

**Phase transitions in finite systems:
from possible signals in heavy ion collisions to
their rigorous theoretical treatment I**

Kyrill Bugaev, A. Ivanytskyi, D. Oliinychenko, V. Sagun, G. Zinovjev

Bogolyubov ITP, Kiev, Ukraine

E. Nikonov

Laboratory for Information Technologies, JINR, Dubna, Russia

D. H. Rischke

ITP, J.W. Goethe University, Frankfurt, Germany

I. Mishustin and L. Satarov

FIAS, J.W. Goethe University, Frankfurt, Germany

Dubna, July 12, 2017

Outline

1. Motivation

2. Problems of single component Hadron Resonance Gas Models

3. Some results of multicomponent Hadron Resonance Gas Models

4. Novel and Old Irregularities at chemical freeze out

5. Shock adiabat model of $A+A$ collisions

6. Newest results and possible evidence for two phase transitions

7. Conclusions

Experiments on A+A Collisions

AGS (BNL)	up to	4.9 GeV	} Completed
SPS (CERN)		6.1 - 17.1 GeV	
RHIC (BNL)		62, 130, 200 GeV	

Ongoing HIC experiments

LHC (CERN) > 1 TeV (high energy)

RHIC (BNL) low energy

SPS (CERN) low energy

Future HIC experiments

NICA (JINR, Dubna)

SIS300 = FAIR (GSI)

J-PARC

Present Status

In 2000 CERN claimed indirect evidence for a creation of new matter

In 2010 RHIC collaborations claimed to have created a quark-gluon plasma/liquid

However, up to now we do not know:

- 1. whether deconfinement is a phase transition**
- 2. where does the onset of deconfinement begin**

In order to answer 2-nd question we need a very accurate tool to analyze data.

Where Is Onset of Deconfinement?

30 years experience tells, that it is not difficult to invent a signal of QGP formation.

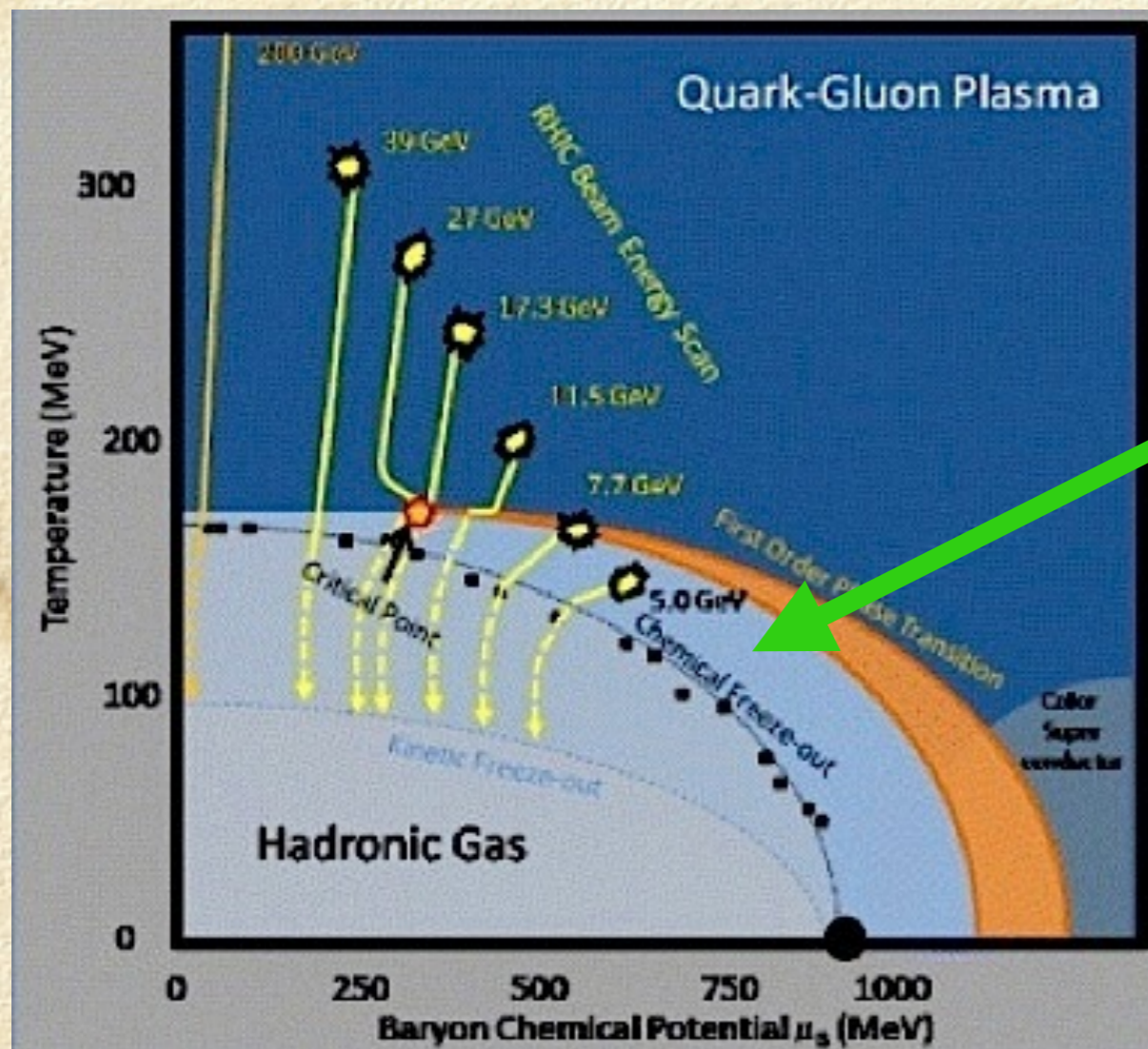
The most difficult part is to justify that it is related to phase transition.

In order to make such relations we need a very accurate tool to analyze data.

HRG: a Multi-component Model

HRG model is a truncated **Statistical Bootstrap Model** with the excluded volume correction a la VdWaals for all hadrons and resonances known from Particle Data Group.

For given temperature T , baryonic chem. potential, strange charge chem. potential, chem. potential of isospin 3-rd projection \Rightarrow thermodynamic quantities \Rightarrow all charge densities, to fit data.



Chemical freeze-out - moment after which hadronic composition is fixed and only strong decays are possible. I.e. there are no inelastic reactions.

HRG: a Multi-component Model

Traditional HRG model: one hard-core radius $R=0.25-0.3$ fm

A. Andronic, P. Braun-Munzinger, J. Stachel, NPA (2006)777

Overall description of data (mid-rapidity or 4π multiplicities) is good!

But there are problems with K^+/π^+ and Λ/π^- ratios at SPS energies!!! \Rightarrow Two component model was suggested

HRG: a Multi-component Model

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A. Andronic, P. Braun-Munzinger, J. Stachel, NPA (2006)777

Overall description of data (mid-rapidity or 4π multiplicities) is good!

Two hard-core radii: $R_{\pi}=0.62$ fm, $R_{\text{other}}=0.8$ fm

G. D. Yen, M. Gorenstein, W. Greiner, S.N. Yang, PRC (1997)56

Or: $R_{\text{mesons}}=0.25$ fm, $R_{\text{baryons}}=0.3$ fm

A. Andronic, P. Braun-Munzinger, J. Stachel, NPA (2006) 777 PLB (2009) 673

**Two component models do not solve the problems!
Hence we need more sophisticated approach.**

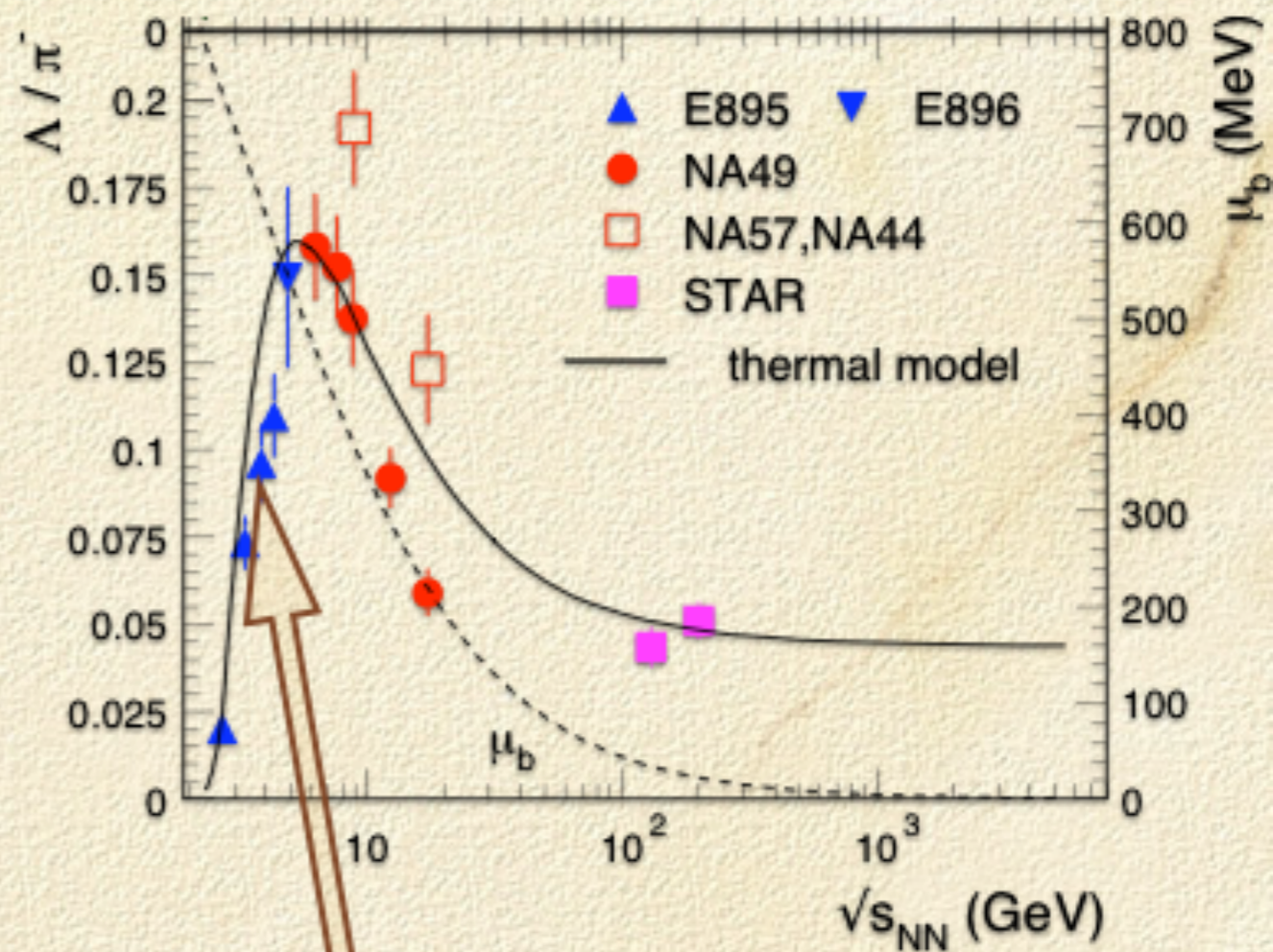
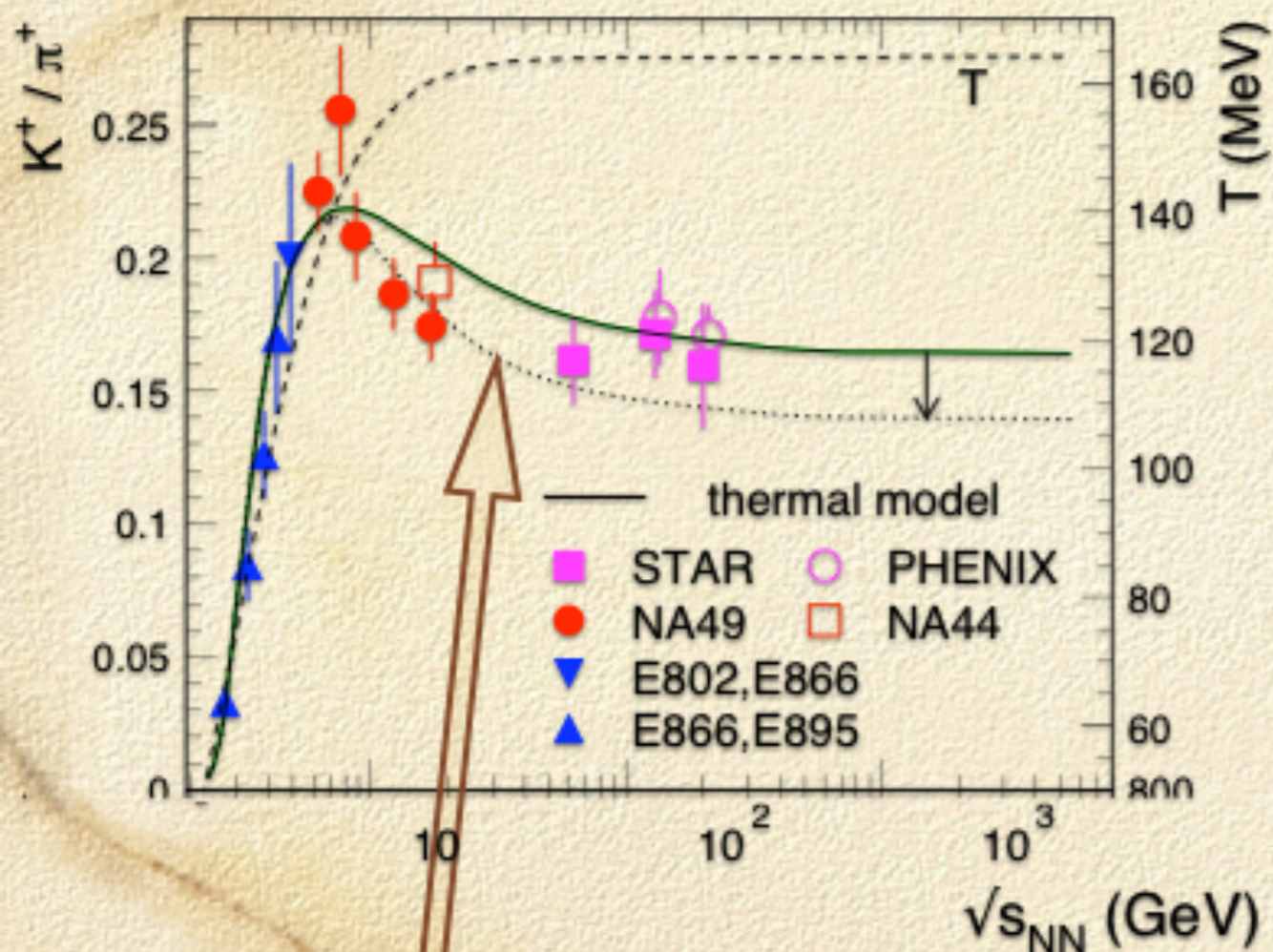
Horns Description in 1-component HRG

Too slow decrease after maximum!

Too steep increase before maximum and too slow decrease after it!

$$\chi^2/dof = 21.8/14$$

$$\chi^2/dof = 79/12$$



Short dashed line: a desired result

Anti Lambda problem!

Simple Solution to Horn Puzzle

Use four hard-core radii: R_{pi} , R_K are fitting parameters;
 $R_{mesons} = 0.4$ fm, $R_{baryons} = 0.2$ fm are fixed

G. Zeeb, K.A. Bugaev, P.T. Reuter and H. Stoecker, Ukr. J. Phys. 53, 279 (2008)

D.R. Oliinychenko, K.A. Bugaev and A.S. Sorin, Ukr. J. Phys. 58, (2013), No. 3, 211-227

p is pressure K -th charge density of i -th hadron sort is n_i^K ($K \in \{B, S, I3\}$)

\mathcal{B} the second virial coefficients matrix $b_{ij} \equiv \frac{2\pi}{3}(R_i + R_j)^3$

$$p = T \sum_{i=1}^N \xi_i, \quad n_i^K = Q_i^K \xi_i \left[1 + \frac{\xi^T \mathcal{B} \xi}{\sum_{j=1}^N \xi_j} \right]^{-1}, \quad \xi = \begin{pmatrix} \xi_1 \\ \xi_2 \\ \dots \\ \xi_s \end{pmatrix},$$

NO strangeness suppression is included!

the variables ξ_i are the solution of the following system:

$$\xi_i = \phi_i(T) \exp \left(\frac{\mu_i}{T} - \sum_{j=1}^N 2\xi_j b_{ij} + \frac{\xi^T \mathcal{B} \xi}{\sum_{j=1}^N \xi_j} \right), \quad \phi_i(T) = \underbrace{\frac{g_i}{(2\pi)^3} \int \exp \left(-\frac{\sqrt{k^2 + m_i^2}}{T} \right) d^3k}_{\text{THERMAL DENSITY}}$$

Chemical potential of i -th hadron sort: $\mu_i \equiv Q_i^B \mu_B + Q_i^S \mu_S + Q_i^{I3} \mu_{I3}$

Q_i^K are charges, m_i is mass and g_i is degeneracy of the i -th hadron sort

Wide Resonances Are Important

The resonance width is taken into account in thermal densities.

In contrast to many other groups we found that wide resonances are VERY important in a thermal model. For instance, description of pions cannot be achieved without

σ meson: $m_\sigma = 484 \pm 24$ MeV, width $\Gamma_\sigma = 510 \pm 20$ MeV

R. Garcia-Martin, J. R. Pelaez and F. J. Yndurain, PRD (2007) 76

$$n_X^{tot} = n_X^{thermal} + n_X^{decay} = n_X^{th} + \sum_Y n_Y^{th} Br(Y \rightarrow X)$$

$Br(Y \rightarrow X)$ is decay branching of Y-th hadron into hadron X

Data and Fitting Parameters

111 independent hadronic ratios measured at AGS, SPS and RHIC energies

of published ratios measured at mid-rapidity depends on energy =>

$\sqrt{s_{NN}}$ (GeV)	N_{rat} FO
2.7	4
3.3	5
3.8	5
4.3	5
4.9	8
6.3	9
7.6	10
8.8	11
9.2	5
12	10
17	13
62.4	5
130	11
200	10
Sum	111

of local fit parameters cannot be larger than 4 (for all energies) or larger than 5 (for energies above 2.7 GeV)

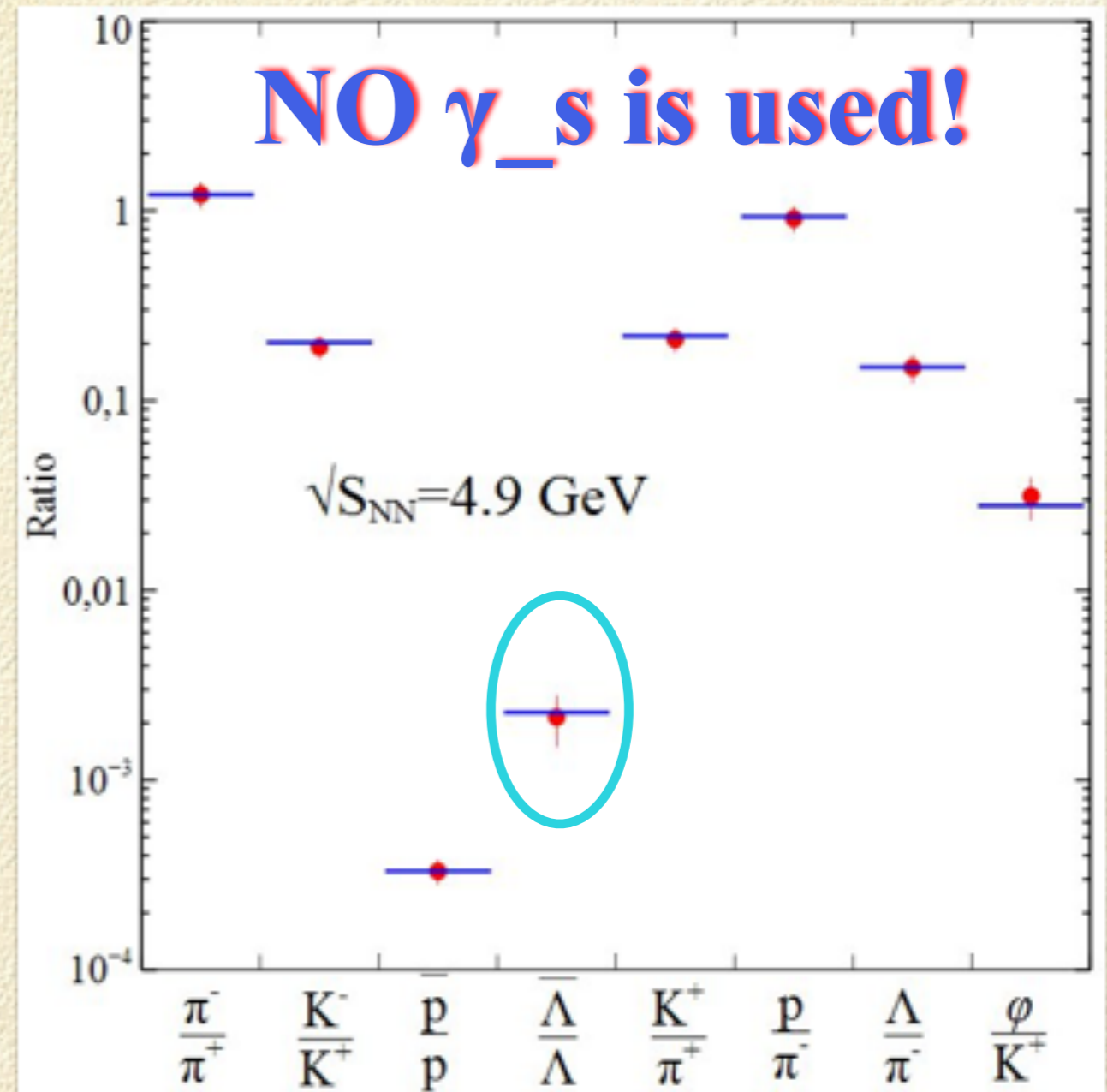
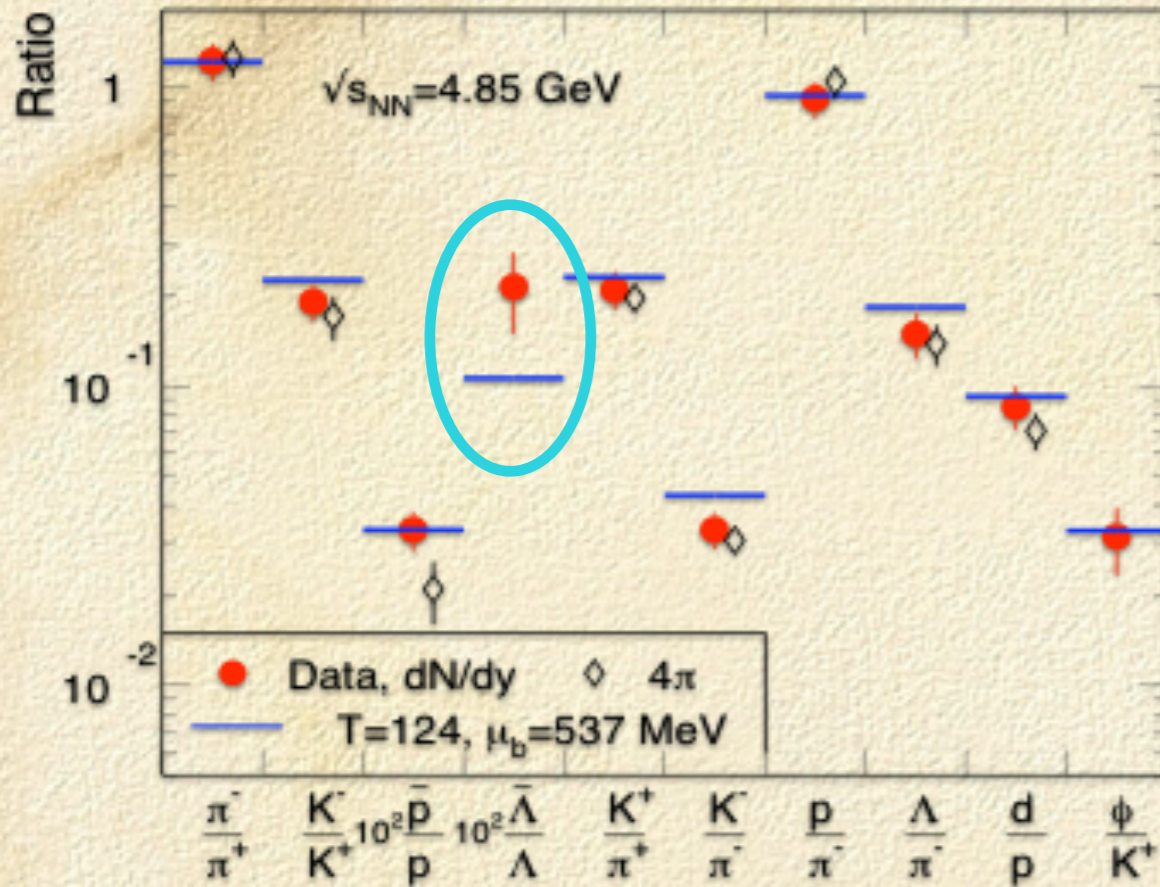
of local fit parameters for each collision energy = 3 (no γ_S factor)
 T, μ_B, μ_{I3}
 Total # for 14 energies = 42

of fit parameters with γ_S factor is 4
 Total # for 14 energies = 56

of global fit parameters = 4
 $R_{\pi}, R_K, R_{\text{mesons}}, R_{\text{baryons}}$

Results for Ratios (AGS)

There is **NO** anti Lambda problem here and all ratios are well described!

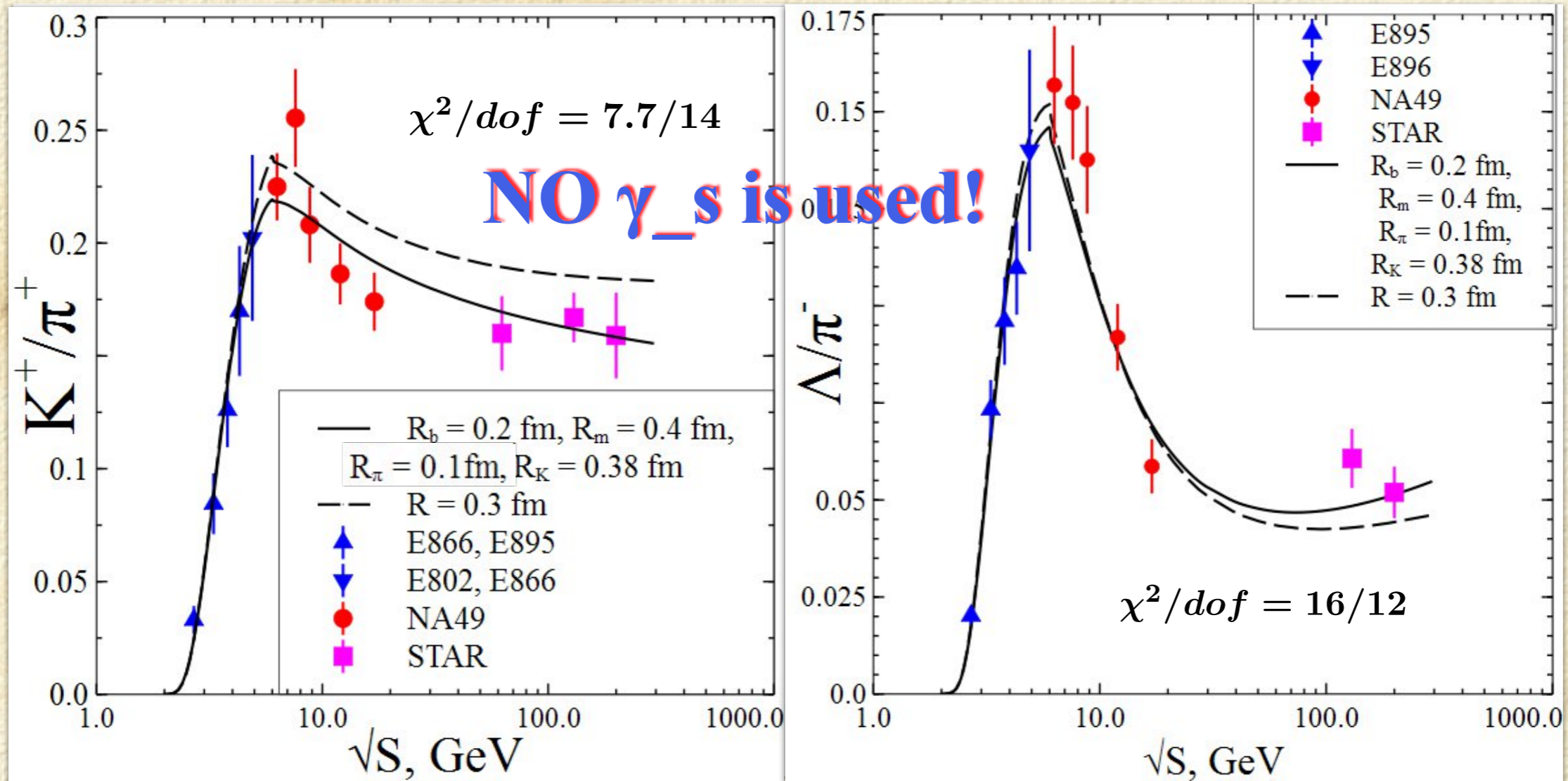


There is an anti Lambda problem!
Also K-/K+ and K/pi and Lambda/pi-
are not well described!

$T \simeq 131 \text{ MeV}$, $\mu_B \simeq 539 \text{ MeV}$, $\mu_{I3} \simeq -16 \text{ MeV}$

A. Andronic, P. Braun-Munzinger, J. Stachel, K.A.B., D.R. Oliinychenko, A.S. Sorin, G.M. Zinovjev,
 NPA (2006)777 Eur. Phys. J. A 49 (2013), 30--1-8.

Description of Horns at SPS



Best global fit of all ratios gives $R_\pi=0.1$ fm, $R_K=0.38$ fm,
 $\chi^2/dof=1.16$ for fixed: $R_{\text{baryons}}=0.2$ fm, $R_{\text{mesons}}=0.4$ fm

Note that Lambda and other hyperons can be described better!

K. A. Bugaev, D. R. Oliinychenko, A. S. Sorin and G. M. Zinovjev, Simple Solution to the Strangeness Horn Description Puzzle, Eur. Phys. J. A 49 (2013), 30--1-8:

Strangeness Enhancement as Deconfinement Signal

In 1982 J. Rafelski and B. Müller predicted that **enhancement of strangeness** production is a signal of deconfinement. **Phys. Rev. Lett. 48(1982)**

In 1991 J. Rafelski introduced strangeness fugacity **γ_S factor** **Phys. Lett. 62(1991)**

which quantifies strange charge chemical **oversaturation** (>1) or
strange charge chemical **undersaturation** (<1)

Idea: if s-(anti)quarks are created at QGP stage, then their number should not be changed during further evolution since s-(anti)quarks number is small and since density decreases \Rightarrow there is no chance for their annihilation!

Hence, we should observe chemical enhancement of strangeness with $\gamma_S > 1$

However, until 2013 the situation with strangeness was unclear:

P. Braun-Munzinger & Co found that γ_S factor is about 1

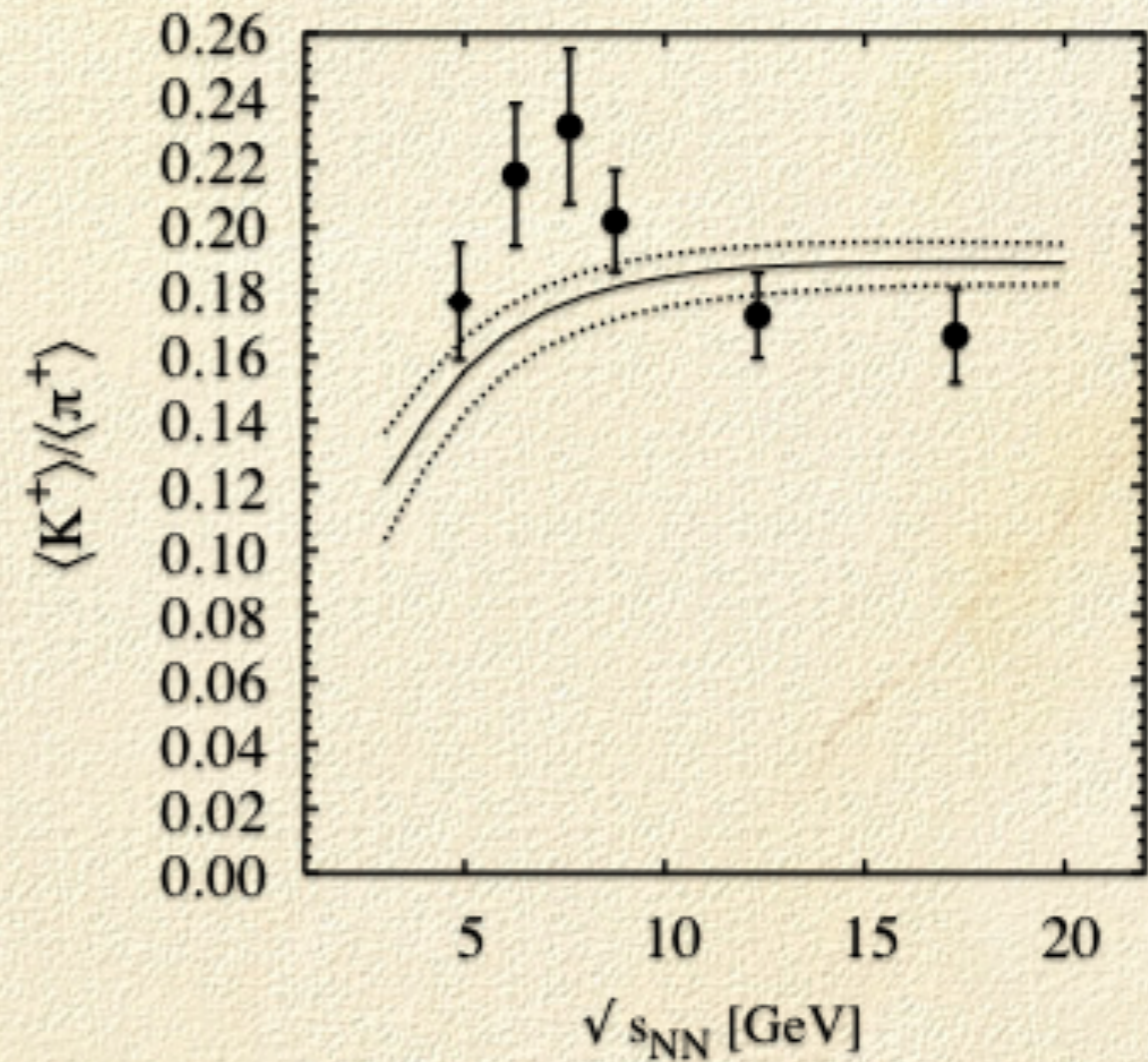
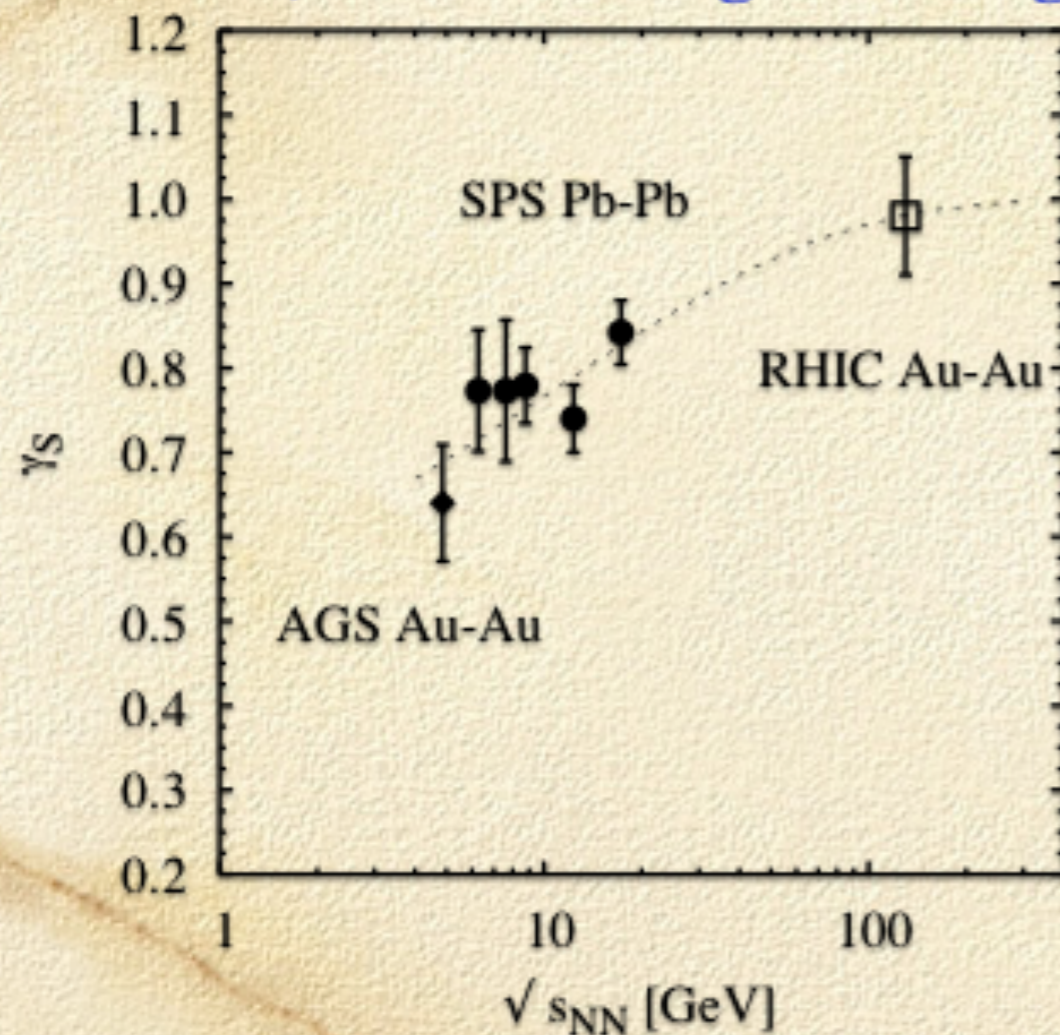
F. Becattini & Co found that γ_S factor is < 1

Systematics of Strangeness Suppression

Include γ_s factor $\phi_i(T) \rightarrow \phi_i(T)\gamma_s^{s_i}$, into thermal density

where s_i is number of strange valence quarks plus number of strange valence anti-quarks.

Thus, it is a strangeness fugacity

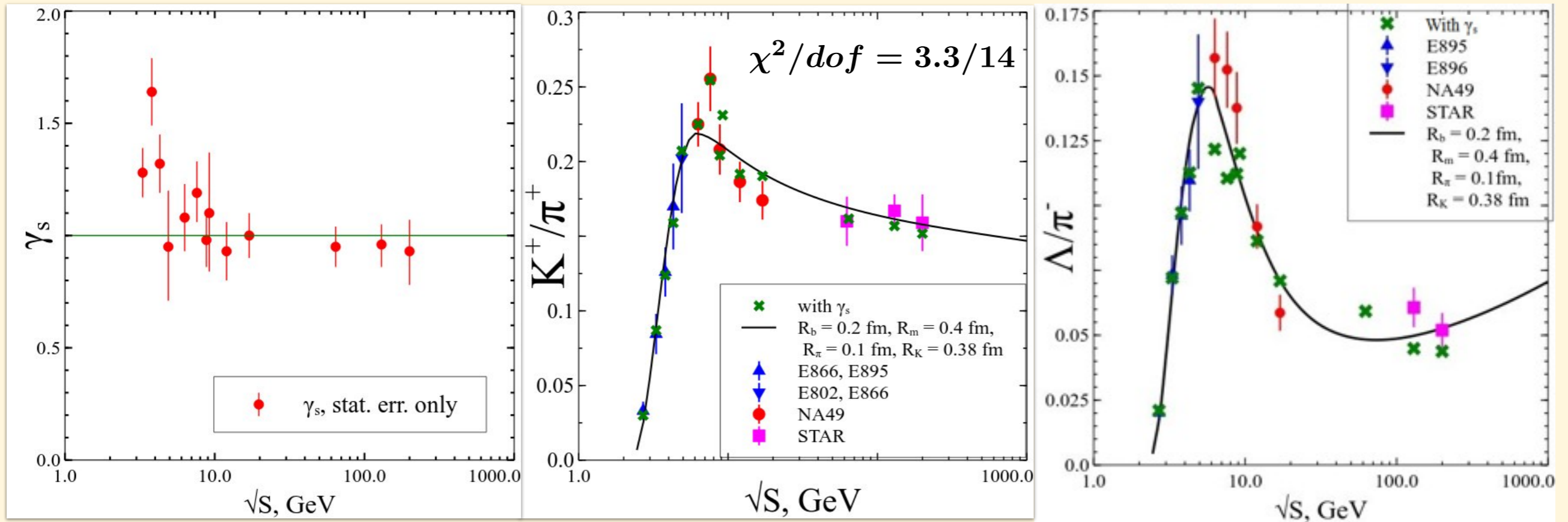


Single component model **F. Becattini, J. Manninen and M. Gazdzicki, PRC 73 (2006) 044905**

Typical values of $\chi^2/\text{dof} > 2$ at given energy!

Our Results on Strangeness Enhancement in 2013

High quality description of hadron multiplicities requires T , μ_B , μ_{I3} and γ_s factor



$\chi^2/dof = 1.15$ for 111 ratios measured for c.m. energies 2.7--200 GeV

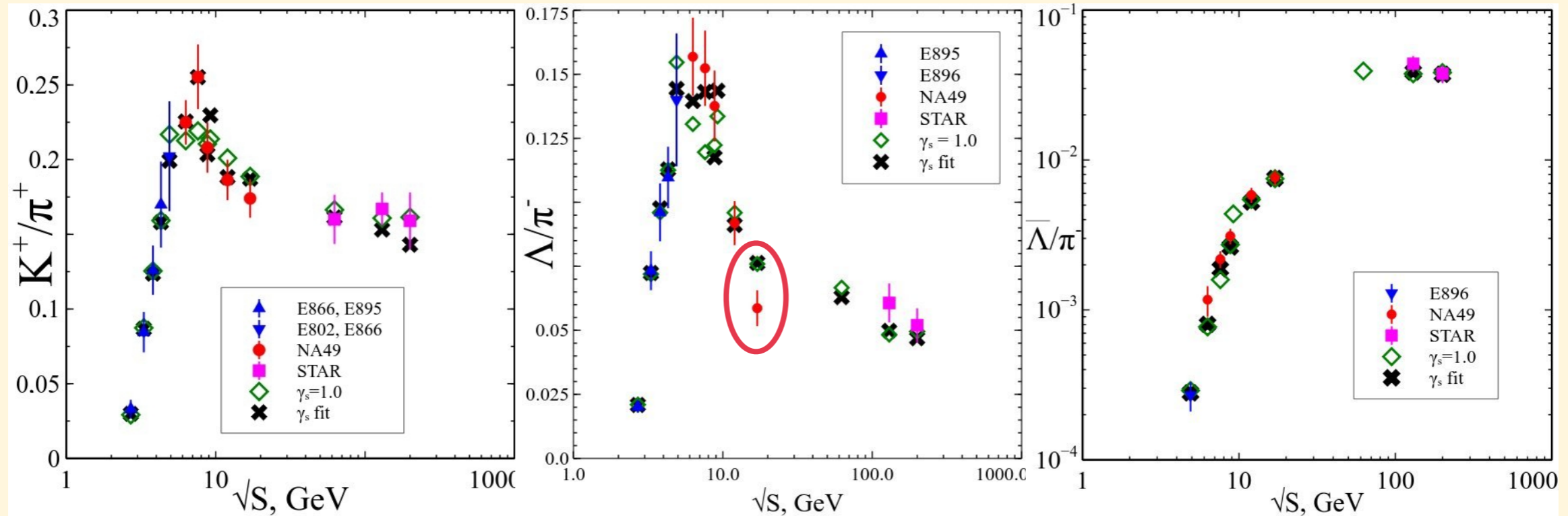
K.A. Bugaev, D. R. Oliinychenko, J. Cleymans, A.I. Ivanytskyi, I.N. Mishustin, E.G. Nikonov and V.V. Sagun, *Europhys. Lett.* 104, 22002, (2013) p.1-6

Strangeness enhancement exists where we do not expect deconfinement!

Solving problem with Kaons leads to (anti) Λ selective suppression!

Strangeness Horn and Λ Horn in 2014

To avoid selective suppression of Λ -hyperons we added their hard-core radius



$$\chi^2/14 = 3.9/14$$

$$\chi^2/12 = 10.22/12$$

$$\chi^2/8 = 6.49/8$$

$R_{\pi} = 0.1$ fm, $R_{\Lambda} = 0.1$ fm, $R_b = 0.36$ fm, $R_K = 0.38$ fm, $R_m = 0.4$ fm

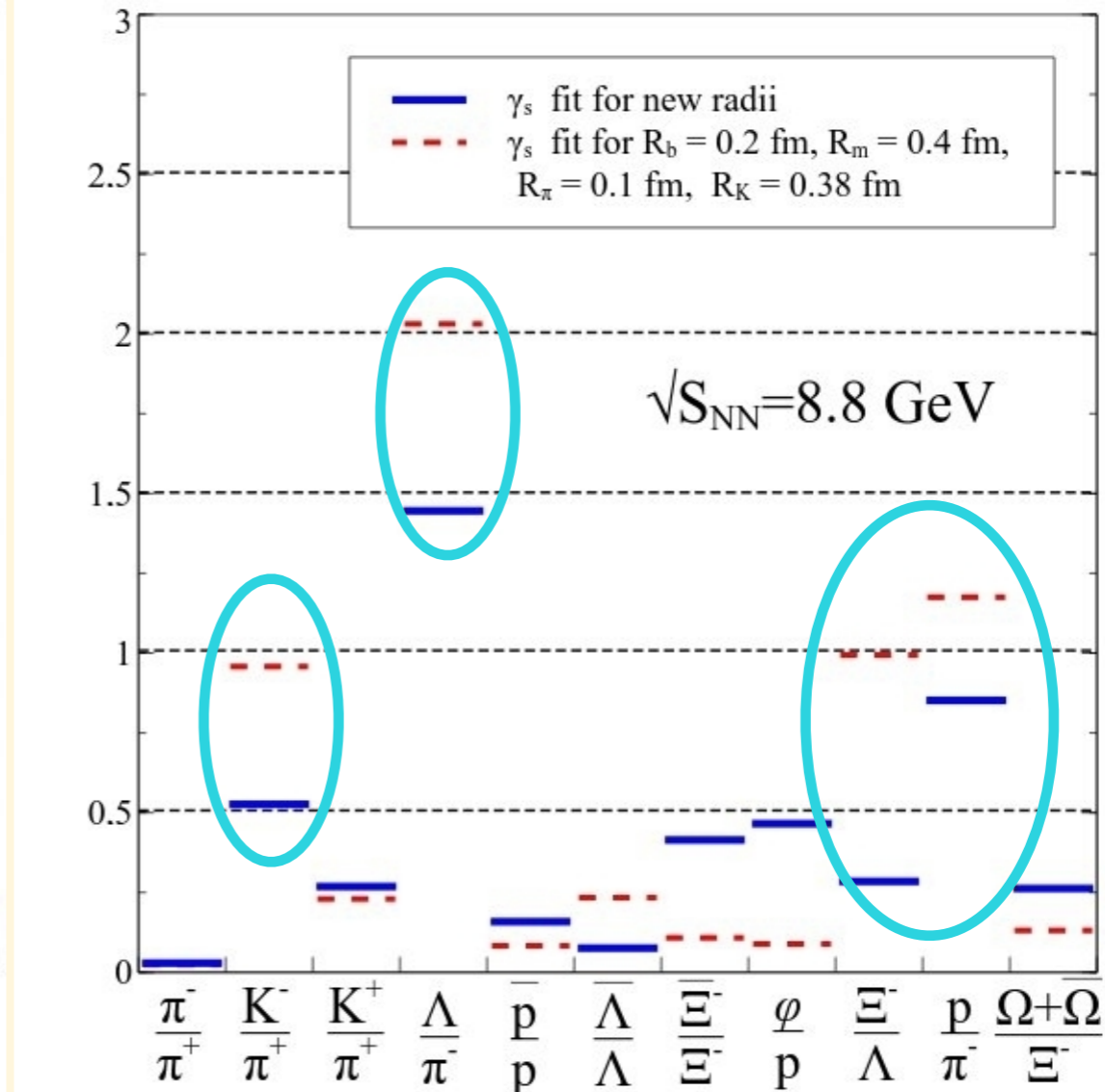
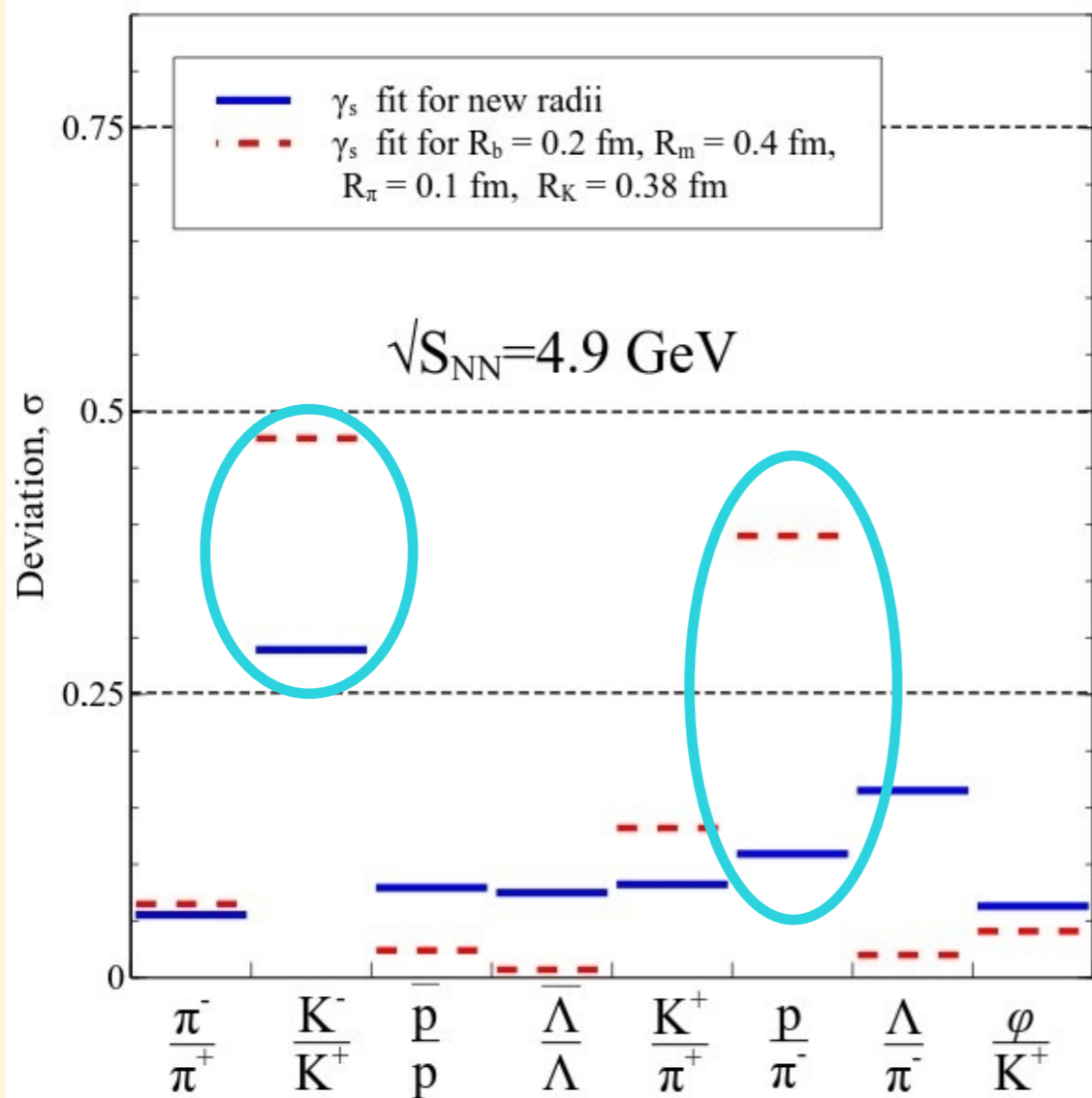
Total fit of 111 independent hadron ratios is the best of existing!

V. V. Sagun, *Ukr. J. Phys.* 59, No 8, 755-763 (2014)

V. V. Sagun et al., *Ukr. J. Phys.* 59, No 11, 1043-1050 (2014)

$$\chi^2 / dof = 52/55 \simeq 0.95.$$

Strangeness Horn and Λ Horn in 2014

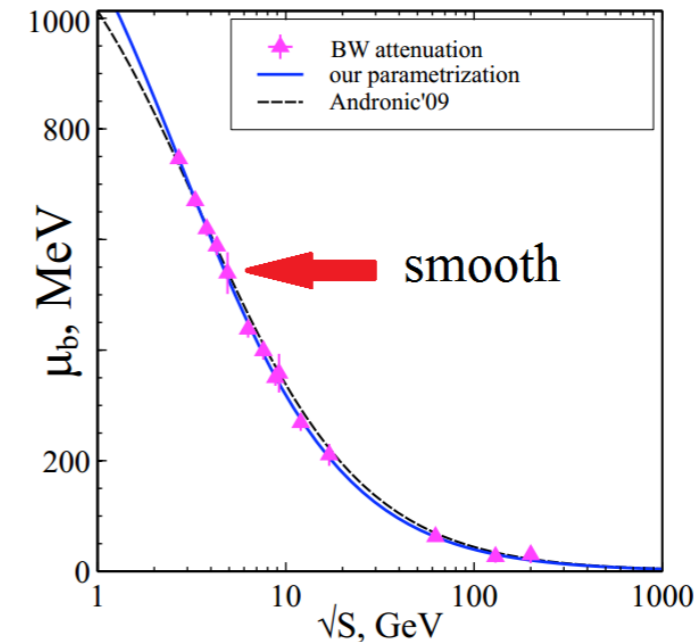
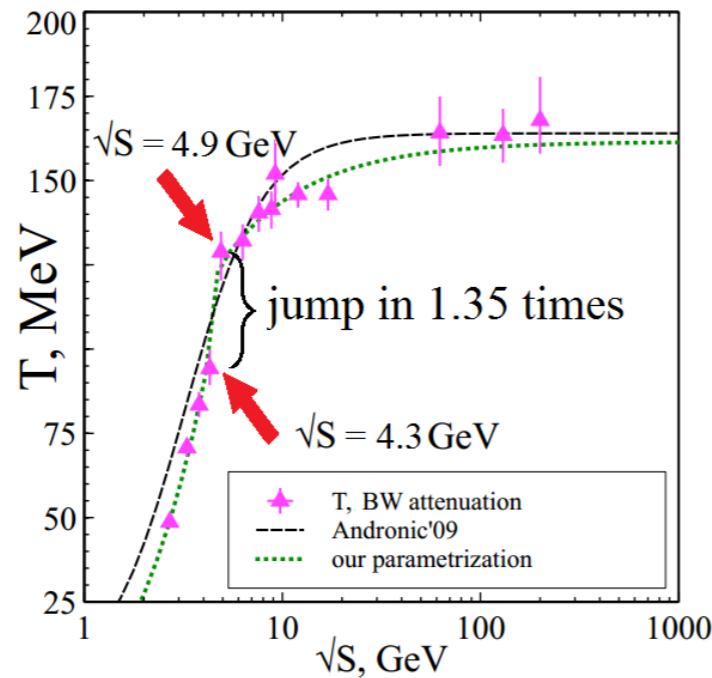


Intermediate Conclusions

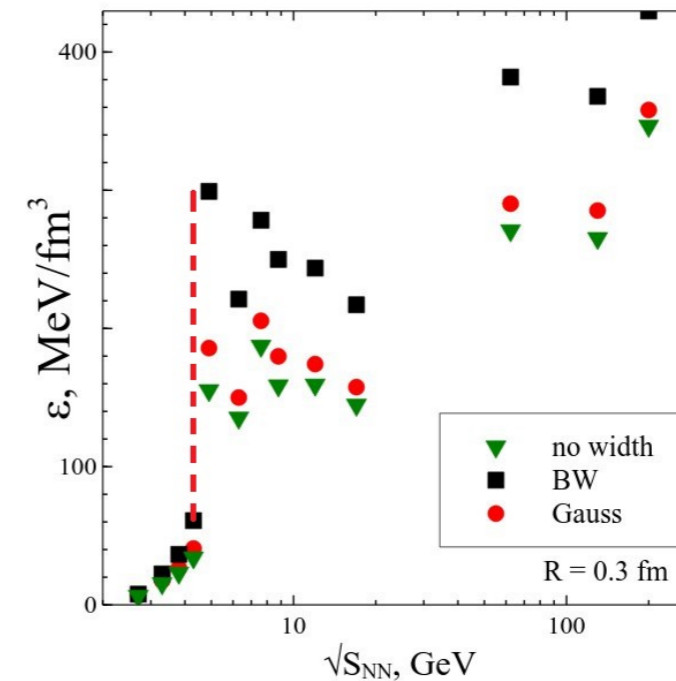
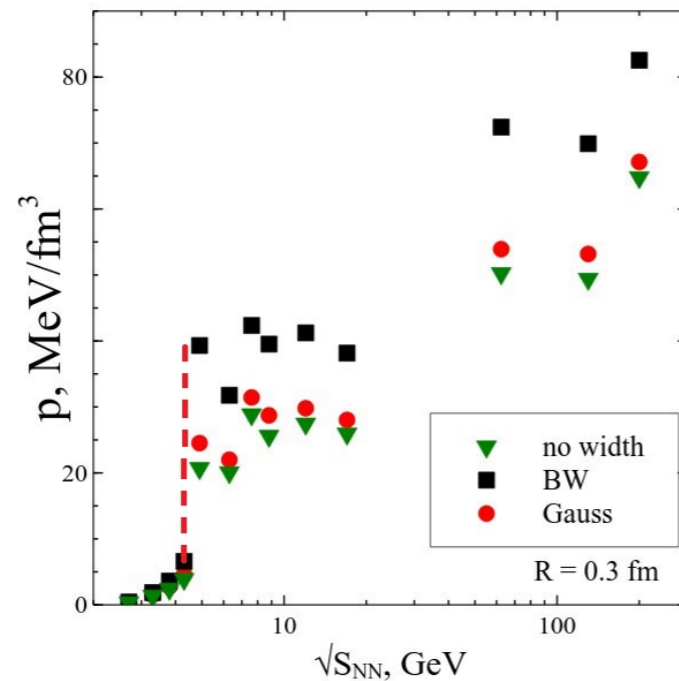
1. The multicomponent HRG model is a precise tool of HIC phenomenology
2. With high confidence we conclude that chemical enhancement of strangeness exists at very low energies where we do not expect deconfinement
3. Studies of chemical freeze-out require that ANY realistic EoS of hadron matter must reproduce HRG model results at chemical freeze-out (**important for NICA & FAIR**)
4. Using multicomponent HRG model we can study thermodynamics at chemical freeze out

Jump of ChFO Pressure at AGS Energies

- Temperature T_{CFO} as a function of collision energy \sqrt{s} is rather non smooth



- Significant jump of pressure ($\simeq 6$ times) and energy density ($\simeq 5$ times)

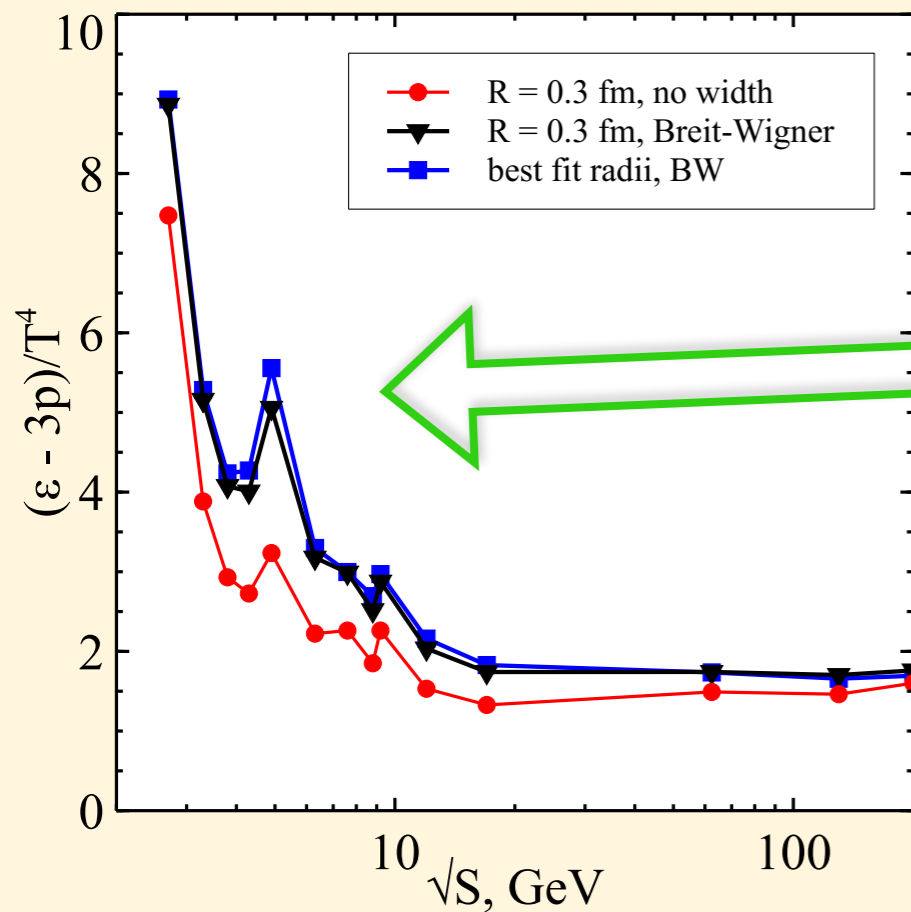


K.A. Bugaev et al., Phys. Part. Nucl. Lett. 12(2015) [arXiv:1405.3575];

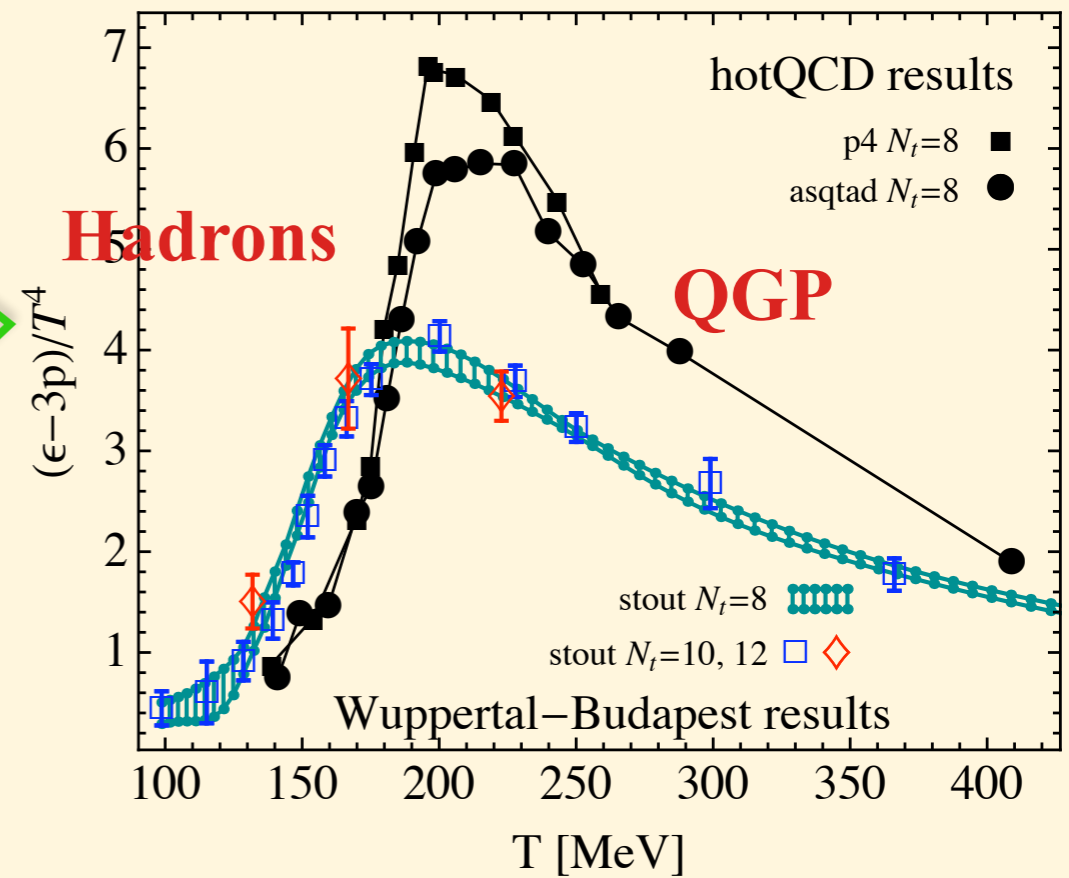
Ukr. J. Phys. 60 (2015) [arXiv:1312.4367]

Trace Anomaly Peaks

At chemical FO (large μ)



Lattice QCD (vanishing μ)



K.A. Bugaev et al., arXiv:1412.0718 [nucl-th]

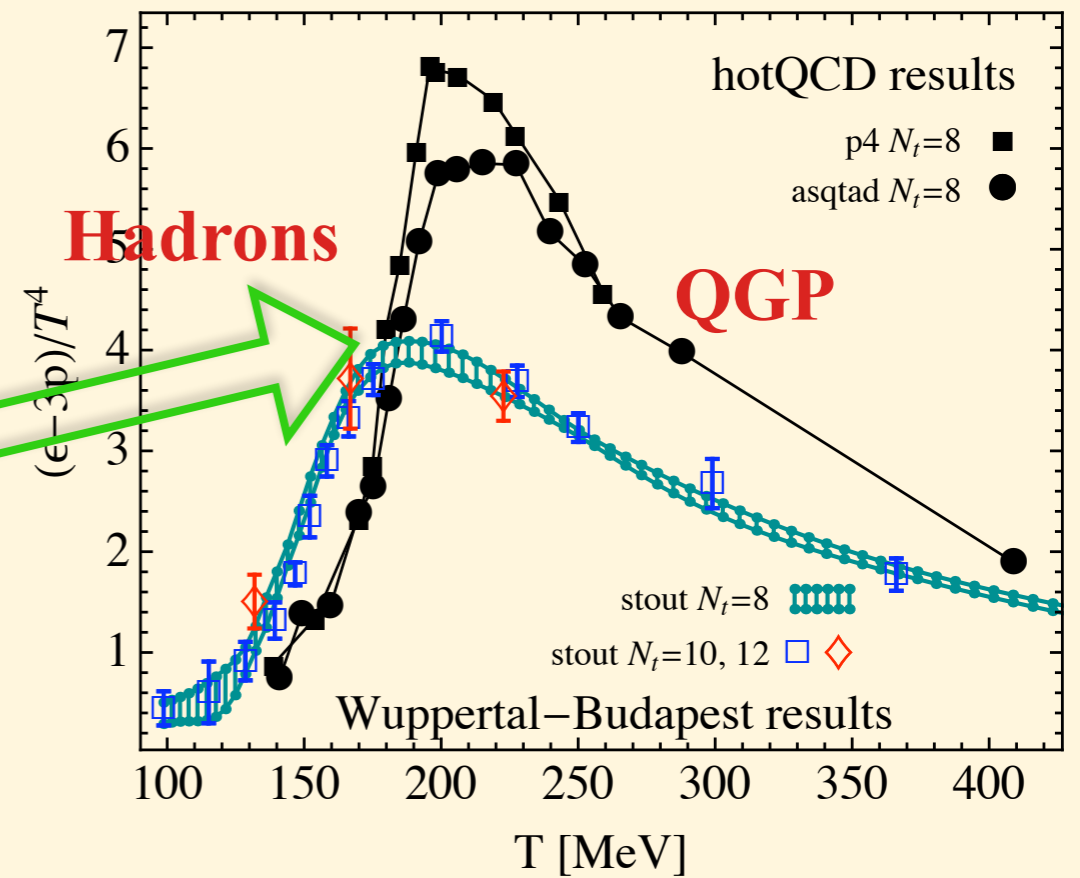
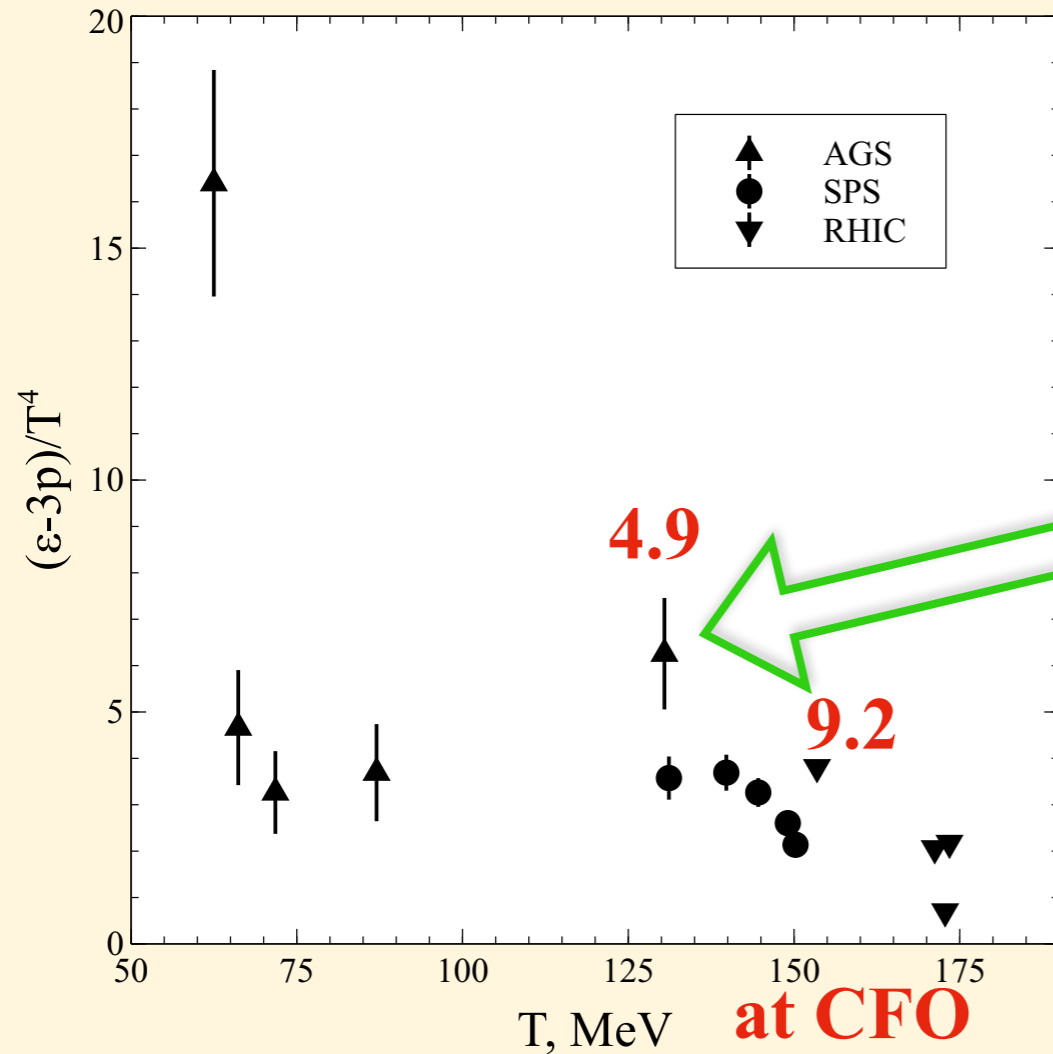
WupBud EOS arxiv: lat 1007.2580

Are these trace anomaly peaks related to each other?

Trace Anomaly Peaks (Most Recent)

At chemical FO (large μ)

Lattice QCD (vanishing μ)



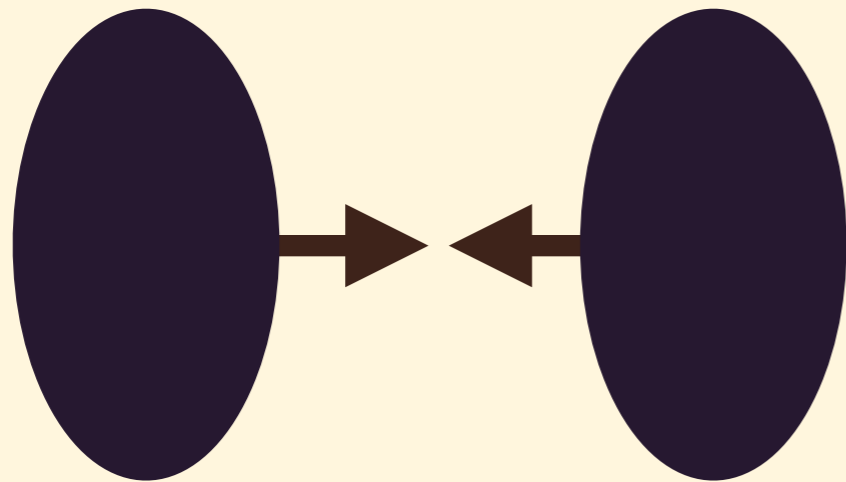
WupBud EOS arXiv: lat 1007.2580

Model from V.V. Sagun et al., arXiv:1703.00009 [hep-ph]

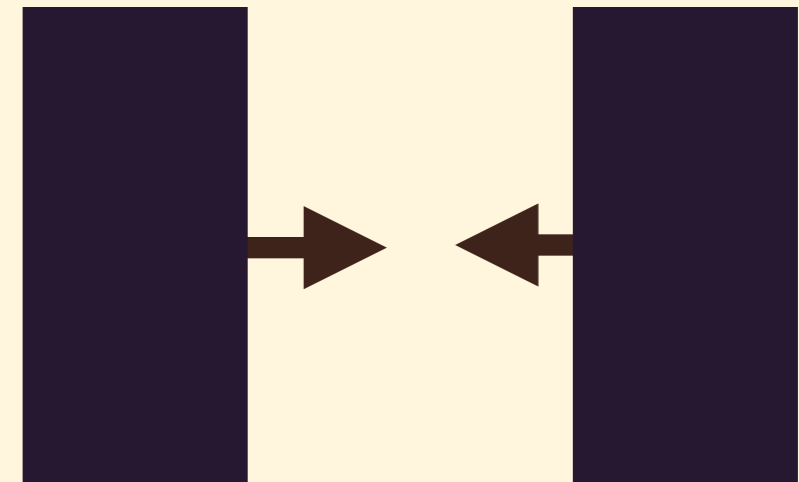
Are these trace anomaly peaks related to each other?

Shock Adiabatic Model for A+A Collisions

A+A central collision at $1 < E_{\text{lab}} < 30$



Its hydrodynamic model



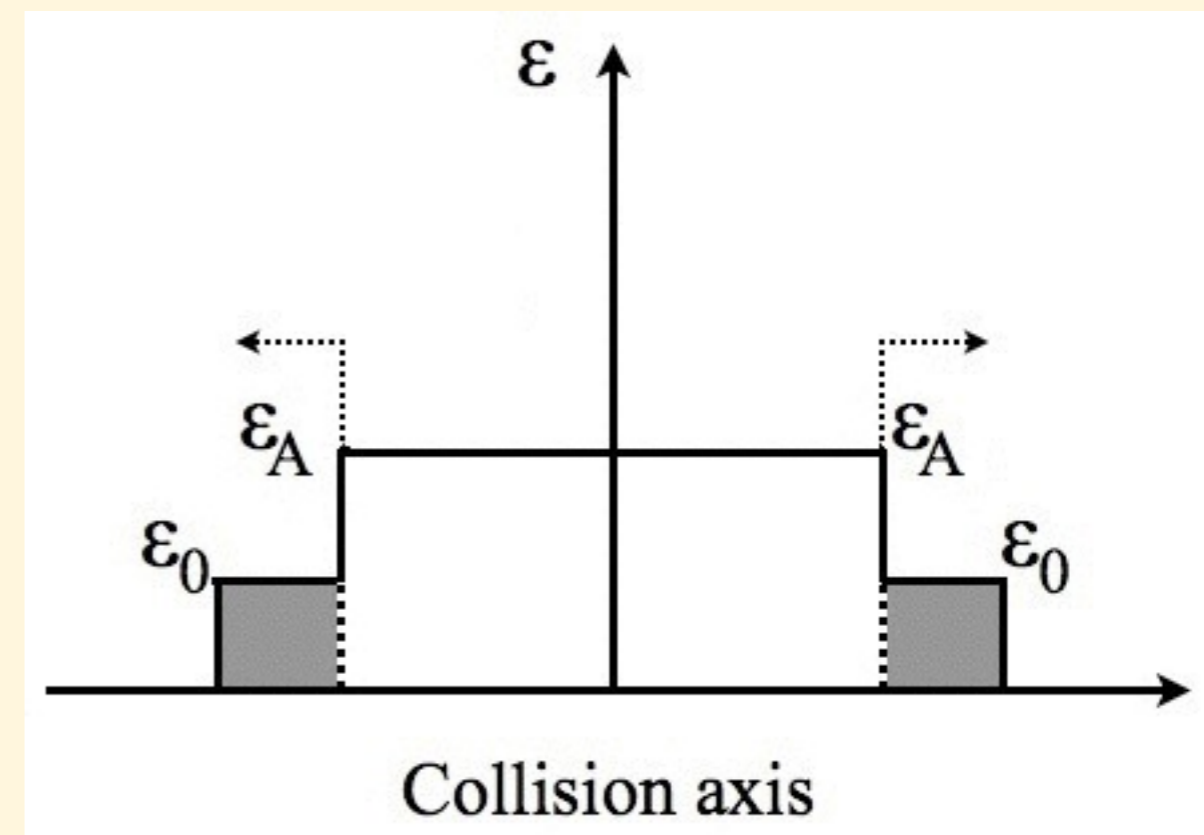
Works reasonably well at these energies.

H. Stoecker and W. Greiner, Phys. Rep. 137 (1986)

Yu.B. Ivanov, V.N. Russkikh, and V.D. Toneev,

Phys. Rev. C 73 (2006)

From hydrodynamic point of view
this is a problem of
arbitrary discontinuity decay:
in normal media there appeared
two shocks moving outwards



Medium with Normal and Anomalous Properties

Normal properties, if $\Sigma \equiv \left(\frac{\partial^2 p}{\partial X^2} \right)_{s/\rho_B}^{-1} > 0$ = convex down:

Usually pure phases (Hadron Gas, QGP)
have normal properties

$X = \frac{\varepsilon + p}{\rho_B^2}$ – generalized specific volume

ε is energy density, p is pressure,

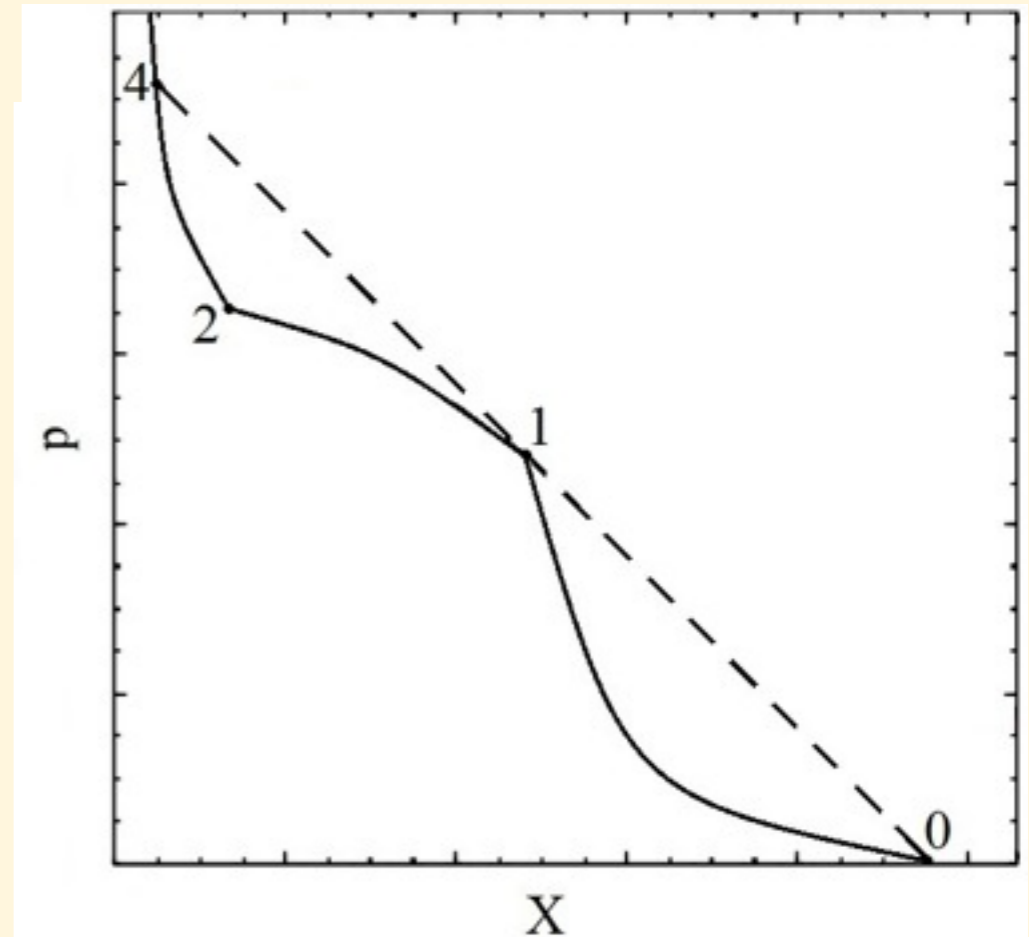
ρ_B is baryonic charge density

Anomalous properties otherwise.

**Almost in all substances
with liquid-gas phase transition
the mixed phase has anomalous properties!**

**Then shock transitions to mixed phase
are unstable and more complicated flows
are possible.**

Shock adiabat example

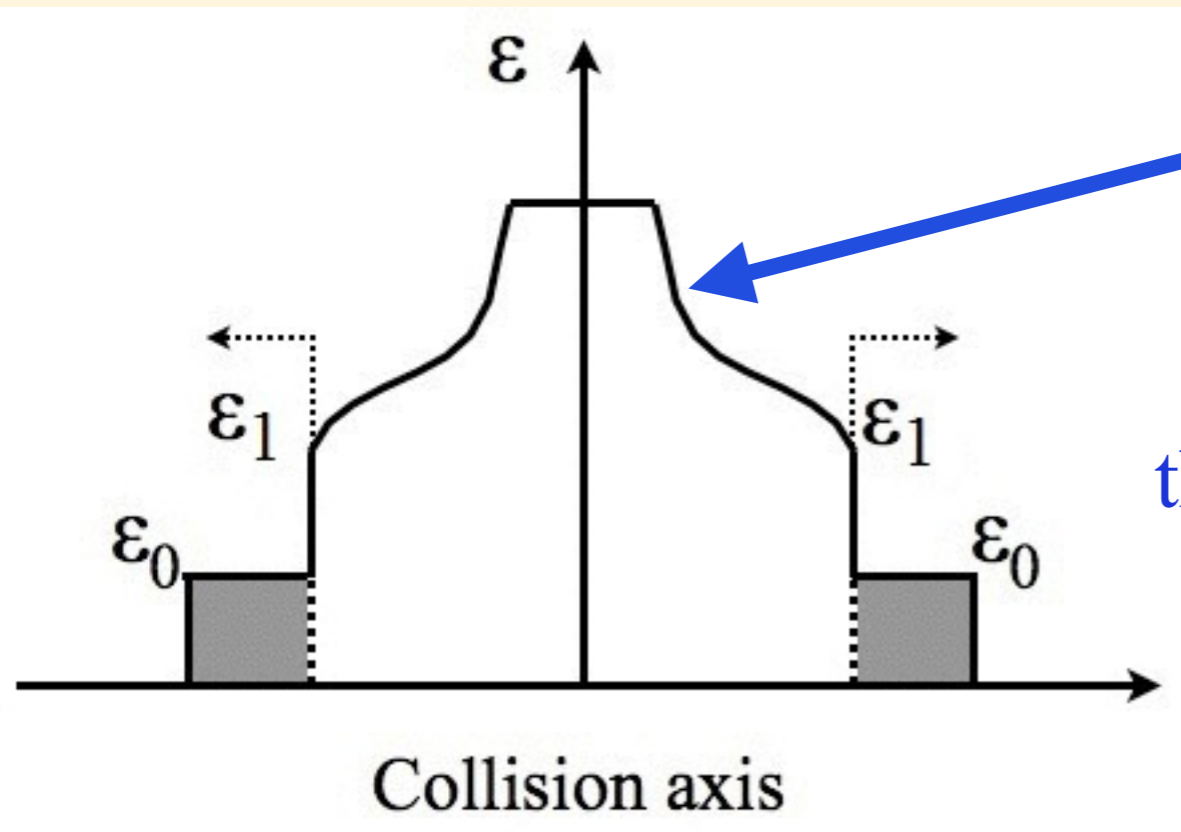


Region 1-2 is mixed
phase with **anomalous
properties.**

Generalized Shock Adiabats Model

In case of unstable shock transitions more complicated flows appear:

K.A. Bugaev, M.I. Gorenstein, B. Kampher, V.I. Zhdanov, Phys. Rev. D 40, 9, (1989)
 K.A. Bugaev, M.I. Gorenstein, D.H. Rischke, Phys. Lett. B 255, 1, 18 (1991)



shock 01 + compression simple wave

In each point of simple wave $\frac{s}{\rho_B} = \text{const}$

If during expansion entropy conserves, then unstable parts lead to entropy plateau!

Remarkably

Z model has stable RHT adiabat, which leads to quasi plateau!

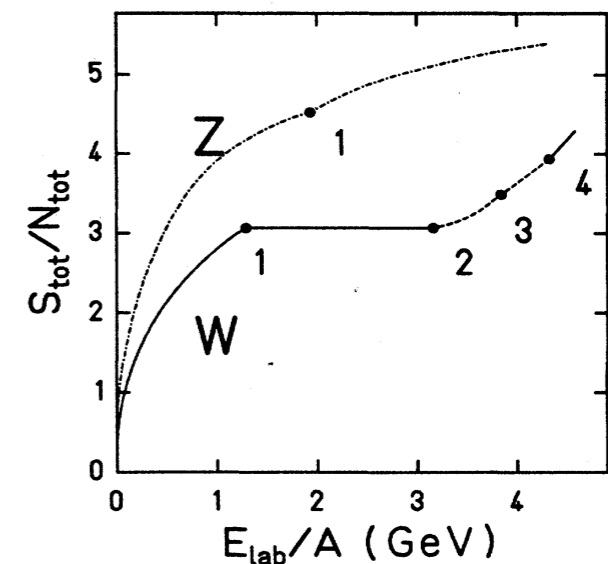


FIG. 9. The entropy per baryon as a function of the bombarding energy per nucleon of the colliding nuclei for models W and Z. The points 1, 2, 3, 4 on curve W correspond to those on the generalized adiabat as displayed in Fig. 7. The point 1 on curve Z marks the boundary to the mixed phase.

Correlated Quasi-Plateaus

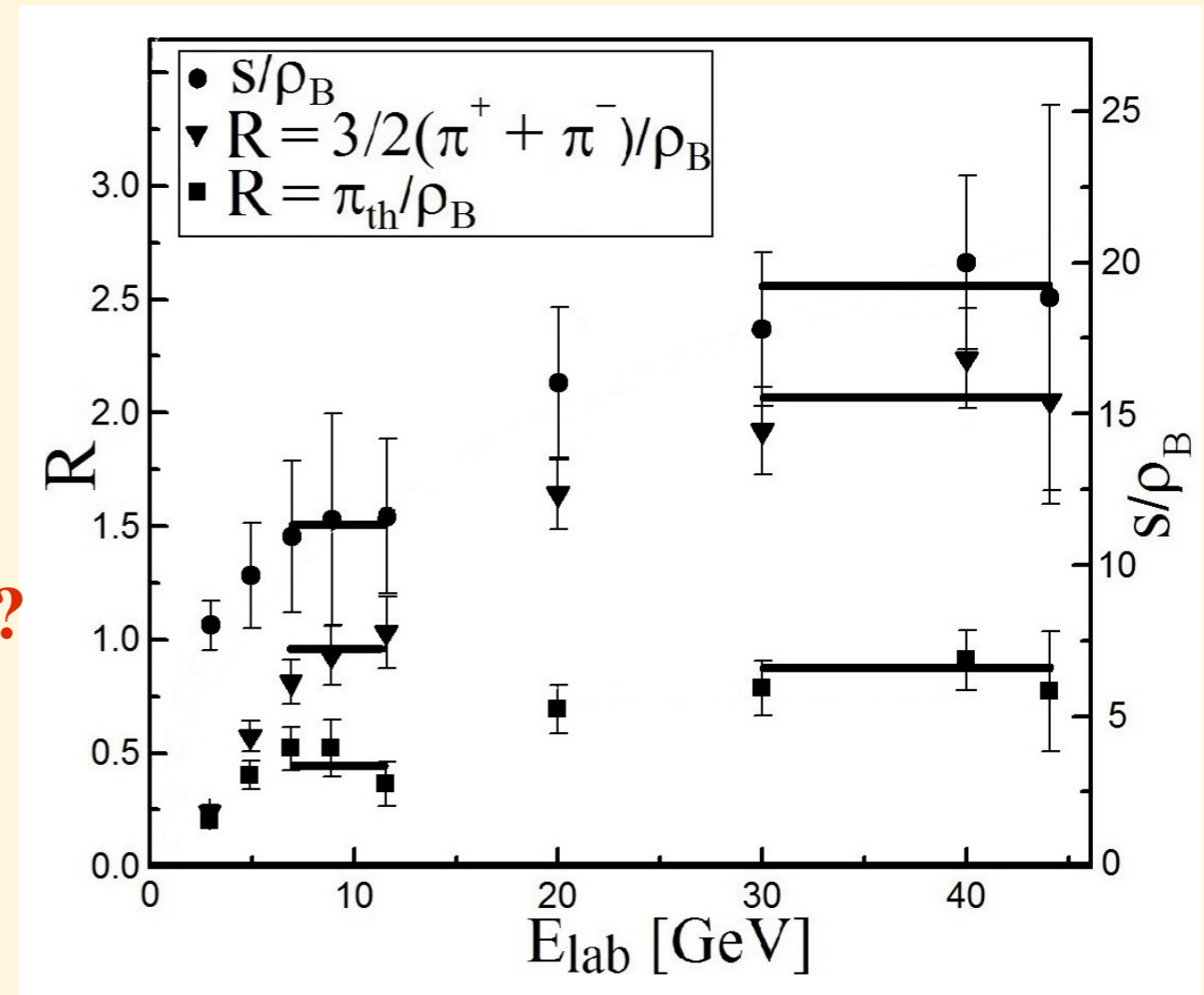
Since the main part of the system entropy is defined by thermal pions =>
thermal pions/baryon should have a plateau!

Also the total number of **pions per baryons should have a (quasi)plateau!**

**Entropy per baryon has wide plateaus
due to large errors**

Quasi-plateau in total pions per baryon ?

Thermal pions demonstrate 2 plateaus

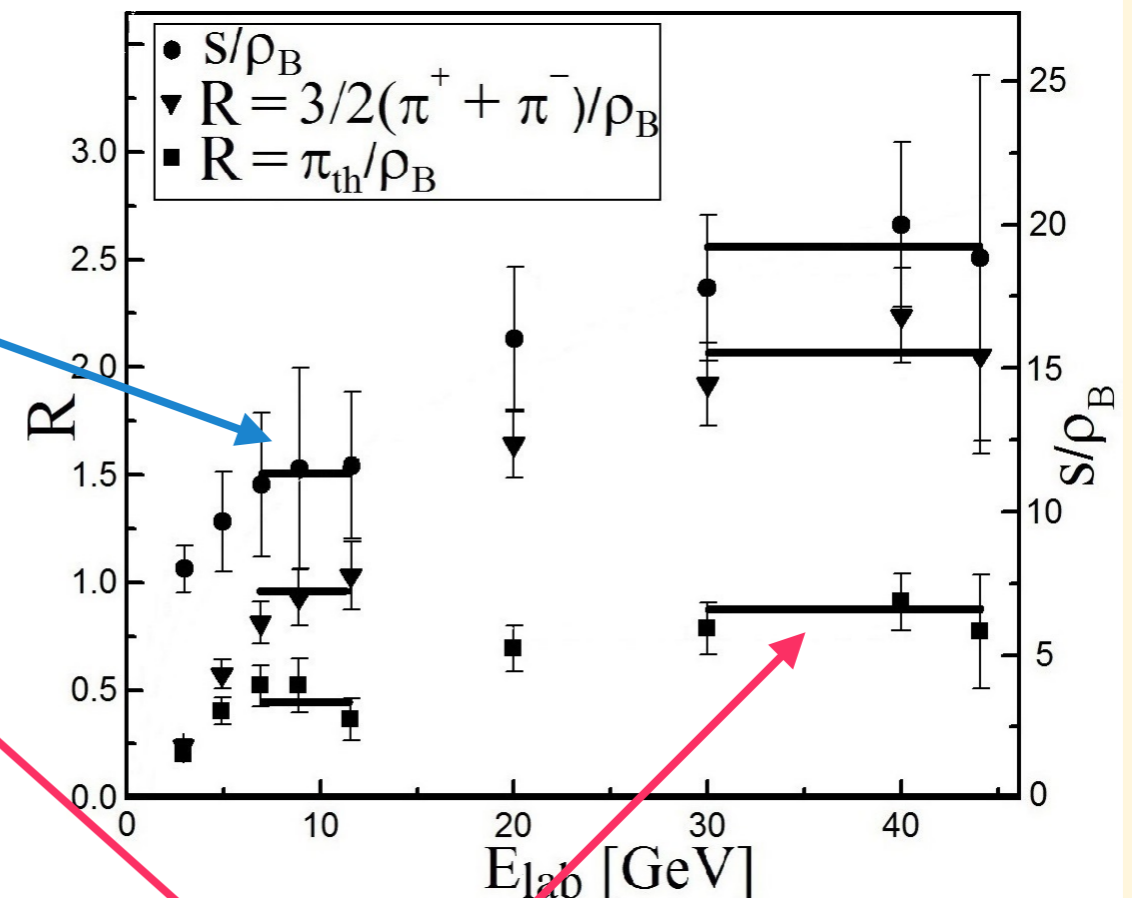


Details on Highly Correlated Quasi-Plateaus

- Common width M – number of points belonging to each plateau
- Common beginning i_0 – first point of each plateau
- For every M , i_0 minimization of χ^2/dof yields $A \in \{s/\rho_B, \rho_\pi^{\text{th}}/\rho_B, \rho_\pi^{\text{tot}}/\rho_B\}$:

$$\chi^2/\text{dof} = \frac{1}{3M-3} \sum_A \sum_{i=i_0}^{i_0+M-1} \left(\frac{A - A_i}{\delta A_i} \right)^2 \Rightarrow A = \frac{\sum_{i=i_0}^{i_0+M-1} \frac{A_i}{(\delta A_i)^2}}{\sum_{i=i_0}^{i_0+M-1} \frac{1}{(\delta A_i)^2}}$$

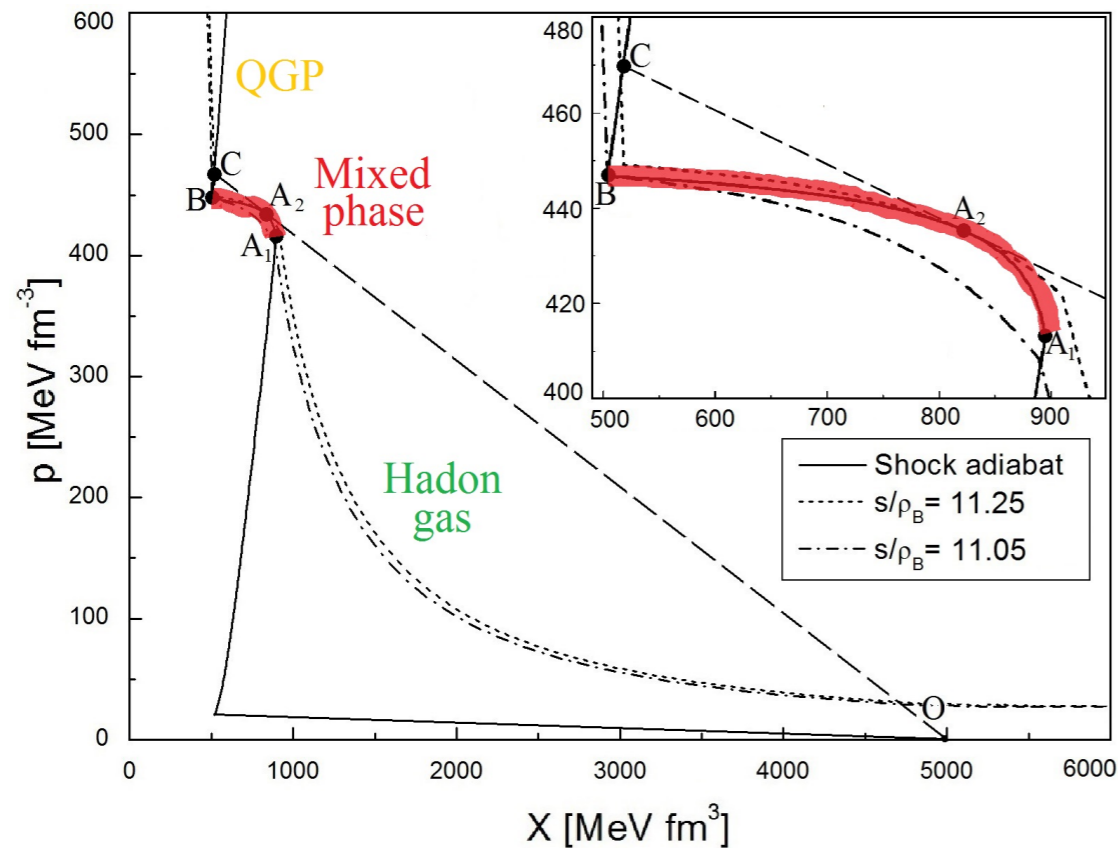
Low energy plateau					
M	i_0	s/ρ_B	$\rho_\pi^{\text{th}}/\rho_B$	$\rho_\pi^{\text{tot}}/\rho_B$	χ^2/dof
2	3	11.12	0.52	0.85	0.17
3	3	11.31	0.46	0.89	0.53
4	2	10.55	0.43	0.72	1.64
5	2	11.53	0.47	0.84	4.45
High energy plateau					
2	8	19.80	0.88	2.20	0.12
3	7	18.77	0.83	2.05	0.34
4	6	17.82	0.77	1.87	0.87
5	5	16.26	0.64	1.62	3.72



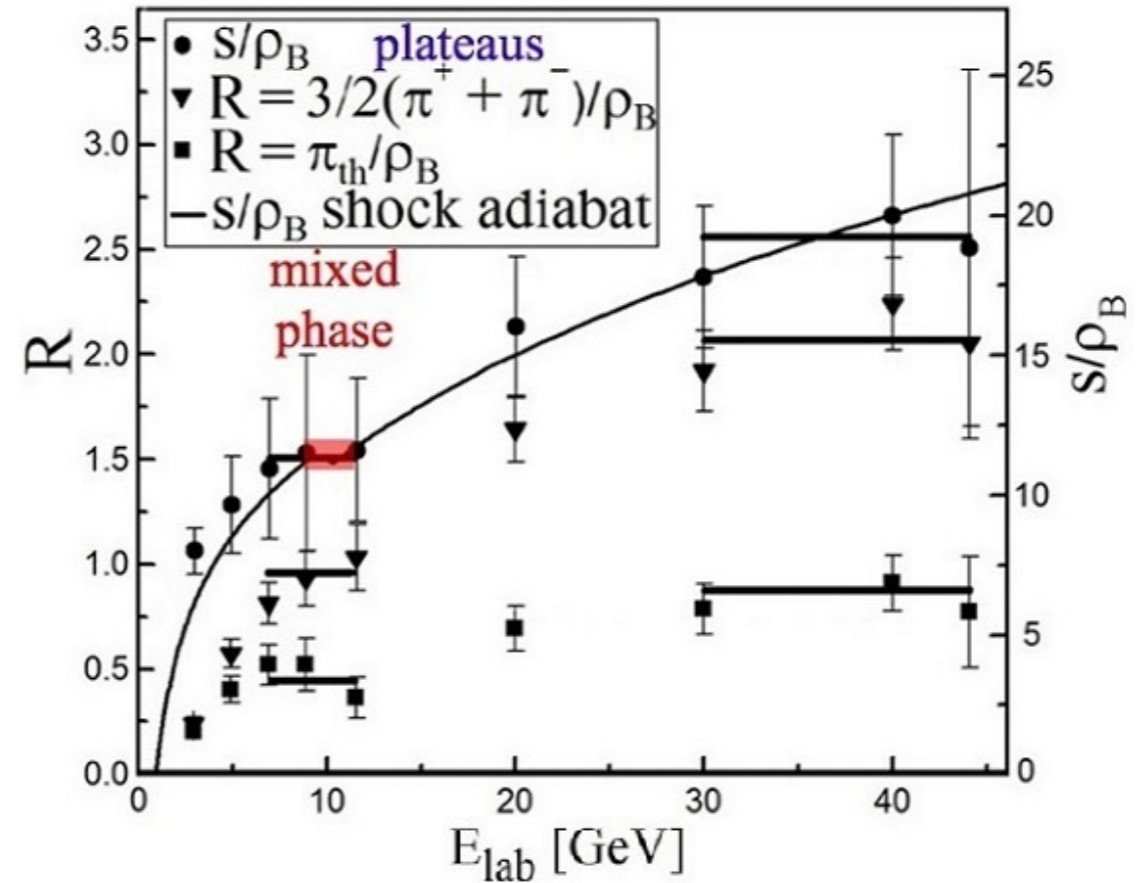
Unstable Transitions to Mixed Phase

$$X = \frac{\varepsilon + p}{\rho_B^2} - \text{generalized specific volume}$$

other PT?



K.A. Bugaev et al., arXiv:1405.3575[hep-ph]



GSA Model explains irregularities at CFO as a signature of mixed phase

QGP EOS is MIT bag model with coefficients been fitted with condition $T_c = 150$ MeV at vanishing baryonic density!

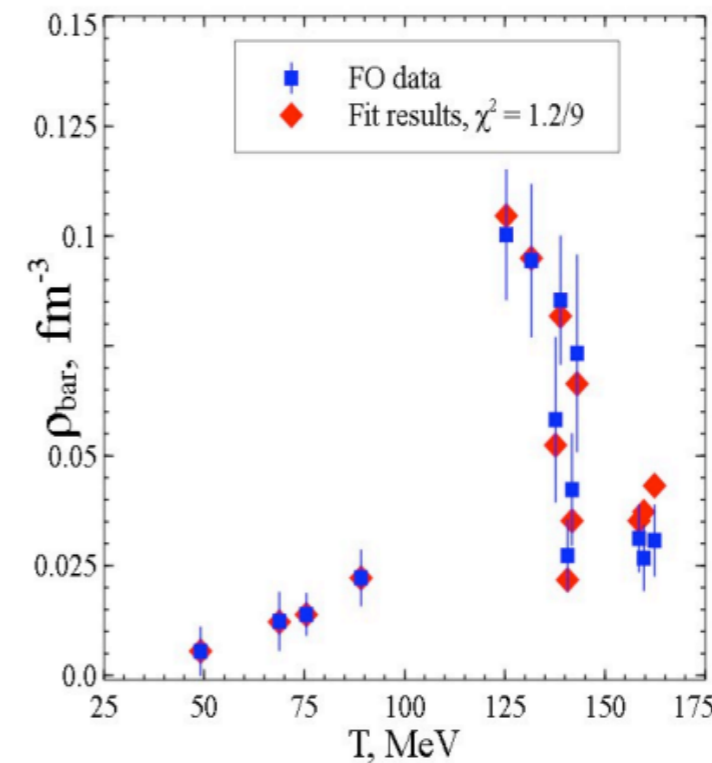
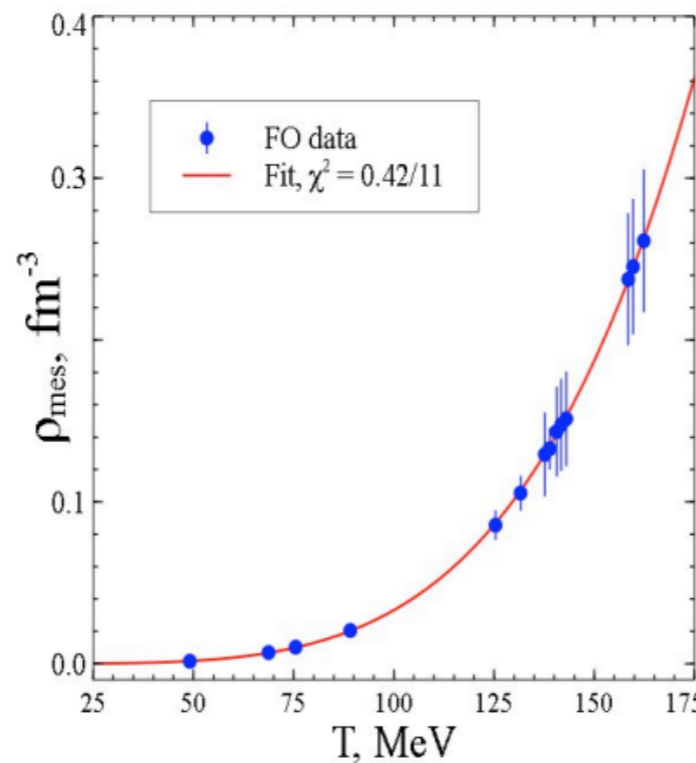
HadronGas EOS is simplified HRGM discussed above.

Details of Hadronic and QGP EOS

- Summation of hadronic spectrum \Rightarrow (anti)baryonic and mesonic contributions

$$p = \left[\overbrace{2C_B T^{A_B} \text{ch} \left(\frac{\mu}{T} \right) e^{-\frac{m_B}{T}}}^{(\text{anti})\text{baryons}} + \overbrace{C_M T^{A_M} e^{-\frac{m_M}{T}}}^{\text{mesons}} \right] e^{-\frac{pV_H}{T}}$$

- Effective EoS describes (anti)baryonic and mesonic densities at CFO

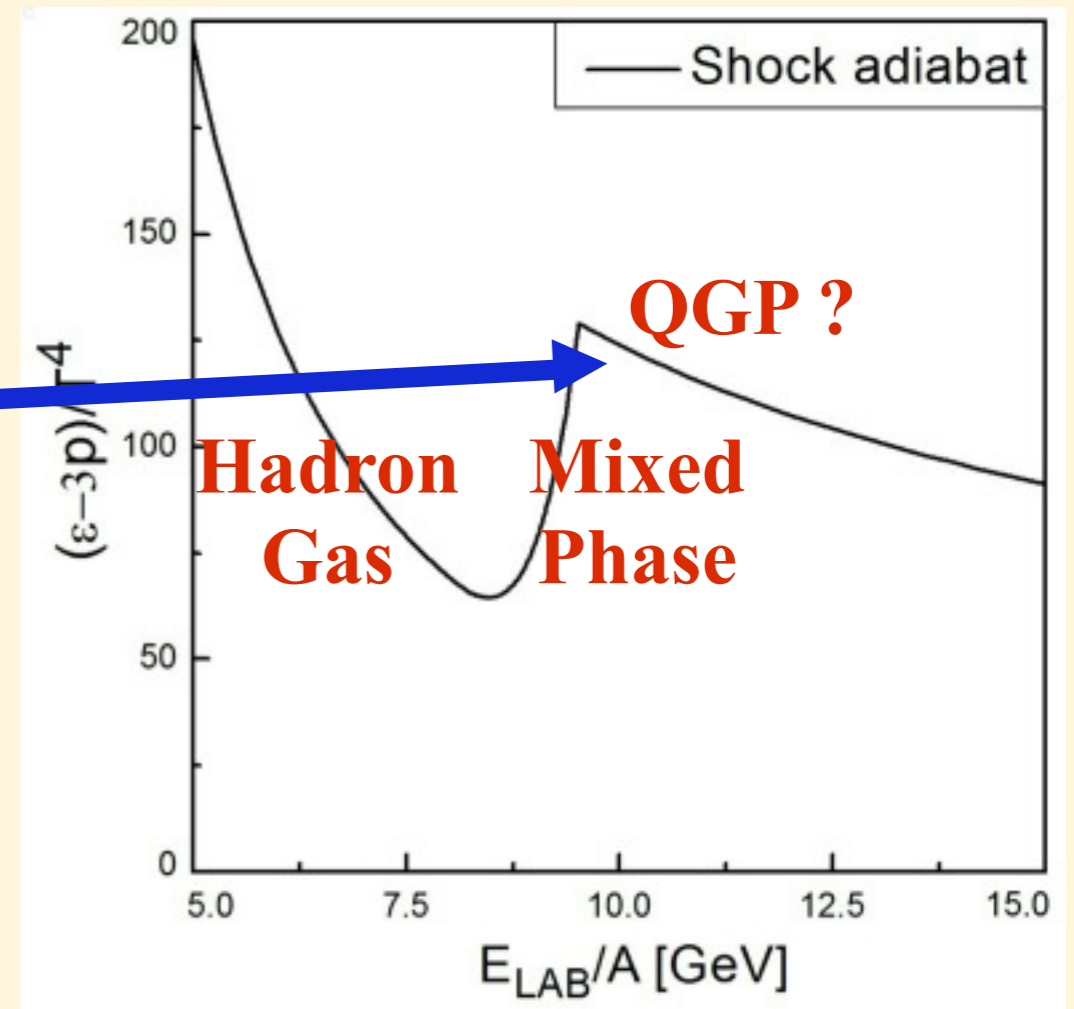
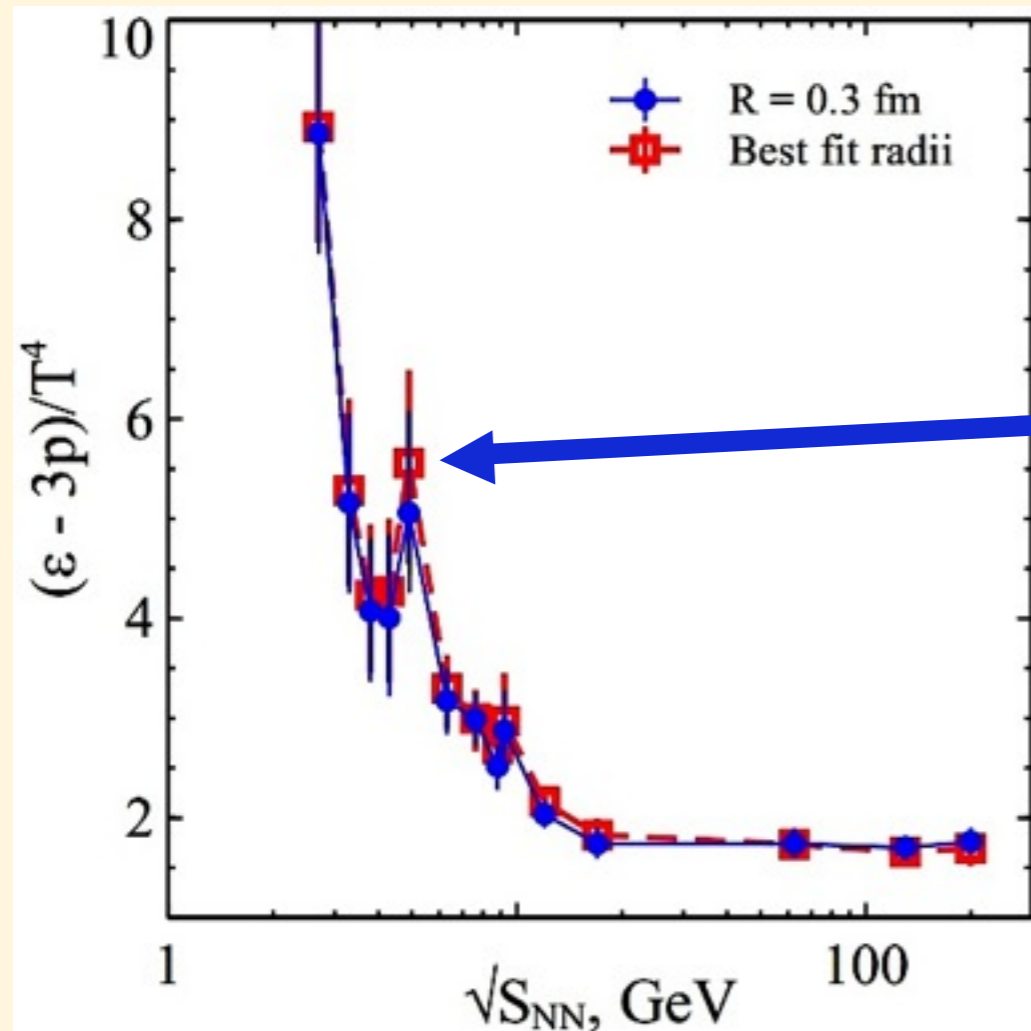


K.Bugaev et al. PoS Baldin ISHEPP XXI (2012) 017, arXiv:1212.0132 [hep-ph]

$$p_{QGP} = \underbrace{A_0 T^4 + A_2 T^2 \mu^2 + A_4 \mu^4 - B}_{\text{fitting}} = \underbrace{A_0^L T^4 + A_2^L T^2 \mu^2 + A_4^L \mu^4 - B_{eff}}_{LQCD}$$

$$B_{eff}(T, \mu_B) = B - (A_0 - A_0^L) T^4 - (A_2 - A_2^L) T^2 \mu^2 - (A_4 - A_4^L) \mu^4$$

Trace Anomaly Along Shock Adiabats 2016



K.A. Bugaev et al., EPJ A (2016)

We found one-to-one correspondence between these two peaks.

Thus, sharp peak of trace anomaly at c.m. energy 4.9 GeV evidences for mixed phase formation. But what is it?

Is second peak at c.m. energy 9.2 GeV due to another PT?

Induced Surface Tension EOS (2017)

Recall A. Ivanytskyi talk on EoS beyond the Van der Waals approximation

pressure $\frac{p}{T} = \sum_i \phi_i \exp\left(\frac{\mu_i - pV_i - \Sigma S_i}{T}\right)$ **new term**

induced surface tension $\frac{\Sigma}{T} = \sum_i R_i \phi_i \exp\left(\frac{\mu_i - pV_i - \Sigma S_i}{T}\right) \cdot \overbrace{\exp\left(\frac{(1 - \alpha)S_i \Sigma}{T}\right)}$

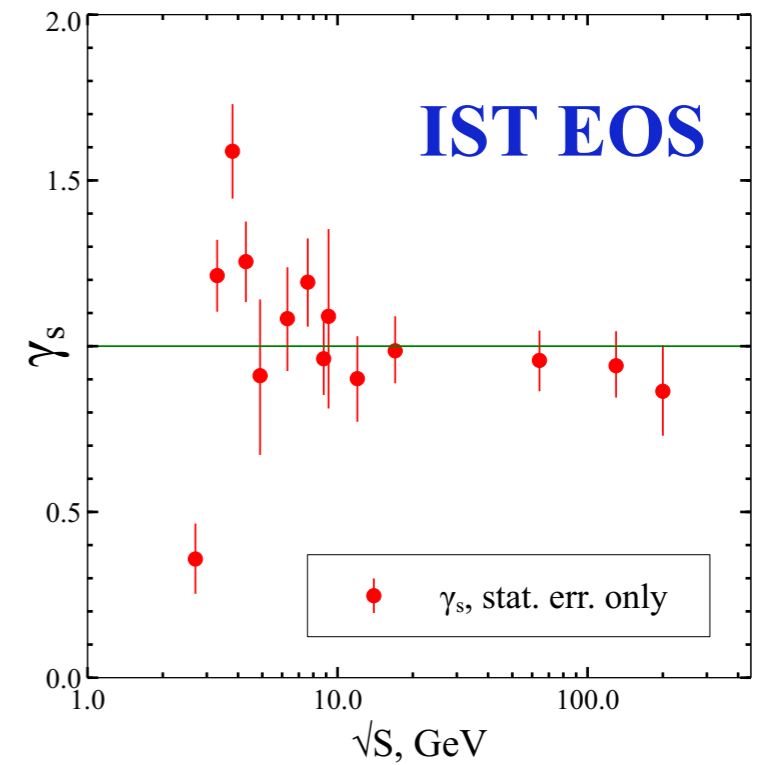
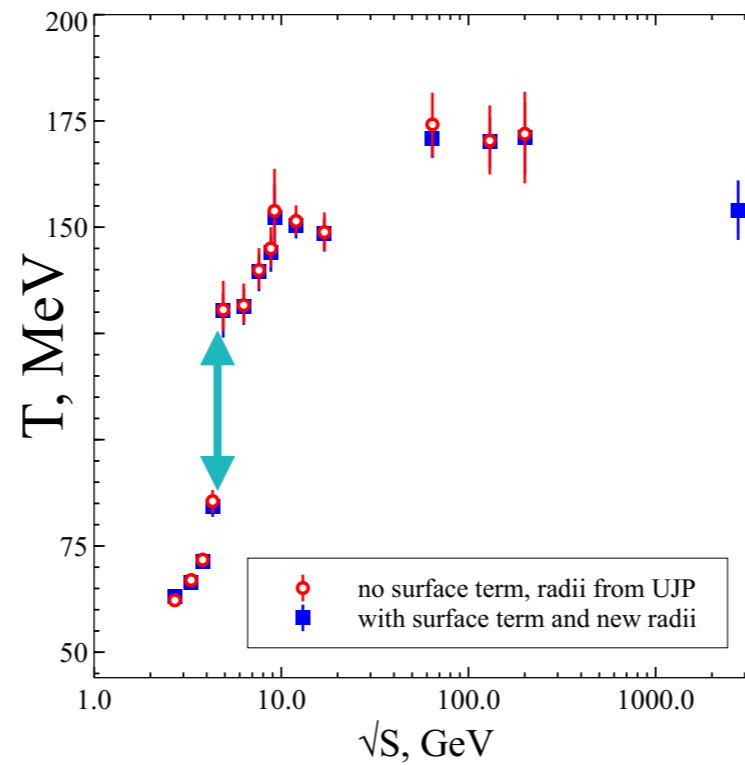
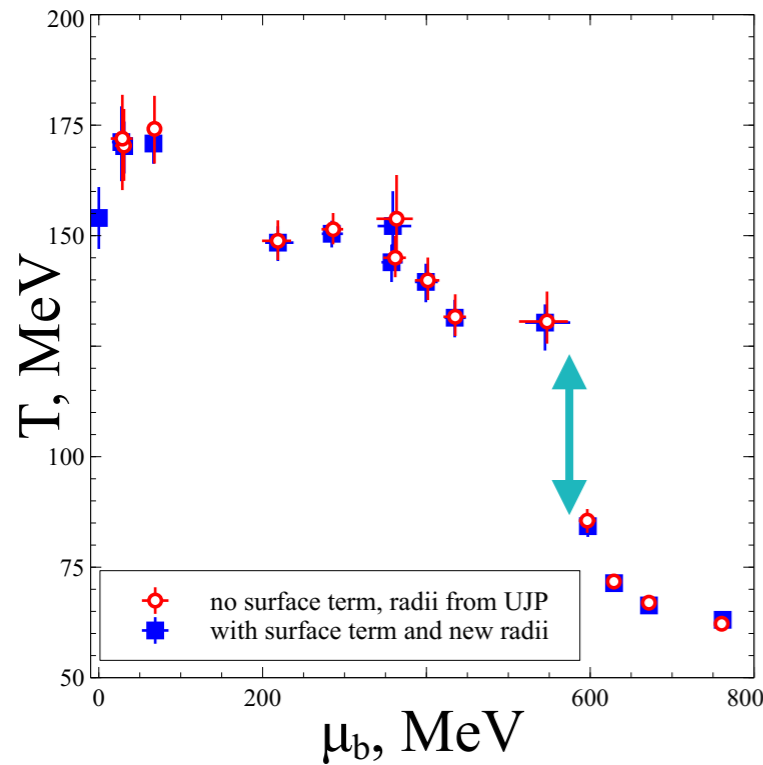
V_k and S_k are eigenvolume and eigensurface of hadron of sort k

α switches excluded and eigen volume regimes
high order virial coefficients?

Advantages

1. Allows to go beyond the Van der Waals approximation
2. Number of equations is 2 and it does not depend on the number different hard-core radii!

Main Results for AGS, SPS and RHIC energies



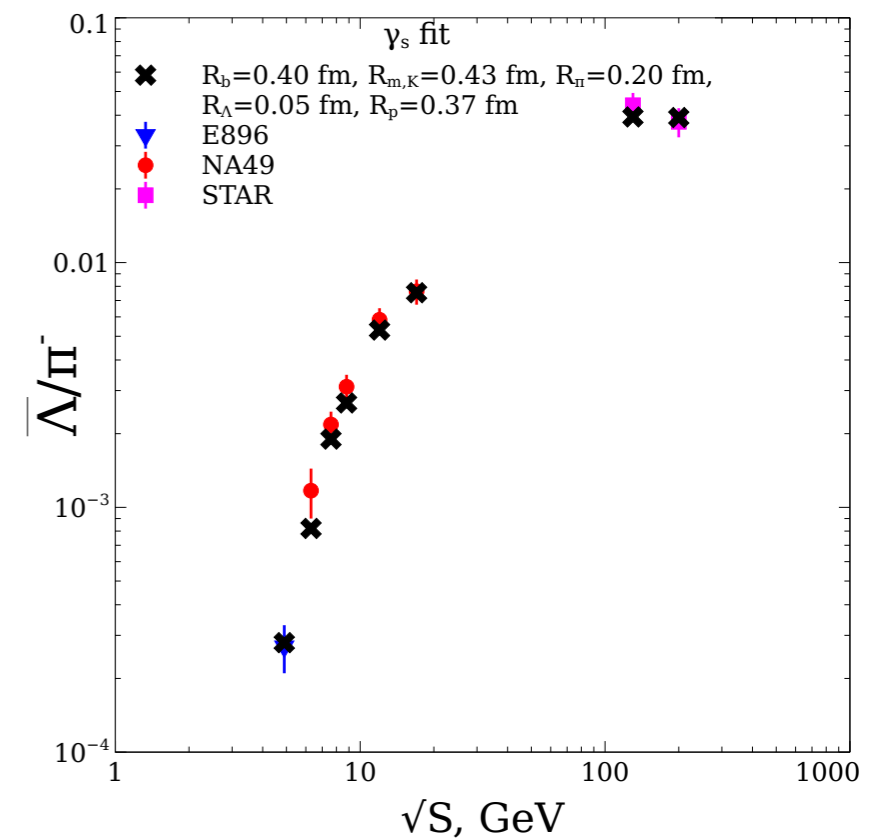
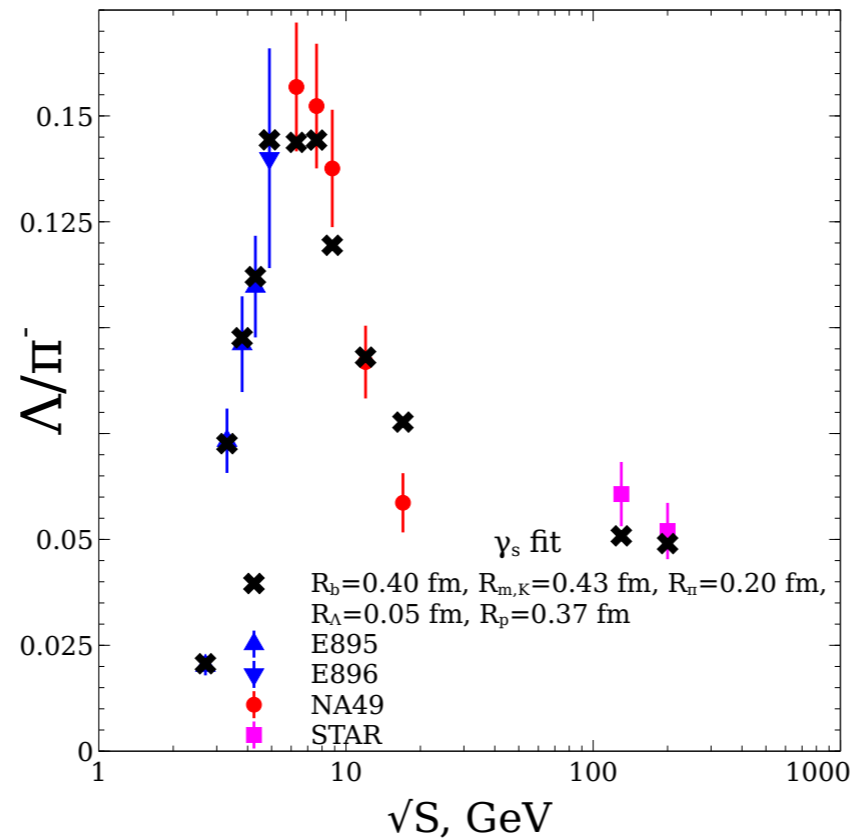
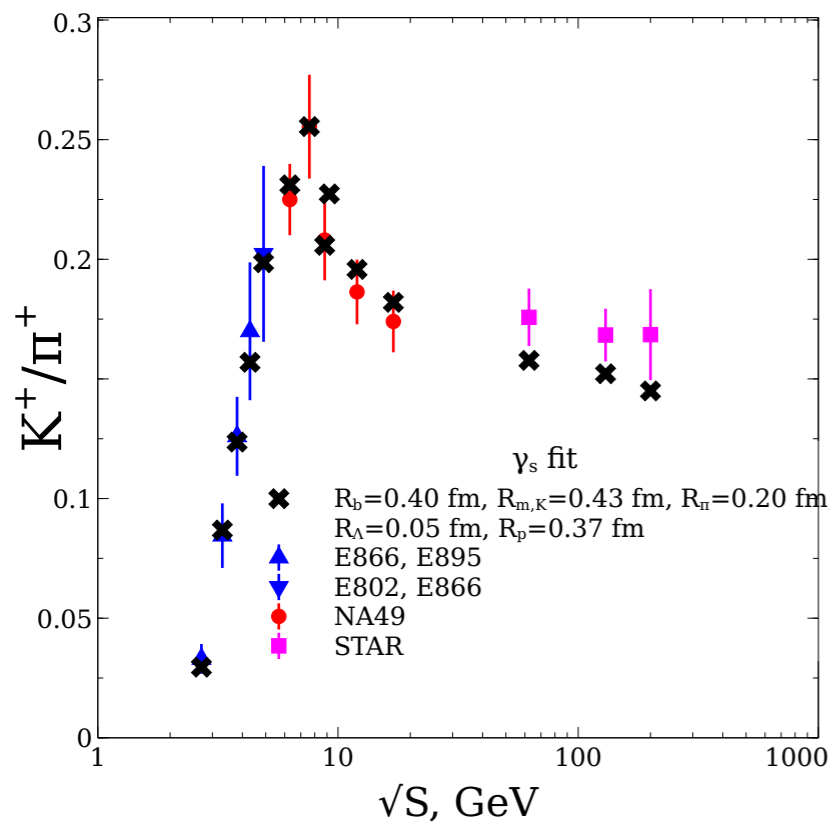
IST EOS (without ALICE): $\chi_1^2/dof = 57.099/55 \simeq 1.04$

$$R_\pi = 0.15 \text{ fm}, \quad R_K = 0.395 \text{ fm}, \quad R_\Lambda = 0.085 \text{ fm}, \quad R_b = 0.365 \text{ fm}, \quad R_m = 0.42 \text{ fm}$$

Only pion and Λ hyperon radii are changed, but no effect on T and μ_B

1. We confirm that there is a **jump** of T_{CFO} between $\sqrt{s} = 4.3 \text{ GeV}$ and $\sqrt{s} = 4.9 \text{ GeV}$
2. We confirm that there is a **strangeness enhancement peak** at $\sqrt{s} = 3.8 \text{ GeV}$

Most Problematic ratios at AGS, SPS and RHIC energies



IST EOS results are very similar to previous ones:

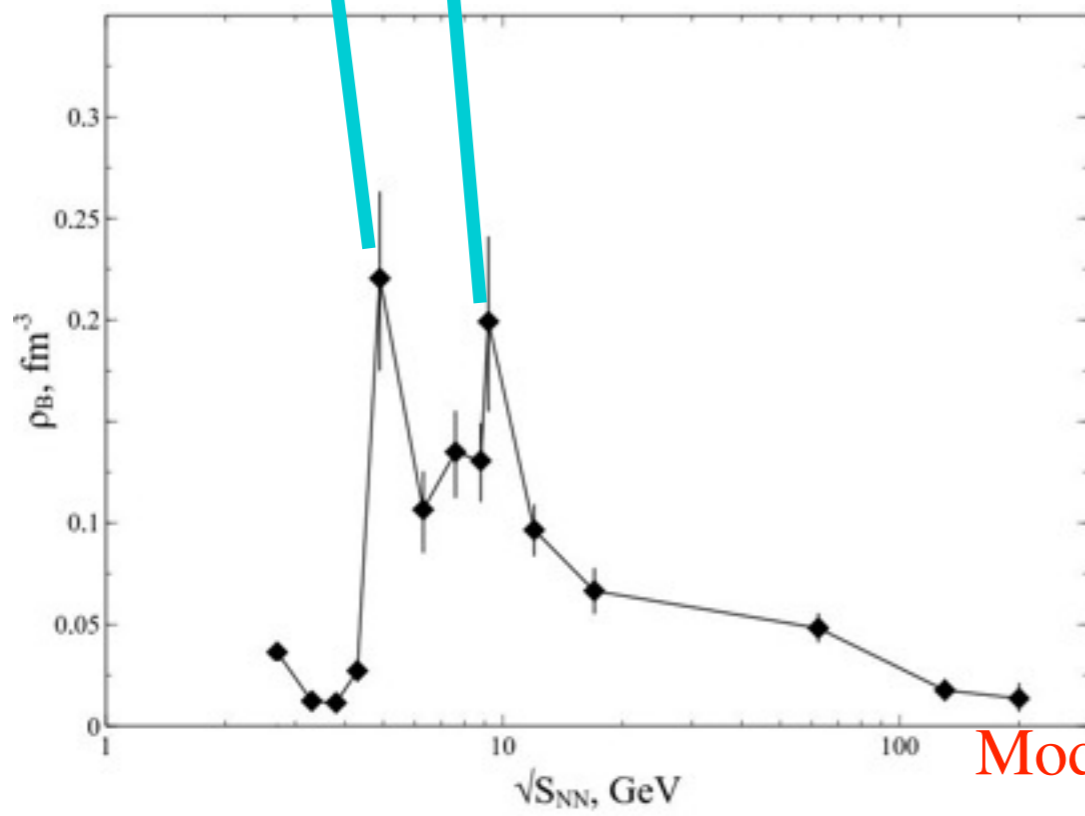
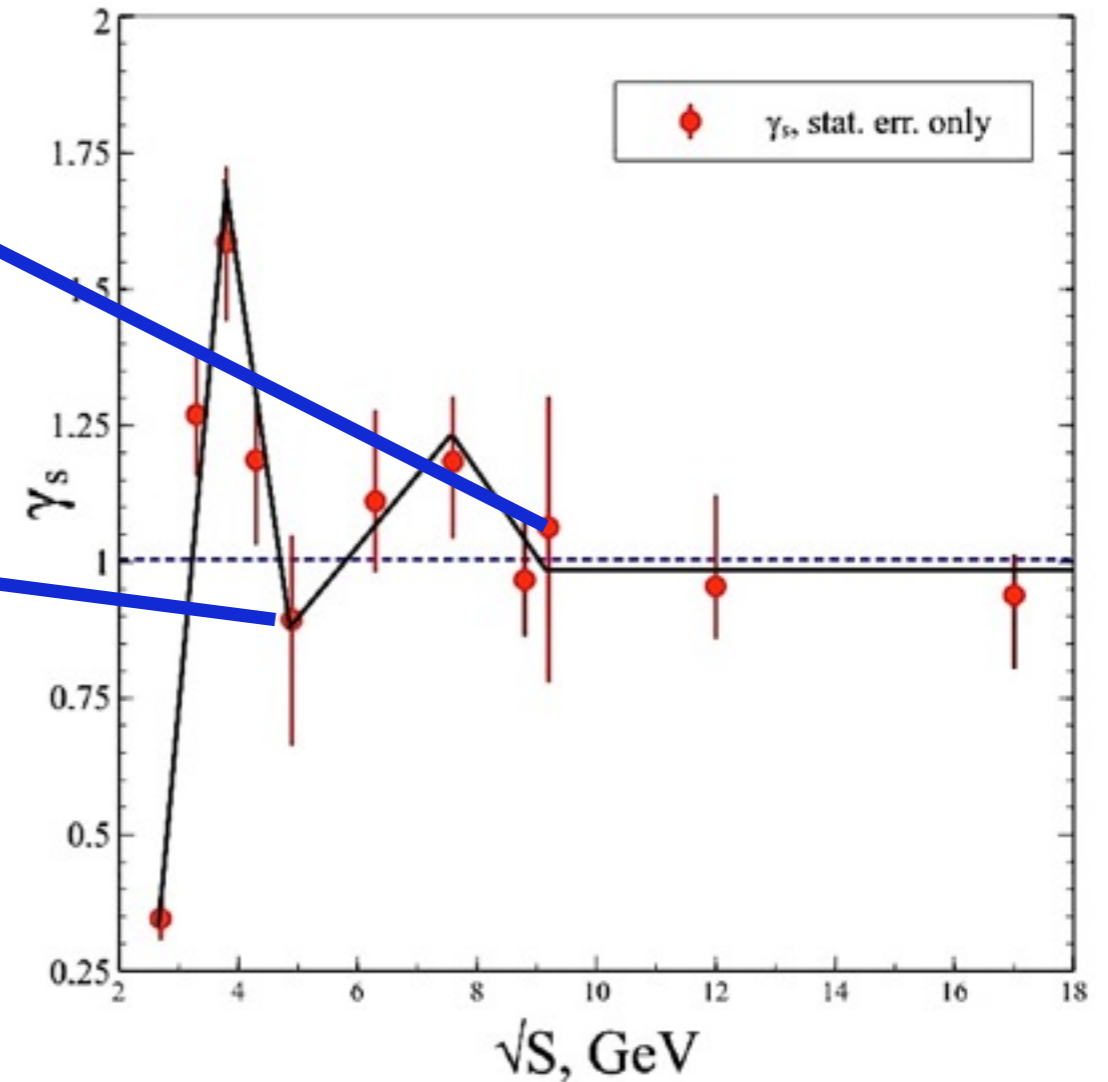
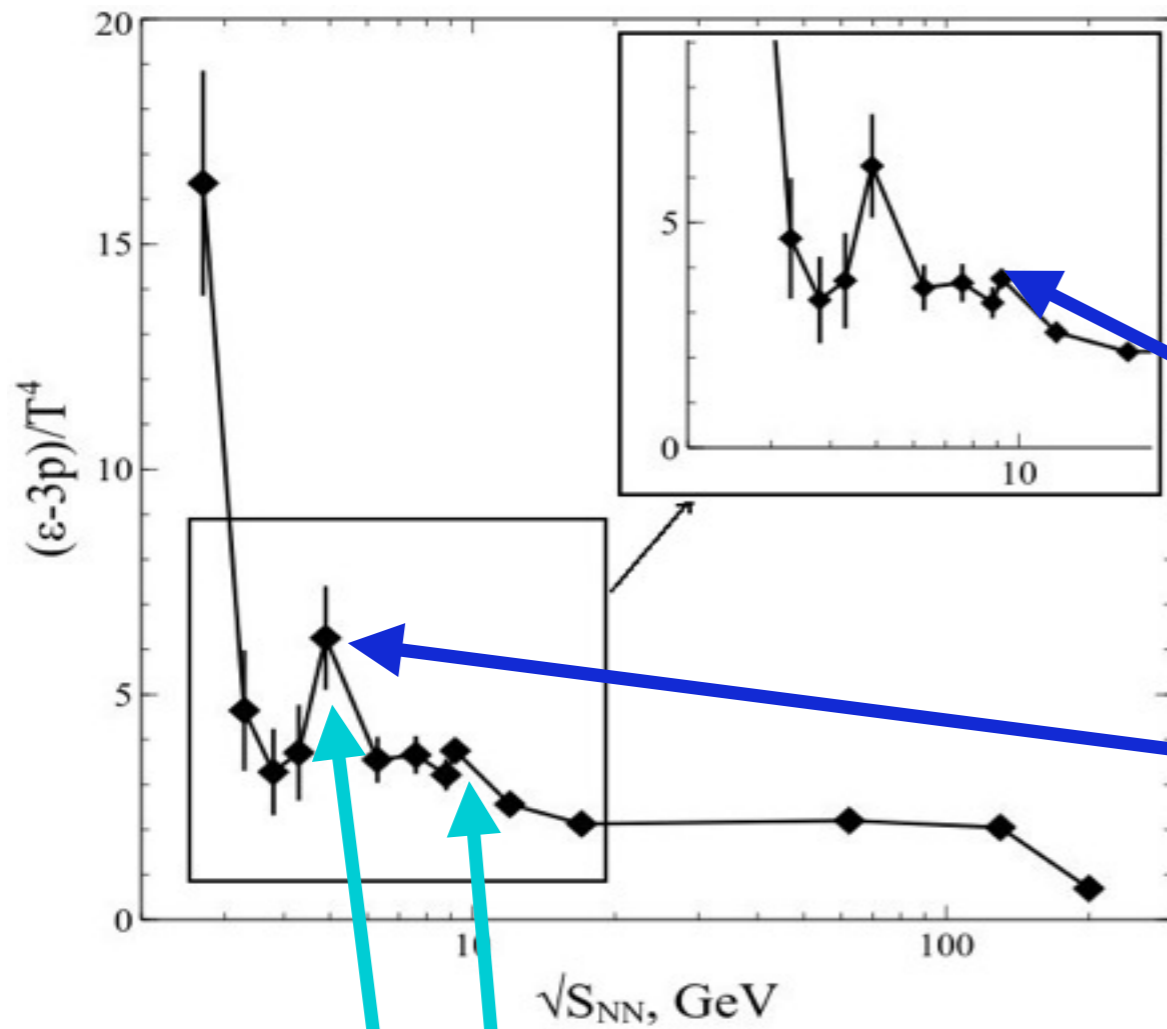
$$\chi^2/dof \simeq 3.29/14$$

$$\chi^2/dof \simeq 11.62/12$$

$$\chi^2/dof \simeq 8.89/8$$

Only few points for Λ (anti)hyperon are improved

Related Peaks (2017)

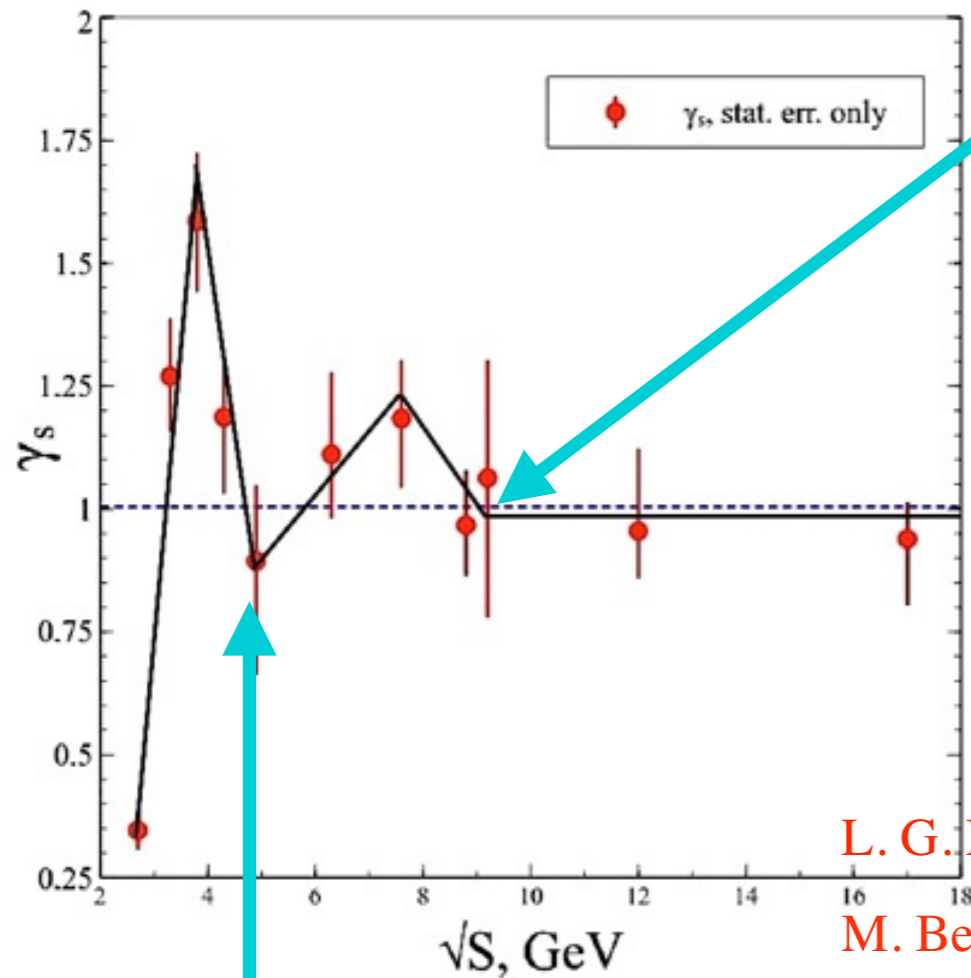


Trace anomaly peaks and baryonic density peaks are related to each other.

Can we relate them to γ_s irregularities?

Model from V.V. Sagun et al., arXiv:1703.00009 [hep-ph]

Strangeness Irregularities



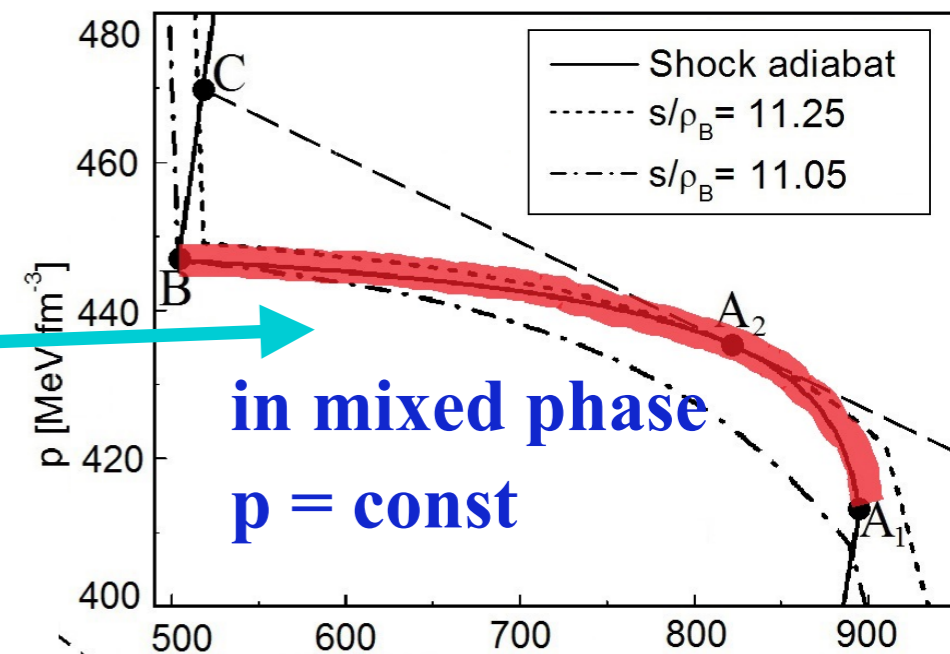
At c.m. energies above 8.8 GeV the strange hadrons are in chemical equilibrium due to formation of QG bags with Hagedorn mass spectrum!

Hagedorn mass spectrum is a perfect thermostat and a perfect particle reservoir! \Rightarrow Hadrons born from such bags will be in a full equilibrium!

L. G. Moretto, K. A. B., J. B. Elliott and L. Phair, Europhys. Lett. 76, 402 (2006)

M. Beitel, K. Gallmeister and C. Greiner, Phys. Rev. C 90, 045203 (2014)

At c.m. energy $\sqrt{S} \approx 4.5$ GeV strange particles are in chemical equilibrium due to formation of mixed phase, since under CONSTANT PRESSURE condition the mixed phase of 1-st order PT is explicit thermostat and explicit particle reservoir!



If There Are 2 Phase Transitions, then

1. What kind of phase exists at $\sqrt{s} = 4.9-9.2$ GeV?

2. Can we get any info about its properties?

We are sure that: **at $\sqrt{s} > 62.4$ GeV there exists sQGP and the partons are massless**

at 1 GeV $< \sqrt{s} < 4.3$ GeV there exists Hadron Gas

Then at $4.9 < \sqrt{s} < 9.2$ GeV they cannot exist!

In our fit of entropy per baryon along the shock adiabat we used the QGP EoS

$$p_{QGP} = \underbrace{A_0 T^4 + A_2 T^2 \mu^2 + A_4 \mu^4 - B}_{\text{fitting}}$$

$$A_0 \simeq 2.53 \cdot 10^{-5} \text{ MeV}^{-3} \text{ fm}^{-3}$$

$$A_2 \simeq 1.51 \cdot 10^{-6} \text{ MeV}^{-3} \text{ fm}^{-3}$$

$$A_4 \simeq 1.001 \cdot 10^{-9} \text{ MeV}^{-3} \text{ fm}^{-3}$$

$$B \simeq 9488 \text{ MeV fm}^{-3}$$

Effective Number of Degrees of Freedom

One look at this EoS:

$$p_{QGP} = \underbrace{A_0 T^4 + A_2 T^2 \mu^2 + A_4 \mu^4 - B}_{\text{fitting}} = \underbrace{A_0^L T^4 + A_2^L T^2 \mu^2 + A_4^L \mu^4}_{LQCD} - B_{eff}$$

$$B_{eff}(T, \mu_B) = B - (A_0 - A_0^L) T^4 - (A_2 - A_2^L) T^2 \mu^2 - (A_4 - A_4^L) \mu^4$$

Another look at this EoS:

$$p_{\text{New phase}} = \underbrace{A_0 T^4 + A_2 T^2 \mu^2 + A_4 \mu^4 - B}_{\text{fitting}}$$

It corresponds to massless particles with strong interaction

Then one can find an effective #dof from A_0 !

For massless particles

$$A_0 = N_{dof} \frac{\pi^2}{90} \quad \text{with} \quad N_{dof} = N_{dof}^{Bosons} + \frac{7}{8} \times 2N_{dof}^{Fermions}$$

$$\Rightarrow N_{dof} = A_0 \hbar^3 \frac{90}{\pi^2} \simeq 1800$$

It's a huge number for QGP!

Possible Interpretations

1. The phase emerging at $\sqrt{s} = 4.9-9.2$ GeV has no Hagedorn mass spectrum, since strange hadrons are not in chemical equilibrium.
2. 1800 of massless dof may evidence either about new phenomena (i.e. unitary/chiral symmetry restoration) in hadronic sector.
3. Or 1800 of massless dof may evidence about tetra-quarks with massive strange quark!?
see Refs. in R.D. Pisarski, 1606.04111 [hep-ph]
4. Or 1800 of massless dof may evidence about quarkyonic phase!?
A. Andronic et. al, Nucl. Phys. A 837, 65 (2010)
5. 1800 of massless dof may evidence about something else...

Consequent Problem and Its Possible Solution

If 1800 of massless dof exist then at high T and same μ_B the QGP cannot exist, since its pressure is too low to dominate!

⇒ Contradiction with Lattice QCD!

The only possibility to avoid the contradiction with LQCD is to assume hard-core repulsion for 1800 of massless dof!

Since they are almost massless ($m \ll T$), then the hard-core repulsion should be formulated for ultra-relativistic particles and include the effect of Lorentz contraction.

see K. A. Bugaev, Nucl. Phys. A 807, 251 (2008).

In the limit $\mu_B / T \ll 1$ and $\text{mass}/T \ll 1$ the pressure of such system is

$$p \simeq \frac{T^2}{V_0^{\frac{2}{3}}} N_{dof}^{\frac{1}{3}} C \quad \text{with} \quad C = \text{Const} \sim 1 \quad \text{here } V_0 \text{ is eigenvolume of hadron}$$

No mass dependence and very weak dependences on T and on #dof: $N_{dof}^{\frac{1}{3}} \simeq 12$

Conclusions

1. High quality description of the chemical FO data allowed us to find few novel irregularities at c.m. energies 4.3-4.9 GeV (pressure, entropy density jumps e.t.c.)
2. HRG model with multicomponent repulsion allowed us to find the correlated (quasi)plateaus at c.m. energies 3.8-4.9 GeV which were predicted about 26 years ago. The second set of plateaus and irregularities may be a signal of another phase transition!
3. Generalized shock adiabat model allowed us to describe entropy per baryon at chemical FO and determine the parameters of the EOS of new phase from the data. **Actually, LQCD can help us to find out what kind of properties exist at cross-over!**
4. Hopefully, FAIR, NICA and J-PARC experiments will allow us to make more definite conclusions

Thank You for Your Attention!

Microcanonical Ensemble

Example #1: 1-d Harmonic Oscillator

- For 1-d Harmonic Oscillator with energy ε in contact with Hagedorn resonance (**just exponential spectrum for simplicity**).
Total energy is E . K.A.B. et al, Europhys. Lett. 76 (2006) 402
- The microcanonical probability of state ε is:

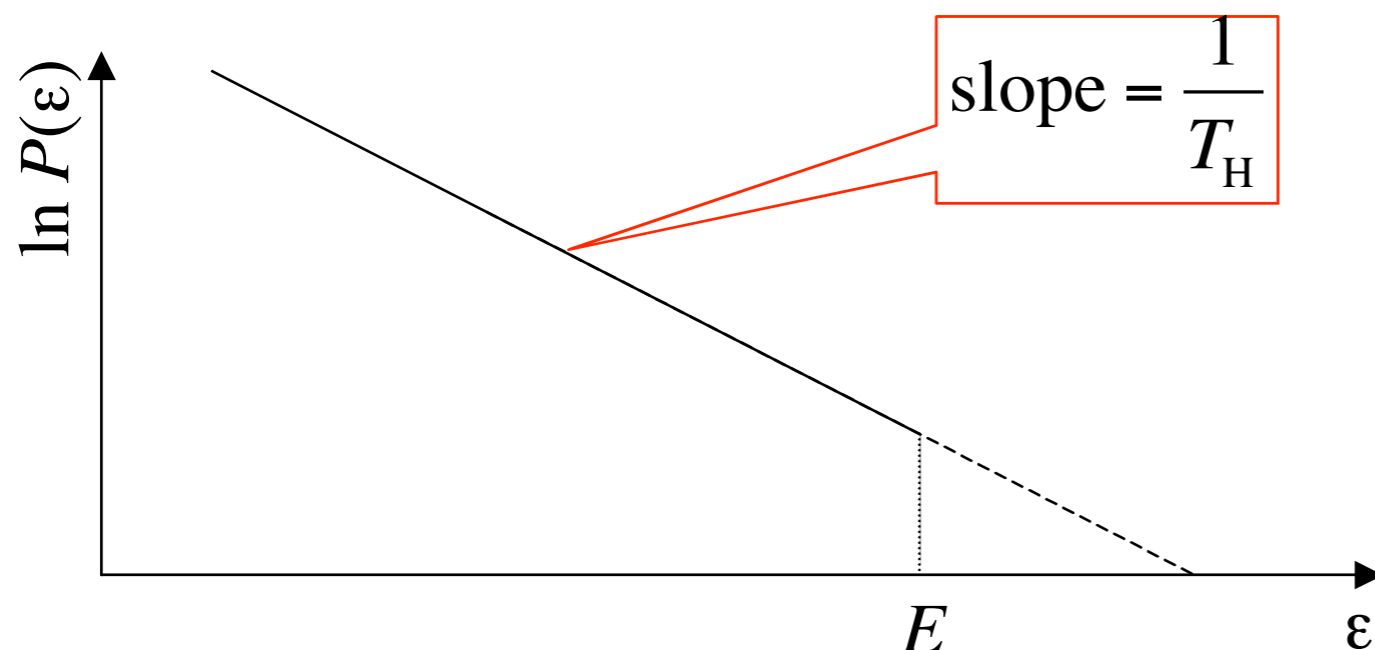
$$P(\varepsilon) = \rho(E - \varepsilon) = \exp\left(\frac{E - \varepsilon}{T_H}\right) = \exp\left(\frac{E}{T_H}\right) \exp\left(-\frac{\varepsilon}{T_H}\right)$$

Exponent is
Grand canonical!
With fixed T !

Average value of ε is

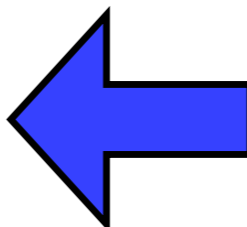
$$\bar{\varepsilon} = T_H \left(1 - \frac{E/T_H}{\exp(E/T_H) - 1} \right)$$

For $E \rightarrow \infty$: $\bar{\varepsilon} \rightarrow T_H$



Example #2: An Ideal Vapor coupled to Hagedorn resonance

- Consider microcanonical partition of N particles of mass m and kin. energy ε . The total level density is

$$P(E, \varepsilon) = \rho_H(E - \varepsilon) \rho_{iv}(\varepsilon) = \frac{V^N}{N! \left(\frac{3}{2}N\right)!} \left(\frac{m\varepsilon}{2\pi}\right)^{\frac{3}{2}N} \exp\left(\frac{E - mN - \varepsilon}{T_H}\right)$$


Exponent is
Grand canonical!
With fixed T !

The most probable energy partition is

$$\frac{\partial \ln P}{\partial \varepsilon} = \frac{3N}{2\varepsilon} - \frac{1}{T_H} = 0 \Rightarrow \frac{\varepsilon}{N} = \frac{3}{2} T_H$$

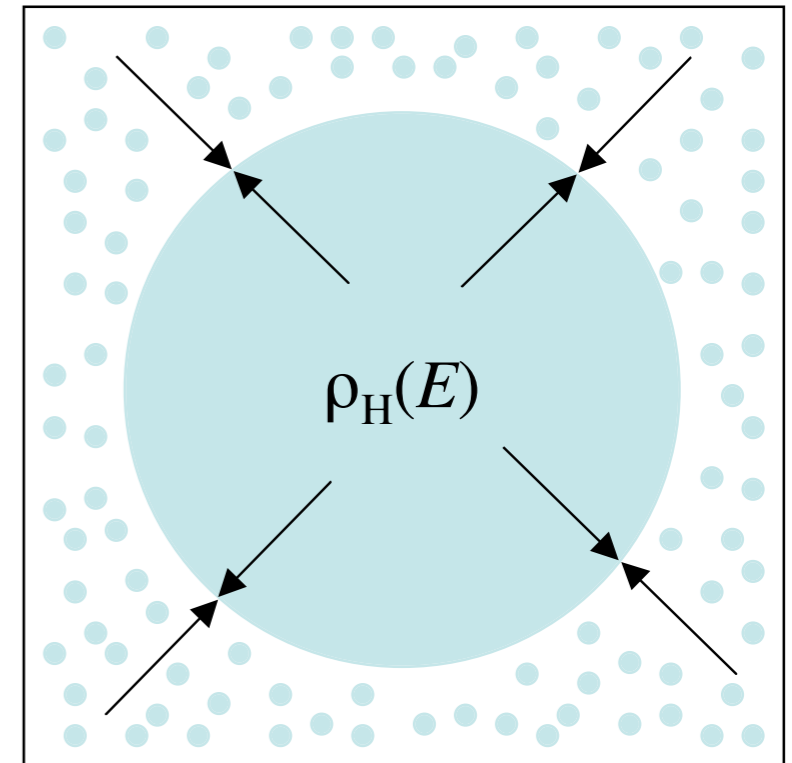
- T_H is the sole temperature characterizing the system:
- A Hagedorn-like system is a perfect thermostat!**

Example #3: An Ideal Particle Reservoir

L.G. Moretto, K.A.B. et al, nucl-th/0601010

- If, in addition, particles are generated by the Hagedorn resonance, their concentration is **volume independent!**

$$\left. \frac{\partial \ln P}{\partial N} \right|_V = -\frac{m}{T_H} + \ln \left[\frac{V}{N} \left(\frac{m T_H}{2\pi} \right)^{\frac{3}{2}} \right] = 0 \Rightarrow \frac{N}{V} = \left(\frac{m T_H}{2\pi} \right)^{\frac{3}{2}} \exp\left(-\frac{m}{T_H}\right)$$



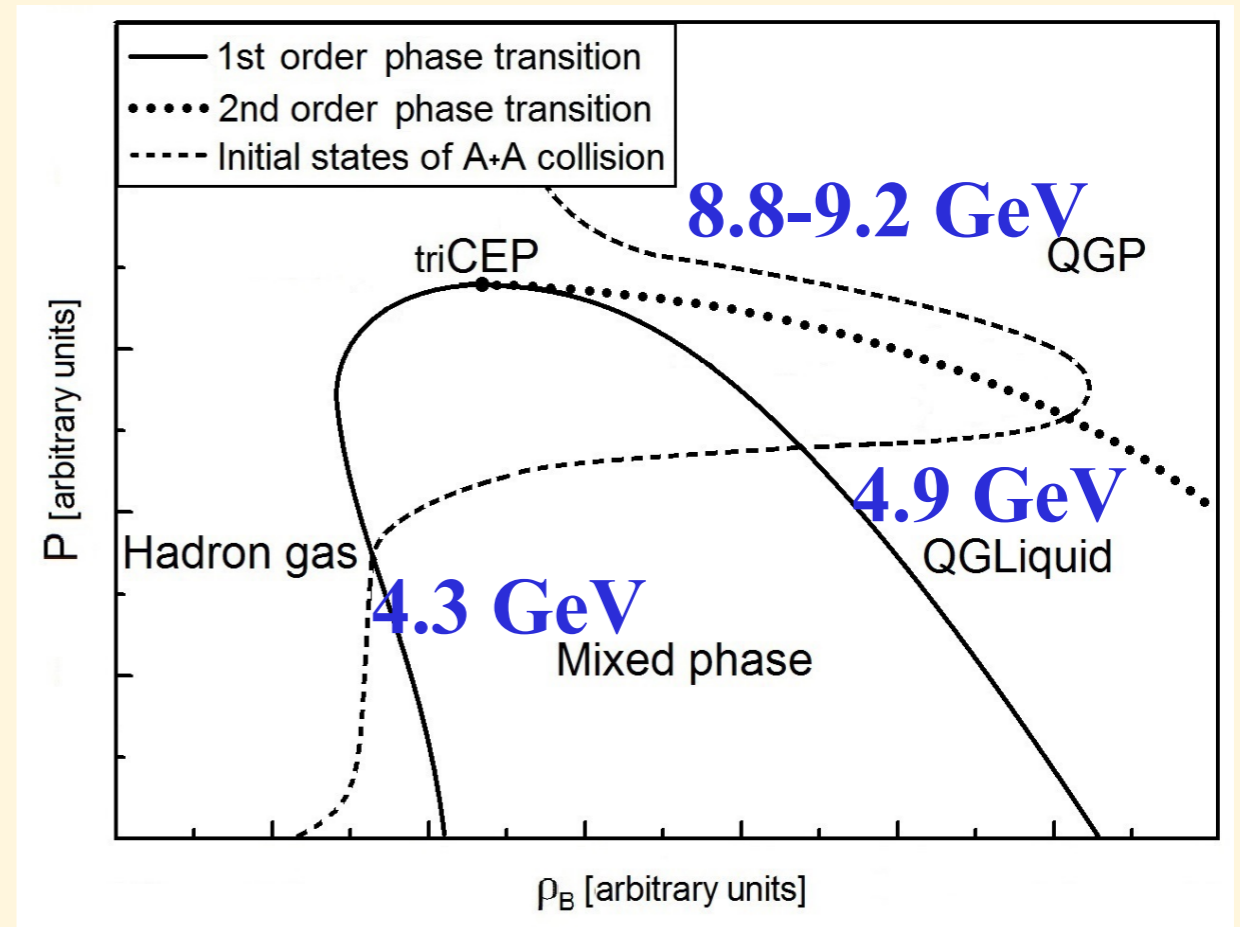
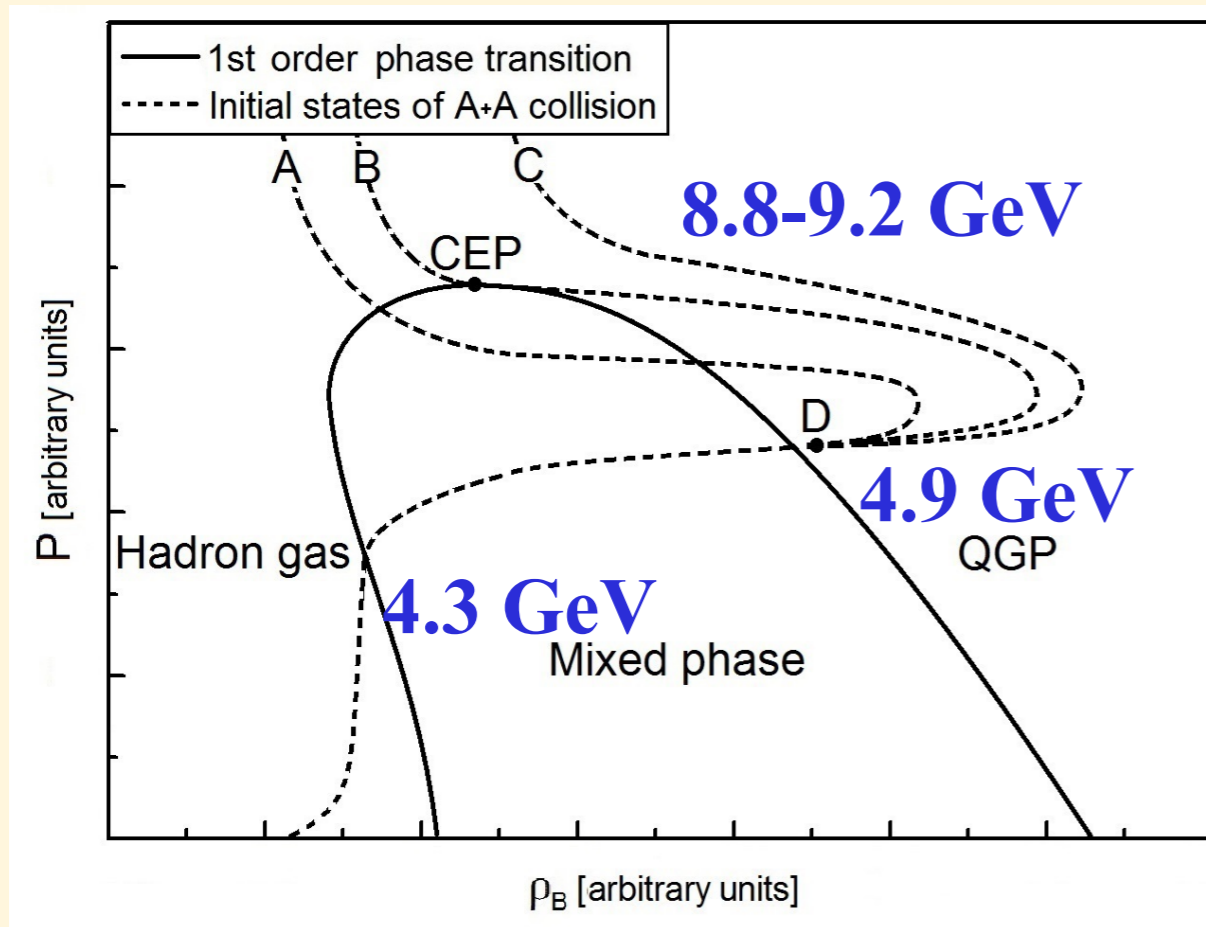
ideal vapor ρ_{iv}

- particle mass = m
- volume = V
- particle number = N
- energy = ϵ

Remarkable result because it mean saturation between gas of particles and Hagedorn thermostat!

Possible Interpretation

Evolution of possible «initial» states with collision energy

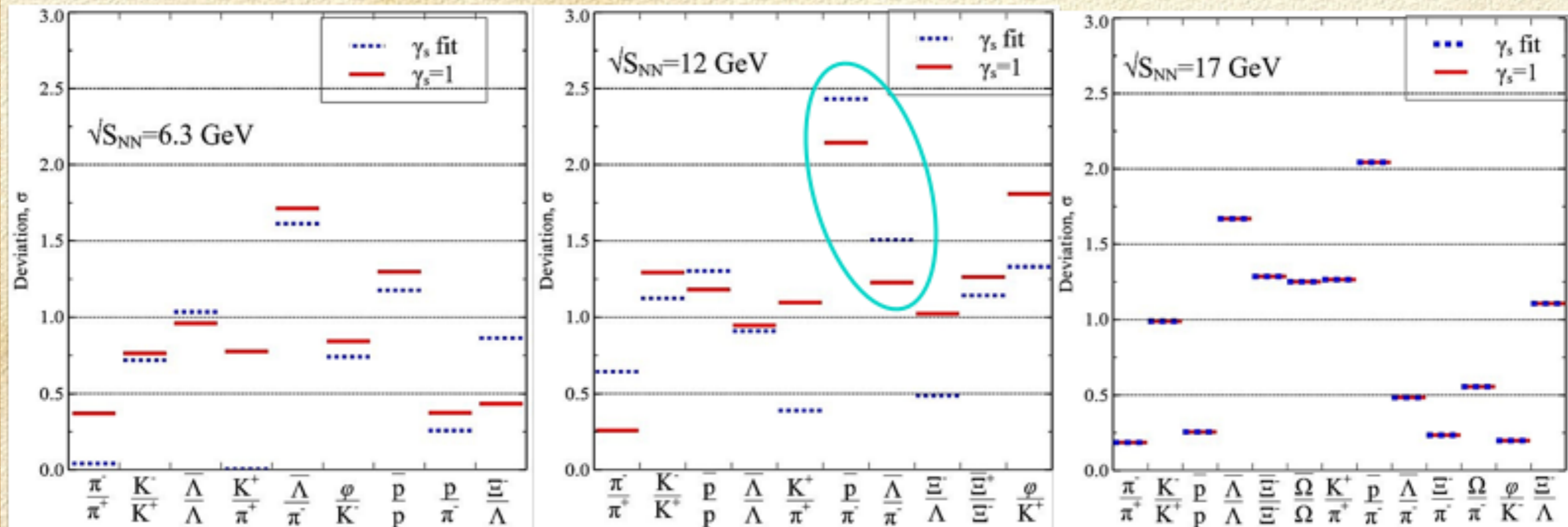


Appearance of 2-nd intersection at c.m. energies 8.8-9.2 GeV

**probably means that trajectory goes
near critical (left) or 3critical (right) endpoint**

To resolve this problem we need data from NICA and FAIR!

Particle Yield Ratios



1. Some ratios are improved while some are not

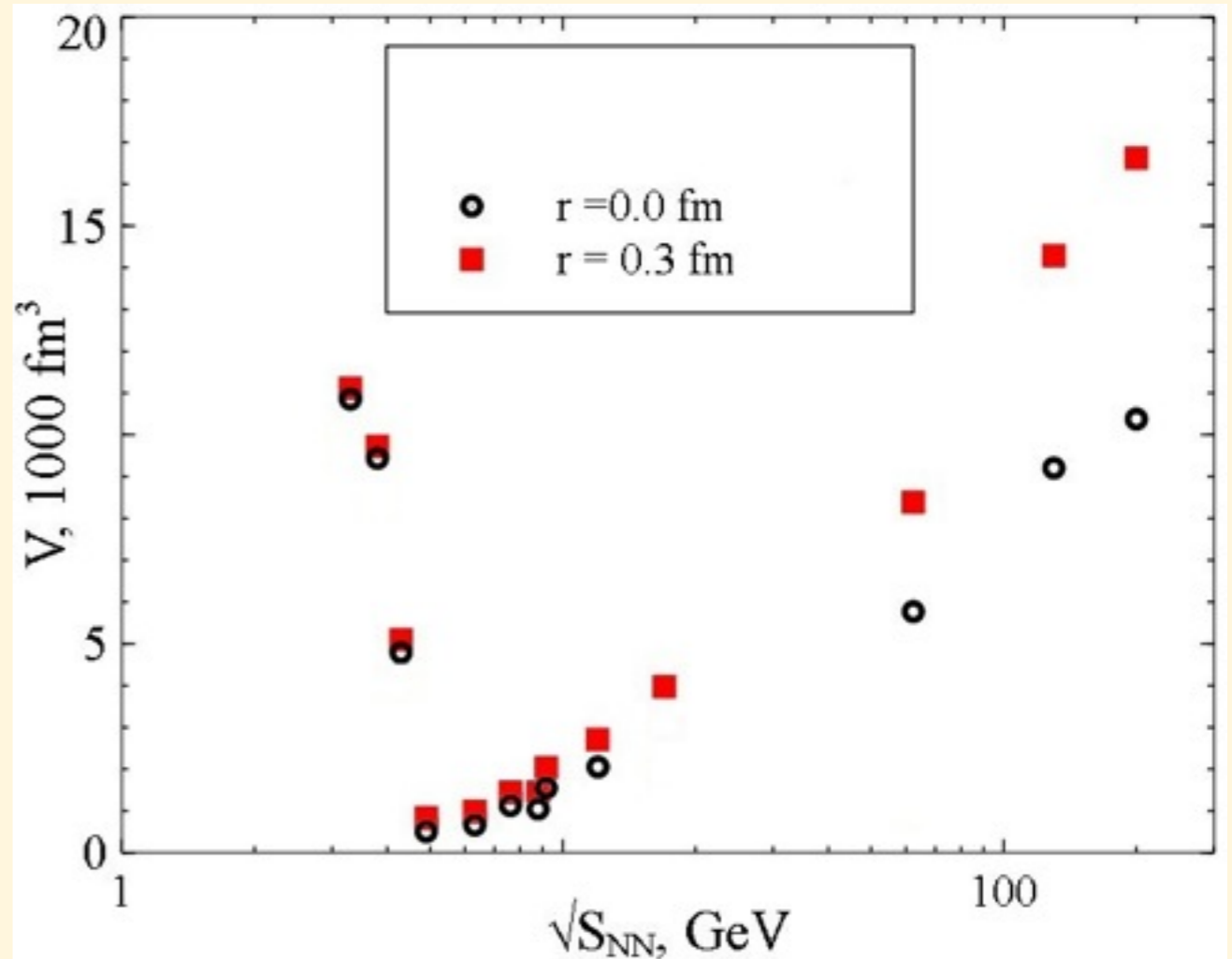
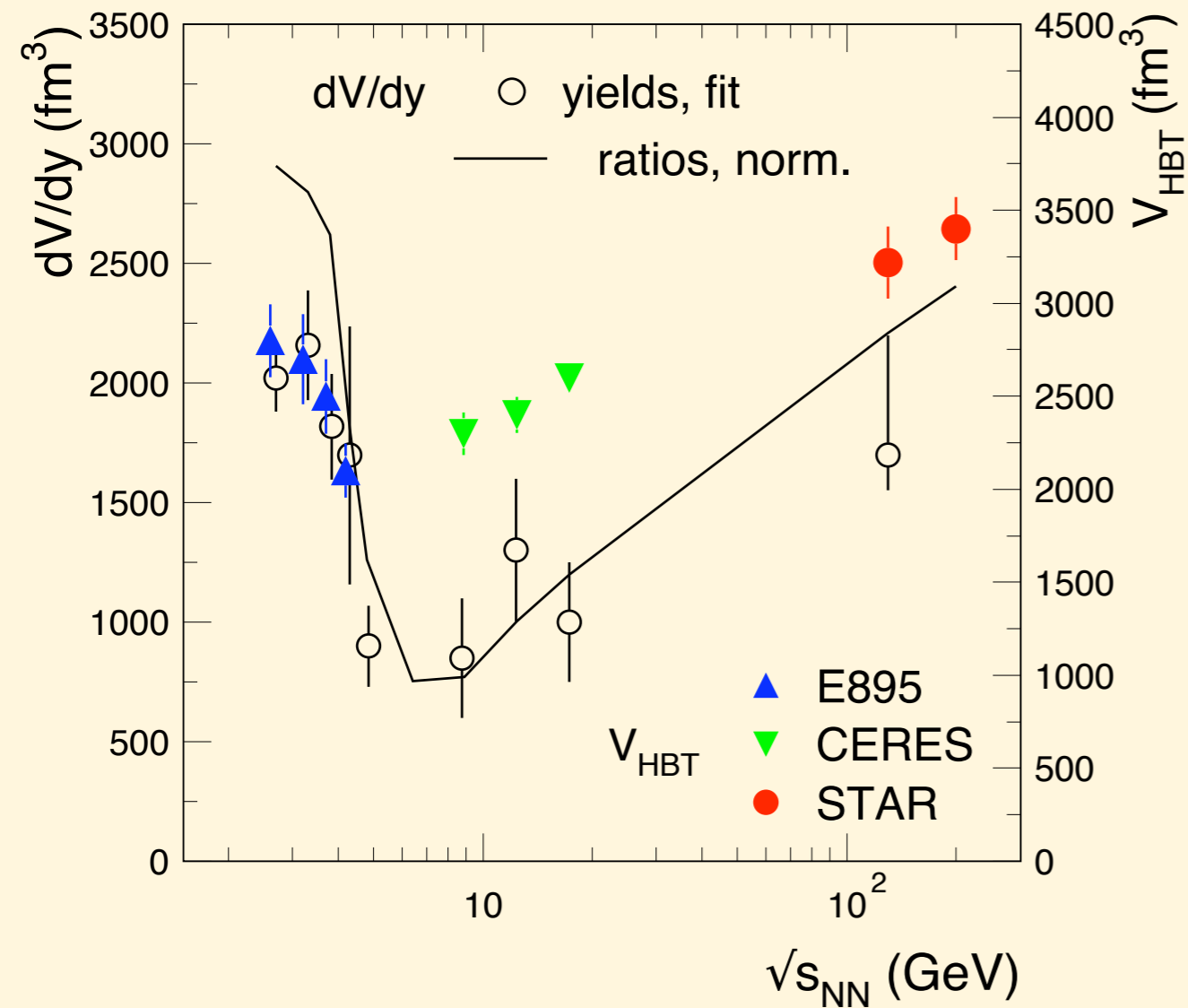
2. At energies < 5 GeV and at 17 GeV there are almost no improvements

3. At low energies there are local minima at $\gamma_s < 1$!

But we took the deepest ones! \Rightarrow Becattini et al took the wrong one!

4. Many wrong results are based on Becattini et al work.

Minimum of ChFO Volume at AGS Energies



A. Andronic, P. Braun-Munzinger, J. Stachel,
 NPA (2006)777

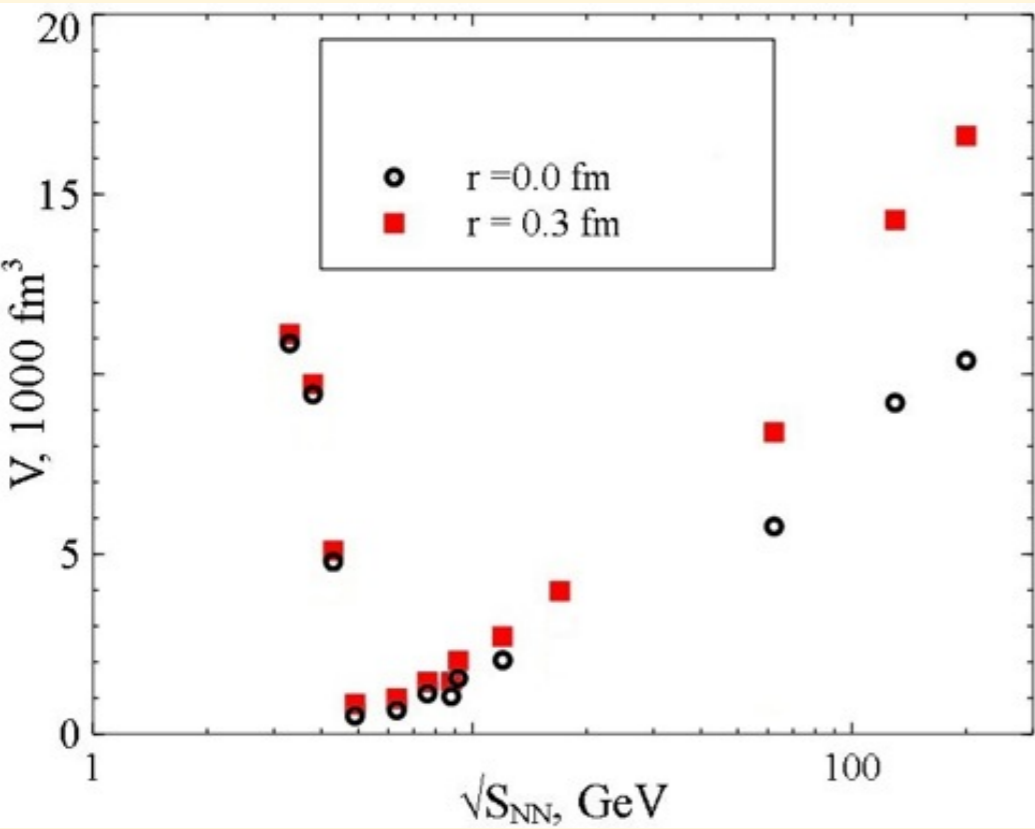
D.R. Oliinychenko, K.A. Bugaev and A.S. Sorin,
 Ukr. J. Phys. 58, (2013)

All these irregularities occur at c.m. energies 4.3-4.9 GeV!

Are these minima related to deconfinement?

Other Minima at AGS Energies

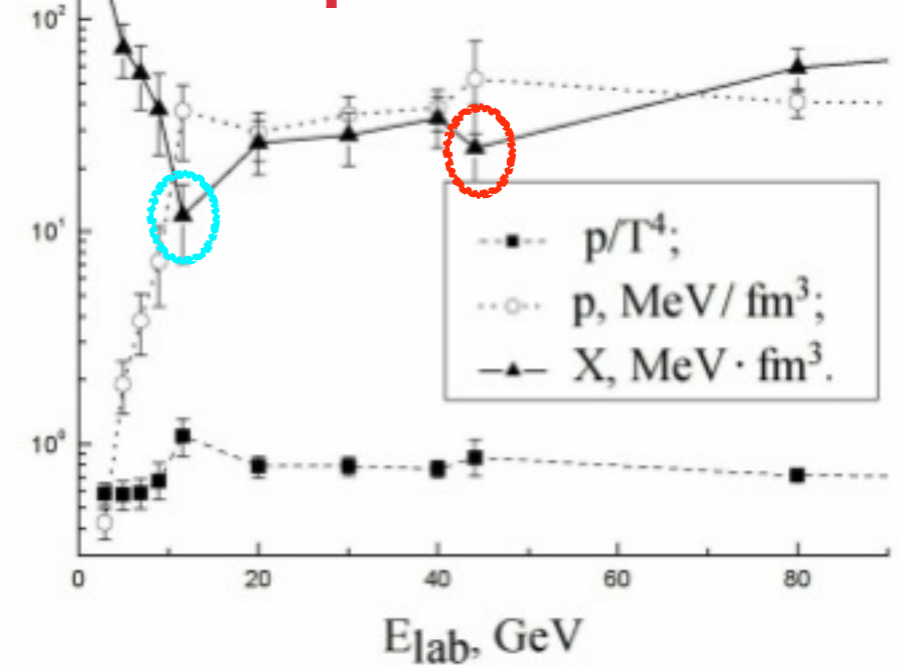
min V at ChFO



SAME energy!

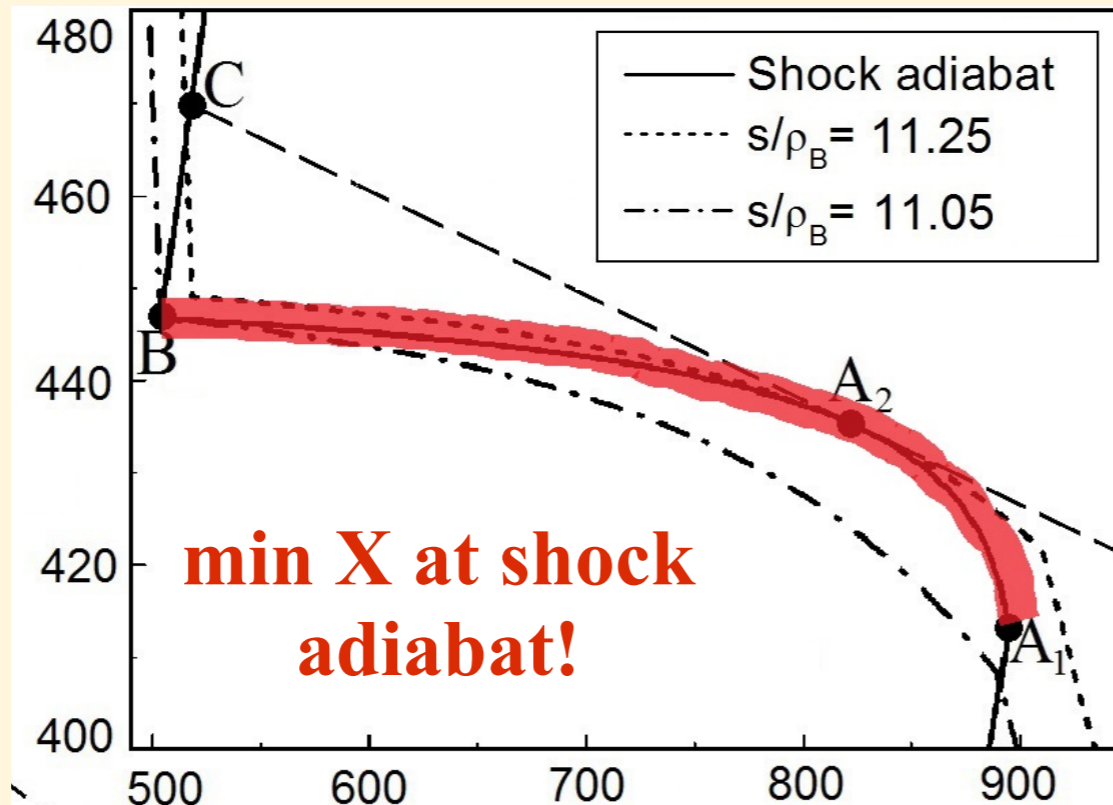
min X at ChFO

X is generalized specific volume
Is second X peak due to other PT?



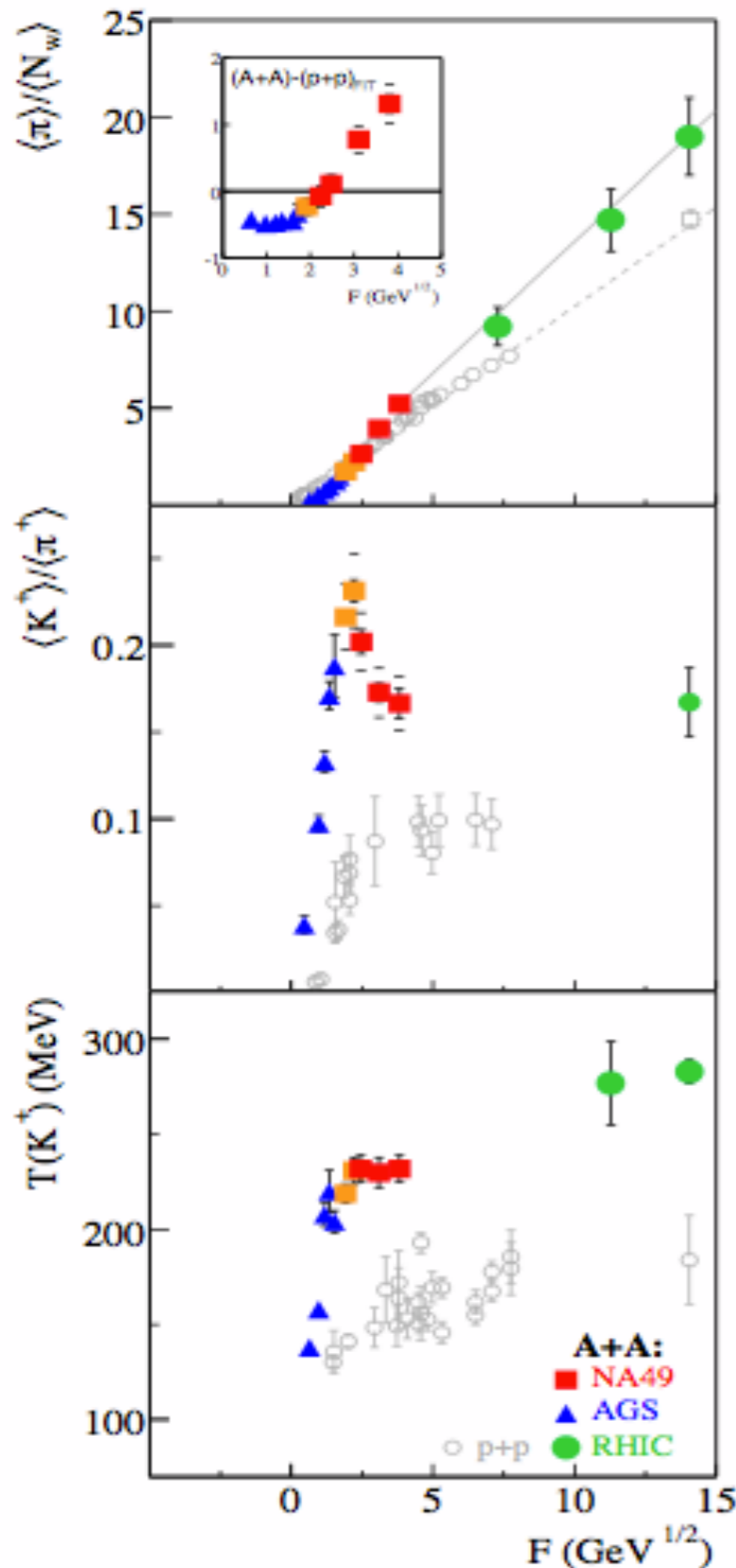
D.R. Oliinychenko, K.A. Bugaev and A.S. Sorin,
Ukr. J. Phys. 58, (2013)

K.A. Bugaev et al., EPJ A (2016)



In this work we gave
a proof that min X
at boundary between
QGP and mixed phase
generates min X at ChFO
which leads to min V
of ChFO!

Popular NA49 "Signals"



Kink in $\frac{\langle \pi \rangle}{\langle N_w \rangle} \approx g^{\frac{1}{4}} F$ shows that the number of d.o.f. g changes at about $E_{lab} = 30$ GeV

It was suggested in

F is Fermi variable $\sim s^{1/4}$

M. Gazdzicki, Z. Phys. C 66 (1995).

Horn in $\frac{\langle K^+ \rangle}{\langle \pi^+ \rangle}$ ratio shows that elementary d.o.f. of strangeness are changing from K^\pm to s_q at about $E_{lab} = 30$ GeV

It was suggested in

Claim that onset of deconfinement is at c.m. energy 7.6 GeV

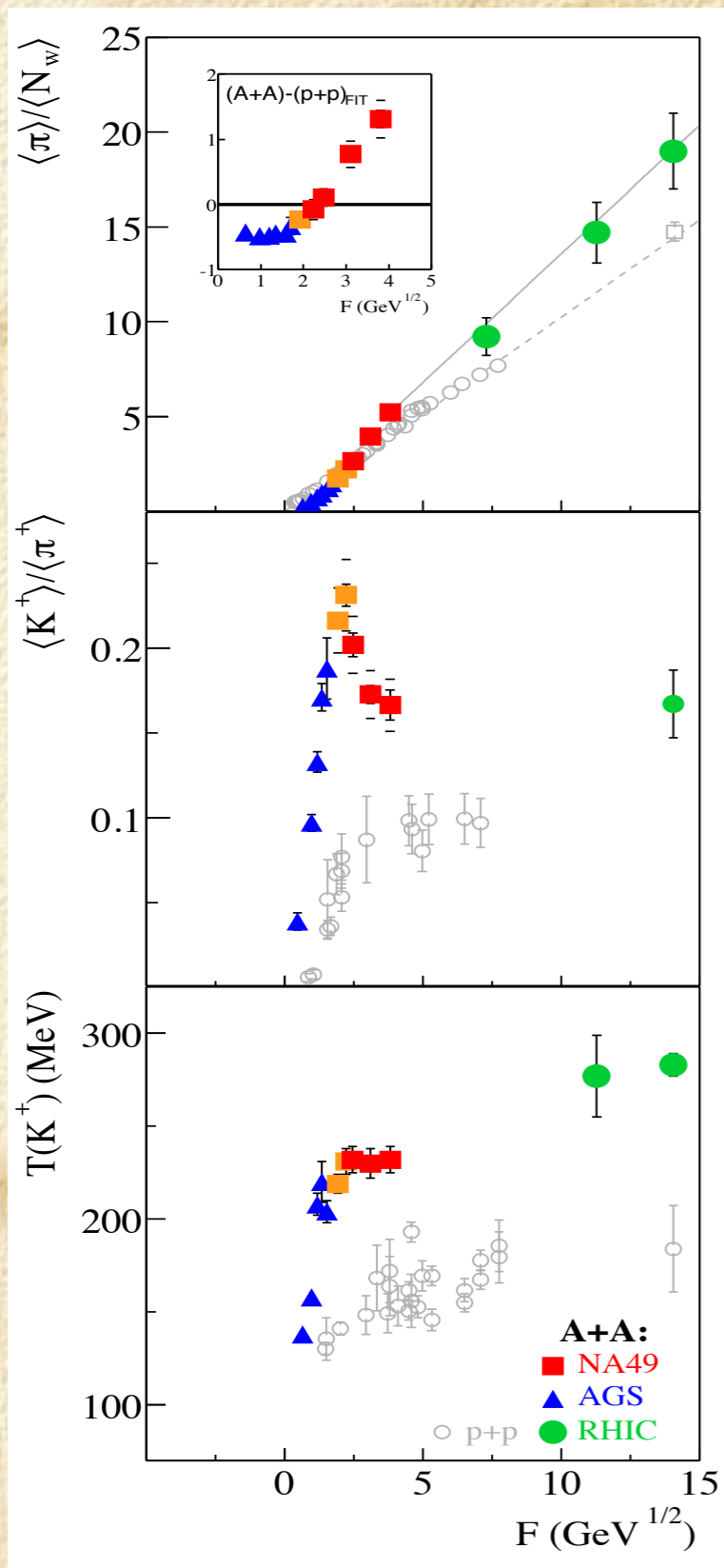
M. Gazdzicki and M.I. Gorenstein, Acta Phys. Polon. B 30 (1999)

Step in K^\pm inverse slopes shows that $\approx F$ independent initial pressure develops at about $E_{lab} = 30$ GeV

It was suggested in

M. Gazdzicki, M.I. Gorenstein and K.A. Bugaev, Phys. Lett. B 567 (2003)

NA49 «Signals» = Irregularities



I. There is **NO** a single model which can simultaneously describe these «signals»!

II. These «signals» cannot be reproduced by existing hydrodynamic and hydro-cascade models with deconfinement phase transition.

Therefore, their relation to deconfinement is unclear!

Hence, these «signals» are irregularities which require an explanation!

Furthermore, it seems that there is also something wrong with our EOS!

Strangeness Enhancement as Deconfinement Signal

In 1982 J. Rafelski and B. Müller predicted that **enhancement of strangeness** production is a signal of deconfinement. Phys. Rev. Lett. 48(1982)

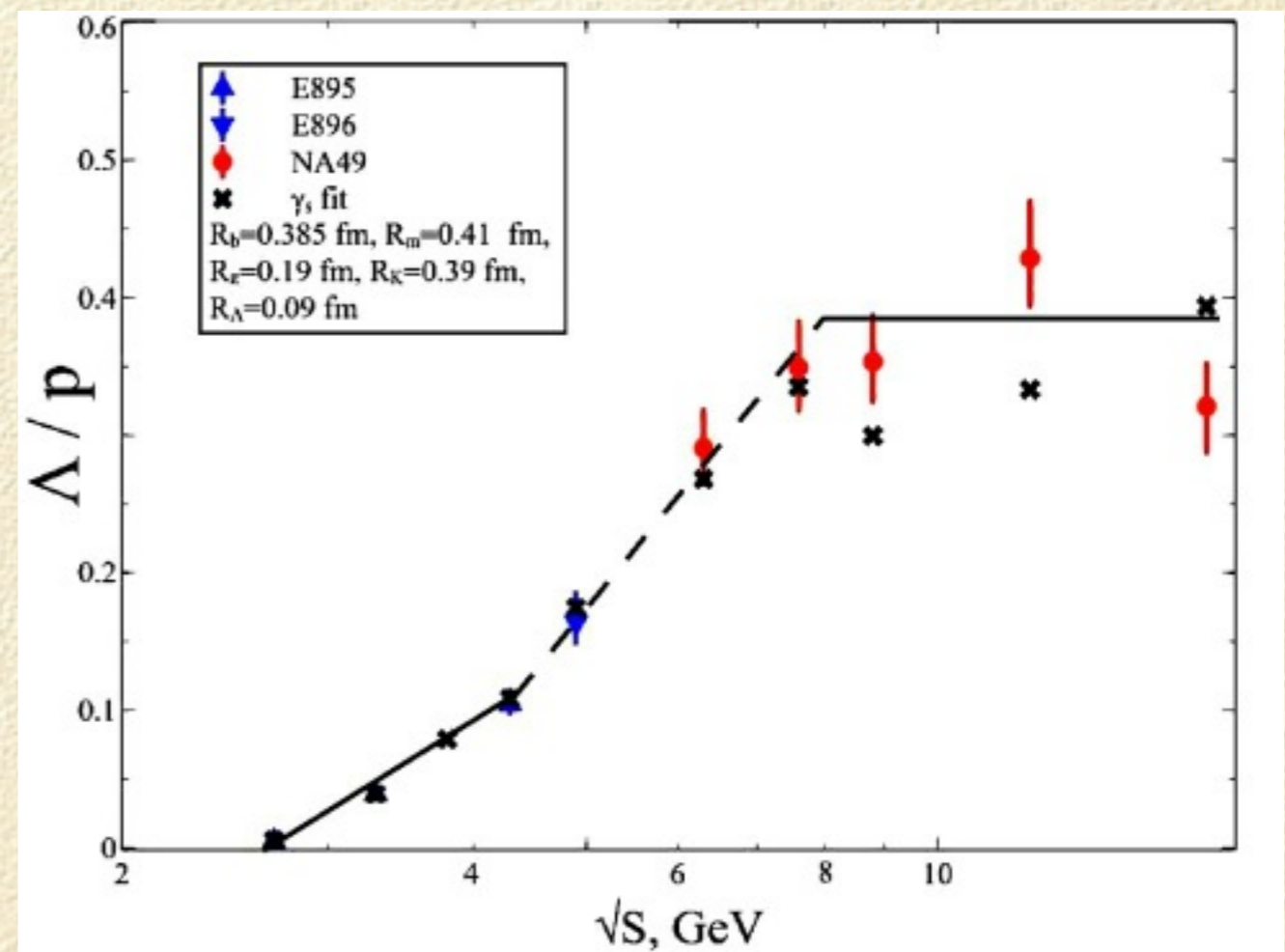
We observe 3 regimes: at c.m. energies 4.3 GeV and ~8 GeV slope of experimental data drastically changes!

Combining **Rafelsky & Muller idea** with our result that mixed phase appears at 4.3 GeV we explain this finding:

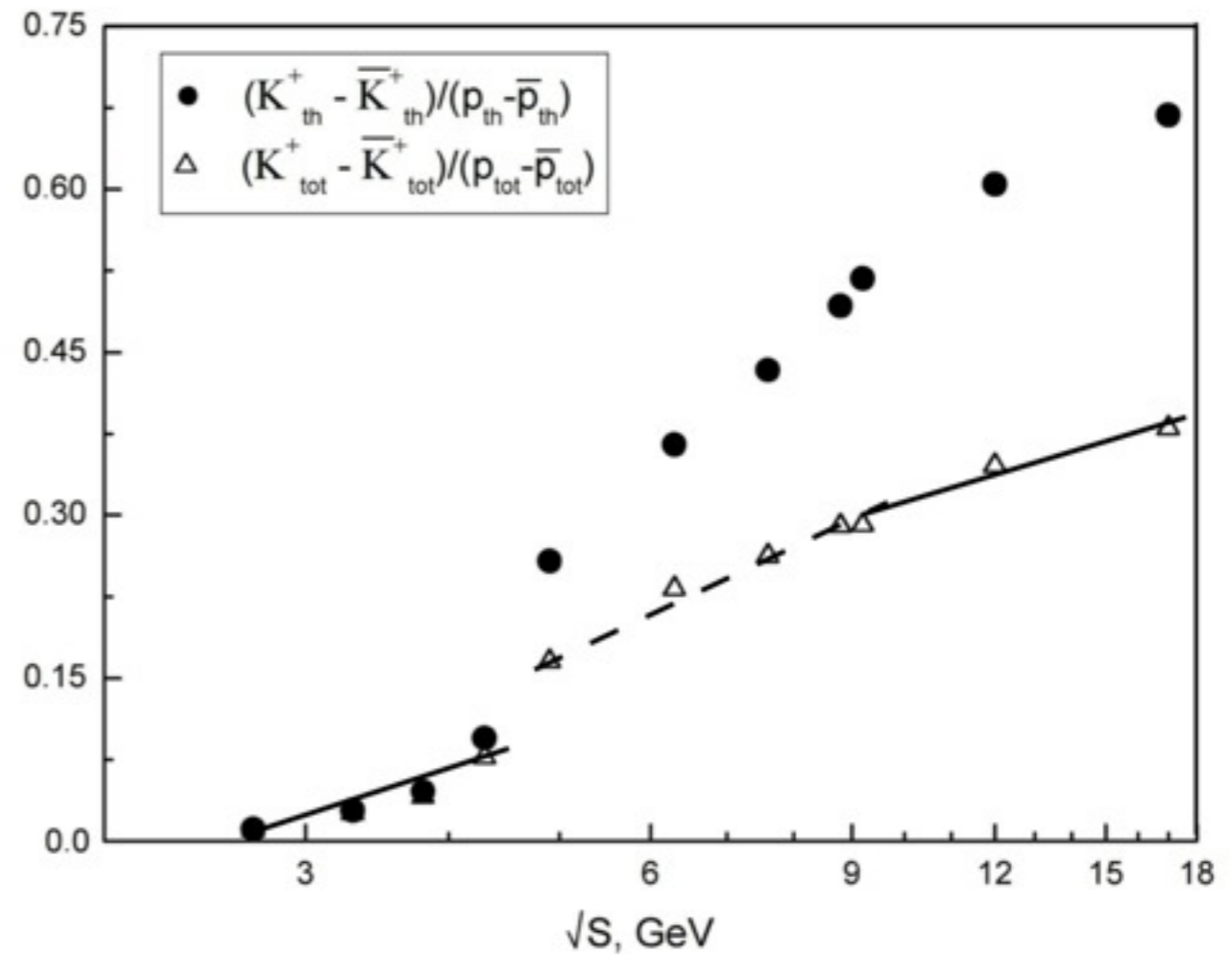
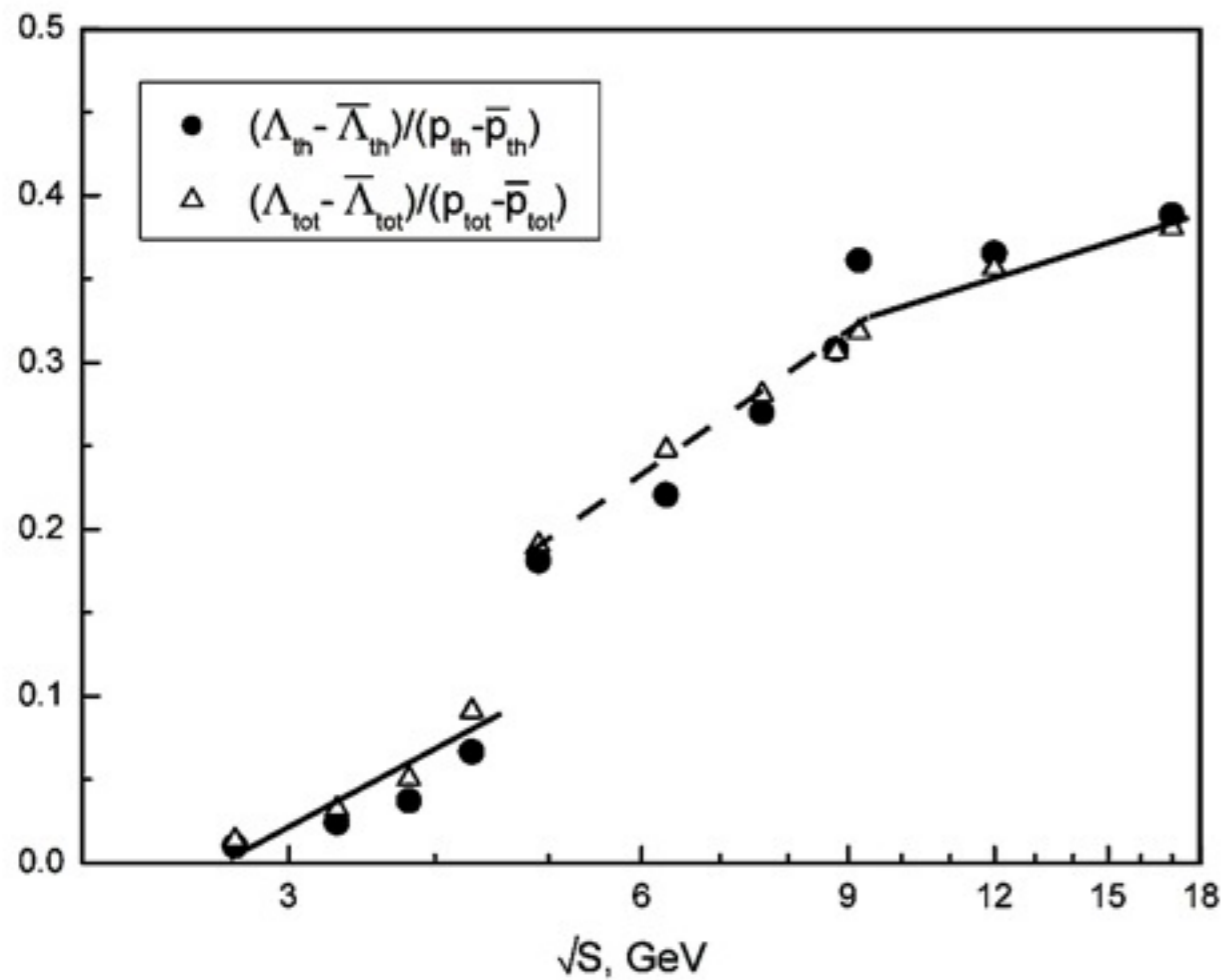
Below 4.3 GeV Lambdas appear in N+N collisions

Above 4.3 GeV and below ~8 GeV formation of QGP produces additional s (anti)s quark pairs

Above ~8 GeV there is saturation due to small baryonic chemical potential



What To Measure at FAIR & NICA ?



We predicted **JUMPS** of these ratios at 4.3 GeV due to 1-st order PT and **CHANGE OF** their **SLOPES** at $\sim 9-12$ GeV due to 2-nd order PT (or weak 1-st order PT?)

To locate the energy of **SLOPE CHANGE** we need **MORE** data at 7-13 GeV