INITIAL CONDITIONS TO NON-PERTURBATIVE KINETIC DESCRIPTION OF THE VACUUM QGP PRODUCTION AT ULTRARELATIVISTIC HEAVY ION COLLISIONS

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Contents

1. Introduction

In the present report I will try to reanimate our old non-perturbative kinetic approach for description of vacuum parton production at collisions of the ultrarelativistic heavy ions

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- This approach is based on the KE (named as the basic KE) obtained for the first time in the framework of the standard QED.

Very coarse assumptions were used here: space homogeneous, linear polarization of an external electric field. These limitations are added by other model assumptions. For example the electrodynamical part of the model is omitted. It is very important, that the strong quasiclassical electric field is set up “by hand” in order to ensure the sufficiently large final multiplicity. So, this statement of the problem is not connected with initial parameters of the colliding nuclei.

Nevertheless, much new results were obtained here for the first time:

- it was showed that the vacuum quarks creation (dynamical Schwinger effect) can be realized on correct dynamical level;
- it was showed that the high out-multiplicity can be ensured if to use very strong field with $E \sim 10^{2-3} E_c$, $E_c = m^2/g$;
- the corresponding mathematical formalism was developed in the strong field kinetic theory of the vacuum particle creation (the Markovian and the low density approximations, renormalized currents and so on).

So, below the question will be about the Abelian projection method which will be equipped by some new important elements:

- strong quantum field (QED and QCD) ground;
- taking into account of the electromagnetic components in evolution of the QGP;
- self consistent description of evolution of the QED an QCD components of QGP;
- formulation of the initial conditions in the terms of the initial distribution functions of quarks; it allows to consider all following evolution QGP depending on the energy and composition of the colliding nuclei;
- evolution of the vacuum polarization of the quark and electromagnetic subsystems are considered also.

2. Construction of the model

2.1 The Abelian sector

Adequate dynamical description of the QGP evolution is impossible without electromagnetic constituent of dynamics. It can expect that strong electromagnetic fields and currents are generated at the same time with strong color fields and currents. We assume that the same Schwinger mechanism acts in QCD as in QED. It leads to the general Abelian picture of the phenomenon of vacuum creation.

Thus, it is necessary to select the Abelian sector in QCD for nonperturbative kinetic description of vacuum quark creation and to consider it simultaneously with the electrodynamical sector of the theory. Absence of the color transmutation in interaction and self-interaction is necessary to this.

The first Abelian condition (the quark sector)

Let us write the initial definitions: the Lagrangian function of the quark sector of QCD in an external classical gluon (B) and electromagnetic (A) fields ($\hat{m}$ is the mass matrix) $\mathcal{L} = i\bar{q}\gamma^{\mu} D_{\mu} q - \bar{q}\hat{m}q,$

equation of motion $(iD_{\mu}\gamma^{\mu} - \hat{m})\, q = 0,$

energy-momentum tensor $T_{\mu\nu} = \frac{i}{2}\left[\bar{q}\gamma_{\mu} D_{\mu} q - (D_{\nu}^{+}\bar{q})\gamma_{\mu} q\right]$

and densities

of electromagnetic $j_{\mu} = \bar{q}\gamma_{\mu} q$ and color $J_{\mu}^{a} = \bar{q}\lambda^{a}\gamma_{\mu} q$ currents.

The electromagnetic A^{μ} and gluons fields B_{a}^{μ} bring in the theory by standard way

$(e > 0, g > 0)$
$$D^{\mu} q^{j} = \sum_{j'} \left[(\partial^{\mu} + ie_{f} A^{\mu})\, \delta_{jj'} - \frac{i}{2} g \sum_{a} B_{a}^{\mu} \lambda_{jj'}^{a} \right] q^{j'}.$$

here $f, f', \ldots$ and flavor and $j, k, \ldots$ are color index; e_f is the electric charge of the quark with flavor f, g is the color charge.

The corresponding Hamiltonian function after exception of the time derivative of the quark field with help of equation of motion is equal

$$\mathcal{H} = T_{00} = \frac{i}{2}\left[\bar{q}\vec{\gamma}\vec{D}q - \left(\vec{D}^{+}\bar{q}\right)\vec{\gamma}q\right].$$

In general case transition in the quasiparticle representation (QPR) implies diaganolization of the Hamiltonian in the Fock representation relative to the quark and antiquark creation and annihilation operators. Bogolubov's canonical transformation method is used for this aim. In QED it is possible for spatial homogeneous time dependent classical electromagnetic fields, which provides locality of the Hamiltonian in the momentum space and does not violate of homogeneity of the quadratic form of the creation and annihilation operators. However the last feature of the QED interaction is violated in QCD, where the nondiagonal color terms in the quark quadratic form in the Hamiltonian are present. So it is necessary to select the terms, which do not change color of quarks in act interaction with the gluon field. It leads to the selection of the diagonal elements of the Gell-Mann matrix λ. Thus, the Hamiltonian function can be represented in the form:

$$\mathcal{H} = \mathcal{H}_Q + \mathcal{H}_{int},$$

where $\mathcal{H}_Q$ is the homogeneous flavor and color quadratic form $\mathcal{H}_Q = \frac{i}{2}[\bar{q}\vec{\gamma}\overrightarrow{\mathfrak{D}}q - (\overrightarrow{\mathfrak{D}}\bar{q})\vec{\gamma}q]$,

where $\mathfrak{D}^{\mu}_{fj}$ contains interaction of the electromagnetic and gluon fields with the quark of a flavor f and a color j only,

$$\mathfrak{D}^{\mu}_{fj} = \partial^{\mu} + ie_f A^{\mu} - \frac{i}{2} g \sum B^{\mu}_{a} \lambda^{a}_{jj} = \partial^{\mu} + ie_f A^{\mu} - \frac{i}{2} g \mathcal{A}^{\mu}_{j}$$

where the gluon fields $\mathcal{A}^{\mu}_{j}$ do not change the quark color,

$$\mathcal{A}^{\mu}_{j} = \begin{cases} B^{\mu}_{3} + \frac{1}{\sqrt{3}} B^{\mu}_{8}, & j = 1; \\ -B^{\mu}_{3} + \frac{1}{\sqrt{3}} B^{\mu}_{8}, & j = 2; \\ -\frac{2}{\sqrt{3}} B^{\mu}_{8}, & j = 3. \end{cases}$$

So, we have the first Abelian condition $\lambda^{a}_{jj'} \to \lambda^{a}_{jj}$

(the nondiagonal components of the Gell-Mann matrix, $j' \neq j$, are taking into account in the Hamiltonian $\mathcal{H}_{int}$)

The second Abelian condition (the gluon sector)

Requirement of absence of the color transmutation kills the non-Abelian structure of the gluon sector: $\mathcal{L}_{YM} = -\frac{1}{4}\sum G_a^{\mu\nu} G_{\mu\nu}^a,$

where $G_a^{\mu\nu} = B_a^{\mu,\nu} - B_a^{\nu,\mu} + g\sum f_{abc} B_b^\mu B_c^\nu,$

and where f_{abc} are the structure constants of the $SU(3)$ group.

The linear polarization field $B_a^\mu = (0,0,0, B_a^3 = B_a)$ is the simplest model which leads to the Abelian gluon fields. Analogous limitations is introduced for electromagnetic field $A^\mu = (0,0,0, A^3 = A)$.

The model limitations:

1) it is suggested that strong quasiclassical electromagnetic and gluon fields are linear polarized;
2) additionally it is suggested, that the system (and fields) are specially homogeneous, i.e. $A = A(t), B = B(t)$. (Fundamental importance !)

2.2 Kinetic equations

In the framework of these model restrictions it can write the kinetic equation (KE) of system for description of vacuum creation of the quark and electric components of QGP (quarks, electrons, positrons etc.).

Such kind KE's were obtained in the works (QED and cosmology):

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Basic KE for D=3+1 QED

$$\dot{f}(\vec{p},t) = \frac{1}{2}\lambda^{\pm}(\vec{p},t)\int_{t_0}^{t} dt'\lambda^{\pm}(\vec{p},t')\left[1 \pm 2f(\vec{p},t')\right]\cos\theta(t,t'),$$

where

$$\theta(t,t') = 2\int_{t'}^{t} d\tau\varepsilon(\vec{p},t),$$

$$\varepsilon(\vec{p},t) = \sqrt{\varepsilon_{\perp}^{2}(\vec{p}) + P^{2}},$$

$$\lambda^{+}(\vec{p},t) = eE(t)P/\varepsilon^{2}(\vec{p},t),$$

$$\varepsilon_{\perp} = \sqrt{m^{2} + p_{\perp}^{2}},$$

$$\lambda^{-}(\vec{p},t) = eE(t)\varepsilon_{\perp}/\varepsilon^{2}(\vec{p},t),$$

$$P = p_{\parallel} - eA(t).$$

Equivalent ODE's system

$$\dot{f} = \frac{1}{2}\lambda^{\pm}u, \quad \dot{u} = \lambda^{\pm}(1 \pm 2f) - 2\varepsilon v, \quad \dot{v} = 2\varepsilon u.$$

Quantum field interpretation:

Foundation: transition in the quasiparticle representation, where the Hamiltonian is diagonal and the states with positive and negative energies are separated.

The creation and annihilation operations of the electrons and positrons are time dependent and are defined under the *t* – dependent vacuum. So it is a theory with unstable vacuum. Then the general distribution function is

$$f(\vec{p},t) = <\mathrm{in}\,|\,a^{+}(\vec{p},t)a(\vec{p},t)\,|\,\mathrm{in}> = <\mathrm{in}\,|\,b^{+}(-\vec{p},t)b(-\vec{p},t)\,|\,\mathrm{in}>$$

under the electroneutrality condition and functions of vacuum polarization

$$u = 2\Re\mathrm{e}\, f^{(+)} = 2\Re\mathrm{e}\, f^{(-)}, \quad v = 2\Im\mathrm{m}\, f^{(+)} = -2\Im\mathrm{m}\, f^{(-)}$$

where

$$f^{(+)}(\vec{p},t) = <\mathrm{in}\,|\,a^{+}(\vec{p},t)b^{+}(-\vec{p},t)\,|\,\mathrm{in}>,$$

$$f^{(-)}(\vec{p},t) = <\mathrm{in}\,|\,b^{-}(-\vec{p},t)a^{-}(-\vec{p},t)\,|\,\mathrm{in}>$$

are the anomalous averages.

2.3 Back reaction problem

So, we have the following picture: a strong external field creates from vacuum some particle-antiparticle plasma and the corresponding current, which in turn generates some inner field. So, evolution of the particle-antiparticle plasma and inner field is self-consistent.

In the framework of the considered model of special homogeneous linear polarized field the corresponding equation of the Maxwell type has the following form

$$\dot{E}_{in} = -2e\int \frac{d^3p}{(2\pi)^3}\frac{p_{\parallel}}{\varepsilon}\left[f + \frac{u}{2}\frac{\varepsilon}{p_{\parallel}} - e\dot{E}\,\frac{\varepsilon_{\perp}^2}{8\varepsilon^4 p_{\parallel}}\right],$$

where $E(t) = E_{ext}(t) + E_{in}(t)$.

The system of KE's and the equation of the Maxwell type form the total closed equation system for the description of evolution of the particle-antiparticle plasma.

3. Adaptation to QCD

3.1 The quark Abelian sector

The general limitations: spatial homogeneous, linear polarization of the color and electric fields.

Specific:

- presence of some (N) flavors with mass m_f and charges e_f (electric) and g (color);
- the Abelian restriction leads to selection of specific combination of the electric and gluon fields in the covariant derivative ($\vec{D} \longrightarrow \vec{\mathfrak{D}}$) and to specific definition of quasimomentum $\vec{P}_{fj} = \left(p^1, p^2, P^3 = p^3 - e_f A(t) + \frac{1}{2} g \mathcal{A}_j(t)\right)$

This selection reflects on structure of the quasienergy

$$\varepsilon_{fj}(\vec{p}, t) = \sqrt{\varepsilon_{\perp f}^2 + P_{3fj}^2}, \qquad \varepsilon_{\perp f}^2 = m_f^2 + p_\perp^2$$

The resulting KE's system in the quark sector is

$$\begin{cases} \dot{f}_{fj}(\vec{p},t) = \frac{1}{2}\lambda_{fj}(\vec{p},t)u_{fj}(\vec{p},t), \\ \dot{u}_{fj}(\vec{p},t) = \lambda_{fj}(\vec{p},t)\left[1 - 2f_{fj}(\vec{p},t)\right] - 2\varepsilon_{fj}(\vec{p},t)v_{fj}(\vec{p},t), \\ \dot{v}_{fj}(\vec{p},t) = 2\varepsilon_{fj}(\vec{p},t)u_{fj}(\vec{p},t) \end{cases}$$

at condition $f_{fj} = \tilde{f}_{fj}$.

Here $\lambda_{fj}(\vec{p},t) = \frac{1}{\varepsilon^2_{fj}(\vec{p},t)}\dot{P}_{3fj} = \frac{1}{\varepsilon^2_{fj}(\vec{p},t)}\left[-e_f\dot{A}(t) + \frac{1}{2}g\dot{\mathcal{A}}_j\,(t)\right]$.

Altogether 27 equations.

3.2 Electromagnetic sector

Presence of strong electric field leads to generation from vacuum of the electron-positron plasma (EPP) and maybe more massive electrodynamical components.

It leads to the following KE's

$$\dot{f} = \frac{1}{2}\lambda u, \qquad \dot{u} = \lambda(1-2f) - 2\varepsilon v, \qquad \dot{v} = 2\varepsilon u,$$

where now $\lambda = \frac{eE(t)\varepsilon_{\perp}}{\varepsilon^2}$, $E = -\dot{A}$, $\varepsilon = \sqrt{\varepsilon_{\perp}^2 + (p_3 - eA(t))^2}$

by the condition $f = \tilde{f}$.

Altogether 3 equations.

3.3 The Maxwell type equations

According to the Abelian limitations, it is necessary to write the Maxwell type equations only for the two components of gluon fields

$B_3 = B_3^{\mu=3}$ and $B_8 = B_8^{\mu=3}$, corresponding to the linear polarization ($j = 3, 8$),

$$\dot{E}_j = -2g \sum_f \int \frac{d^3p}{(2\pi)^3} \frac{P_{fj}^{(3)}}{\varepsilon_{fj}} \left[f_{fj} + \frac{u_{fj}\varepsilon_{f\perp}}{2P_{fj}^{(3)}} - g\dot{E}_j \frac{\varepsilon_{f\perp}^2}{8\varepsilon_{fj}^4 P_{fj}^{(3)}} \right] = -4\pi \sum_f j_{fj} \,.$$

Analogous equation defines the electric field

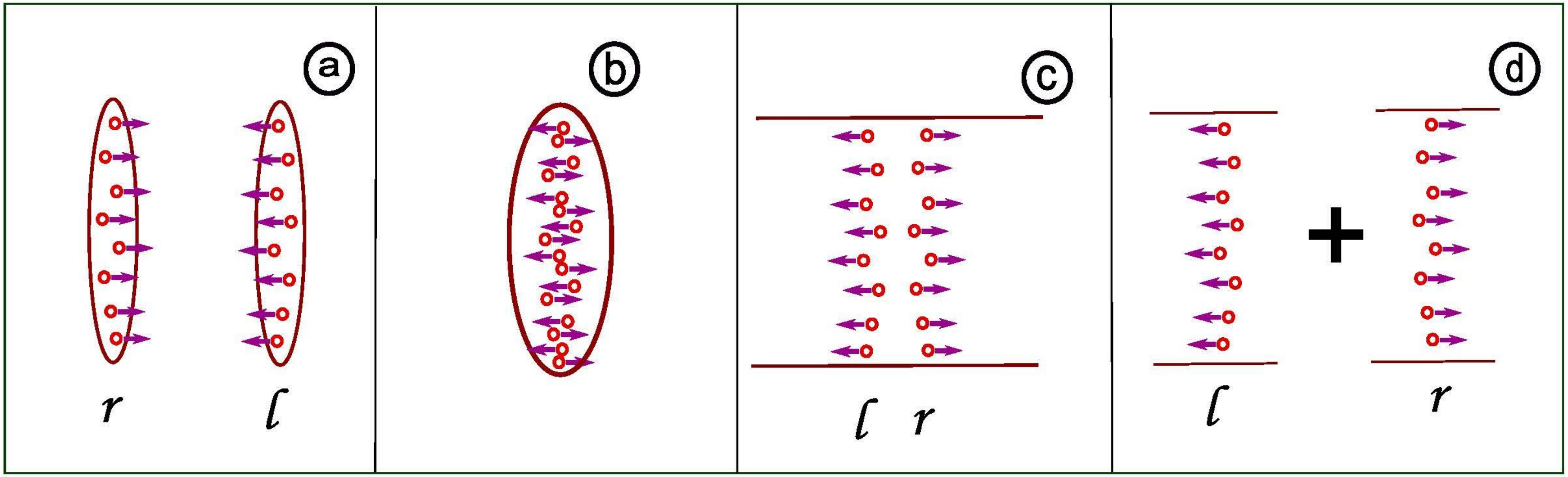
$$\dot{E} = -2e \int \frac{d^3p}{(2\pi)^3} \frac{P_3}{\varepsilon} \left[f + \frac{u}{2} \frac{\varepsilon}{P_3} - e\dot{E} \frac{\varepsilon_\perp^2}{8\varepsilon^4 P_3} \right] - \frac{4\pi}{g} \sum_{f,j} e_f j_{fj} \,.$$

Altogether 3 equations.

3.4 Initial conditions

This total closed system of the 33 ODE’s needs in the initial conditions, which are selected in order to correspond as much as possible to the conditions of ultrarelativistic heavy ion collisions and which would be compatible with spatial homogeneity of the model.

In our understanding, it corresponds to model of the color tube, in which the initial state of QGP is defined by parameters arising in the moment of deconfiment of the colliding nuclei.



In this point of time it is assumed, that

- all quasiclassical fields are absent, $A^{in} = B_3^{in} = B_8^{in} = 0$;
- and the quark composition of the nuclear matter is well known and characterized by the quark density n_{fj} and chemical potential μ_f;
- it is naturally to introduce the initial quark distribution function

$$f_{0f}(\vec{p}) = \left\{\exp\beta\left[\varepsilon_{0f} \pm VP_{\parallel} - \mu_f\right] + 1\right\}^{-1}$$

and the equation of state $n_f = \int \frac{d^3p}{(2\pi)^3} f_{0f}(\vec{p})$;

- it is assumed also that electrons are absent in the initial state, $n_e = 0$;
- all quarks are moving in one direction with velocity V in laboratory reference system.

So, it can give the initial color and electric currents:

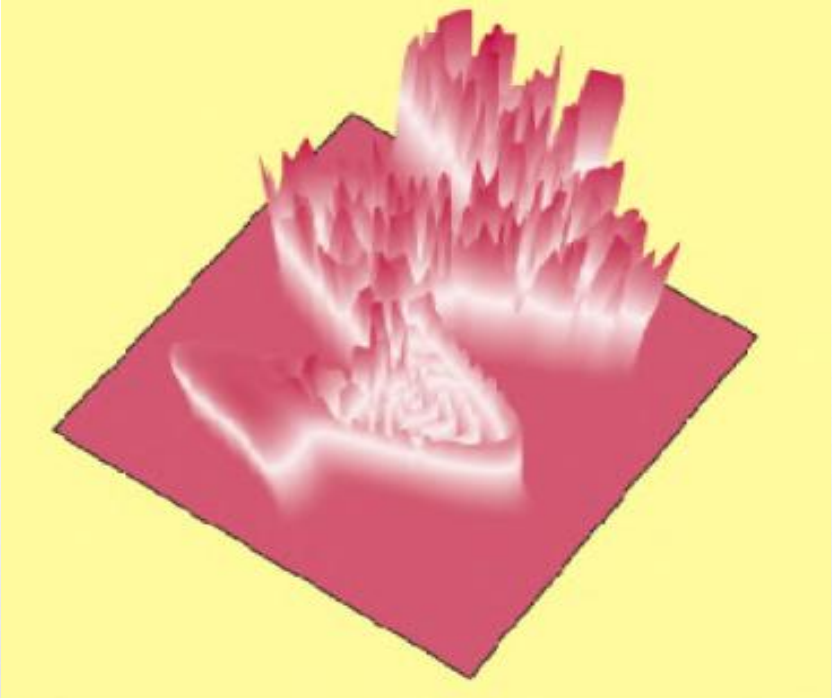
$$j_j^{in} = gV \sum_f n_{fj}, \qquad j^{in} = V \sum_{f,j} e_j n_{fj}.$$

4. Conclusion

We have formulated the statement of the problem about description of the vacuum creation and evolution of QGP in strong gluon and electric fields on the foundation of the correct nonperturbative QCD approach in the framework of some strong limitations (spatial homogeneous, linear polarization of the gluon and electric fields, the Abelian sector of QCD). The obtained ODE system is accessible for the numerical investigation. Undoubtedly, this model is rather rough. But it is based on the system of accurate assumptions, that allows to make better the model by means of elimination of one or other restrictions.

For example, the next step is the most perspective: transition to the case of arbitrary polarization of the gluon and electric fields. The corresponding generalization in QED is known (Pervushin, Skokov; Prozorkevich, Smolyansky). This step will bring at once to the non-Abelian generalization of the considered model.

At last, the model opens a row new mathematical and physical problems, which are important for the strong field physics (e.g., for the problem of observation of the electron-positron plasma in super strong laser fields).



THANK YOU VERY MUCH FOR ATTENTION !