Modern aspects of quark-hadron matter EoS

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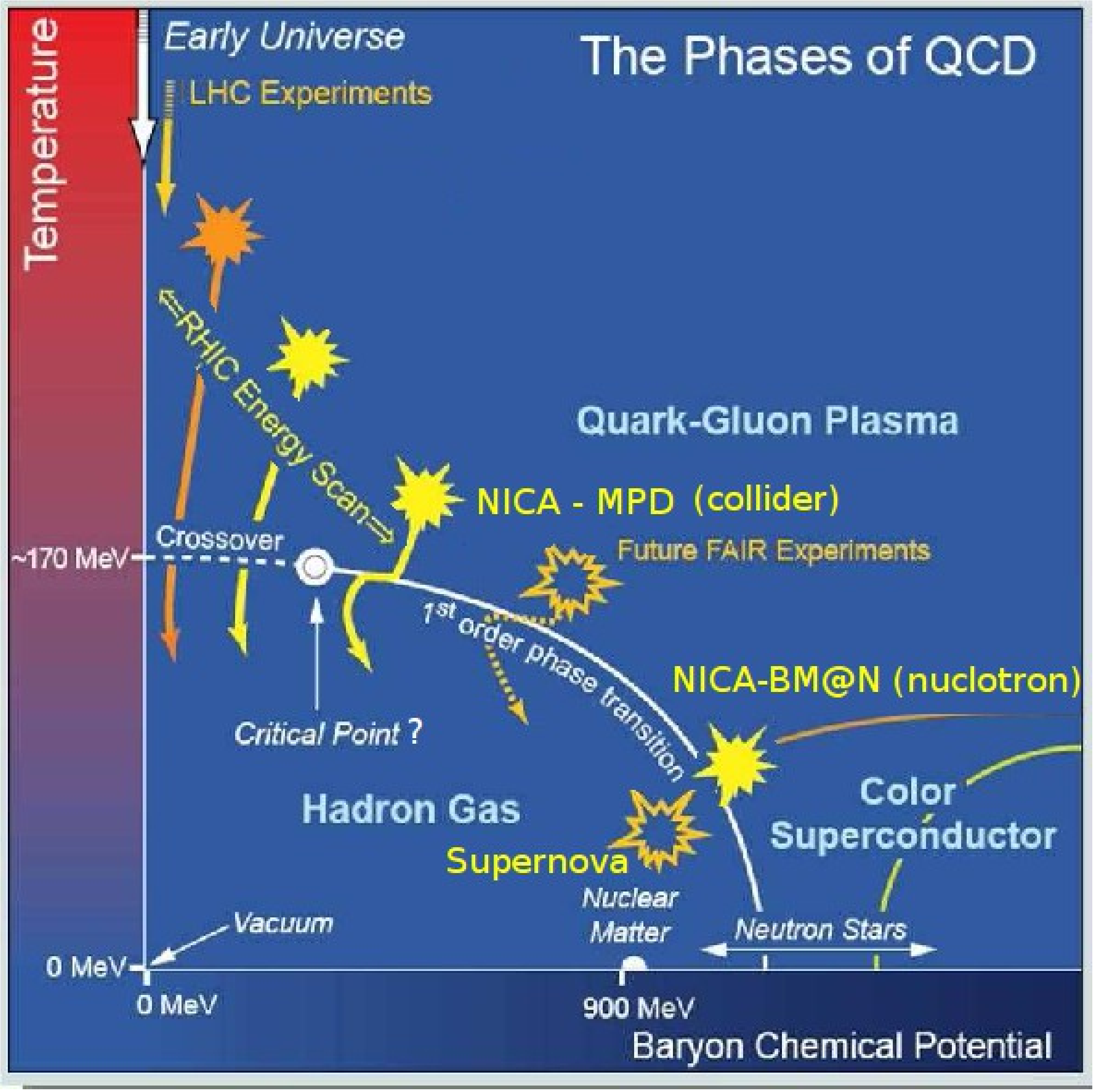
1. **Mott-Anderson hadronisation: chemical freezeout traces the QCD transition(s) ?**
2. **Quark Pauli blocking and excluded volume: Flow constraint ?**
3. **GBU EoS for 2+1 flavor quark-hadron matter in (P)NJL-type models: K+/pi+ "horn" vs. "tooth", signal of the CEP ?**

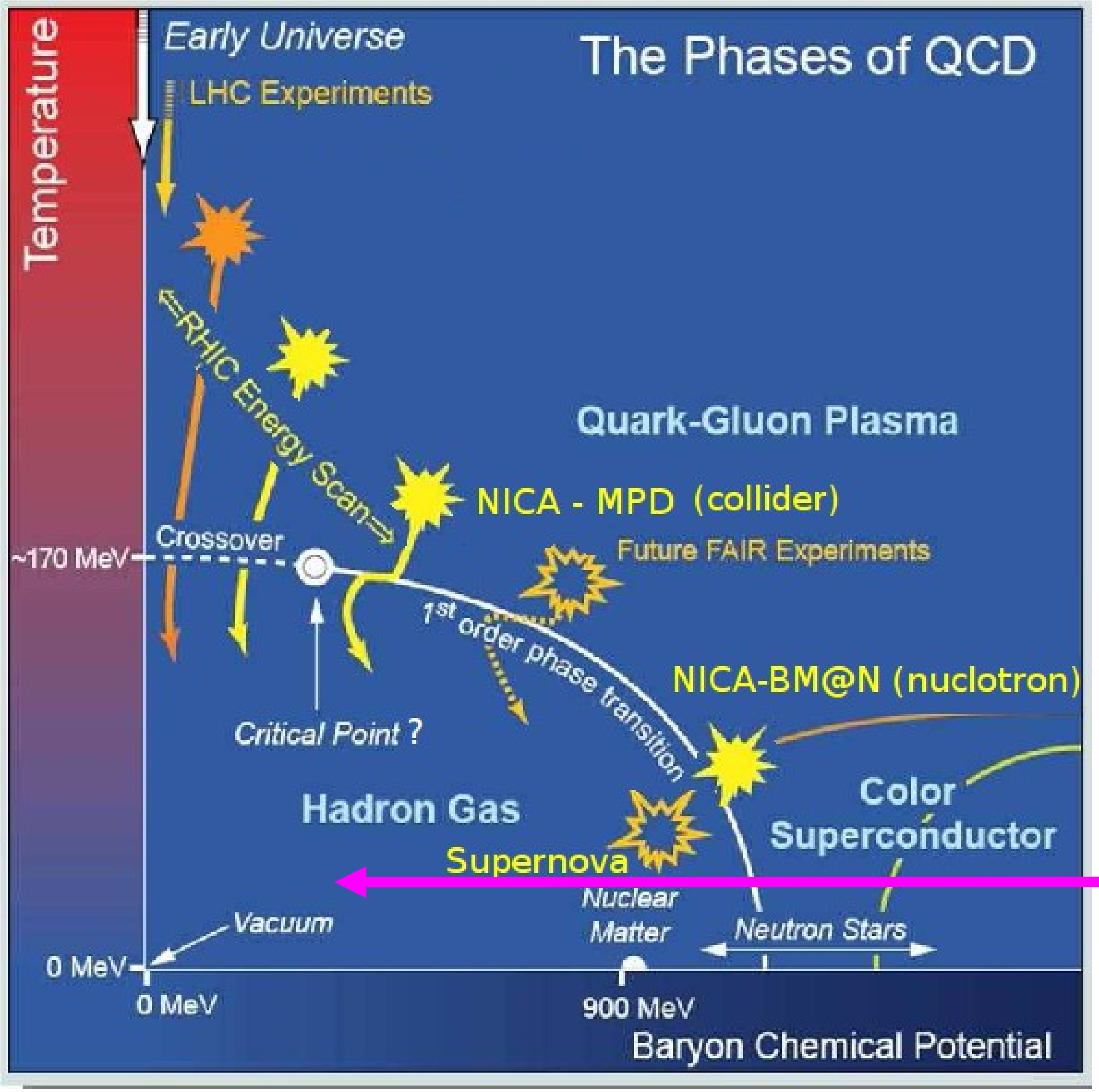
Mini-Workshop "Simulations at NICA Energies", JINR Dubna, 10.04.2017

DUBNA

The Goal: Theory of the QCD Phase Diagram



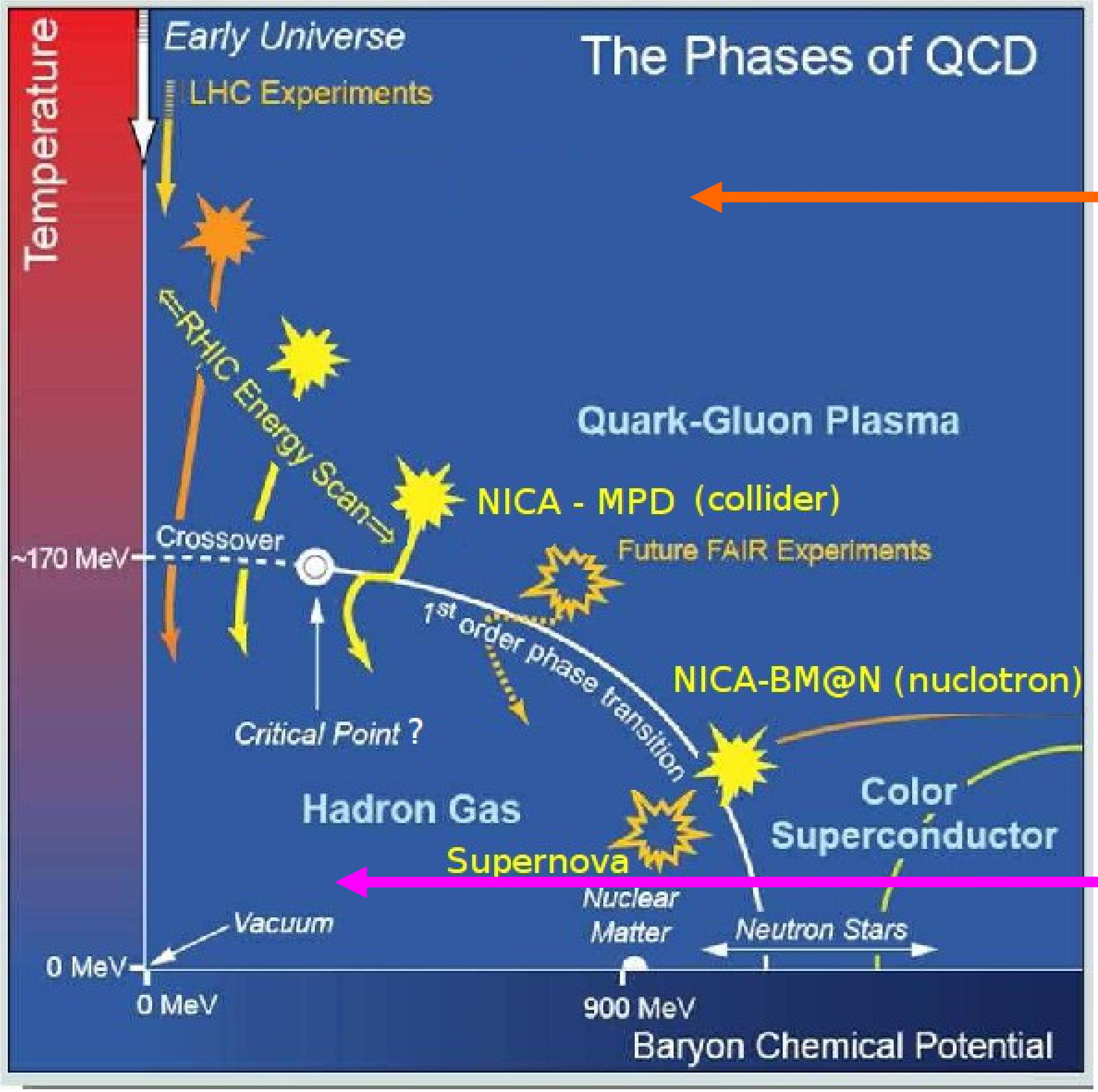
The Goal: Theory of the QCD Phase Diagram



Statistical Model of Hadron Resonance Gas

Well established for Description of chemical freezeout

The Goal: Theory of the QCD Phase Diagram



Perturbative QCD

Approximately selfconsistent HTL resummation
(T > 2.5 Tc , μ > 1500 MeV)

Statistical Model of Hadron Resonance Gas

Well established for Description of chemical freezeout

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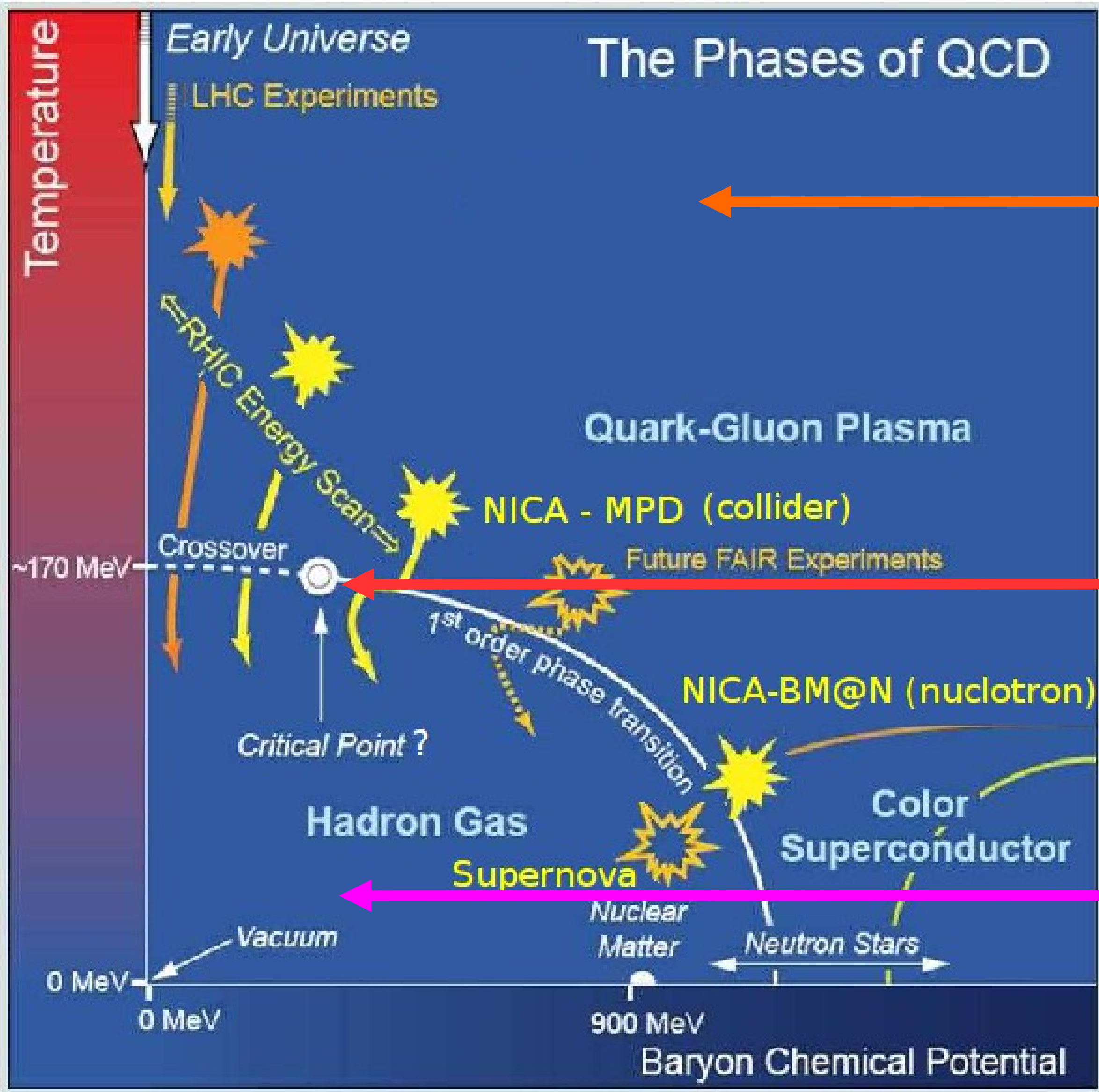


Perturbative QCD

Approximately selfconsistent HTL resummation (T > 2.5 Tc , μ > 1500 MeV)

QCD Phase transition(s)

Mott dissociation of hadrons, Deconfinement, χSR

Statistical Model of Hadron Resonance Gas

Well established for Description of chemical freezeout

1. Mott-Anderson localization model for chemical freeze-out

DB, J. Berdermann, J. Cleymans, K. Redlich, Phys. Part. Nucl. Lett. 8 (2011) 811

The basic idea: Localization of (certain) multiquark states (“cluster”) = hadronization;
Reverse process = delocalization by quark exchange between hadrons

Freeze-out criterion:

$$H_{\rm exp}(\tau) = \frac{\dot{R}(\tau)}{R(\tau)} = \tau^{-1}_{{\rm coll},i}(T,\mu)\ ,$$

$$\tau^{-1}_{{\rm coll},i}(T,\mu) = \sum_j \sigma_{ij} v n_j(T,\mu)$$

Povh-Huefner law, PRC 46 (1992) 990

$$\sigma_{ij} = \lambda \langle r_i^2 \rangle \langle r_j^2 \rangle$$

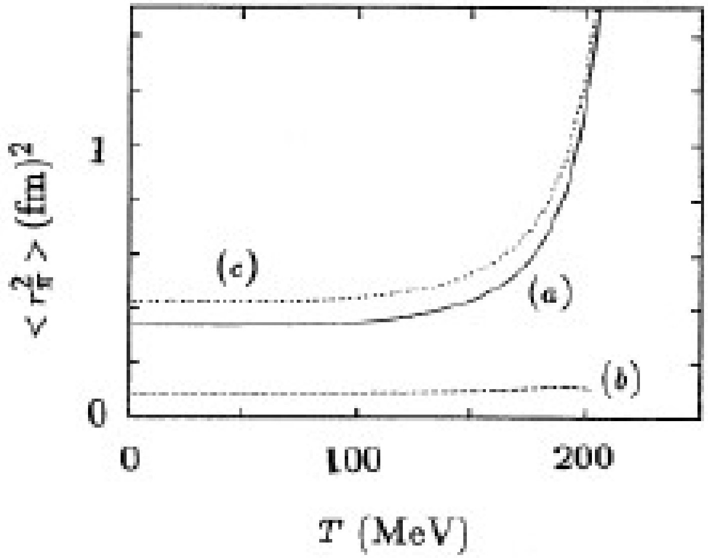
$$r_\pi^2(T,\mu) = \frac{3}{4\pi^2} f_\pi^{-2}(T,\mu)$$

$$f_\pi^2(T,\mu) = -m_0 \langle \bar{q}q \rangle_{T,\mu} / M_\pi^2$$

$$r_\pi^2(T,\mu) = \frac{3M_\pi^2}{4\pi^2 m_q} |\langle \bar{q}q \rangle_{T,\mu}|^{-1}$$

$$\langle \bar{q}q \rangle = \langle \bar{q}q \rangle_{\rm MF} \left[1 - \frac{T^2}{8 f_\pi^2(T,\mu)} - \frac{\sigma_N n_{s,N}(T,\mu)}{M_\pi^2 f_\pi^2(T,\mu)} \right]$$

Hippe & Klevansky, PRC 52 (1995) 2172



1. Mott-Anderson localization model for chemical freeze-out

DB, J. Berdermann, J. Cleymans, K. Redlich, Phys. Part. Nucl. Lett. 8 (2011) 811

Povh-Huefner law behaviour for quark exchange between hadrons

PHYSICAL REVIEW C VOLUME 51, NUMBER 5 MAY 1995

Quark exchange model for charmonium dissociation in hot hadronic matter

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E. Quack‡
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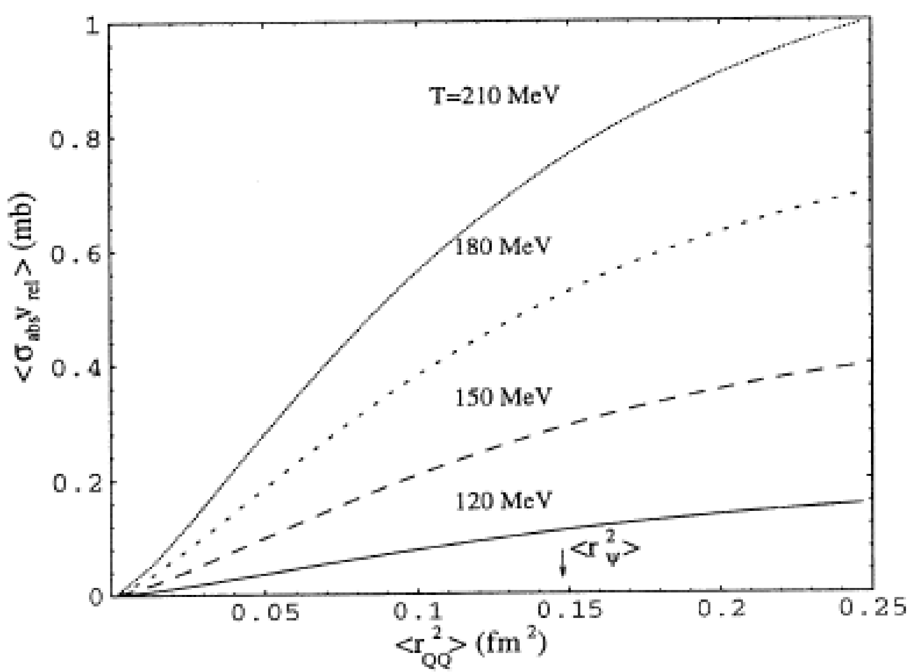
(Received 15 November 1994)



$$\langle \sigma_{\rm abs} v_{\rm rel} \rangle \propto \langle r^2 \rangle_{Q\bar{Q}} \langle r^2 \rangle_{q\bar{q}}$$

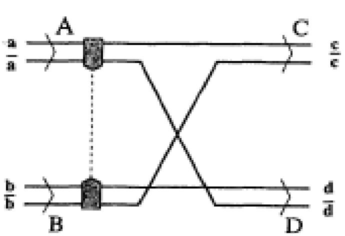
Flavor exchange processes

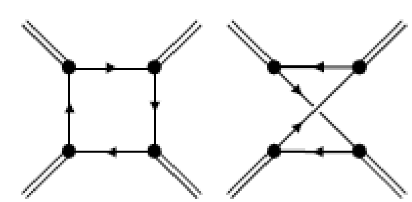
$$\phi + \pi \to K + \bar{K},$$

$$K^- + p \to \Lambda + X,$$
$$K^+ + p \to \Lambda + X,$$

Nonrelativistic → rel. quark loop integrals

M_{fi} =





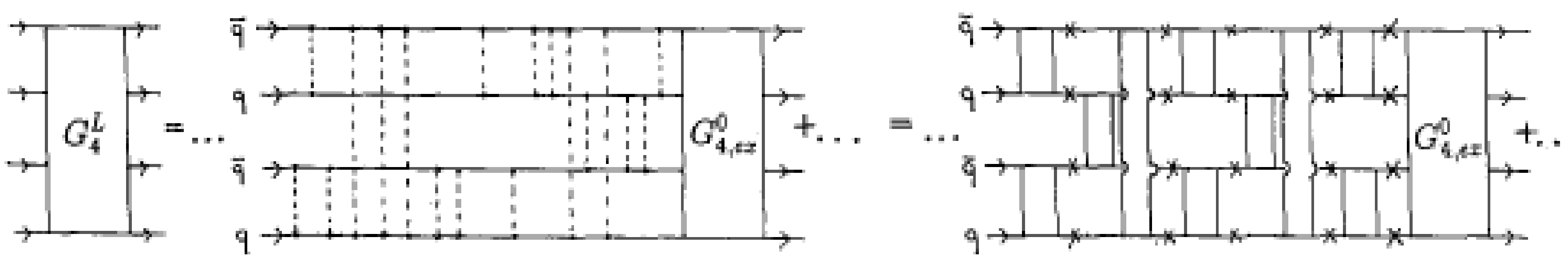
1. Quark exchange in meson-meson scattering

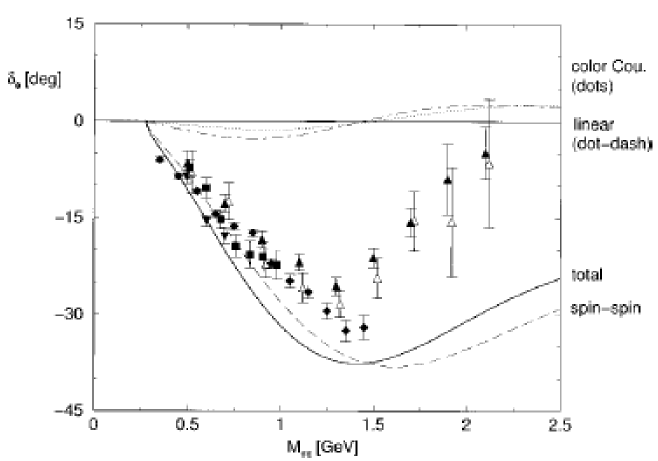
DB, G. Roepke, Phys. Lett. B 299 (1993) 332; T. Barnes et al., PRC 63 (2001) 025204

Povh-Huefner law behaviour for quark exchange between hadrons ?

$$\sigma_{ij} = \lambda \langle r_i^2 \rangle \langle r_j^2 \rangle \qquad r_\pi^2(T,\mu) = \frac{3M_\pi^2}{4\pi^2 m_q} \left| \langle \bar{q}q \rangle_{T,\mu} \right|^{-1}$$

$$\mathscr{V}^{ss}(12,1'2') = \frac{16}{3\sqrt{3}} C_{SFC}(12,1'2') \frac{(2\pi)^3}{\Omega_0} \frac{\alpha_s}{3\pi^2 m_q^2} \exp\left(-\frac{1}{4b^2}\left(k'^2 + \tfrac{1}{3}k^2\right) \right) \delta_{K,K'}$$

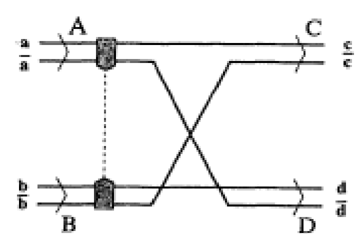


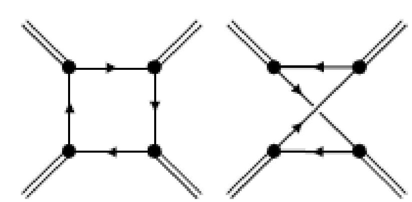


Quark exchange process in M-M scattering

Nonrelativistic → rel. quark loop integrals

$M_{fi} =$





1. Mott-Anderson localization model for chemical freeze-out

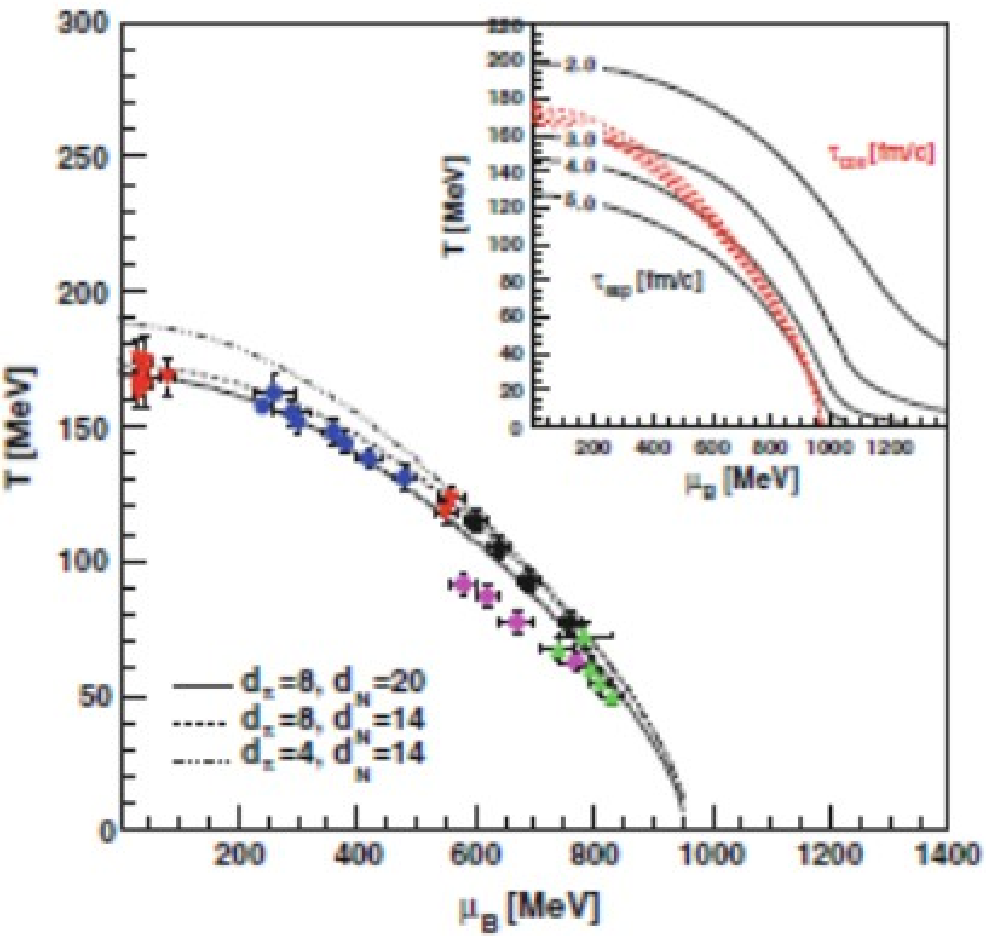
DB, J. Berdermann, J. Cleymans, K. Redlich, Phys. Part. Nucl. Lett. 8 (2011) 811

Model results:

$$\tau_{exp}(T, \mu) = \tau_{coll}(T, \mu)$$

Collision time strongly T, mu dependent !

Schematic resonance gas: dp pions, dN nucleons

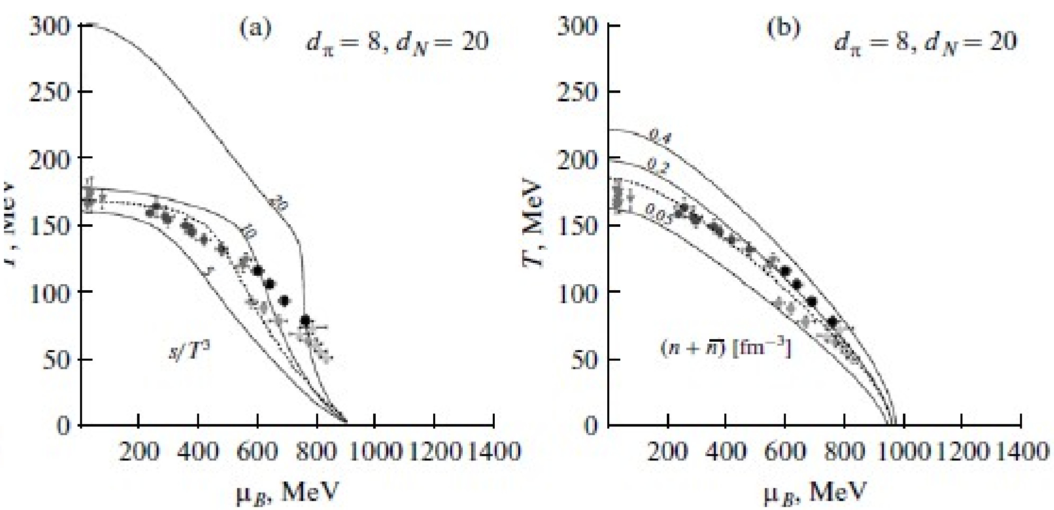


Expansion time scale from entropy conservation:

$$s(T, \mu)\, V(\tau_{exp}) = \text{const}$$

$$\tau_{exp}(T, \mu) = a s^{-1/3}(T, \mu),$$

Thermodynamics consistent with phenomenological Freeze-out rules:



1. Mott-Anderson localization model for chemical freeze-out

DB, J. Berdermann, J. Cleymans, K. Redlich, Few Body Syst. 53 (2012) 99

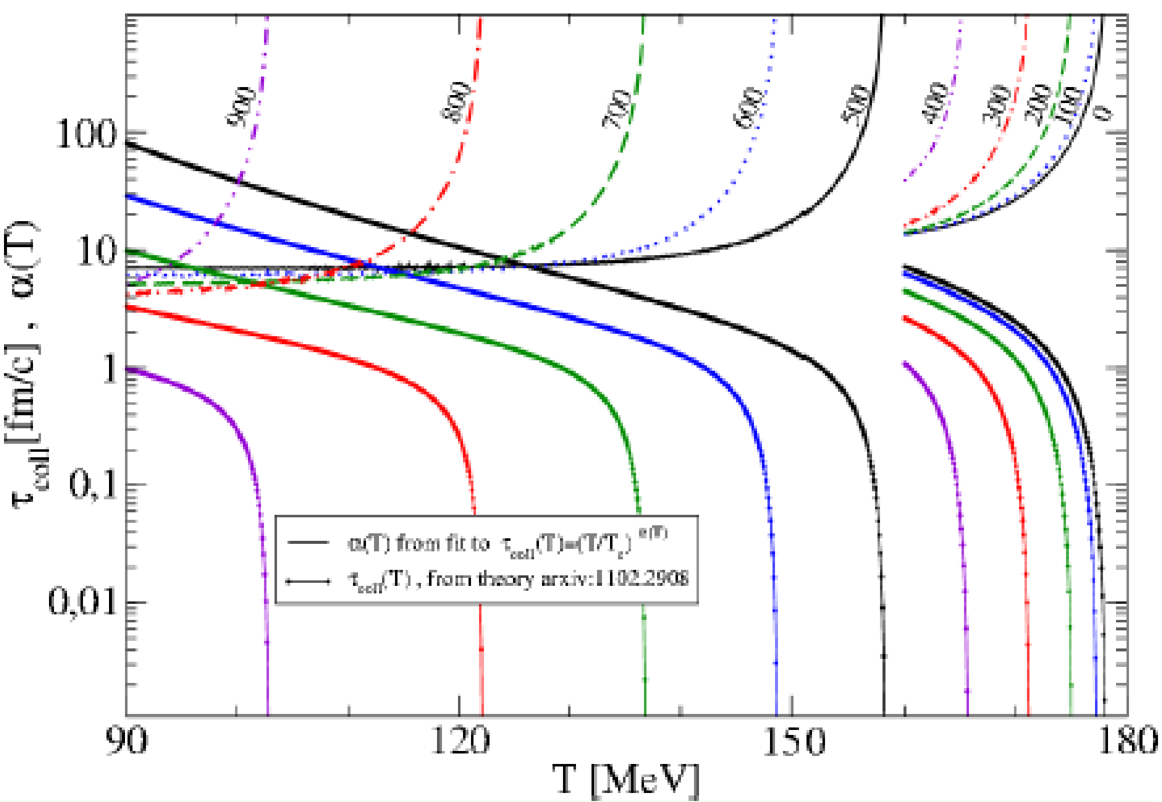
<u>Model results:</u>
Full hadron resonance gas model

$$\sigma_{ij} = \lambda \langle r_i^2 \rangle \langle r_j^2 \rangle \ ;$$

$$r_\pi^2(T,\mu) = \frac{3 M_\pi^2}{4\pi^2 m_q} \left| \langle \bar{q} q \rangle_{T,\mu} \right|^{-1}$$

$$r_N^2(T,\mu) = r_0^2 + r_\pi^2(T,\mu)$$

$$\frac{\langle \bar{q}q \rangle}{\langle \bar{q}q \rangle_{\rm vac}} = 1 - \frac{m_0}{F_\pi^2 m_\pi^2} \Bigg\{ 4N_c \int \frac{dp\, p^2}{2\pi^2} \frac{m}{\varepsilon_p} [f_\Phi^+ + f_\Phi^-]$$
$$+ \sum_{M=f_0,\omega,\dots} d_M (2 - N_s) \int \frac{dp\, p^2}{2\pi^2} \frac{m_M}{E_M(p)} f_M(E_M(p))$$
$$+ \sum_{B=N,\Lambda,\dots} d_B (3 - N_s) \int \frac{dp\, p^2}{2\pi^2} \frac{m_B}{E_B(p)} [f_B^+(E_B(p)) + f_B^-(E_B(p))] \Bigg\}$$
$$- \sum_{G=\pi,K,\eta,\eta'} \frac{d_G r_G}{4\pi^2 F_G^2} \int dp \frac{p^2}{E_G(p)} f_G(E_G(p)).$$



Collision time follows a power law
t_coll ~ (T/Tc)^a
with a large exponent a ~ 20

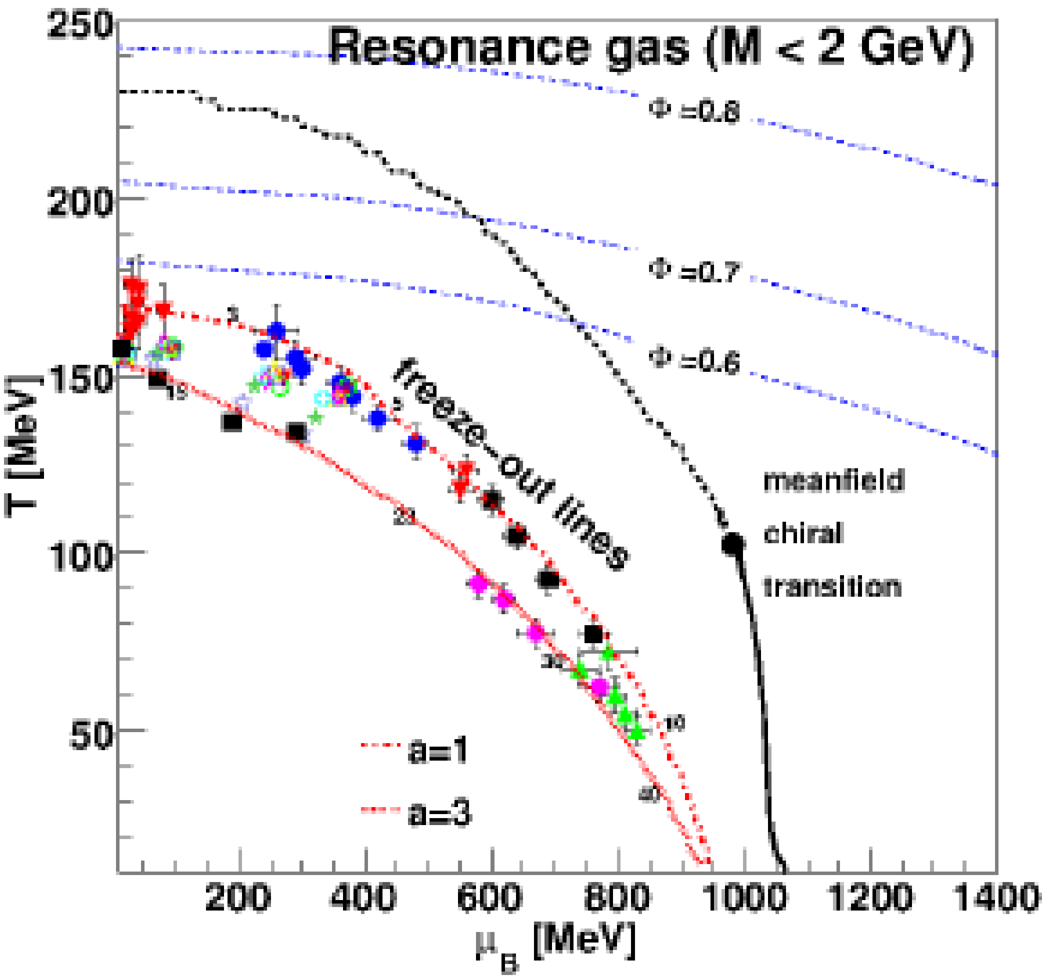
See also: P. Braun-Munzinger, J. Stachel, C. Wetterich, PLB (2004)

1. Mott-Anderson localization model for chemical freeze-out

DB, J. Berdermann, J. Cleymans, K. Redlich, Few Body Syst. 53 (2012) 99

Model results:
Full hadron resonance gas model

See also: S. Leupold, J. Phys. G (2006)



$$\frac{\langle\bar{q}q\rangle}{\langle\bar{q}q\rangle_{\rm vac}} = 1 - \frac{m_0}{F_\pi^2 m_\pi^2}\Big\{4N_c\int\frac{dp\,p^2}{2\pi^2}\frac{m}{\varepsilon_p}[f_\Phi^+ + f_\Phi^-]$$
$$+ \sum_{M=f_0,\omega,\ldots} d_M(2-N_s)\int\frac{dp\,p^2}{2\pi^2}\frac{m_M}{E_M(p)}f_M(E_M(p))$$
$$+ \sum_{B=N,\Lambda,\ldots} d_B(3-N_s)\int\frac{dp\,p^2}{2\pi^2}\frac{m_B}{E_B(p)}[f_B^+(E_B(p)) + f_B^-(E_B(p))]\Big\}$$
$$- \sum_{G=\pi,K,\eta,\eta'}\frac{d_G r_G}{4\pi^2 F_G^2}\int dp\,\frac{p^2}{E_G(p)}f_G(E_G(p)).$$

$$\sigma_{ij} = \lambda\langle r_i^2\rangle\langle r_j^2\rangle \quad ; \quad r_N^2(T,\mu) = r_0^2 + r_\pi^2(T,\mu)$$

$$r_\pi^2(T,\mu) = \frac{3M_\pi^2}{4\pi^2 m_q}\left|\langle\bar{q}q\rangle_{T,\mu}\right|^{-1}$$

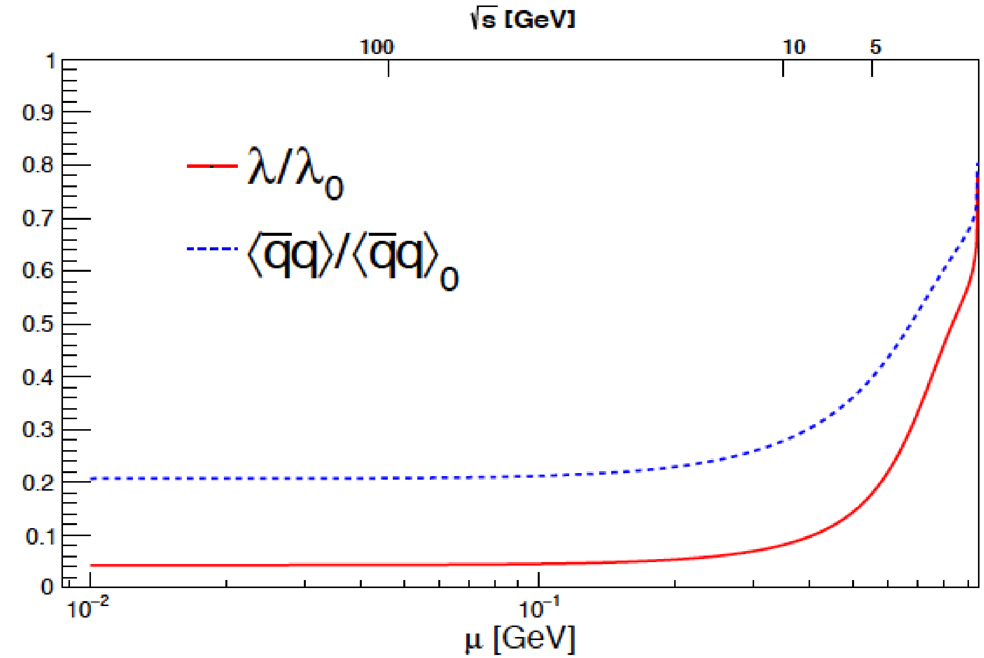
The factor a stands for the inverse system size in the formula

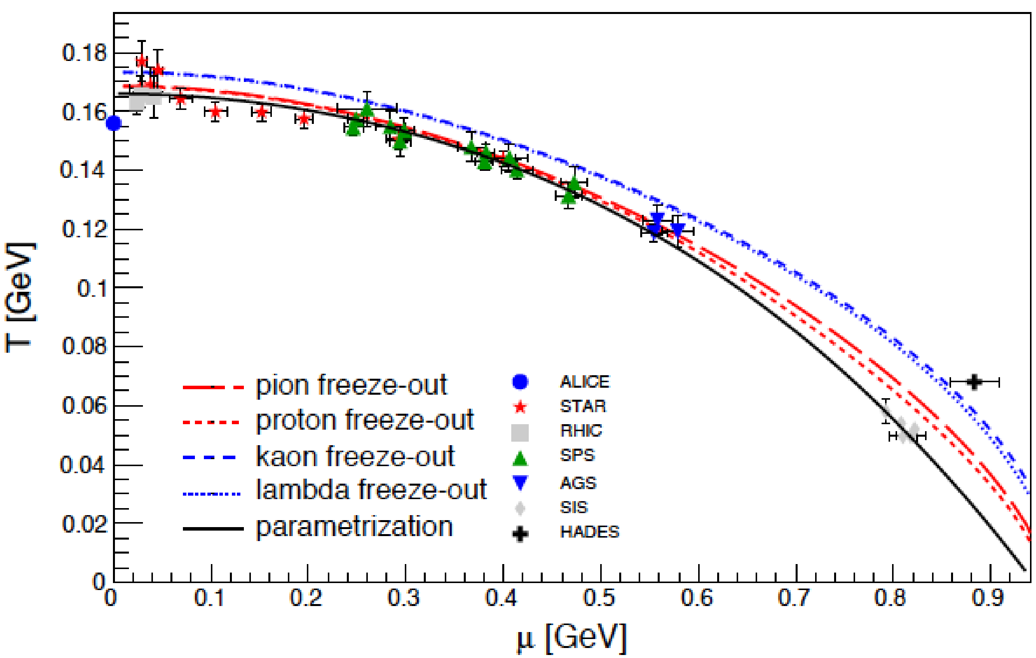
$$\tau_{\rm exp}(T,\mu) = \tau_{\rm coll}(T,\mu)$$

for the 3D expansion time scale assuming entropy conservation

1. Mott-Anderson localization model for K+/π+ “horn”

DB, J. Jankowski, M. Naskret, in prep. (2017)





Full HRG model condensate;
J. Jankowski et al., Phys. Rev. D (2013)

$$\langle \bar q q\rangle_{T,\mu} = \langle \bar q q\rangle^{MF}_{T,\mu} + \sum_{h=M,B} \frac{\sigma_q^h}{m_q} n_h(T,\mu) ,$$

$$n_h(T,\mu) = \frac{d_h}{2\pi^2}\int_0^\infty dk\, k^2 \frac{m_h}{E_h}\frac{1}{e^{(E_h-\mu_h)/T}\mp 1} .$$

$$\tau^{-1}_{\mathrm{coll},i}(T,\mu) = \sum_j \sigma_{ij} v n_j(T,\mu) \ ; \quad \sigma_{ij} = \lambda \langle r_i^2\rangle \langle r_j^2\rangle$$

$$\langle r_\pi^2\rangle_{T,\mu} \simeq \frac{3}{4\pi^2} f_\pi^{-2}(T,\mu) = \frac{3M_\pi^2}{4\pi^2 m_q}\left|\langle\bar q q\rangle_{T,\mu}\right|^{-1}$$

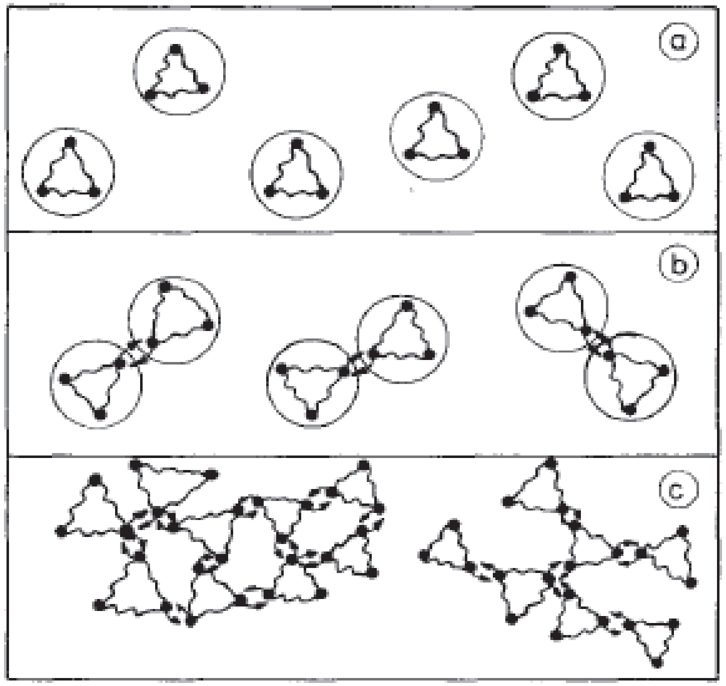
$$\langle r_K^2\rangle_{T,\mu} \simeq \frac{3M_K^2}{\pi^2(m_q+m_s)}\left|\langle\bar q q\rangle_{T,\mu}+\langle \bar s s\rangle_{T,\mu}\right|^{-1}$$

The factor a stands for the inverse system size in the formula

$$\tau_{exp}(T,\mu) = \tau_{coll}(T,\mu)$$

for the 3D expansion time scale assuming entropy conservation

2. Pauli blocking among baryons

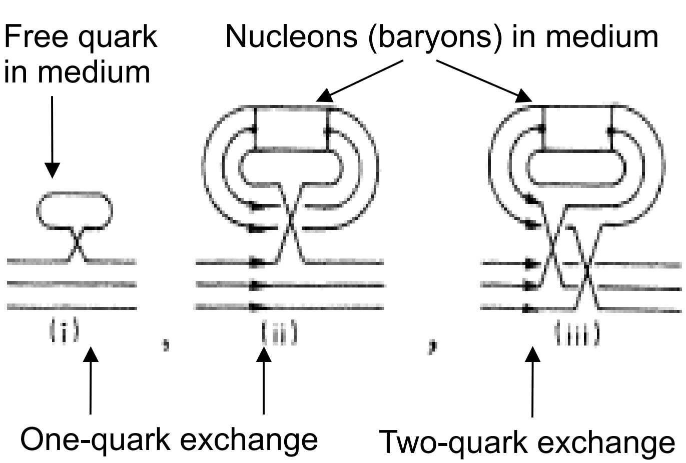


a) Low density: Fermi gas of nucleons (baryons)

b) ~ saturation: Quark exchange interaction and Pauli blocking among nucleons (baryons)

c) high density: Quark cluster matter (string-flip model ...)

Roepke & Schulz, Z. Phys. C 35, 379 (1987); Roepke, DB, Schulz, PRD 34, 3499 (1986)



Nucleon (baryon) self-energy --> Energy shift

$$\Delta E^{\mathrm{Pauli}}_{\nu P} = \sum_{123} |\psi_{\nu P}(123)|^2 [E(1)+E(2)+E(3)-E^0_{\nu P}][f_{\alpha_1}(1)+f_{\alpha_2}(2)+f_{\alpha_3}(3)]$$

$$+\sum_{123}\sum_{456}\sum_{\nu' P'} \psi^*_{\nu P}(123)\psi_{\nu' P'}(456) f_3(E^0_{\nu' P'})\{\delta_{36}\psi_{\nu P}(123)\psi^*_{\nu' P'}(456)-\psi_{\nu P}(453)\psi^*_{\nu' P'}(126)\}$$

$$\times[E(1)+E(2)+E(3)+E(4)+E(5)+E(6)-E^0_{\nu P}-E^0_{\nu' P'}]$$

$$=\Delta E^{\mathrm{Pauli,free}}_{\nu P}+\Delta E^{\mathrm{Pauli,bound}}_{\nu P} .$$

PHYSICAL REVIEW D VOLUME 34, NUMBER 11 1 DECEMBER 1986

Pauli quenching effects in a simple string model of quark/nuclear matter

G. Röpke and D. Blaschke
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H. Schulz
Central Institute for Nuclear Research, Rossendorf, 8051 Dresden, German Democratic Republic
and The Niels Bohr Institute, 2100 Copenhagen, Denmark

(Received 16 December 1985)

2. Pauli blocking among baryons - details

$$\Sigma_\nu(p, p_{Fn}, p_{Fp}) = \sum_{\nu'=\{n,p\}} \sum_{\alpha=1,2} C^{(\alpha)}_{\nu\nu'} W_\alpha(p_{F\nu'}, p)$$

$$W_\alpha(p_{F\nu'}, p) = \frac{\Omega}{2\pi^2} \int_0^{p_{F\nu'}} p'^2 \bar{V}^{(\alpha)}(p, p') dp';$$

$$\bar{V}^{(\alpha)}(p, p') = \frac{1}{2} \int_{-1}^{1} V^{(\alpha)}(\vec{p}, \vec{p}') dz;$$

$$V^{(\alpha)}(\vec{p}, \vec{p}') = \frac{V_0 b}{\Omega m} \left(\frac{15}{2} - \lambda_\alpha^2 (\vec{p} - \vec{p}')^2 \right) \exp(-\lambda_\alpha^2 (\vec{p} - \vec{p}')^2).$$

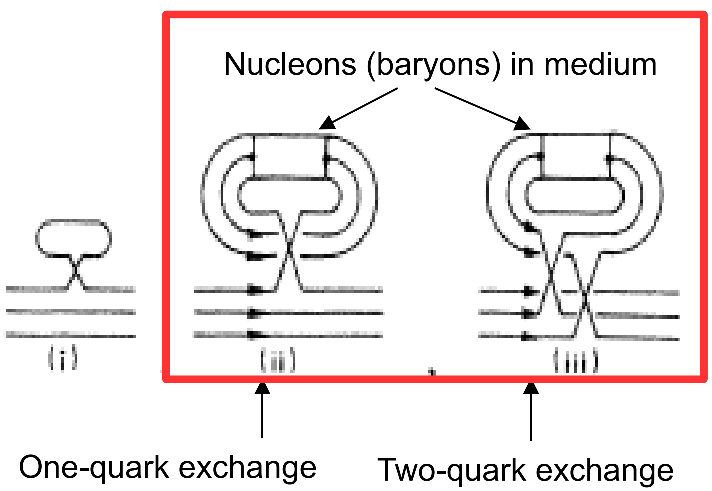
	$C^{(1)}_{n\ \nu}$	$C^{(2)}_{n\ \nu}$
neutron	$-\frac{96}{243}$	$\frac{10}{27}$
proton	$-\frac{28}{81}$	$\frac{8}{27}$

$$b^{-2} = \sqrt{3} m\omega$$

$$\omega = 178.425 \text{ MeV}$$

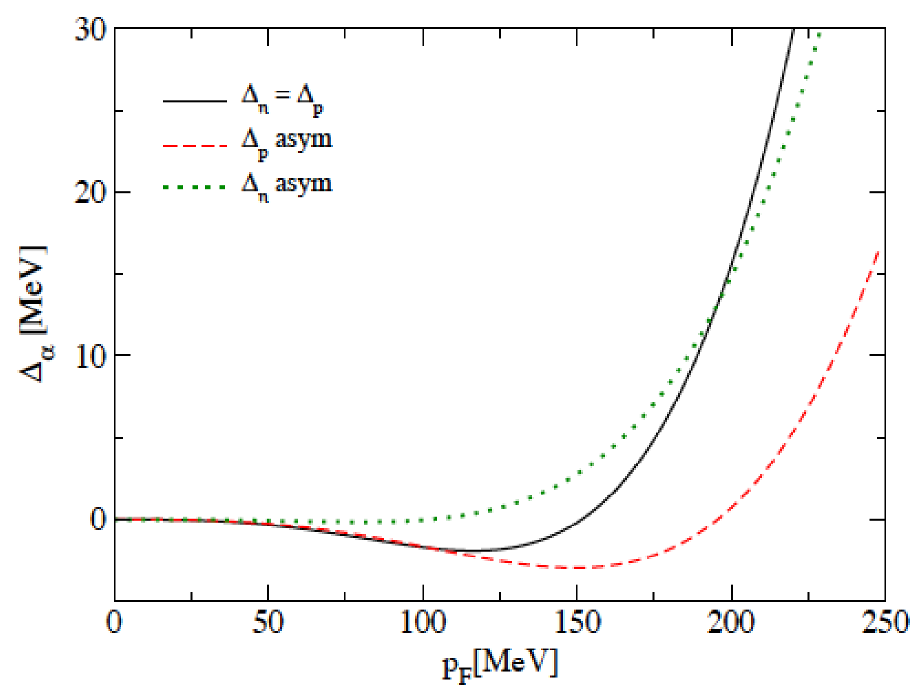
$$m = 350 \text{ MeV} \qquad b = 0.6 \text{ fm}$$

$$V_0 = \frac{9\sqrt{3}\pi^{3/2}}{2} \text{ and } \lambda_\alpha = \frac{b}{\sqrt{3}\alpha}.$$



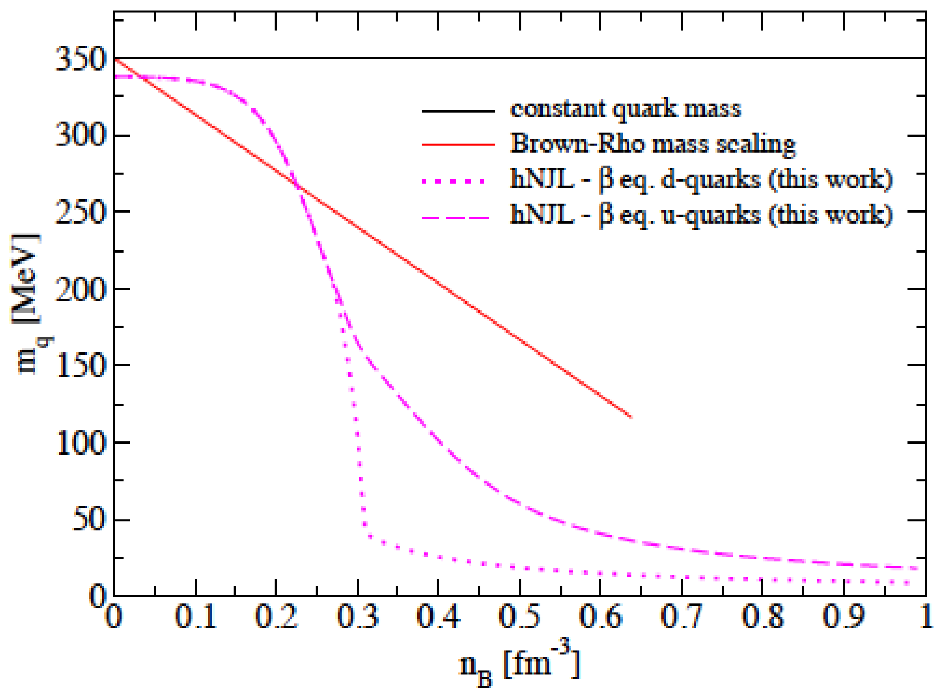
$$W_\alpha(p_{F\nu'}, p) = \frac{V_0 b}{32\pi^2 \lambda_\alpha^4 m} \{12\lambda_\alpha \sqrt{\pi} [\text{erf}(\lambda_\alpha(p_{F\nu'} - p)) + \text{erf}(\lambda_\alpha(p_{F\nu'} + p))]$$
$$+ \frac{1}{p}[(11 - 2\lambda_\alpha^2 p_{F\nu'}(p_{F\nu'} + p)) e^{-\lambda_\alpha^2 (p_{F\nu'} + p)^2}$$
$$+ (11 - 2\lambda_\alpha^2 p_{F\nu'}(p_{F\nu'} - p)) e^{-\lambda_\alpha^2 (p_{F\nu'} - p)^2}]\}$$

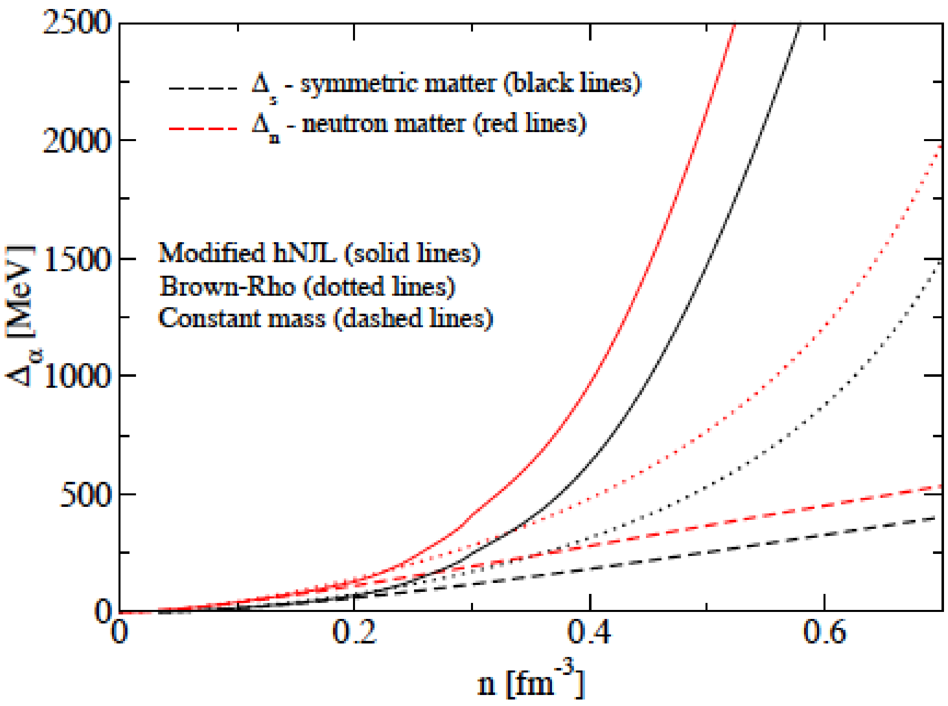
$$\Delta^{Pauli}_{\nu_A, P} = \frac{1}{24\sqrt{3\pi}} \frac{b}{m} \sum_{\nu'} [15 a_{\nu,\nu'} P_F(\nu')^3 + \frac{17}{12} b_{\nu,\nu'} b^2 (P^2 + P_F(\nu')^2) P_F(\nu')^3]$$



2. Pauli blocking in NM – details

New aspect: chiral restoration --> dropping quark mass





Increased baryon swelling at supersaturation densities:
--> dramatic enhancement of the Pauli repulsion !!

D.B., H. Grigorian, G. Roepke: “Quark exchange effects in dense nuclear matter”, in prep. (2017)

2. Pauli blocking among baryons – results

New EoS: Joining RMF (Linear Walecka) for pointlike baryons with chiral Pauli blocking

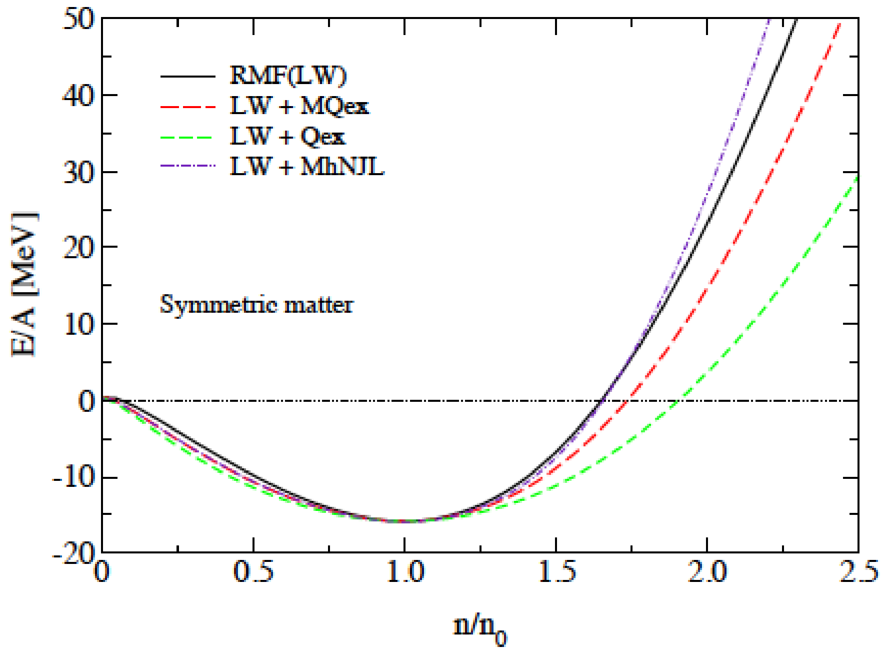
$$p = \frac{1}{4\pi^2}\sum_\nu \left[-E_\nu^* m_\nu^{*2} p_{F\nu} + \frac{2}{3}E_\nu^* p_{F\nu}^3 + m_\nu^{*4}\log\left(\frac{E_\nu^* + p_{F\nu}}{m_\nu^*}\right)\right]$$
$$+ \frac{1}{2}\left(\frac{g_\omega}{m_\omega}\right)^2 n^2 - \frac{1}{2}\left(\frac{g_\sigma}{m_\sigma}\right)^2 n_s^2 + p_{ex};$$

$$\epsilon = \frac{1}{4\pi^2}\sum_\nu \left[2\,E_\nu^{*3} p_{F\nu} - E_\nu^* m_\nu^{*2} p_{F\nu} - m_\nu^{*4}\log\left(\frac{E_I^* + p_{F\nu}}{m_\nu^*}\right)\right]$$
$$+ \frac{1}{2}\left(\frac{g_\omega}{m_\omega}\right)^2 n^2 + \frac{1}{2}\left(\frac{g_\sigma}{m_\sigma}\right)^2 n_s^2 + \epsilon_{ex},$$

$$n_{s,\nu} = \frac{m_\nu^*}{\pi^2}\left[E_\nu^* p_{F\nu} - m_\nu^{*2}\log\left(\frac{E_\nu^* + p_{F\nu}}{m_\nu^*}\right)\right],$$
$$E_\nu^* = \sqrt{m_\nu^{*2} + p_{F\nu}^2}$$
$$n_\nu = \frac{p_{F\nu}^3}{3\pi^2},$$
$$m_\nu^* = m_\nu - \left(\frac{g_\sigma}{m_\sigma}\right)^2 n_{s,\nu},$$
$$\mu_\nu = E_\nu^* + \left(\frac{g_\omega}{m_\omega}\right)^2 n_\nu + \mu_{ex,\nu}.$$

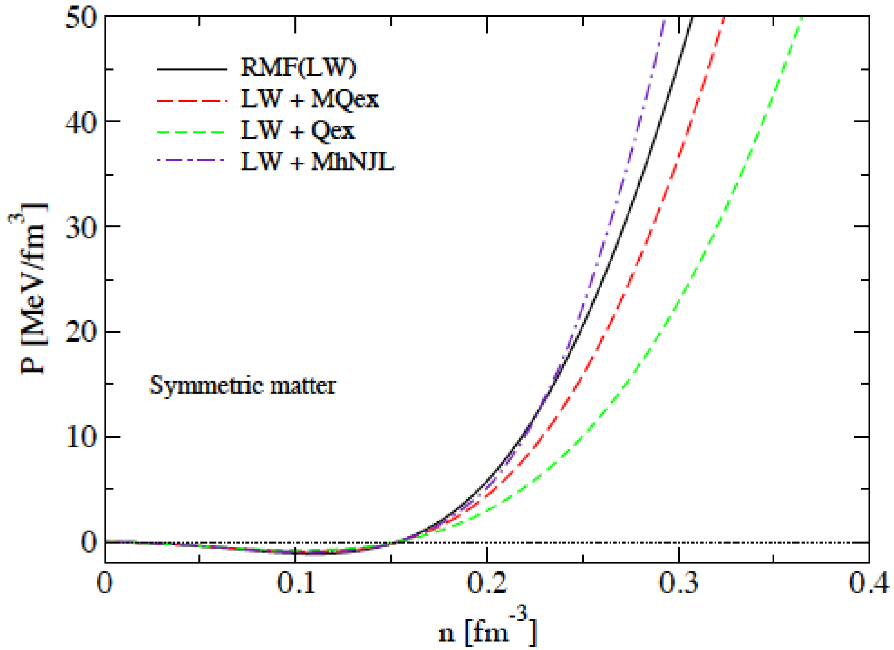
$$\mu_{ex,\nu} = \Delta_\nu(n,x) = \Sigma_\nu(p_{F,\nu}; p_{Fn,}, p_{Fp}),$$
$$\epsilon_{ex} = \sum_\nu \int_0^n dn'\{x\Delta_p(n',x) + (1-x)\Delta_n(n',x)\},$$
$$p_{ex} = \sum_\nu \mu_{ex,\nu}\, n_\nu - \epsilon_{ex},$$

2. Pauli blocking among baryons – results

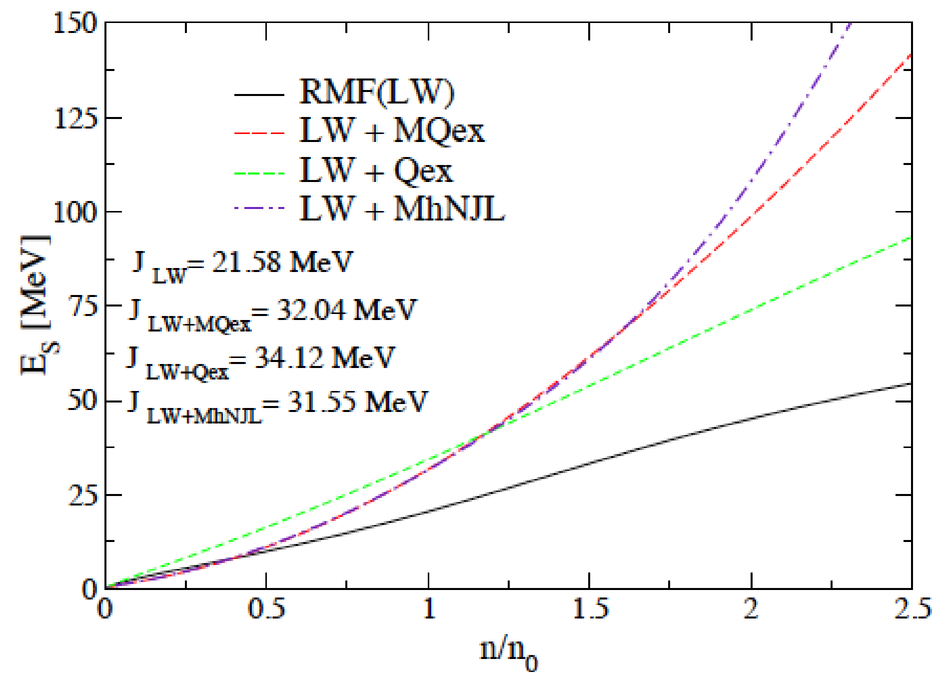


Parametrization: from saturation properties

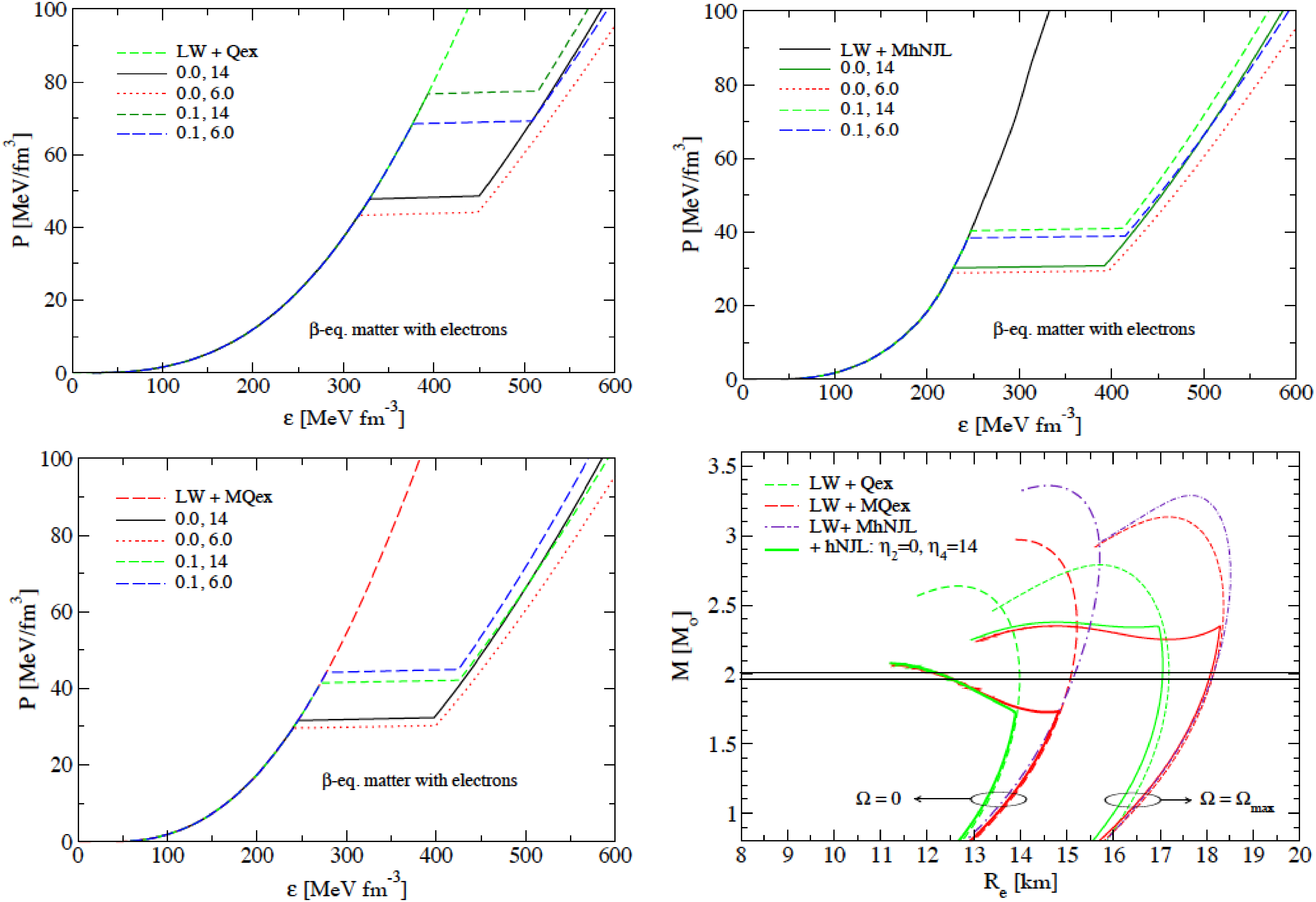
	$(g_\omega/m_\omega)^2$[fm^2]	$(g_\sigma/m_\sigma)^2$ [fm^2]
RMF (LW)	11.6582	15.2883
LW+Qex	6.11035	9.91197
LW+MQex	6.59170	13.29118
LW+MhNJL	9.25683	13.9474



Prediction: symmetry energy

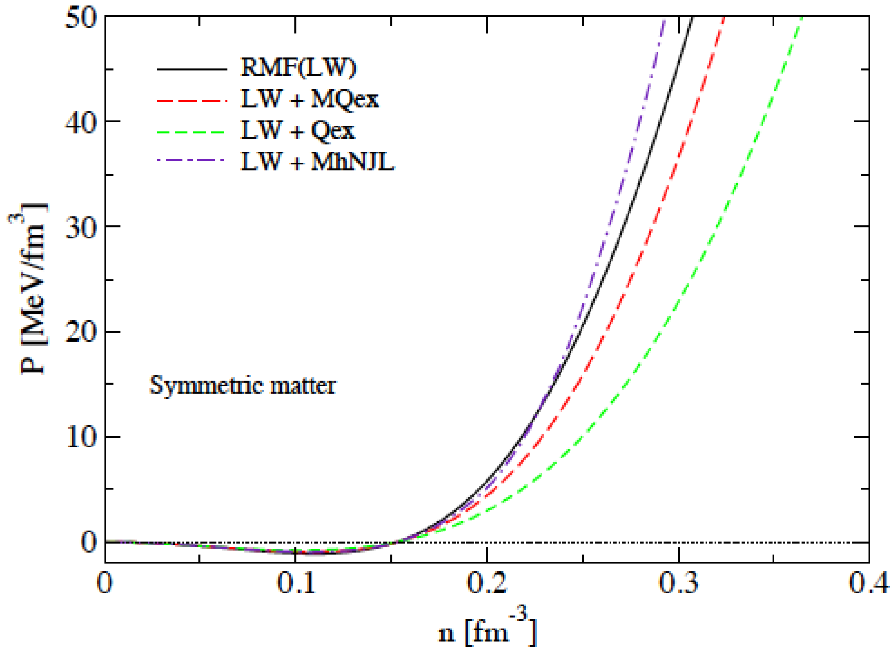


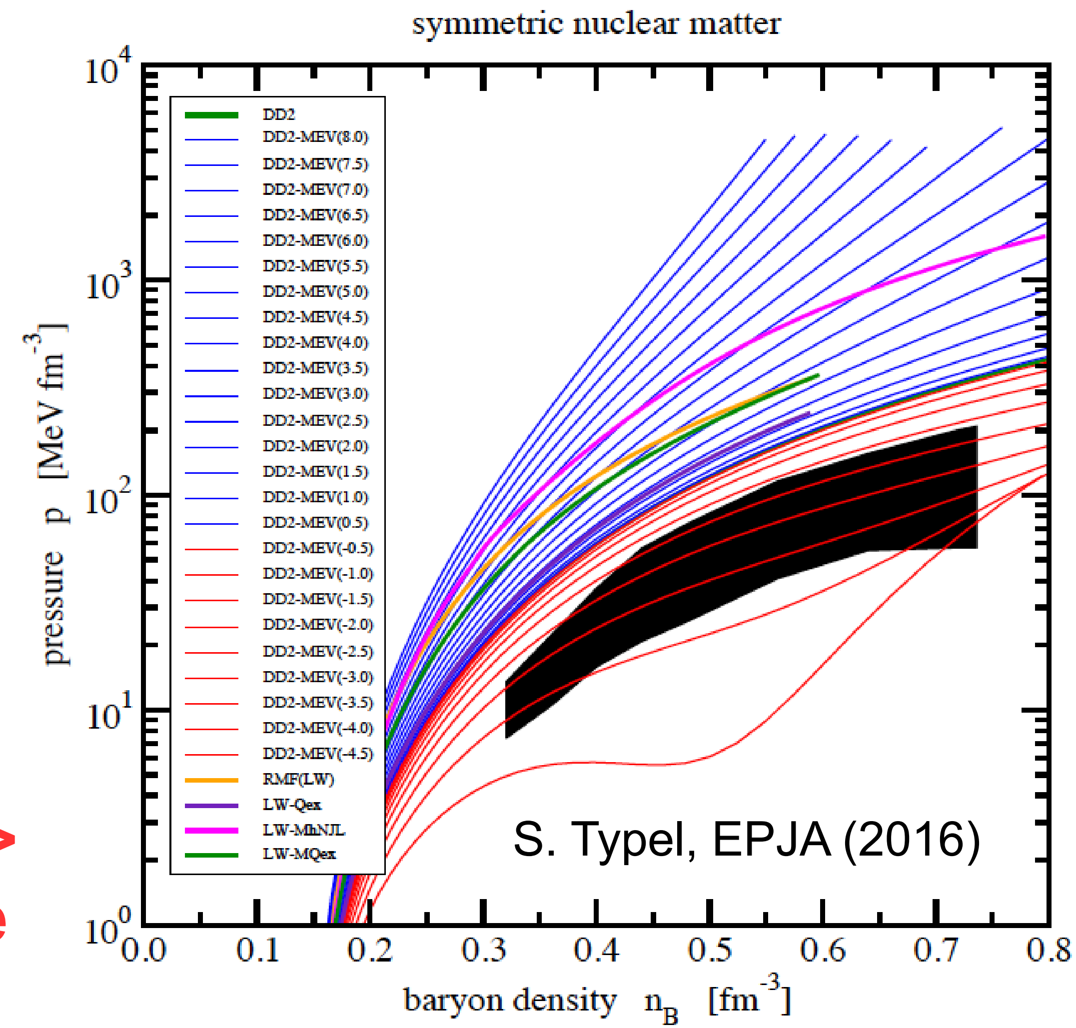
2. Pauli blocking in NM – results neutron stars



2. Pauli blocking in NM – nucleon excluded V.

New aspect: chiral restoration --> dropping quark mass





Increased baryon swelling at supersaturation densities: --> dramatic enhancement of the Pauli repulsion !!

D.B., H. Grigorian, G. Roepke: “Quark exchange effects in dense nuclear matter”, in prep. (2017)

2. Pauli blocking in NM – Summary

Pauli blocking selfenergy (cluster meanfield) calculable in potential models for baryon structure

Partial replacement of other short-range repulsion mechanisms (vector meson exchange)

Modern aspects:
- onset of chiral symmetry restoration enhances nucleon swelling and Pauli blocking at high n
- quark exchange among baryons -> six-quark wavefunction -> "bag melting" -> deconfinement

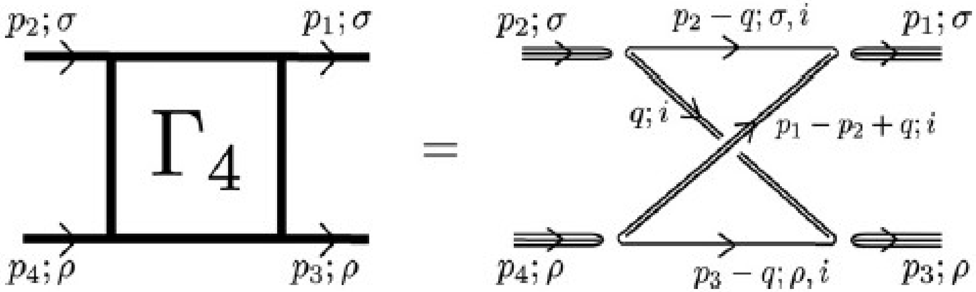
Chiral stiffening of nuclear matter --> reduces onset density for deconfinement

Hybrid EoS:
Convenient generalization of RMF models,
Take care: eventually aspects of quark exchange already in density dependent vertices!

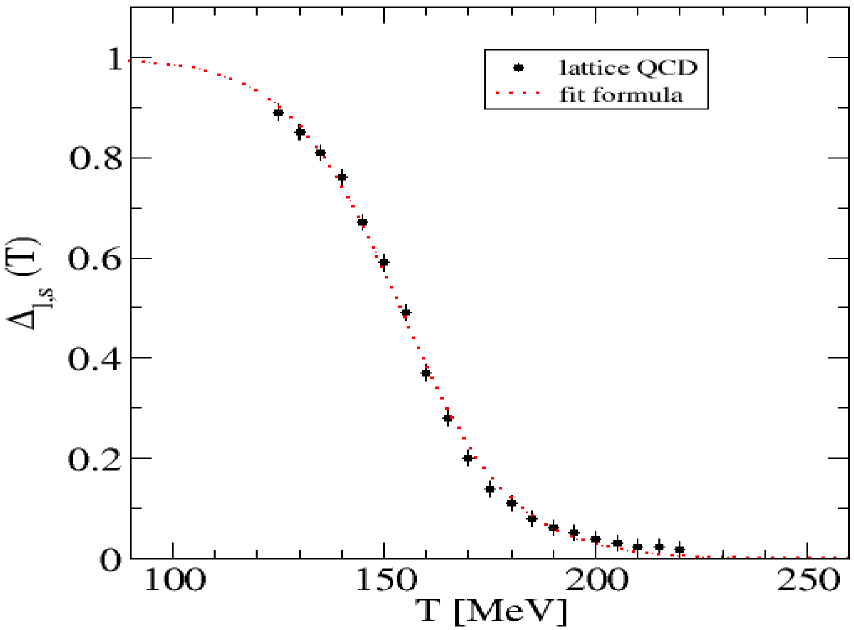
Other baryons:
- hyperons
- deltas
Again calculable, partially done in nonrelativistic quark exchange models, chiral effects not yet!

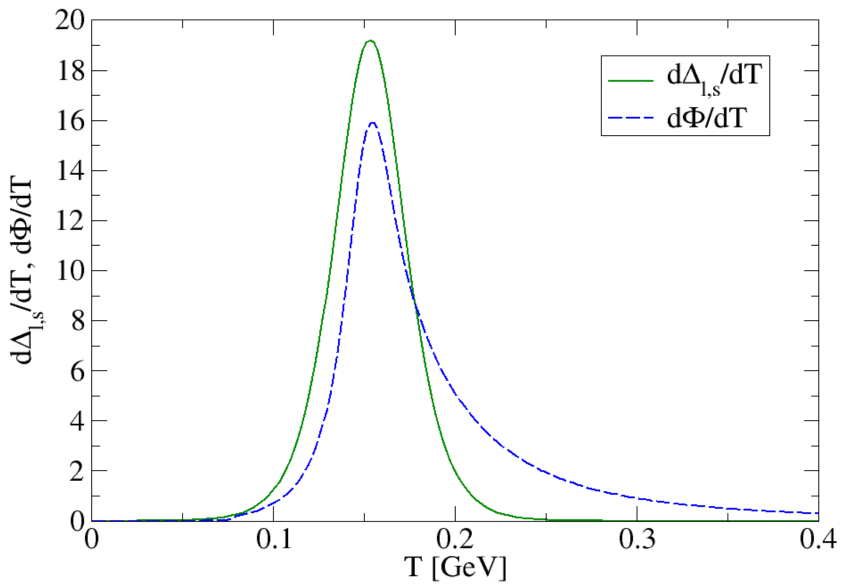
Relativistic generalization:
Box diagrams of quark-diquark model ...

K. Maeda, Ann. Phys. 326 (2011) 1032



3. Mott HRG / PNJL – effective model





$$P_{\rm PNJL}(T) = P_{\rm FG}(T) + \mathcal{U}[\Phi;T] \ ,$$

$$P_{\rm FG}(T) = 4 \sum_{q=u,d,s} \int \frac{d^3p}{(2\pi)^3} T \ln\left[1 + 3\Phi(Y + Y^2) + Y^3\right]$$

$$Y(E_p) \ = \ \exp(-E_p/T)$$

$$\mathcal{U}[\Phi;T] = -\frac{a(T)}{2}\Phi^2 + b(T)\ln(1 - 6\Phi^2 + 8\Phi^3 - 3\Phi^4)$$

T-dependent quark masses from fit to LQCD

$$m(T) = [m(0) - m_0]\Delta_{l,s}(T) + m_0 \ ,$$

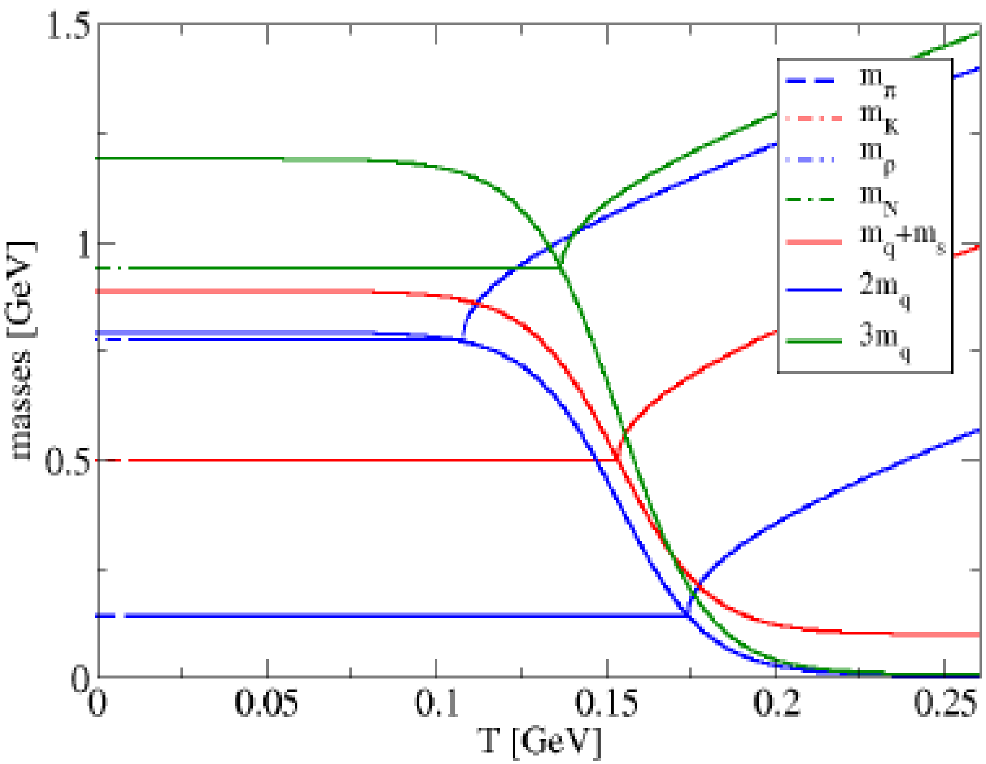
$$m_s(T) = m(T) + m_s - m_0 \ ,$$

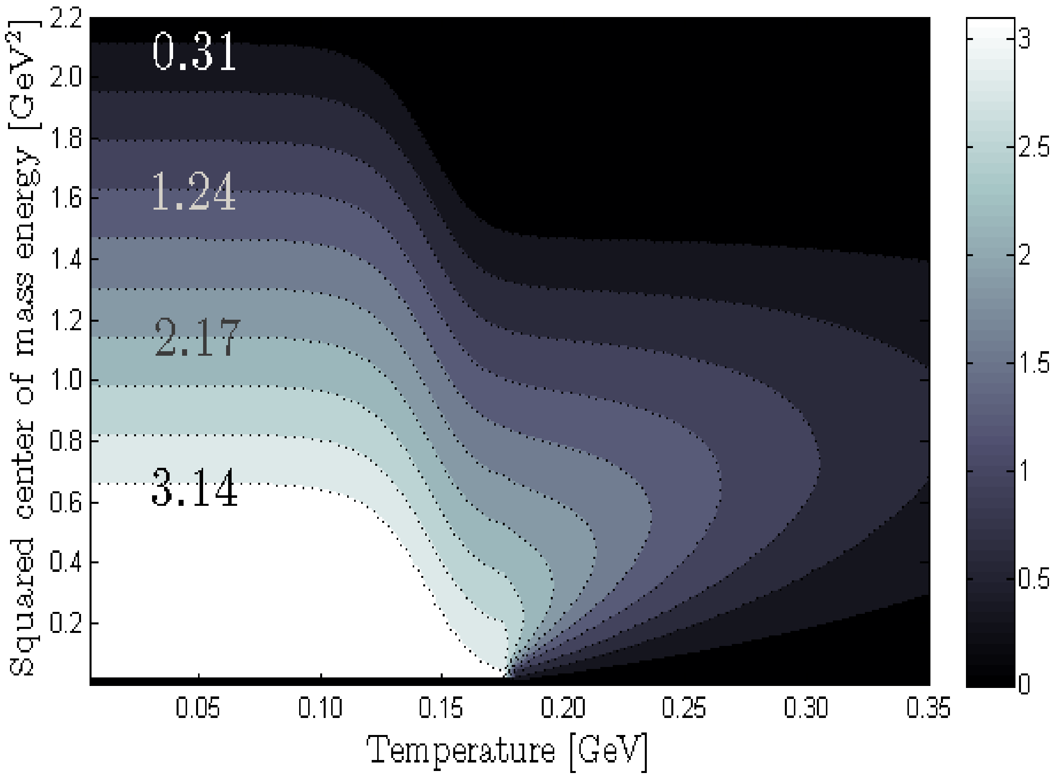
$$\Delta_{l,s}(T) = \frac{1}{2}\left[1 - \tanh\left(\frac{T - T_c}{\delta_T}\right)\right]$$

$$T_c = 154 \ \mathrm{MeV} \qquad \delta_T = 26 \ \mathrm{MeV}$$

D. Blaschke, A. Dubinin, L. Turko, arxiv:1611.09845

3. Mott HRG / PNJL – effective model





Hadrons + Mott effect

$$P_i(T) = \mp d_i \int_0^\infty \frac{dp\, p^2}{2\pi^2} \int_0^\infty dM\, T \ln\left(1 \mp e^{-\sqrt{p^2+M^2}/T}\right) \frac{2}{\pi} \sin^2 \delta_i(M^2;T) \frac{d\delta_i(M^2;T)}{dM}$$

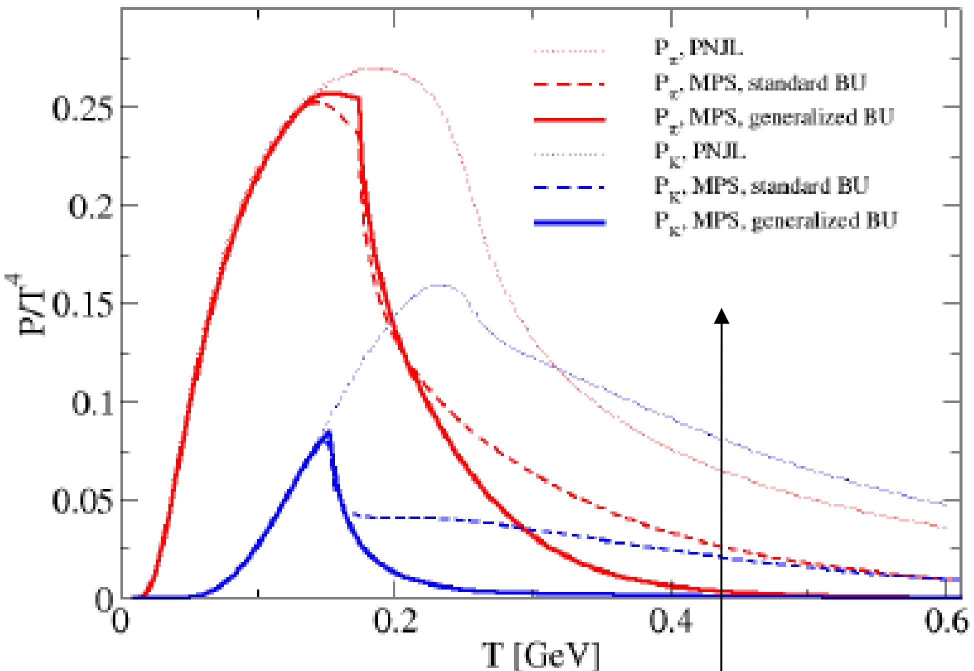
Quarks + rescattering effects

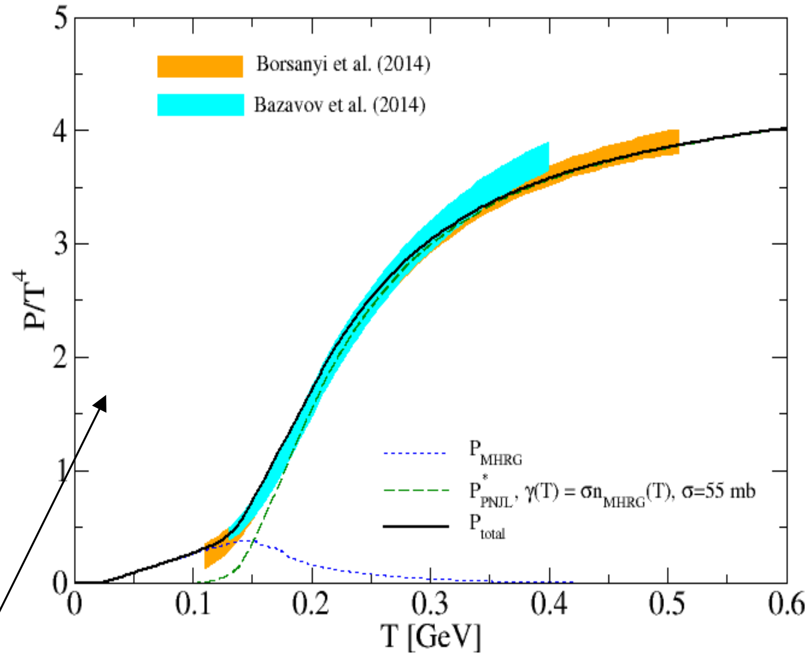
$$f_\Phi(\omega) = \frac{\Phi(1+2Y)Y + Y^3}{1 + 3\Phi(1+Y)Y + Y^3},$$

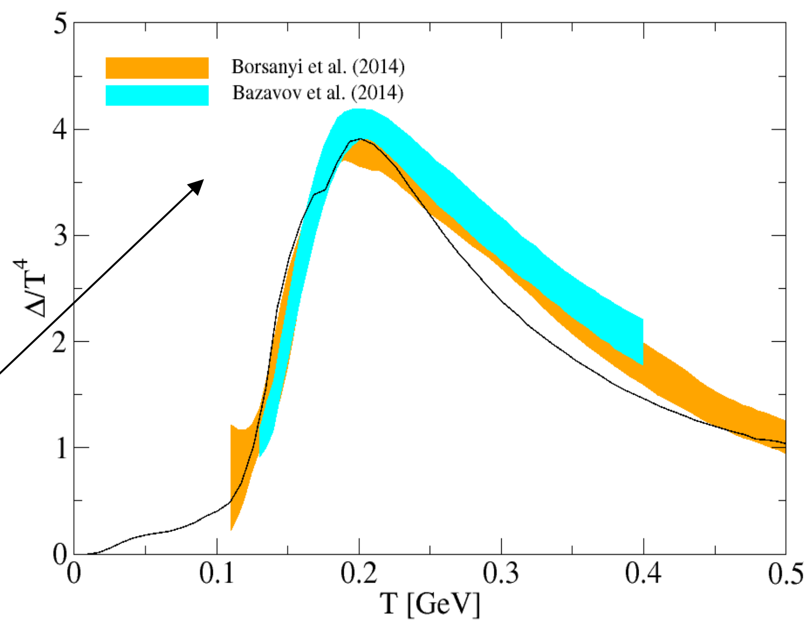
$$P^*_{\rm FG}(T) = 4N_c \sum_{q=u,d,s} \int \frac{dp\, p^2}{2\pi^2} \int \frac{d\omega}{\pi} f_\Phi(\omega)\delta_q(\omega;\gamma),$$

$$\delta_q(\omega;\gamma) = \frac{\pi}{2} + \arctan\left[\frac{\omega - \sqrt{p^2+m_q^2}}{\gamma}\right]$$

3. Mott HRG / PNJL – effective model







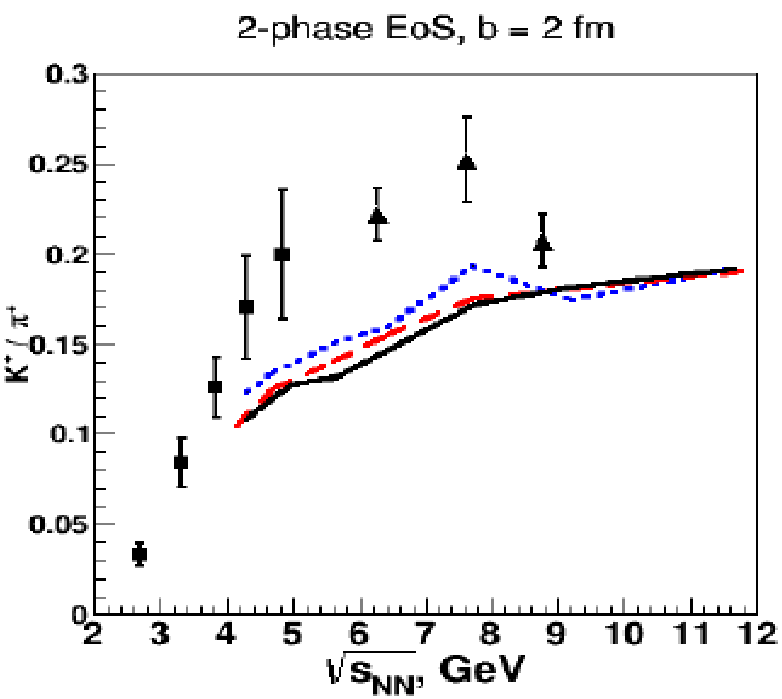
- Mott dissociation of hadrons (here pi, K) at the Chiral restoration temperature T_c = 153 MeV

- Asymptotic behaviour of quark-gluon Pressure can be adjusted with rescattering Parameter gamma

- Very good correspondence between lattice QCD Thermodynamics and improved MHRG/PNJL model; Hadronic and partonic contributions quantified

D.B., A. Dubinin, L. Turko, arxiv:1612.09556

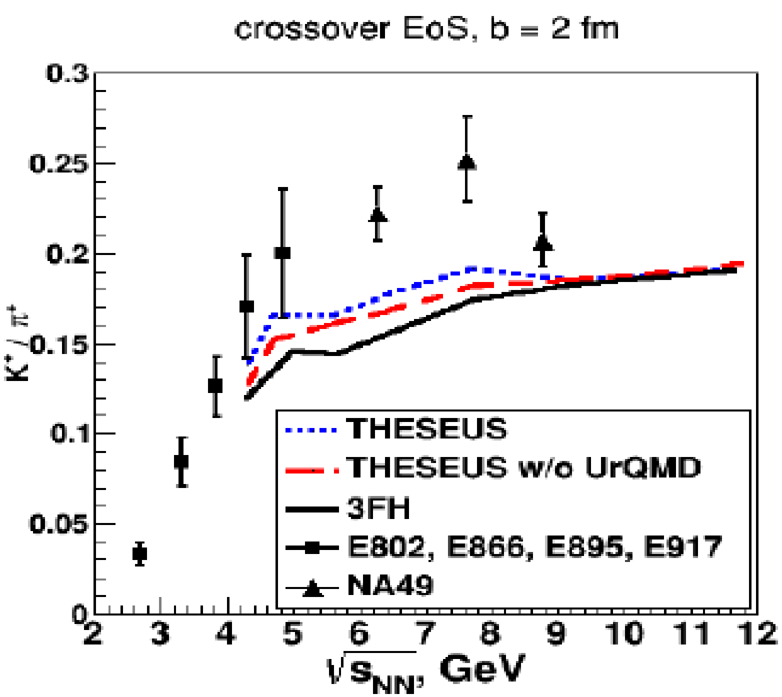
3. What about K+/π+ (Marek's horn) in THESEUS ?



THESEUS simulation reproduces 3FH result,
Thus it has the same discrepancy with experiment

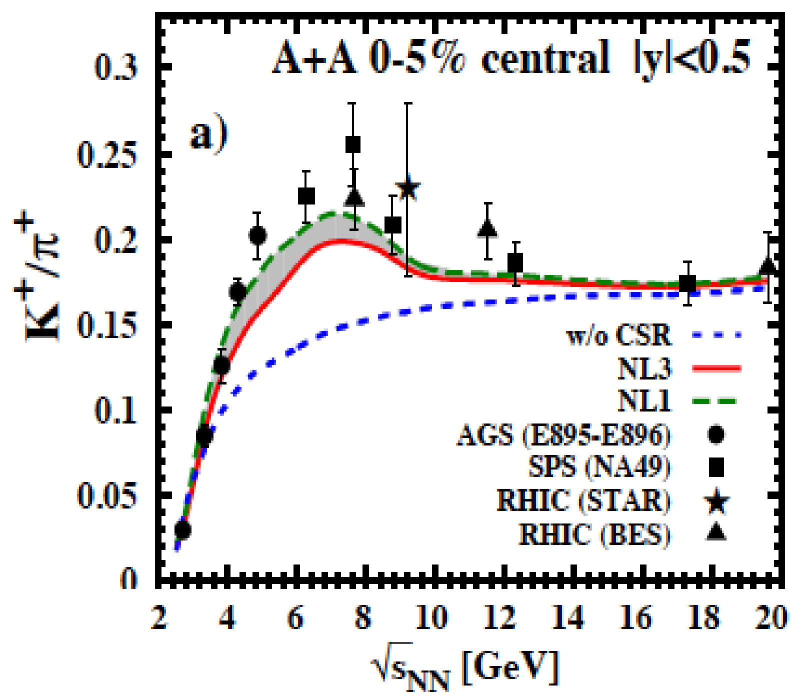
--> some key element still missing in the program

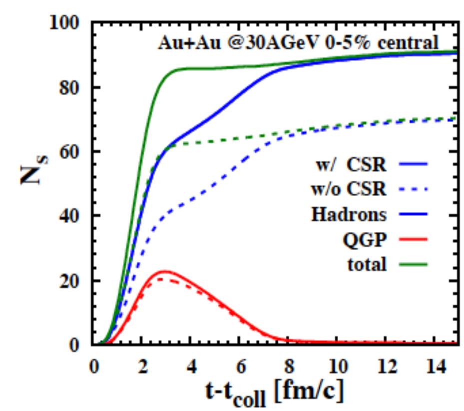
Batyuk, D.B., Bleicher, et al., PRC 94, 044917 (2016)



Recent new development in PHSD

Chiral symmetry restoration in HIC at intermediate ..."
A. Palmese et al., arxiv: 1607.04073; PRC 94, 044912





Strange particle number increase by CSR

3. Mott dissociation of π and K in hot, dense quark matter

D. Blaschke, A. Dubinin, A. Radzhabov, A. Wergieluk, arxiv:1608.05383

Andrey Radzhabov in front of the University of Wroclaw

3. PNJL model for N_f=2+1 quark matter with π and K

$$\mathcal{L} = \bar{q}\,(i\gamma^\mu D_\mu + \hat{m}_0)\,q + G_S \sum_{a=0}^{8} \left[(\bar{q}\lambda^a q)^2 + (\bar{q}i\gamma_5\lambda^a q)^2 \right] - \mathcal{U}\,(\Phi[A], \bar{\Phi}[A]; T)$$

$$\Pi^{M^a}_{ff'}(q_0, \mathbf{q}) = 2N_c T \sum_n \int \frac{d^3p}{(2\pi)^3} \mathrm{tr}_D \left[S_f(p_n, \mathbf{p}) \Gamma^{M^a}_{ff'} S_{f'}(p_n + q_0, \mathbf{p} + \mathbf{q}) \Gamma^{M^a}_{ff'} \right]$$

$$\Gamma^{P^a}_{ff'} = i\gamma_5\, T^a_{ff'} \,, \quad \Gamma^{S^a}_{ff'} = T^a_{ff'} \,, \quad T^a_{ff'} = \begin{cases} (\lambda_3)_{ff'}, \\ (\lambda_1 \pm i\lambda_2)_{ff'}/\sqrt{2}, \\ (\lambda_4 \pm i\lambda_5)_{ff'}/\sqrt{2}, \\ (\lambda_6 \pm i\lambda_7)_{ff'}/\sqrt{2}, \end{cases}$$

$$P^a = \pi^0, \pi^\pm, K^\pm, K^0, \bar{K}^0$$

$$\Pi^{P^a,S^a}_{ff'}(q_0 + i\eta, \mathbf{0}) = 4\{ I^f_1(T, \mu_f) + I^{f'}_1(T, \mu_{f'}) \mp [(q_0 + \mu_{ff'})^2 - (m_f \mp m_{f'})^2]\, I^{ff'}_2(z, T, \mu_{ff'}) \}$$

$$I^f_1(T, \mu_f) = \frac{N_c}{4\pi^2} \int_0^\Lambda \frac{dp\, p^2}{E_f} \left(n^-_f - n^+_f \right),$$

$$I^{ff'}_2(z, T, \mu_{ff'}) = \frac{N_c}{4\pi^2} \int_0^\Lambda \frac{dp\, p^2}{E_f E_{f'}} \left[\frac{E_{f'}}{(z - E_f - \mu_{ff'})^2 - E^2_{f'}}\, n^-_f - \frac{E_{f'}}{(z + E_f - \mu_{ff'})^2 - E^2_{f'}}\, n^+_f + \frac{E_f}{(z + E_{f'} - \mu_{ff'})^2 - E^2_f}\, n^-_{f'} - \frac{E_f}{(z - E_{f'} - \mu_{ff'})^2 - E^2_f}\, n^+_{f'} \right]$$

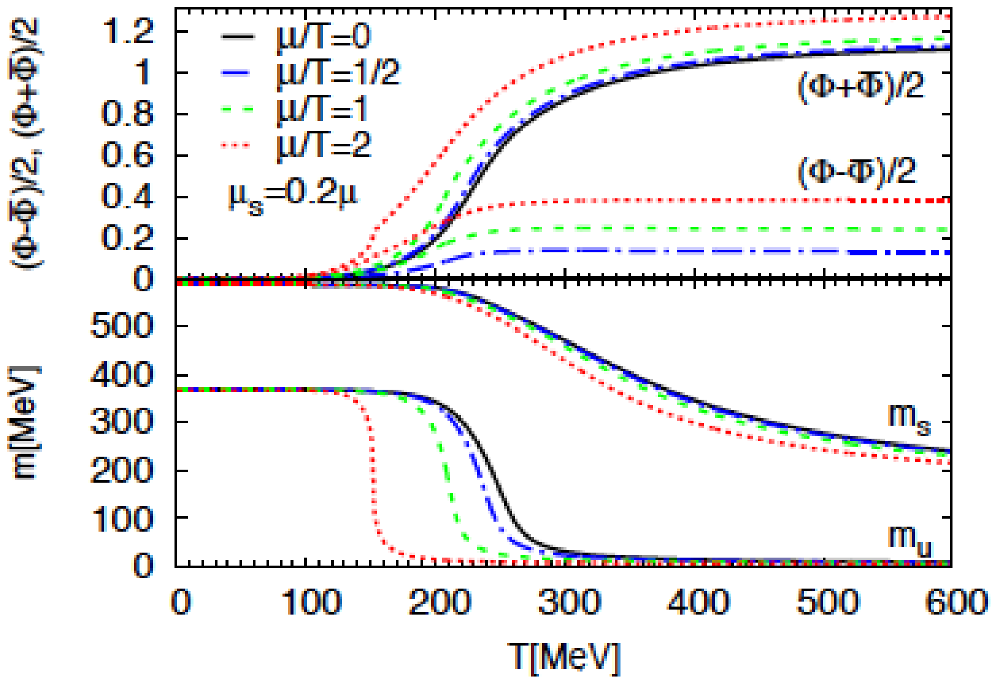
3. PNJL model for N_f=2+1 quark matter with π and K

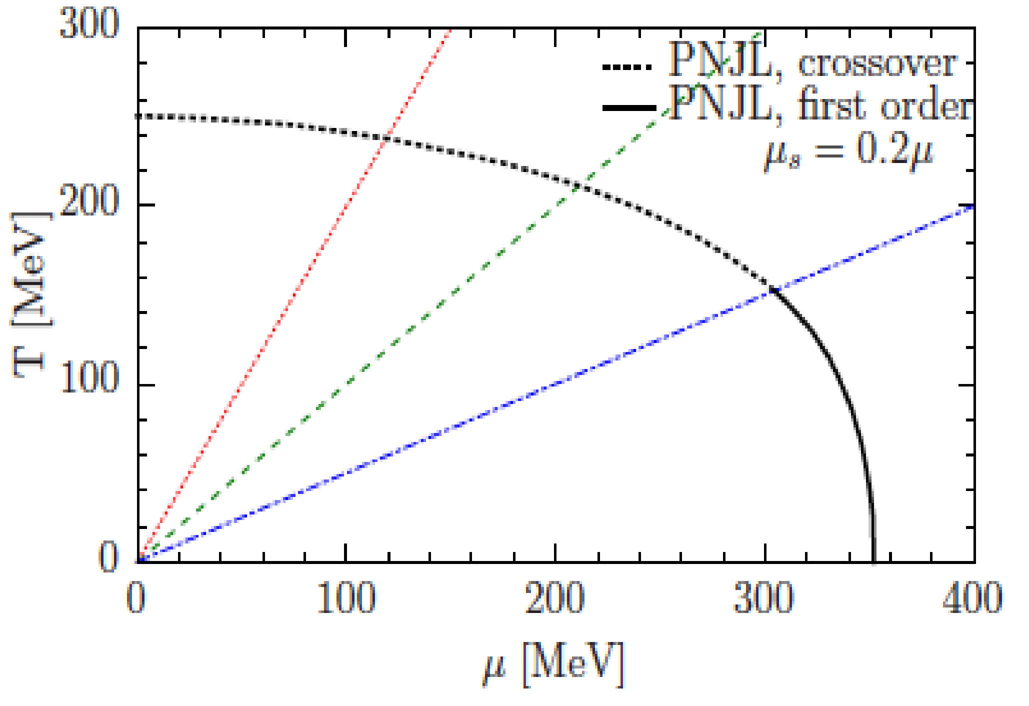
$$m_f = m_{0,f} + 16\, m_f G_S I_1^f(T,\mu), \quad \mathcal{P}_{ff'}^{M^a}(M_{M^a} + i\eta, \mathbf{0}) = 1 - 2G_S \Pi_{ff'}^{M^a}(M_{M^a} + i\eta, \mathbf{0}) = 0.$$

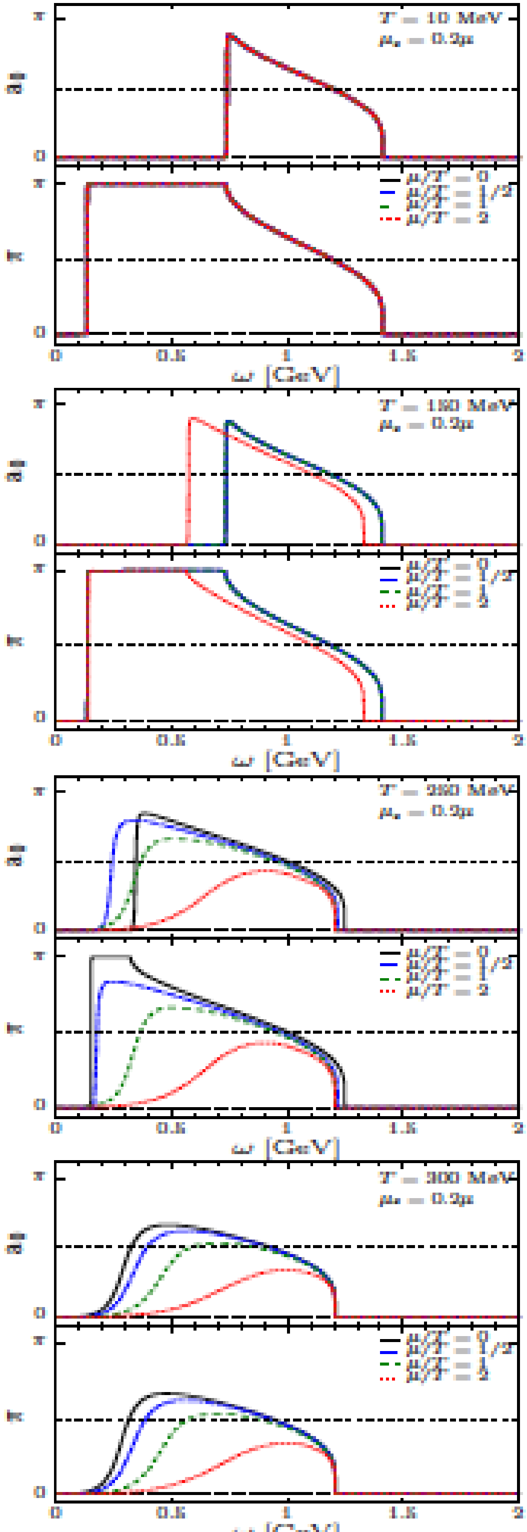
$$P_f = -\frac{(m_f - m_{0,f})^2}{8G} + \frac{N_c}{\pi^2}\int_0^\Lambda dp\, p^2 E_f + \frac{N_c}{3\pi^2}\int_0^\infty \frac{dp\, p^4}{E_f}\left[f_\Phi^+(E_f) + f_\Phi^-(E_f)\right]$$

$$P_M = d_M \int \frac{d^3q}{(2\pi)^3} \int_0^\infty \frac{d\omega}{2\pi} \left\{ g(\omega - \mu_M) + g(\omega + \mu_M) \right\} \delta_M(\omega, \mathbf{q})$$

$$\delta_M(\omega,\mathbf{q}) = -\arctan\left\{\frac{\mathrm{Im}\left(\mathcal{P}_{ff'}^M(\omega - i\eta, \mathbf{q})\right)}{\mathrm{Re}\left(\mathcal{P}_{ff'}^M(\omega + i\eta, \mathbf{q})\right)}\right\}$$







3. Mott dissociation of pions and kaons in the Beth-Uhlenbeck approach ...

D.B., A. Dubinin, A. Radzhabov, A. Wergieluk, arxiv:1608.05383
D.B., M. Buballa, A. Dubinin, G. Ropke, D. Zablocki, Ann. Phys. (2014)

Thermodynamics of resonances (M) via phase shifts

$$P_M = d_M \int \frac{d^3q}{(2\pi)^3} \int_0^\infty \frac{ds}{4\pi} \frac{1}{\sqrt{s+q^2}} \left\{ g(\sqrt{s+q^2} - \mu_M) \right\} \delta_M(\sqrt{s};T,\mu)$$

Polyakov-loop Nambu – Jona-Lasinio modell

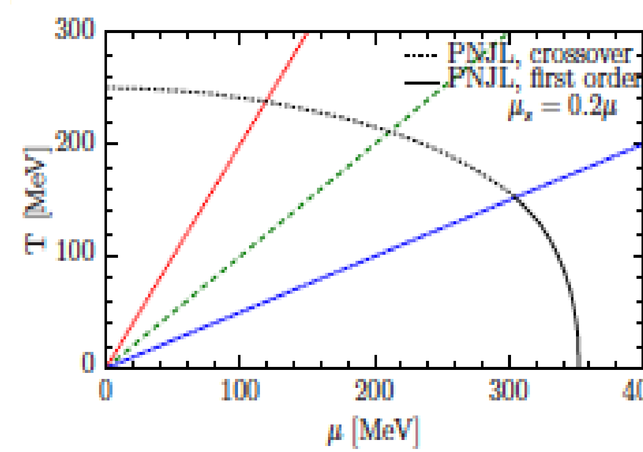
$$\Pi^{M^*}_{ff'}(q_0,\mathbf{q}) = 2N_c T \sum_n \int \frac{d^3p}{(2\pi)^3} \mathrm{tr}_D \left[S_f(p_n,\mathbf{p}) \Gamma^{M^*}_{ff'} S_{f'}(p_n + q_0, \mathbf{p}+\mathbf{q}) \Gamma^{M^*}_{ff'} \right],$$

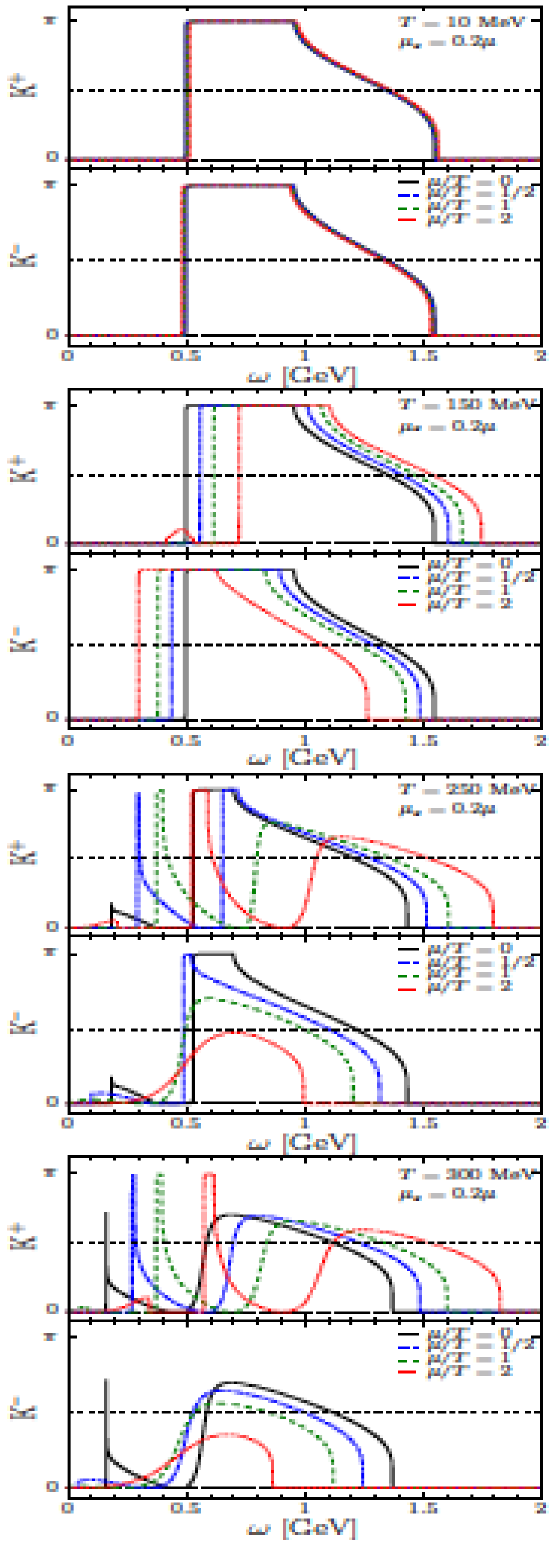
$$\mathcal{P}^{M^*}_{ff'}(M_{M^*} + i\eta, 0) = 1 - 2G_S \Pi^{M^*}_{ff'}(M_{M^*} + i\eta, 0)$$

$$\delta_M(\omega,\mathbf{q}) = -\arctan\left\{ \frac{\mathrm{Im}\left(\mathcal{P}^M_{ff'}(\omega - i\eta,\mathbf{q})\right)}{\mathrm{Re}\left(\mathcal{P}^M_{ff'}(\omega + i\eta,\mathbf{q})\right)} \right\}$$

Evaluation along trajectories μ/T=const in the phase diagram:

- Pion and a0 as partner states,
- Chiral symmetry restoration,
- Mott dissociation of bound states,
- Levinson theorem





3. Mott dissociation of pions and kaons in the Beth-Uhlenbeck approach ...

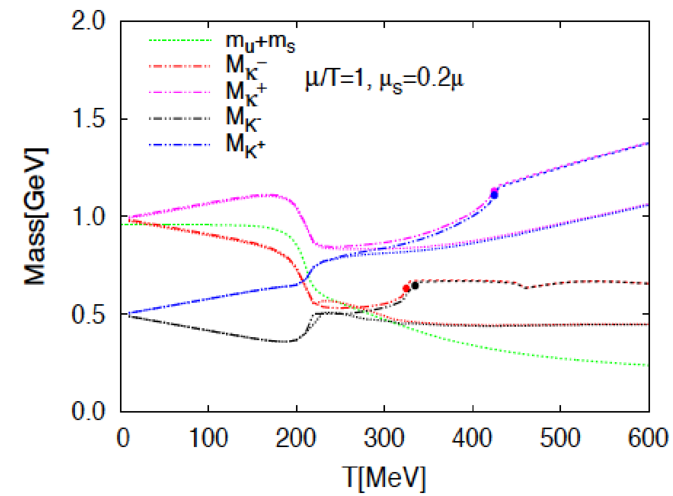
D.B., A. Dubinin, A. Radzhabov, A. Wergieluk, arxiv:1608.05383

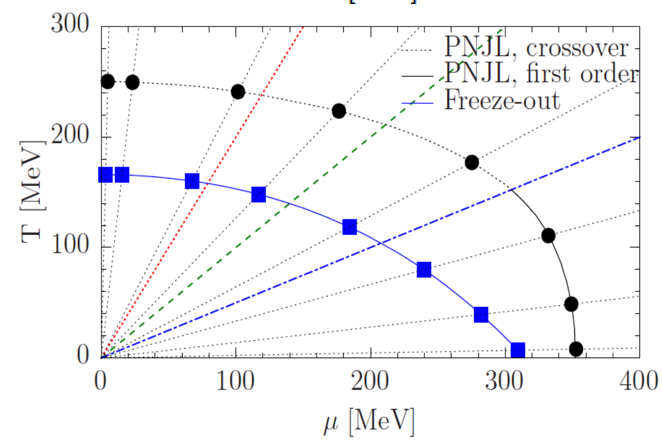
Polarization loop in Polyakov-loop Nambu – Jona-Lasinio model

$$\Pi^{P^a,S^a}_{ff'}(q_0+i\eta,\mathbf{0}) = 4\{I_1^f(T,\mu_f)+I_1^{f'}(T,\mu_{f'}) \mp \left[(q_0+\mu_{ff'})^2-(m_f\mp m_{f'})^2\right] I_2^{ff'}(z,T,\mu_{ff'})\}$$

$$I_1^f(T,\mu_f) = \frac{N_c}{4\pi^2}\int_0^\Lambda \frac{dp\,p^2}{E_f}\left(n_f^- - n_f^+\right),$$

$$I_2^{ff'}(z,T,\mu_{ff'}) = \frac{N_c}{4\pi^2}\int_0^\Lambda \frac{dp\,p^2}{E_f E_{f'}} \left[\frac{E_{f'}}{(z-E_f-\mu_{ff'})^2-E_{f'}^2}\,n_f^- - \frac{E_{f'}}{(z+E_f-\mu_{ff'})^2-E_{f'}^2}\,n_f^+ + \frac{E_f}{(z+E_{f'}-\mu_{ff'})^2-E_f^2}\,n_{f'}^- - \frac{E_f}{(z-E_{f'}-\mu_{ff'})^2-E_f^2}\,n_{f'}^+\right]$$



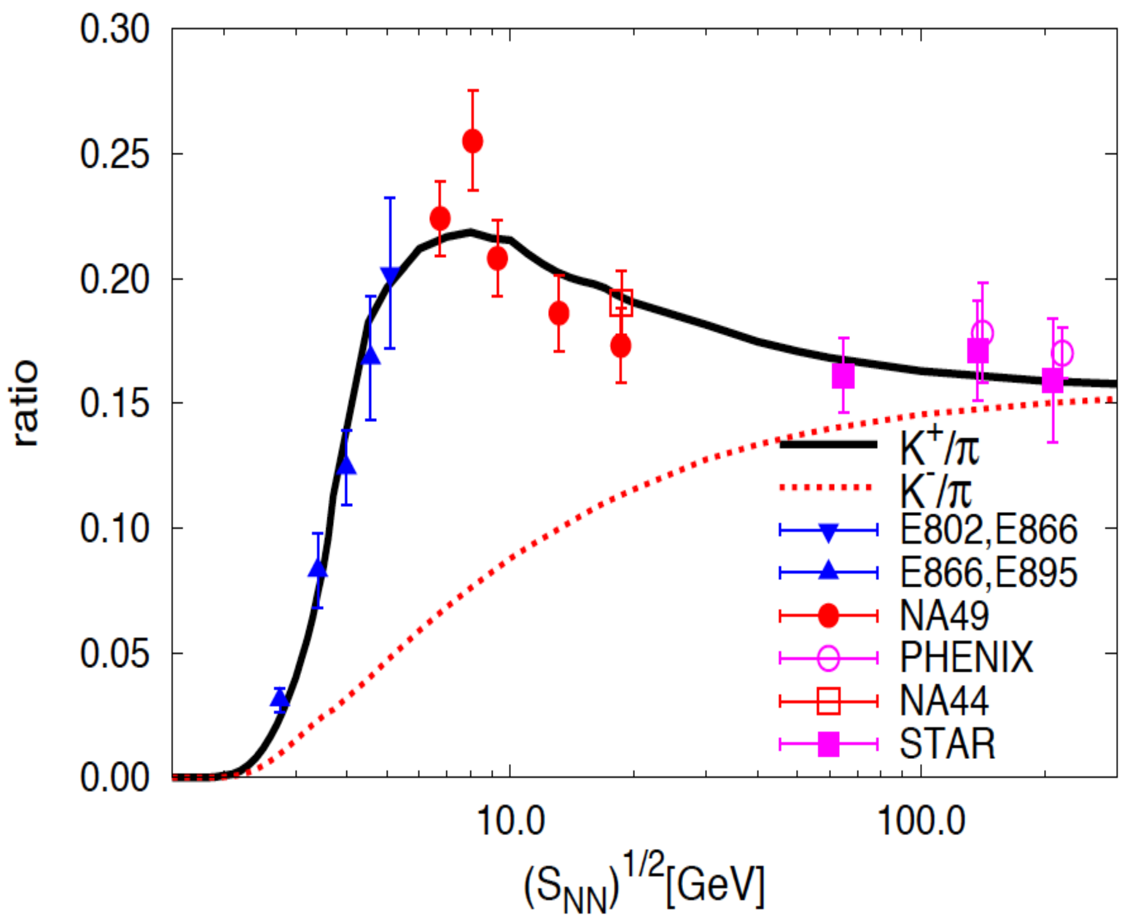


Anomalous low-mass mode for K+ in the dense medium !!

3. Mott dissociation of pions and kaons in Beth-Uhlenbeck: Explanation of the “horn” effect for K+/π+ in HIC?

Ratio of yields in BU approach defined via phase shifts:

$$\frac{n_{K^\pm}}{n_{\pi^\pm}} = \frac{\int dM \int d^3p \, (M/E) g_{K^\pm}(E)[1+g_{K^\pm}(E)]\delta_{K^\pm}(M)}{\int dM \int d^3p \, (M/E) g_{\pi^\pm}(E)[1+g_{\pi^\pm}(E)]\delta_{\pi^\pm}(M)}$$



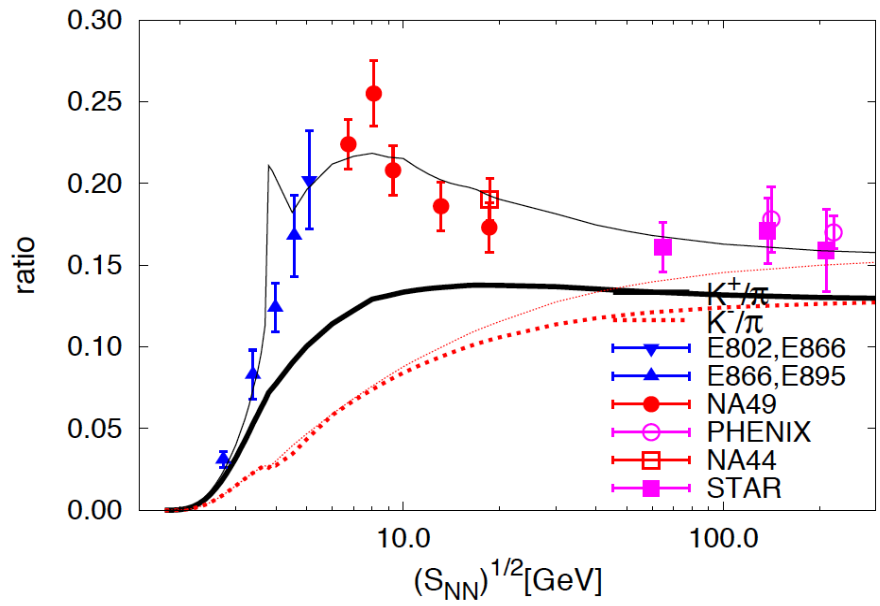
Evaluation along the freeze-out Curve parametrized by Cleymans et al.

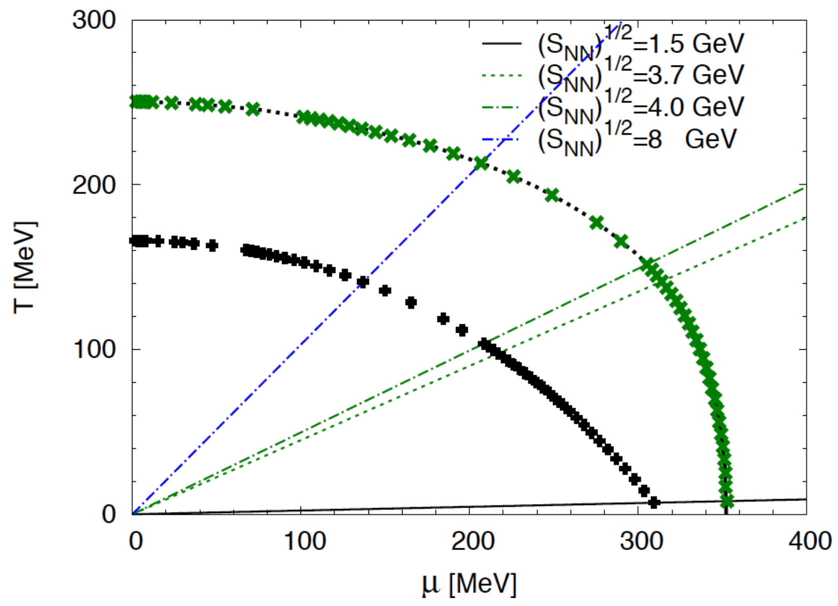
- enhancement for K+ due to anomalous in-medium bound state mode
- no such enhancement for K- or pions

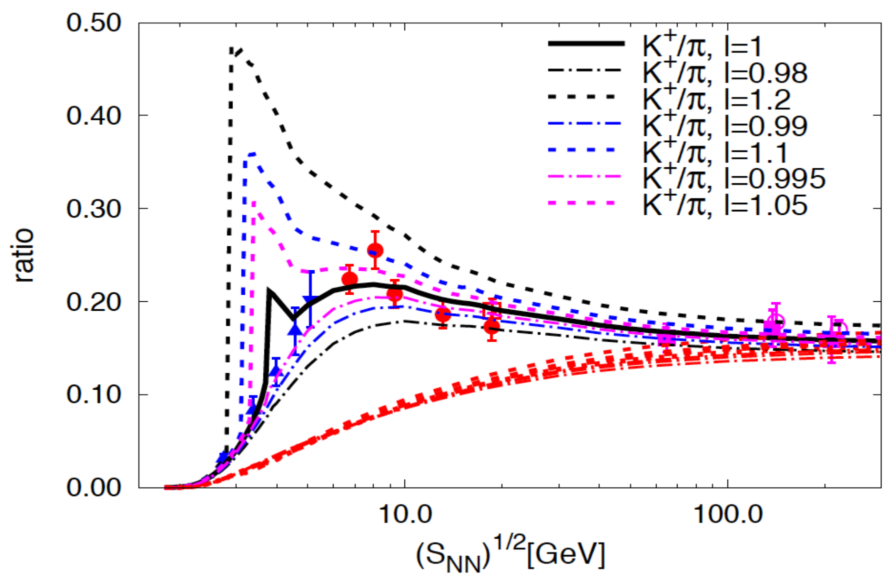
- explore the effect in thermal statistical models and in THESEUS ...

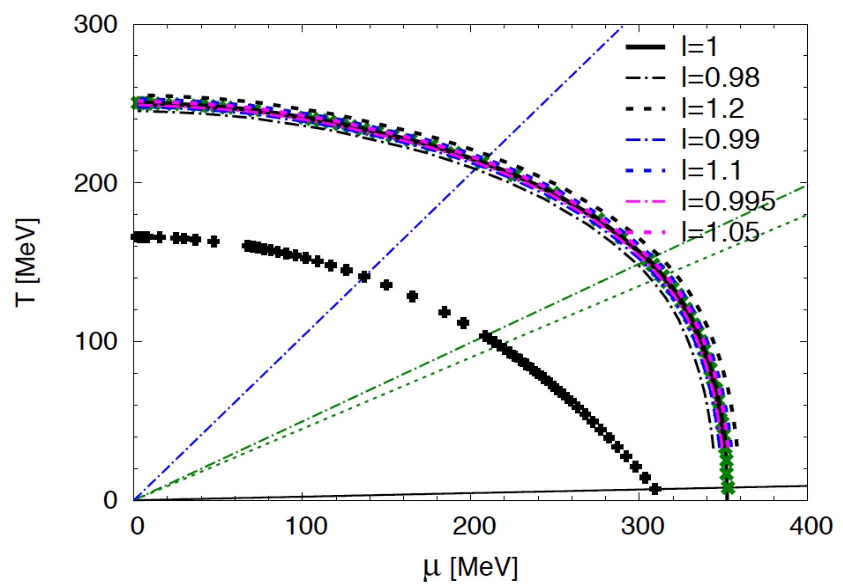
D.B., A. Dubinin, A. Radzhabov, A. Wergieluk, arxiv:1608.05383

3. “Tooth” on the “horn” due to anomalous K+; sign of CEP?





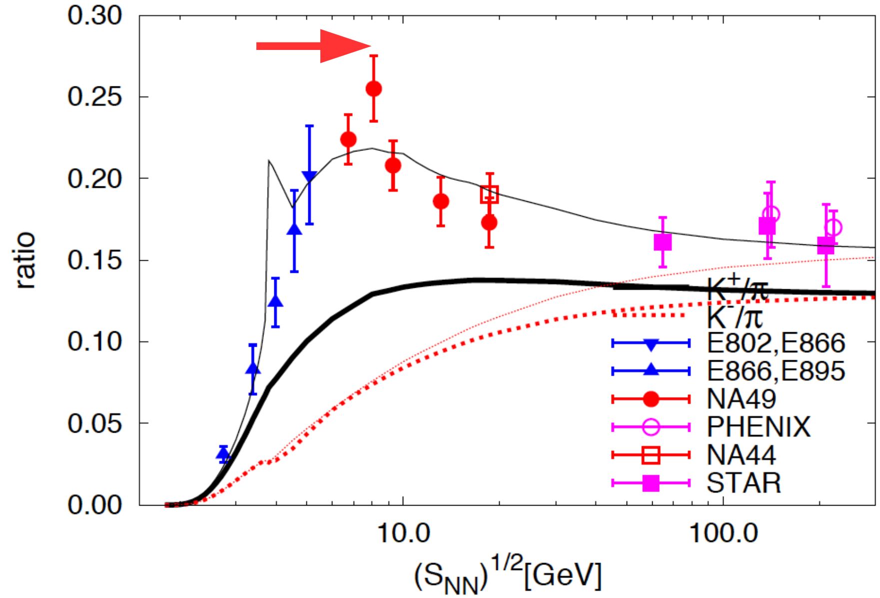


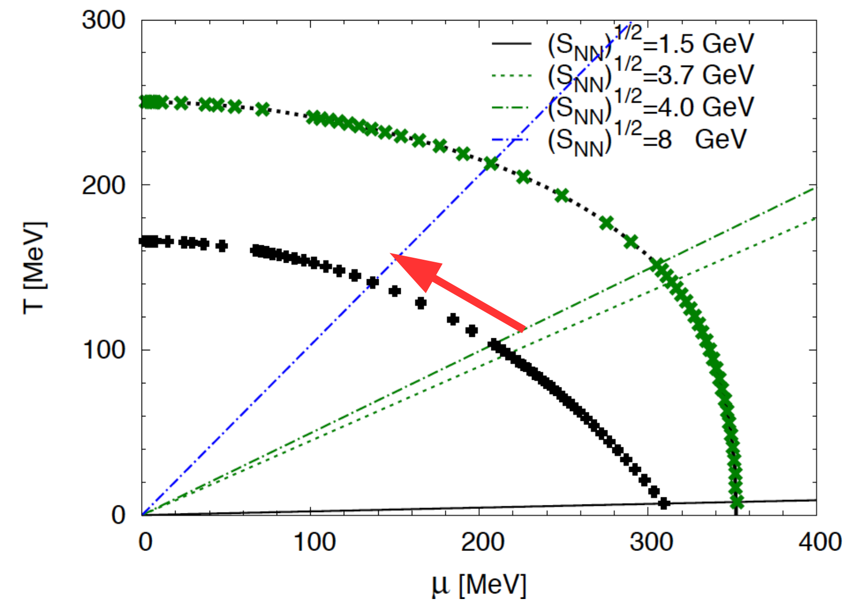


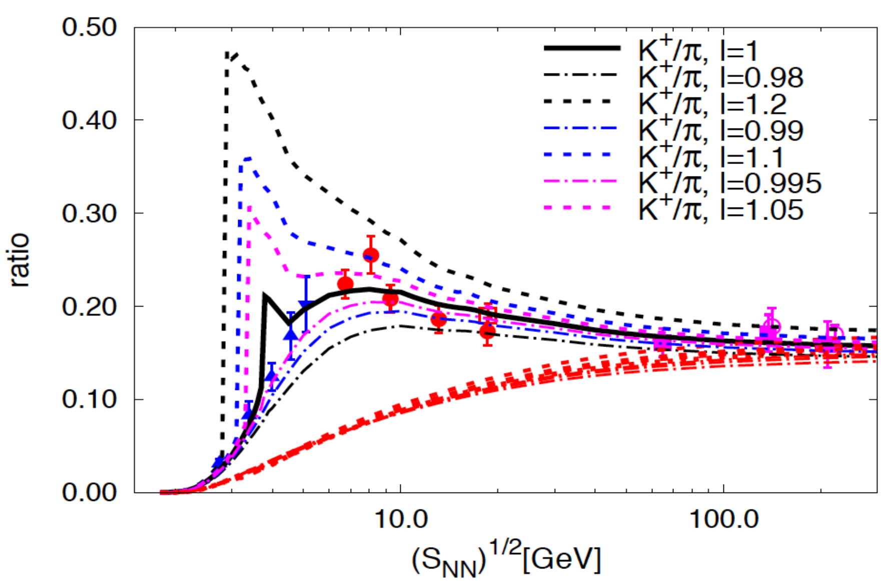
- enhancement for K+ due to anomalous in-medium bound state mode

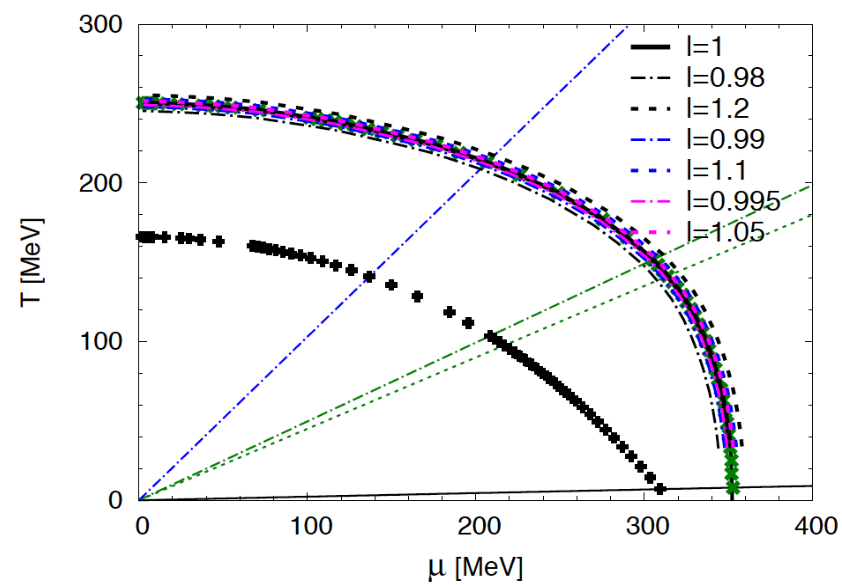
D.B., A. Radzhabov, in prep. (2017)

3. “Tooth” on the “horn” due to anomalous K+; sign of CEP?









- “tooth” correlated to the CEP → indicator for CEP !!

D.B., A. Radzhabov, in prep. (2017)

Summary:

- GBU accounts consistently for hadron formation and dissociation (Mott effect)
  Inverse: Mott-Anderson Hadronization !!

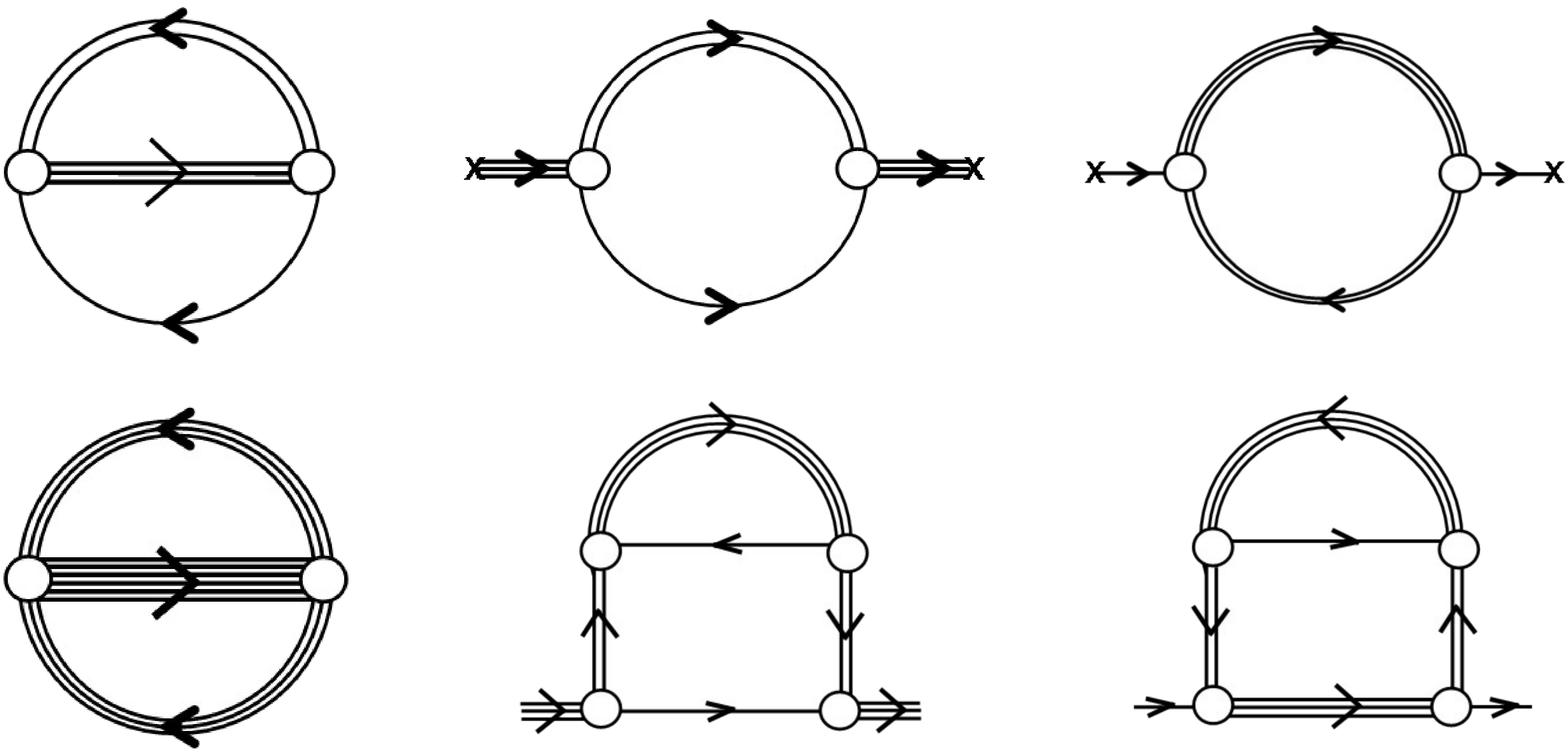
- Quark Pauli blocking leads to stiffening hadronic EoS, precursor of deconfinement

- New modes in medium due to BSE dynamics (e.g., K+)

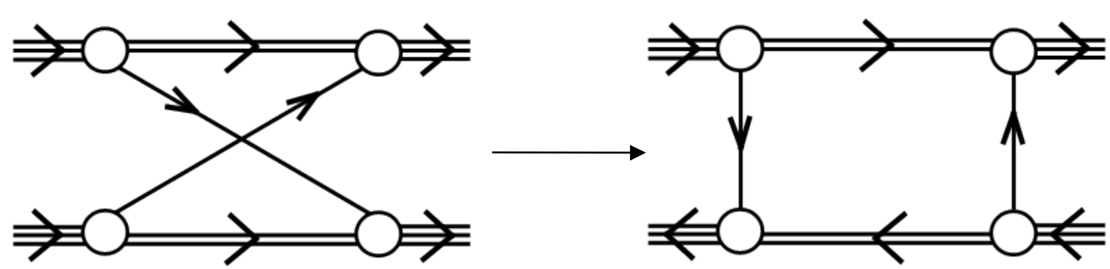
Additional Slides

Example C: Nucleons in quark matter

Φ-functional | nucleon selfenergy | quark selfenergy

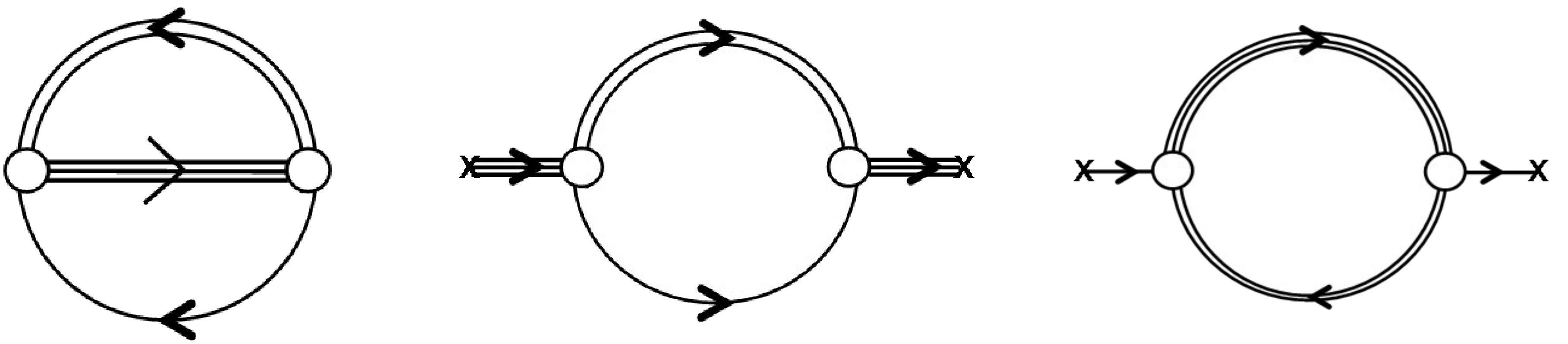


quark exchange interaction
between nucleons:

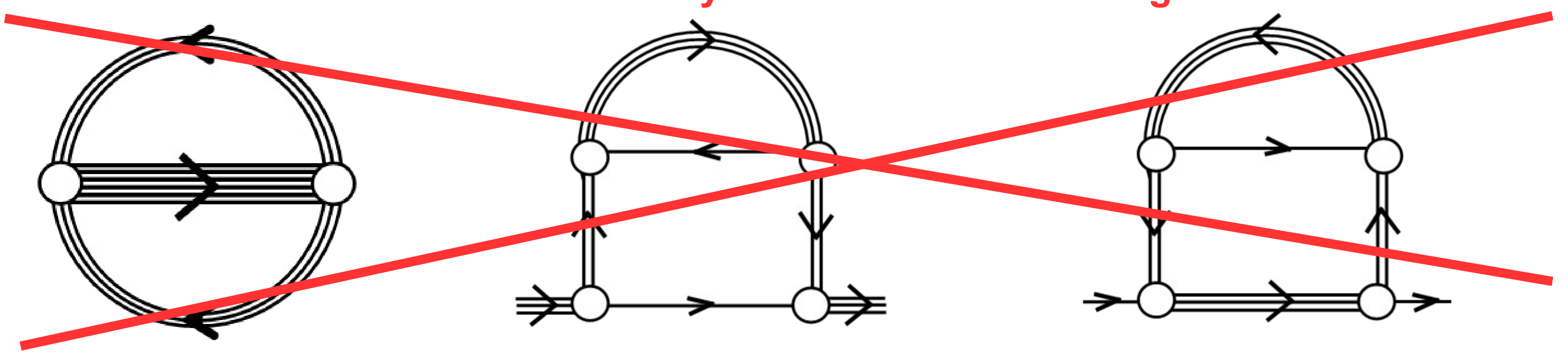


Example C: Nucleons in quark matter

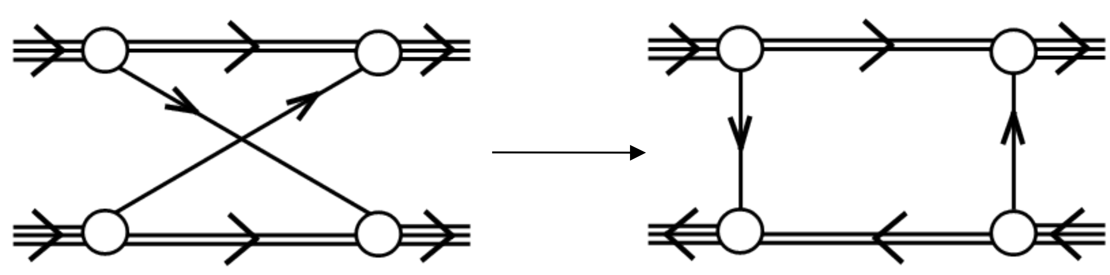
Φ-functional | nucleon selfenergy | quark selfenergy



Not new! Already contained in above diagrams!

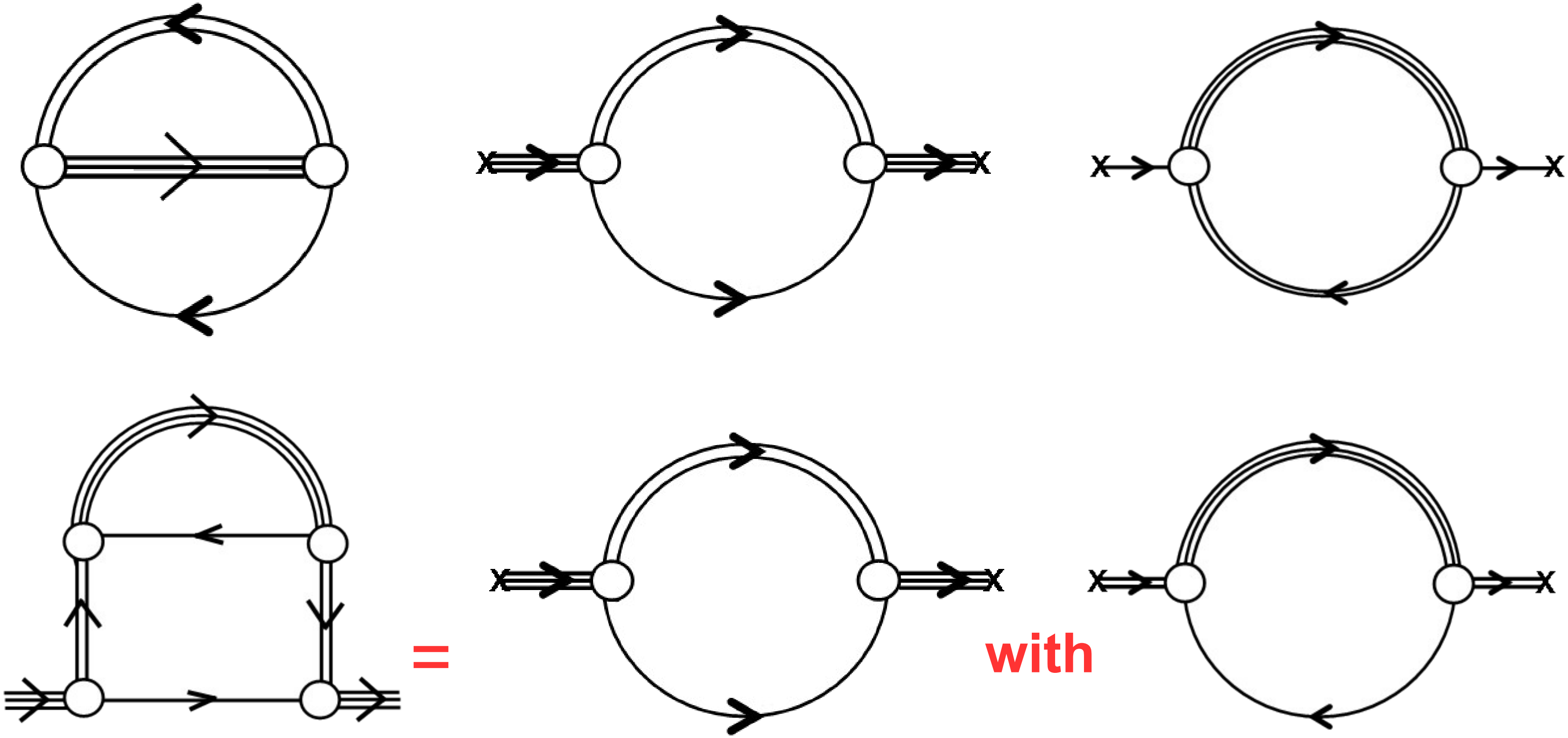


quark exchange interaction between nucleons:



Example C: Nucleons in quark matter

Φ-functional | nucleon selfenergy | quark selfenergy



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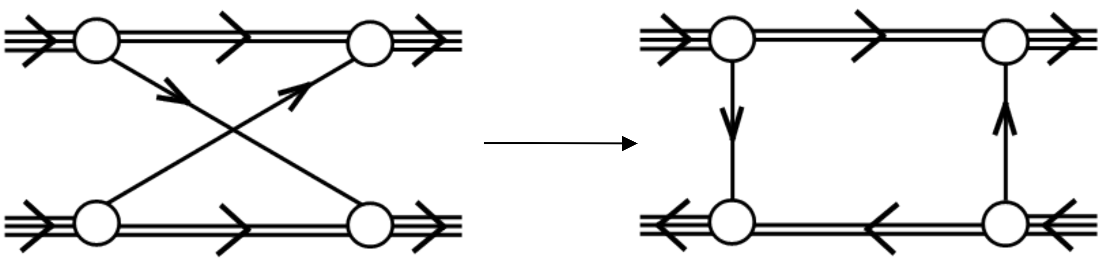
quark exchange interaction
between nucleons:



Intermezzo: Structure of the baryon?

12-Apostle Church, Kars

Intermezzo: Structure of the baryon?

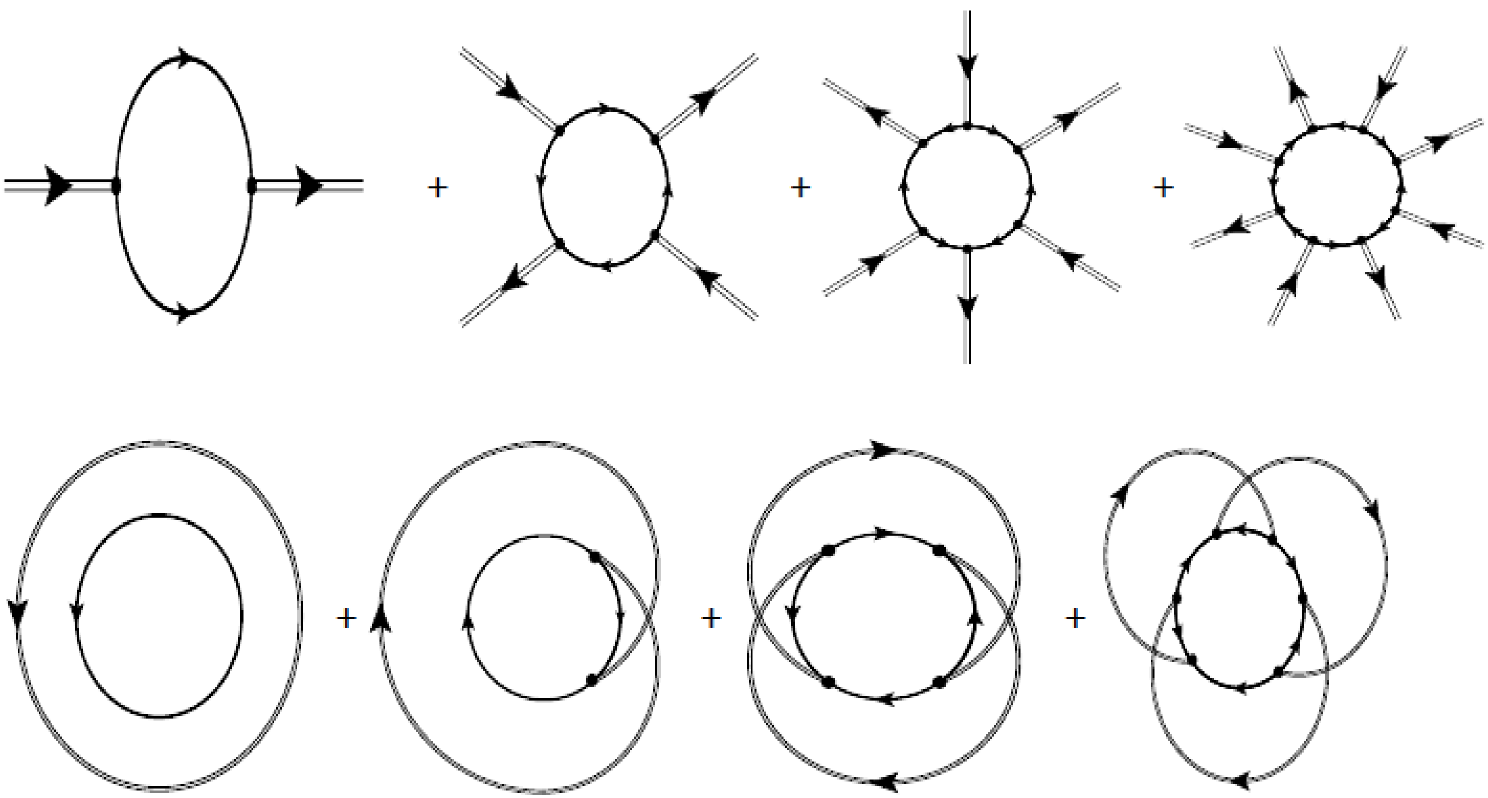
12-Apostle
Church,
Kars

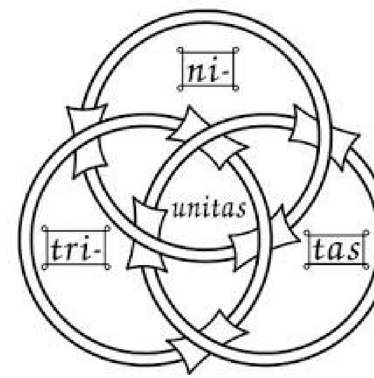
Intermezzo: Structure of the baryon?

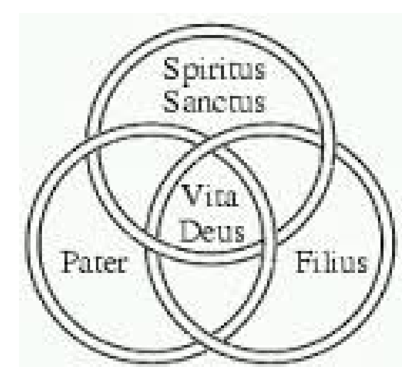
Borromean ? !!

$$Z_{\rm fluct} = \int D\Delta^\dagger D\Delta D\phi \exp\{-\frac{|\Delta|^2}{4G_D} - \frac{\phi^2}{4G} - Tr\, ln S^{-1}[\Delta, \Delta^\dagger, \phi]\}$$

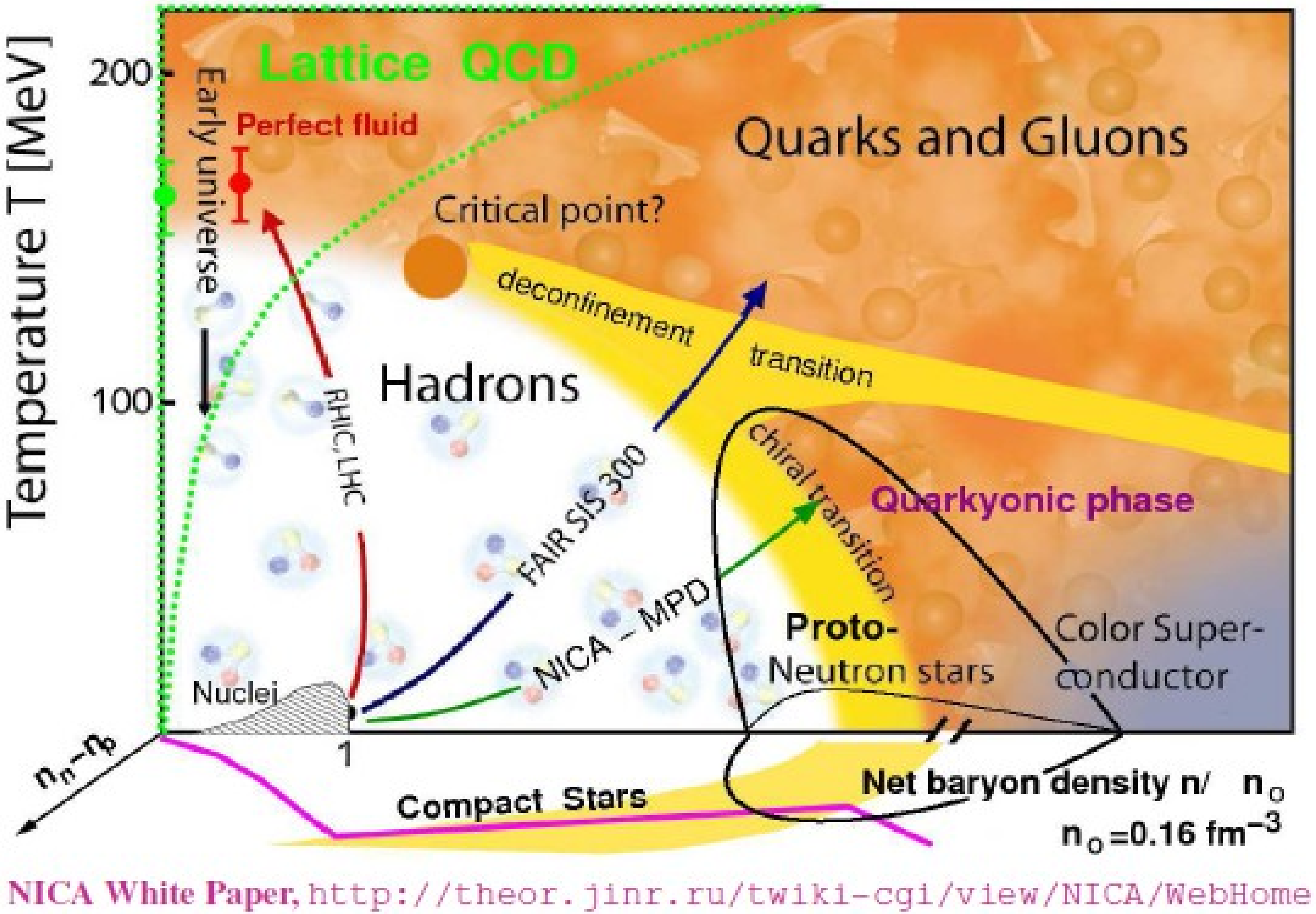
Cahill, Roberts, Prashifka: Aust. J. Phys. 42 (1989) 129, 161
Cahill, ibid, 171; Reinhardt: PLB 244 (1990) 316; Buck, Alkofer, Reinhardt: PLB 286 (1992) 29







Support a CEP in QCD phase diagram with Astrophysics?



S. Benic et al., A&A 577, A40 (2015)

Crossover at finite T (Lattice QCD) + First order at zero T (Astrophysics) = Critical endpoint exists!

Introduction: Beth-Uhlenbeck vs. Generalized BU

Beth-Uhlenbeck: 2nd virial coefficient B(T)

$$pV = NkT\left(1 + \frac{B(T)}{V} + \frac{C(T)}{V^2} + \ldots\right)$$

BU for virial expansion of density:

$$n(\mu, T) = n_{\rm free}(\mu, T) + 2n_{\rm corr}(\mu, T)$$

$$n_{\rm free}(\mu, T) = 4\int \frac{d^3p}{h^3} e^{-(p^2/2m-\mu)/T} = \frac{4}{\lambda^3} e^{\mu/T}$$

$$n_{\rm corr}(\mu, T) = \int \frac{d^3\mathbf{P}}{h^3} e^{-(P^2/4m-2\mu)/T} \int_{-\infty}^{\infty} \frac{dE}{\pi} e^{-E/T} D(E)$$

$$= \frac{2^{3/2}}{\lambda^3} e^{2\mu/T} \int_{-\infty}^{\infty} \frac{dE}{\pi} e^{-E/T} D(E).$$

Density of states: bound and scattering part

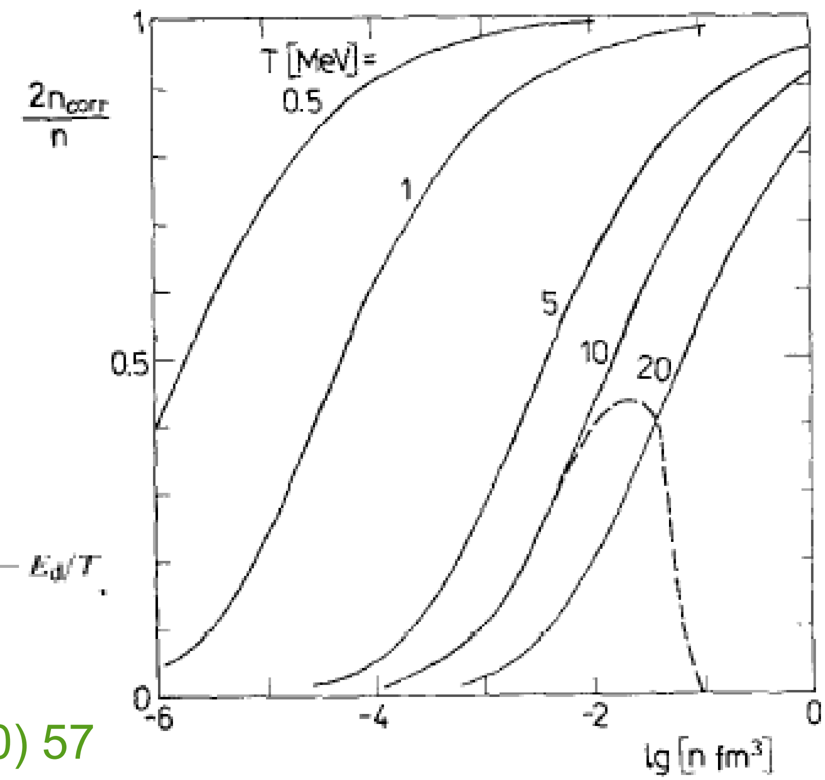
$$D(E) = \sum_\alpha c_\alpha \left[\pi\delta(E - E_\alpha) + \frac{d}{dE}\delta_\alpha(E)\right],$$

Example: Deuterons in nuclear matter

$$n = n_{\rm free} + 2n_{\rm free}^2 I(T)$$

$$I(T) = \lambda^3 \frac{2^{1/2}}{8}\left[3(e^{-E_d/T} - 1) + \int_0^\infty \frac{dE}{\pi T} e^{-E/T} \sum_\alpha c_\alpha \delta_\alpha(E)\right].$$

For T<<E$_d$: $n = n_{\rm free} + 2n_{\rm deut}\,, \quad n_{\rm deut} = n_{\rm free}^2 \lambda^3 3\,\frac{2^{1/2}}{8} e^{-E_d/T}.$



E. Beth and G.E. Uhlenbeck, Physica IV (1937) 915;
S. Schmidt, G. Roepke, H. Schulz, Ann. Phys. 202 (1990) 57

Introduction: Beth-Uhlenbeck vs. Generalized BU

Thermodynamic Greens function approach:

$$n(1, \mu_1, T) = \int \frac{dE}{2\pi} f_1(E)\, A(1, E)$$

$$A(1, E) = \frac{2\Sigma_I(1, E - i0)}{(E - E(1) - \Sigma_R(1, E))^2 + \Sigma_I(1, E - i0)^2} \approx \frac{2\pi\, \delta(E - e(1))}{1 - ((d/dz)\,\Sigma_R(1, z))|_{z = e(1) + i0}} - 2\Sigma_I(1, E + i0) \frac{d}{dE} \frac{\mathbf{P}}{E - e(1)}$$

Density formula
(free and corr. Quasiparticles):

$$n(\mu, T) = n_{free}(\mu, T) + 2n_{corr}(\mu, T),$$

$$\Sigma(1, z_\nu) = T \sum_2 \sum_{z_{\nu'}} [T(1212, z_\nu + z_{\nu'}) - \mathrm{ex}]\, G(2, z_{\nu'}).$$

$$n_{corr}(\mu, T) = \int \frac{dE}{2\pi} g(E)\, F(E)$$

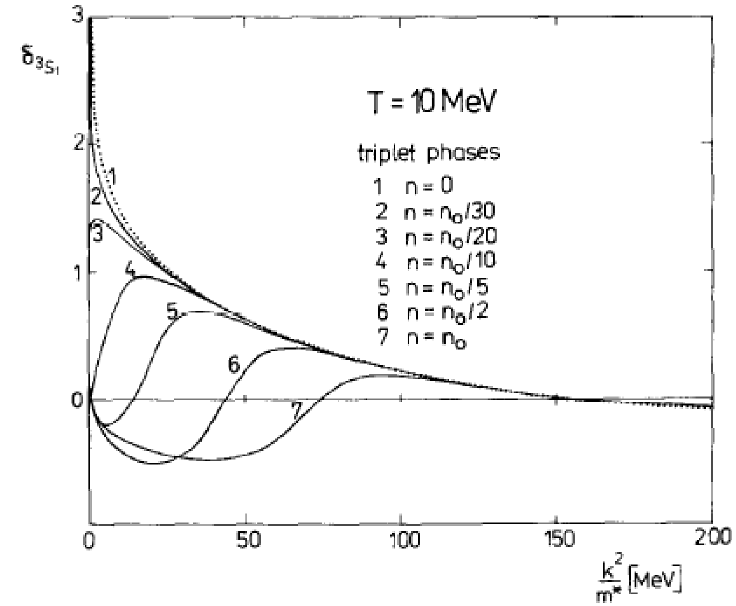
$$F(E) = F_{deut}(E) + \frac{2}{4\pi} \sum_\alpha c_\alpha F_\alpha(E),$$

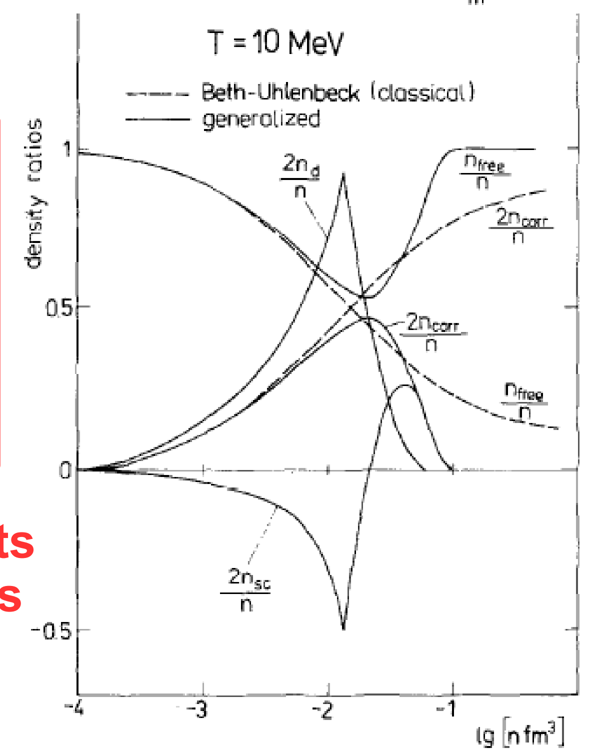
$$F_{deut}(E) = 6 \sum_{K > K^{Mott}} \pi\delta(E - E_b(K, \mu, T)).$$

$$F(E) = \sum_{12} [1 - f(e(1)) - f(e(2))] \cdot \left[(T_I(1212, E + i0) - \mathrm{ex}) \frac{d}{dE} \frac{\mathbf{P}}{e(1) + e(2) - E} - \pi\delta(E - e(1) - e(2)) \frac{d}{dE} (T_R(1212, E + i0) - \mathrm{ex}) \right].$$

$$F_\alpha(E) = 8\pi \sum_K \sin^2 \delta_\alpha(E, K, \mu, T) \frac{d}{dE} \delta_\alpha(E, K, \mu, T).$$

The $\sin^2\delta$ term accounts for quasiparticle effects





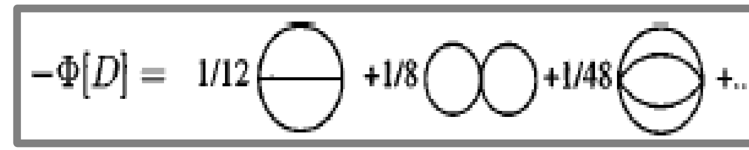
S. Schmidt, G. Röpke, H. Schulz, Ann. Phys. 202 (1990) 57

Φ-derivable approach, 2-loop approximation

J.-P. Blaizot, E. Iancu, A. Rebhan, Phys. Rev. D 63 (2001) 065003

Skeleton expansion for thermodynamic potential and entropy

$$\beta\Omega[D] = -\log Z = \frac{1}{2}\mathrm{Tr}\log D^{-1} - \frac{1}{2}\mathrm{Tr}\,\Pi D + \Phi[D]$$



Inv. Temp: 1/T   trace in conf. Space   self-energy related to D

Dyson equation: $D^{-1} = D_0^{-1} + \Pi$ Free propagator Do is known

Essential property of Ω[D] is Stationarity under variation of D: $\delta\,\Omega[D] / \delta\,D = 0$

This implies $\delta\,\Phi[D] / \delta\,D = 1/2\ \Pi$

Physical propagator and selfenergy are defined self-consistently !

Self-consistent approximations are defined by the choice of Φ

→ **Φ – derivable theories**

G. Baym, Phys. Rev. 127 (1962) 1391; Vanderheyden & Baym; J. Stat. Phys. 93, 843 (1998)

Approximately selfconsistent thermodynamics

Matsubara summation:

$$\Omega/V = \int \frac{d^4k}{(2\pi)^4} n(\omega)[\mathrm{Im}\log(-\omega^2+k^2+\Pi) - \mathrm{Im}\,\Pi D] \; + T\Phi[D]/V$$

Analytic properties:

$$D(\omega,k) = \int_{-\infty}^{\infty} \frac{dk_0}{2\pi} \frac{\rho(k_0,k)}{k_0-\omega}, \qquad \mathrm{Im}\,D(\omega,k) \equiv \mathrm{Im}\,D(\omega+i\epsilon,k) = \frac{\rho(\omega,k)}{2}.$$

Thermodynamics from entropy density: $\mathcal{S} = -\partial(\Omega/V)/\partial T$

$$\mathcal{S} = -\int \frac{d^4k}{(2\pi)^4} \frac{\partial n(\omega)}{\partial T} \mathrm{Im}\log D^{-1}(\omega,k) \; + \int \frac{d^4k}{(2\pi)^4} \frac{\partial n(\omega)}{\partial T} \mathrm{Im}\,\Pi(\omega,k)\mathrm{Re}\,D(\omega,k) + \mathcal{S}'$$

$$\mathcal{S}' \equiv -\left.\frac{\partial(T\Phi/V)}{\partial T}\right|_D + \int \frac{d^4k}{(2\pi)^4} \frac{\partial n(\omega)}{\partial T} \mathrm{Re}\,\Pi\,\mathrm{Im}\,D \longrightarrow 0$$

for two-loop skeleton diagrams

Loosely speaking: S' accounts for residual interactions of "independent quasiparticles"

$d/d\omega\,[\,\mathrm{Im}\log D^{-1} + \mathrm{Im}\Pi\ \mathrm{Re}D\,] = 2\,\mathrm{Im}\,[\,D\,\mathrm{Im}\Pi\,(d/d\omega\, D^*)\,\mathrm{Im}\Pi] = 2\sin^2\delta\; d\delta/d\omega$, for $D = |D|e^{i\delta}$

D. B., in preparation (2017)

Proof of cancellations resulting in S'=0 (I)

$$S' \equiv -\frac{\partial(T\Phi/V)}{\partial T}\bigg|_D + \int \frac{d^4k}{(2\pi)^4}\left\{\frac{\partial n(\omega)}{\partial T}\mathrm{Re}\Pi\,\mathrm{Im}D\right\}$$

First term

$$-\frac{T}{V}\Phi = \frac{g^2}{12}T^2\sum_{\omega_1,\omega_2}\int\frac{d^3k_1d^3k_2}{(2\pi)^6}D(\omega_1,|k_1|)D(\omega_2,|k_2|)D(-\omega_1-\omega_2,|-k_1-k_2|)$$

Spectral representation

$$D(\omega,k) = \int_{-\infty}^{\infty}\frac{dk_0}{2\pi}\frac{\rho(k_0,k)}{k_0-\omega}$$

Matsubara sums

$$-\frac{T}{V}\Phi = \frac{g^2}{12}T^2\sum_{\omega_1,\omega_2}\int\frac{d^4kd^4k'd^4k"}{(2\pi)^9}\delta^{(3)}(\mathbf{k}+\mathbf{k}'+\mathbf{k}'')\rho(k)\rho(k')\rho(k'')\frac{-1}{\omega_1-k_0}\frac{-1}{\omega_2-k_0'}\frac{1}{\omega_1+\omega_2+k_0''}$$

Partial fraction decomposition of the three energy denominators and Matsubara summation over ω_1, ω_2 yields:

$$\frac{1}{k_0+k_0'+k_0''}\left\{[n(k_0'')+1][n(k_0)+n(k_0'')+1]+n(k_0)n(k_0'')\right\}$$

Temperature derivative and renaming variables under the integrals

$$\partial_T\left[n(k_0+n(k_0')+n(k_0'')+n(k_0')n(k_0)+n(k_0')n(k_0'')+n(k_0)n(k_0'')\right]\rightarrow 3\partial_T n(k_0)\left[1+n(k_0')+n(k_0'')\right]$$

Proof of cancellations resulting in S'=0 (II)

Second term:

$$\begin{aligned} \mathrm{Re}\Pi(\omega,q) &= -\frac{g^2}{2}\int\frac{d^3k}{(2\pi)^3}\int\frac{dk_0}{2\pi}\int\frac{dk_0'}{2\pi}\rho(k_0,|k|)\rho(k_0',|\mathbf{k}+\mathbf{q}|)\sum_{\omega_1}\frac{1}{\omega_1-k_0}\,\frac{1}{\omega_1+\omega-k_0'} \\ &= -\frac{g^2}{2}\int\frac{d^3k}{(2\pi)^3}\int\frac{dk_0}{2\pi}\int\frac{dk_0'}{2\pi}\rho(k_0,|k|)\rho(k_0',|\mathbf{k}+\mathbf{q}|)\frac{1+n(k_0)+n(k_0')}{\omega+k_0+k_0'} \end{aligned} \tag{7}$$

$$\int\frac{d^4q}{(2\pi)^4}\frac{\partial n(k_0)}{\partial T}\mathrm{Re}\Pi(\omega,q)\mathrm{Im}D(\omega,q) =$$

$$= -\frac{g^2}{2\cdot 2}\int\frac{d^4q}{(2\pi)^4}\int\frac{d^k}{(2\pi)^4}\int\frac{d^4k'}{2\pi}\delta^{(3)}(\mathbf{q}+\mathbf{k}+\mathbf{k}')\rho(q)\rho(k)\rho(k')\partial_T n(q_0)\left[1+n(k_0)+n(k_0')\right]\frac{1}{q_0+k_0+k_0'} \tag{8}$$

This proves the cancellation of $\mathcal{S}'$ for the scalar theory with cubic selfinteraction in the 2-loop approximation (sunset diagram) for the $\Phi-$ functional.

This cancellation holds as well for the pressure and the density!

For the pressure we obtain

$$p(T) = -\int\frac{d^4q}{(2\pi)^4}n(q_0)\left[\delta(q)-\sin\delta(q)\cos\delta(q)\right] = -\int\frac{d^4q}{(2\pi)^4}T\ln\left(1-\mathrm{e}^{-q_0/T}\right)\frac{\partial\delta(q)}{\partial q_0}\,2\sin^2\delta(q) \tag{9}$$

Note that in the approximation $\delta(q_0,q) = -\arctan[\omega\gamma/(q_0^2-\omega^2)]$ the "spectral distribution" does not correspond to a Lorentzian (Breit-Wigner) function as naïvely expected, but to a "squared Lorentzian"

$$\frac{q_0(\omega\gamma)^3}{[(q_0^2-\omega^2)^2+(\omega\gamma)^2]^2} \tag{10}$$

See, e.g., Vanderheyden & Baym (1998); Morozov & Röpke, Ann. Phys. 324 (2009) 1261

Approximately selfconsistent HTL resumm. QCD

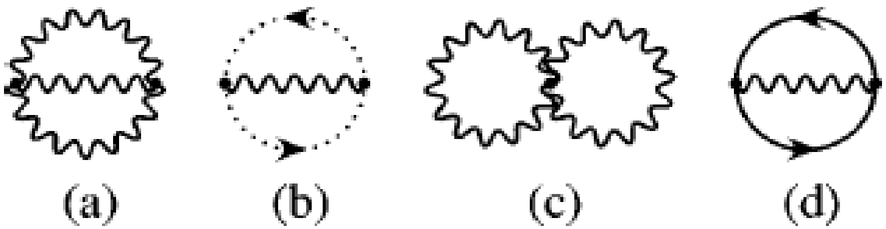
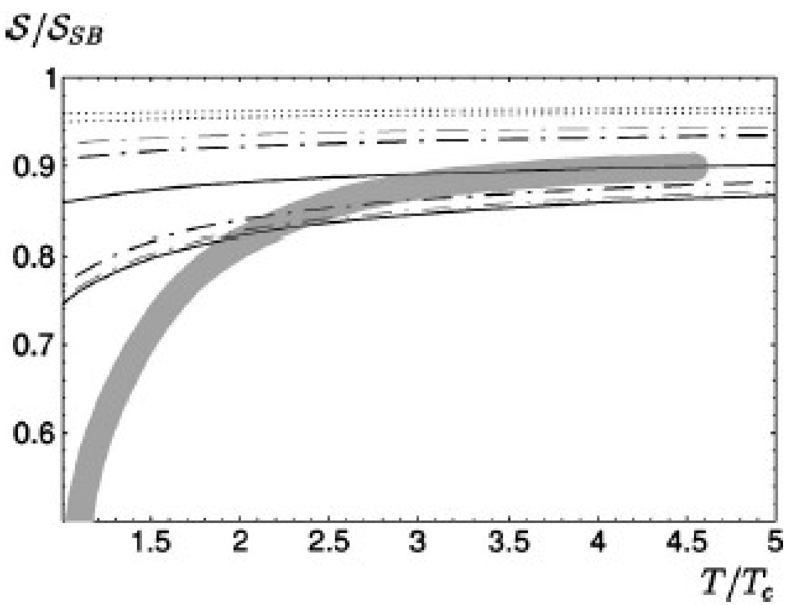


FIG. 3. Diagrams for Φ at 2-loop order in QCD. Wiggly, plain, and dotted lines refer respectively to gluons, quarks, and ghosts.



In ghost-free gauge, HTL resummed QCD thermodyn.

$$\mathcal{S}_2 = -\frac{g^2 N_g T}{48}\left\{\frac{4N+5N_f}{3}T^2 + \frac{3N_f}{\pi^2}\mu^2\right\},$$

$$\mathcal{N}_2 = -\frac{g^2 \mu N_g N_f}{16\pi^2}\left(T^2 + \frac{\mu^2}{\pi^2}\right),$$

$$P_2 = -\frac{g^2 N_g}{32}\left\{\frac{4N+5N_f}{18}T^4 + \frac{N_f}{\pi^2}\mu^2 T^2 + \frac{N_f}{2\pi^4}\mu^4\right\}$$

J.-P. Blaizot, E. Iancu, A. Rebhan, Phys. Rev. D 63 (2001) 065003

Generalized Optical Theorems

See derivations for T-matrices by R. Zimmermann & H. Stolz, pss (b) 131, 151 (1985)
Here we consider the analogue of $T^{-1} = V^{-1} - G_2^0$, the propagator $S^{-1} = G^{-1} – \Pi$, G real, static

Assuming the inverse exists we have two identities: $S = S^* S^{*-1} S$ and $S^* = S^* S^{-1} S$

$$\begin{aligned} S_R + iS_I &= S^*(S_R^{-1} - iS_I^{-1})S \,, \\ S_R - iS_I &= S^*(S_R^{-1} + iS_I^{-1})S \,. \end{aligned} \longrightarrow \begin{aligned} S_R &= S^* S_R^{-1} S \,, \\ S_I &= -S^* S_I^{-1} S \,, \end{aligned}$$

With definition $S^{-1} = G^{-1} – \Pi$ follows off-shell optical theorem: $S_I = S^* \Pi_I S = S \Pi_I S^*$

Using the fact that G is a real constant, we have: $(S_R^{-1})' = -\Pi_R'$ and $S_I^{-1} = -\Pi_I$

$$\begin{aligned} S_R' &= S^{*\prime} S_R^{-1} S + S^*(S_R^{-1})' S + S^* S_R^{-1} S' \\ &= S^{*\prime}(\underbrace{S_R^{-1} + iS_I^{-1}}_{S^{-1}} - iS_I^{-1})S + S^*(S_R^{-1})'S + S^*(\underbrace{S_R^{-1} - iS_I^{-1}}_{S^{*-1}} + iS_I^{-1})S' \\ &= \underbrace{S^{*\prime} + S'}_{2S_R'} - iS^{*\prime} S_I^{-1} S + iS^* S_I^{-1} S' + S^*(S_R^{-1})' S \\ &= S^* \Pi_R' S - iS^{*\prime} \Pi_I S + iS^* \Pi_I S' \,, \end{aligned}$$

Derivative optical theorem:

$$S_R' \Pi_I = \underbrace{S^* \Pi_R' S \Pi_I}_{\Pi_R' S_I} + \underbrace{iS^* \Pi_I S' \Pi_I - iS^{*\prime} \Pi_I S \Pi_I}_{2\,\mathrm{Im}[\Pi_I S \Pi_I S^{*\prime}]} \,, \longrightarrow \boxed{S_R' \Pi_I - \Pi_R' S_I = 2\,\mathrm{Im}\,[\Pi_I S \Pi_I S^{*\prime}]}$$

Wir fahren aufs Land
Eva Scherbarth
Ravensburger

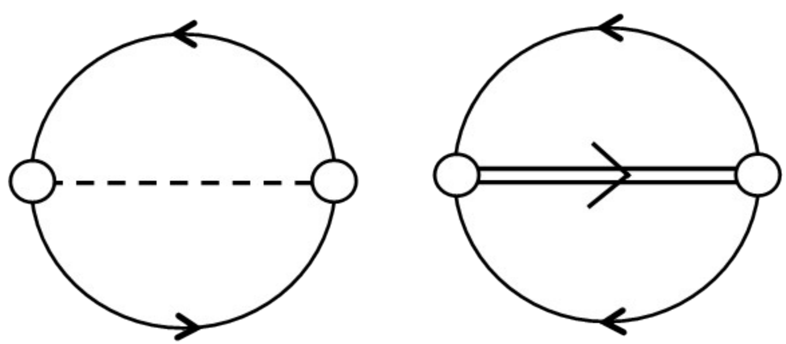
Φ-derivable Q-M-D PNJL model, 2-loop approximation

$$\Omega = \frac{1}{2}\frac{T}{V}\sum_{i=Q,M,D} c_i \,\mathrm{Tr}\left\{\ln\left[S_i^{-1}\right] + \left[S_i \Pi_i\right]\right\} + \Phi\left[S_Q, S_M, S_D\right] \ ,$$

$$S_i^{-1}(\mathrm{i}z_n, \mathbf{q}) = S_{i,0}^{-1}(\mathrm{i}z_n, \mathbf{q}) - \Pi_i(\mathrm{i}z_n, \mathbf{q}) \ , \qquad \frac{\delta\Omega}{\delta S_i} = 0 \ , \quad \text{if} \quad \Pi_i = \frac{\delta\Omega}{\delta S_i} \ .$$

$$\Omega \ = \ \frac{1}{2}T \sum_{i=Q,M,D} \int \frac{d^3q}{(2\pi)^3} \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} f_i(\omega)\, \mathrm{Tr}\left\{\mathrm{Im}\ln\left[S_i^{-1}\right] + \left[\mathrm{Re}\, S_i\, \mathrm{Im}\,\Pi_i\right]\right\} + \tilde{\Omega}$$

$$\tilde{\Omega} \ = \ \Phi\left[S_Q, S_M, S_D\right] - \frac{1}{2}T \sum_{i=Q,M,D} \int \frac{d^3q}{(2\pi)^3} \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} f_i(\omega)\, \mathrm{Tr}\left\{\left[\mathrm{Im}\, S_i\, \mathrm{Re}\,\Pi_i\right]\right\} \ ,$$

$\Phi =$ 

$$\mathcal{S} = -\frac{\partial\Omega}{\partial T} = \sum_i \mathcal{S}_i + \tilde{\mathcal{S}}$$

$$\mathcal{N} = -\frac{\partial\Omega}{\partial\mu} = \sum_i \mathcal{N}_i + \tilde{\mathcal{N}} \ .$$

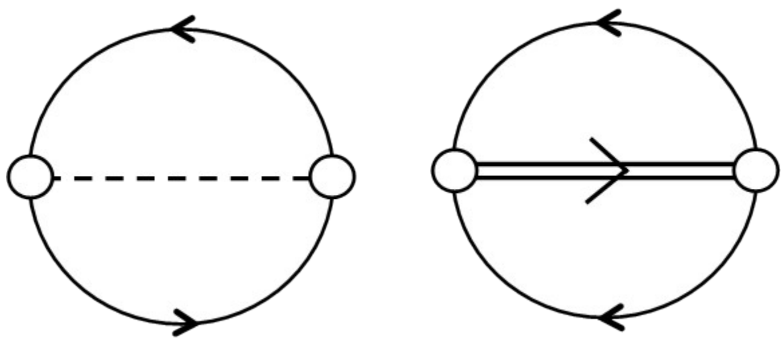
Φ-derivable Q-M-D PNJL model, 2-loop approximation

$$\Omega = \frac{1}{2}\frac{T}{V}\sum_{i=Q,M,D} c_i \,\mathrm{Tr}\left\{\ln\left[S_i^{-1}\right] + \left[S_i \Pi_i\right]\right\} + \Phi\left[S_Q, S_M, S_D\right] \;,$$

$$S_i^{-1}(\mathrm{i}z_n, \mathbf{q}) = S_{i,0}^{-1}(\mathrm{i}z_n, \mathbf{q}) - \Pi_i(\mathrm{i}z_n, \mathbf{q}) \;, \qquad \frac{\delta\Omega}{\delta S_i} = 0 \;, \quad \text{if} \quad \Pi_i = \frac{\delta\Omega}{\delta S_i} \;.$$

$$\Omega = \frac{1}{2}T\sum_{i=Q,M,D}\int\frac{d^3q}{(2\pi)^3}\int_{-\infty}^{\infty}\frac{d\omega}{2\pi} f_i(\omega)\,\mathrm{Tr}\left\{\mathrm{Im}\ln\left[S_i^{-1}\right] + \left[\mathrm{Re}\,S_i\,\mathrm{Im}\,\Pi_i\right]\right\} + \tilde{\Omega}$$

$$\tilde{\Omega} = \Phi\left[S_Q, S_M, S_D\right] - \frac{1}{2}T\sum_{i=Q,M,D}\int\frac{d^3q}{(2\pi)^3}\int_{-\infty}^{\infty}\frac{d\omega}{2\pi} f_i(\omega)\,\mathrm{Tr}\left\{\left[\mathrm{Im}\,S_i\,\mathrm{Re}\,\Pi_i\right]\right\} \;,$$

$\Phi =$ 

$$\mathcal{S} = -\frac{\partial\Omega}{\partial T} = \sum_i \mathcal{S}_i + \tilde{\mathcal{S}}$$

$$\mathcal{N} = -\frac{\partial\Omega}{\partial\mu} = \sum_i \mathcal{N}_i + \tilde{\mathcal{N}} \,.$$

Φ-derivable Q-M-D PNJL model, 2-loop approximation

$$\left(\mathrm{Im}\ln S^{-1}\right)' = -\,\mathrm{Im}\,(S\Pi') = \underbrace{S_R'\Pi_I - S_I\Pi_R'}_{2\,\mathrm{Im}(S\Pi_I S^{*\prime}\Pi_I)} - \underbrace{(\Pi_I S_R' + S_R\Pi_I')}_{(\Pi_I S_R)'} \;,$$

Use optical theorems ...

$$S\Pi_I \;=\; \sin\delta\, \mathrm{e}^{\mathrm{i}\delta} \;,\quad S^{*\prime}\Pi_I = -\mathrm{i}\delta'\sin\delta\,\mathrm{e}^{-\mathrm{i}\delta}\;,\quad 2\,\mathrm{Im}(S\Pi_I S^{*\prime}\Pi_I) = -2\delta'\sin^2\delta\;.$$

Generalized Beth-Uhlenbeck EoS

$$\Omega \;=\; -\sum_{i=Q,M,D} d_i \int \frac{\mathrm{d}^3q}{(2\pi)^3} \int_{-\infty}^{\infty} \frac{\mathrm{d}\omega}{2\pi}\, T\ln[1-\mathrm{e}^{-(\omega-\mu_i)/T}]\,\sin^2\delta_i(\omega,\mathbf{q})\frac{\partial\delta_i(\omega,\mathbf{q})}{\partial\omega}$$

Effect of the sin^2 term ... example: Breit-Wigner ...

$$\delta_i(\omega) = -\arctan\left[\frac{\omega_i\Gamma_i}{\omega^2-\omega_i^2}\right]\;,\quad \frac{\partial\delta_i(\omega)}{\partial\omega} = \frac{2\omega\omega_i\Gamma_i}{(\omega^2-\omega_i^2)^2+\omega_i^2\Gamma_i^2}\;,$$

$$\sin^2\delta_i(\omega)\frac{\partial\delta_i(\omega)}{\partial\omega} \;=\; \frac{2\omega(\omega_i\Gamma_i)^3}{[(\omega^2-\omega_i^2)^2+\omega_i^2\Gamma_i^2]^2}\;.$$

"Squared Lorentzian" ...
Vanderheyden & Baym (1998)
Morozov & Roepke (2009)

1. Cluster expansion in the 2PI formalism

- $\Phi-$ derivable approach to the grand canonical thermodynamic potential
  [Baym, Phys. Rev. 127 (1962) 139]

$$J = -\mathrm{Tr}\left\{\ln(-G_1)\right\} - \mathrm{Tr}\{\Sigma_1 G_1\} + \mathrm{Tr}\left\{\ln(-G_2)\right\} + \mathrm{Tr}\{\Sigma_2 G_2\} + \Phi[G_1, G_2]$$

with full propagators:
$G_1^{-1}(1,z) = z - E_1(p_1) - \Sigma_1(1,z)$; $G_2^{-1}(12,1'2',z) = z - E_1(p_1) - E_2(p_2) - \Sigma_2(12,1'2',z)$
and selfenergies

$$\Sigma_1(1,1') = \frac{\delta\Phi}{\delta G_1(1,1')}\ ; \Sigma_2(12,1'2',z) = \frac{\delta\Phi}{\delta G_2^{-1}(12,1'2',z)}.$$

Because of stationarity equivalent to

$$n = -\frac{1}{\Omega}\frac{\partial J}{\partial\mu} = \frac{1}{\Omega}\sum_1 \int_{-\infty}^{\infty}\frac{d\omega}{\pi} f_1(\omega) S_1(1,\omega)\,,$$

(baryon number conservation)

- Generalization to A-nucleon clusters in nuclear matter

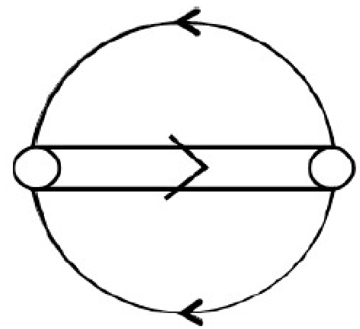
$$\Omega \;=\; \sum_A (-1)^A \left[\mathrm{Tr}\ln\left(-G_A^{-1}\right) + \mathrm{Tr}\left(\Sigma_A\, G_A\right)\right] + \Phi\,,$$

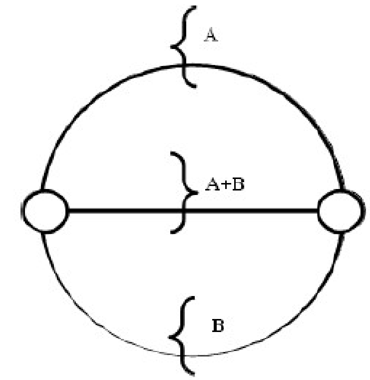
$$G_A^{-1} \;=\; G_A^{(0)-1} - \Sigma_A\,,\ \ \Sigma_A(1\ldots A, 1'\ldots A', z_A) = \frac{\delta\Phi}{\delta G_A(1\ldots A, 1'\ldots A', z_A)}\,.$$

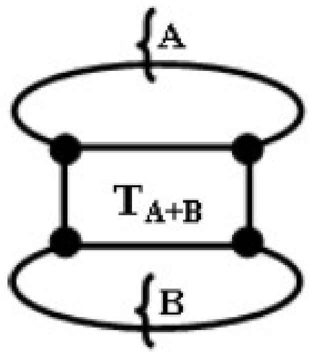
1. Cluster expansion in the 2PI formalism

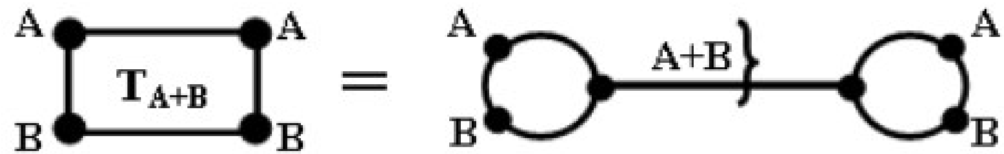
A) Choice of the Φ-functional:

- 2-particle irreducible diagrams
- closed 2-loop diagram involving 3 cluster propagators (A, B, A+B) and 2 vertices
- equivalent to 1 T-matrix + 2 propagators









B) Ansatz for thermodynamic potential:

$$\Omega = \sum_{A} (-1)^{A} \left[\mathrm{Tr} \ln \left(-G_A^{-1} \right) + \mathrm{Tr} \left(\Sigma_A \, G_A \right) \right] + \sum_{A,B} \Phi[G_A, G_B, G_{A+B}] \, ,$$

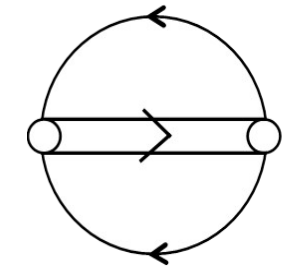
$$G_A^{-1} = G_A^{(0)^{-1}} - \Sigma_A \, , \; \Sigma_A(1 \ldots A, 1' \ldots A', z_A) = \frac{\delta \Phi}{\delta G_A(1 \ldots A, 1' \ldots A', z_A)} \, .$$

C) Check: conservation laws, e.g.:
(correspondence to GF formalism)

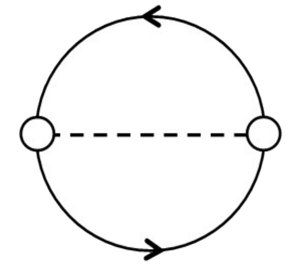
$$n = -\frac{1}{V} \frac{\partial \Omega}{\partial \mu} = \frac{1}{V} \sum_{1} \int_{-\infty}^{\infty} \frac{d\omega}{\pi} f_1(\omega) A_1(1, \omega)$$

Cluster virial expansion in the 2PI formalism, Examples:

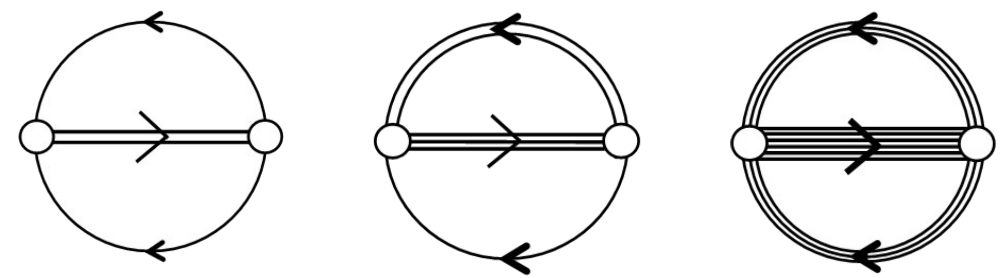
A) Deuterons in nuclear matter:



B) Mesons in quark matter:

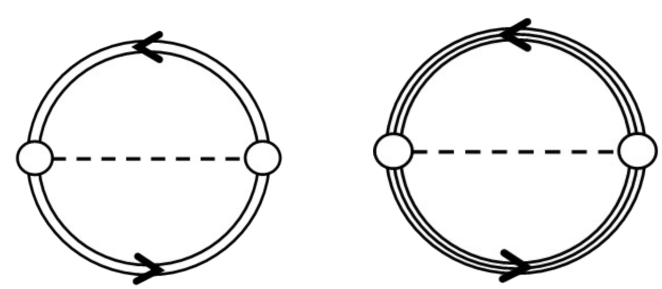


C) Nucleons in quark matter:



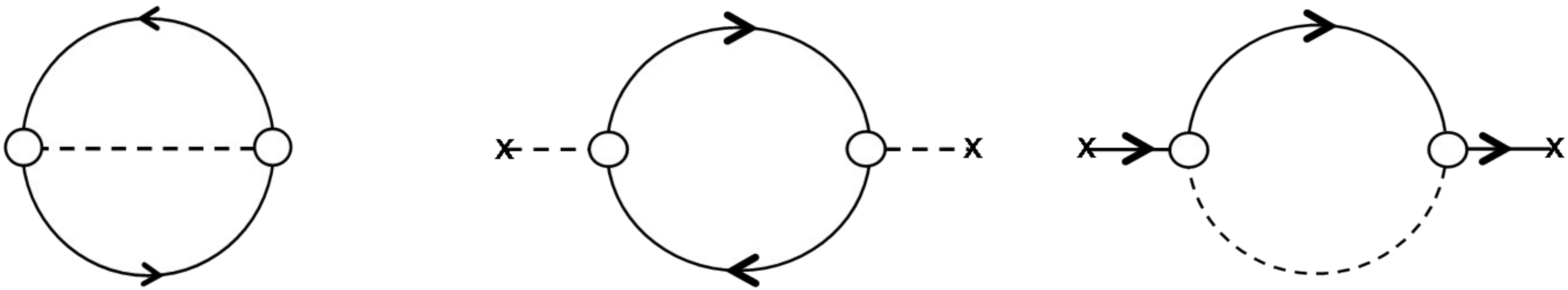
D) Nucleons and mesons (hadron resonance gas) in quark matter:

B) + C) +



Example B: Mesons in quark matter

Φ-functional | Meson selfenergy (RPA) | Quark selfenergy



$$T_M^{-1}(q,\omega+i\eta) = G_S^{-1} - \Pi_M(q,\omega+i\eta) = |T_M(q,\omega)|^{-1}\mathrm{e}^{-i\delta_M(q,\omega)}\ , \quad \delta_M(q,\omega) = \arctan(\Im T_M/\Re T_M)$$

$$\Omega = \Omega_{\mathrm{MF}} + \Omega_M\ , \qquad \sigma_{\mathrm{MF}} = 2N_fN_cG_S\int\frac{d^3p}{(2\pi)^3}\frac{m}{E_p}[1-f_-(E_p)-f_+(E_p)]\ ,$$

$$\Omega_{\mathrm{MF}} = \frac{\sigma_{\mathrm{MF}}^2}{4G_S} - 2N_cN_f\int\frac{d^3p}{(2\pi)^3}\left[E_p + T\ln\left(1+\mathrm{e}^{-(E_p-\Sigma_+-\mu)/T}\right) + T\ln\left(1+\mathrm{e}^{-(E_p+\Sigma_-+\mu)/T}\right)\right]\ ,$$

$$\Omega_M = d_M\int\frac{d^3k}{(2\pi)^3}\int\frac{d\omega}{2\pi}\left\{\omega + 2T\ln\left[1-\mathrm{e}^{-\omega/T}\right]2\sin^2\delta_M(k,\omega)\,\frac{\delta_M(k,\omega)}{d\omega}\right\}\ ,$$

$$\Sigma_M(0,p_0) = d_M\int\frac{d^4q}{(2\pi)^4}\pi\rho_M(\mathbf{q},q_0)\left\{\frac{(\gamma_0+m/E_q)[1+g(q_0)-f_-(E_q)]}{q_0-p_0+E_q-\mu-i\eta} + \frac{(\gamma_0-m/E_q)[g(q_0)+f_+(E_q)]}{q_0-p_0-E_q-\mu-i\eta}\right\}\ ,$$

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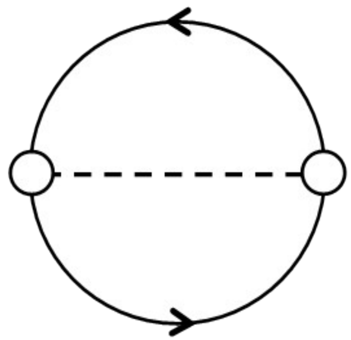
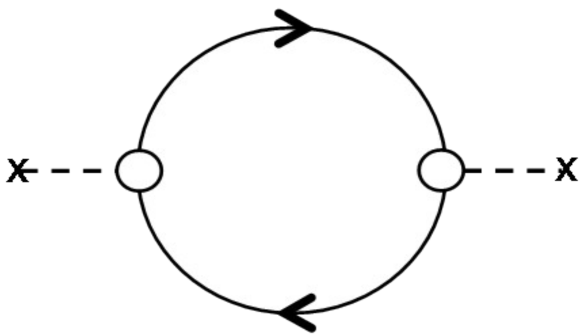
Example B: Mesons in quark matter

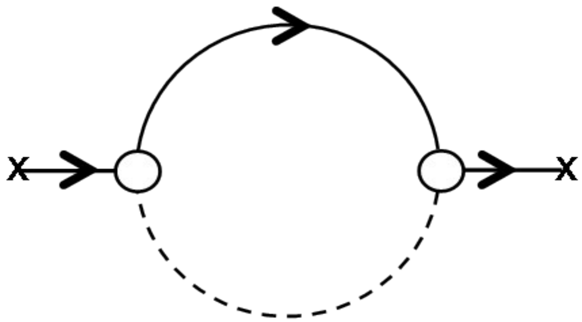
Φ-functional



Meson selfenergy (RPA)



Quark selfenergy



$$T_M^{-1}(q,\omega+i\eta) = G_S^{-1} - \Pi_M(q,\omega+i\eta) = |T_M(q,\omega)|^{-1}\mathrm{e}^{-i\delta_M(q,\omega)}\ , \quad \delta_M(q,\omega) = \arctan(\Im T_M/\Re T_M)$$

$$\Omega = \Omega_{\mathrm{MF}} + \Omega_M\ , \qquad \sigma_{\mathrm{MF}} = 2N_f N_c G_S \int \frac{d^3p}{(2\pi)^3}\frac{m}{E_p}[1 - f_-(E_p) - f_+(E_p)]\ ,$$

$$\Omega_{\mathrm{MF}} = \frac{\sigma_{\mathrm{MF}}^2}{4G_S} - 2N_cN_f \int \frac{d^3p}{(2\pi)^3}\left[E_p + T\ln\left(1+\mathrm{e}^{-(E_p-\Sigma_+-\mu)/T}\right) + T\ln\left(1+\mathrm{e}^{-(E_p+\Sigma_-+\mu)/T}\right)\right]\ ,$$

$$\Omega_M = d_M \int \frac{d^3k}{(2\pi)^3}\int \frac{d\omega}{2\pi}\left\{\omega + 2T\ln\left[1-\mathrm{e}^{-\omega/T}\right] \boxed{2\sin^2\delta_M(k,\omega)} \frac{\delta_M(k,\omega)}{d\omega}\right\},$$

new !

$$\Sigma_M(0,p_0) = d_M \int \frac{d^4q}{(2\pi)^4}\pi\rho_M(\mathbf{q},q_0)\left\{\frac{(\gamma_0+m/E_q)[1+g(q_0)-f_-(E_q)]}{q_0-p_0+E_q-\mu-i\eta} + \frac{(\gamma_0-m/E_q)[g(q_0)+f_+(E_q)]}{q_0-p_0-E_q-\mu-i\eta}\right\},$$

D. Blaschke, M. Buballa, A. Dubinin, G. Roepke, D. Zablocki: Ann. Phys. 348 (2014) 228
D. Blaschke, PoS Baldin ISHEPXXII (2015) 113; arxiv:1502.06279

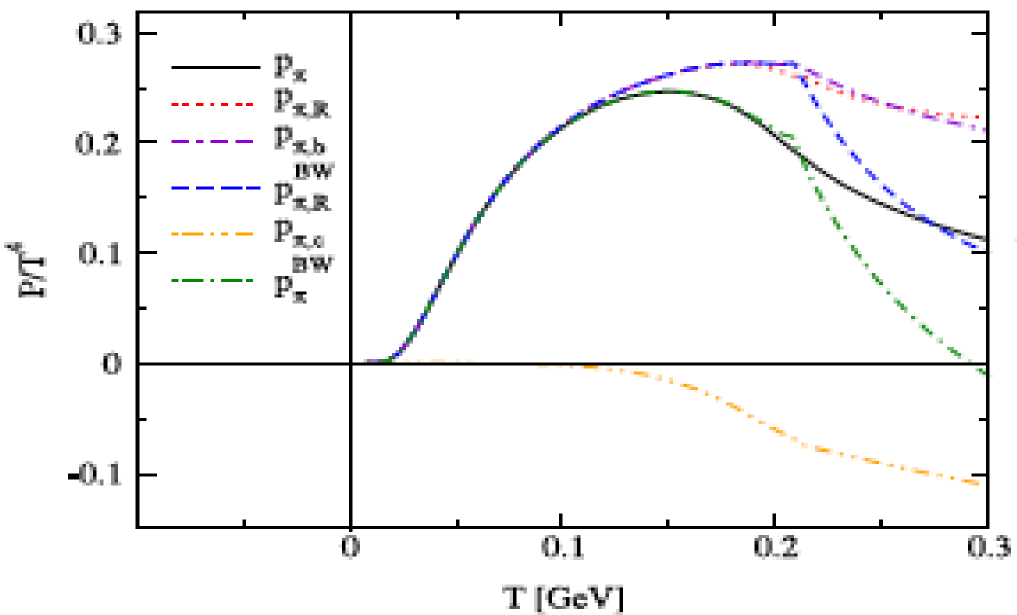
Example B: Mesons in quark matter

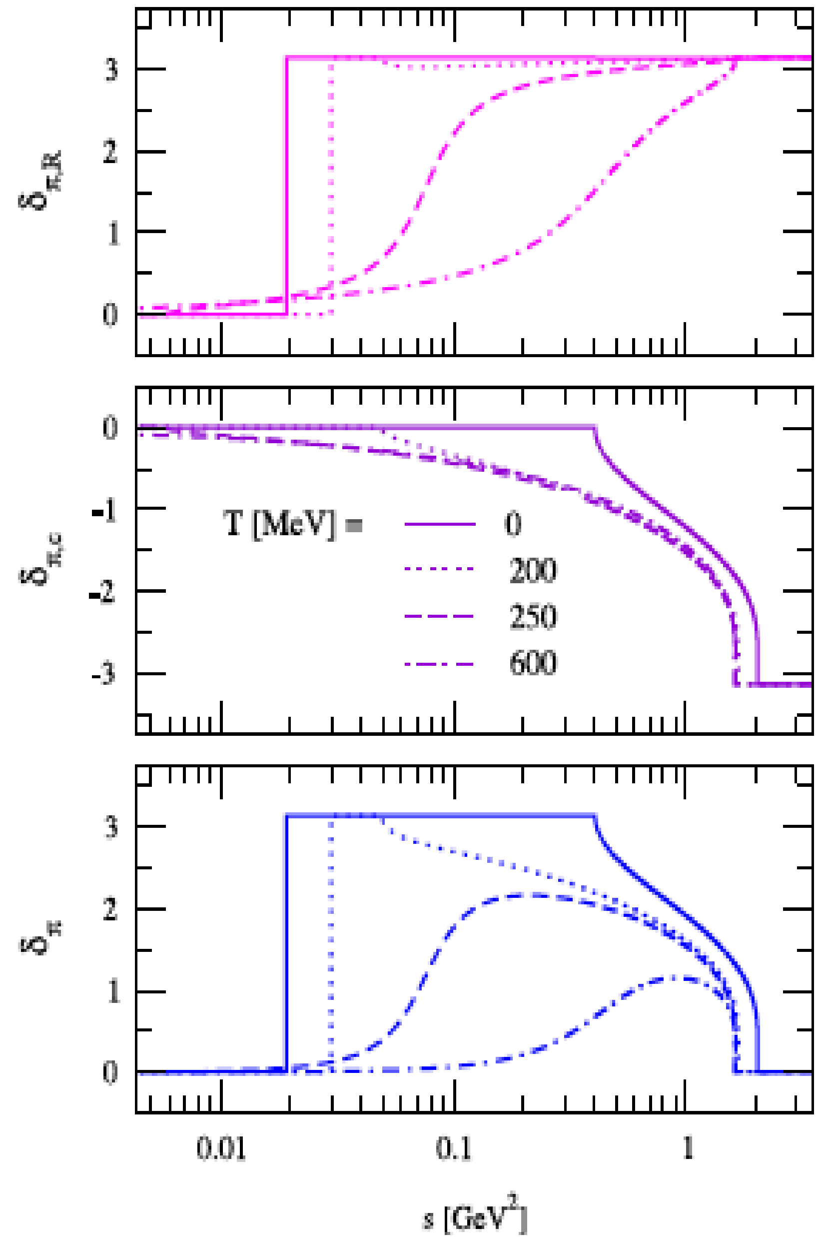
$$\Omega_X(T,\mu) = -d_X \int \frac{d^3q}{(2\pi)^3} \int_{-\infty}^{\infty} \frac{d\omega}{2\pi}\, n_X^-(\omega)\,\delta_X(\omega,\mathbf{q}),$$

$$\delta_\pi \doteq \delta_{\pi,c} + \delta_{\pi,R}$$

$$\int_0^\infty d\omega \frac{1}{\pi}\frac{d\delta_X(\omega;T)}{d\omega} = 0 = \underbrace{\int_0^{\omega_{thr}(T)} d\omega \frac{1}{\pi}\frac{d\delta_X(\omega;T)}{d\omega}}_{n_{B,X}(T)} + \underbrace{\frac{1}{\pi}\int_{\omega_{thr}(T)}^{\infty} d\omega \frac{d\delta_X(\omega;T)}{d\omega}}_{\frac{1}{\pi}[\delta_X(\infty;T)-\delta_X(\omega_{thr};T)]},$$

$$p_\pi(T) = -d_\pi T \int \frac{d^3q}{(2\pi)^3} \int_0^\infty \frac{d\omega}{\pi} \ln\left(1 - e^{-\omega/T}\right) \frac{d\delta_\pi(\omega,\mathbf{q})}{d\omega}$$





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Example B*: Mesons+diquarks in quark matter

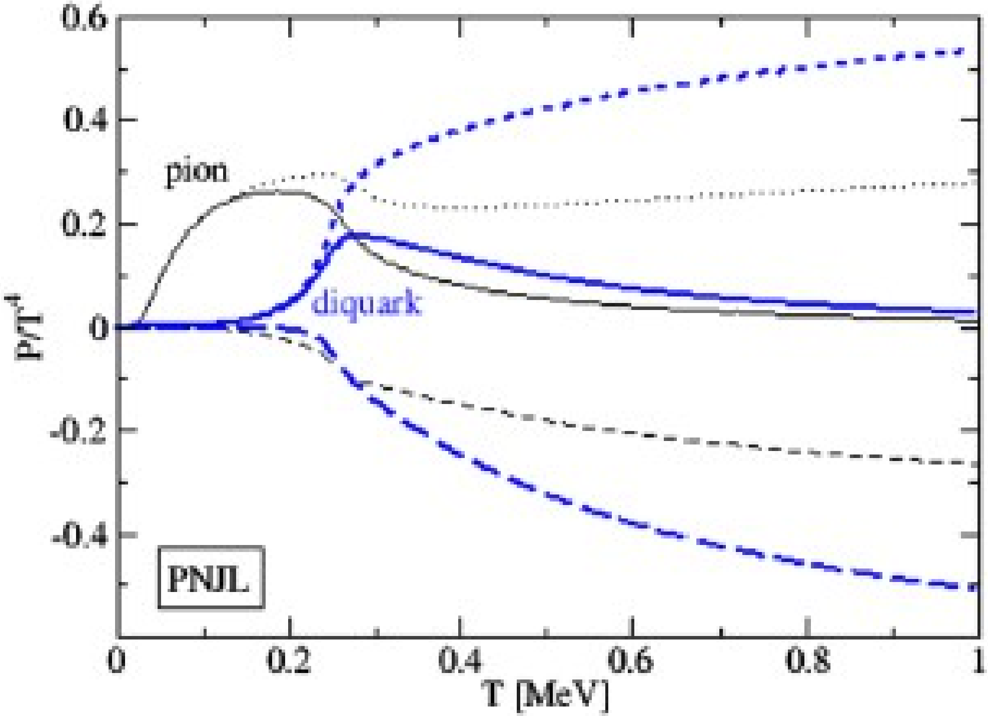
$$\Omega_{\mathrm{Q}} = -\frac{2N_c N_f}{3}\int \frac{dp}{2\pi^2}\frac{p^4}{E_p}[f_\Phi^+(E_p) + f_{\bar\Phi}^-(E_p)],\ \ f_\Phi^+(E_p) = \frac{(\bar\Phi + 2\Phi Y)Y + Y^3}{1 + 3(\bar\Phi + \Phi Y)Y + Y^3},\ \ Y = \mathrm{e}^{-(E_p-\mu)/T}$$

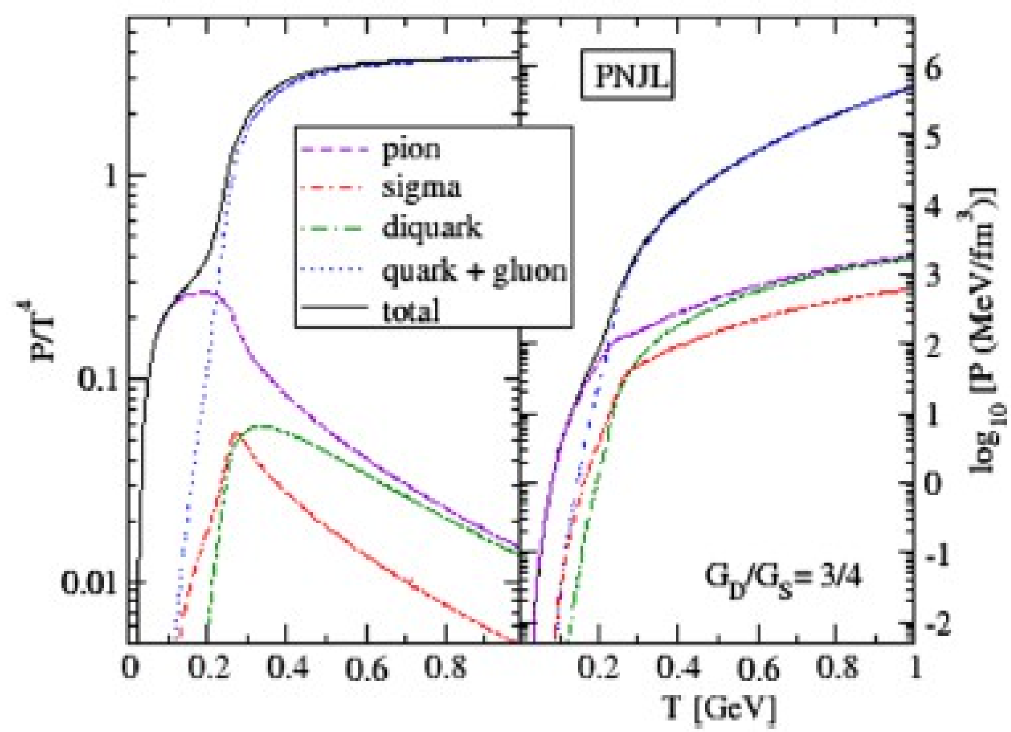
$$X = \mathrm{e}^{-(\omega-2\mu)/T}$$

$$\Omega_{\mathrm{D}} = -3\int \frac{d^3p}{(2\pi)^3}\int \frac{d\omega}{2\pi}[g_\Phi^+(\omega) + g_{\bar\Phi}^-(\omega)]\delta_D(\omega),\ \ g_\Phi^+(\omega) = \frac{(\Phi - 2\bar\Phi X)X + X^3}{1 - 3(\Phi - \bar\Phi X)X - X^3}$$

Suppression of colored states by Polyakov-loop Φ

Confinement: Φ=0





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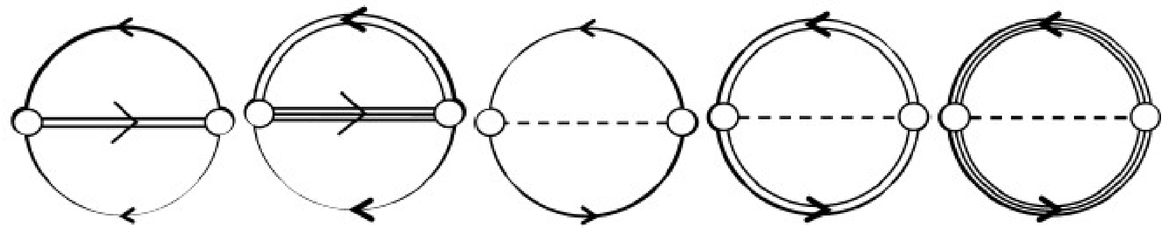
Example D: Hadron resonance gas – effect. model

Φ-functional:



Selfenergies: