Monte-Carlo generation of final state hadrons

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Simulations of HIC for NICA energies
JINR Dubna

11.4.2017

Motivation

Two approaches to see signals of various processes and effects in nuclear collisions:

1. full collision simulation, identify observables, test their signals
   - direct connection to first (or second, third, ...) principles
   - usually computationally expensive
   - transport codes, hydrodynamic simulations, or hybrid approaches

2. Monte Carlo generation of final-state particles, signals put in by hand
   - can be used for designing observables for various signals and testing their sensitivity
   - no direct connection with first (or second, third, ...) principles
   - cheap(!) evaluation

Outline

1. DRAGON (DRoplet and hAdron GeneratOr for Nuclear collisions)
2. REGGAE: generating particles with energy and momentum conservation
3. open issues/questions
4. (Event Shape Sorting)
5. Conclusions

DRAGON

DRoplet and hAdron GeneratOr for Nuclear collisions

- generate hadrons from the blast-wave model
- includes resonances (baryons up to 2 GeV, mesons up to 1.5 GeV)
- chemical equilibrium given by T_{ch} and μ_b different from T_{kin}
- inofficial version with individual chemical potentials for all stable species
- fast MC generation of particles
- elliptic flow and elliptic deformation of the fireball
- provides the possibility to emit hadrons from droplets
- no calculation of multiplicity, it is an input parameter

[B. Tomášik, Comp. Phys. Commun. **180** (2008) 1642, *ibid.* **207** (2016) 545]

The blast-wave model

given by the emission function:

$$S(x,p)d^4x = r\,dr\,d\phi\,d\eta\,d\tau\,\delta(\tau-\tau_0)$$
$$\times \frac{\tau m_t \cosh(\eta - y)}{(2\pi)^3}\Theta(R(\phi)-r)H(\eta)$$
$$\times \left(\exp\left(p^\mu u_\mu(x)/T\right) \pm 1\right)^{-1}$$

transverse expansion velocity:

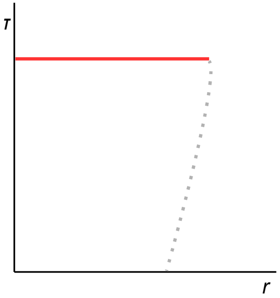
$$v_t = \tanh\left(\frac{r}{R}\sqrt{2}\rho_0\left(1+2\rho_2\cos(\phi)\right)\right)$$

elliptic shape

$$R_x = aR \quad R_y = \frac{R}{a}$$

$H(\eta)$ can be:

- Gaussian (low energies)
- uniform (high energies)

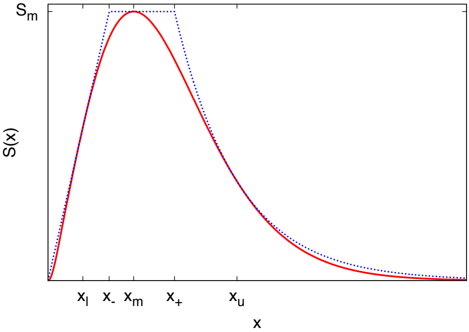


freeze-out hypersurface:
constant $\tau = \sqrt{t^2 - z^2}$

Fast MC generation of relativistic quantum-statistical distribution

rejection method

log-concave tail of the distribution compared to exponential distribution



effectivity > 0.9

[B. Tomášik, I. Melo, J. Cimerman, arXiv:1602.08233]

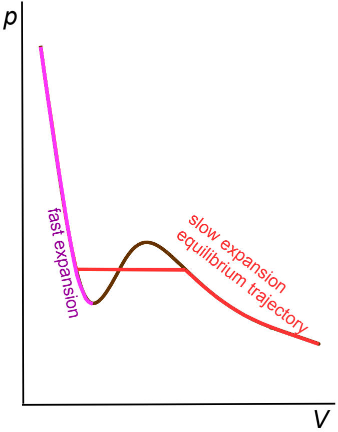
First order phase transition: fragmentation

Rapid passage through
the phase transition leads to
spinodal decomposition
(known also in classical physics)

Scenario possible if
nucleation rate $<$ expansion rate

- may be relevant for nuclear collisions
- observed in multifragmentation

Example:
isotherm in van der Waals EoS



Droplets in DRAGON

- tunable share of hadrons produced from droplets (0–100%)
- droplets move with the local flow velocity where the are produced
- droplet volumes can be chosen constant or distributed according to

$$\mathcal{P}_2(V) = \frac{V}{b^2} e^{-V/b}$$

  with parameter b

- droplets decay exponentially in time

$$\mathcal{P}_\tau(\tau_d) = \frac{1}{R_d} e^{-\tau_d/R_d}$$

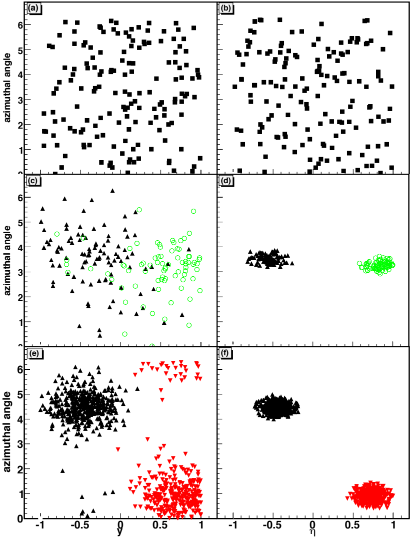
  where R_d is droplet radius

- each droplet can have their own temperature which determines the energies of their decay particles

Hadrons from droplets: an illustration

distribution of hadrons in position space and momentum space

momentum | position



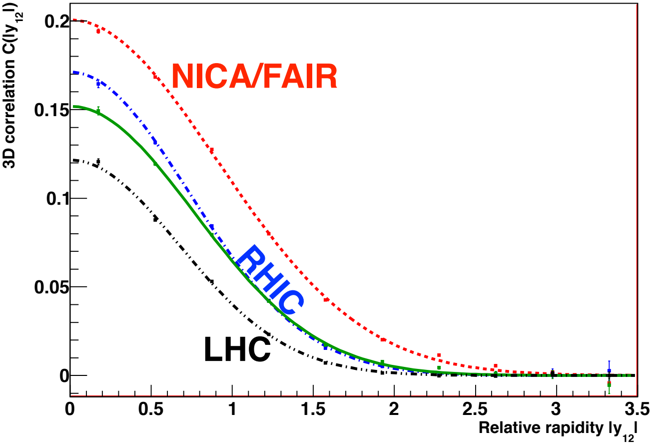
hadrons from droplets high temperature →

hadrons from droplets low temperature →

Signature of droplets: rapidity correlations

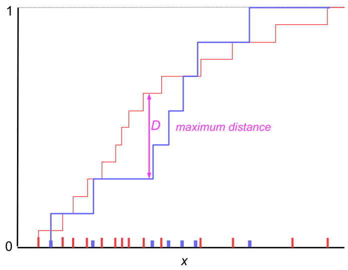
Protons produced from droplets would have correlated rapidities.
With their large mass they best copy the droplet velocity.

Correlation function in relative rapidity $C(y_{12}) = \frac{\Phi_2(y_{12})}{\Phi_{2,\mathrm{mixed}}(y_{12})}$



$$
\begin{aligned}
b &= 50\,\mathrm{fm}^3 \\
T_{NICA} &= 140\,\mathrm{MeV} \\
T_{RHIC} &= 150\,\mathrm{MeV} \\
T_{LHC} &= 150\,\mathrm{MeV}
\end{aligned}
$$

Signature of droplets: Kolmogorov-Smirnov test

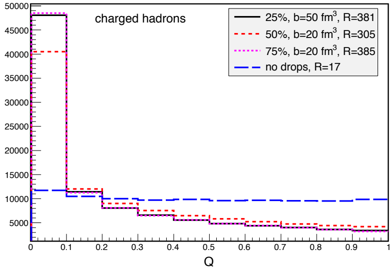


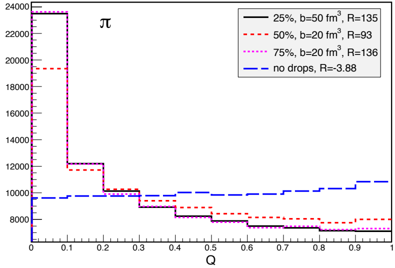
define scaled distance

$$d = \sqrt{\frac{n_1 n_2}{n_1 + n_2}} D$$

plot histogram in

$$Q(d) = P(d' > d)$$





Particle generation with E and $\vec{P}$ conservation

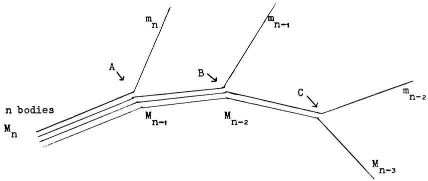
Find the momenta and energies of particles from multiparticle decay which has no additional information.
Maximum entropy obtained in the final distribution of particles.
Available other treatments (to my best knowledge):

- sequential decay: GENBOD [Kopylov, Raubold, Lynch, James]
- RAMBO [Kleis, Stirling, Ellis], NUPHAZ [Block]: designed for high energies ($p \gg m$)
- match the momentum of the last particle and rescale energies of all particles [Toneev]
  or calculate momenta and energies of the last two particles [Ferroni]: works, but may fail sometimes for large number of particles; generally not clean

GENBOD: sequential decay

series of 2-body decays:

$$E = M_n$$
$$\rightarrow m_1,\, m_2,\, m_3,\, \ldots,\, m_n$$



[F. James: CERN-68-15]

masses must satisfy:

$$\sum_{i=1}^{j} m_j < M_j < M_n - \sum_{i=j+1}^{n} m_i$$

they are generated as

$$M_j = r_j \left(M_n - \sum_{i=j+1}^{n} m_i \right) + \sum_{i=1}^{j} m_j$$

with uniform random deviates

$$0 < r_1 < \cdots < r_j < r_{j+1} < \cdots < r_{n-2} < 1$$

REGGAE: REscattering-after-Genbod GenerAtor of Events

The idea: collisions lead to thermalisation

Generation of particles with masses (m_1, m_2, ..., m_n) with total 4-momentum P

REGGAE:

1. boost to the rest frame, calculate E^*.
2. run GENBOD
3. Rescattering
    1. choose random pair of particles
    2. boost into the CM system of the pair and rotate momenta into new random direction
    3. repeat for each particle N_c times
4. boost to the original frame

[M. Mereš, *et al.*, Comp. Phys. Commun. **182** (2011) 2561]

The approach to most likely configurations

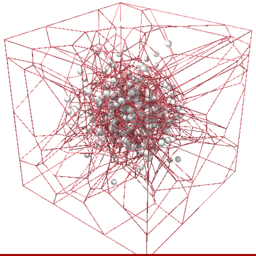
Estimate the entropy

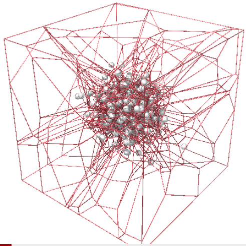
$$S = -\int_{\Sigma} \rho(\vec{p}) \ln \rho(\vec{p})\, d^3\vec{p}$$

with the estimator

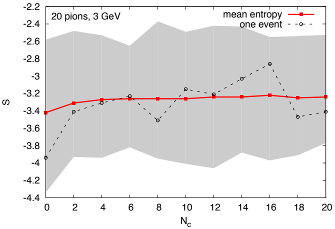
$$\hat{S} = \frac{1}{n} \sum_{i=1}^{N} \ln(nV_i)$$

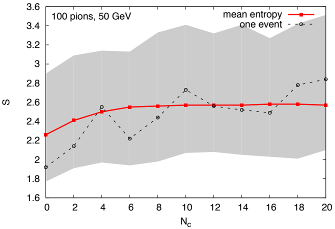
V_i: Voronoi regions in **momentum** space.





Dependence of $\hat{S}$ on N_c





Evaluation of phase-space integrals

test: integrate over phase-space

$$f_5(p_1, p_2, p_3, p_4, p_5) = \frac{p_1 \cdot p_2 + p_3 \cdot p_4}{M^2 + (p_4 + p_5)^2}$$

$M^2 = 4$ GeV^2, $E^* = 100$ GeV

Comparison of evaluation times:

$n = 5,\ m = 1$ GeV

N	REGGAE ($N_c = 6$)	NUPHAZ	RAMBO
10^4	195.4	270.48	181.60
10^5	214.5	223.62	199.24
10^6	211.2	212.56	211.63
10^7	209.8	207.71	208.97
time	26 min	5 min	1 min

$n = 60,\ m = 1$ GeV

N	REGGAE ($N_c = 12$)	NUPHAZ
10^4	0.6339	0.6185
10^5	0.6185	0.6315
time for 10^5	6 min	63 min

REGGAE is the best for many particles, where large part of energy is taken by masses.

Open issues / to do list

- merge REGGAE into DRAGON
- construct DRAGON for more general hypersurfaces, maybe use as freeze-out generator
- include momentum distribution with viscous correction
- couple DRAGON to transport code
- how to implement momentum conservation only in transverse space?

Conclusions 1

- DRAGON: a tool for easy generation of hadrons from a fragmented fireball
- DRAGON can be used also without droplets
- REGGAE can be used for generation of particle momenta which conserve total energy and momentum.

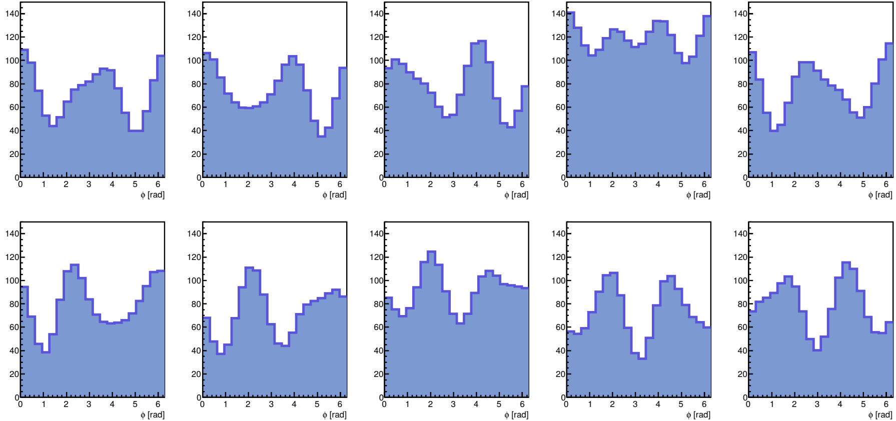
Event Shape Sorting

ESS is a method ...

- good for organising events with multi-particle production, e.g. in ultra-relativistic heavy-ion collisions.
- that allows to select events which have similar distributions of hadron momenta.
- that is self-organised and does not require the user to specify any sorting variable.

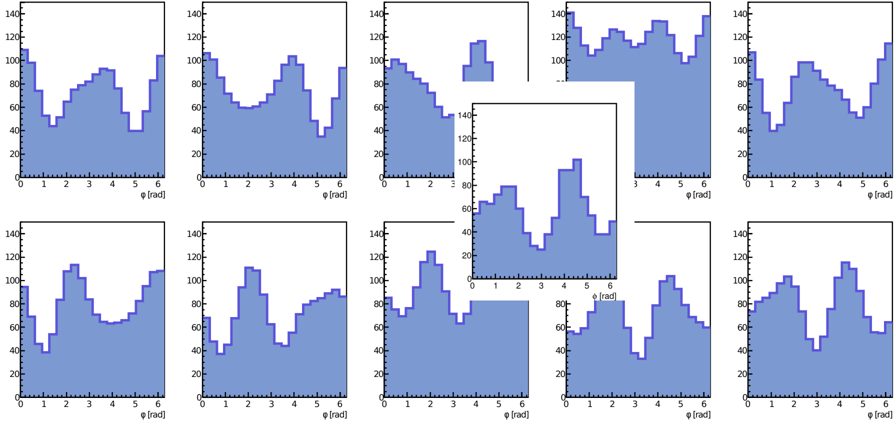
How does it work

2000 AMPT events, Pb+Pb at $\sqrt{s_{NN}} = 2.76$ TeV, centr. 0–20%, sorted



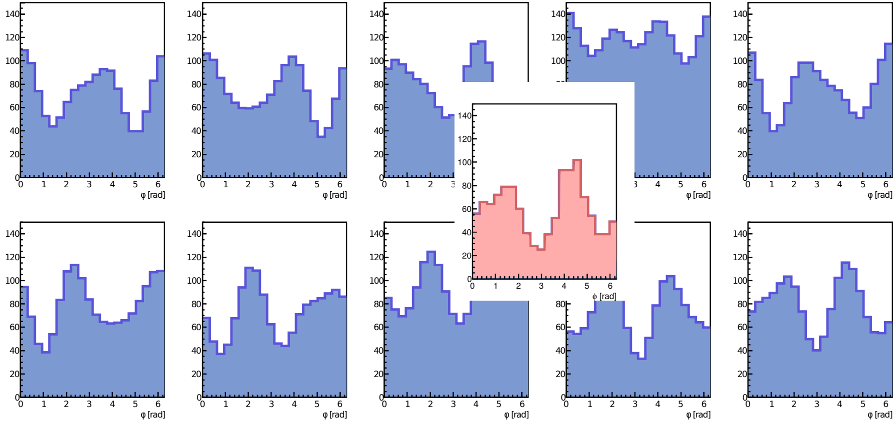
How does it work

To which event bin is this event similar?



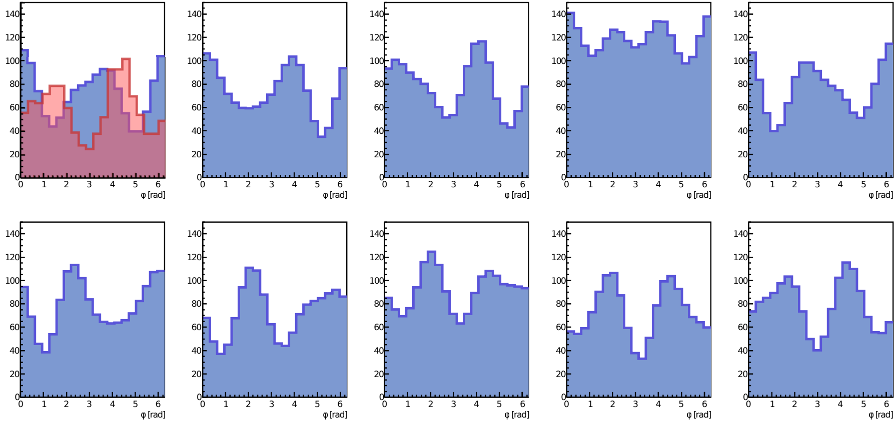
How does it work

Calculate Bayesian probability that the event belong to each event bin



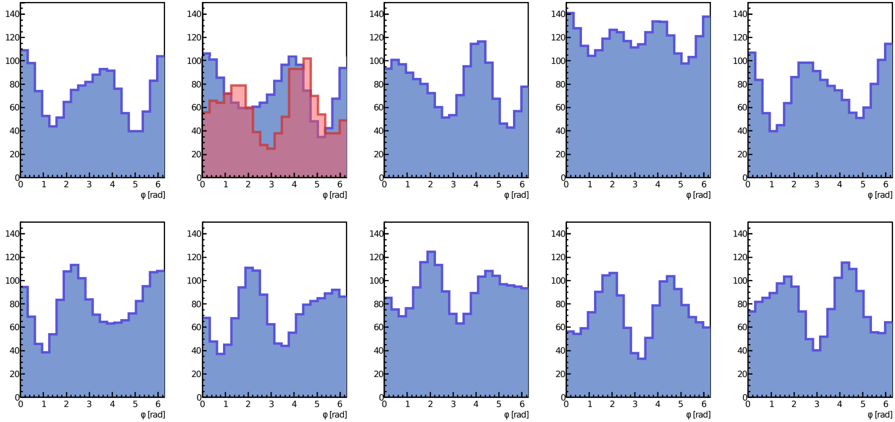
How does it work

Calculate Bayesian probability that the event belong to event bin 1



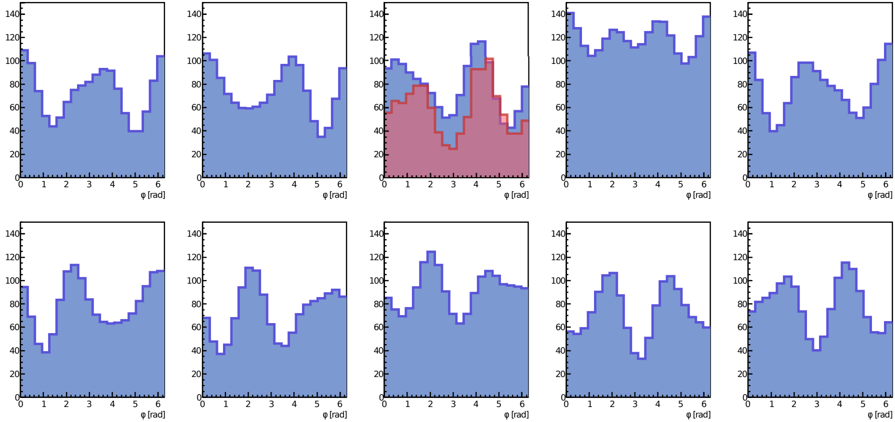
How does it work

Calculate Bayesian probability that the event belong to event bin 2



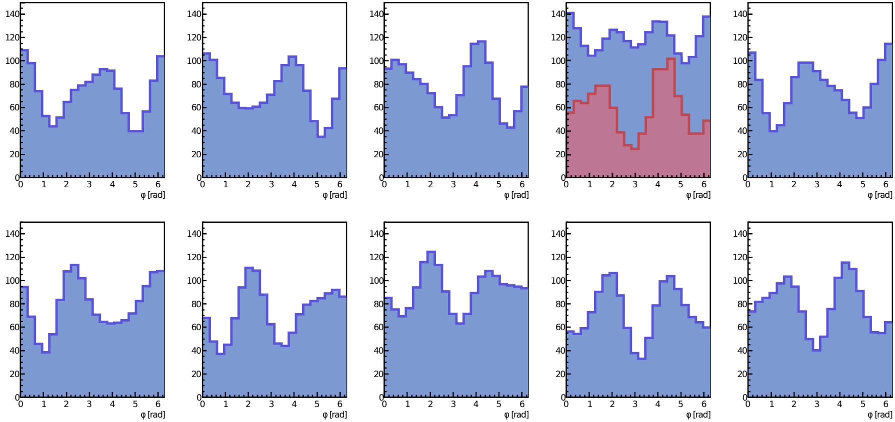
How does it work

Calculate Bayesian probability that the event belong to event bin 3



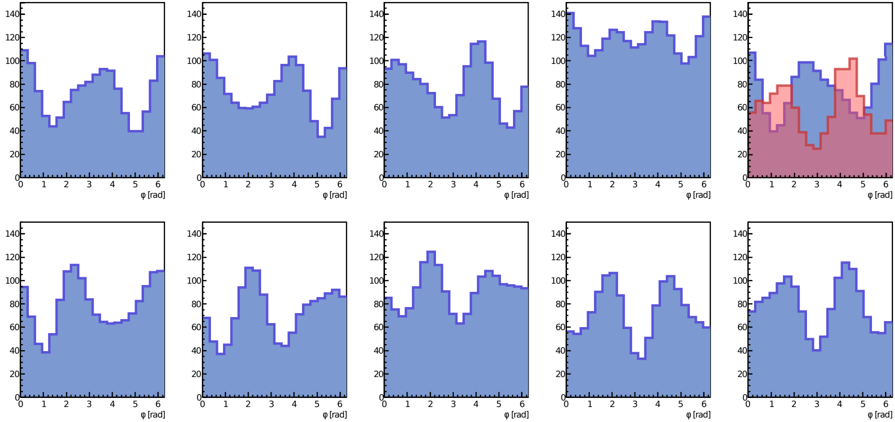
How does it work

Calculate Bayesian probability that the event belong to event bin 4



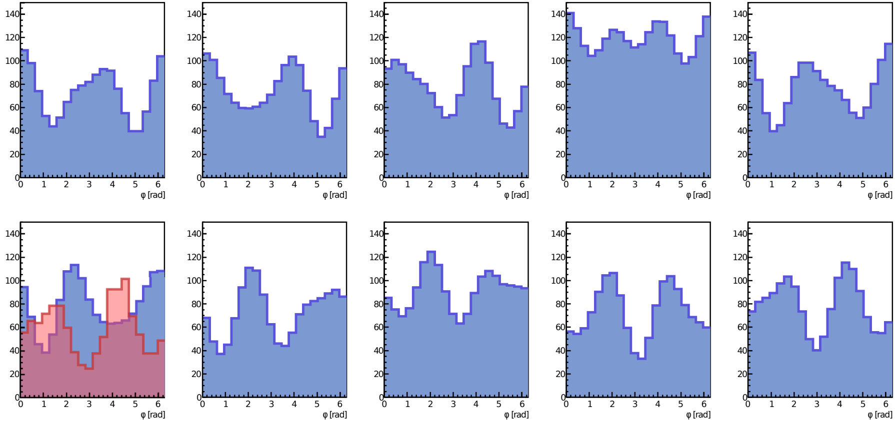
How does it work

Calculate Bayesian probability that the event belong to event bin 5



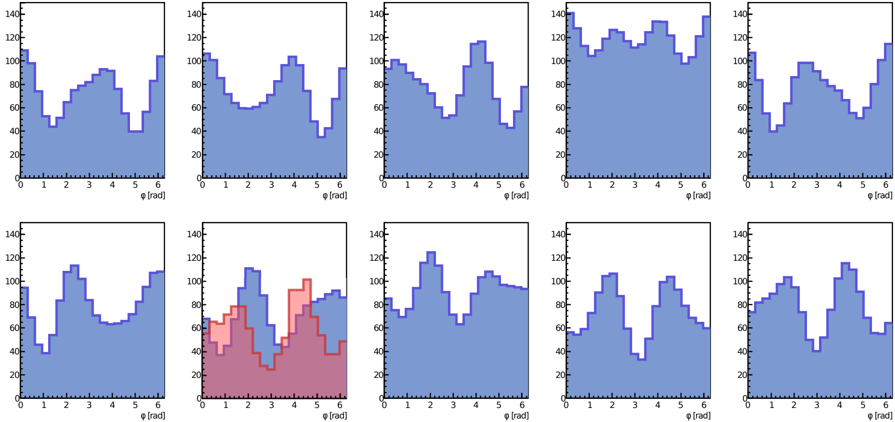
How does it work

Calculate Bayesian probability that the event belong to event bin 6



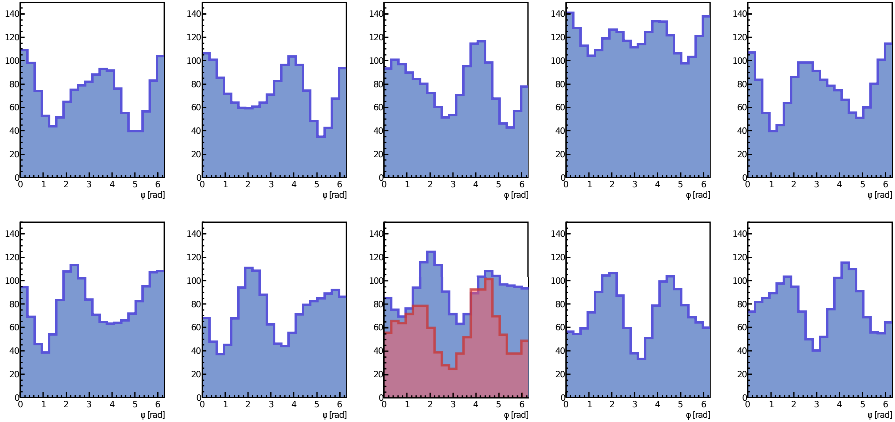
How does it work

Calculate Bayesian probability that the event belong to event bin 7



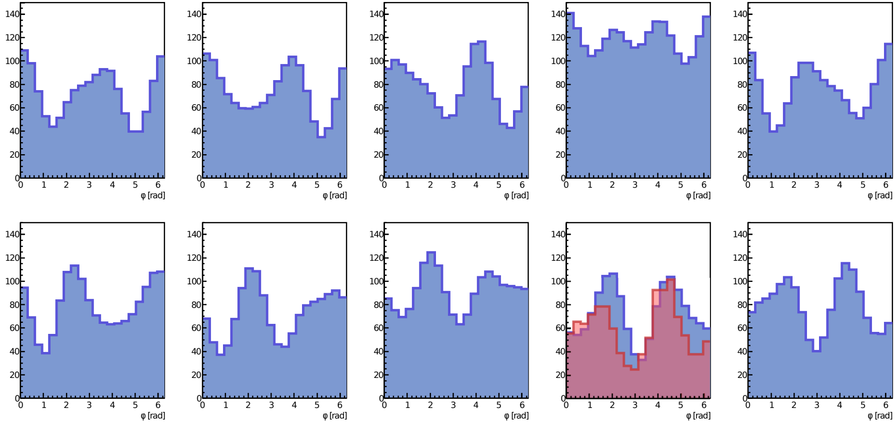
How does it work

Calculate Bayesian probability that the event belong to event bin 8



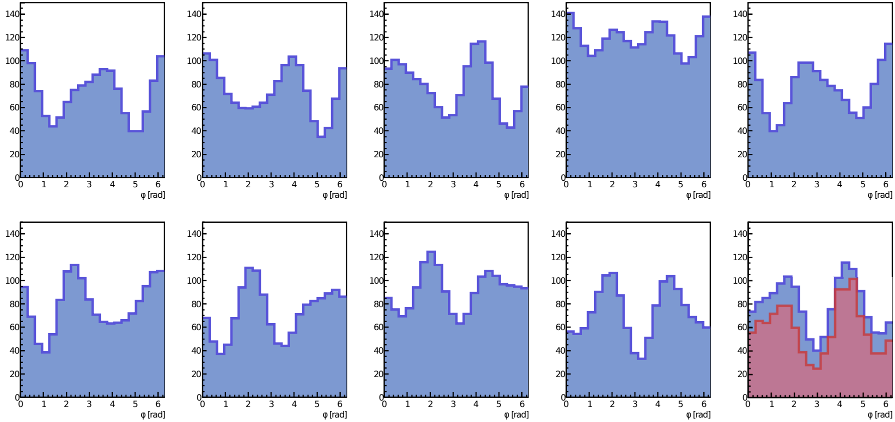
How does it work

Calculate Bayesian probability that the event belong to event bin 9



How does it work

Calculate Bayesian probability that the event belong to event bin 10



After sorting . . .

Similar events end up close to each other.

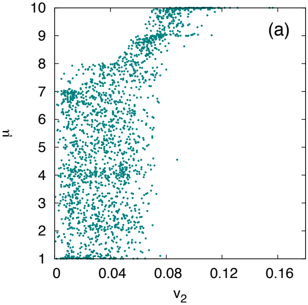
Different events are far away from each other.

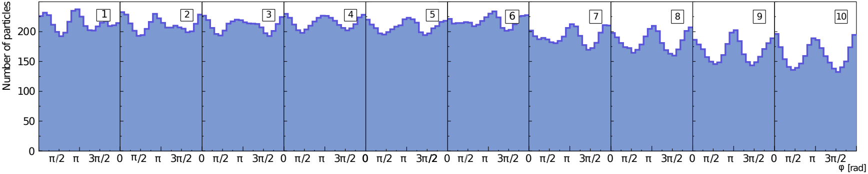
The method always converges to the same sorting.

Sorted AMPT events

Event shape sorting goes beyond characterisation of events according to single variable (e.g. v_2 or q_2)

- simulated 2000 central 0–20% events from AMPT for $\sqrt{s_{NN}} = 2.76$ TeV
- correlation between sorting variable μ and elliptic flow v_2

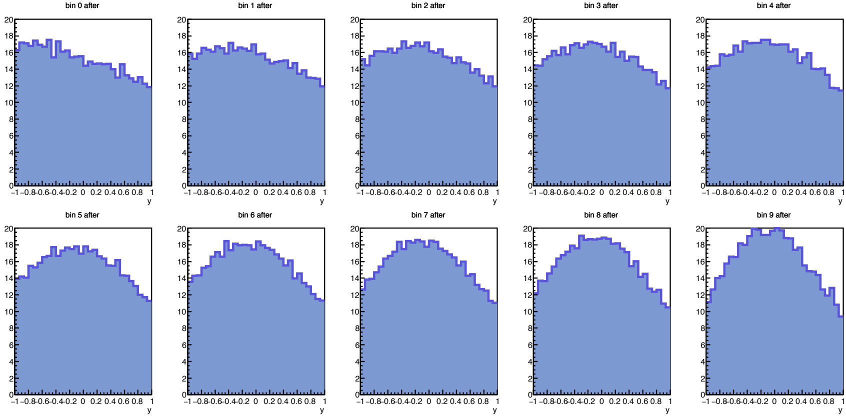




R. Kopečná, B. Tomášik: Eur. Phys. J. A **52** (2016) 115.

A use on rapidity distributions

Simulation with DRAGON, $b = 50\ \mathrm{fm}^3$, 25% hadrons from droplets



Differences between event bins are bigger than in case of no droplets.
More investigation needed.

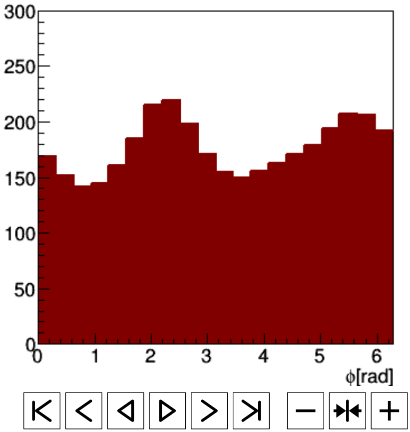
[B. Tomášik *et al.*: Eur. Phys. J. A **52** (2016) 236]

What is Event Shape Sorting good for?

- More selective comparison of data to theory.
- Construction of mixed-events background for correlation functions.
- Allows single-event femtoscopy?
- Recognise large e-by-e fluctuations.
- . . . ideas welcome!

[R. Kopečná, B. Tomášik: Eur. Phys. J. A **52** (2016) 115]

Sorted events: Gradual change of event shape



- 2000 events, AMPT centrality 0–20%, $\sqrt{s_{NN}} = 2.76$ TeV
- each frame averaged over 50 events and shifted by 10 events wrt previous frame
- change of colour = change of event bin

Conclusions

- DRAGON: a tool for easy generation of hadrons from a fragmented fireball
- DRAGON can be used also without droplets
- REGGAE can be used for generation of particle momenta which conserve total energy and momentum.
- Try Event Shape Sorting for more exclusive selection of events.

Event Shape Sorting: the algorithm

We will sort events according to their histograms in azimuthal angle.

1. (Rotate the events appropriately)
2. Sort your events as you wish
3. Divide sorted events into quantiles (we'll do deciles)
4. Determine average histograms in each quantiles
5. For each event i calculate Bayesian probability $P(i|\mu)$ that it belongs to quantile μ
6. For each event calculate average $\bar{\mu} = \sum_\mu \mu P(i|\mu)$
7. Sort events according to their values of $\bar{\mu}$
8. If order of events changed, return to 3. Otherwise sorting converged.

S. Lehmann, A.D. Jackson, B. Lautrup, arXiv:physics/0512238
S. Lehmann, A. D. Jackson and B. E. Lautrup, Scientometrics **76** (2008) 369 [physics/0701311 [physics.soc-ph]]