# Simulations of vorticity, hydrodynamical helicity and handedness

International Mini-Workshop on Simulations of HIC for NICA energies,

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**Phys.Rev. C88 (2013) 061901, C92 (2015) 014906, C93 (2016) 031902, C95 (2017) 011902 , ArXiv 1701.00923 and work in progress**

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# Main Topics

- Rotation in heavy-ion collisions
- Angular momentum and vortical structures
- Hydrodynamical helicity and its separation
- Polarization: from nucleons to ions
- Anomalous mechanism: 4-velocity as a gauge field
- Chemical potential and Energy dependence
- Polarization enhancement for antihyperons
- Handedness in HIC
- Conslusions

# Microworld: where is the fastest possible rotation?

- Non-central heavy ion collisions (Angular velocity ~ c/Compton wavelength)
- ~25 orders of magnitude faster than Earth's rotation
- Differential rotation – vorticity
- P-odd :May lead to various P-odd effects
- Calculation in kinetic quark - gluon string model (DCM/QGSM) – Boltzmann type eqns + phenomenological string amplitudes): Baznat,Gudima,Sorin,OT, PRC'13,16; HSD-OT,Usubov'15

# Rotation in HIC and related quantities

- Non-central collisions – orbital angular momentum
- L=Σ r x p
- Differential pseudovector characteristics – vorticity
- ϖ = curl v
- Pseudoscalar – helicity
- H ~ <(v curl v)>
- Maximal helicity – Beltrami chaotic flows
  v || curl v

# Simulation in QGSM (Kinetics -> HD)

$50 \times 50 \times 100$ cells $\quad dx = dy = 0.6$ fm, $dz = 0.6/\gamma$ fm

- Velocity

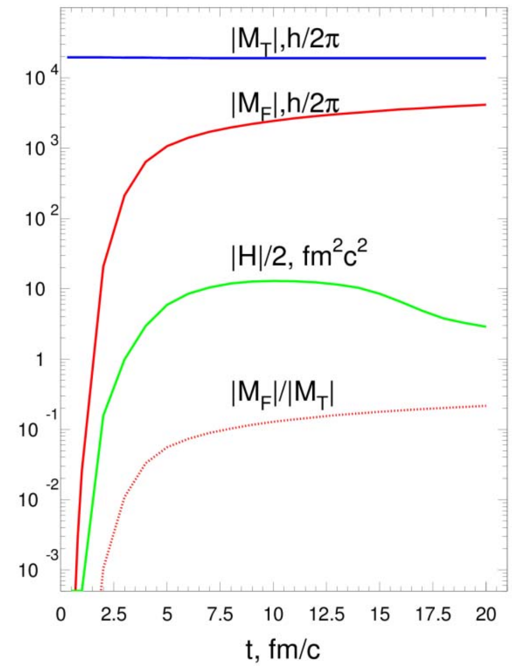
$$\vec{v}(x, y, z, t) = \frac{\sum_i \sum_j \vec{P}_{ij}}{\sum_i \sum_j E_{ij}}$$

- Vorticity – from discrete partial derivatives

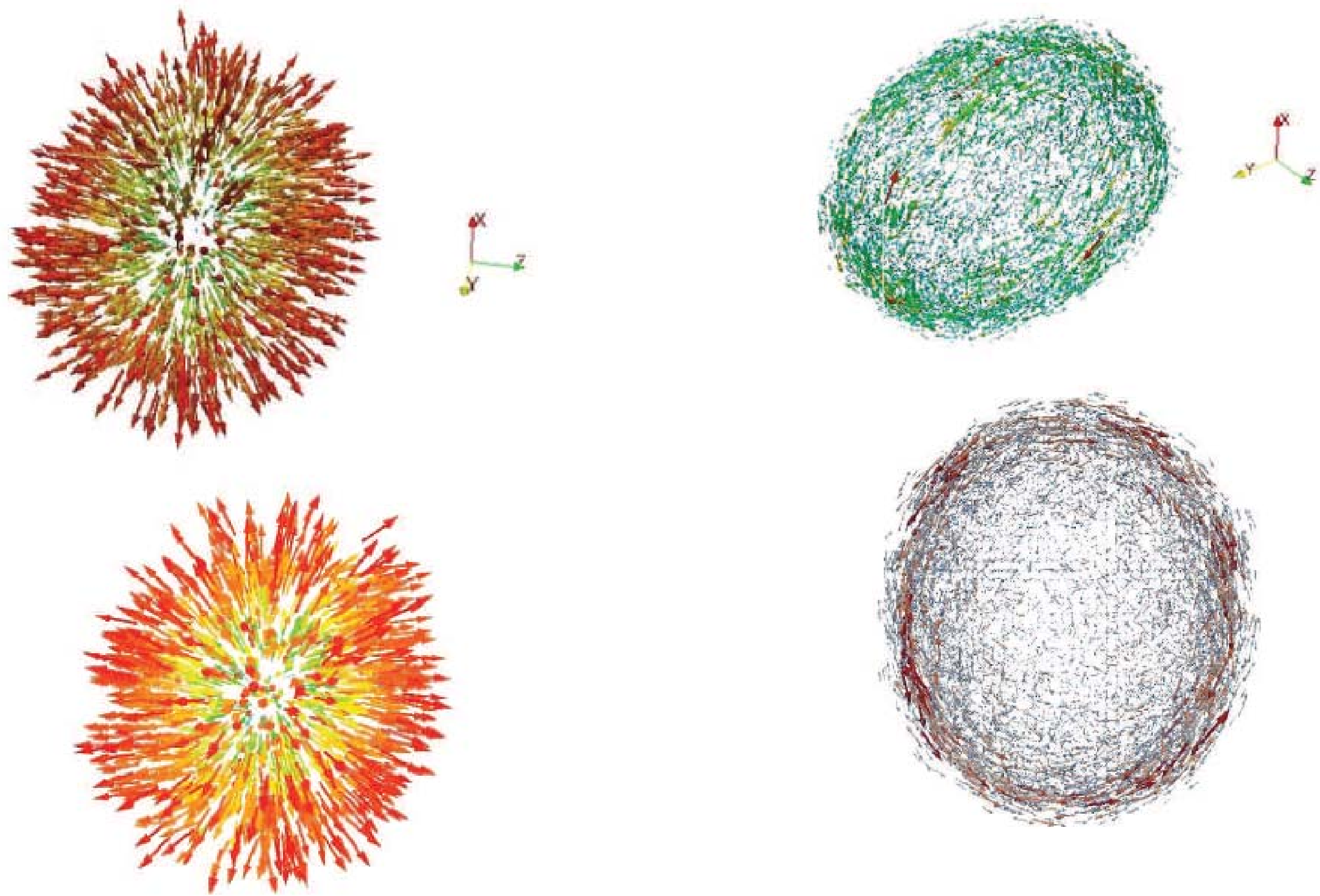
# Angular momentum conservation and helicity

- Helicity vs orbital angular momentum (OAM) of fireball
- (~10% of total)

- Conservation of OAM with a good accuracy!

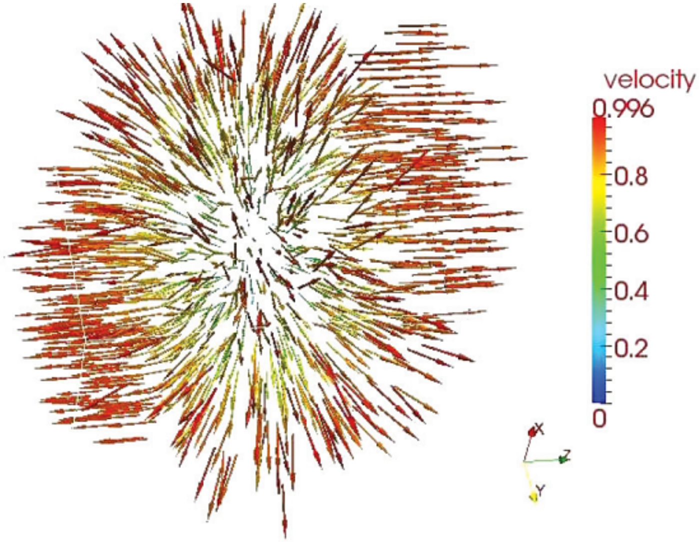


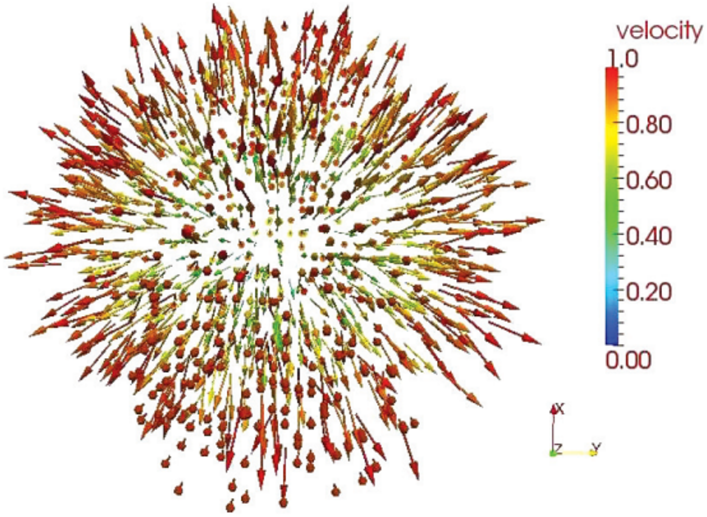
# Structure of velocity and vorticity fields (NICA@JINR-5 GeV/c)



# Distribution of velocity ("Small Bang")

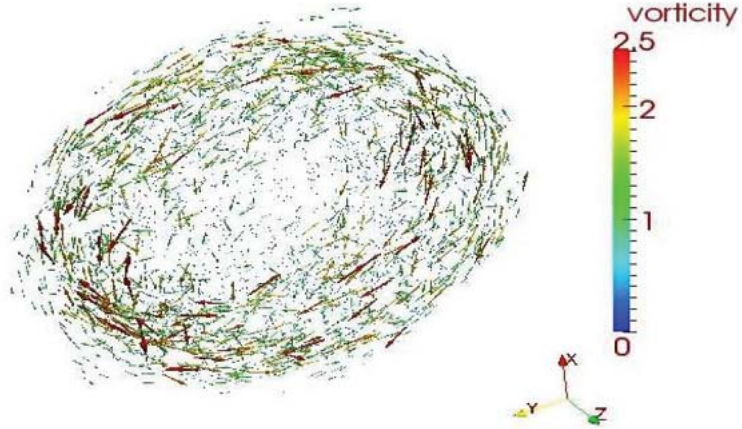
- 3D/2D projection
- z-beams direction
- x-impact paramater

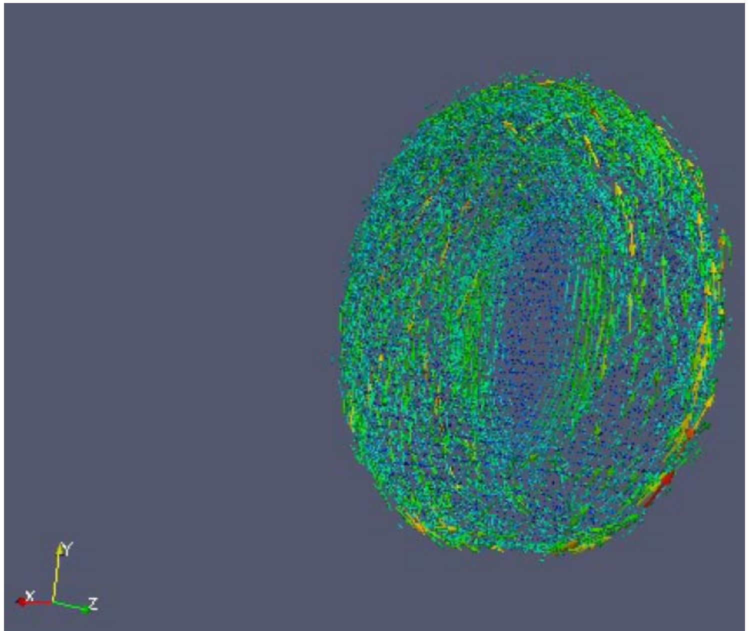


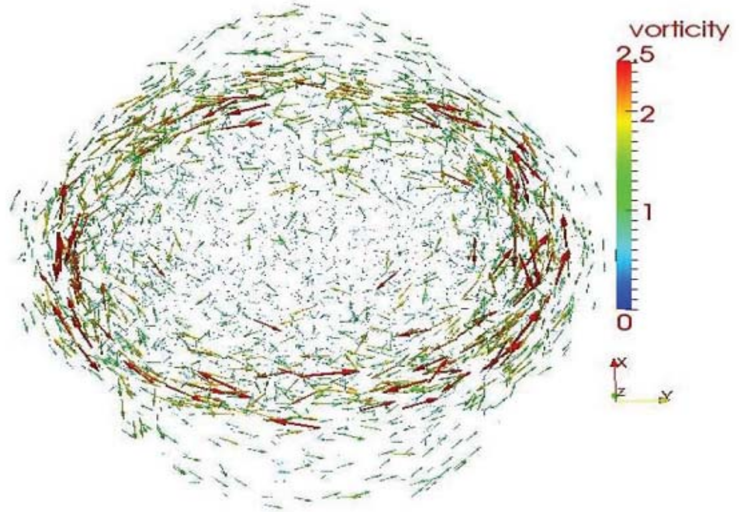



# Distribution of vorticity ("small galaxies")

- Layer (on core - corona borderline) patterns

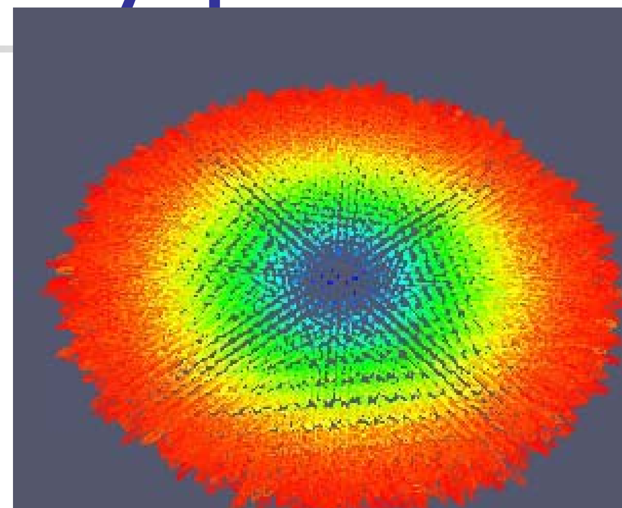


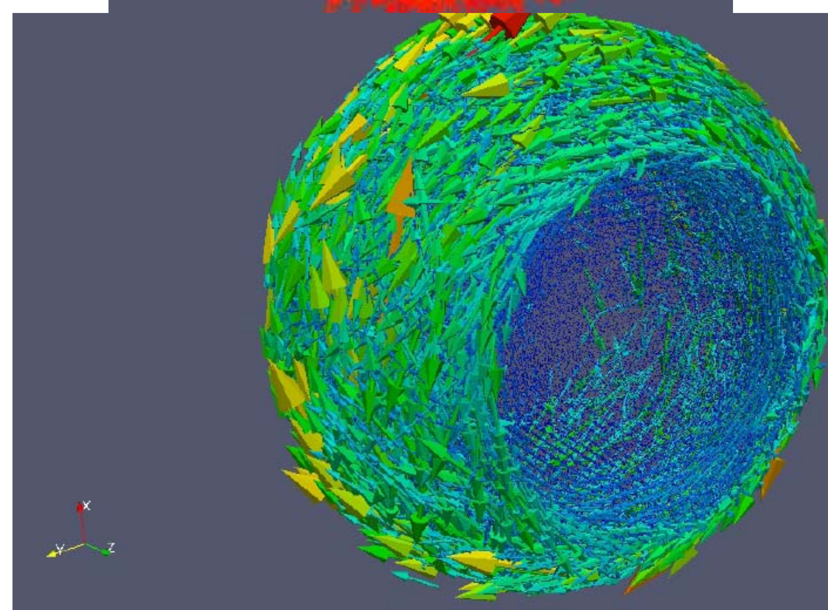



# Velocity and vorticity patterns

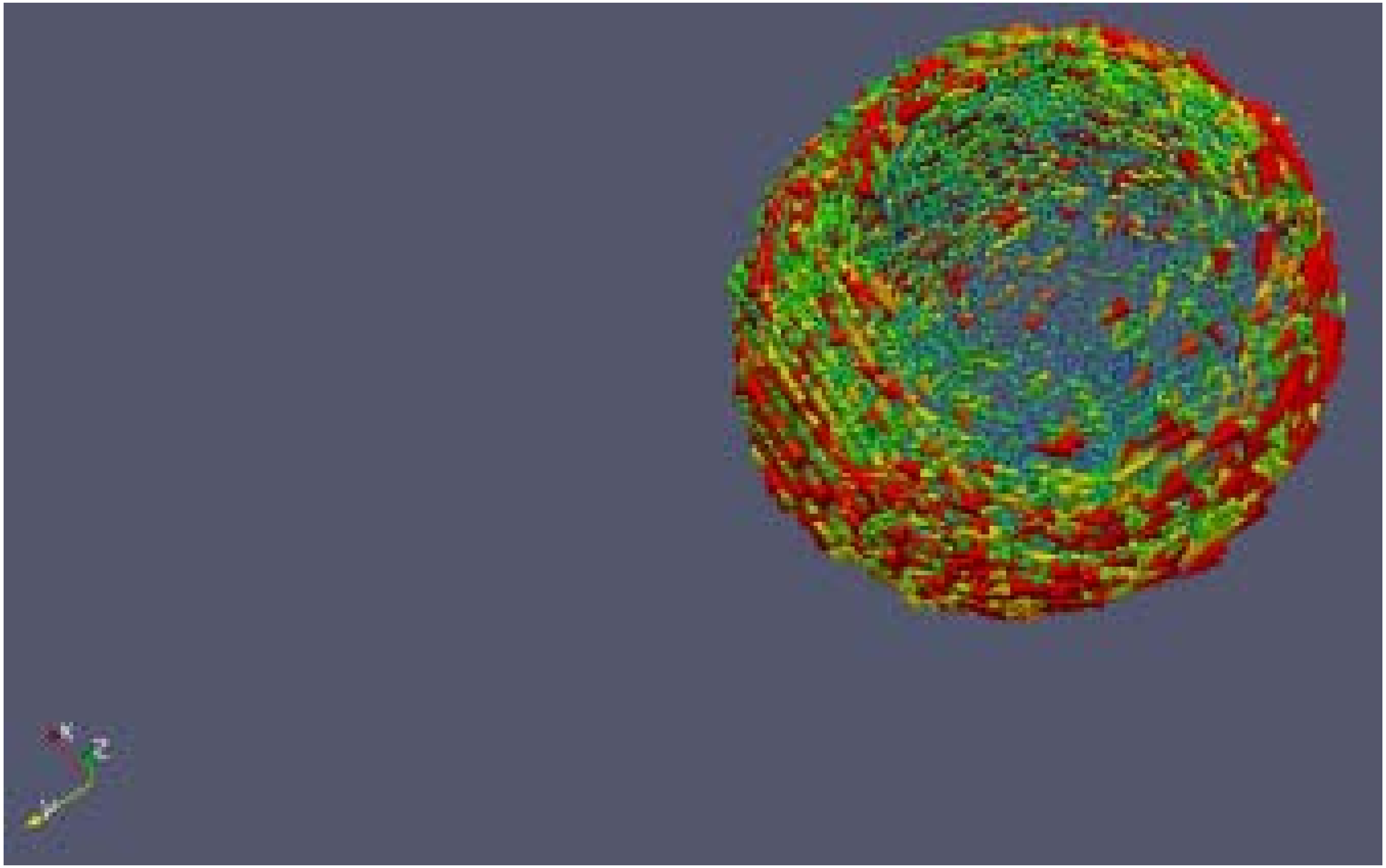
- Velocity



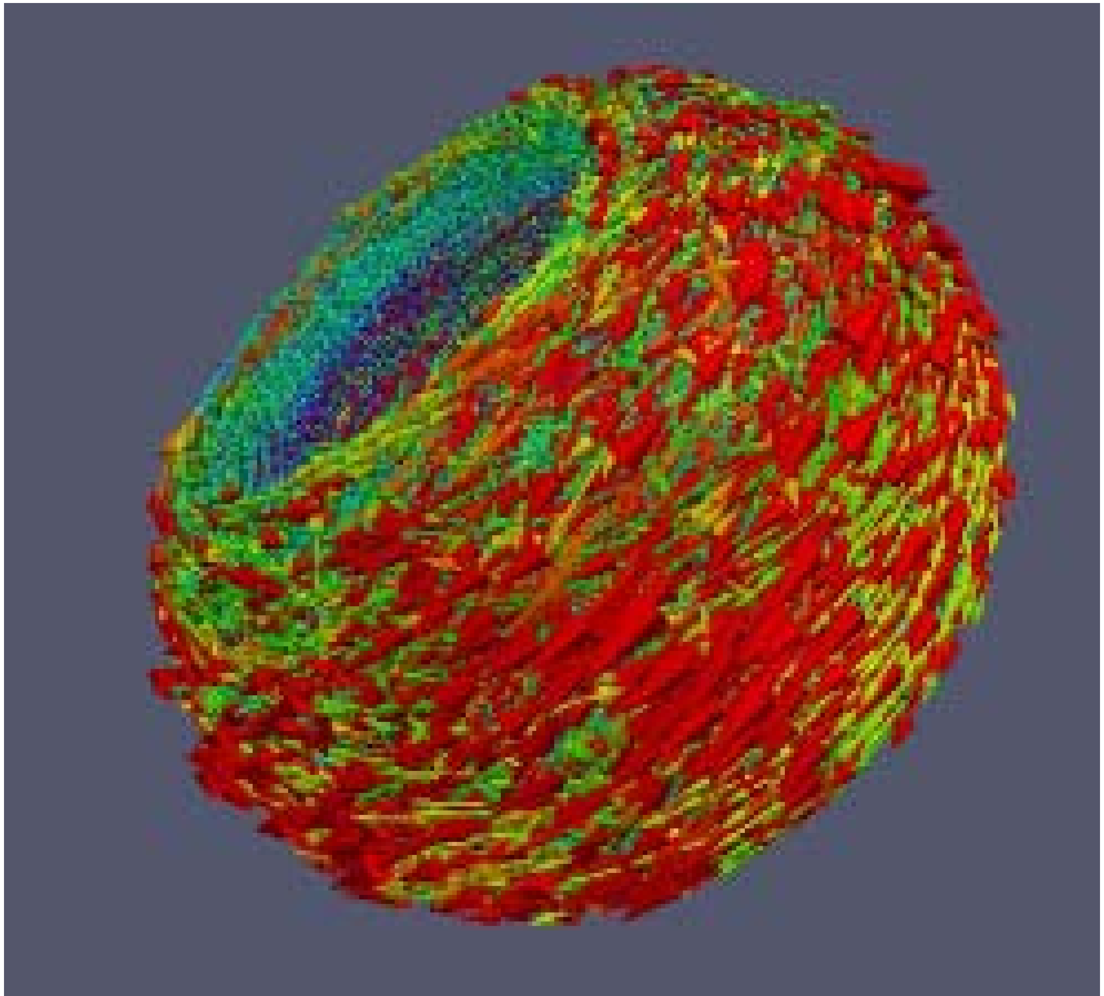
- Vorticity pattern – vortex sheets - due to L BUT cylinder symmetry!



# Vortex sheet (fixed direction of L)

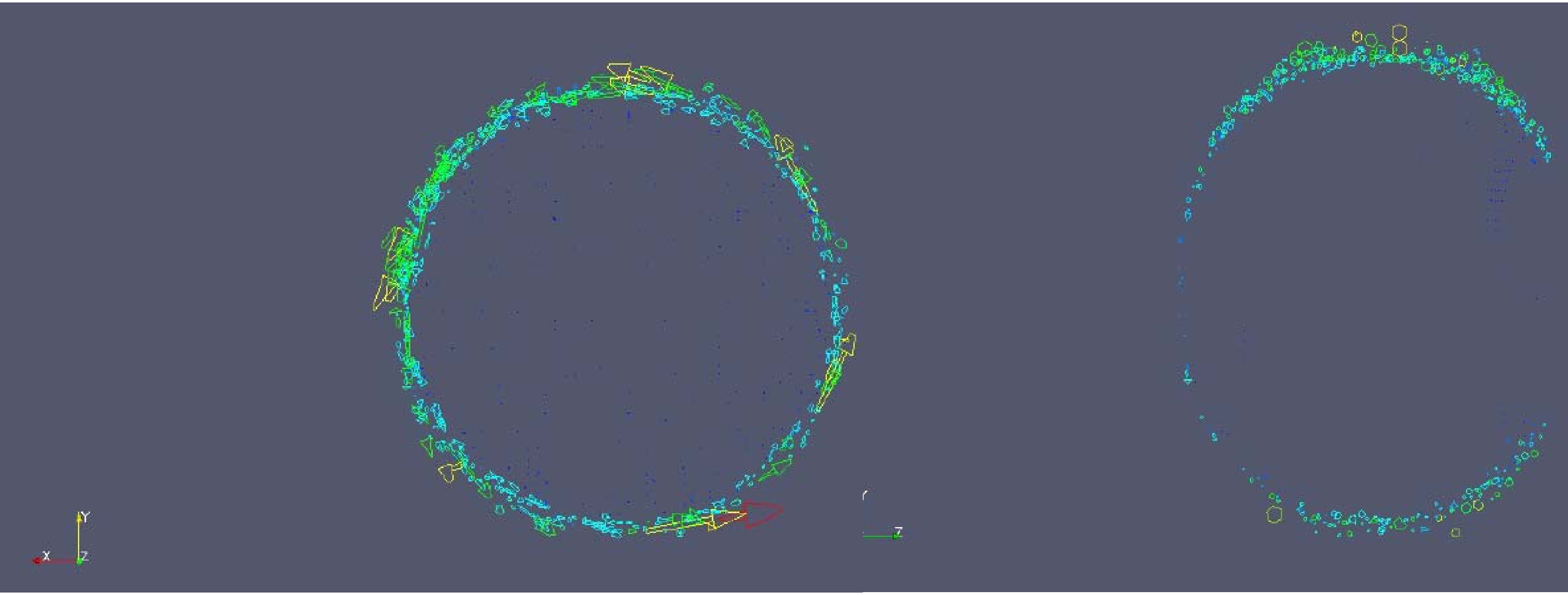


# Vortex sheet （Average over L directions）



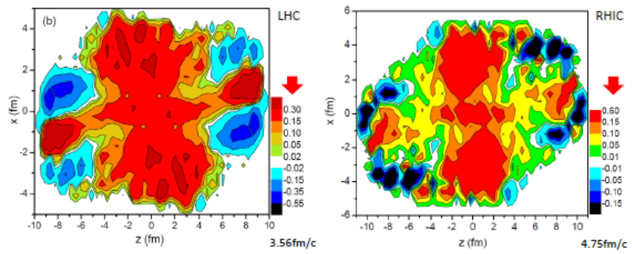
# Sections of vorticity patterns

- Front and side views

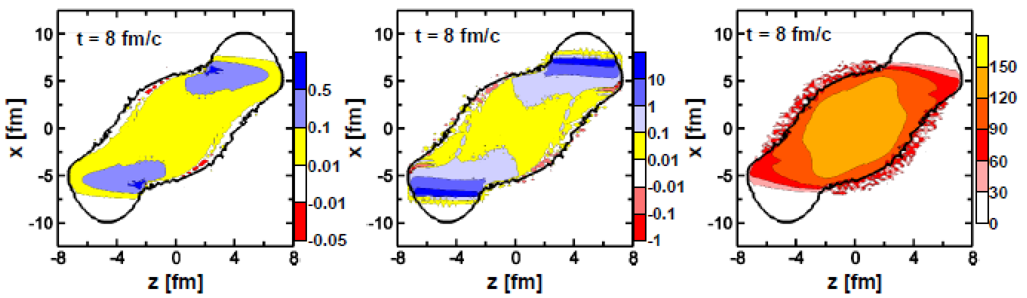


# Vortex sheets

- Naturally appears in kinetic models
- Absent in viscous HD (L. Csernai et al)

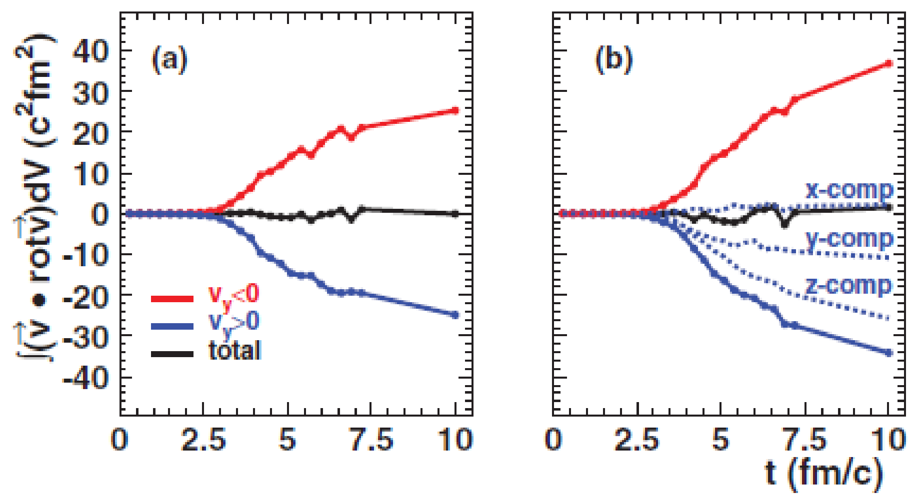



- Appears in 3 fluid dynamics model : Yu. Ivanov (Talk), A. Soldatov, [arXiv:1701.01319]



# Helicity separation in QGSM **PRC88 (2013) 061901**

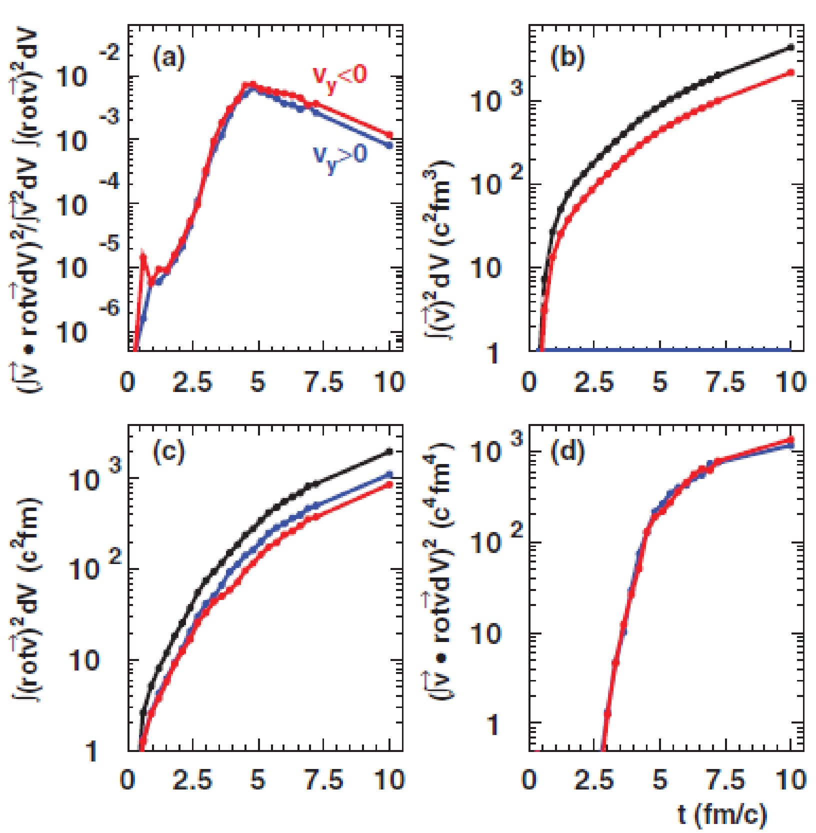
- Total helicity integrates to zero BUT
- Mirror helicities below and above the reaction plane
- Confirmed in HSD (OT, Usubov, PR**C92 (2015) 014906**



# What is the relative orientation of velocity and vorticity?

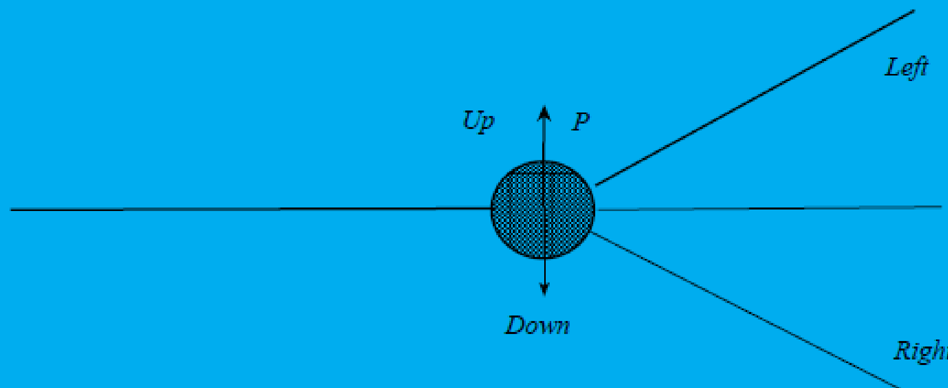
- Measure – Cauchy-Schwarz inequality
- Small but non-negligible correlation
- Maximal correlation - Beltrami flows

PHYSICAL REVIEW C 88, 061901(R) (2013)



# Single Spin Asymmetries: indirect probe of vorticity

Simplest example - (non-relativistic) elastic pion-nucleon scattering $\pi\vec{N} \to \pi N$




$M = a + ib(\vec{\sigma}\vec{n})$ $\vec{n}$ is the normal to the scattering plane.

Density matrix: $\rho = \frac{1}{2}(1 + \vec{\sigma}\vec{P})$,

Differential cross-section: $d\sigma \sim 1 + A(\vec{P}\vec{n}), A = \frac{2Im(ab^*)}{|a|^2+|b|^2}$

# Λ-polarisation

- Self-analyzing in weak decay
- Directly related to s-quarks polarization: complementary probe of strangeness
- Widely explored in hadronic processes
- Disappearance-probe of QCD matter formation (Hoyer; Jacob, Rafelsky: ’87): Randomization – smearing – no direction normal to the scattering plane

# Global polarization

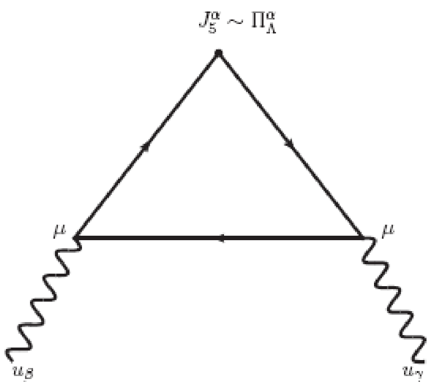
- Global polarization normal to REACTION plane
- Predictions (Z.-T.Liang et al.): large orbital angular momentum -> large polarization
- Search by STAR (Selyuzhenkov et al.'07) : polarization NOT found at % level!
- Maybe due to locality of LS coupling while large orbital angular momentum is distributed
- How to transform rotation to spin?

# Anomalous mechanism – polarization similar to CM(V)E

- 4-Velocity is also a GAUGE FIELD (V.I. Zakharov)

$$e_j A_\alpha J^\alpha \Rightarrow \mu_j V_\alpha J^\alpha$$

- Triangle anomaly leads to polarization of quarks and hyperons (Rogachevsky, Sorin, OT '10)
- Analogous to anomalous gluon contribution to nucleon spin (Efremov,OT'88)
- 4-velocity instead of gluon field!



# Anomaly for polarization

- Induced axial charge

$$c_V = \frac{\mu_s^2 + \mu_A^2}{2\pi^2} + \frac{T^2}{6}, \quad Q_5^s = N_c \int d^3x \, c_V \gamma^2 \epsilon^{ijk} v_i \partial_j v_k$$

- Neglect axial chemical potential
- T-dependent term- related to gravitational anomaly
- Lattice simulation: suppressed due to collective effects

# Energy dependence

- Coupling -> chemical potential

$$Q_5^s = \frac{N_c}{2\pi^2} \int d^3x \, \mu_s^2(x) \gamma^2 \epsilon^{ijk} v_i \partial_j v_k$$

- Field -> velocity; (Color) magnetic field strength -> vorticity;
- Topological current -> hydrodynamical helicity
- Large chemical potential: appropriate for NICA/FAIR energies

**M. Lisa, for the STAR collaboration , QCD Chirality Workshop, UCLA, February 2016;**
**SQM2016, Berkeley, June 2016**

## *One might compare the prediction below with the right panel figures*

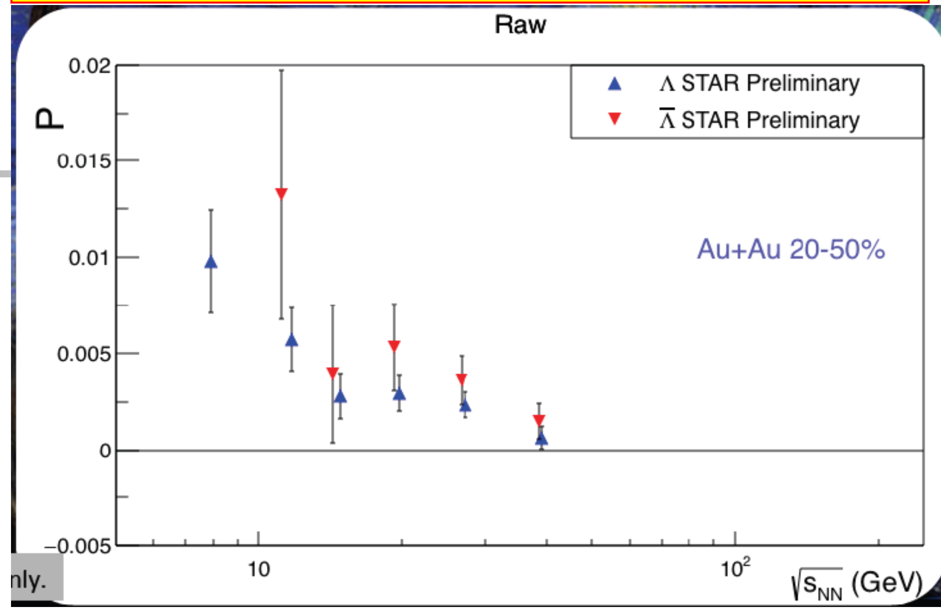
**O. Rogachevsky, A. Sorin, O. Teryaev**
***Chiral vortaic effect and neutron asymmetries in heavy-ion collisions***
**PHYSICAL REVIEW C 82, 054910 (2010)**

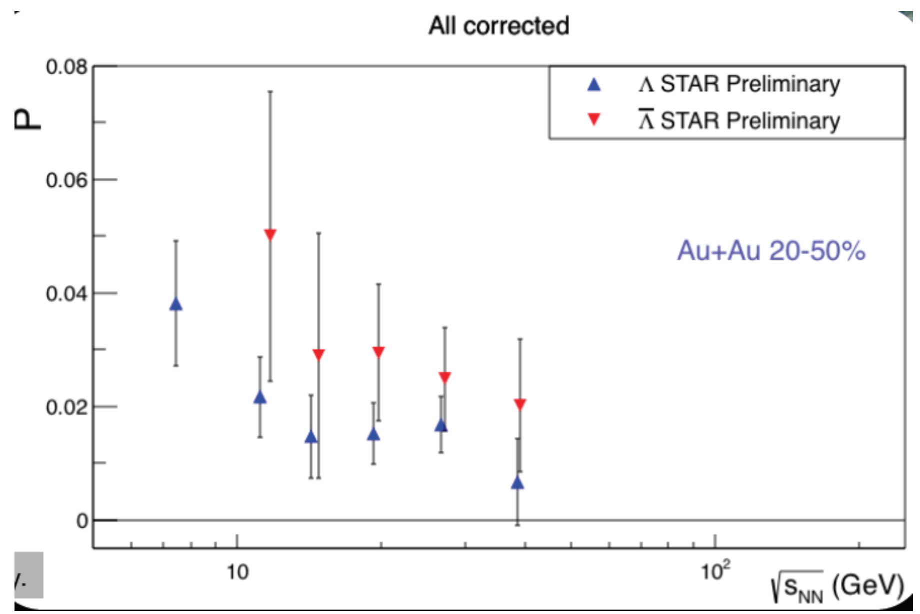
One would expect that polarization is proportional to the anomalously induced axial current [7]

$$j_A^\mu \sim \mu^2 \left(1 - \frac{2\mu n}{3(\epsilon + P)}\right) \epsilon^{\mu\nu\lambda\rho} V_\nu \partial_\lambda V_\rho, \qquad (6)$$

where $n$ and $\epsilon$ are the corresponding charge and energy densities and $P$ is the pressure. Therefore, the $\mu$ dependence of polarization must be stronger than that of the CVE, leading to the effect's increasing rapidly with decreasing energy.

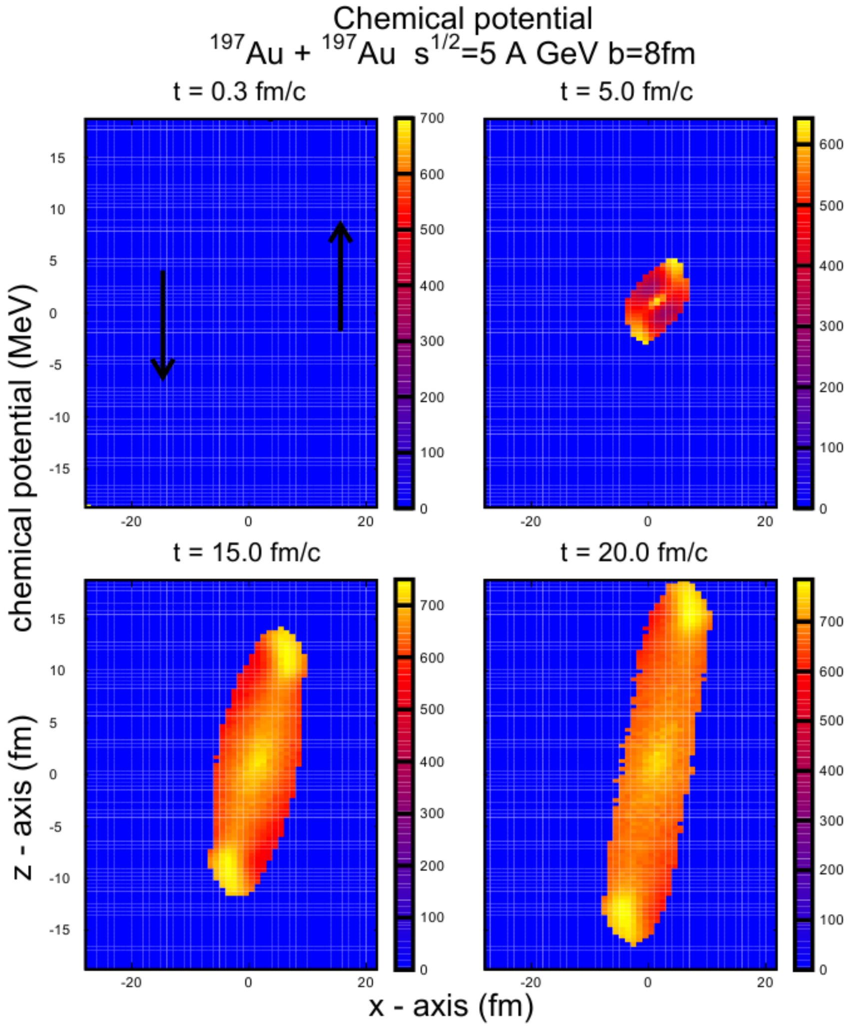
This option may be explored in the framework of the program of polarization studies at the NICA [17] performed at collision points as well as within the low-energy scan program at the RHIC.



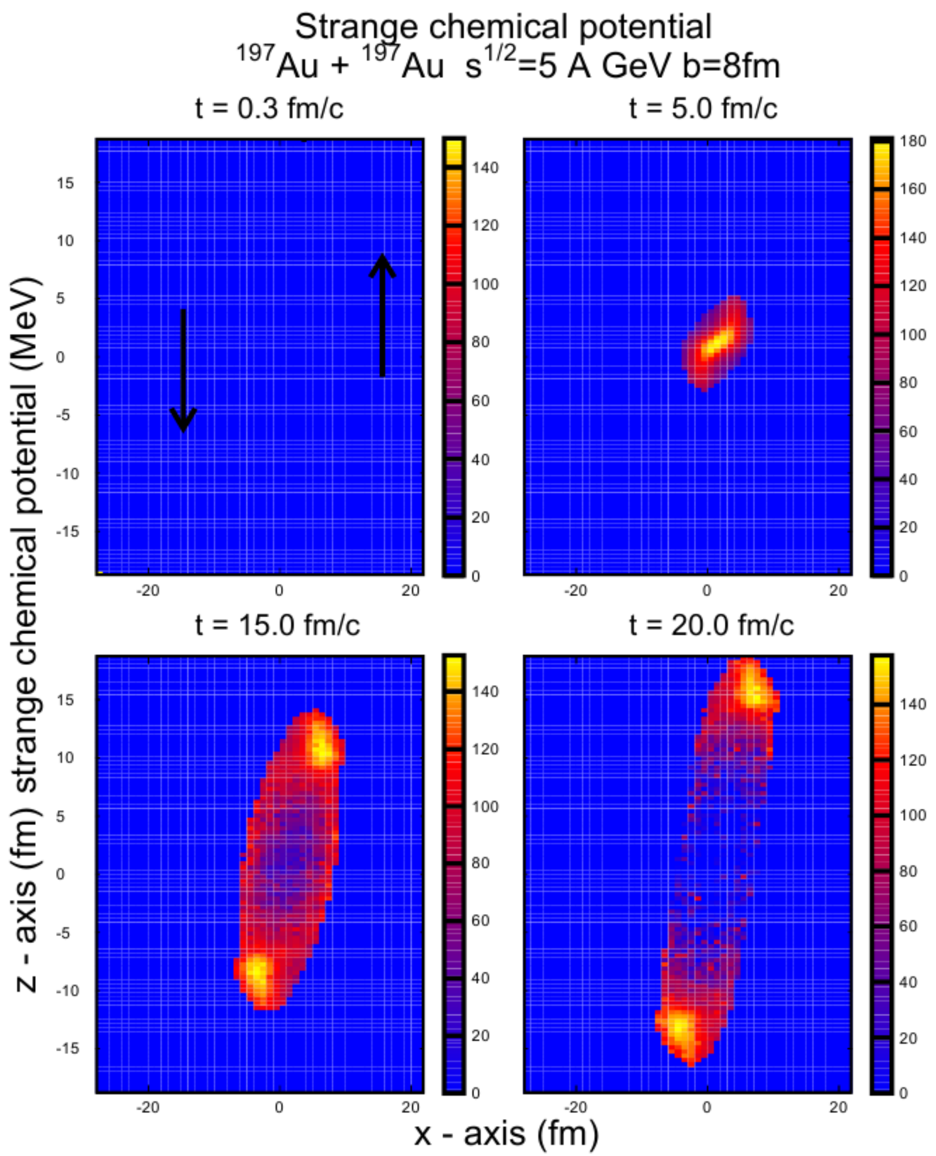
# Chemical potential : Kinetics -> TD

- TD and chemical equilibrium
- Conservation laws
- Chemical potential from equilibrium distribution functions
- 2d section: y=0

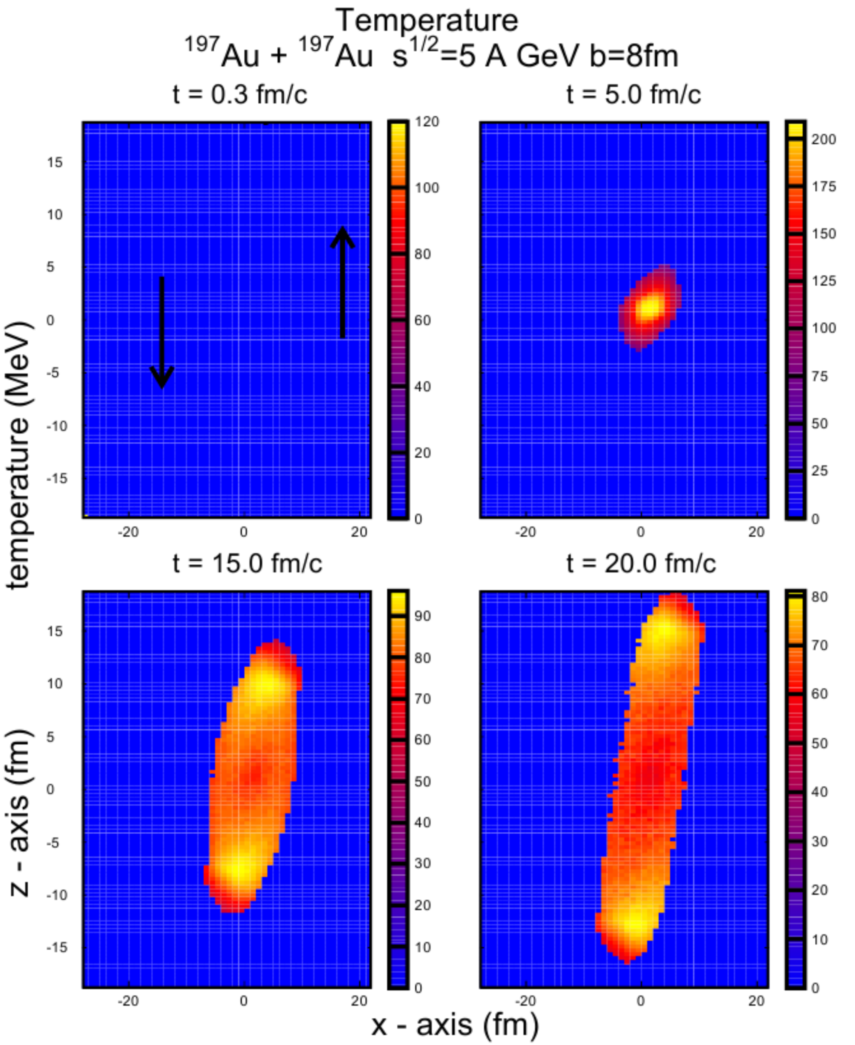


# Strange chemical potential (polarization of Lambda is carried by strange quark!)

- Non-uniform in space and time



# Temperature



# From axial charge to polarization

- Analogy of matrix elements and classical averages

$$< p_n | j^0(0) | p_n >= 2p_n^0 Q_n \qquad < Q >\equiv \frac{\sum_{n=1}^{N} Q_n}{N} = \frac{\int d^3x \, j^0_{class}(x)}{N}$$

- Lorentz boost: compensate the sign of helicity

$$\Pi^{\Lambda,lab} = (\Pi_0^{\Lambda,lab}, \Pi_x^{\Lambda,lab}, \Pi_y^{\Lambda,lab}, \Pi_z^{\Lambda,lab}) = \frac{\Pi_0^{\Lambda}}{m_\Lambda}(p_y, 0, p_0, 0)$$

$$< \Pi_0^{\Lambda} >= \frac{m_\Lambda \Pi_0^{\Lambda,lab}}{p_y} =< \frac{m_\Lambda}{N_\Lambda p_y} > Q_5^s \equiv < \frac{m_\Lambda}{N_\Lambda p_y} > \frac{N_c}{2\pi^2} \int d^3x \, \mu_s^2(x) \gamma^2 \epsilon^{ijk} v_i \partial_j v_k$$

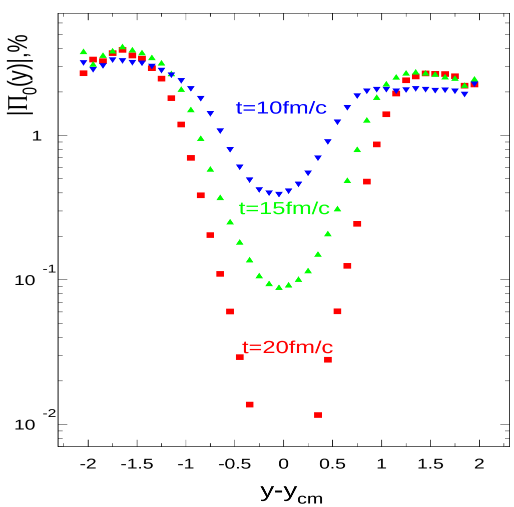
- Antihyperons (smaller N) : same sign and larger value (confirmed by STAR)

# Helicity -> rest frame polarization

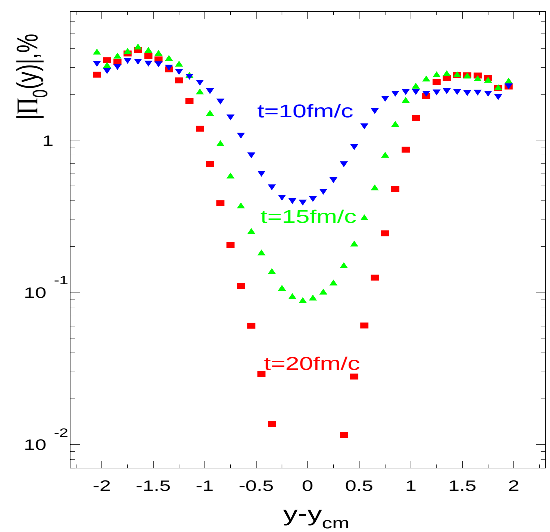
- Helicity ~ 0th component of polarization in lab. frame – effect of boost to Lambda rest frame – various options

$$\Pi_0(y)=1/(4\pi^2)\int \gamma^2(x)\mu_s^2(x)|v\cdot rot(v)|n_\Lambda(y,x)w_1d^3x \Big/ \int n_\Lambda(y,x)w_2d^3x$$

$w_1=1$, $w2= 1$

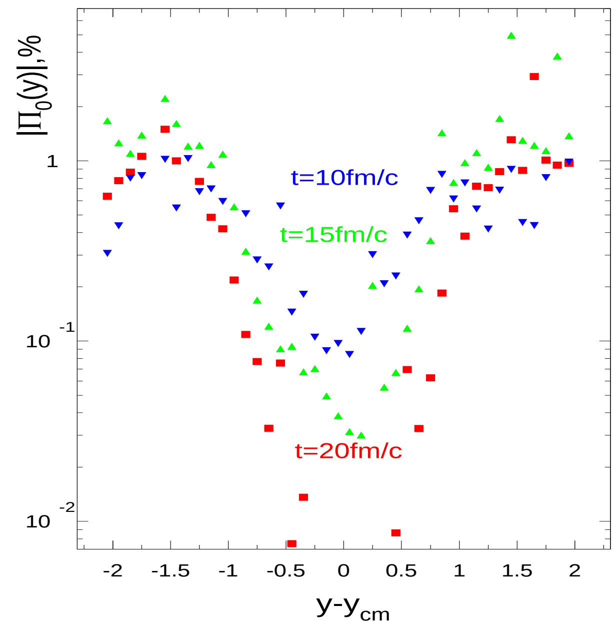


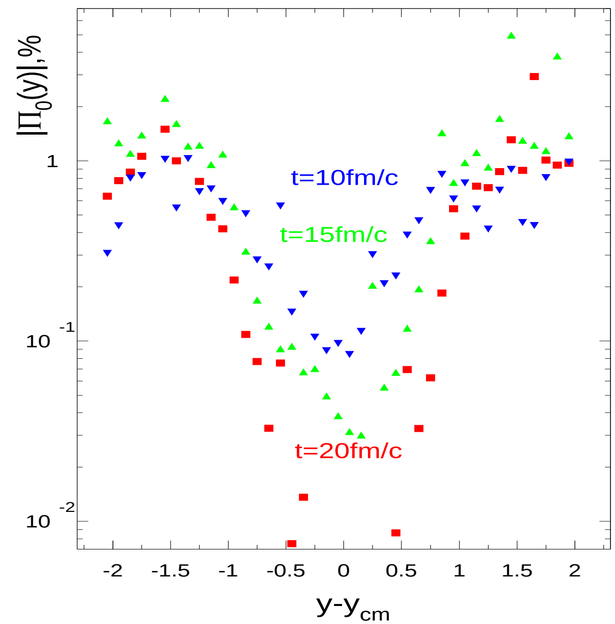

$w_1=1$, $w_2=p_y/m$



# Various methods of boost implementation

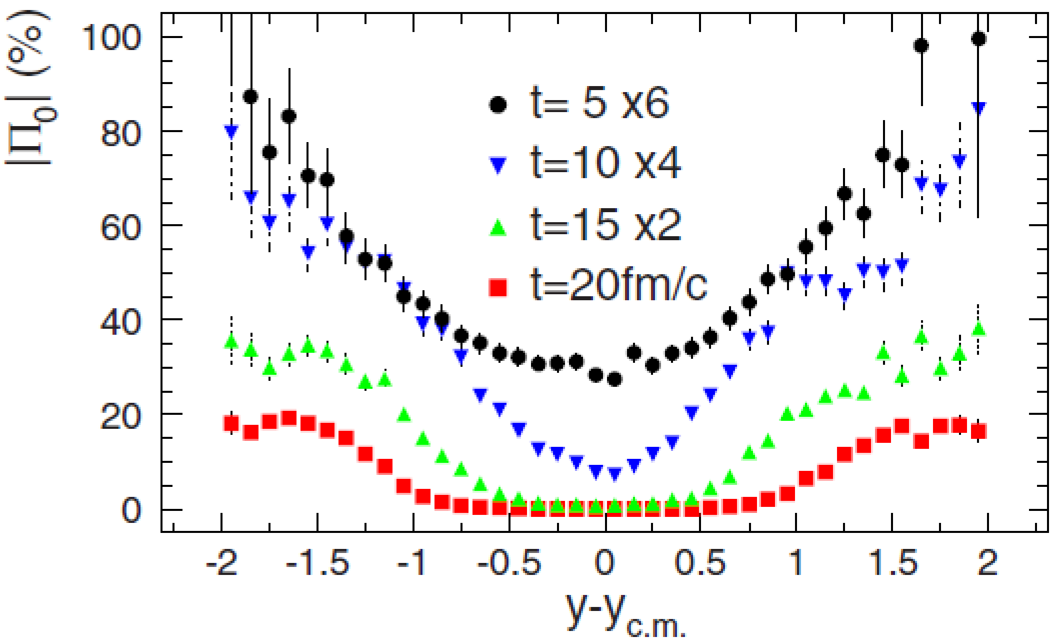
- $w_1=m/p_y$, $w_2=1$ $w_1=m/p_y$ , $w_2=p_y/m$



# Combining QGSM (thermal)vorticity with TD mechanism (F. Becattini et al., talk of Iu. Karpenko)

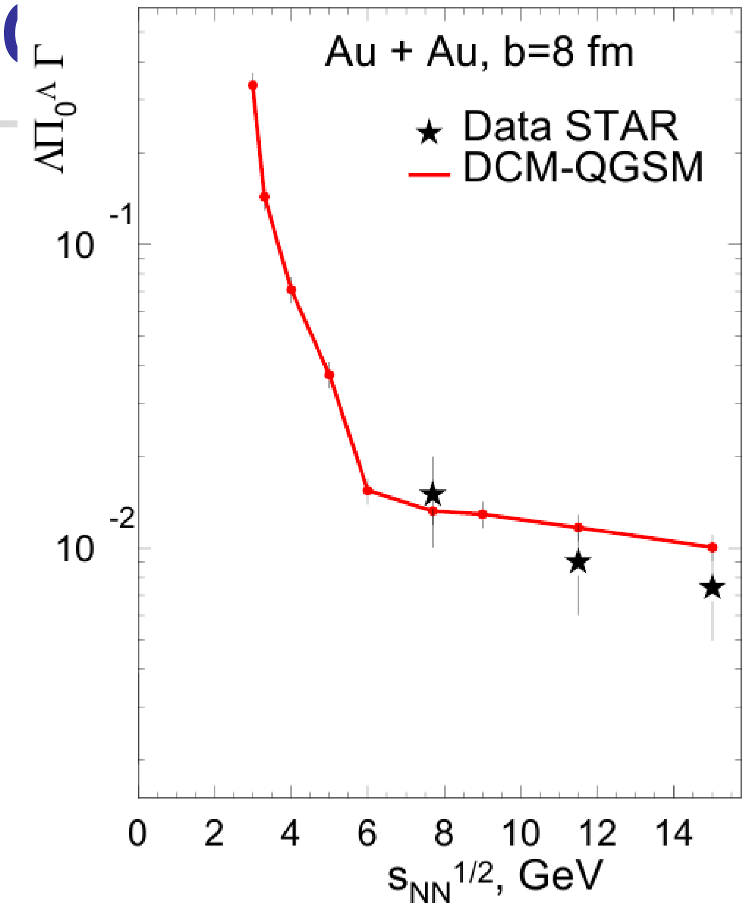
- Temperature – calculated analogously to chemical potential




- Similar polarization pattern

# Energy dependenc

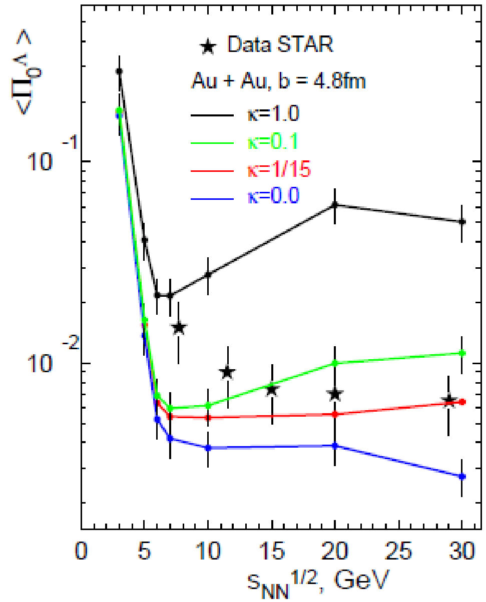
- Growth at low energy




- Surprisingly close to STAR data!
- Structure – may be due to fluctuation for low particles number

# The role of (gravitational anomaly related) $T^2$ term

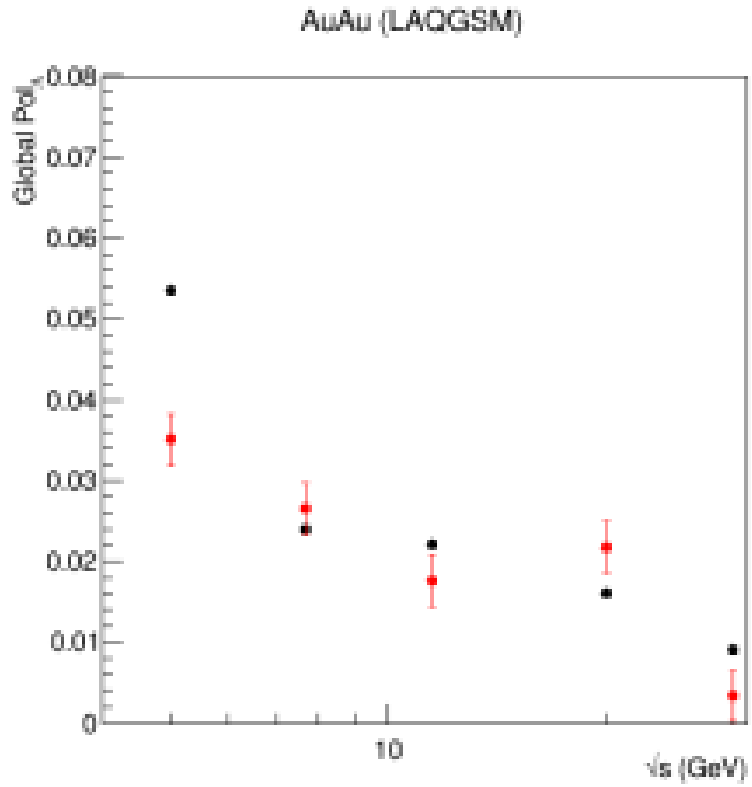
- Different values of coefficient probed




- LQCD suppression by collective effects supported

# Polarization at NICA/MPD (A. Kechechyan)

- QGSM Simulations and recovery accounting for MPD acceptance effects



# Role of vector mesons

- Strange axial charge may be also carried by K* mesons
- Λ - accompanied by (+,anti 0) K* mesons with two sea quarks – small corrections
- Anti Λ – more numerous (-,0) K* mesons with single (sea) strange antiquark –reduction of enhancement
- Vector polarization implies also tensor polarization – anisotropy measurable in dilepton angular distributions

# Handedness: directly observable P-odd momentum correlations

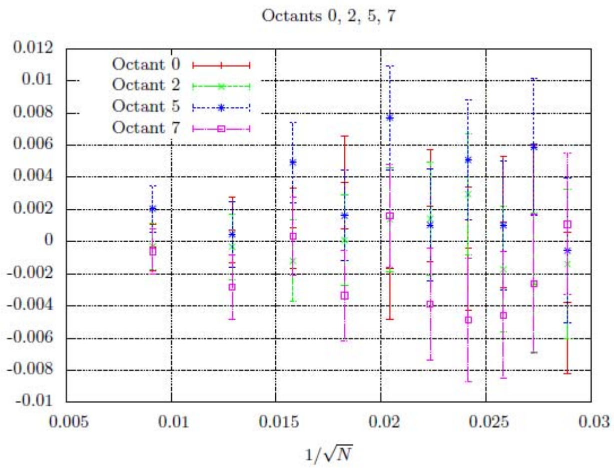
- Suggetsed for jets by A.V. Efremov et al '92,
- Found in jets in $e^+e^-$ annihilation (LEP, BELLE)
- First attempt in HIC: OT, Usubov, **arXiv:1406.4451** (PR**C92 (2015) 1, 014906**)
- Average =0: Phase space – 8 octants

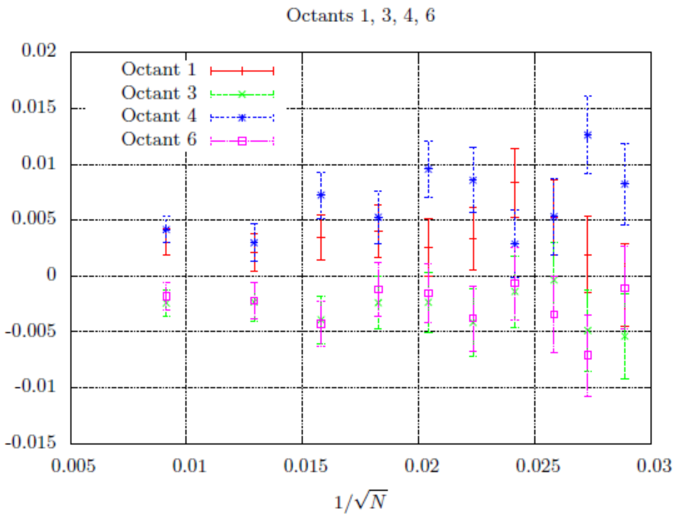
| Octant | Momentum |
|---|---|
| 0 | $p_x > 0, p_y > 0, p_z > 0$ |
| 1 | $p_x > 0, p_y > 0, p_z \leq 0$ |
| 2 | $p_x > 0, p_y \leq 0, p_z > 0$ |
| 3 | $p_x > 0, p_y \leq 0, p_z \leq 0$ |
| 4 | $p_x \leq 0, p_y > 0, p_z > 0$ |
| 5 | $p_x \leq 0, p_y > 0, p_z \leq 0$ |
| 6 | $p_x \leq 0, p_y \leq 0, p_z > 0$ |
| 7 | $p_x \leq 0, p_y \leq 0, p_z \leq 0$ |

$$\eta = \frac{\sum(\vec{p_3}, \vec{p_2}, \vec{p_1})}{\sum|(\vec{p_3}, \vec{p_2}, \vec{p_1})|}$$

# Handedness separation

- Indication for small separation effect in some of the octants

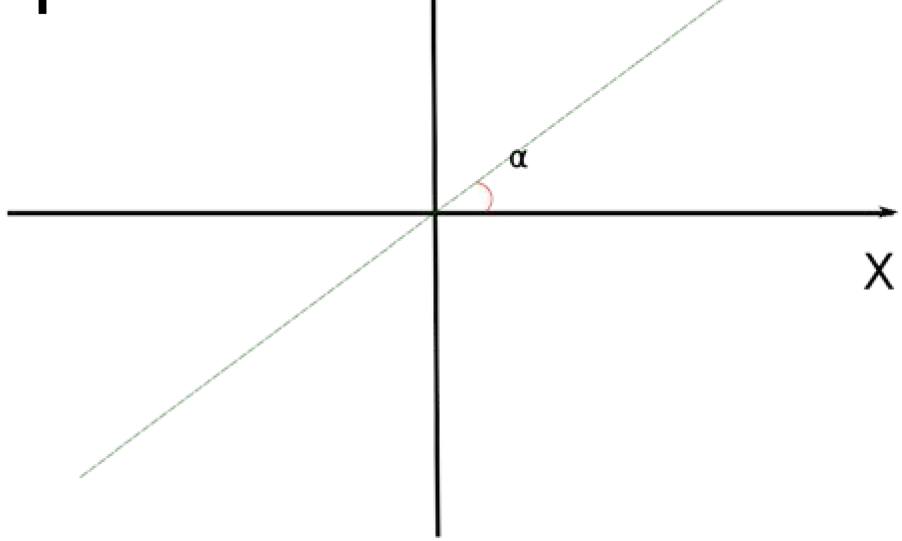


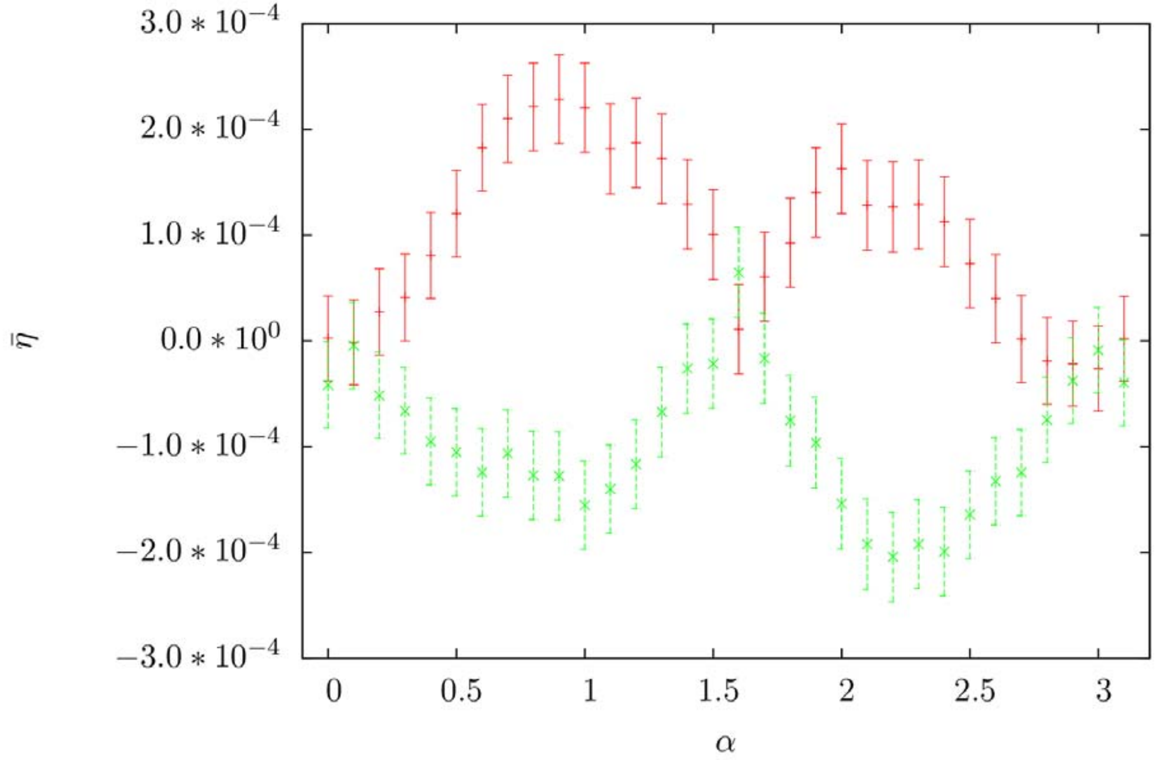

| Octant | Momentum |
|---|---|
| 0 | $p_x > 0, p_y > 0, p_z > 0$ |
| 1 | $p_x > 0, p_y > 0, p_z \leq 0$ |
| 2 | $p_x > 0, p_y \leq 0, p_z > 0$ |
| 3 | $p_x > 0, p_y \leq 0, p_z \leq 0$ |
| 4 | $p_x \leq 0, p_y > 0, p_z > 0$ |
| 5 | $p_x \leq 0, p_y > 0, p_z \leq 0$ |
| 6 | $p_x \leq 0, p_y \leq 0, p_z > 0$ |
| 7 | $p_x \leq 0, p_y \leq 0, p_z \leq 0$ |

# Angular dependence (OT, Usubov, work in progress)

- Handedness is calculated in two half-spaces divided by a plane.
- Dependence of handedness on the angle between the subdividing plane and reaction plane is studied:



# Handedness angular dependence in PHSD model (preliminary)



# Conclusions/Outlook

- Vorticity – femto-vortex sheet formation(QGSM+; HSD?) , helicity separation (QGSM+,HSD+)
- New simulations in various models required
- Polarization – new probe of anomaly in quark-gluon matter (to be studied at NICA!?)
- Energy dependence predicted and confirmed
- Same sign and larger magnitude of antihyperon polarization
- T-dependent term due to gravitational anomaly may be extracted from the data
- Handedness in HIC – directly observable momentum correlations

# BACKUP

# Properties of SSA

The same for the case of initial or final state polarization.

Various possibilities to measure the effects: change sign of $\vec{n}$ or $\vec{P}$: left-right or up-down asymmetry.

Qualitative features of the asymmetry

Transverse momentum required (to have $\vec{n}$)

Transverse polarization (to maximize $(\vec{P}\vec{n})$)

Interference of amplitudes

IMAGINARY phase between amplitudes - absent in Born approximation

# Phases and T-oddness

Clearly seen in relativistic approach:

$\rho = \frac{1}{2}(\hat{p} + m)(1 + \hat{s}\gamma_5)$

Than: $d\sigma \sim Tr[\gamma_5 ....] \sim i m \varepsilon_{s p_1 p_2 p_3} ...$

Imaginary parts (loop amplitudes) are required to produce real observable.

$\varepsilon_{abcd} \equiv \varepsilon^{\alpha\beta\gamma\delta} a_\alpha b_\beta c_\gamma d_\delta$ each index appears once: $P-$ (compensate $S$) and $T-$ odd.

However: no real $T-$violation: interchange $|i> \leftrightarrow |f>$ is the nontrivial operation in the case of nonzero phases of $<f|S|i>^* = <i|S|f>$.

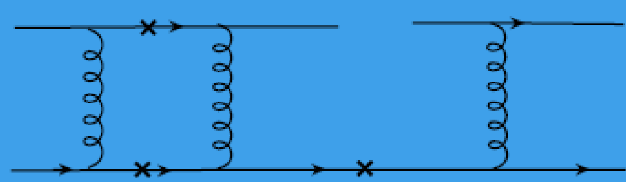
SSA - either T-violation or the phases.

DIS - no phases ($Q^2 < 0$)- real T-violation.

# Perturbative PHASES IN QCD

QCD factorization: where to borrow imaginary parts?

Simplest way: from short distances - loops in partonic subprocess. Quarks elastic scattering (like $q-e$ scattering in DIS):

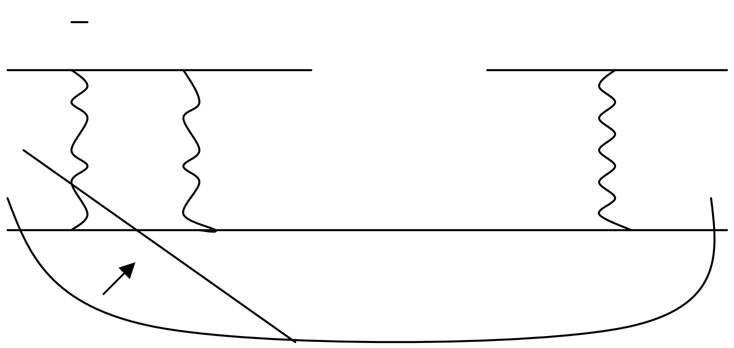


$$A \sim \frac{\alpha_S m p_T}{p_T^2 + m^2}$$

Large SSA "...contradict QCD or its applicability"

# Short+ large overlap– twist 3

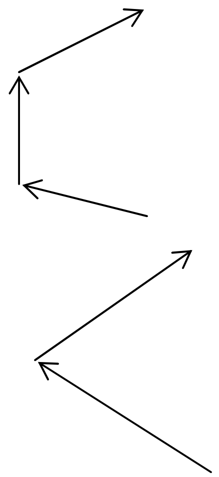
- Quarks – only from hadrons
- Various options for factorization – shift of SH separation



- New option for SSA: Instead of 1-loop twist 2 – Born twist 3 (quark-gluon correlator): Efremov, OT (85, Fermionic poles); Qiu, Sterman (91, GLUONIC poles)
- Further shift to large distances – T-odd fragmentation functions (Collins, dihadron, handedness)

# Correlations of jets handedness

- LEP – quarks are polarized due to weak interaction
- BUT – how to ditinguish quark/antiquark jets?
- 2 jets - correlation of helicities – correlation of handedness
- Hadronic collisions – for jets from the same quark-antiquark pair



# CONCLUSIONS (fast rotation)

- HIC: Lambda polarization of % order predominantly in forward/backward regions
- Correlation of quark jet handedness – sensitive to production mechanisms
- Correlation of handedness in HIC – measure of angular momentum?

# Spin-gravity/rotation (~ 25 orders of magnitude slower!) interactions

- How to describe hadron spin/gravity(inertia) couplings?
- Matrix elements of Energy-Momentum Tensor
- May be studied in non-gravitational experiments/theory
- Simple interpretation in comparison to EM field case

# Gravitational Formfactors

$$\langle p'|T^{\mu\nu}_{q,g}|p\rangle = \bar{u}(p')\Big[A_{q,g}(\Delta^2)\gamma^{(\mu}p^{\nu)} + B_{q,g}(\Delta^2)P^{(\mu}i\sigma^{\nu)\alpha}\Delta_\alpha/2M]u(p)$$

- Conservation laws - zero Anomalous Gravitomagnetic Moment : $\mu_G = J$ (g=2)

$$P_{q,g} = A_{q,g}(0) \qquad A_q(0) + A_g(0) = 1$$

$$J_{q,g} = \frac{1}{2}\left[A_{q,g}(0) + B_{q,g}(0)\right] \qquad A_q(0) + B_q(0) + A_g(0) + B_g(0) = 1$$

- May be extracted from high-energy experiments/NPQCD calculations
- Describe the partition of angular momentum between quarks and gluons
- Describe interaction with both classical and TeV gravity

# Generalized Parton Diistributions (related to matrix elements of non local operators ) – models for both EM and Gravitational Formfactors (Selyugin,OT '09)

- Smaller mass square radius (attraction vs repulsion!?)

$$\rho(b) = \sum_q e_q \int dx q(x,b) \qquad = \int d^2 q F_1(Q^2 = q^2) e^{i\vec{q}\vec{b}}$$

$$= \int_0^\infty \frac{q dq}{2\pi} J_0(qb) \frac{G_E(q^2) + \tau G_M(q^2)}{1+\tau}$$

$$\rho_0^{\mathrm{Gr}}(b) = \frac{1}{2\pi} \int_\infty^0 dq q J_0(qb) A(q^2)$$

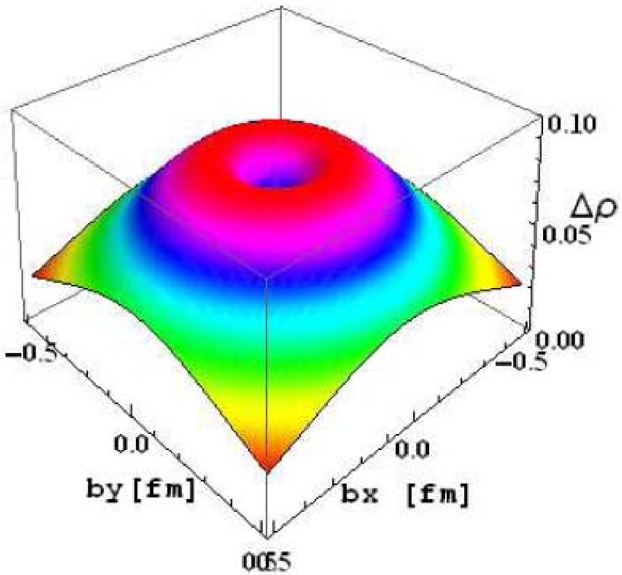



FIG. 17: Difference in the forms of charge density $F_1^P$ and "matter" density $(A)$

# Electromagnetism vs Gravity

- Interaction – field vs metric deviation

$$M = \langle P' | J_q^\mu | P \rangle A_\mu(q) \qquad M = \frac{1}{2} \sum_{q,G} \langle P' | T_{q,G}^{\mu\nu} | P \rangle h_{\mu\nu}(q)$$

- Static limit

$$\langle P | J_q^\mu | P \rangle = 2 e_q P^\mu \qquad \sum_{q,G} \langle P | T_i^{\mu\nu} | P \rangle = 2 P^\mu P^\nu$$

$$h_{00} = 2\phi(x)$$

$$M_0 = \langle P | J_q^\mu | P \rangle A_\mu = 2 e_q M \phi(q) \qquad M_0 = \frac{1}{2} \sum_{q,G} \langle P | T_i^{\mu\nu} | P \rangle h_{\mu\nu} = 2M \cdot M \phi(q)$$

- Mass as charge – equivalence principle (Einstein ’10-11, Praha)

# Equivalence principle

- Newtonian – "Falling elevator" – well known and checked with high accuracy (also for elementary particles)
- Post-Newtonian – gravity action on SPIN – known since 1962 (Kobzarev and Okun' ZhETF paper contains acknowledgment to Landau: probably his last contribution to theoretical physics before car accident); rederived from conservarion laws - Kobzarev and Zakharov
- Anomalous gravitomagnetic (and electric-CP-odd) moment iz ZERO or
- Classical and QUANTUM rotators behave in the SAME way

- For GEDM –checked with sometimes controversial results
- For AGM not checked on purpose but in fact checked in the same atomic spins experiments at % level (Silenko,OT'07)

# Gravitomagnetism

- Gravitomagnetic field (weak, except in gravity waves) – action on spin from $M = \frac{1}{2}\sum_{q,G}\langle P'|T^{\mu\nu}_{q,G}|P\rangle h_{\mu\nu}(q)$

$$\vec{H}_J = \frac{1}{2} rot\vec{g};\ \vec{g}_i \equiv g_{0i}$$

spin dragging twice smaller than EM

- Lorentz force – similar to EM case: factor ½ cancelled with 2 from $h_{00} = 2\phi(x)$ Larmor frequency same as EM

$$\omega_J = \frac{\mu_G}{J} H_J = \frac{H_L}{2} = \omega_L\ \vec{H}_L = rot\vec{g}$$

- Orbital and Spin momenta dragging – the same - Equivalence principle

# Experimental test of PNEP

- Reinterpretation of the data on G(EDM) search

PHYSICAL REVIEW LETTERS

VOLUME 68 13 JANUARY 1992 NUMBER 2

**Search for a Coupling of the Earth's Gravitational Field to Nuclear Spins in Atomic Mercury**

B. J. Venema, P. K. Majumder, S. K. Lamoreaux, B. R. Heckel, and E. N. Fortson

*Physics Department, FM-15, University of Washington, Seattle, Washington 98195*

(Received 25 September 1991)

- If (CP-odd!) GEDM=0 -> constraint for AGM (Silenko, OT'07) from Earth rotation – was considered as obvious (but it is just EP!) background

$$\mathcal{H} = -g\mu_N \boldsymbol{B}\cdot\boldsymbol{S} - \zeta\hbar\boldsymbol{\omega}\cdot\boldsymbol{S}, \quad \zeta = 1+\chi$$

$$|\chi(^{201}\mathrm{Hg}) + 0.369\chi(^{199}\mathrm{Hg})| < 0.042 \quad (95\%\mathrm{C.L.})$$

# Equivalence principle for moving particles

- Compare gravity and acceleration: gravity provides EXTRA space components of metrics $h_{zz} = h_{xx} = h_{yy} = h_{00}$
- Matrix elements DIFFER

$$\mathcal{M}_g = (\epsilon^2 + p^2) h_{00}(q), \qquad \mathcal{M}_a = \epsilon^2 h_{00}(q)$$

- Ratio of accelerations: $R = \frac{\epsilon^2 + p^2}{\epsilon^2}$ - confirmed by explicit solution of Dirac equation (Silenko, OT, '05)
- Arbitrary fields – Obukhov, Silenko, OT '09,'11,'13

# Gravity vs accelerated frame for spin and helicity

- Spin precession – well known factor 3 (Probe B; spin at satellite – probe of PNEP!) – smallness of relativistic correction ($\sim \mathbf{P}^2$) is compensated by $1/\mathbf{P}^2$ in the momentum direction precession frequency
- Helicity flip – the same!
- No helicity flip in gravitomagnetic field – another formulation of PNEP (OT'99)

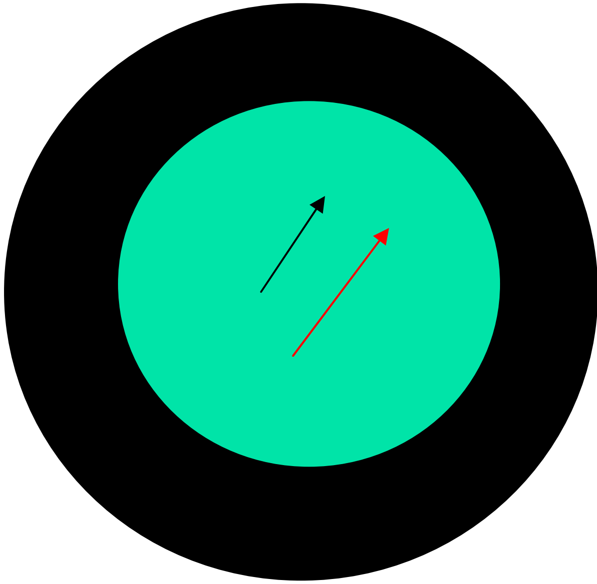
# Gyromagnetic and Gravigyromagnetic ratios

- Free particles – coincide
- $\langle P+q|T^{mn}|P-q\rangle = P^{\{m}\langle P+q|J^{n\}}|P-q\rangle/e$ up to the terms linear in q
- Special role of g=2 for any spin (asymptotic freedom for vector bosons)

- Should Einstein know about PNEP, the outcome of his and de Haas experiment would not be so surprising
- Recall also g=2 for Black Holes. Indication of "quantum" nature?!

# Cosmological implications of PNEP

- Necessary condition for Mach's Principle (in the spirit of Weinberg's textbook) -
- Lense-Thirring inside massive rotating empty shell (=model of Universe)
- For flat "Universe" - precession frequency equal to that of shell rotation
- Simple observation-Must be the same for classical and quantum rotators – PNEP!
- More elaborate models - Tests for cosmology ?!



# Torsion – acts only on spin (violates EP )

Dirac eq+FW transformation-Obukhov,Silenko,OT, **arXiv:1410.6197**

- Hermitian Dirac Hamiltonian

$$e_i^{\hat{0}} = V\,\delta_i^0, \qquad e_i^{\hat{a}} = W^{\hat{a}}{}_b\left(\delta_i^b - cK^b\,\delta_i^0\right)$$

$$\mathcal{H} = \beta mc^2 V + q\Phi + \frac{c}{2}\left(\pi_b \mathcal{F}^b{}_a\alpha^a + \alpha^a\mathcal{F}^b{}_a\pi_b\right)$$

$$ds^2 = V^2c^2dt^2 - \delta_{\hat{a}\hat{b}}W^{\hat{a}}{}_cW^{\hat{b}}{}_d\,(dx^c - K^c cdt)\,(dx^d - K^d cdt)$$

$$+\frac{c}{2}\left(\boldsymbol{K}\cdot\boldsymbol{\pi} + \boldsymbol{\pi}\cdot\boldsymbol{K}\right) + \frac{\hbar c}{4}\left(\Xi\cdot\boldsymbol{\Sigma} - \Upsilon\gamma_5\right),$$

$$\mathcal{F}^b{}_a = VW^b{}_{\hat{a}}, \qquad \Upsilon = V\epsilon^{\hat{a}\hat{b}\hat{c}}\Gamma_{\hat{a}\hat{b}\hat{c}}, \qquad \Xi^a = \frac{V}{c}\epsilon^{\hat{a}\hat{b}\hat{c}}\left(\Gamma_{\hat{0}\hat{b}\hat{c}} + \Gamma_{\hat{b}\hat{c}\hat{0}} + \Gamma_{\hat{c}\hat{0}\hat{b}}\right)$$

- Spin-torsion coupling

$$-\frac{\hbar cV}{4}\left(\boldsymbol{\Sigma}\cdot\check{\boldsymbol{T}} + c\gamma_5\check{T}^{\hat{0}}\right)$$

$$\check{T}^\alpha = -\frac{1}{2}\eta^{\alpha\mu\nu\lambda}T_{\mu\nu\lambda}$$

- FW – semiclassical limit - precession

$$\Omega^{(T)} = -\frac{c}{2}\check{\boldsymbol{T}} + \beta\frac{c^3}{8}\left\{\frac{1}{\epsilon'}, \left\{\boldsymbol{p}, \check{T}^{\hat{0}}\right\}\right\} + \frac{c}{8}\left\{\frac{c^2}{\epsilon'(\epsilon' + mc^2)}, \left(\{\boldsymbol{p}^2, \check{\boldsymbol{T}}\} - \{\boldsymbol{p}, (\boldsymbol{p}\cdot\check{\boldsymbol{T}})\}\right)\right\}$$

# Experimental bounds for torsion

- Magnetic field+rotation+torsion

$$H = -g_N \frac{\mu_N}{\hbar} \boldsymbol{B} \cdot \boldsymbol{s} - \boldsymbol{\omega} \cdot \boldsymbol{s} - \frac{c}{2} \check{\boldsymbol{T}} \cdot \boldsymbol{s}$$

- Same '92 EDM experiment

$$\frac{\hbar c}{4} |\check{T}| \cdot |\cos\Theta| < 2.2 \times 10^{-21}\,\mathrm{eV}, \qquad |\check{T}| \cdot |\cos\Theta| < 4.3 \times 10^{-14}\,\mathrm{m}^{-1}$$

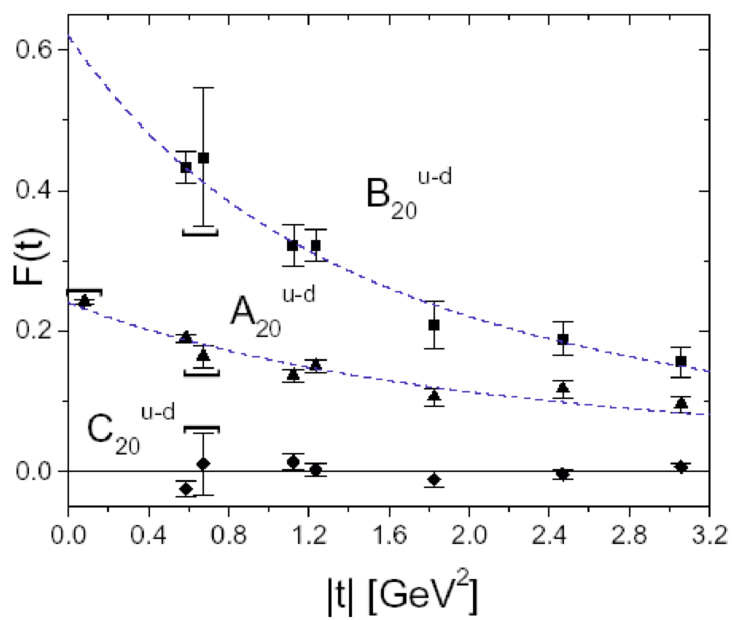
- New(based on Gemmel et al '10)

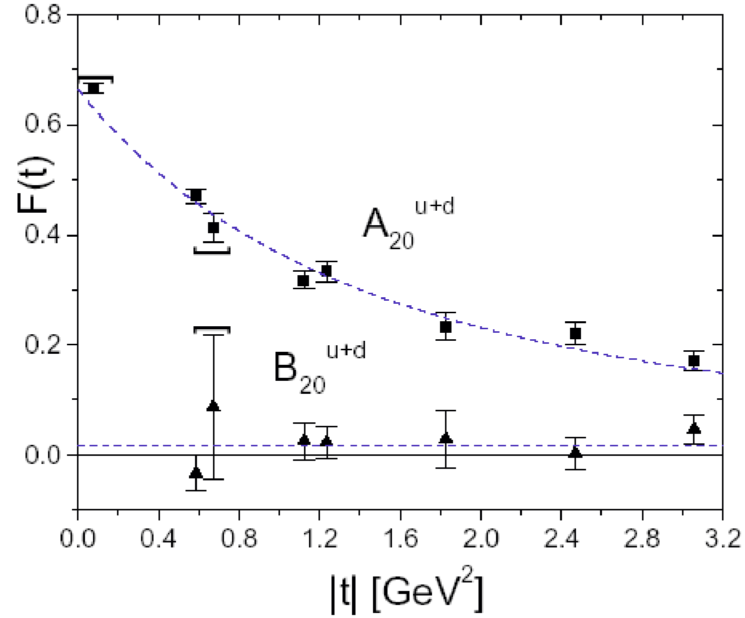
$$\frac{\hbar c}{2} |\check{T}| \cdot |(1-\mathcal{G})\cos\Theta| < 4.1 \times 10^{-22}\,\mathrm{eV}, \qquad |\check{T}| \cdot |\cos\Theta| < 2.4 \times 10^{-15}\,\mathrm{m}^{-1}$$

$$\mathcal{G} = g_{He}/g_{Xe}$$

# Generalization of Equivalence principle

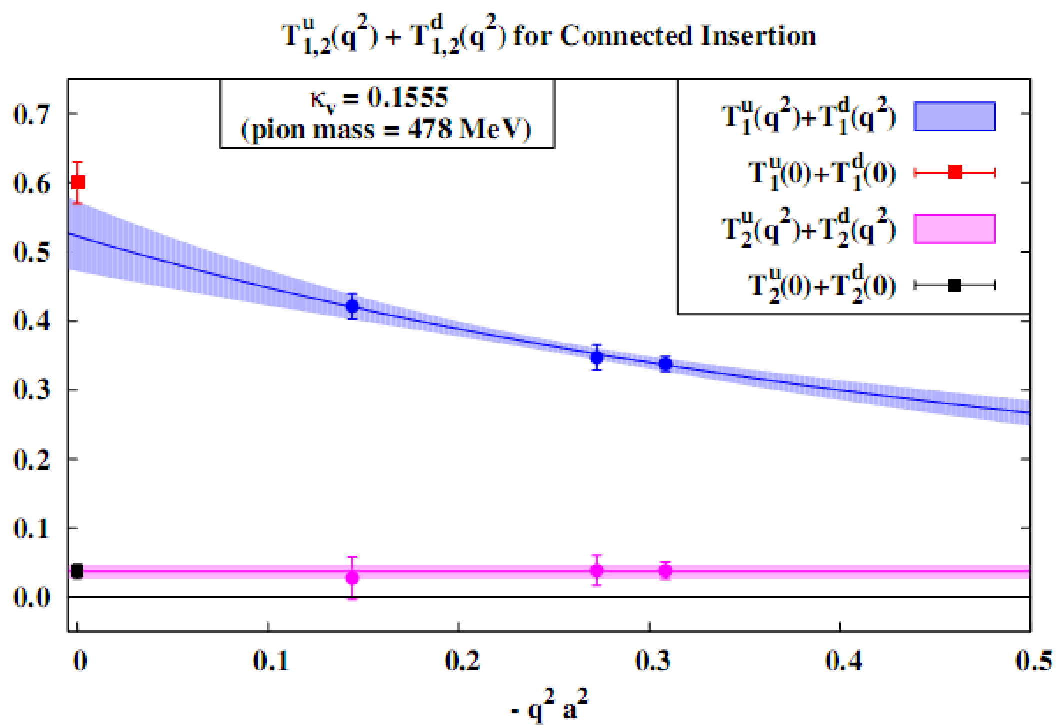
- Various arguments: AGM $\approx 0$ separately for quarks and gluons – most clear from the lattice (LHPC/SESAM)



# Recent lattice study (M. Deka et al. **arXiv:1312.4816**)

- Sum of u and d for Dirac (T1) and Pauli (T2) FFs



# Extended Equivalence Principle=Exact EquiPartition

- In pQCD – violated
- Reason – in the case of ExEP- no smooth transition for zero fermion mass limit (Milton, 73)
- Conjecture (O.T., 2001 – prior to lattice data) – valid in NP QCD – zero quark mass limit is safe due to chiral symmetry breaking
- Gravity-proof confinement (should the hadrons survive enetering Black Hole?)?!

# Another manifestation of post-Newtonian (E)EP for spin 1 hadrons

- Tensor polarization - coupling of gravity to spin in forward matrix elements - inclusive processes
- Second moments of tensor distributions should sum to zero

$$\langle P,S|\bar{\psi}(0)\gamma^{\nu}D^{\nu_1}...D^{\nu_n}\psi(0)|P,S\rangle_{\mu^2} = i^{-n}M^2 S^{\nu\nu_1}P^{\nu_2}...P\nu_n \int_0^1 C_q^T(x)x^n dx$$

$$\sum_q \langle P,S|T_i^{\mu\nu}|P,S\rangle_{\mu^2} = 2P^{\mu}P^{\nu}(1-\delta(\mu^2)) + 2M^2 S^{\mu\nu}\delta_1(\mu^2)$$

$$\langle P,S|T_g^{\mu\nu}|P,S\rangle_{\mu^2} = 2P^{\mu}P^{\nu}\delta(\mu^2) - 2M^2 S^{\mu\nu}\delta_1(\mu^2)$$

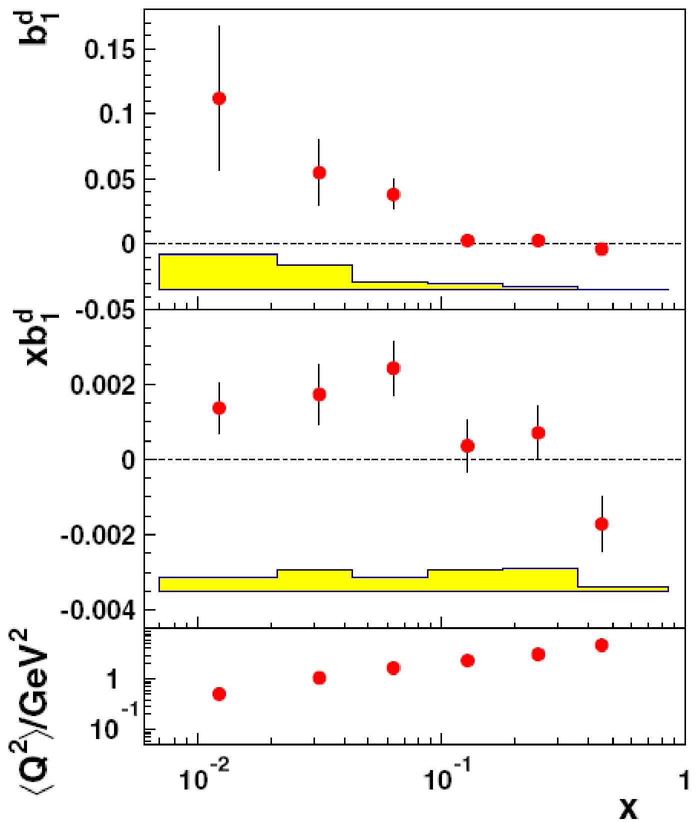
$$\sum_q \int_0^1 C_i^T(x)x dx = \delta_1(\mu^2)$$ =0 for ExEP

# HERMES – data on tensor spin structure function

PRL **95**, 242001 (2005)

- Isoscalar target – proportional to the sum of u and d quarks – combination required by EEP
- Second moments – compatible to zero better than the first one (collective glue << sea) – for valence:

$$\int_0^1 C_i^T(x)dx = 0$$



# Conclusions (slow rotation)

- Probe of equivalence principle for spin
- May be tested in EDM search experiments
- Extension of EP –validity separately for quarks and gluons

- BACKUP SLIDES

# Sum rules for EMT (and OAM)

- First (seminal) example: X. Ji’s sum rule (’96). Gravity counterpart – OT’99
- Burkardt sum rule – looks similar: can it be derived from EMT?
- Yes, if provide correct prescription to gluonic pole (OT’14)

# Pole prescription and Burkardt SR

- Pole prescription (dynamics!) provides ("T-odd") symmetric part!
$$\sum \int \int dx_1 dx_2 \frac{T(x_1,x_2)}{x_1 - x_2 + i\varepsilon} = 0$$
- SR: $\sum \int dx T(x,x) = 0$ (but relation of gluon Sivers to twist 3 still not founs – prediction!)
- Can it be valid separately for each quark flavour: nodes (related to "sign problem")?
- Valid if structures forbidden for TOTAL EMT do not appear for each flavour
- Structure contains besides S gauge vector n: If GI separation of EMT – forbidden: SR valid separately!

# Are more accurate data possible?

- HERMES – unlikely

- JLab may provide information about collective sea and glue in deuteron and indirect new test of Equivalence Principle

# CONCLUSIONS

- Spin-gravity interactions may be probed directly in gravitational (inertial) experiments and indirectly – studing EMT matrix element
- Torsion and EP are tested in EDM experiments
- SR's for deuteron tensor polarization- indirectly probe  EP and its extension separately for quarks and gluons

# EEP and AdS/QCD

- Recent development – calculation of Rho formfactors in Holographic QCD (Grigoryan, Radyushkin)
- Provides g=2 identically!
- Experimental test at time –like region possible