Modeling heavy ion collisions
with different time scales

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Dubna, July 13, 2017

Outline

- **Thermostatic properties of Hagedorn mass spectrum**
- **Source of negative eigen surface tension in QCD**
- **Finite width model (FWM) of QG bags**
- **Conclusions**

Hagedorn Mass Spectrum

GBM contains the Hagedorn mass (volume) spectrum of bags

PARADOXICAL SITUATION WITH THE HAGEDORN MASS SPECTRUM:

$$\rho(m)\Big|_{m \gg T_H} \sim \exp\left[\frac{m}{T_H}\right]$$

It was predicted for $m \gg 1$ GeV by Hagedorn in 1965

It follows from the statistical bootstrap model (Frautschi, 1971);

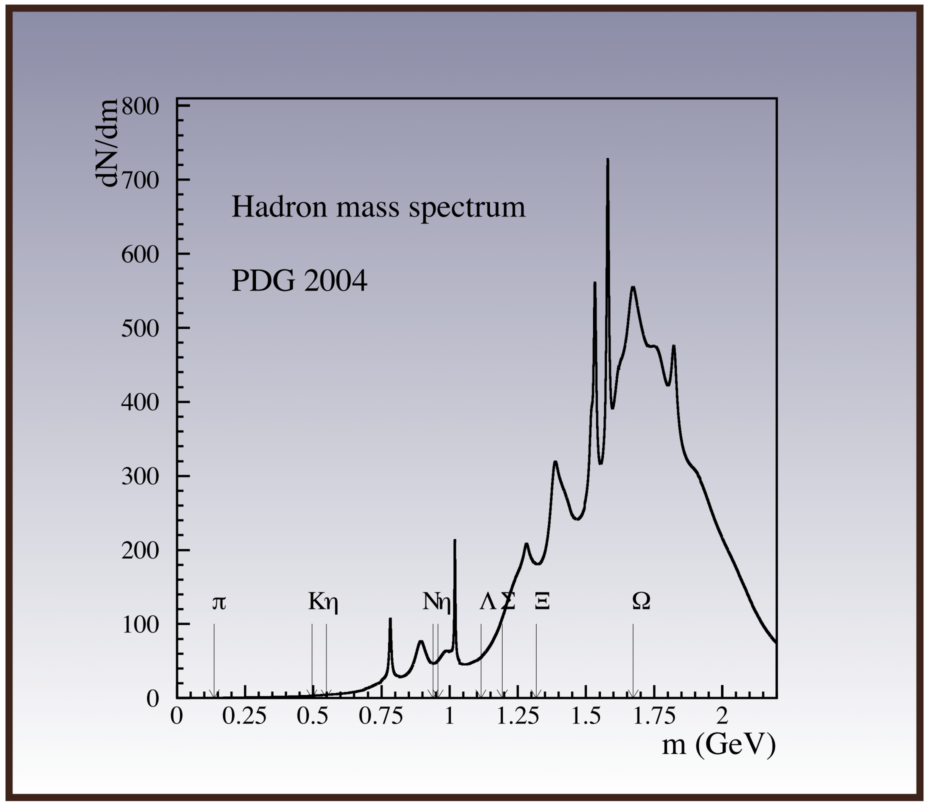
from Veneziano model (1970), from Bag Model (Kapusta, 1981);

from large N_c limit of 3+1 QCD (Cohen, 2009)

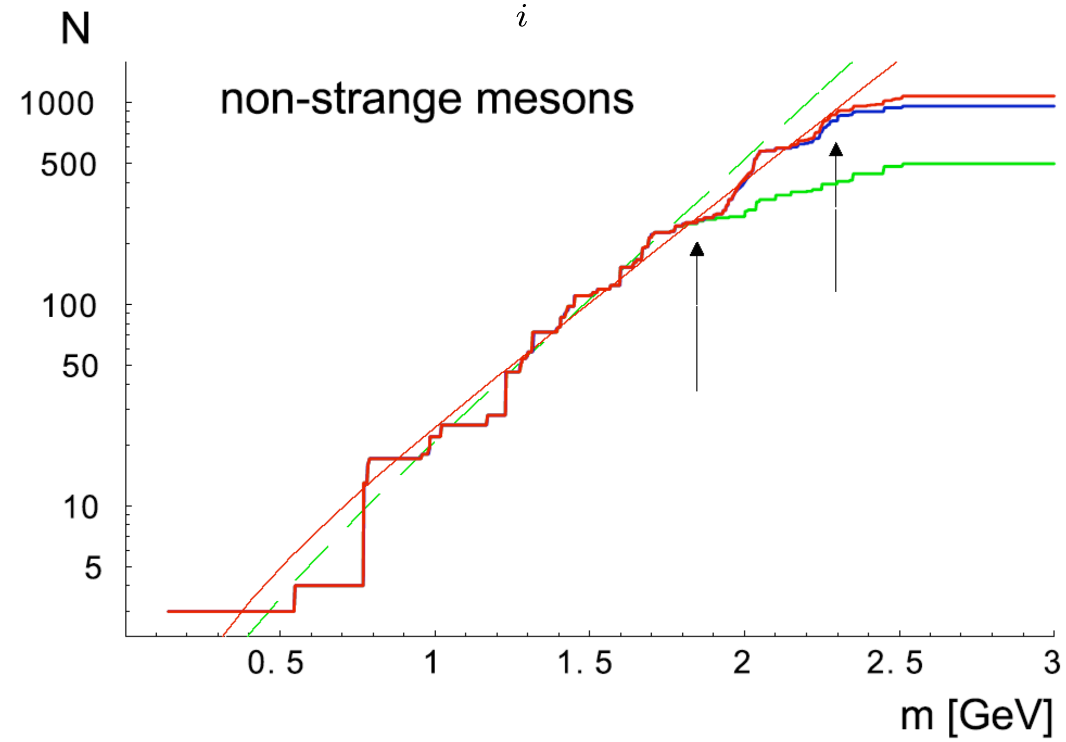
Also the Hagedorn mass spectrum is observed experimentally,

BUT

Second Conceptual Problem



$$N_{\rm exp}(m) = \sum_i g_i \Theta(m - m_i),$$



It is observed for 1.3 **GeV** $< m < 2.5$ **GeV only,**

i.e. NOT WHERE IT WAS PREDICTED!

$\Rightarrow$ **There is a huge deficit of heavy hadrons predicted by stat. bootstrap model!**

It is believed that heavy resonances are not observed due to their large width.

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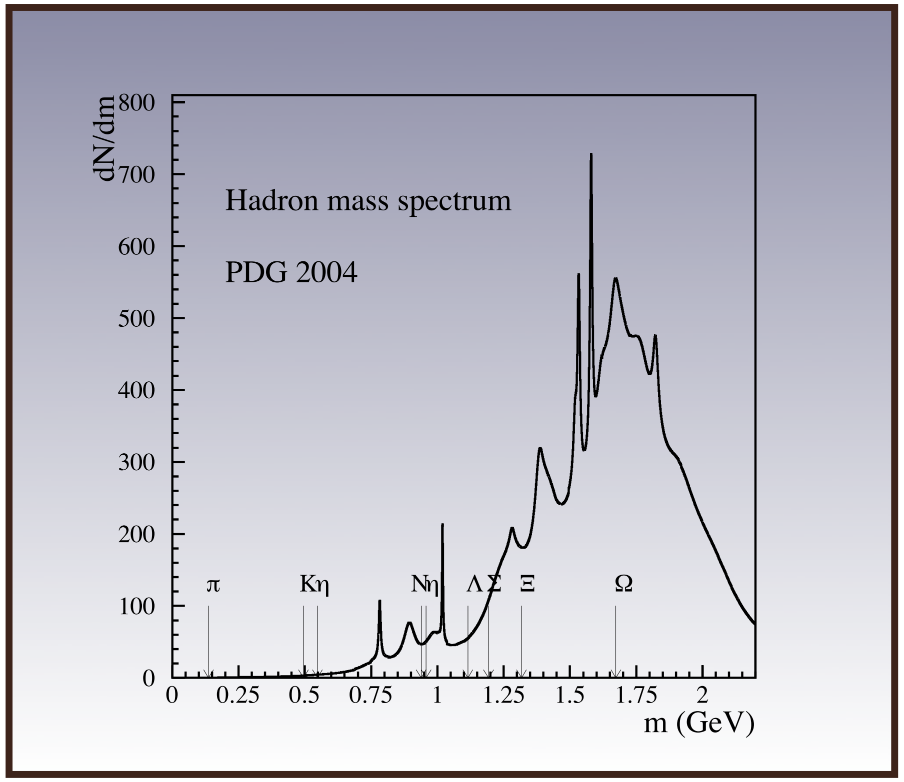

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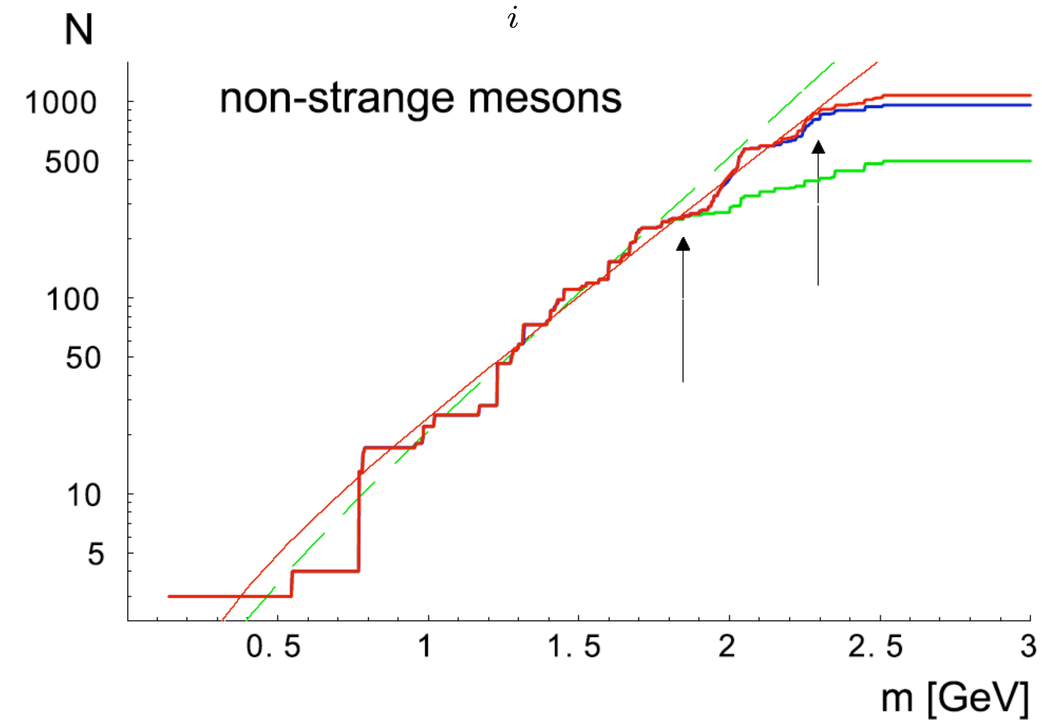

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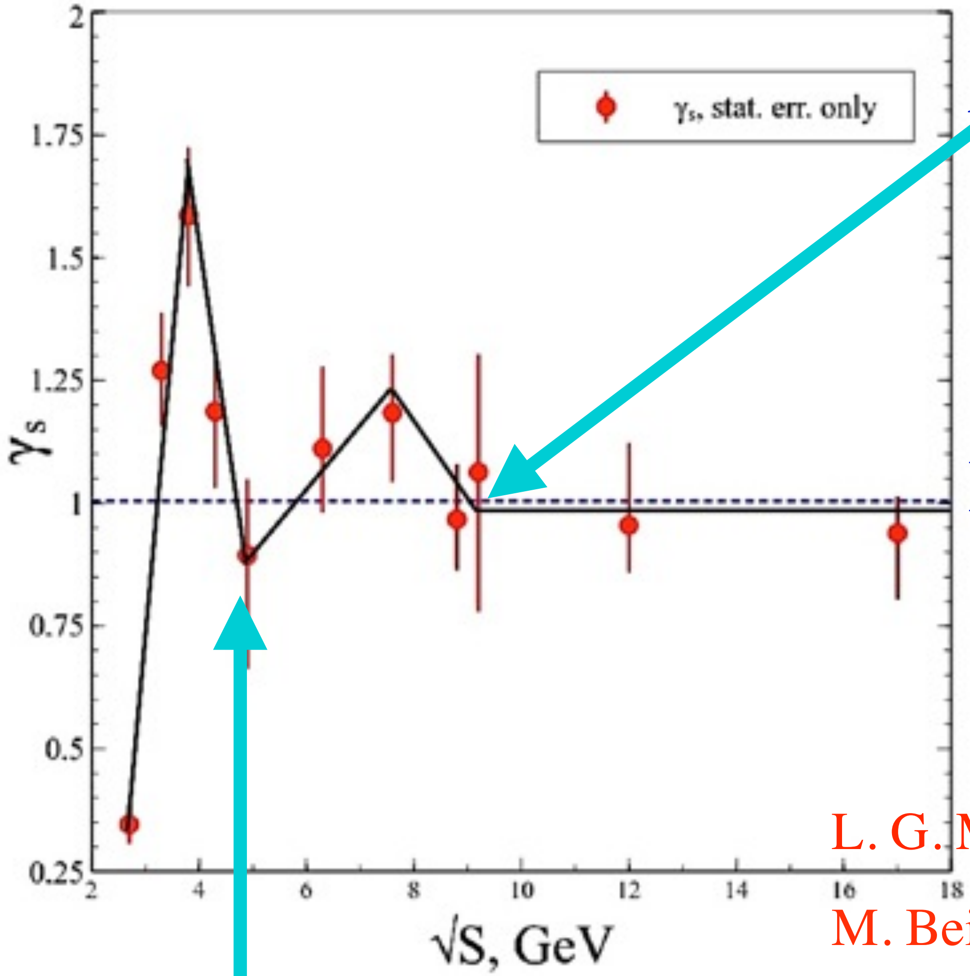
$\Rightarrow$ **There is a huge deficit of heavy hadrons predicted by stat. bootstrap model!**

It is believed that heavy resonances are not observed due to their large width.

However, the full Hagedorn mass spectrum is used in ALL realistic statistical models like Gas of Bags Model (GBM) and NO width is accounted for!

For width of QGP bags see D.Blaschke & K.A.B. in 2003-2005

Strangeness Irregularities



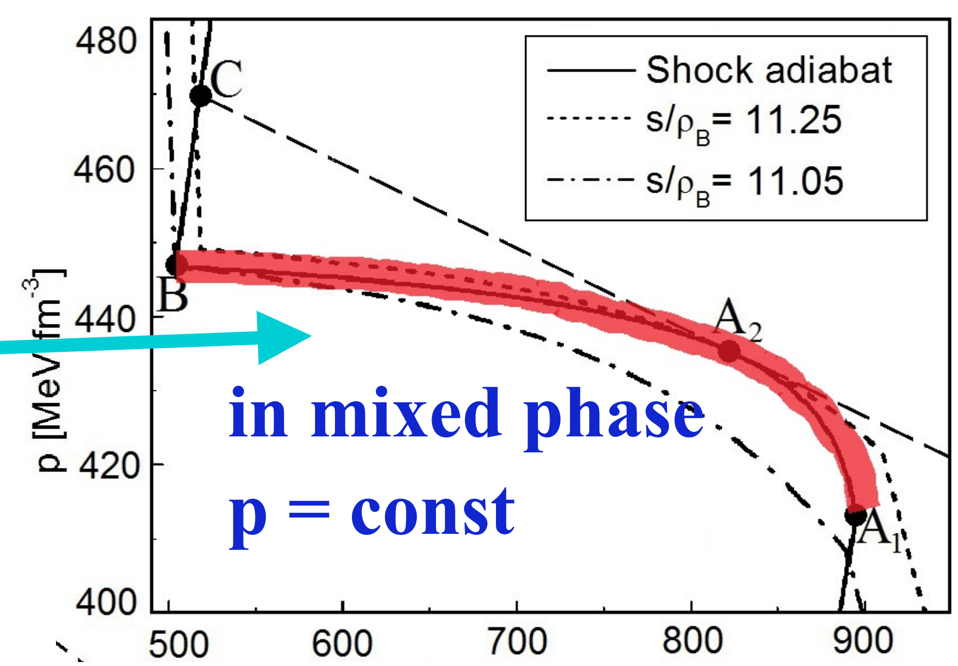
At c.m. energies above 8.8 GeV the strange hadrons are in chemical equilibrium due to formation of QG bags with Hagedorn mass spectrum!

Hagedorn mass spectrum is a perfect thermostat and a perfect particle reservoir! => Hadrons born from such bags will be in a full equilibrium!

L. G. Moretto, K. A. B., J. B. Elliott and L. Phair, Europhys. Lett. 76, 402 (2006)

M. Beitel, K. Gallmeister and C. Greiner, Phys. Rev. C 90, 045203 (2014)

At c.m. energy GeV strange particles are in chemical equilibrium due to formation of mixed phase, since under CONSTANT PRESSURE condition the mixed phase of 1-st order PT is explicit thermostat and explicit particle reservoir!

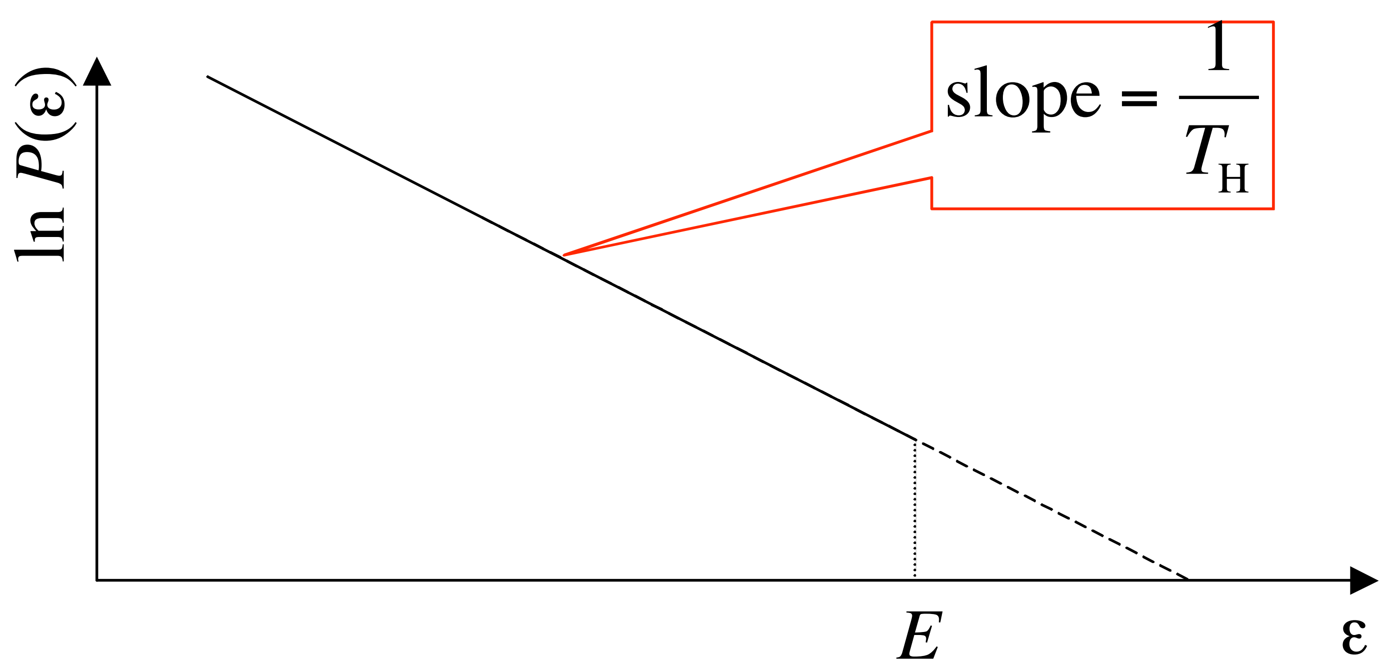


Microcanonical Ensemble
Example #1: 1-d Harmonic Oscillator

- For 1-d Harmonic Oscillator with energy ε in contact with Hagedorn resonance (just exponential spectrum for simplicity). Total energy is E. K.A.B.et al, Europhys. Lett. 76 (2006) 402
- The microcanonical probability of state & is:

$$P(\varepsilon) = \rho(E-\varepsilon) = \exp\left(\frac{E-\varepsilon}{T_{\mathrm{H}}}\right) = \exp\left(\frac{E}{T_{\mathrm{H}}}\right)\exp\left(-\frac{\varepsilon}{T_{\mathrm{H}}}\right)$$

Exponent is Grand canonical! With fixed T!



Average value of & is

$$\bar{\varepsilon} = T_{\mathrm{H}}\left(1 - \frac{E/T_{\mathrm{H}}}{\exp(E/T_{\mathrm{H}}) - 1}\right)$$

For $E \to \infty$ **:** $\bar{\varepsilon} \to T_H$

Example #2: An Ideal Vapor coupled to Hagedorn resonance

- Consider microcanonical partition of N particles of mass m and kin. energy ε. The total level density is

$$P(E,\varepsilon)=\rho_{\mathrm{H}}(E-\varepsilon)\rho_{\mathrm{iv}}(\varepsilon)=\boxed{\frac{V^{N}}{N!\left(\frac{3}{2}N\right)!}\left(\frac{\mathrm{m}\varepsilon}{2\pi}\right)^{\frac{3}{2}N}}\boxed{\exp\left(\frac{E-\mathrm{m}N-\varepsilon}{\mathrm{T_H}}\right)}$$

Exponent is Grand canonical! With fixed T!

The most probable energy partition is

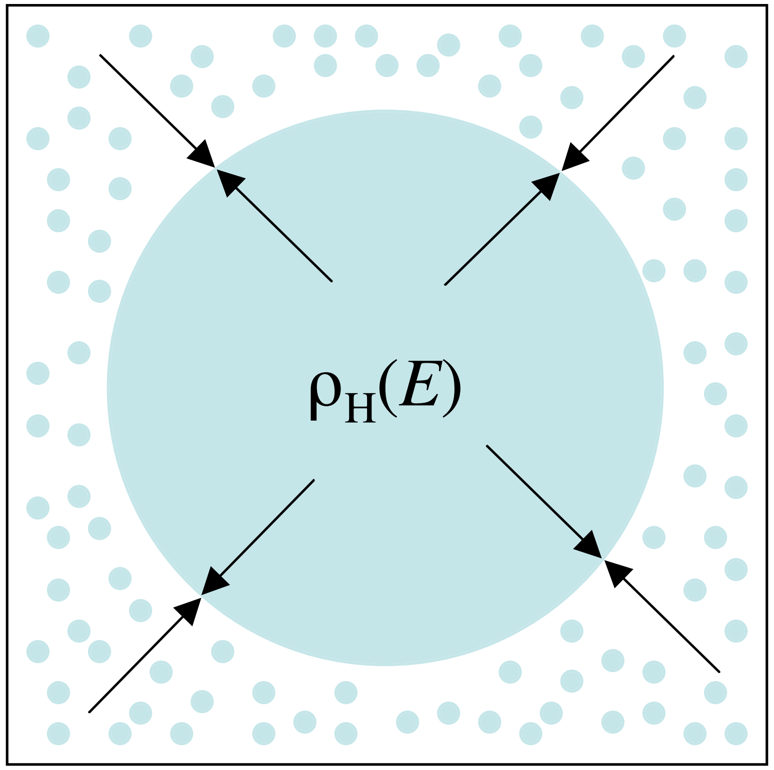
$$\frac{\partial \ln P}{\partial \varepsilon}=\frac{3N}{2\varepsilon}-\frac{1}{T_{\mathrm{H}}}=0 \Rightarrow \boxed{\frac{\varepsilon}{N}=\frac{3}{2}T_{\mathrm{H}}}$$

- T_H is the sole temperature characterizing the system:
- **A Hagedorn-like system is a perfect thermostat!**
  - **as long as there is enough energy to provide such a solution!**

Example #3: An Ideal Particle Reservoir

L.G. Moretto, K.A.B. et al, nucl-th/0601010

- If, in addition, particles are generated by the Hagedorn resonance, their concentration is volume independent!



ideal vapor ρ_{iv}

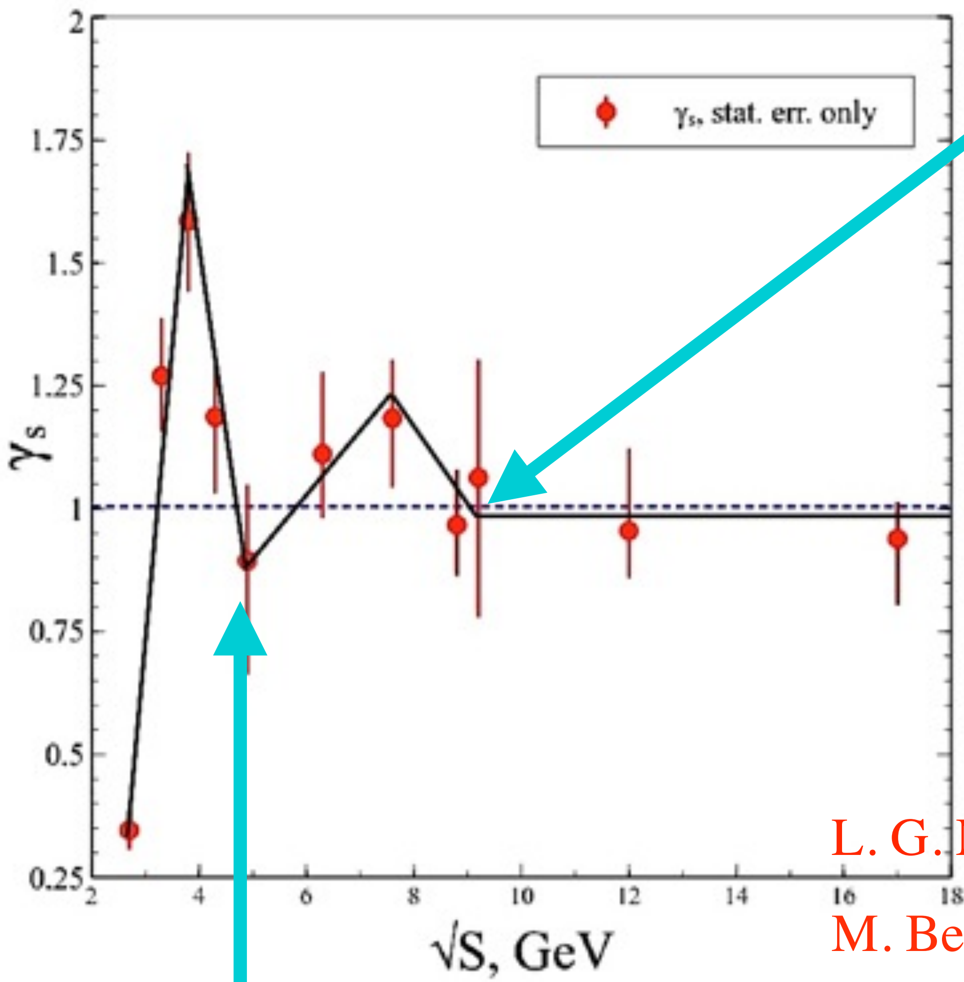
- particle mass = m
- volume = V
- particle number = N
- energy = ε

$$\left.\frac{\partial \ln P}{\partial N}\right|_V = -\frac{m}{T_H} + \ln\left[\frac{V}{N}\left(\frac{mT_H}{2\pi}\right)^{\frac{3}{2}}\right] = 0 \Rightarrow \boxed{\frac{N}{V} = \left(\frac{mT_H}{2\pi}\right)^{\frac{3}{2}} \exp\left(-\frac{m}{T_H}\right)}$$

Remarkable result because it mean saturation between gas of particles and Hagedorn thermostat!

- **as long as there is enough energy to provide such a solution!**

Strangeness Irregularities



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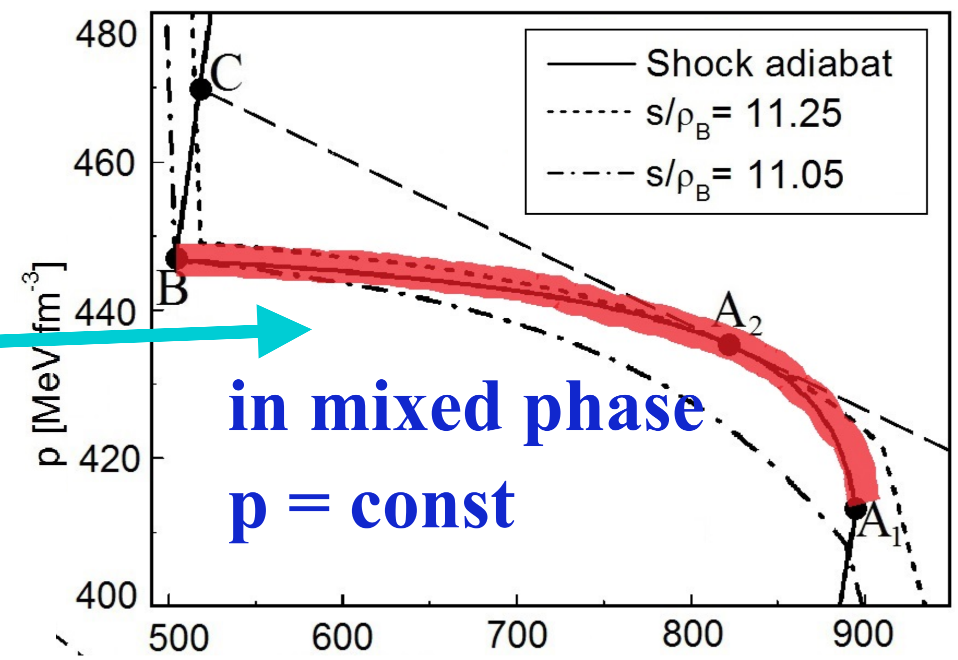
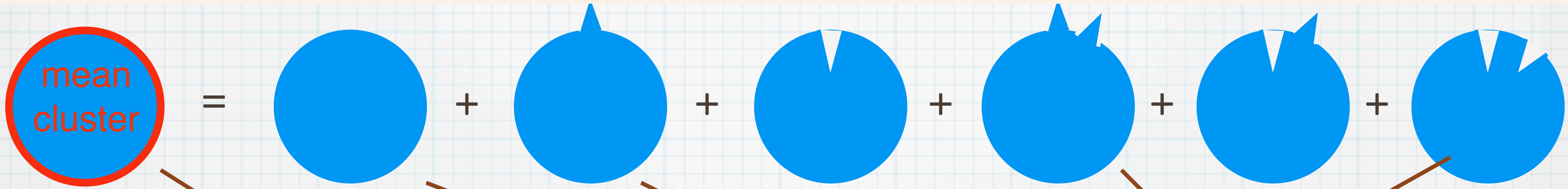


Eigen Surface Free Energy: F = E -TS

To find eigen surface F one has to count for ALL surface deformations together with energy costs

Can be exactly done within Hills and Dales Model for v-volume cluster:

K.A.B. et al, PRE 72 (2005)



$$\underbrace{\exp\left[-\frac{\sigma_o v^{2/3}}{T}\right]}_{Energy\ part}\underbrace{\exp(S)}_{Entropy\ part} = \underbrace{\exp\left[-\frac{\sigma_o v^{2/3}}{T}\right]}_{Sphere's\ Energy}\times\left\{1 + \left(\underbrace{w_H N_H}_{1\ Hill}+\underbrace{w_D N_D}_{1\ Dale}\right)\exp\left[-\frac{\sigma_o \Delta S_1}{T}\right] + 2,\ 3,\ \text{etc deformations}\right\}$$

$$= \underbrace{\exp\left[-\frac{\sigma_o v^{2/3}}{T}\right]}_{Energy\ part}\underbrace{\exp\left[+\frac{\sigma_o v^{2/3}}{T_c}\right]}_{Entropy\ part}$$

Simplest case (M. Fisher)

Also one can find supremum and infimum for surface F and surface partition

$$\sigma_0(1-\lambda_L T)\, v^{\frac{2}{3}} \quad \geq \quad F \quad \geq \quad \sigma_0(1-\lambda_U T)\, v^{\frac{2}{3}}\,, \quad \lambda_L \approx 0.28\, T_c^{-1}, \quad \lambda_U \approx 1.06\, T_c^{-1}$$

K.A.B. & Elliott, UJP 52 (2007)

Thus, there is NOTHING wrong, if surface F < 0 above critical T!
This means only that entropy dominates!

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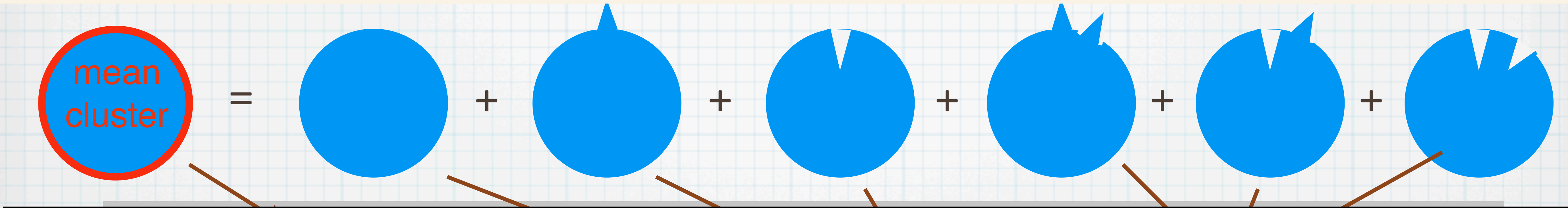

Can we find the surface tension of QG bags from lattice QCD?

$$\sigma_0(1-\lambda_L T)\, v^{\frac{2}{3}} \quad \geq \quad F \quad \geq \quad \sigma_0(1-\lambda_U T)\, v^{\frac{2}{3}}\,, \quad \lambda_L \approx 0.28\, T_c^{-1}, \quad \lambda_U \approx 1.06\, T_c^{-1}$$

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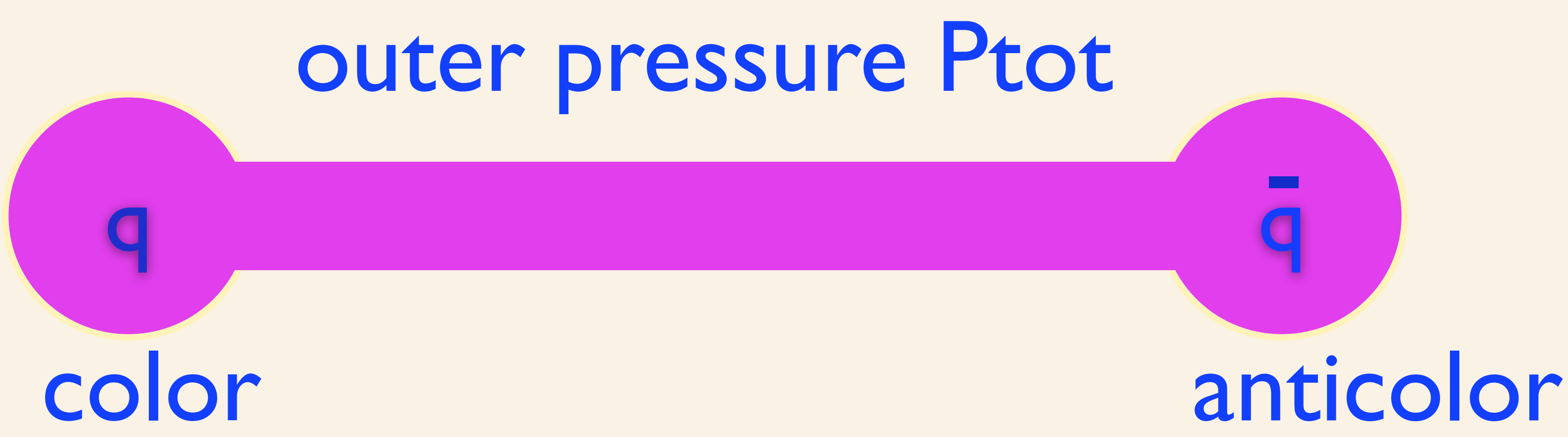
Confining String = Color Tube

Consider confining string between static q & anti q of length L and radius R<<L

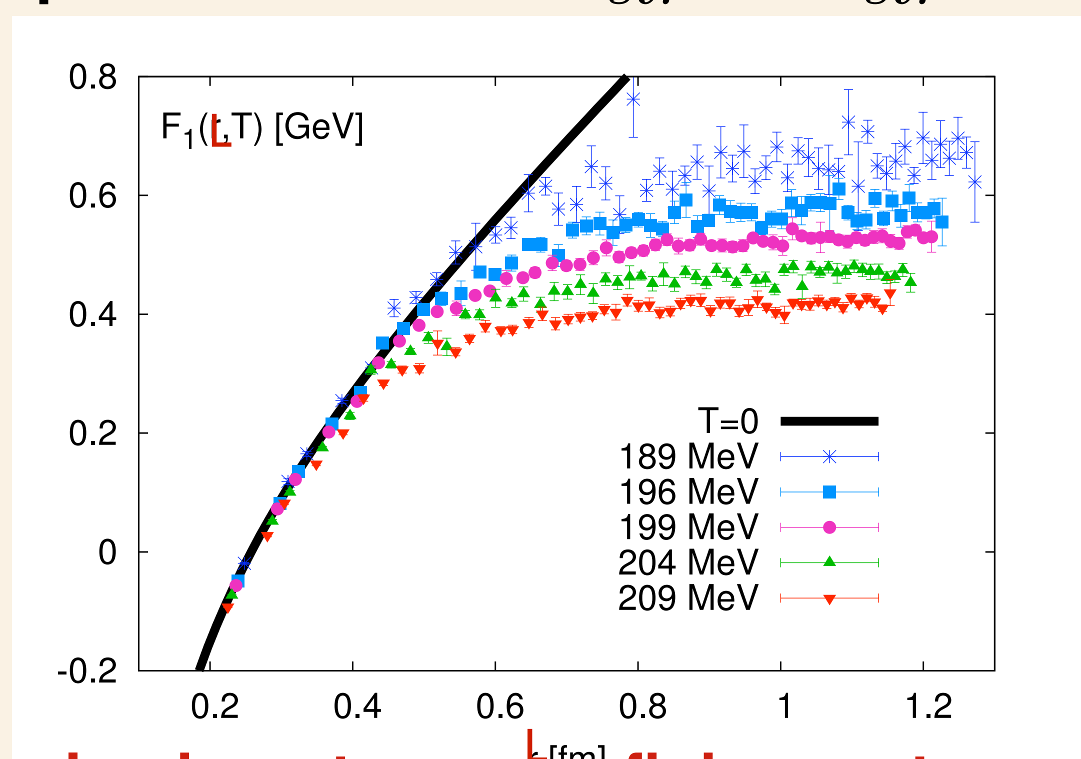


Its free energy measured from Polyakov loop correlator is $F_{str} = \sigma_{str} L$

Confinement means infinite free energy for infinite L

Deconfinement means that string tension vanishes

Can be rigorously found by Lattice QCD

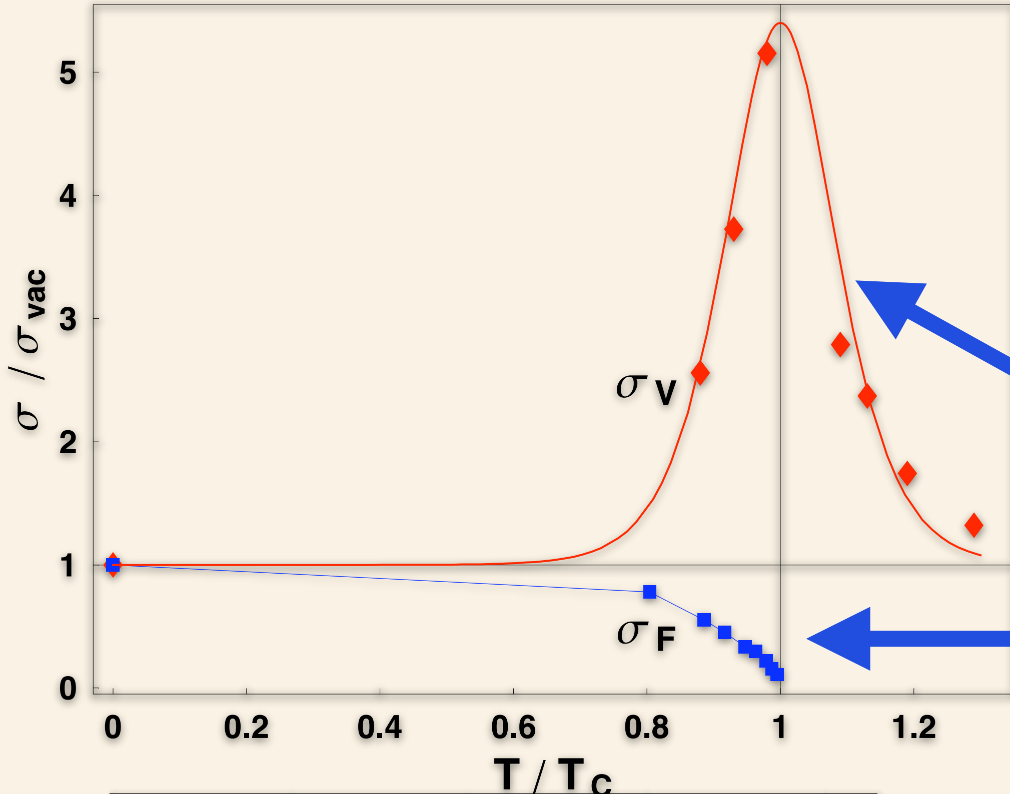


Coulomb part confining part

Confinement by Color String within sQGP

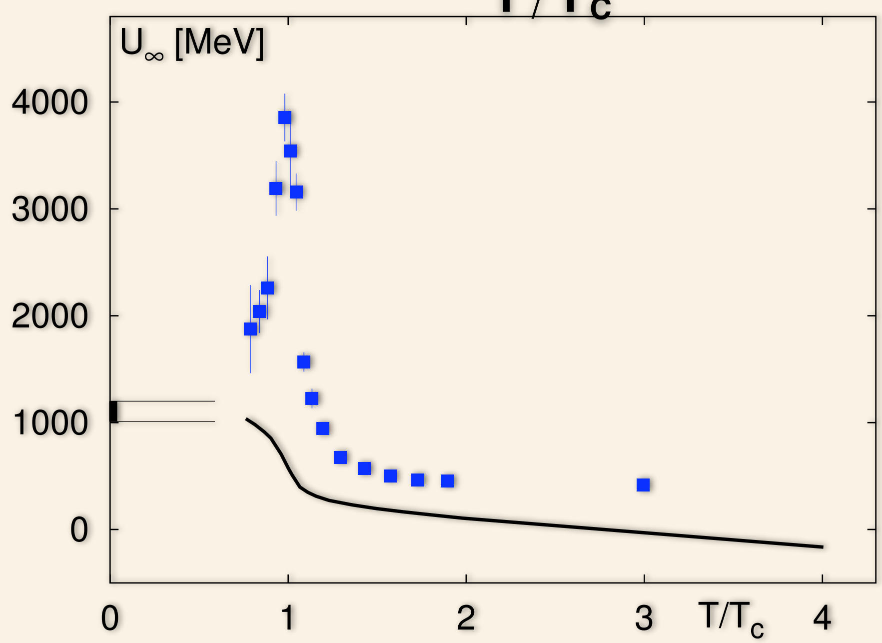
Internal energy U, entropy S

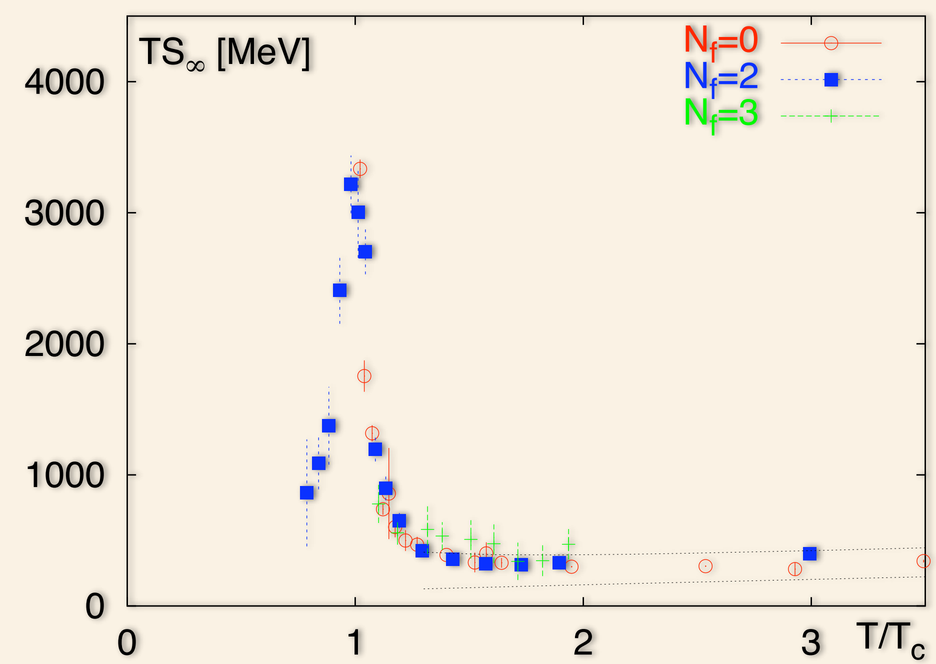
$$U(T,r) = F - TdF/dT = F + TS$$



String tension for internal energy (V)

String tension for free energy (F) —> 0





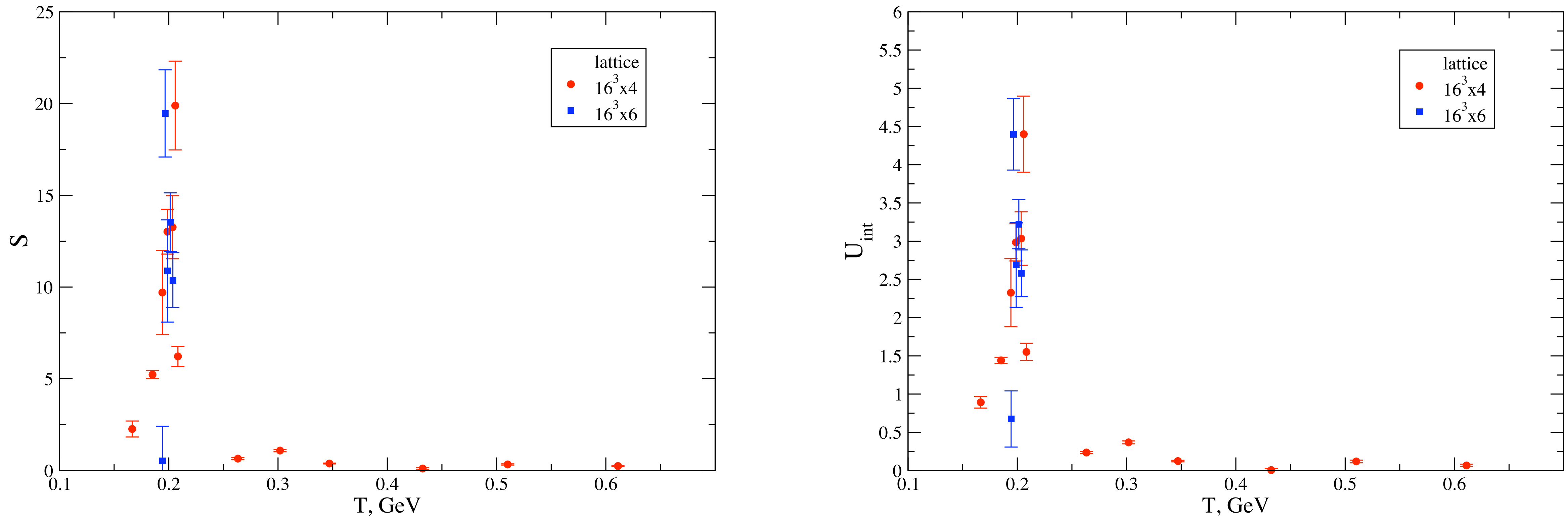
Very strong interaction! => No color charge separation!

Plots are taken from O. Kaczmarek and F. Zantow, PoS LAT2005, 192 (2006)

Mysterious Maximum

Entropy and Internal Energy

In Edward Shuryak lectures **Prog. Part. Nucl. Phys. 62:48-101 (2009)**



HUGE maximum in color tube entropy S was called Mysterious

because it was unclear what are the dof with #dof = exp (S=20) = 485 000 000

String Tension vs Surface Tension

K.A.B., G.M. Zinovjev, Nucl. Phys. A848 (2010)

Consider now this tube as the cylindrical bag of length L and radius R<<L

Neglect effects of color sources and get cylinder FREE ENERGY as:

$$F_{cyl}(T,L,R) \equiv -\underbrace{p_v(T)\pi R^2 L}_{thermal} + \underbrace{\sigma_{surf}(T) 2\pi R L}_{surface} + \underbrace{T\tau \ln \frac{V}{V_0}}_{small}$$

Equating the cylinder FREE ENERGY to string free energy $F_{str} = \sigma_{str} L$

$$\boxed{\sigma_{str}(T) = \sigma_{surf}(T)\, 2\pi R \; - \; p_v(T)\pi R^2} \cancel{+ \frac{T\tau}{L} \ln \left[\frac{\pi R^2 L}{V_0}\right]}$$

We got a new possibility to determine QGP bag surface tension directly from LQCD!

From bag model pressure $p_v(T=0) = -(0.25)^4$ GeV^4, $R = 0.5$ fm and $\sigma_{str}(T=0) = (0.42)^2$ GeV^2 $\Rightarrow$

$$\sigma_{surf}(T=0) = (0.2229 \text{ GeV})^3 + 0.5\, p_v\, R \approx \boxed{(0.183 \text{ GeV})^3} \approx 157.4 \text{ MeV fm}^{-2}.$$

Surface Tension at Cross-over

For vanishing σ_{str} one has $\quad \sigma_{str}^{LQCD} \approx \frac{\ln(L/L_0)}{R^2} C$

This is due to increase of surface fluctuations $\quad \Rightarrow \quad$ in general

$\sigma_{str}(T)\, R^k \longrightarrow \omega_k > 0 \quad$ for $\quad k > 0$

Parametrize $\sigma_{str} = \sigma_{str}^0\, t^\nu$, $\quad$ where $\quad t \equiv \frac{T_{tr}(\mu) - T}{T_{tr}(\mu)} \longrightarrow +0$

and find total pressure and total entropy density

for $\mu = 0$ (baryonic chemical potential)

$$p_{tot} = p_v(T) - \frac{\sigma_{surf}(T)}{R} \equiv \frac{\sigma_{surf}(T)}{R} - \frac{\sigma_{str}}{\pi R^2} \longrightarrow \left[\frac{\sigma_{str}}{\omega_k}\right]^{\frac{1}{k}} \left[\sigma_{surf} - \frac{\omega_k}{\pi}\left[\frac{\sigma_{str}}{\omega_k}\right]^{\frac{k+1}{k}}\right]$$

$$s_{tot} = \left(\frac{\partial\, p_{tot}}{\partial\, T}\right)_\mu \longrightarrow \underbrace{\frac{1}{k\,\sigma_{str}}\left[\frac{\sigma_{str}}{\omega_k}\right]^{\frac{1}{k}} \frac{\partial\, \sigma_{str}}{\partial\, T}\, \sigma_{surf}}_{dominant \quad since \quad \sigma_{str} \rightarrow\ 0} + \left[\frac{\sigma_{str}}{\omega_k}\right]^{\frac{1}{k}} \frac{\partial\, \sigma_{surf}}{\partial\, T} - \frac{k+2}{\pi\, k}\left[\frac{\sigma_{str}}{\omega_k}\right]^{\frac{2}{k}} \frac{\partial\, \sigma_{str}}{\partial\, T}$$

For finite σ_{surf} and $\frac{\partial\, \sigma_{str}}{\partial\, T} < 0$ $\quad \Rightarrow \quad \sigma_{surf} < 0 \quad$ since $\quad s_{tot} > 0$

Comparison with LQCD

$\Rightarrow$ Assume: we can apply our results to LQCD data with $L \gg R$

For $\sigma_{str} \to 0 \quad \Rightarrow \quad R \to \frac{2\,\sigma_{surf}}{p_v}$ and lattice entropy is

$$\frac{S_{lat}}{L} = -\frac{1}{L}\,\frac{\partial F_{lat}}{\partial T} \;\to\; -\;\frac{s_{tot}\,k\,\sigma_{str} R}{\sigma_{surf}} \;=\; -\;\frac{s_{tot}\,k\,\omega_k}{\sigma_{surf} R^{k-1}} \;\to\; t^{\nu-1}$$

***t* is reduced temperature**

$$t \equiv \frac{T_{tr}(\mu)-T}{T_{tr}(\mu)} \to +0$$

$\Rightarrow$ again $\sigma_{surf} < 0$

$\Rightarrow$ S_{lat} diverges for $\nu < 1$ and $R \to \infty$

$\Rightarrow$ S_{lat} has a sharp inclease for $\nu < 1$ and $R \to R_{lat} < \infty$

Physics: for negative surface tension coefficient there must appear the FRACTAL ripples on the surface of color tube.

This explains a huge # of dof

The ripples must disappear when tube occupies the whole volume and there is no free surface!

This explains why the Mysterious Maximum vanishes

So far everything looks fine

Recall that in Ginzburg-Landau theory of the type II superconductors the surface tension is negative

2.8 Superconductors of the Second Kind

Results of the previous section explain the existence of two kinds of superconductors: superconductors of the first and second kinds or type I and type II. Superconductors of the first kind have a positive surface energy σ_{sn}. Superconductors with $\sigma_{sn} < 0$ are superconductors of the second kind. Therefore, near T_c superconductors with $\kappa < 1/\sqrt{2}$ are of the first kind and those with $\kappa > 1/\sqrt{2}$ are of the second kind.

The main difference between these two types of superconductors is in the character of the phase transition. A sharp boundary between two phases is possible only if the surface tension of the interface is positive. Thus, our discussion of the phase transition in previous sections relates only to type I superconductors.

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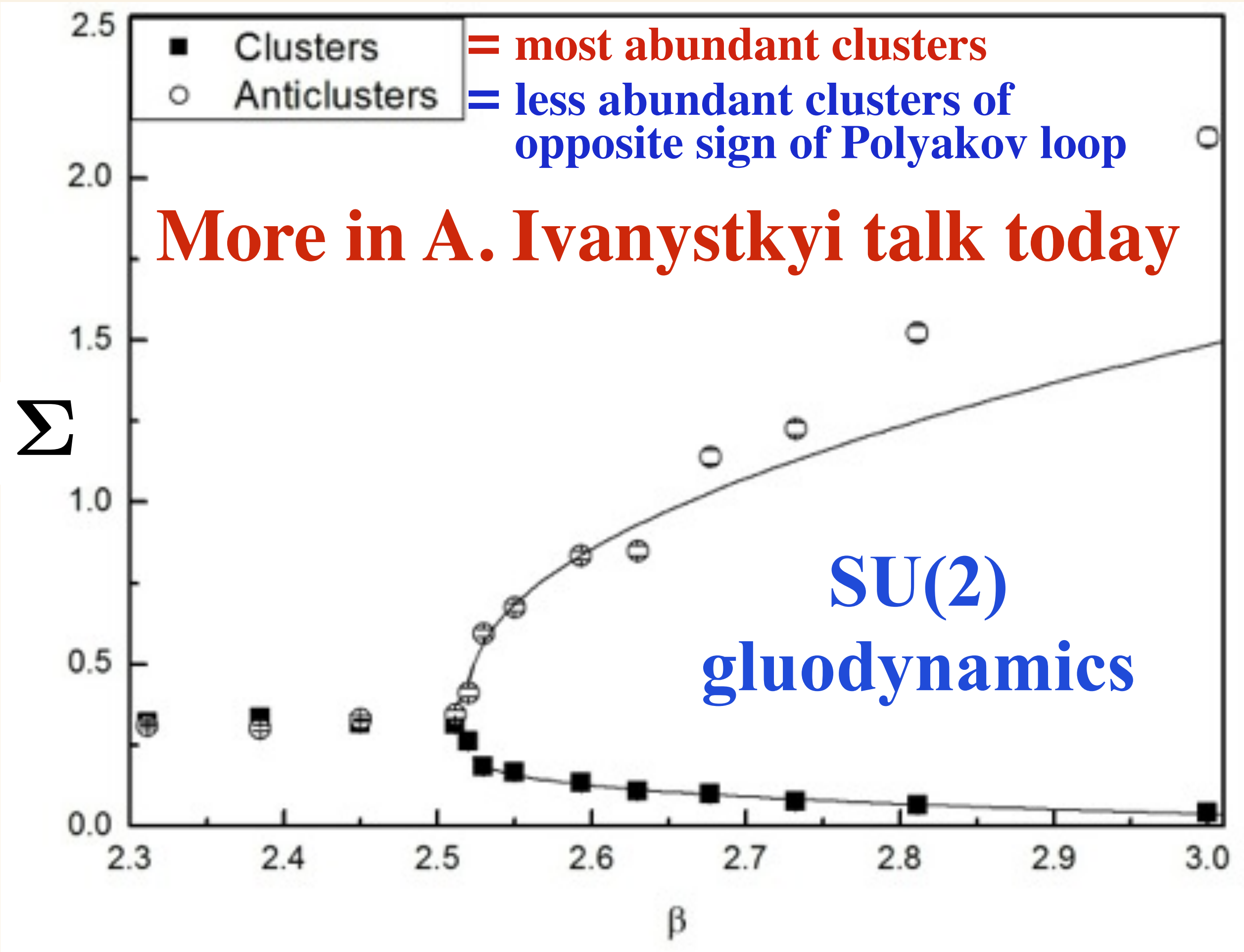
However, the surface tension of geometrical clusters formed by Polyakov loops in SU(2) gluodynamics is positive or 0!

see A.I. Ivanytskyi et al., Nucl. Phys. A 960 (2017) 90



Did We Miss Something ?

The Hills and Dales Model of mean cluster and Cylindrical Bag Model are dealing with Eigen Surface Tension of a separated mean cluster!

However, in a medium the clusters should an additional Surface Tension Induced by the hard-core repulsion between them!

Recall A. Ivanytskyi talk on EoS beyond Van der Waals approximation!

Thus, if we knew the Induced Surface Tension of geometrical clusters in SU(2) gluodynamics, then we could get negative Eigen Surface Tension

Besides, I do not know alternative mechanism for a cross-over in ordinary liquids and in full QCD.

Hence, the total Surface Tension in LQCD with quarks should be measured!

Practical Conclusions

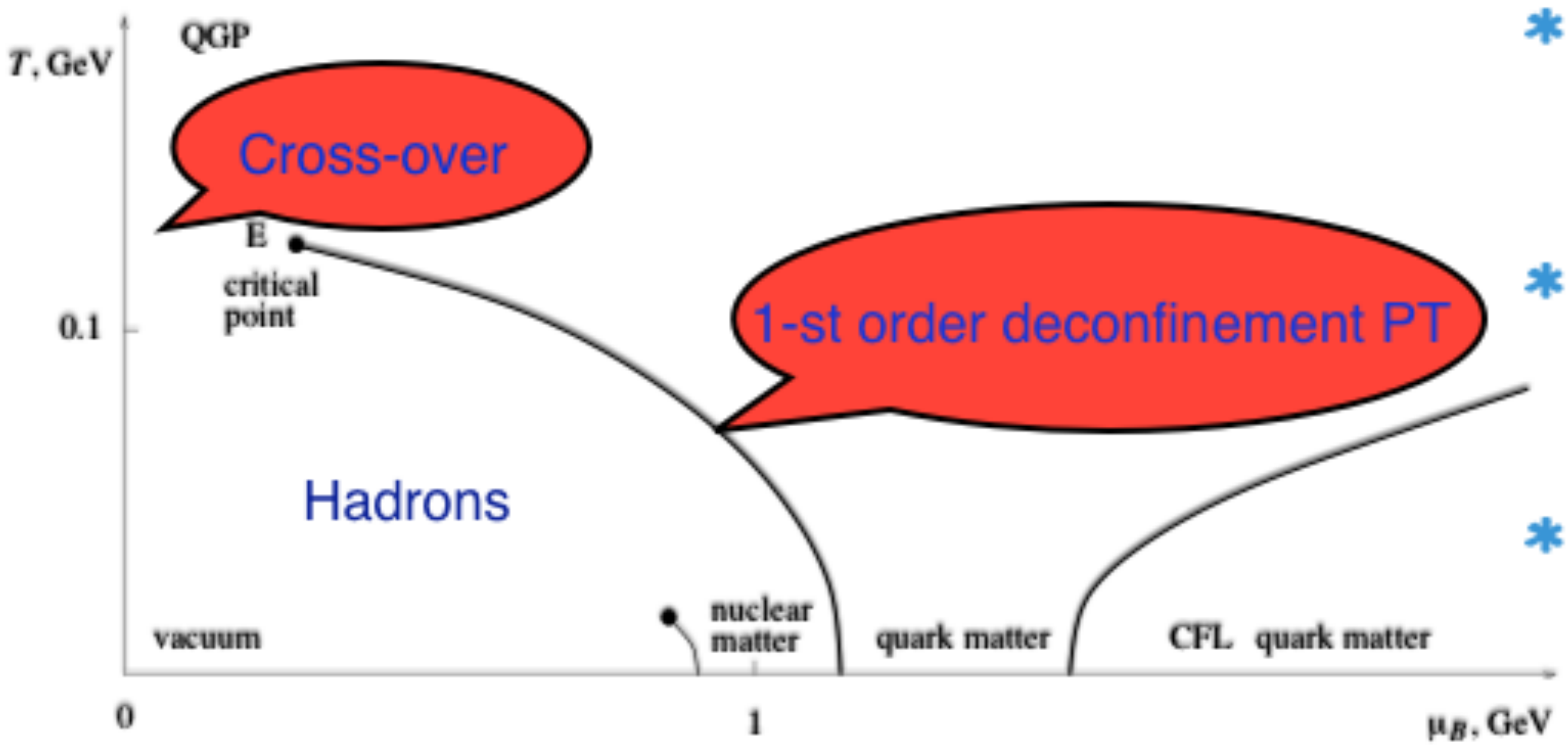
Rigorous theory of surface tension of ordinary liquids and QG bags should be developed!

We have to search for other mechanisms of (3)CEP generation and find relation between relation between the PT induced by Surface Tension and Chiral Symmetry Restoration

Modeling heavy ion collisions with different time scales

~30 Years of QGP Searches

- AGS (BNL), SPS (CERN) & RHIC (BNL) - no mixed phase found...
- RHIC low energy program (BNL) - searches for the (tri)critical endpoint of QCD phase diagram;
- NICA (JINR, Dubna) - searches for the mixed phase (hadrons+QGP);
- FAIR (GSI, Darmstadt) - searches for the densest state of nuclear matter



Phase diagram major elements:

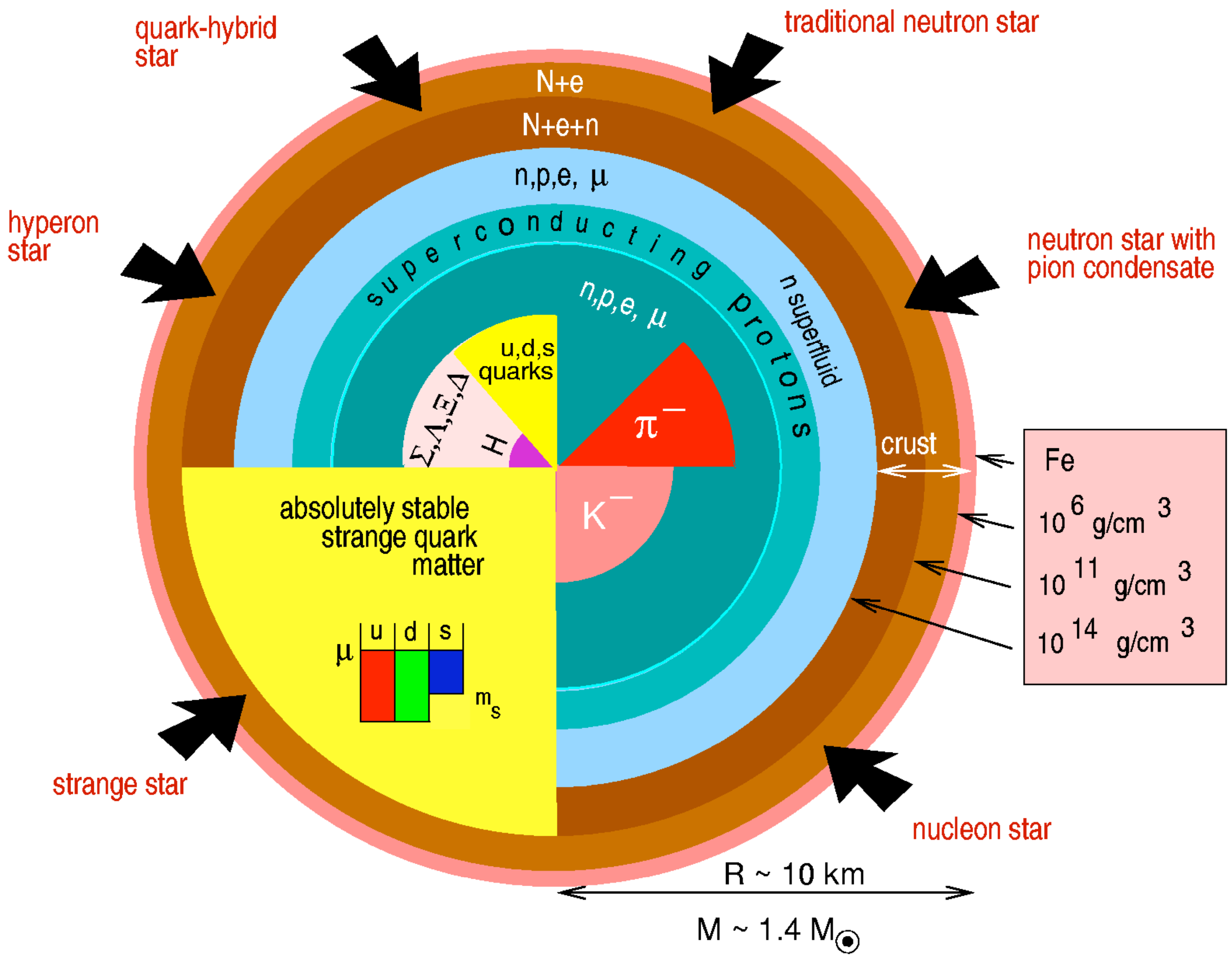
- 1-st order deconfinement PT (low T, large μ)
- cross-over transition (low μ, large T)
- (tri)critical endpoint in between. **Exact location is unknown!**

Astro & Cosmic QGP Searches Programs

* Quark (core) stars, neutron stars, **stable strange stars, …**

- **Strangelet** - {see Bodmer (1971), Witten(1984), Jaffe (1984)} finite drop of strange matter with **LARGE** baryonic charge and **small** electric charge.
  May be stable at high densities (few normal nuclear densities) due to Chiral Symmetry (CS) Restoration {Buballa(1996}:
  In CS **Restored** phase s-quark mass << Fermi energy of u & d quarks => u & d quarks weakly decay into s-quarks!

Properties of neutron stars



They can be formed is A+A HIC, in QCD phase transition in early Universe, in collisions of compact stars with large strangeness, in cosmic rays e.t.c.

First Conceptual Problem

- Why the small and not too heavy QGP bags with mass of 10–20 GeV have not been observed in A+A or in elementary particle collisions at low T?

- Why the strangelets were never observed at low T?

* Usual concept: QGP bags cannot exist inside hadronic phase because of PT or strong cross-over. They should be extremely suppressed statistically.

Typical form of bag spectrum: the **discrete mass-volume spectrum** $F_H(s,T)$ of hadrons lighter than M_0 and the **continuous volume spectrum** $F_Q(s,T)$

$$F(s,T) \equiv F_H(s,T) + F_Q(s,T) =$$

$$\sum_{j=1}^{n} g_j e^{-v_j s} \phi(T, m_j) + \int_{V_0}^{\infty} dv \int_{M_0}^{\infty} dm \int \frac{d^3k}{(2\pi)^3}\, \rho(m,v)\, e^{-sv - \frac{\sqrt{k^2+m^2}}{T}} =$$

Hard core repulsion

Hard core repulsion

$$\sum_{j=1}^{n} g_j e^{-v_j s} \phi(T, m_j) + u(T) \int_{V_0}^{\infty} dv\, \frac{\exp\left[(s_Q(T) - s)\, v\right]}{v^{\tau}}\,, \qquad \mathbf{V_0 \approx 1\ fm^3}$$

$s_Q(T) = \frac{p_Q(T)}{T}$ is defined via the QGP pressure $p_Q(T)$ (MIT Bag Model). $\mathbf{M_0 \approx 2.5\ GeV}$

First Conceptual Problem

- Why the small and not too heavy QGP bags with mass of 10–20 GeV have not been observed in A+A or in elementary particle collisions at low T?
- Why the strangelets were never observed at low T?

The GCE partition can be written as

$$\boxed{\mathcal{Z}(V,T,\mu) = \sum_{\{\lambda_n\}} e^{\lambda_n V} \left[1 - \frac{\partial \mathcal{F}(\lambda_n, V)}{\partial \lambda_n}\right]^{-1}}$$

For finite V one has to account for ALL singularities λ_n in a complex plane!

If volume V is not large then a few metastable states have to contribute into GCE partition!

And one can find a set of parameters for which the QG bags have nonvanishing probability to appear.

Not Completely True in a Finite System!

However, this is true for an infinite system only!

In finite systems the suppression is not of Avogadro number order,

but 1/1000000 – 1/10000000 only!

Then such QGP bags (and strangelets!) should have been observed

as any METASTABLE STATE!

If they are absent, then there must be a reason for this!

• **Finite volume solution** of GBM, **K.A.B., Phys. Part. Nucl. 38 (2007),** allows one

To show that for small V the finite QGP bags are **not suppressed anymore!**

To estimate a DECAY (FORMATION) TIME of metastable states $(n = 1, 2, 3, ...)$

$$\tau_n \approx \frac{V_{max} \hbar}{\pi n \, V_0 \, T} \approx \frac{60 \, V_{max} [\mathrm{fm} \cdot \mathrm{MeV}]}{n V_0 \, T [\mathrm{MeV}]}$$

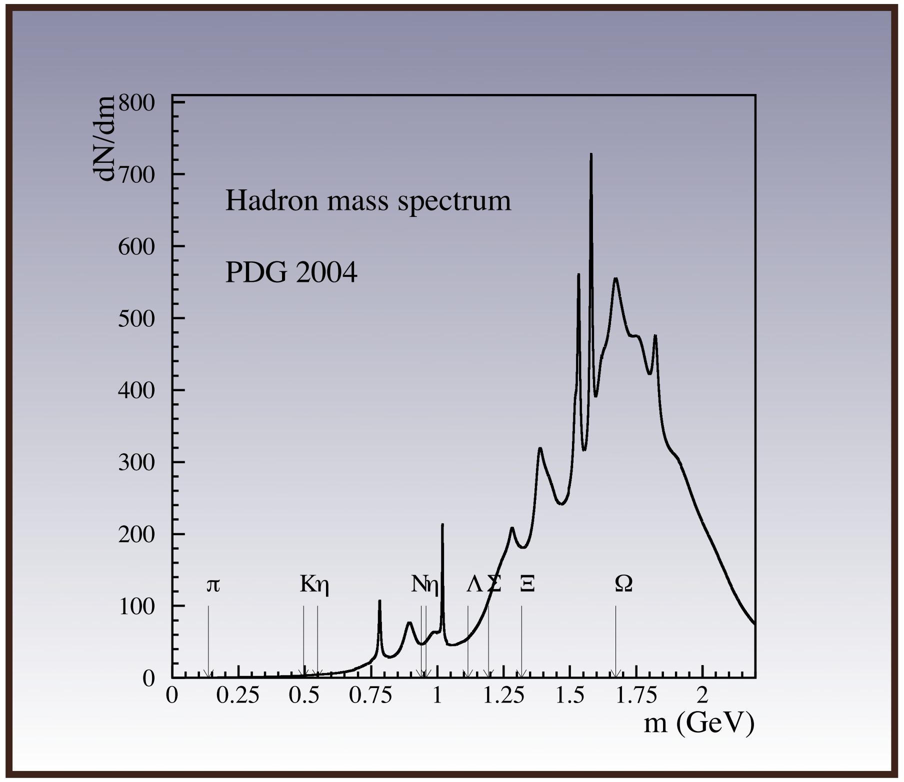
Typical example: strangelets

$\Longrightarrow$ at low $T \ll T_c$ the $n \geq 1$ states with **finite QGP bags could exist very long time!**

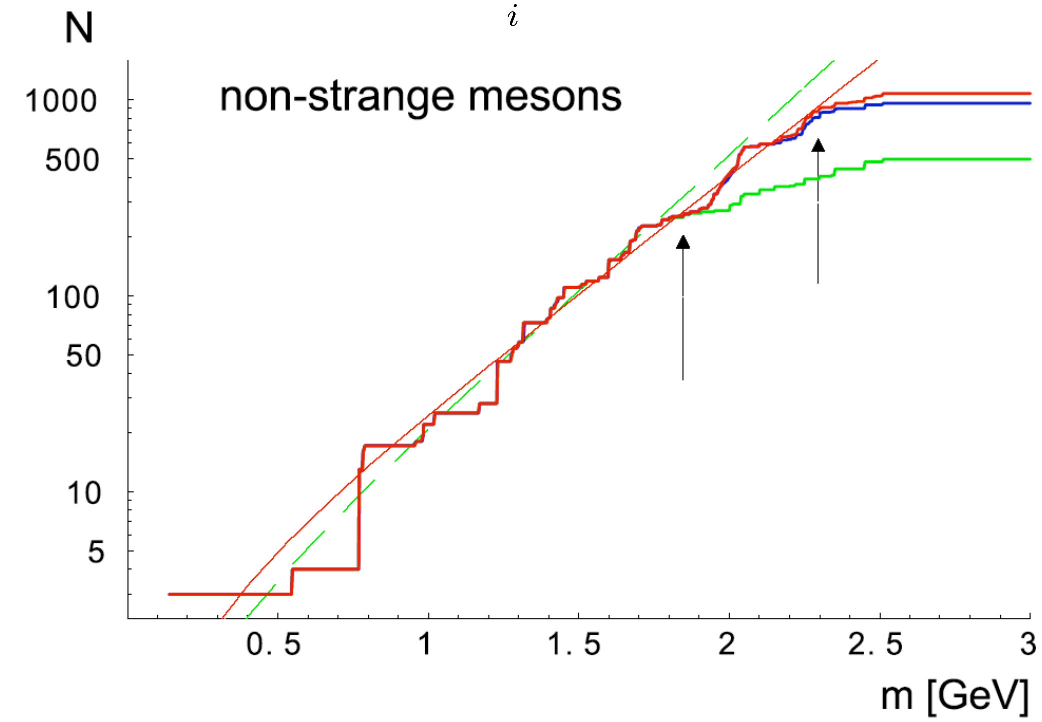
$\Longrightarrow$ **for small V the finite QGP bags could coexist with hadrons!**

• Moreover, since initial stage of collision is not equilibrated $\Longrightarrow$ **nothing can prevent** THE FORMATION OF METASTABLE QGP BAGS in the hadronic phase!

Second Conceptual Problem



$$N_{\exp}(m) = \sum_i g_i \Theta(m - m_i),$$



It is observed for 1.3 GeV $< m <$ 2.5 GeV only,

i.e. NOT WHERE IT WAS PREDICTED!

$\Rightarrow$ **There is a huge deficit of heavy hadrons predicted by stat. bootstrap model!**

It is believed that heavy resonances are not observed due to their large width.

However, the full Hagedorn mass spectrum is used in ALL realistic statistical models like Gas of Bags Model (GBM) and NO width is accounted for!

For width of QGP bags see D.Blaschke & K.A.B. in 2003-2005

Finite Width Model

Major aims are:

1) to include the finite medium dependent width into statistical model in the most general fashion (FWM).

2) to resolve these two conceptual problems and to derive a general form of EOS from the clear physical assumptions.

3) to compare the obtained EOS with the lattice QCD results and to find out the width of heavy QGP bags.

In fact, we want to make a firm bridge between the lattice QCD thermodynamics and hadronic phenomenology via the statistical approach.

Width Estimate Sensitivity

TABLE I: The values of the resonance width for different models. Model A corresponds to the $SU(2)_C$ pure gluodynamics of Ref. [45]. Model B describes the $SU(3)_C$ LQCD with 2 quark flavors [46] and Model C is the $SU(3)_C$ LQCD with 3 quark flavors [50].

$$V_0 = 1\ \mathrm{fm}^3$$

	Model Ref.	$\frac{90\sigma}{\pi^2}$ d.o.f.	T_c (MeV)	$\Gamma_R(V_0, 0)$ (MeV) at T=0	$\Gamma_R(V_0, T_H)$ (MeV) at T=Th	
SU(2)c pure gluodynamics	A	6	170	410	1420	Bielefeld data, finite-size effects are accounted for
	A	6	$\boxed{200}$	$\boxed{616}$	$\boxed{2133}$	
SU(3)c LQCD 2 q flavors	B	37	170	391	1355	Bielefeld data, finite-size effects are accounted for
	B	37	$\boxed{200}$	$\boxed{587}$	$\boxed{2034}$	
SU(3)c LQCD 3 q flavors	C	$\frac{95}{2}$	$\boxed{196}$	$\boxed{596}$	$\boxed{2066}$	Bielefeld+BNL+ Copenhagen data, no FSE, but large lattices!

K. A. Bugaev, V. K. Petrov and G. M. Zinovjev, Phys. Rev. C 79, No 5, (2009) 054913

Width Estimate Sensitivity

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$$V_0 = 1 \text{ fm}^3$$

SU(2)c pure gluodynamics — Bielefeld data, finite-size effects are accounted for

SU(3)c LQCD 2 q flavors — Bielefeld data, finite-size effects are accounted for

SU(3)c LQCD 3 q flavors — Bielefeld+BNL+ Copenhagen data, ... ices!

- Width of QGP bags is very stable against dof number! Strong argument in favor of B-ansatz.
- Strongly depends on T and on Tc!
- QGP bags with so large width cannot be observed!

Finite Width Model II

The medium-dependent finite width is introduced into an exactly solvable model with the general mass-volume spectrum of the QGP bags.

The model allows us to estimate the minimal value of the QGP bags' width from the lattice QCD data.

The large width of the QGP bags not only explains the observed deficit in the number of hadronic resonances comparing to the Hagedorn mass spectrum, but also clarifies the reason why the heavy QGP bags cannot be directly observed as metastable states in a hadronic phase.

K. A. Bugaev, V. K. Petrov and G. M. Zinovjev, Europhys. Lett. 85, (2009) 22002

K. A. Bugaev, V. K. Petrov and G. M. Zinovjev, Phys. Rev. C 79, No 5, (2009) 054913

Thank You for Your Attention!

Finite Width Model Spectrum

- To make the bridge from hadronic phenomenology **we need the Hagedorn-like mass spectrum!**

Experimental input

- To introduce the **width Γ we need the Gaussian attenuation,**
  since **the Breit-Wigner one does not work** for the Hagedorn-like mass spectrum!

To have convergent partition

- To get a realistic model **we need to introduce the surface tension for QGP bags!**

To have (tri)critical endpoint

$\Rightarrow$ **the simplest parameterization of the spectrum $\rho(m,v)$ is**

$$\rho(m,v) = \frac{\rho_1(v)\; N_\Gamma}{\Gamma(v)\; m^{a+\frac{3}{2}}} \exp\underbrace{\left[\frac{m}{T_H} - \frac{(m-Bv)^2}{2\Gamma^2(v)}\right]}_{\text{Hagedorn \& Gaussian terms}}, \quad \text{with} \quad \rho_1(v) = f(T)\, v^{-b} \exp\underbrace{\left[-\frac{\sigma(T)}{T}\, v^{\varkappa}\right]}_{\text{Surface tension}},$$

K.A.B. PRC76(2007)

Important:

- **Gaussian width $\Gamma(v)$ depends on bag's volume v, on T, but not on mass m!**
- Gaussian width $\Gamma(v)$ is related to the true resonance width as $\Gamma_R = 2\sqrt{2\ln 2}\,\Gamma(v) \approx 2.355\,\Gamma(v)$
- The **most probable mass in a vacuum** must be positive $\mathbf{B > 0}$

$\Rightarrow$ Normalization factor is

$$N_\Gamma^{-1} = \int\limits_{M_0}^{\infty} \frac{dm}{\Gamma(v)} \exp\left[-\frac{(m-Bv)^2}{2\Gamma^2(v)}\right] \approx \Big|_{v \gg V_0, M_0/B} \approx \sqrt{2\,\pi}$$

Analysis of the FWM Spectrum

- For simplicity let's consider **only two choices** for Gaussian width $\Gamma(v)$:

v-**independent width** $\Gamma(v) = \mathbf{\Gamma_0} \equiv Const$ and v-**dependent width** $\Gamma(v) = \mathbf{\Gamma_1} \equiv \gamma \mathbf{v}^{\frac{1}{2}}$

Ignoring the hard-core repulsion and thermostate in $F_Q(s,T)$: $\Rightarrow$

$$\rho(m) \equiv \int_{V_0}^{\infty} dv\, \rho(m,v) = \int_{V_0}^{\infty} dv\, \frac{\rho_1(v)\ N_\Gamma}{\Gamma(v)\ m^{a+\frac{3}{2}}} \exp\left[\frac{m}{T_H} - \frac{(m-Bv)^2}{2\Gamma^2(v)}\right] \approx \boxed{\Big|_{m \gg M_0} \approx \frac{\rho_1(\frac{m}{B})}{B\ m^{a+\frac{3}{2}}} \exp\left[\frac{m}{T_H}\right]}$$

Can be derived, if for $v \gg V_0$ the **width grows slower than** $\mathbf{v^{(1-\varkappa/2)} = v^{2/3}}$

This is so, since for $\Gamma(v) = \mathbf{\Gamma_0}$ or $\Gamma(v) = \mathbf{\Gamma_1}$ the Gaussian width acts like the Dirac δ-function!

$\Rightarrow$ **The FWM spectrum corresponds to the Hagedorn mass spectrum**

modified by the surface tension!

$\Rightarrow$ Similarly, the mean width $\overline{\Gamma(v)} \approx \Gamma(m/B)$

$\Rightarrow$ for $\Gamma(v) = \mathbf{\Gamma_1}$ one gets **the large mean width** $\Gamma_1(m/B) = \gamma\sqrt{m/B}$

$\Rightarrow$ **for $\mathbf{\Gamma_1(m/B) = \gamma\sqrt{m/B}}$ the heavy resonances are hard to be observed!**

THE SECOND CONCEPTUAL PROBLEM IS RESOLVED.

High T Behavior of FWM Spectrum

Let's calculate $F(s,T)$. Depending on the sign of the **most probable mass**

$$\boxed{\underbrace{\langle m \rangle}_{\textbf{most probable mass}} \equiv Bv + \Gamma^2(v)\beta,} \quad \text{with} \qquad \beta \equiv T_H^{-1} - T^{-1}$$

there are two distinct cases : $\langle m \rangle > 0$ and $\langle m \rangle < 0$

Let's calculate $F(s,T)$ for $T \geq T_H$ $\Rightarrow$ $\boldsymbol{\langle m \rangle > 0}$ for $v \gg V_0$ **by saddle point**

For $M_0 \gg T$ one can use the nonrelativistic approximation for momentum $\Rightarrow$

$$F_Q^+(s,T) = \int\limits_{V_0}^{\infty} dv \int\limits_{M_0}^{\infty} dm \int \frac{d^3k}{(2\pi)^3}\, \rho(m,v)\, e^{-\mathbf{s}v - \frac{\sqrt{k^2+m^2}}{T}} = \left[\frac{T}{2\pi}\right]^{\frac{3}{2}} \int\limits_{V_0}^{\infty} dv \int\limits_{M_0}^{\infty} dm\, \frac{\rho_1(v)\ N_\Gamma}{\Gamma(v)\ m^a} \exp\left[\beta m - \tfrac{(m-Bv)^2}{2\Gamma^2(v)} - sv\right]$$

$$F_Q^+(s,T) \approx \left[\frac{T}{2\pi}\right]^{\frac{3}{2}} \int\limits_{V_0}^{\infty} dv\ \frac{\rho_1(v)}{\langle m \rangle^a}\ \exp\left[\tfrac{(p^+ - sT)v}{T}\right],$$ **with the pressure**

$$\boxed{p^+ \equiv T\left(\beta B + \frac{\Gamma^2(v)}{2v}\beta^2\right)}$$

In terms of $\boldsymbol{\langle m \rangle > 0}$ it reads as: $\boldsymbol{p^+ \equiv \frac{T\beta}{v}\left[\langle m \rangle - \frac{1}{2}\Gamma^2(v)\beta\right]}$

$\Rightarrow$ **In general, the pressure of large QGP bags is due to the mass density and the width!**

Note that width may CORRECTLY contribute into $\boldsymbol{\langle m \rangle > 0}$ and $\boldsymbol{p^+}$ for $\boldsymbol{\Gamma(v) \leq \Gamma_1}$ only!

High T Behavior of FWM Spectrum

Let's calculate $F(s,T)$. Depending on the sign of the **most probable mass**

$$\boxed{\underbrace{\langle m \rangle \equiv Bv + \Gamma^2(v)\beta}_{\textbf{most probable mass}},} \quad \text{with} \quad \beta \equiv T_H^{-1} - T^{-1}$$

there are two distinct cases : $\langle m \rangle > 0$ and $\langle m \rangle < 0$

Let's calculate $F(s,T)$ for $T \geq T_H \Rightarrow \boldsymbol{\langle m \rangle > 0}$ for $v \gg V_0$ **by saddle point**

* However, this case does not resolve the first problem!

$$F_Q^+(s,T) = \int_{V_0}^{\infty} dv \int_{M_0}^{\infty} dm \int \frac{d^3k}{(2\pi)^3}\, \rho(m,v)\, e^{-\mathbf{s}v - \frac{\sqrt{k^2+m^2}}{T}} = \left[\frac{T}{2\pi}\right]^{\frac{3}{2}} \int_{V_0}^{\infty} dv \int_{M_0}^{\infty} dm\, \frac{\rho_1(v)\ N_\Gamma}{\Gamma(v)\ m^a} \exp\left[\beta m - \frac{(m-Bv)^2}{2\Gamma^2(v)} - sv\right]$$

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$\Rightarrow$ **In general, the pressure of large QGP bags is due to the mass density and the width!**

Note that width may CORRECTLY contribute into $\langle m \rangle > 0$ and p^+ for $\Gamma(v) \leq \Gamma_1$ only!

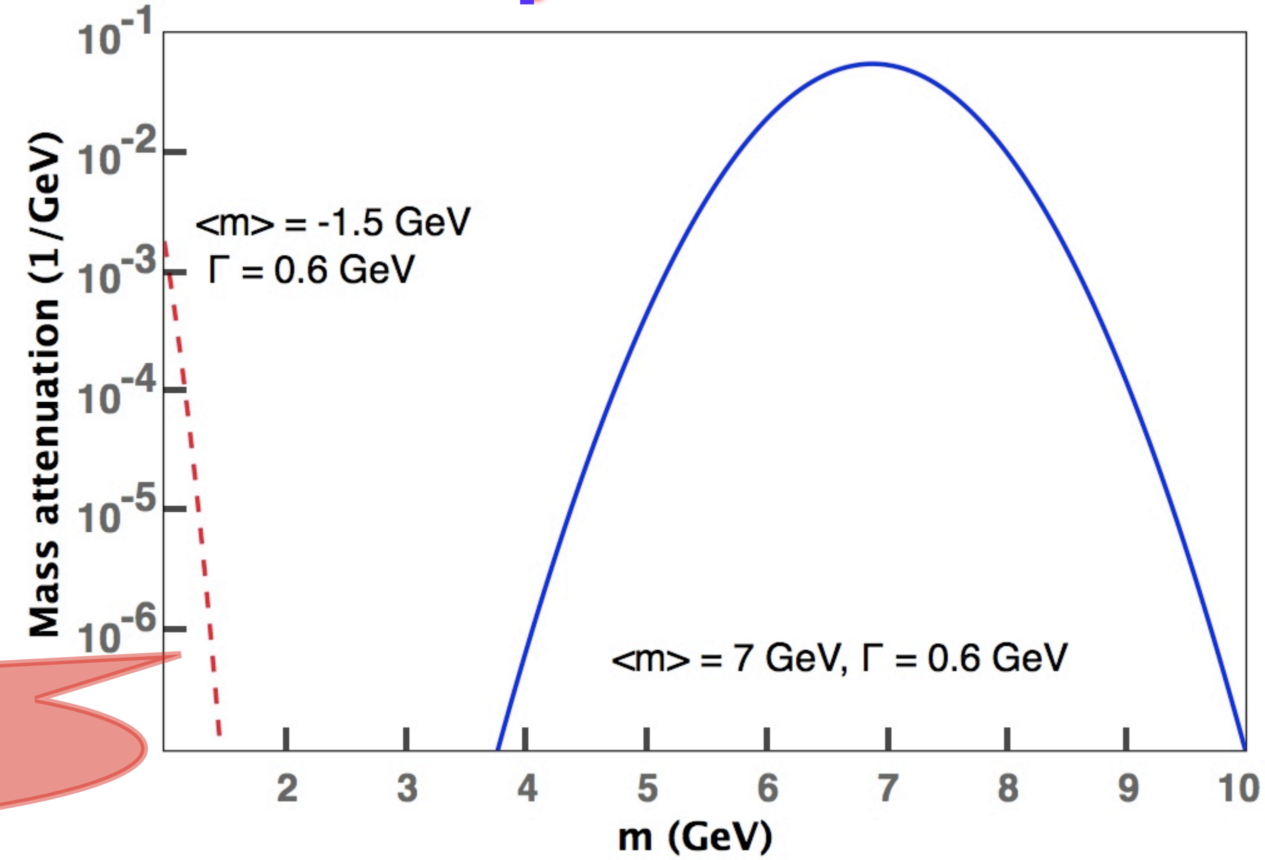
Low T Behavior of FWM Spectrum

For $T \ll T_H$ and $0 < B < \infty \quad \Rightarrow$

$\langle \boldsymbol{m} \rangle < \mathbf{0}$ for $v \gg V_0$: **by steepest descent!**

The maximum is below M_0 and ,hence,

the tail of distribution contributes only!



$\Rightarrow$ **Subthreshold Suppression of QGP bags**

$$F_Q^-(s,T) = \int_{V_0}^{\infty} dv \int_{M_0}^{\infty} dm \int \frac{d^3k}{(2\pi)^3}\, \rho(m,v)\, e^{-\mathbf{s}v - \frac{\sqrt{k^2+m^2}}{T}} = \left[\frac{T}{2\pi}\right]^{\frac{3}{2}} \int_{V_0}^{\infty} dv \int_{M_0}^{\infty} dm\, \frac{\rho_1(v)\; N_\Gamma}{\Gamma(v)\; m^a} \exp\left[\beta m - \frac{(m-Bv)^2}{2\Gamma^2(v)} - sv\right]$$

$$F_Q^-(s,T) \approx \left[\frac{T}{2\pi}\right]^{\frac{3}{2}} \int_{V_0}^{\infty} dv\, \frac{\rho_1(v) N_\Gamma\, \Gamma(v)\, \exp\left[\frac{(p^- - sT)v}{T}\right]}{M_0^a\, [M_0 - \langle m \rangle + a\, \Gamma^2(v)/M_0]}\,,$$

with the pressure $\boxed{p^- = \frac{T}{v}\left[\beta M_0 - \frac{(M_0 - Bv)^2}{2\,\Gamma^2(v)}\right] \approx \Big|_{v \gg V_0} \approx -T\,\frac{B^2}{2\,\gamma^2}\,.}$

Important: • Can be derived for $\boldsymbol{B} > \mathbf{0}$ only!

if $\boldsymbol{B} < \mathbf{0}$, then $N_\Gamma \approx [M_0 - \langle m \rangle]\, \Gamma^{-1}(v) \exp\left[\frac{(M_0 - Bv)^2}{2\,\Gamma^2(v)}\right]$ would cancel the leading term in $\boldsymbol{p}^-$

• Can be derived for $\boldsymbol{\Gamma(v) = \Gamma_1(v)}$, since only in this case $\langle m \rangle \;\equiv\; Bv + \Gamma^2(v)\beta < 0$ for $B > 0$ at low T!

Volume Dependence of Width

- **The case** $\langle m \rangle \equiv Bv + \Gamma^2(v)\beta > 0$ exists for $T \geq \alpha T_H$ with $\alpha < 1$

It generates the QGP bag pressure $p^+ \equiv T\left(\beta B + \frac{\Gamma^2(v)}{2v}\beta^2\right) = \frac{T\beta}{v}\left[\langle m \rangle - \frac{1}{2}\Gamma^2(v)\beta\right]$

for any $\Gamma(v)$, if **width grows slower than** $\mathbf{v}^{(1-\varkappa/2)} = \mathbf{v}^{2/3}$, but is **meaningful for** $\Gamma(\mathbf{v}) \leq \Gamma_1(\mathbf{v})$

Has the same phase structure as the QGBSTM with $\tau = a + b$.

K.A.B. PRC76(2007)

- **The case** $\langle m \rangle \equiv Bv + \Gamma^2(v)\beta < 0$ exists for $T < \alpha T_H$ with $\alpha < 1$

It generates the QGP bag pressure $p^- = -T\frac{B^2}{2\gamma^2}$ and **can be derived for** $\Gamma(v) = \Gamma_1(v)$ **only!**

$\Rightarrow$ **This is truly nonperturbative effect because for stable hadrons it does not exist!**

Closely resembles low T pressure known from lattice QCD!!!

$\Rightarrow$ **Finite pressure of large QGP bags with nonzero width exists**

for $\Gamma(v) = \Gamma_1(v) = \gamma\sqrt{v}$ **only!**

First Conceptual Problem is Resolved

For $T < \alpha T_H$ with $\alpha < 1$ exists Subthreshold Suppression of QGP bags $\Rightarrow$

only the bags with mass $\sim M_0 \approx 2.5$ GeV and volume $\sim V_0 \approx 1$ fm^3

could contribute into partition, but they are HIGHLY SUPPRESSED!

$\Rightarrow$ Case $\langle m \rangle < 0$ resolves the first conceptual problem for finite systems for $T < \alpha T_H$.

What about $\alpha T_H \leq T \leq T_H$?

For several EoS of QGP it was shown that $\alpha \approx 0.5$.

Then for this T range

$\Rightarrow$ they are indistinguishable from the usual short-living hadrons!

What is confinement in terms of width?

Confinement means: **large/heavy QGP bags can exist for very short time (< 0.5 fm/c at best!) then they decay into stable (=long living) hadrons**

Important: **considered QGP bags are colorless!**

Reminder: **in a box surrounded by thermostat the loss of decaying QGP bags is compensated by the reactions and thermostat, but in expanding matter created in A+A collisions this is not the case!**

Conclusion: **in A+A collisions the QGP phase transition into hadrons means the decay of large/heavy but unstable bags!**

Conclusions

- **New mechanisms of PT and (3)CEP models for QCD are required**

- **Rigorous theory of surface tension of ordinary liquids and QCD clusters is necessary**

- **Statistical thermodynamics of finite systems should be developed**

- **A lot of interesting work related to NICA and FAIR experiments awaits for us!**

Back Up Slides

More Realistic EoS

• The model pressure $\boxed{p_a = \sigma T^4 - A_1 T \quad (A_1 > 0)}$

describes LQCD data well: C. G. Kallman, Phys. Lett. B 134, 363 (1984),

M. I. Gorenstein, O. A. Mogilevsky, Z. Phys. C 38 (1988)

But these are OLD LQCD data!

For new data analysis see K.A.B. et al. PRC 79 (2009)

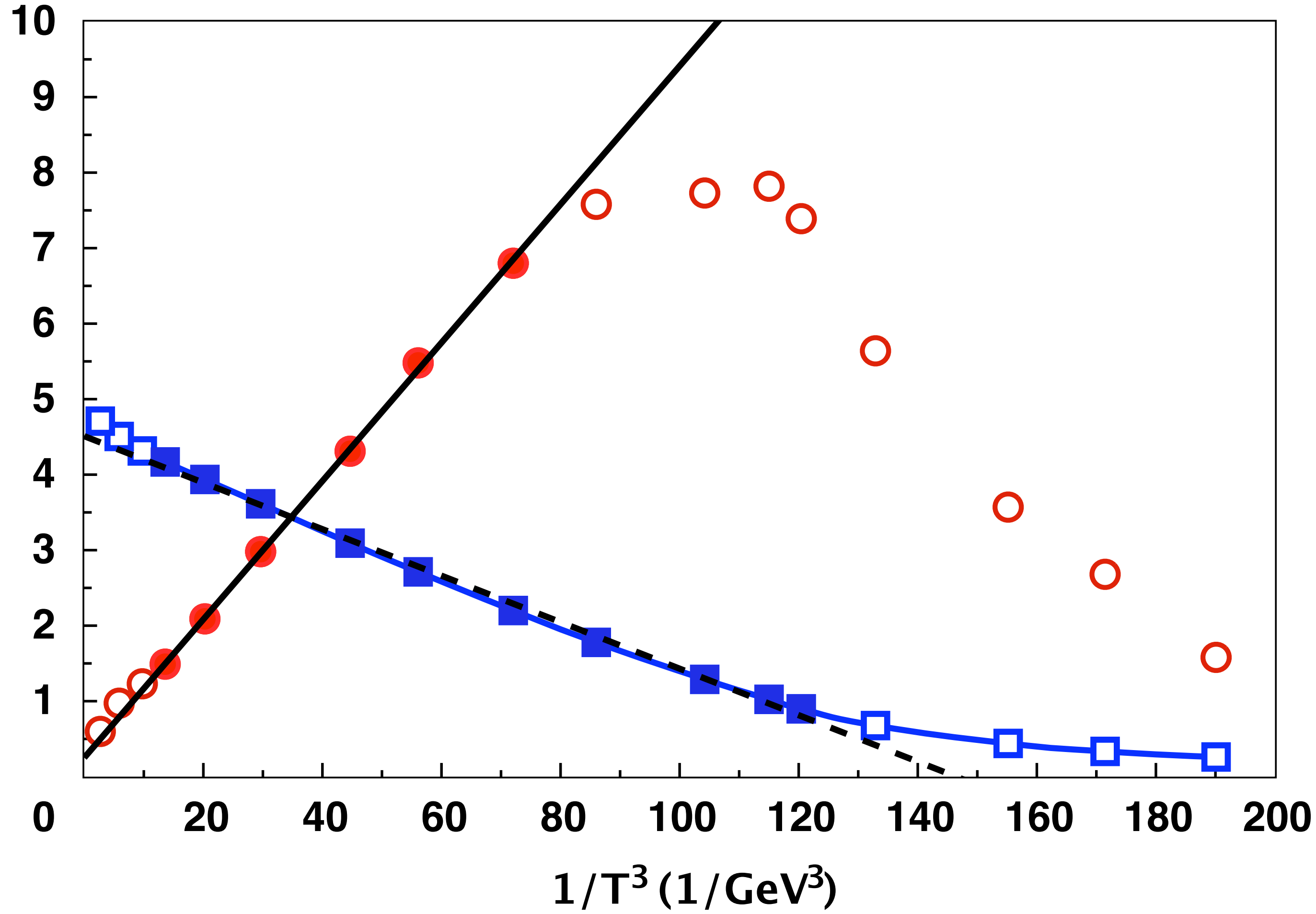
Entropy density: $s = \frac{dp}{dT} = 4\sigma T^3 - A_1$;

and energy density: $\varepsilon = Ts - p = 3\sigma T^4$ $\Rightarrow$ NO T-linear term in ε!

$$\Rightarrow \frac{p}{T^4} = \sigma - \frac{A_1}{T^3}, \qquad \frac{\delta}{T^4} = \frac{\varepsilon - 3\,p}{T^4} = \frac{3A_1}{T^3}$$

$\Rightarrow$ are linear in $\frac{1}{T^3}$

Width Estimate from Lattice QCD



recent LQCD data:
$SU(3)_C$ with 3 flavors
Cheng et al, arXiv:0710.0354

Red symbols:
Trace anomaly $\delta/T^4 = (\varepsilon - 3\,p)/T^4$
$\chi^2/\mathrm{d.o.f} \approx 0.062$

OUR fit: filled symbols

Blue symbols: p/T^4

$\Rightarrow$ LQCD pressure has σT^4, $-\sigma T_H^3 T$ and $T^4 \ln \frac{T}{T_H}$ terms!

Obtained by LQCD data fit

$$p_{QGP} = \sigma\, T^4 \boxed{-\,\sigma T_H^3\, T} + \underbrace{\tilde{a}_0 T^4 \ln\left[\frac{T}{T_H}\right]}_{small}\,, \qquad \text{for} \qquad 240\,\mathrm{MeV} \le T \le 420\,\mathrm{MeV}$$

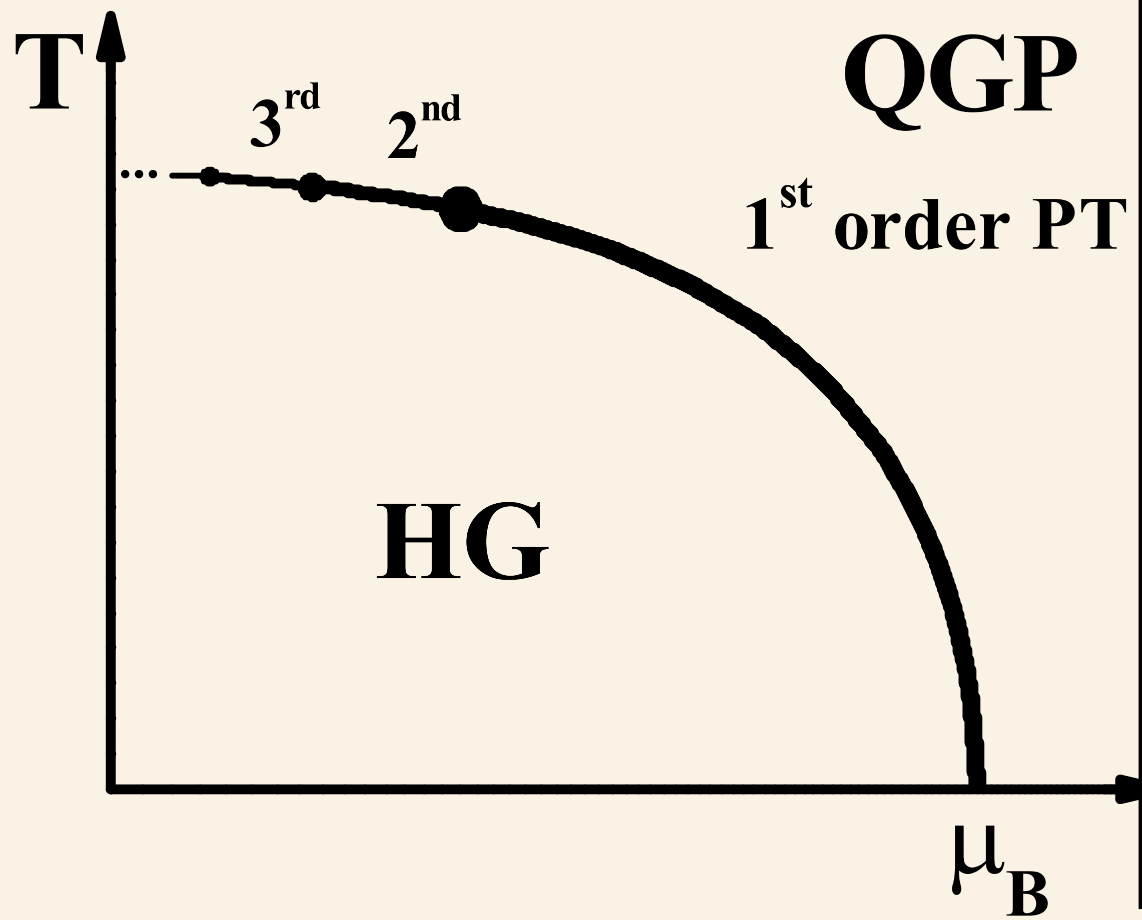
COMPARE T-linear terms!

$$p^- = \boxed{-T\,\frac{B^2}{2\,\gamma^2}}$$

Derived by FWM at low T!

Problems of the Gas of Bags Model

* 2005 A new and EXTRAVAGANT idea to revitalize the GBM: in order to get the CEP and cross-over M.I. Gorenstein, M. Gazdzicki and W. Greiner, Phys. Rev. C 72 (2005) 024909, suggested a line along which the PT order gradually decreases.



There are tenth of thousand substances with extremely complicated phase diagrams known in physics, but such a pathological diagram has never been seen! There are no causes known for such a line! It contradicts to the whole concept of critical phenomena!

- Consequently, such a formulation of GBM lacks an important physical input and has to be modified.