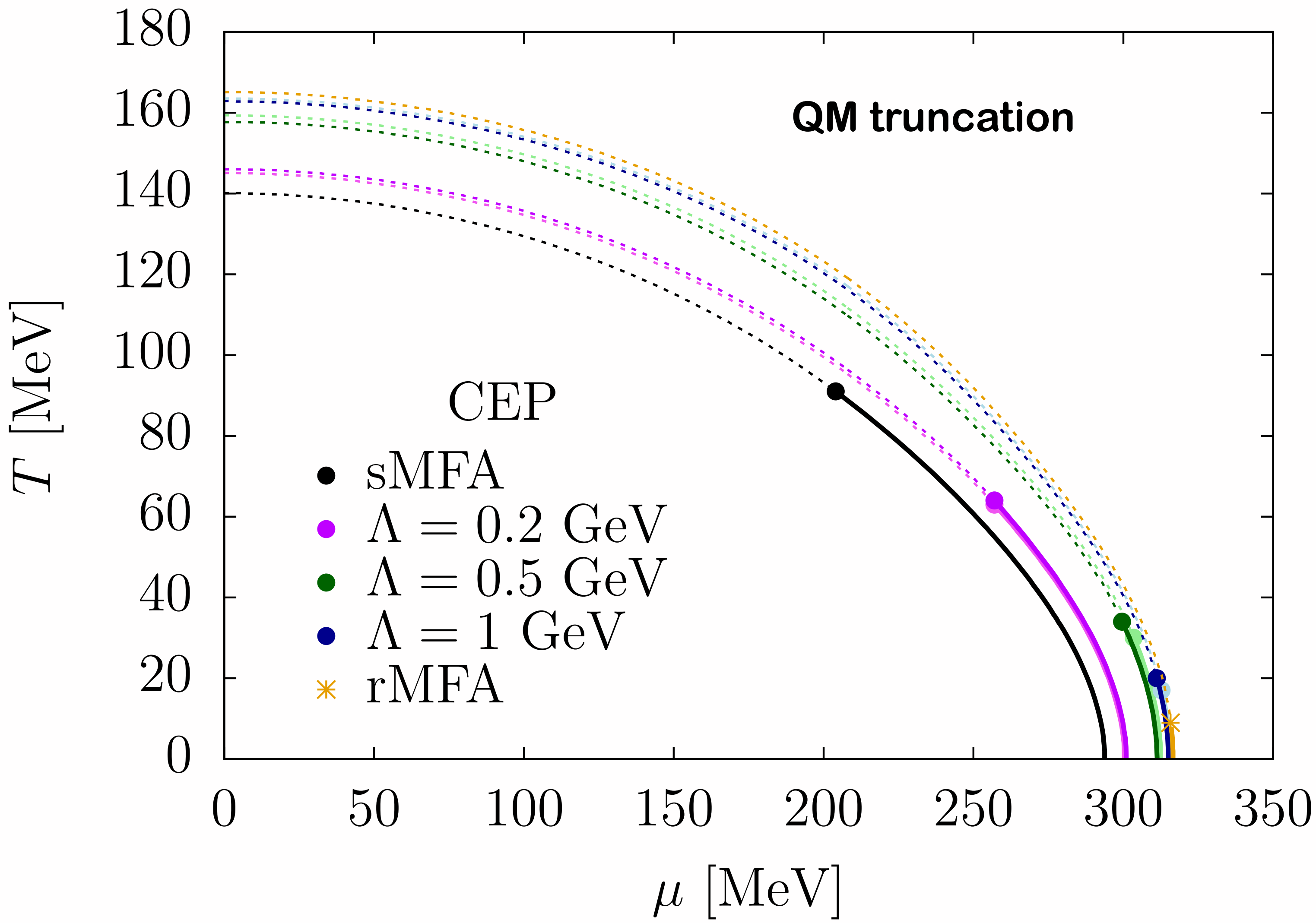


Yang-Mills flow is replaced by effective Polyakov-loop potential $\to \mathcal{U}_{\rm Pol}(\Phi)$ (different implementations for the potential)

fitted to lattice Yang-Mills thermodynamics

Infinite volume

[A Juricic, BJS arXiv:1611.03653]



sMFA: no vacuum fluctuations

⬇

rMFA: renormalized MFA

scheme independence verified

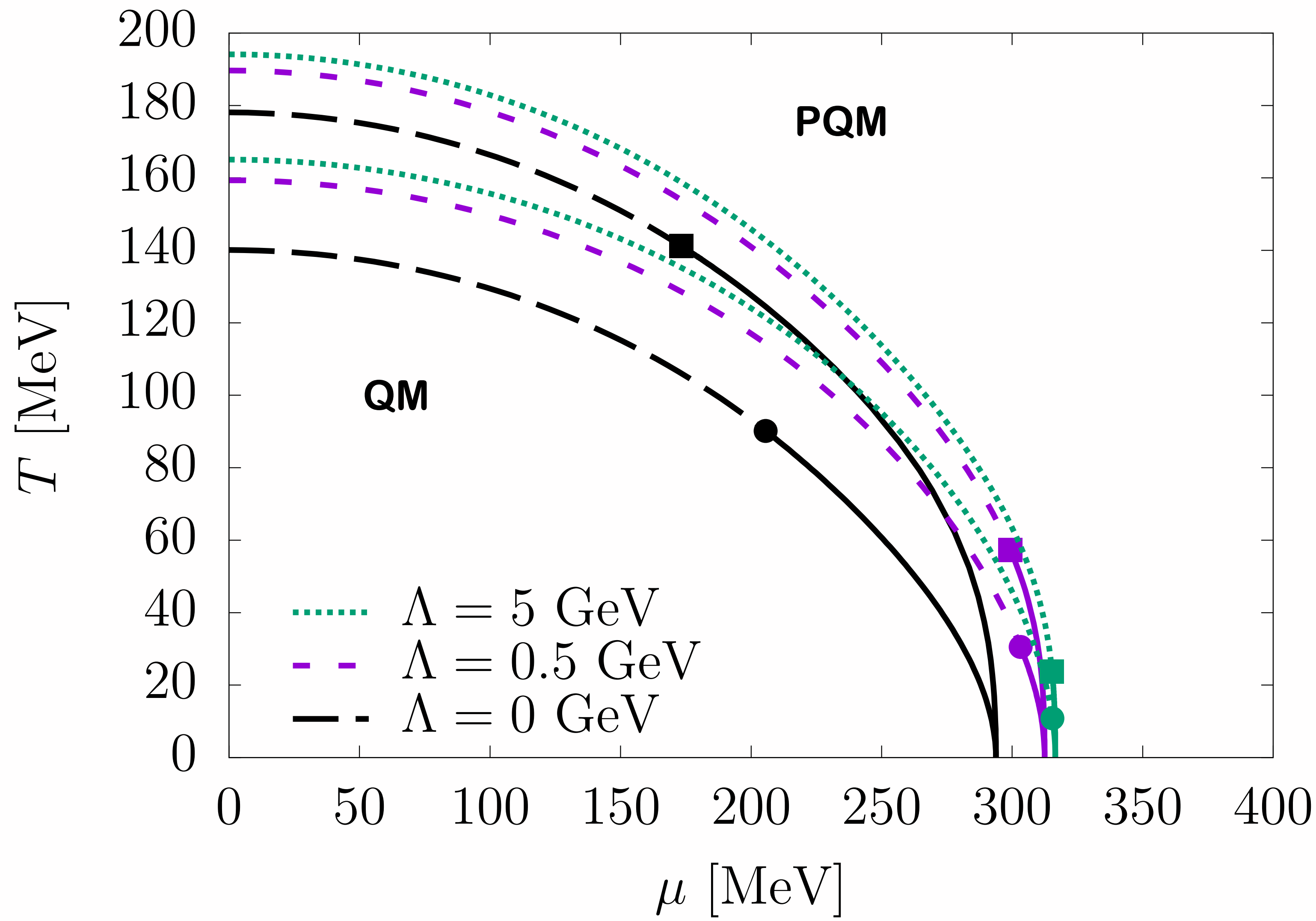
Pauli-Villars regularization

sharp O(3)-momentum cutoff

proper-time regularization

Infinite volume

[S. Resch, BJS to be published]



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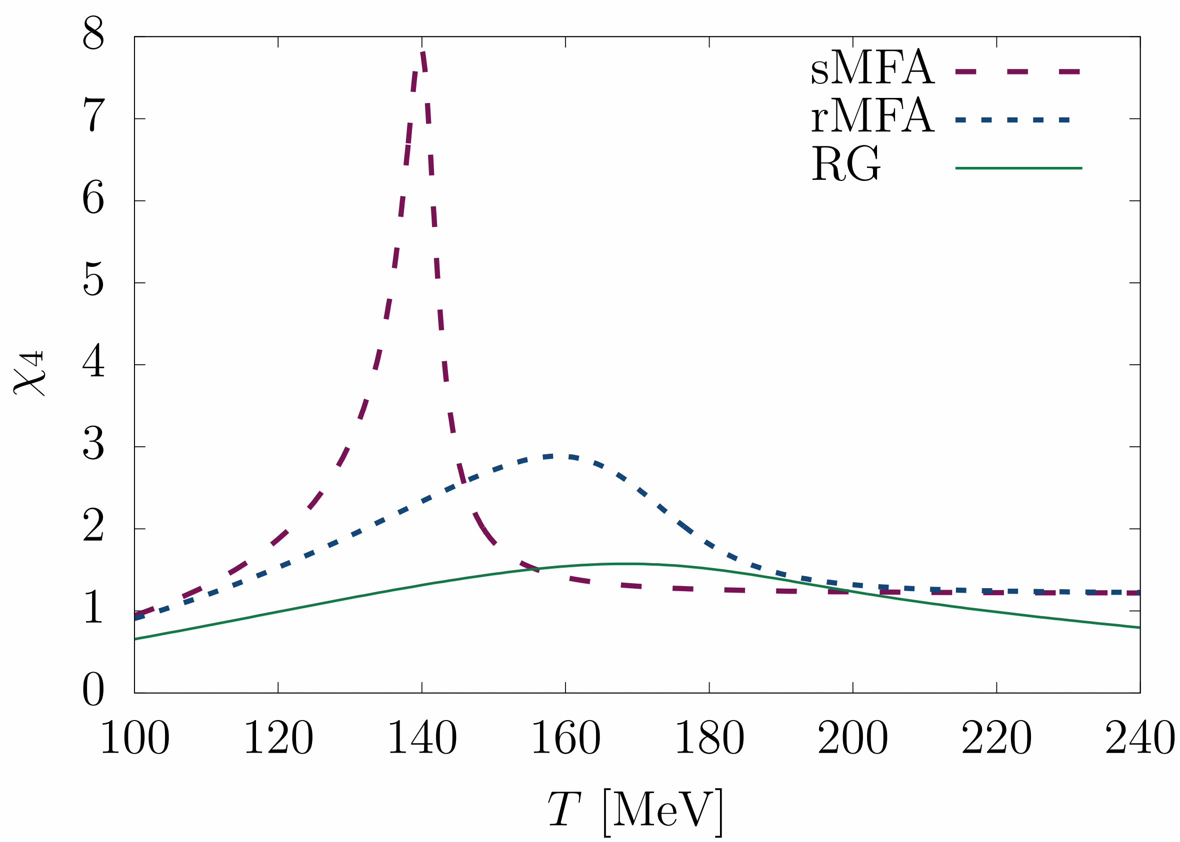
Infinite volume

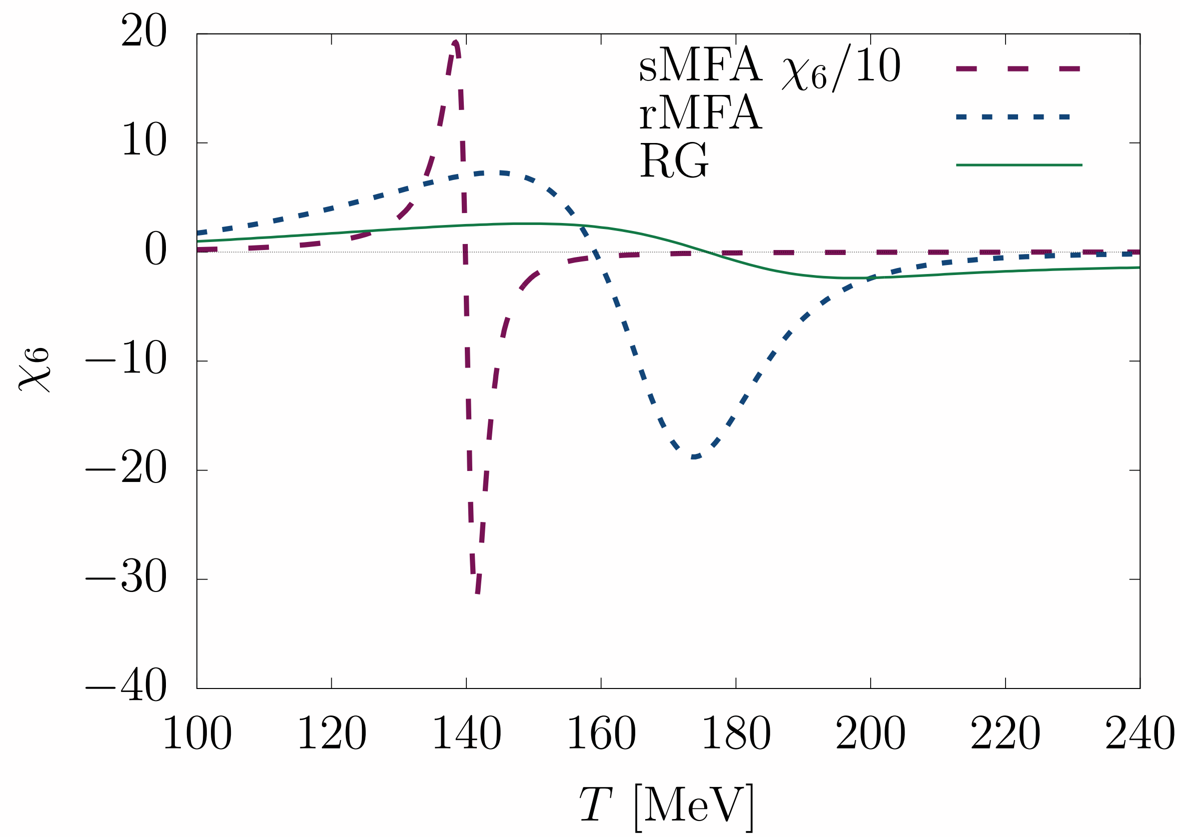
generalized susceptibilities see also [Fu, Pawlowski, Rennecke, BJS 2016]

standard MFA:
no quark vacuum fluctuations

renormalized MFA:
including quark vacuum fluctuations

RG:
quark + meson fluctuations





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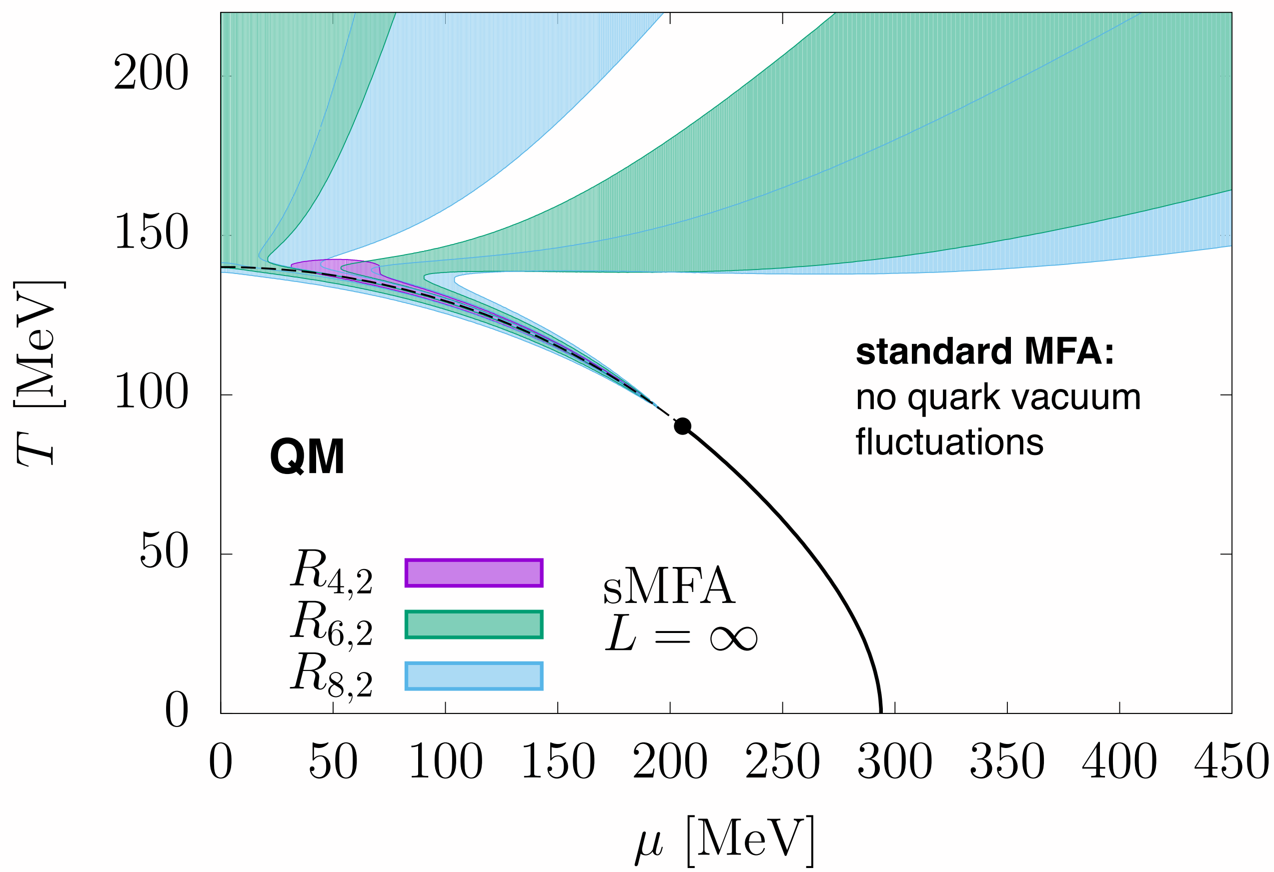
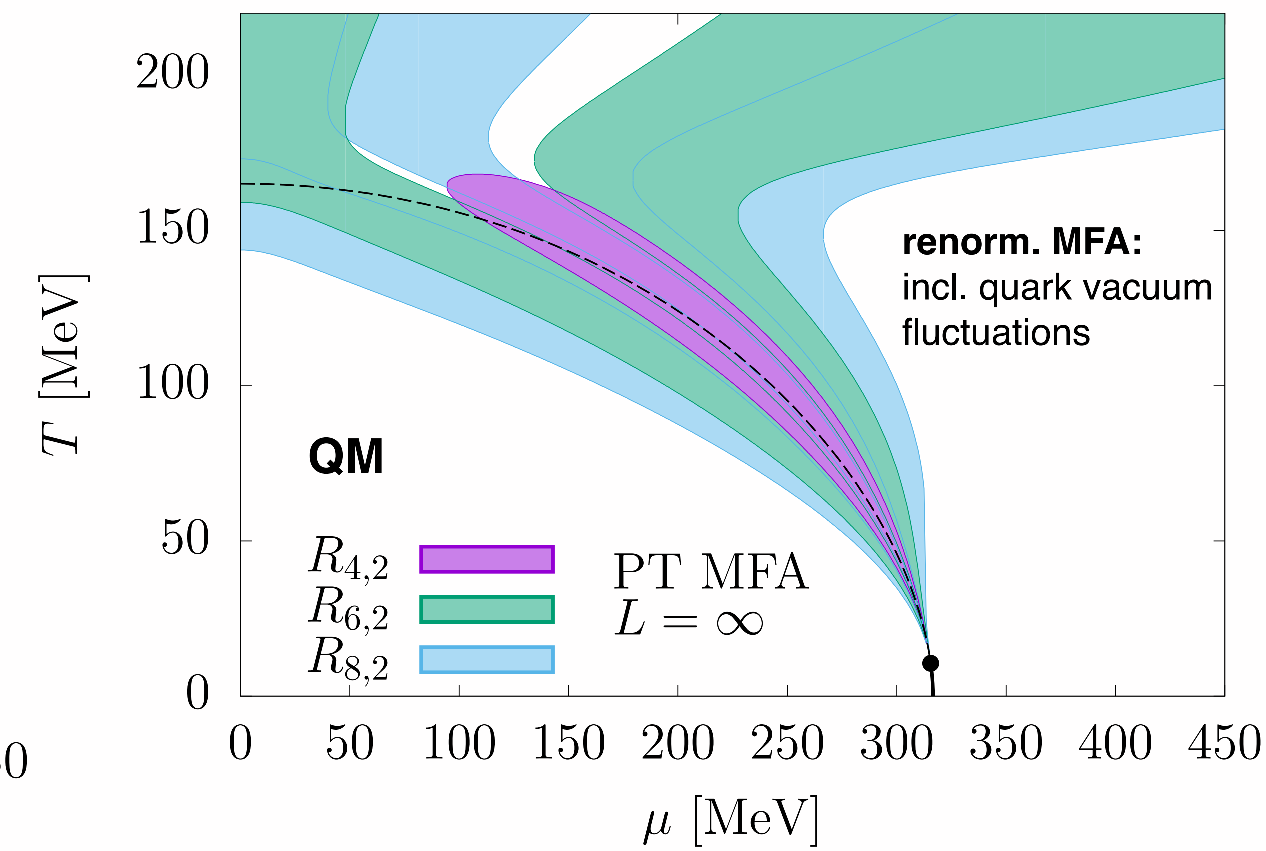
Higher cumulants

[S. Resch, BJS to be published]

infinite volume: influence of fluctuations

(PT = proper time regularization)





findings:

(quark) fluctuations pushes CEP to smaller T and bigger µ

Fluctuations wash out phase transition ➜ broader negative regions

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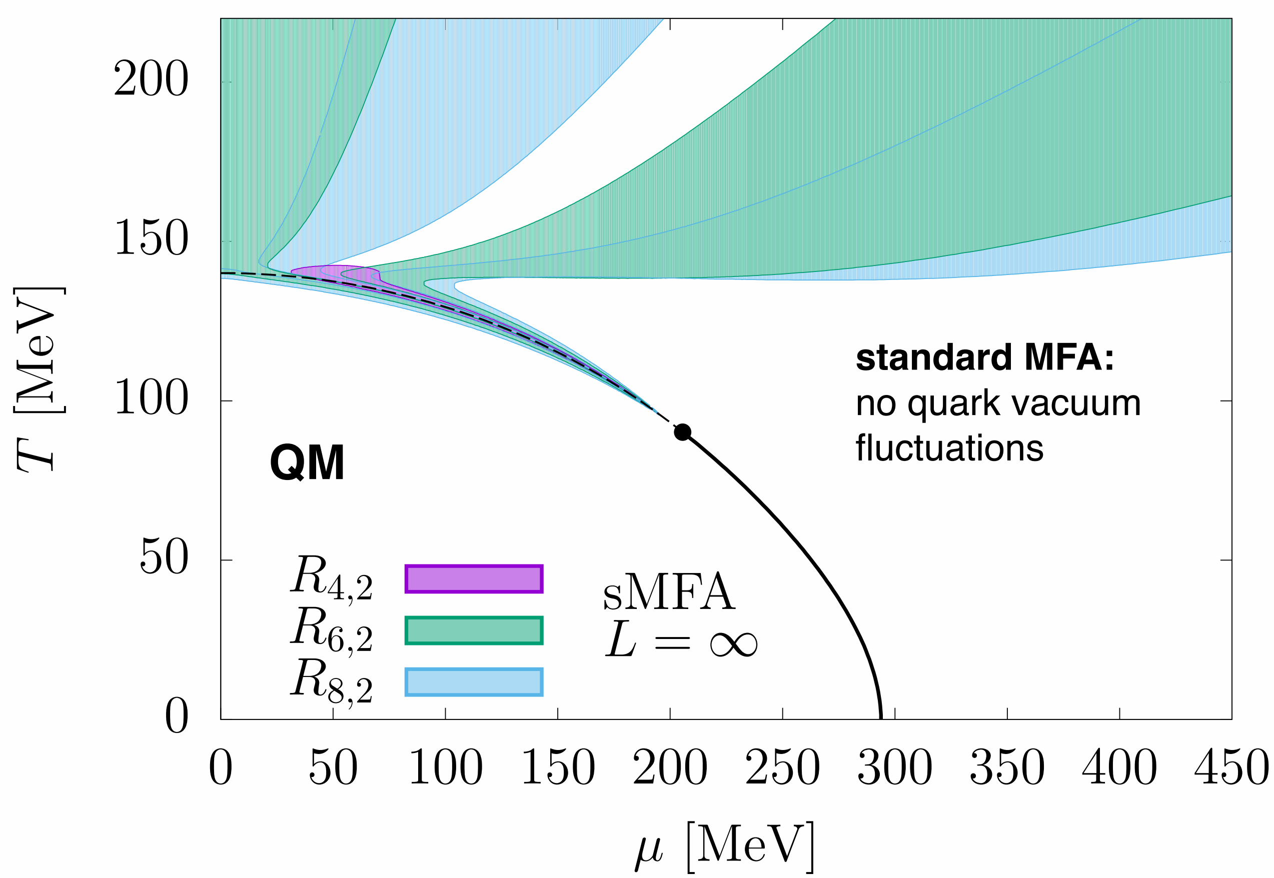

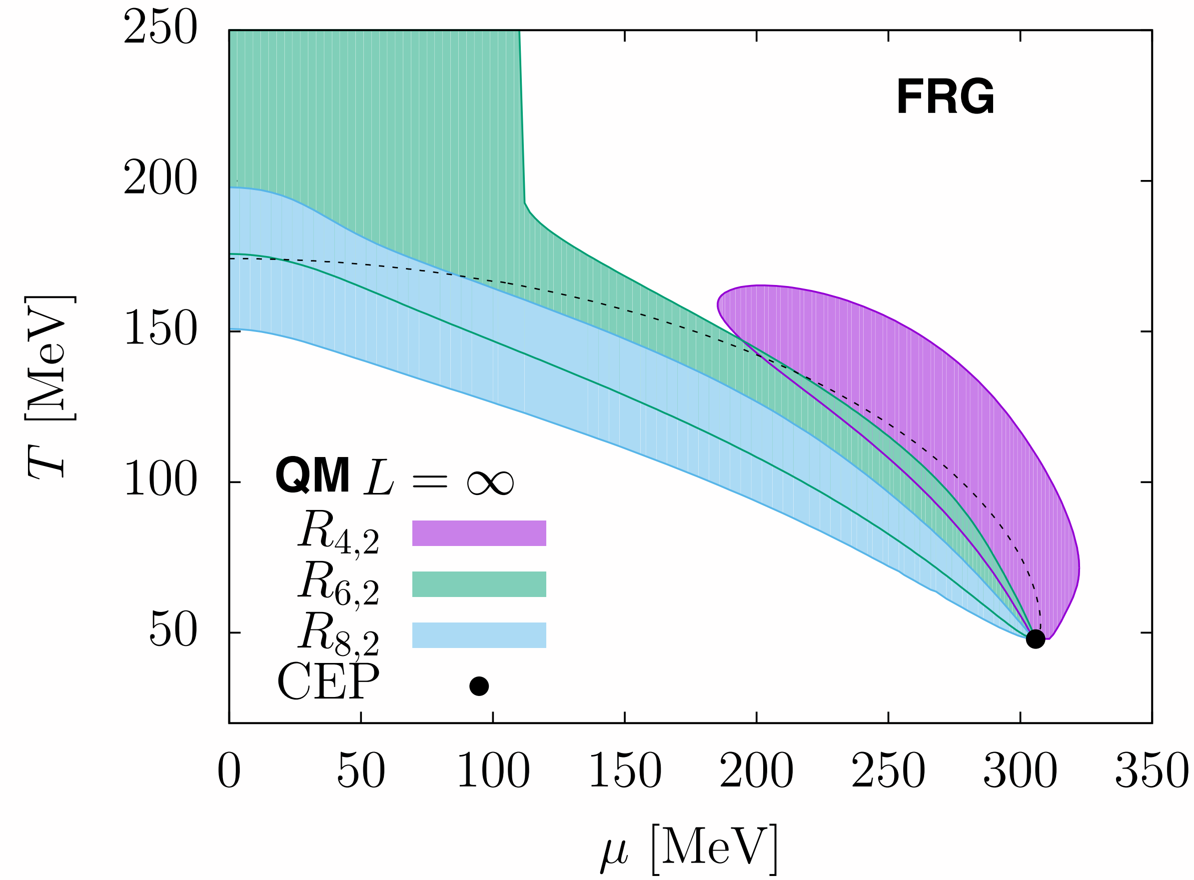

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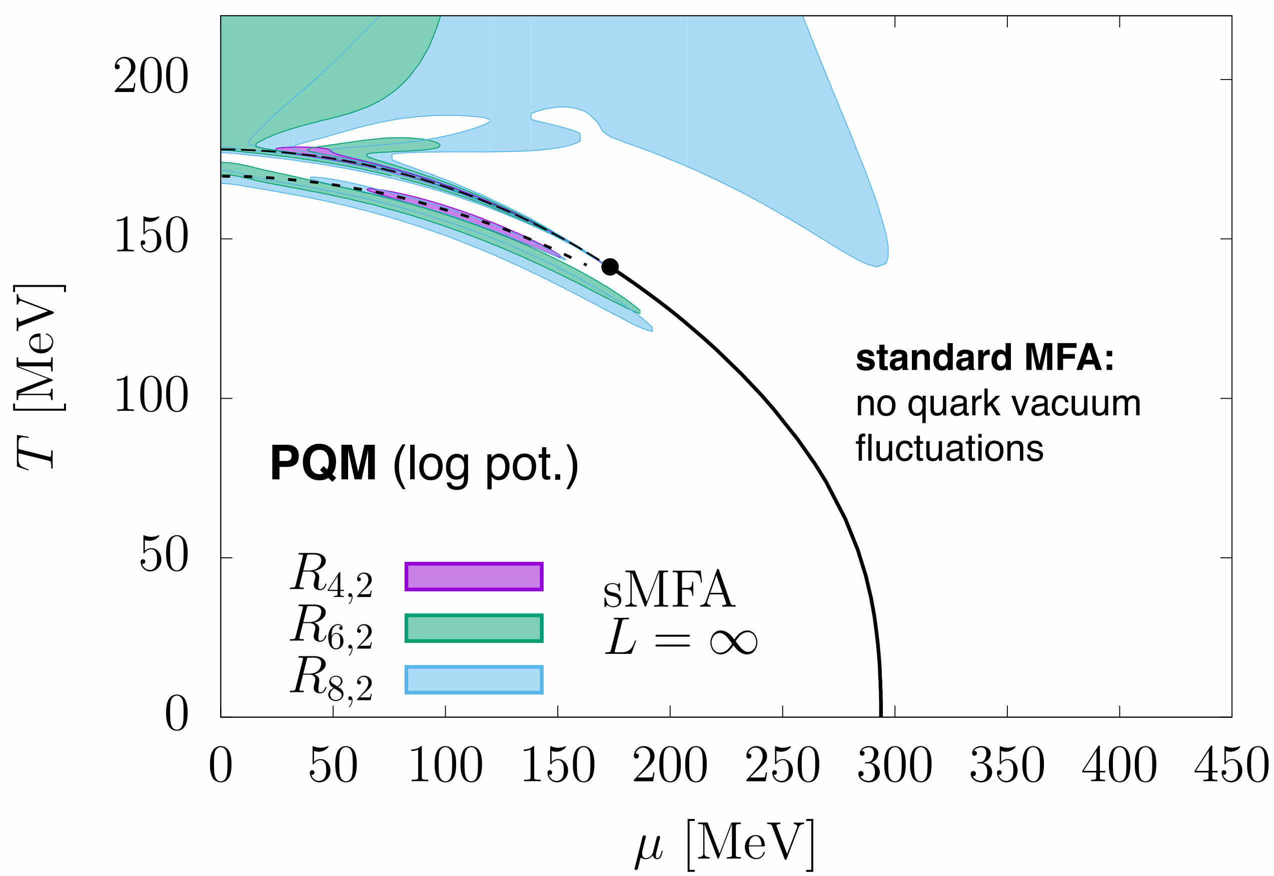

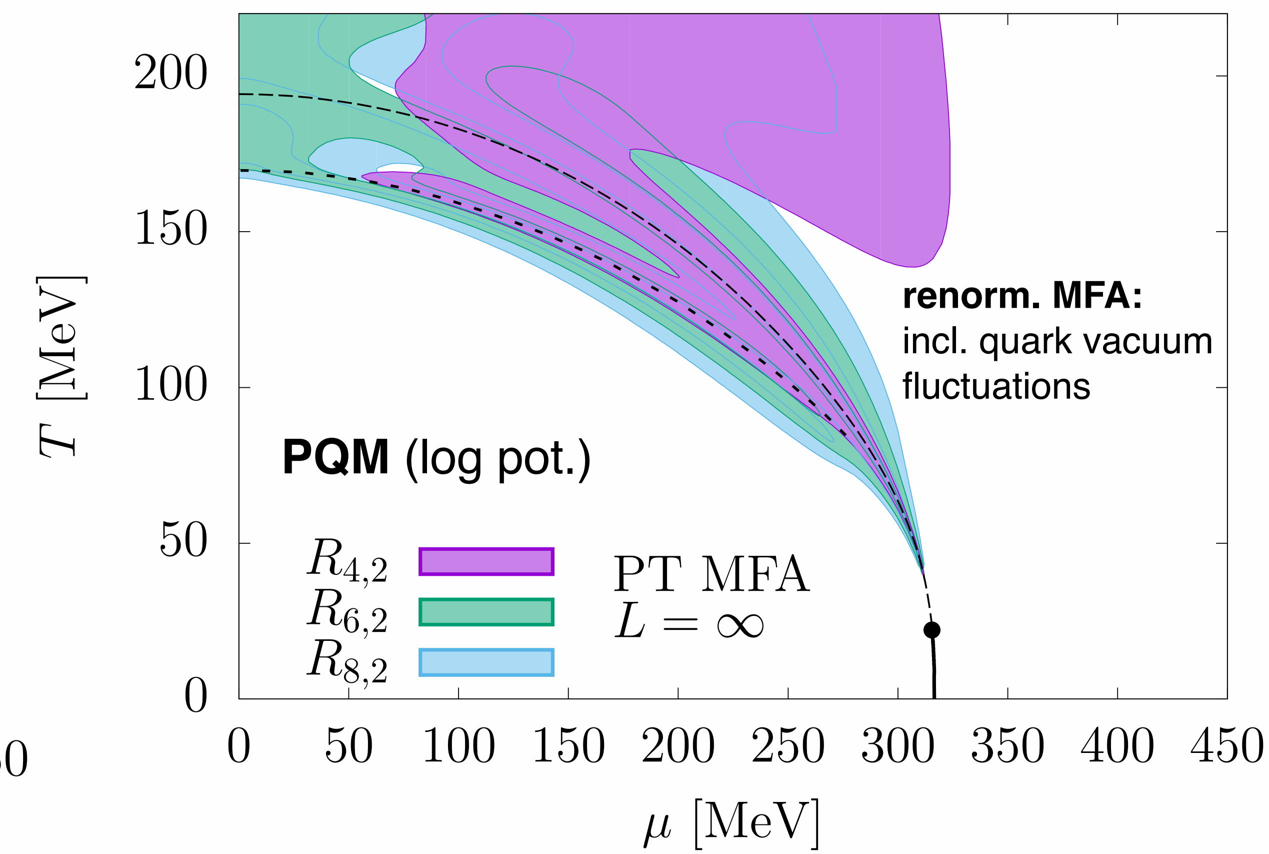

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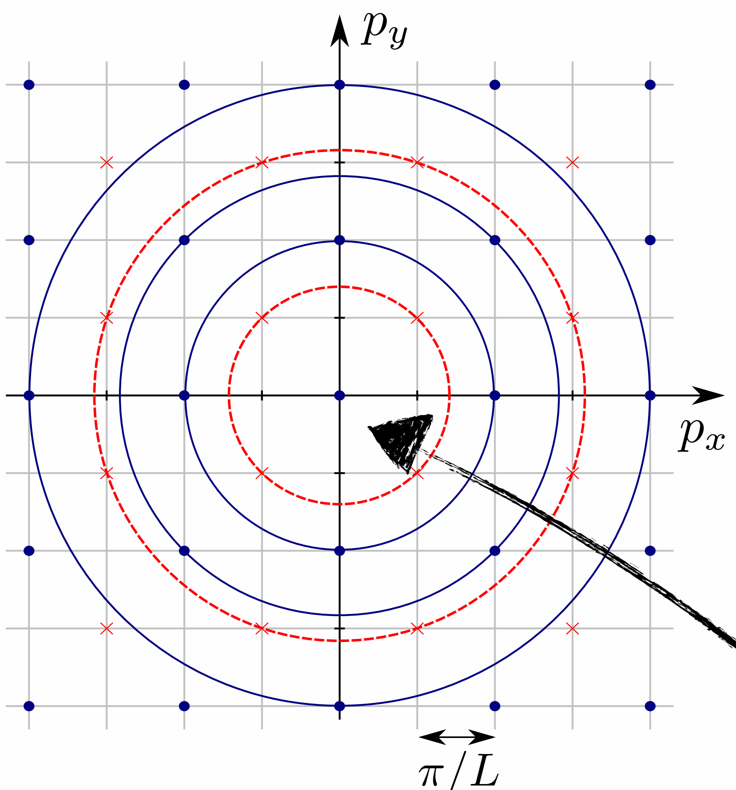
Finite volume



• periodic

× antiperiodic boundary conditions

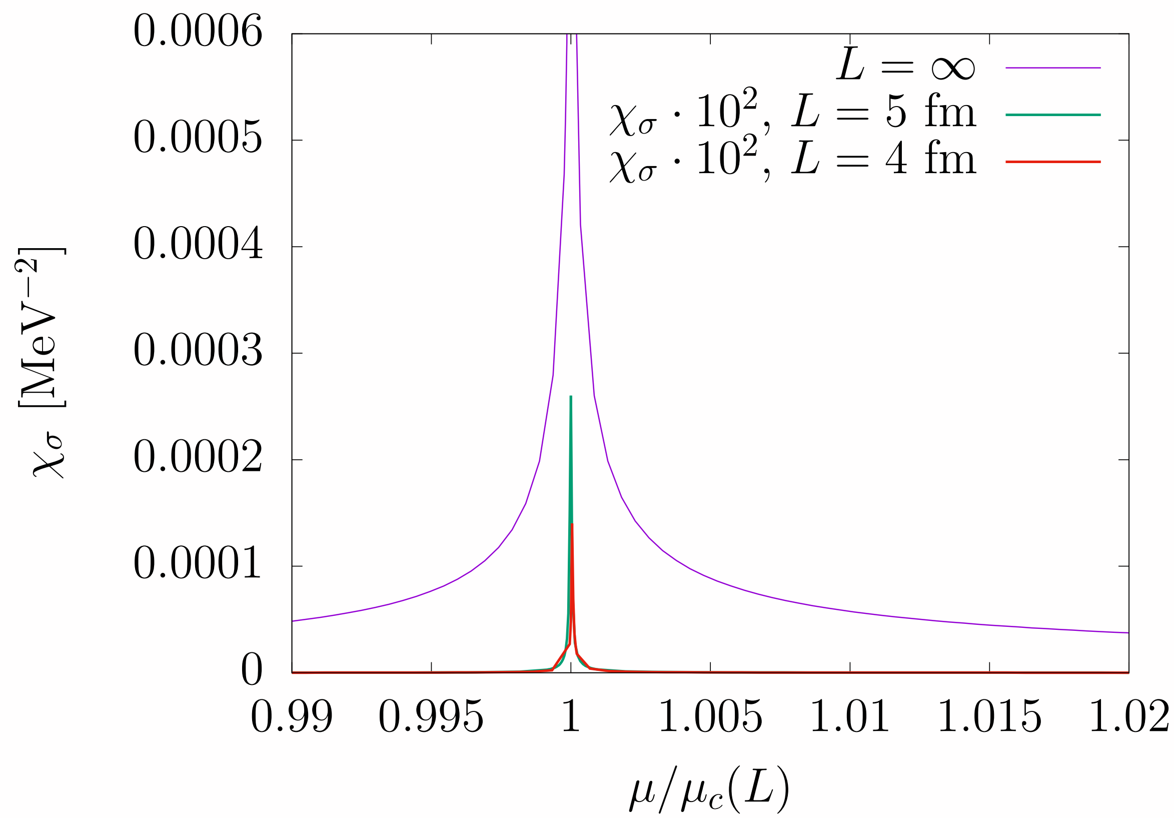
$$\int_{-\infty}^{\infty} \frac{dp_a}{2\pi} \dots \to \frac{1}{L} \sum_{n_a}$$

$$p_i \equiv \begin{cases} 2\pi T n_i \\ 2\pi T (n_i + \frac{1}{2}) + i\mu \end{cases}$$

$$T \leftrightarrow 1/L$$

Longitudinal susceptibility:

$$\chi_\sigma = \frac{1}{m_\sigma^2} \sim \frac{\partial \langle \bar{q} q \rangle}{\partial m_q}$$



<u>Boundary conditions (BC)</u>

PBC: periodic including zero mode

PBC*: star means without zero mode

ABC: antiperiodic

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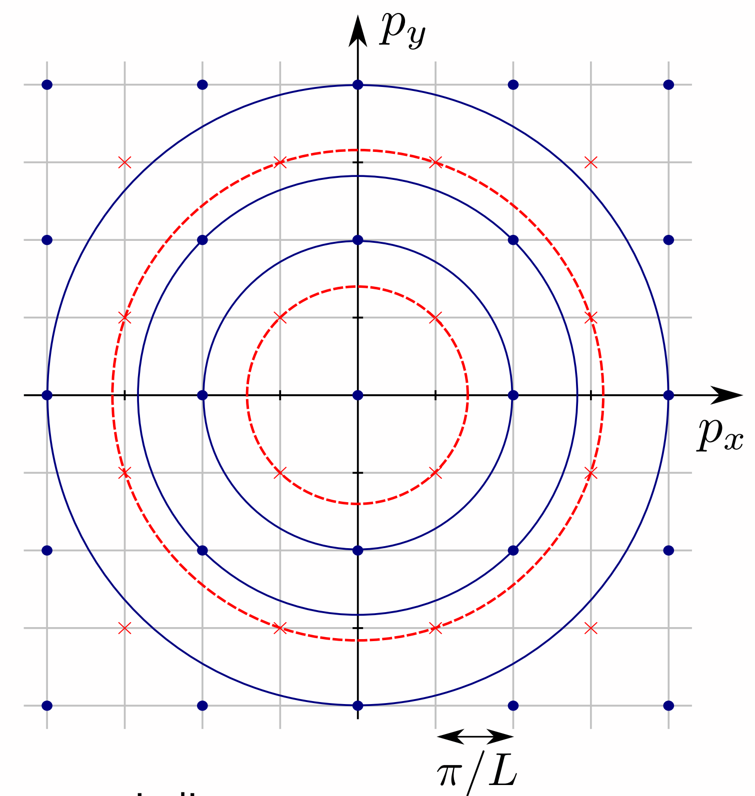
Finite volume



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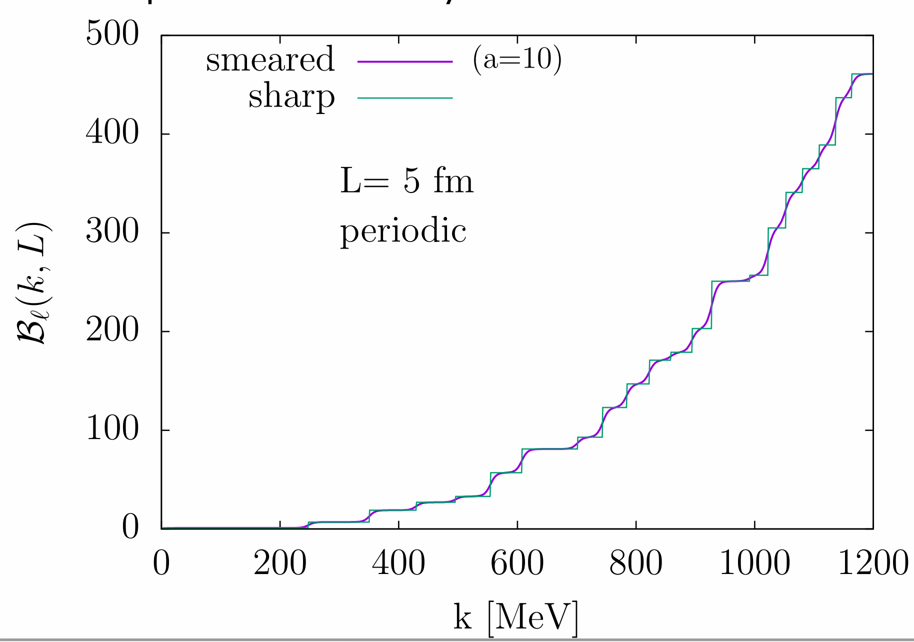

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Flow for sharp Litim regulator (not suitable for finite volume)

$$\partial_k U_k(T,L) \sim \mathcal{B}_\ell \cdot \partial_k U_k(T,\infty)$$

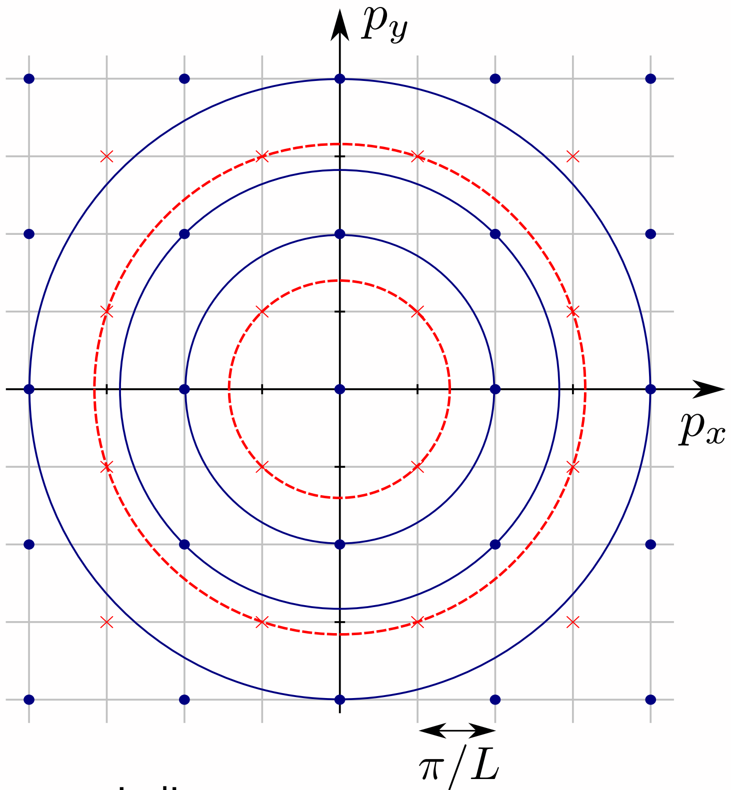
$$\mathcal{B}_\ell(k,L) = \frac{6\pi^2}{(kL)^3} \sum_{\vec{n}} \Theta(k^2 - \vec{p}_\ell^{\,2})$$

➜ use smeared regulator

[Fister, Pawlowski 2015]

[Tripolt, Braun, Klein, BJS 2012, 2014]

Finite volume



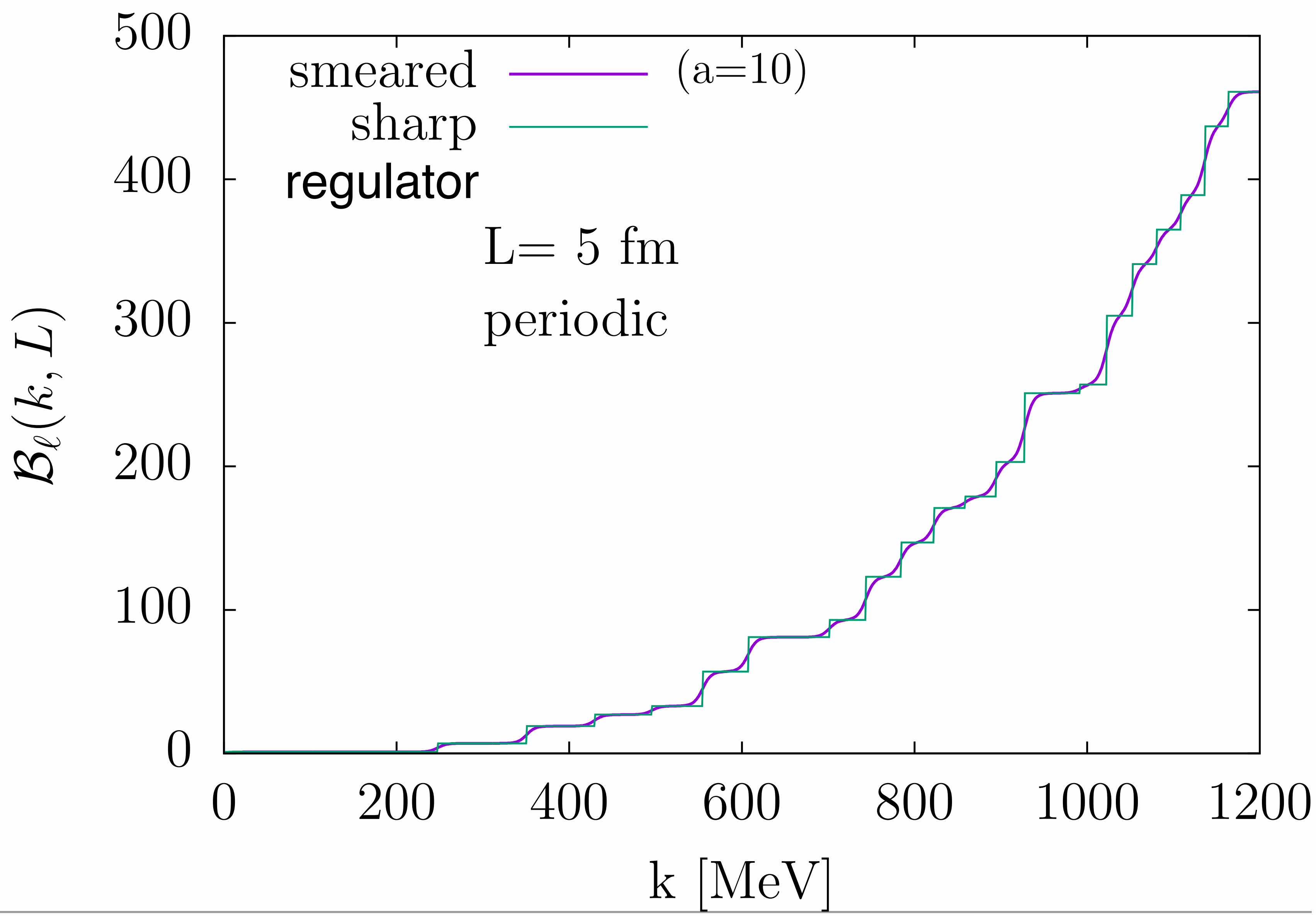
• periodic
× antiperiodic boundary conditions

work in progress

$$\partial_k U_k(T,L) \sim \mathcal{B}_\ell \cdot \partial_k U_k(T,\infty)$$

mode counting function:

$$\mathcal{B}_\ell(k,L) = \frac{6\pi^2}{(kL)^3} \sum_{\vec{n}} \Theta(k^2 - \vec{p}_\ell^{\,2})$$



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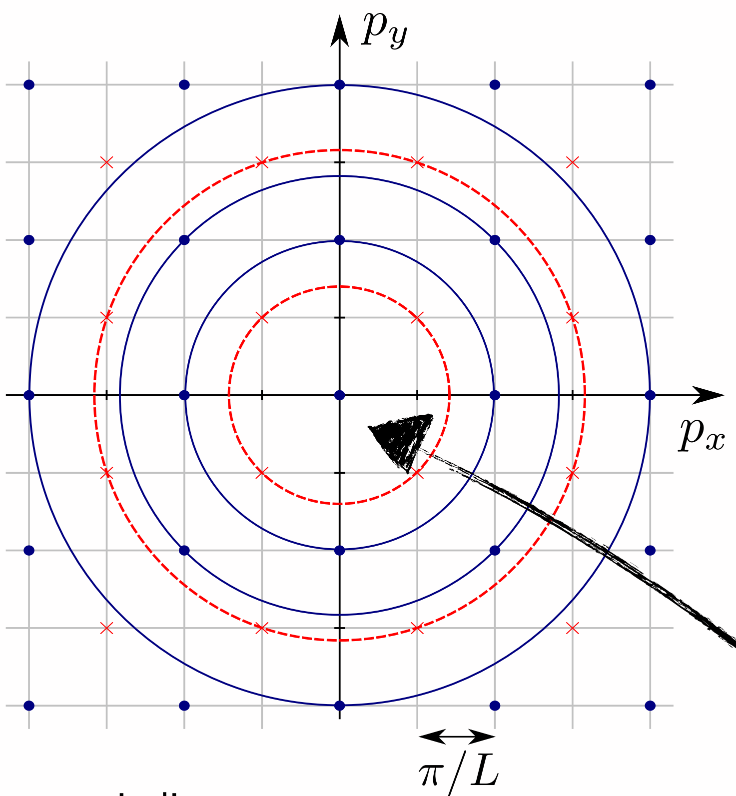

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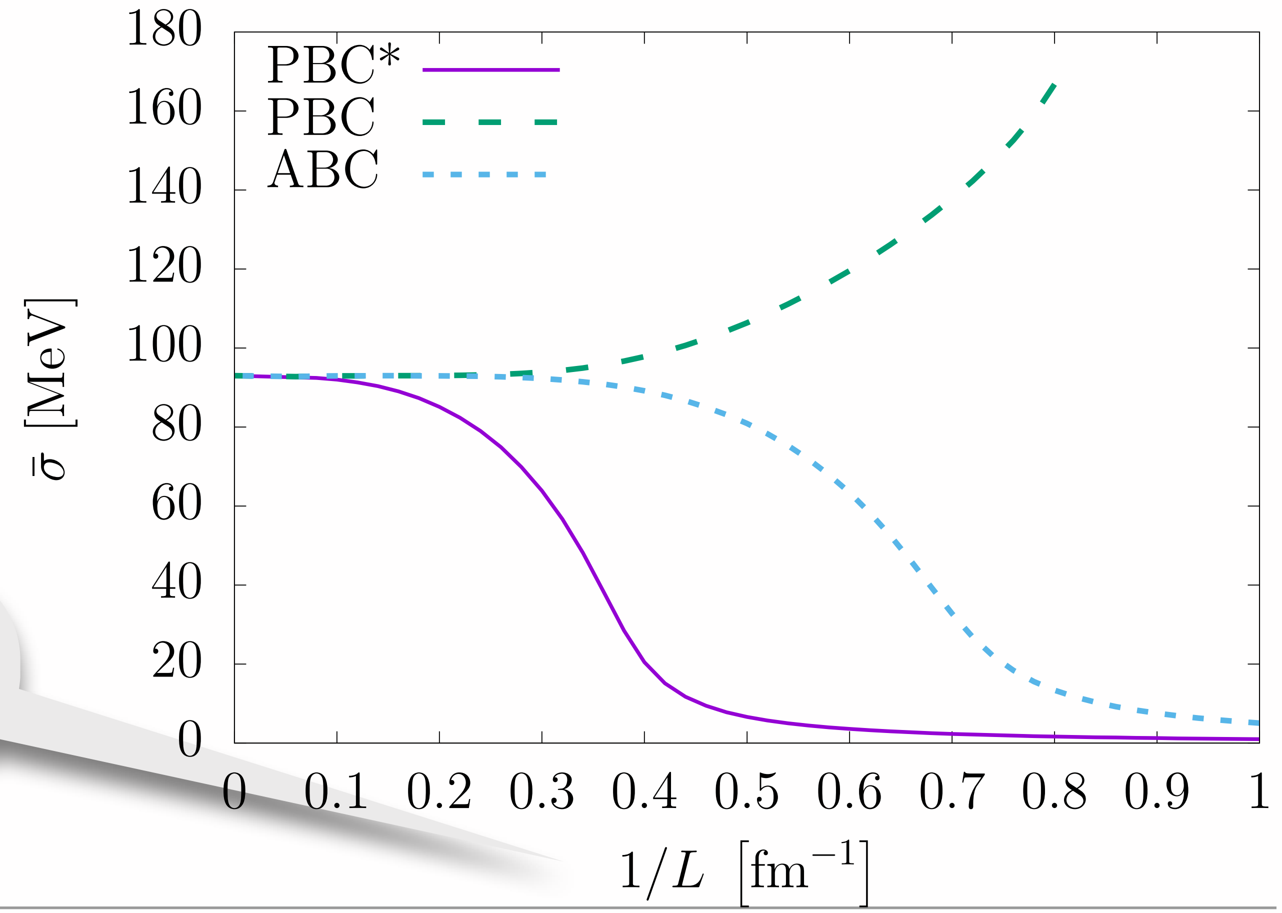
chiral condensate (PQM)



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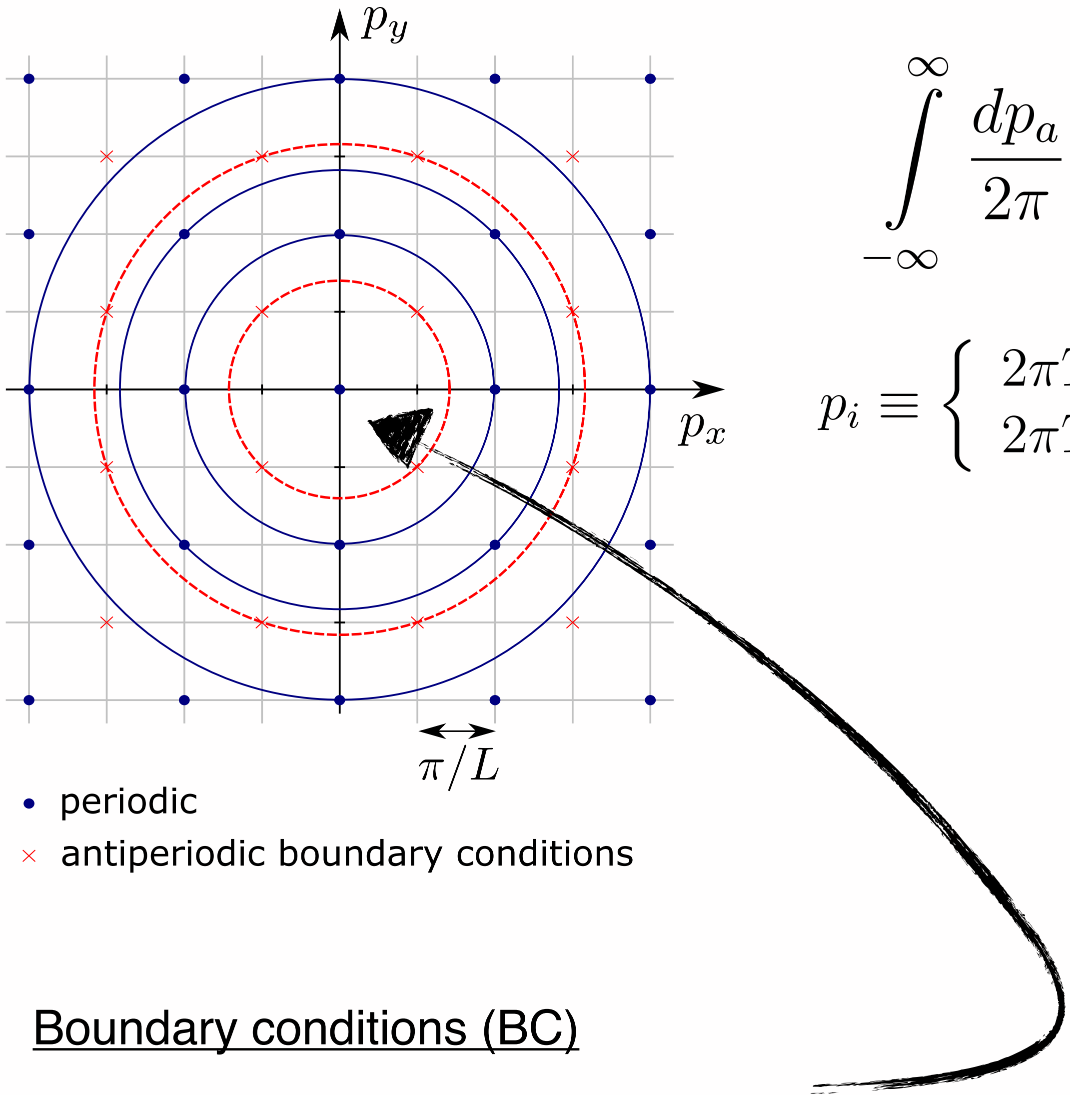

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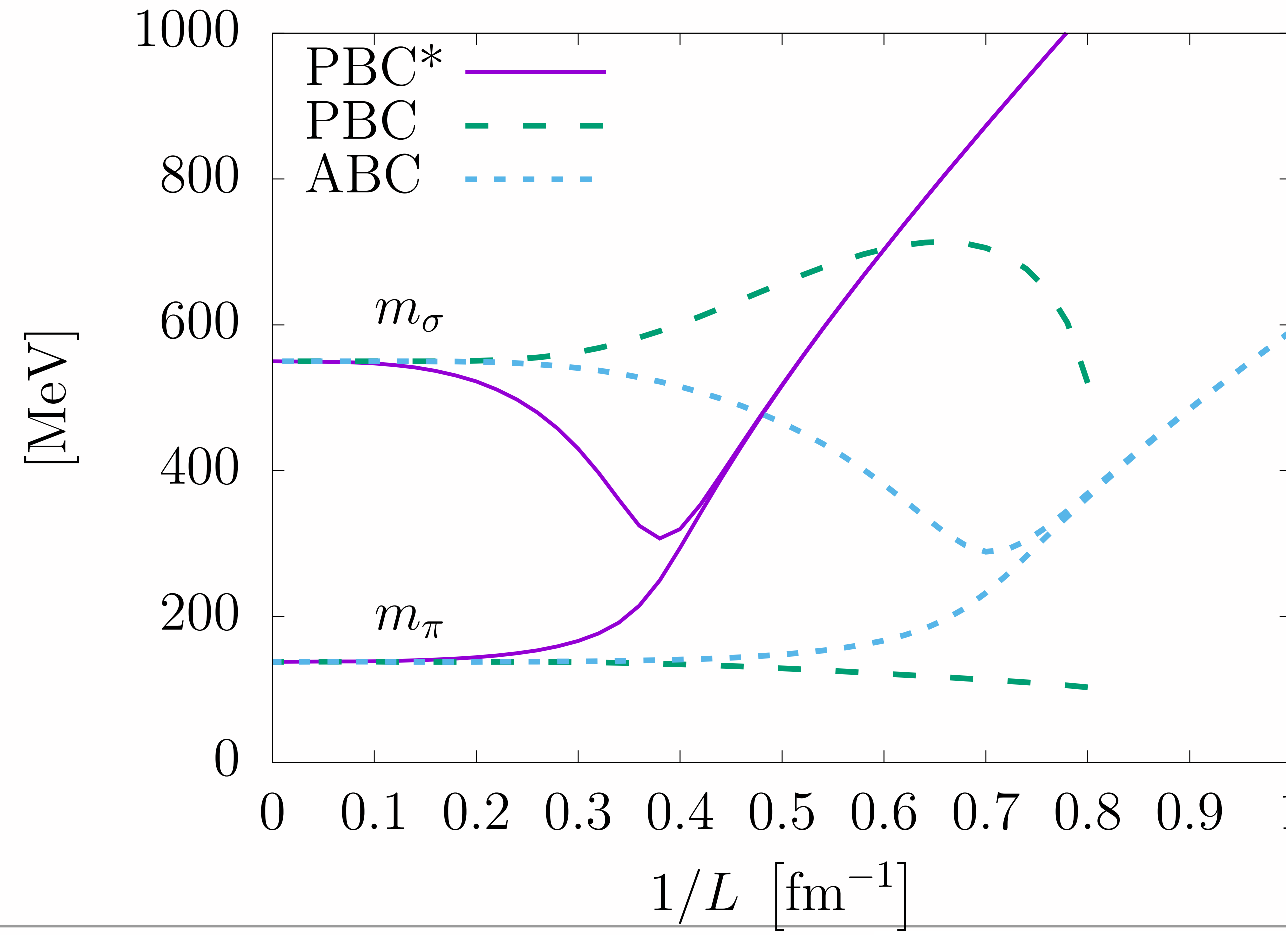
meson (curvature) masses (PQM)



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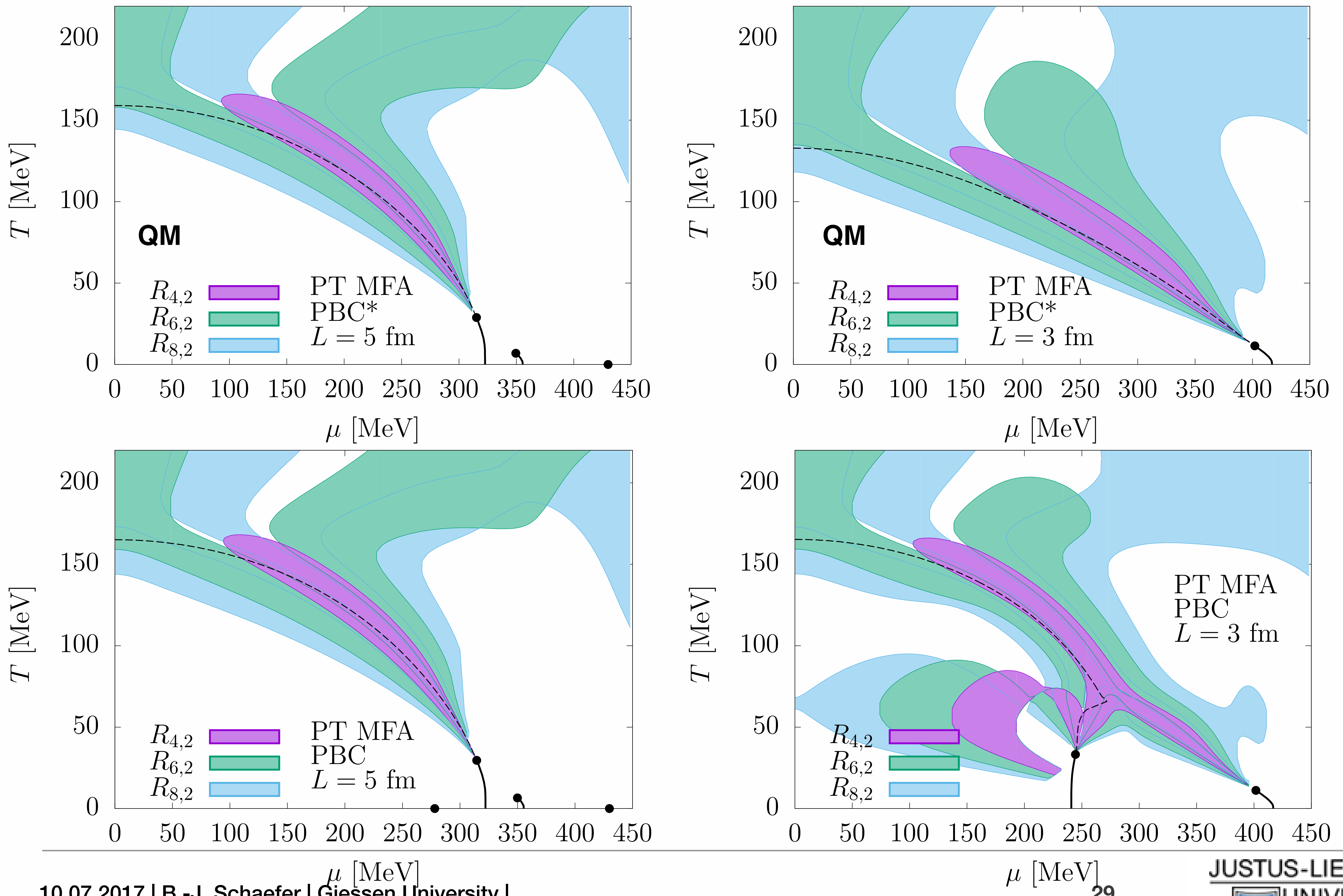
Higher cumulants

[S. Resch, BJS to be published]



Higher cumulants

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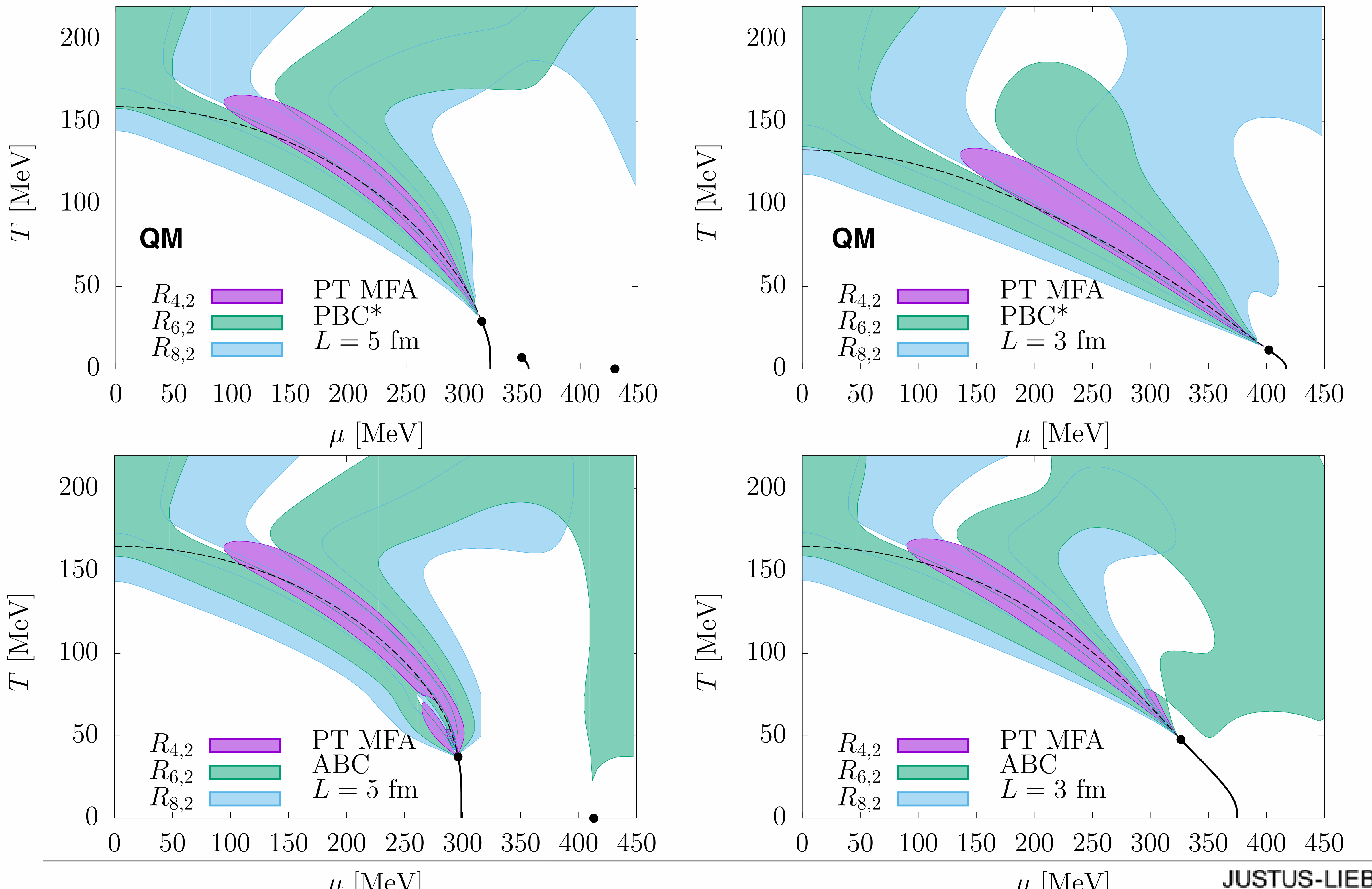

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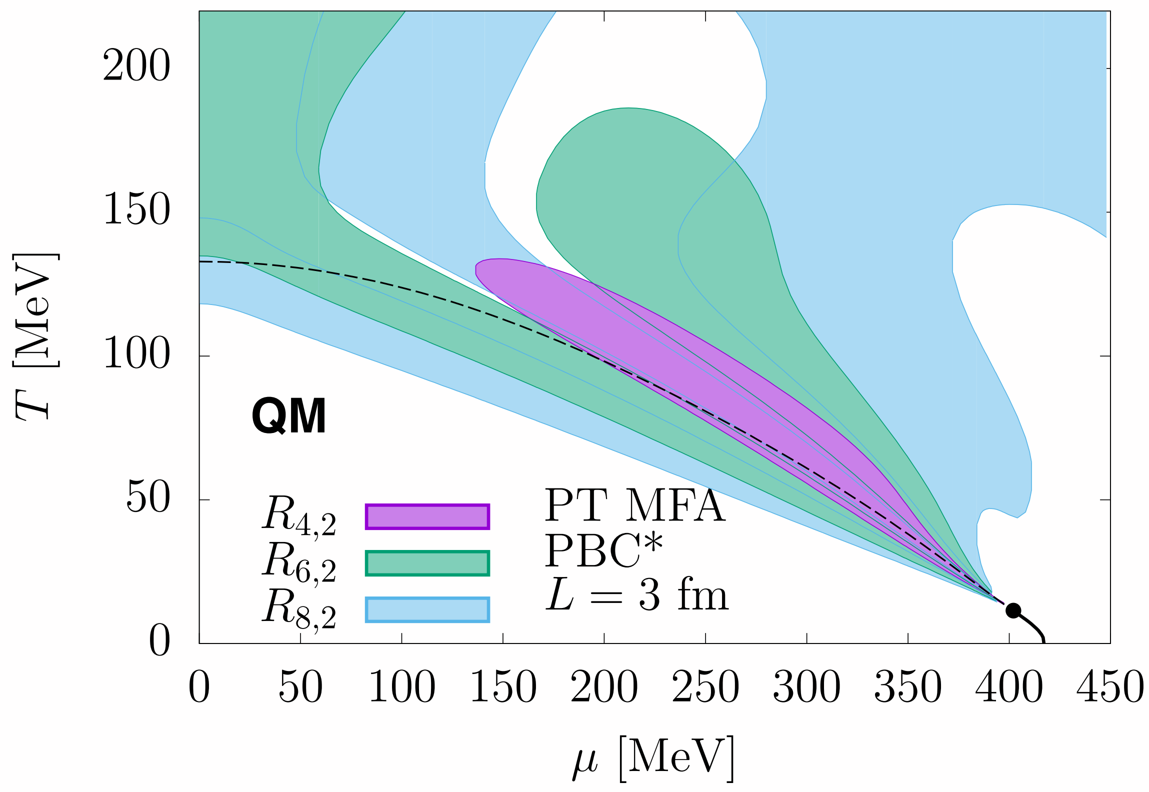
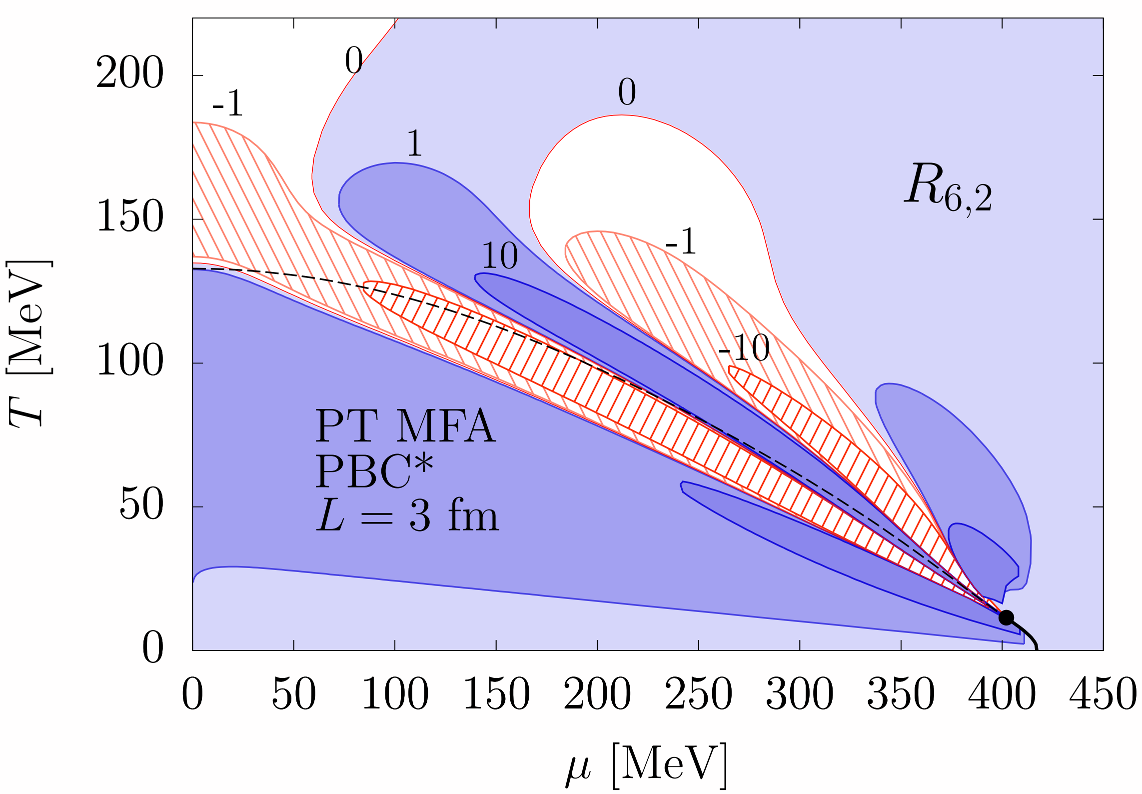
Higher cumulants

[S. Resch, BJS to be published]

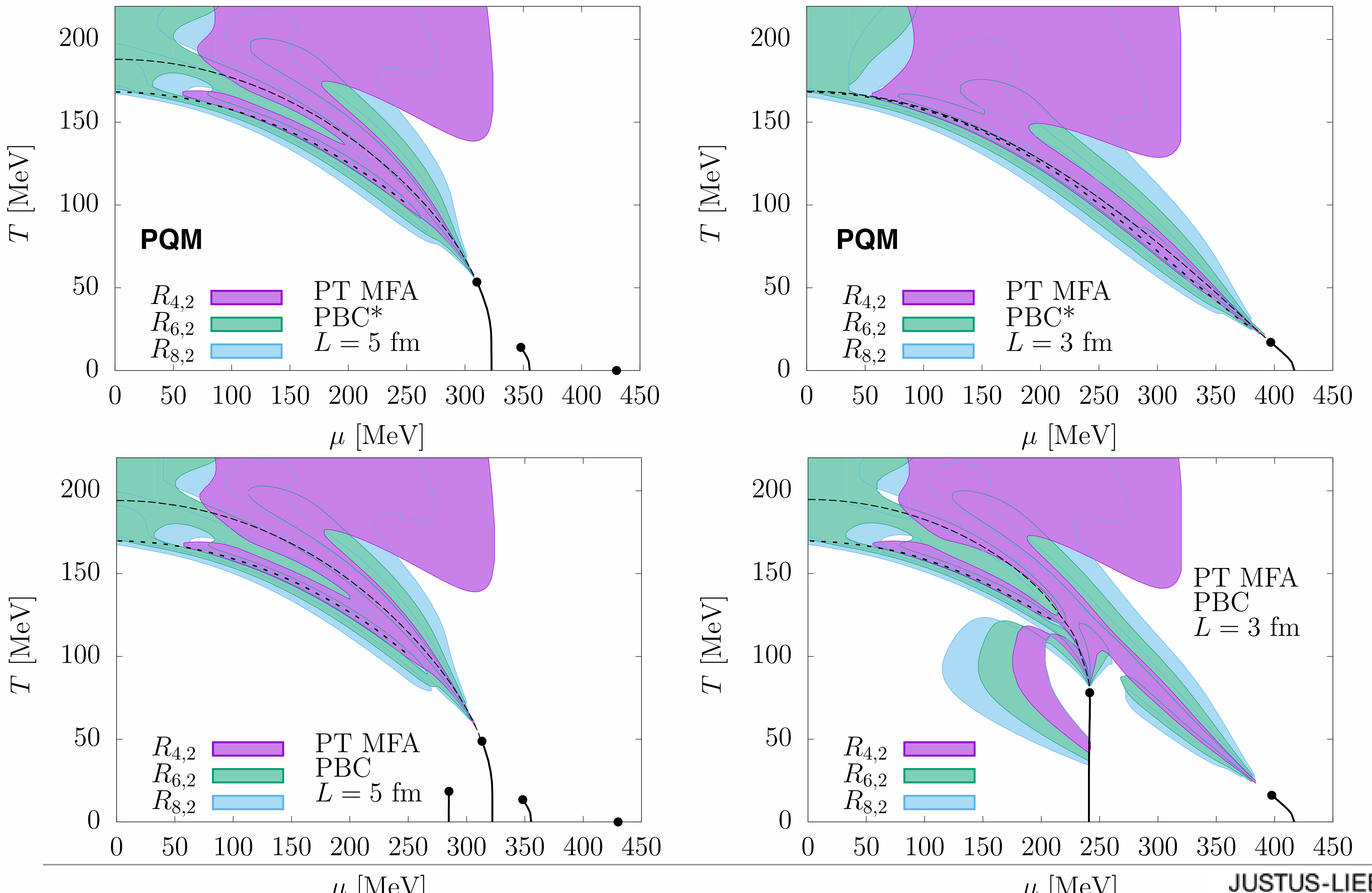


different contours



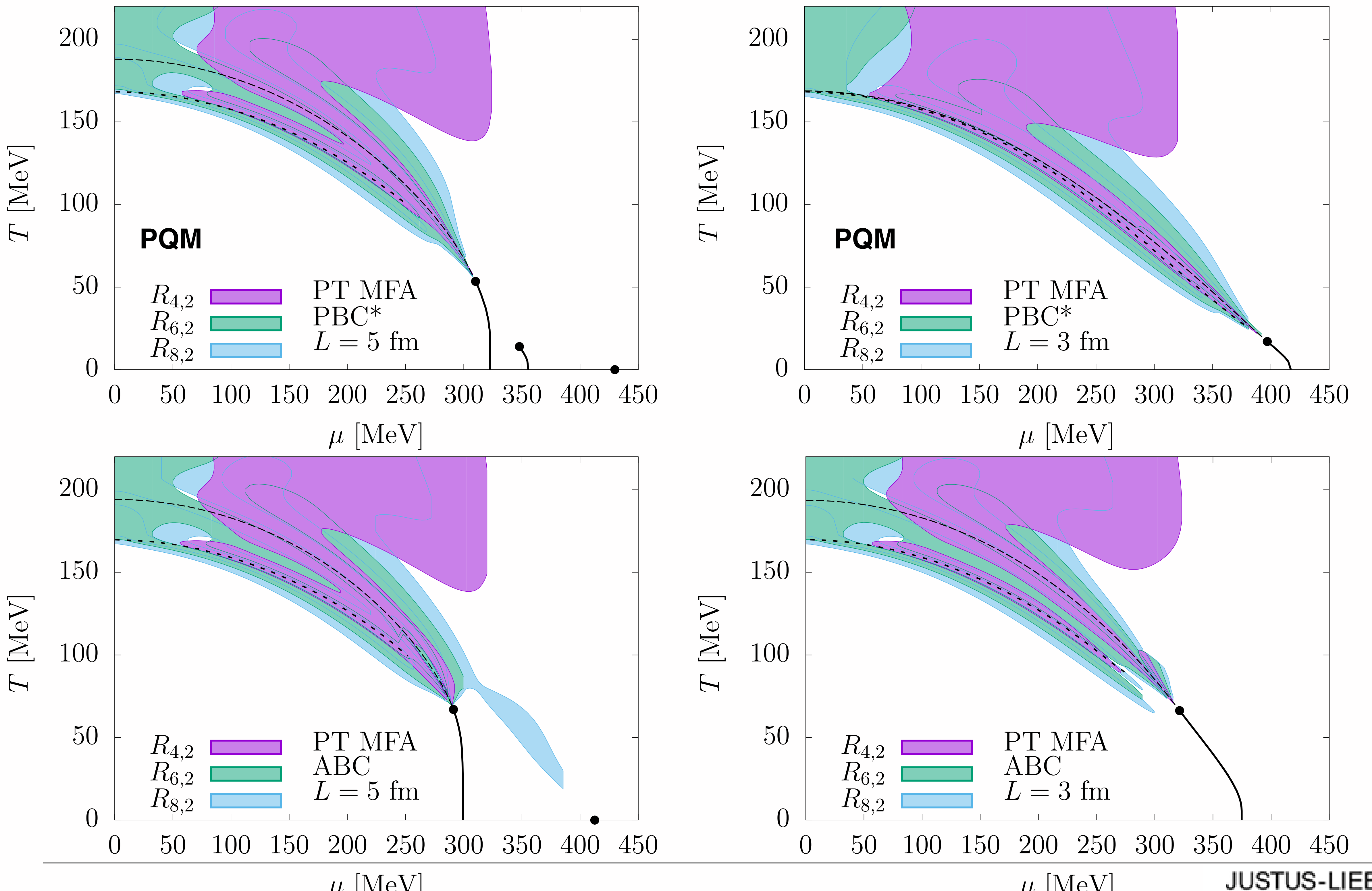
Higher cumulants

[S. Resch, BJS to be published]



Higher cumulants

[S. Resch, BJS to be published]



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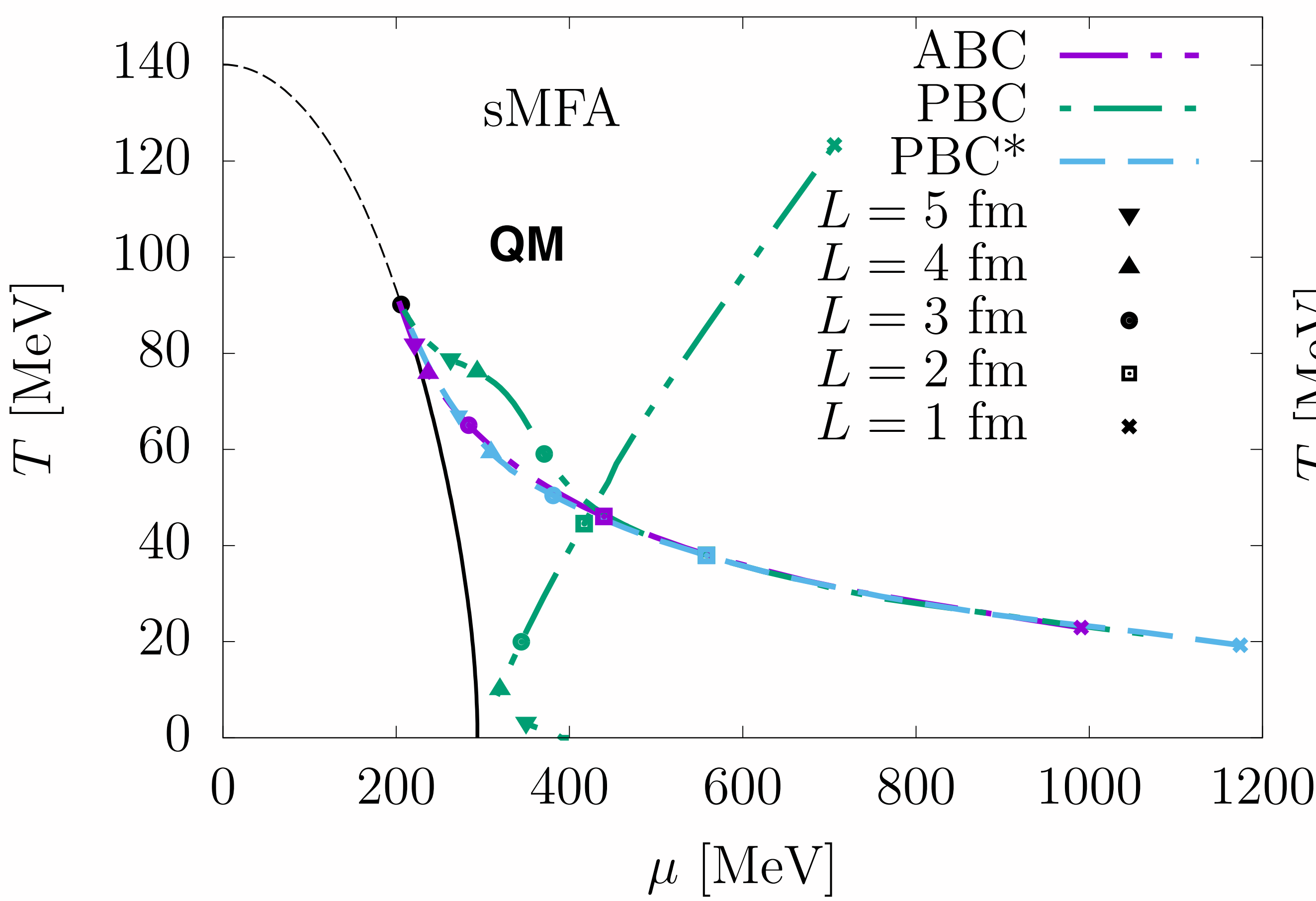
Dienstag, 11. Juli 2017; KW

effect of fluctuations

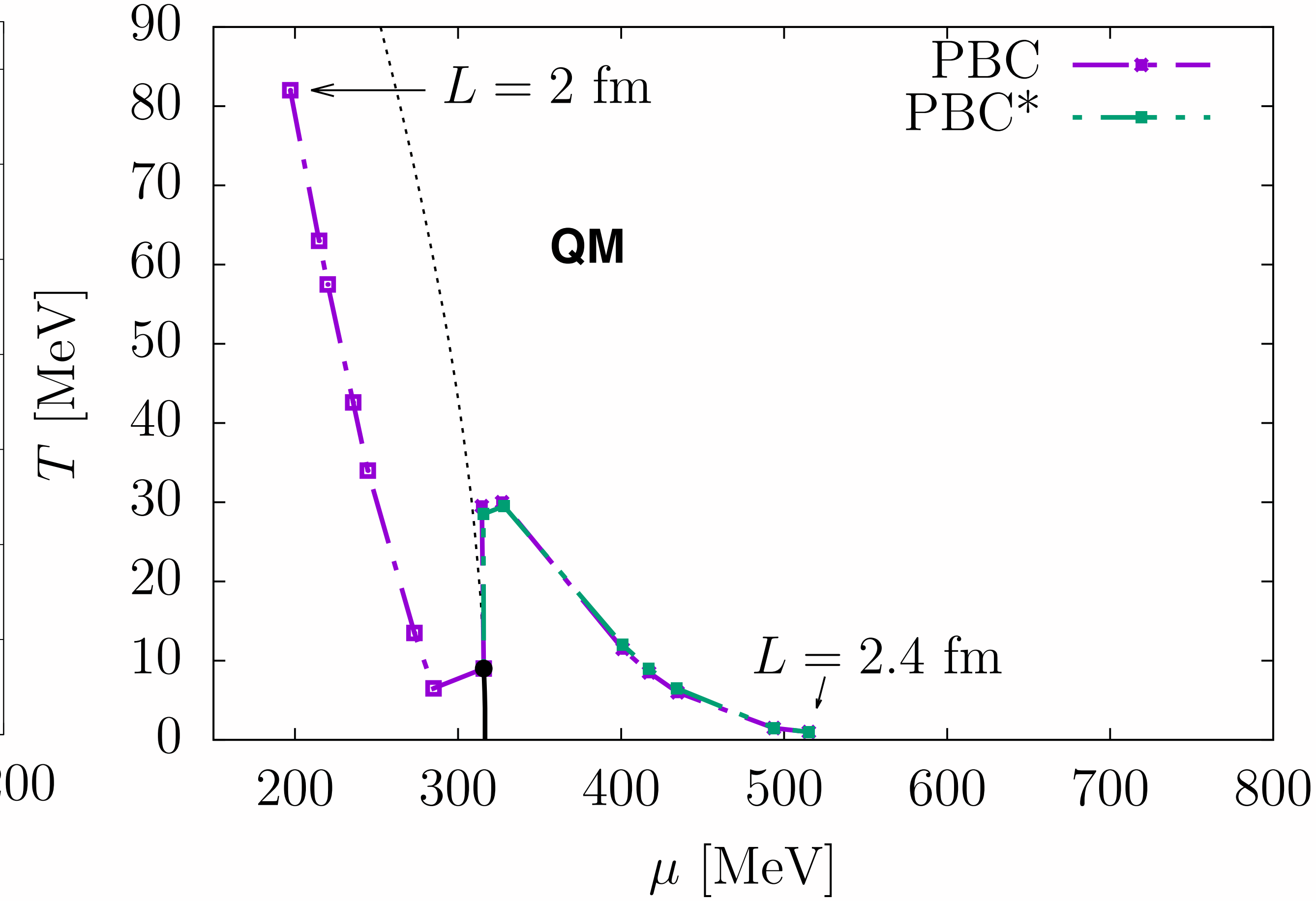
[A Juricic, BJS arXiv:1611.03653]

movement of the CEP's

standard MFA



renormalizes MFA



effect of fluctuations

[A Juricic, BJS arXiv:1611.03653]

movement of the CEP's

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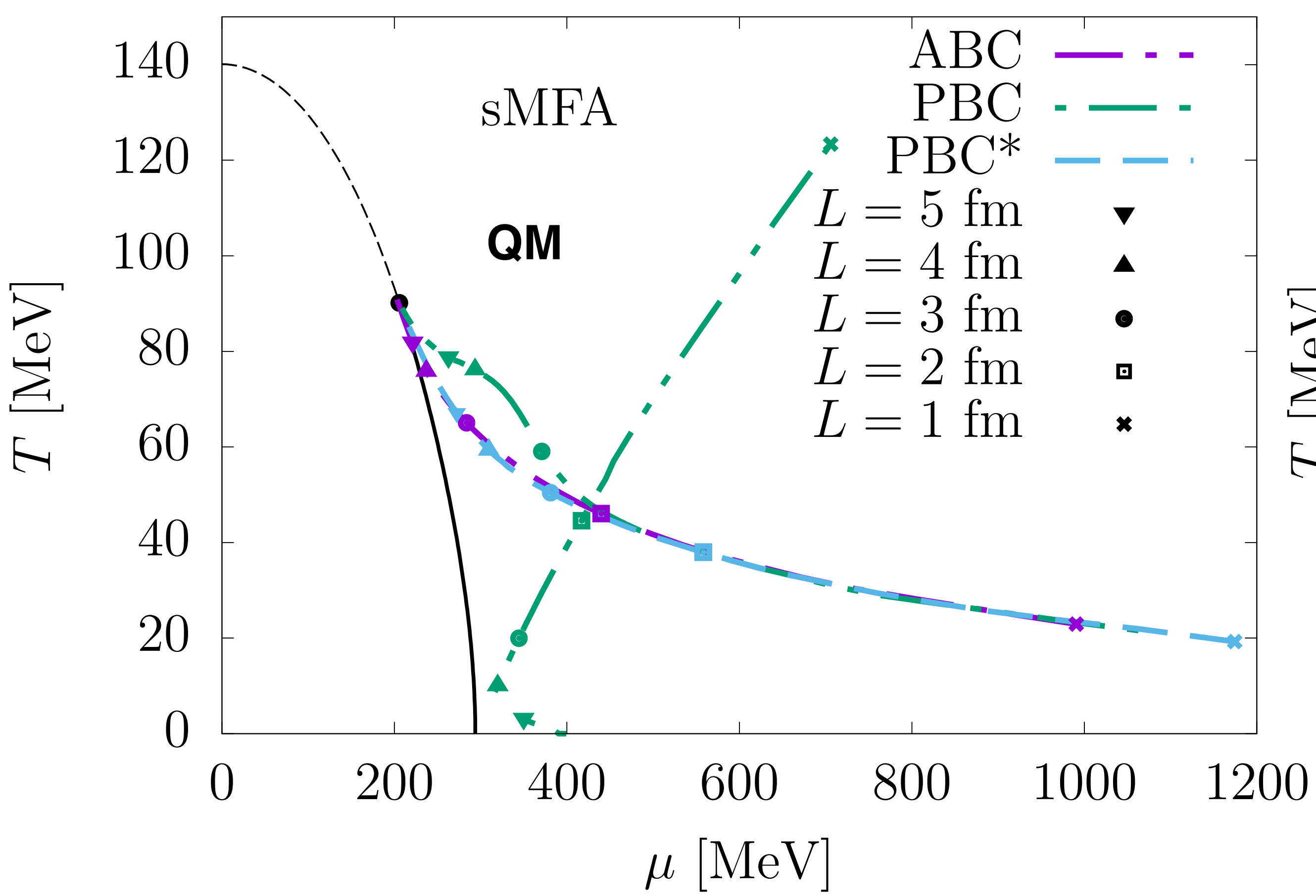
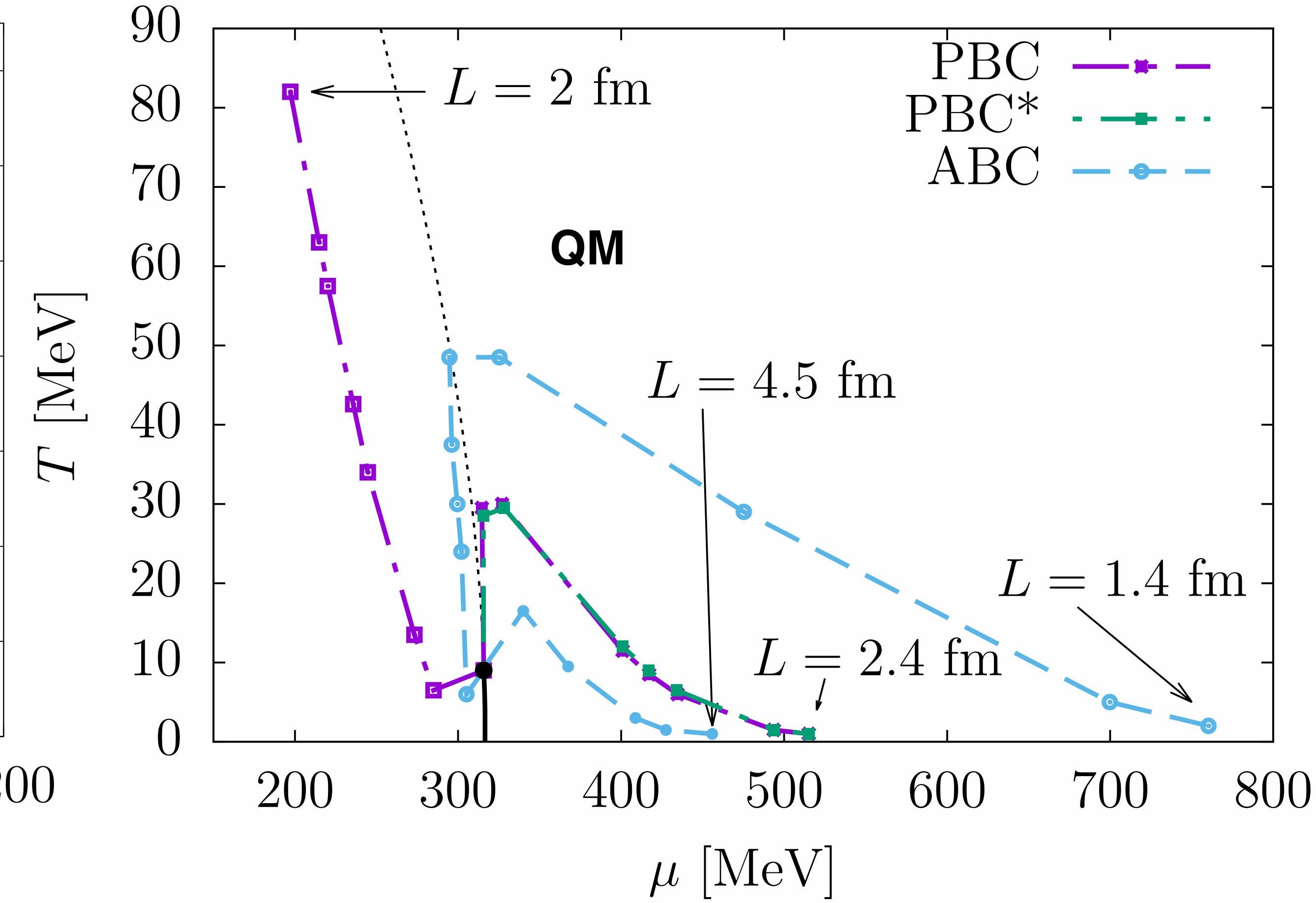
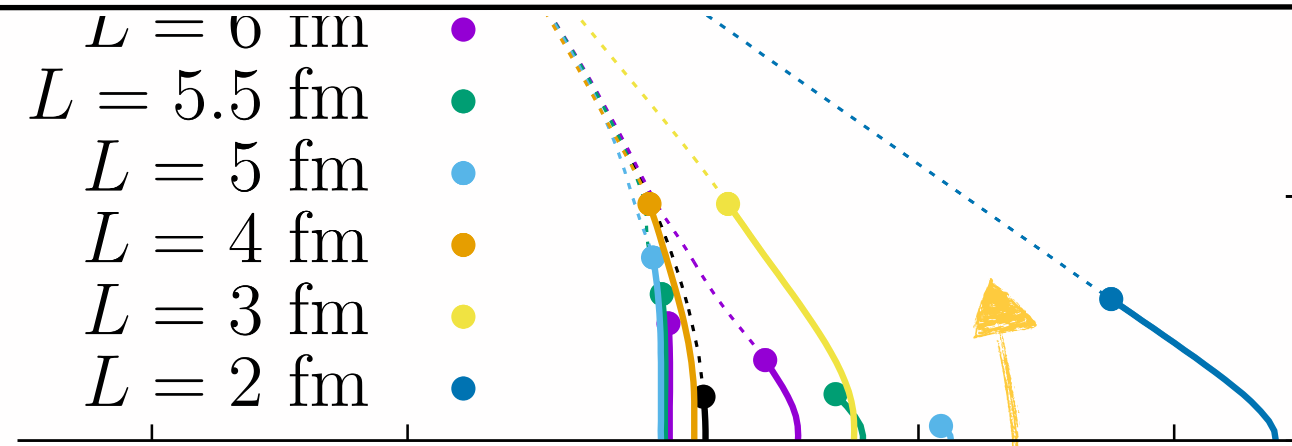


renormalizes MFA

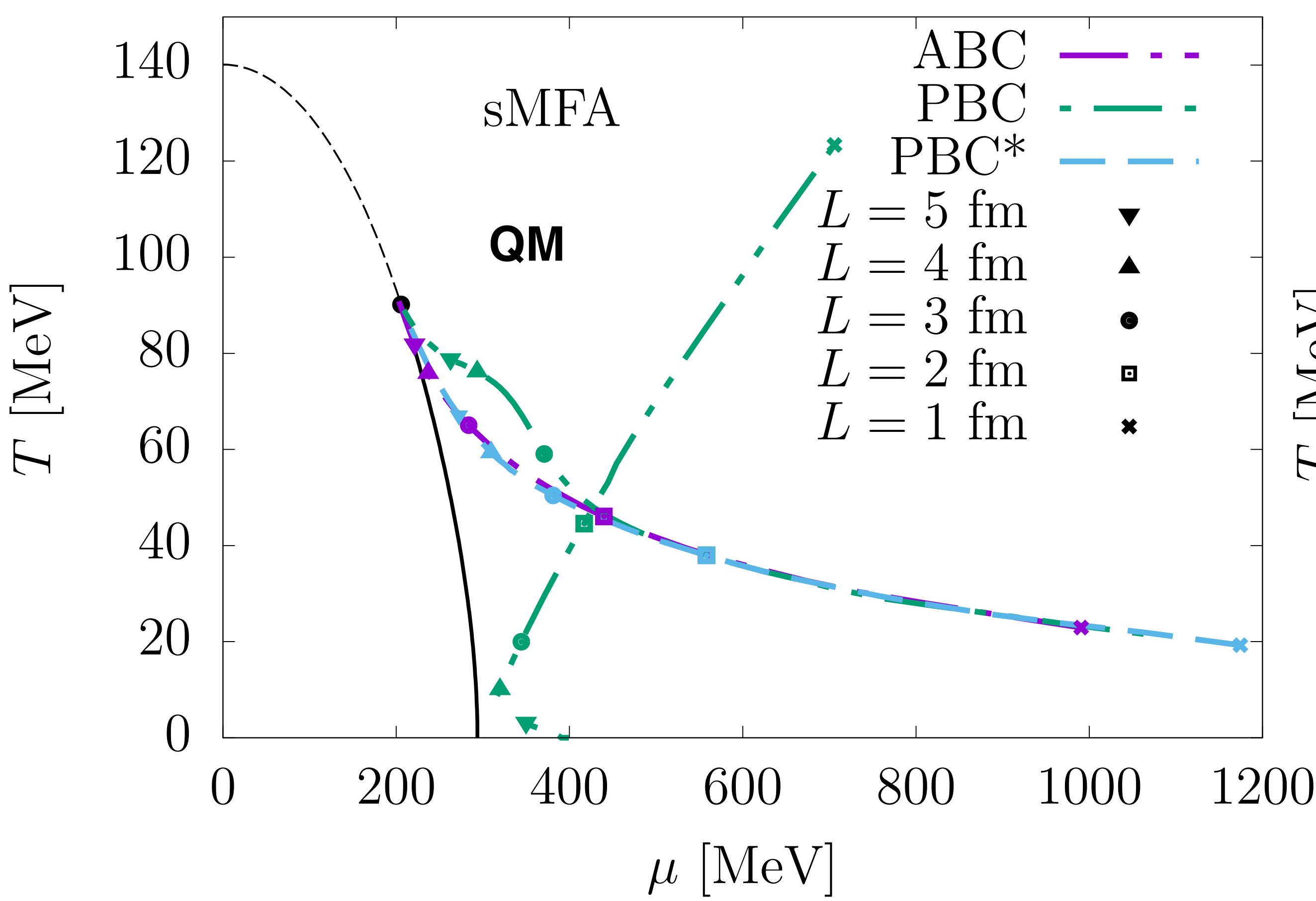


effect of fluctuations

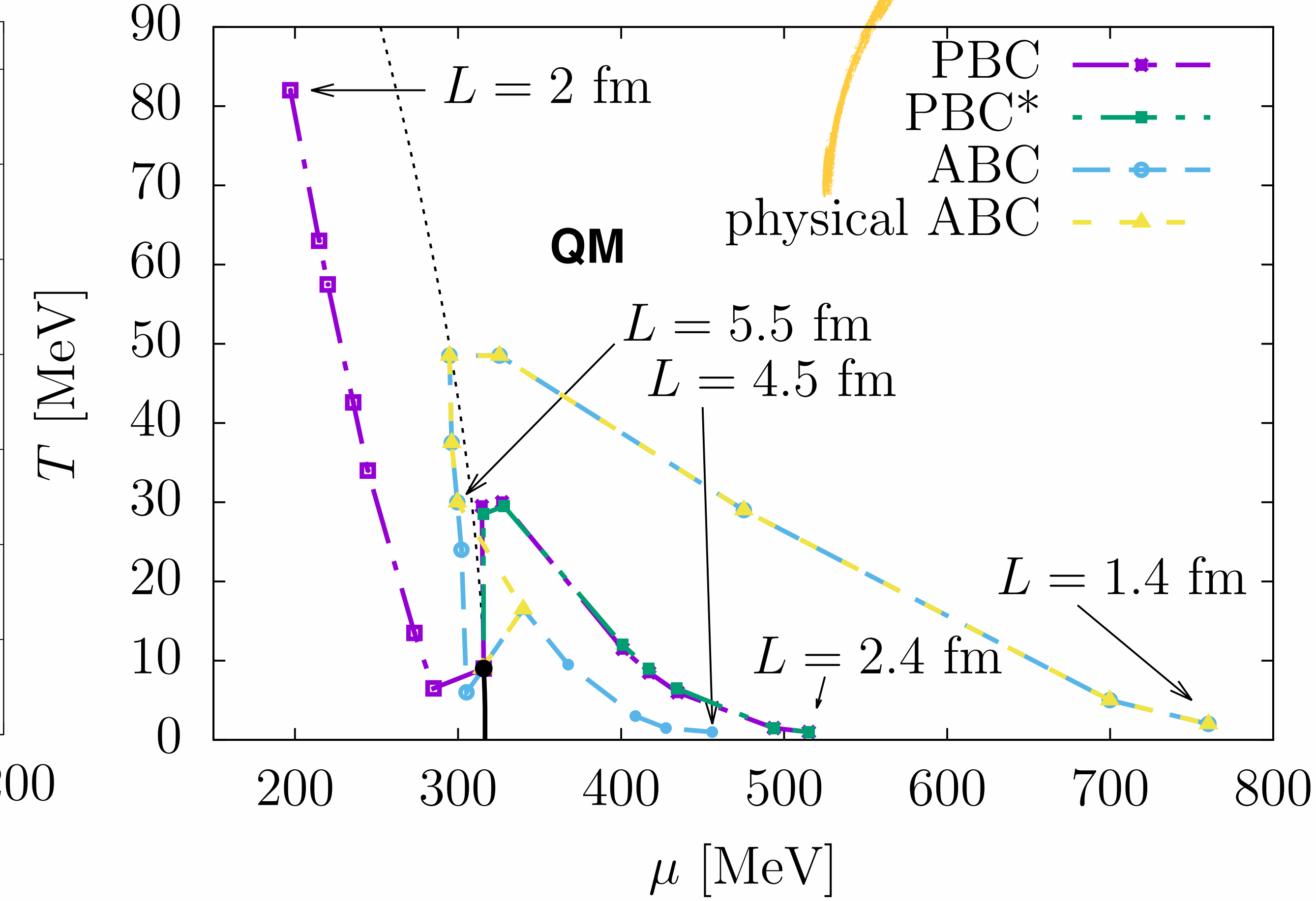
movement of the CEP's



standard MFA



renormalizes MFA



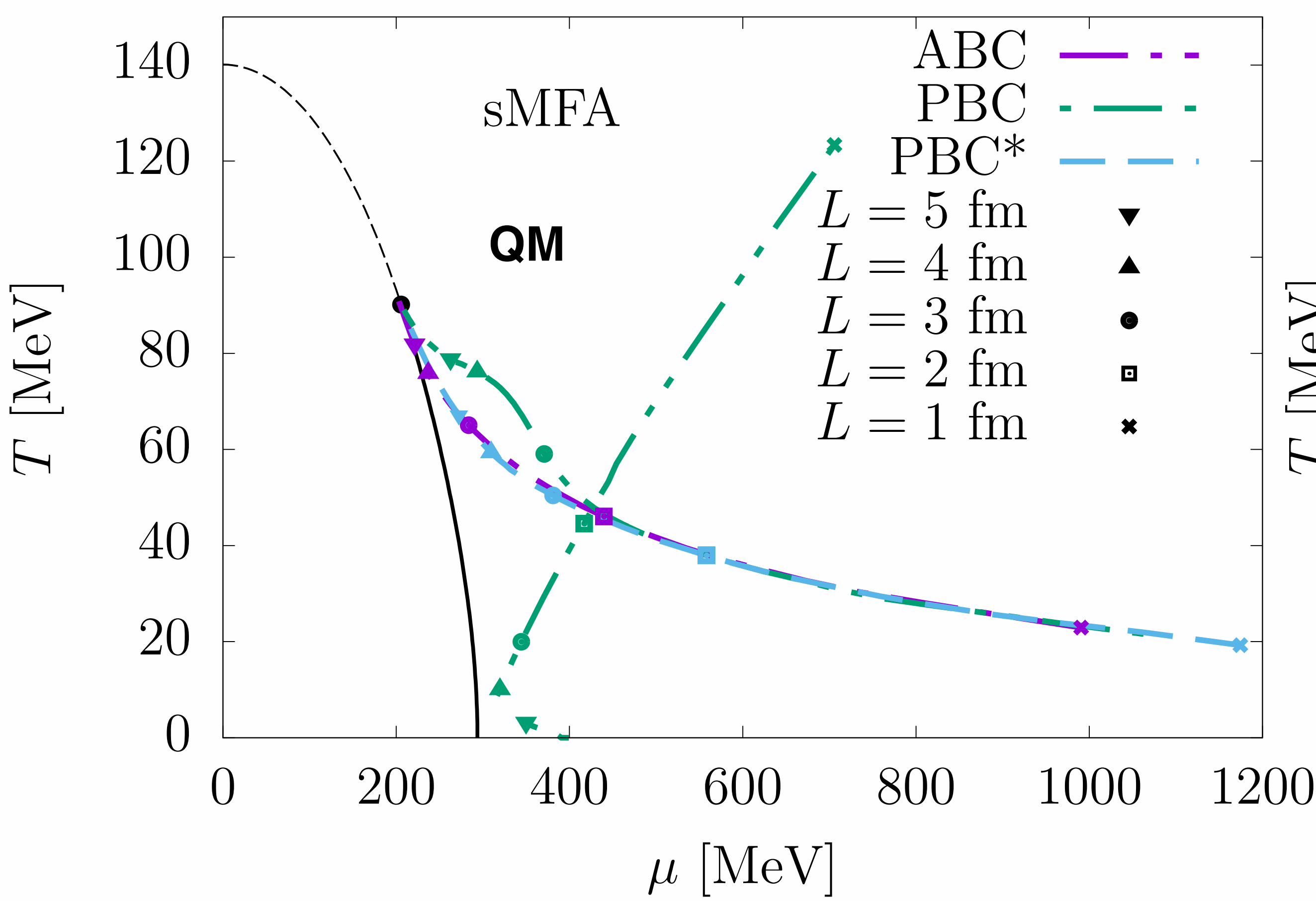
effect of fluctuations

[S. Resch, BJS to be published]

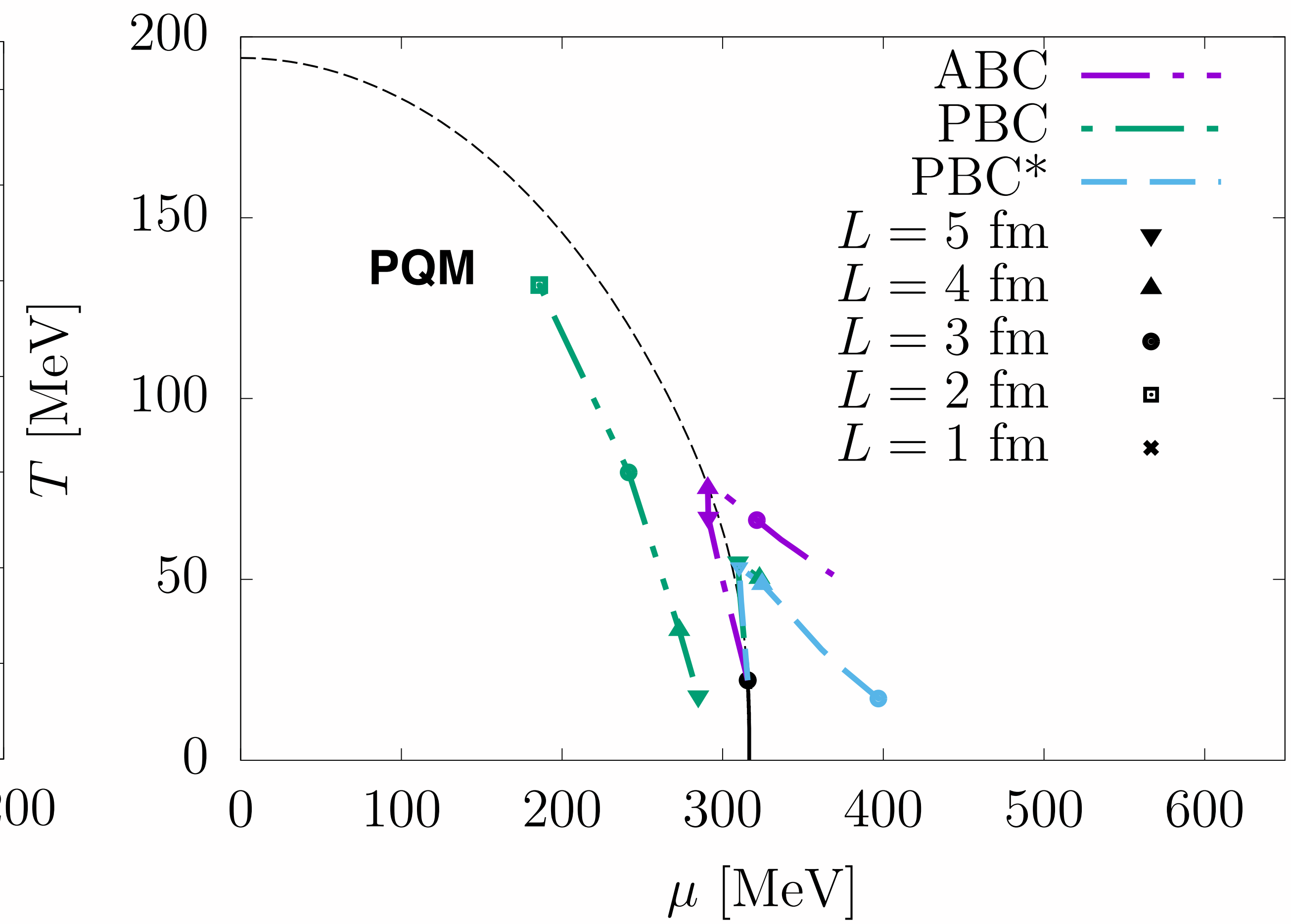
CEP vanishes for small volumes

movement of the CEP's

standard MFA



renormalizes MFA **$T_0(\mu)$** in log. potential



Summary & Conclusions

- effects of quantum and thermal fluctuations in a finite volume

  comparison: sMFA, rMFA, RG

  ➜ fluctuations wash out the phase transition

- existence/movements of critical endpoints in phase diagram in finite volume

  ➜ role of fluctuations: CEP vanishes for smaller volumes

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