# QCD phase diagram in the vector meson extended PQM model

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12.04.2017

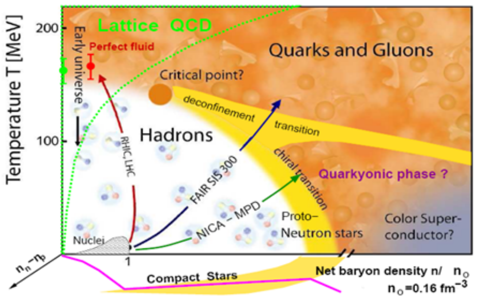
*Mini-Workshop on Simulations of HIC for NICA energies*
*Dubna*

Based on: P. Kovács, Zs. Szép, Gy. Wolf, PRD93 (2016) 114014

# Overview

# QCD phase diagram

Phase diagram in the $T - \mu_\mathrm{B} - \mu_\mathrm{I}$ space




- At $\mu_\mathrm{B} = 0$ $T_c = 151$ MeV Y. Aoki, *et al.*, PLB **643**, 46 (2006)
- Is there a CP? ($T_{CP}$=162 MeV, $\mu_{CP}$=360 MeV, Fodor-Katz)
- At $T = 0$ in $\mu_\mathrm{B}$ where is the phase boundary?

Details of the phase diagram are heavily studied theoretically (Lattice, EFT), and experimentally (RHIC, LHC, FAIR, NICA)

# Motivation

- At $\mu = 0$ we know the properties of strong interactions from the lattice in theoretical side and from STAR/PHENIX and from ALICE in the experimental side. On the other hand, for $\mu >> 0$ at the moment no theory and no experiment provide reasonable information.
- What is the order of phase transition on the T=0 line? Is there a CEP?
- Equation of state for neutron stars.
- How the masses change in medium?

Idea

- Build an effective model having the right global symmetry pattern.
- Compare the thermodynamics of the model with lattice at $\mu = 0$
- Extrapolate to high $\mu$.

# Effective models

Since QCD is very hard to solve $\longrightarrow$ low energy effective models were set up $\longrightarrow$ reflecting the global symmetries of QCD

- Nambu-Jona-Lasinio model (+Kobayashi-Maskawa-t'Hooft)
- Chiral perturbation theory
- Linear and nonlinear (it does not contain degrees of freedom relevant at high T) sigma model
- To study the phase diagram, we introduced the constituent quarks
- For mimicing confinement, we add the Polyakov loops.

extended Polyakov-Quark-Meson model

Similar model e.g.: Pisarski,Skokov, Phys.Rev. D94 (2016) 034015

# Chiral symmetry

If the quark masses are zero (chiral limit) $\Longrightarrow$ QCD invariant under the following global transformation (chiral symmetry):

$q_L = (1-\gamma_5)/2q, \quad q_R = (1+\gamma_5)/2q$ only the mass term mixes

$U(3)_V q = exp(-i\alpha t)q \quad U(3)_A q = exp(-i\beta\gamma_5 t)q$

$U(3)_L \times U(3)_R \simeq U(3)_V \times U(3)_A =$
$SU(3)_V \times SU(3)_A \times U(1)_V \times U(1)_A$

by any quark mass $SU(3)_V \times U(1)_V \times U(1)_A$ remains

$U(1)_V$ term $\longrightarrow$ baryon number conservation

$U(1)_A$ term $\longrightarrow$ broken through axial anomaly

$SU(3)_V$ term $\longrightarrow$ broken down to $SU(2)_V$ if $m_u = m_d \neq m_s$

$\longrightarrow$ totally broken if $m_u \neq m_d \neq m_s$ (in nature)

# Meson fields - pseudoscalar and scalar meson nonets

$$\Phi_{PS} = \sum_{i=0}^{8} \pi_i T_i = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{\eta_N+\pi^0}{\sqrt{2}} & \pi^+ & K^+ \\ \pi^- & \frac{\eta_N-\pi^0}{\sqrt{2}} & K^0 \\ K^- & \bar{K}^0 & \eta_S \end{pmatrix} (\sim \bar{q}_i \gamma_5 q_j)$$

$$\Phi_S = \sum_{i=0}^{8} \sigma_i T_i = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{\sigma_N+a_0^0}{\sqrt{2}} & a_0^+ & K_S^+ \\ a_0^- & \frac{\sigma_N-a_0^0}{\sqrt{2}} & K_S^0 \\ K_S^- & \bar{K}_S^0 & \sigma_S \end{pmatrix} (\sim \bar{q}_i q_j)$$

Particle content:

Pseudoscalars: $\pi(138), K(495), \eta(548), \eta'(958)$

Scalars: $a_0(980 \text{ or } 1450), K_0^\star(800 \text{ or } 1430)$,

$(\sigma_N, \sigma_S)$ : 2 of $f_0(500, 980, 1370, 1500, 1710)$

# Structure of scalar mesons

| | Mass (MeV) | width (MeV) | decays |
|---|---|---|---|
| $A_0(980)$ | $980 \pm 20$ | $50 - 100$ | $\pi\pi$ dominant |
| $A_0(1450)$ | $1474 \pm 19$ | $265 \pm 13$ | $\pi\eta$, $\pi\eta'$, $K\bar{K}$ |
| $K_s(800) = \kappa$ | $682 \pm 29$ | $547 \pm 24$ | $K\pi$ |
| $K_s(1430)$ | $1425 \pm 50$ | $270 \pm 80$ | $K\pi$ dominant |
| $f_0(500) = \sigma$ | 400–550 | $400 - 700$ | $\pi\pi$ dominant |
| $f_0(980)$ | $980 \pm 20$ | $40 - 100$ | $\pi\pi$ dominant |
| $f_0(1370)$ | 1200–1500 | $200 - 500$ | $\pi\pi \approx 250$, $K\bar{K} \approx 150$ |
| $f_0(1500)$ | $1505 \pm 6$ | $109 \pm 7$ | $\pi\pi \approx 38$, $K\bar{K} \approx 9.4$ |
| $f_0(1710)$ | $1722 \pm 6$ | $135 \pm 7$ | $\pi\pi \approx 30$, $K\bar{K} \approx 71$ |

Possible scalar states: $\bar{q}q$, $\bar{q}\bar{q}qq$, meson-meson molecules, glueballs

pseudoscalar nonet: $\pi, K, \eta, \eta'$, scalar nonet: $A_0$, $K_0$, 2 $f_0$

multiquark states: $f_0(980)$,$A_0(980)$ $f_0(600)$, $K_0(800)$ ???

meson-meson bound state $(K\bar{K})$: $f_0(980)$ ???

glueballs: $f_0(1500)$ (weak coupling to $\gamma\gamma$), $f_0(1710)$ ???

# Included fields - vector meson nonets

$$V^\mu = \sum_{i=0}^{8} \rho_i^\mu T_i = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{\omega_N+\rho^0}{\sqrt{2}} & \rho^+ & K^{\star+} \\ \rho^- & \frac{\omega_N-\rho^0}{\sqrt{2}} & K^{\star 0} \\ K^{\star-} & \bar{K}^{\star 0} & \omega_S \end{pmatrix}^\mu$$

$$A^\mu = \sum_{i=0}^{8} b_i^\mu T_i = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{f_{1N}+a_1^0}{\sqrt{2}} & a_1^+ & K_1^+ \\ a_1^- & \frac{f_{1N}-a_1^0}{\sqrt{2}} & K_1^0 \\ K_1^- & \bar{K}_1^0 & f_{1S} \end{pmatrix}^\mu$$

Particle content:

Vector mesons: $\rho(770), K^\star(894), \omega_N = \omega(782), \omega_S = \phi(1020)$

Axial vectors: $a_1(1230), K_1(1270), f_{1N}(1280), f_{1S}(1426)$

# Lagrangian (2/1)

$$
\begin{aligned}
\mathcal{L}_{\text{Tot}} = {} & \mathrm{Tr}[(D_\mu \Phi)^\dagger (D_\mu \Phi)] - m_0^2 \mathrm{Tr}(\Phi^\dagger \Phi) - \lambda_1 [\mathrm{Tr}(\Phi^\dagger \Phi)]^2 - \lambda_2 \mathrm{Tr}(\Phi^\dagger \Phi)^2 \\
& - \frac{1}{4} \mathrm{Tr}(L_{\mu\nu}^2 + R_{\mu\nu}^2) + \mathrm{Tr}\left[\left(\frac{m_1^2}{2} + \Delta\right)(L_\mu^2 + R_\mu^2)\right] + \mathrm{Tr}[H(\Phi + \Phi^\dagger)] \\
& + c_1 (\det \Phi + \det \Phi^\dagger) + i \frac{g_2}{2} (\mathrm{Tr}\{L_{\mu\nu}[L^\mu, L^\nu]\} + \mathrm{Tr}\{R_{\mu\nu}[R^\mu, R^\nu]\}) \\
& + \frac{h_1}{2} \mathrm{Tr}(\Phi^\dagger \Phi) \mathrm{Tr}(L_\mu^2 + R_\mu^2) + h_2 \mathrm{Tr}[(L_\mu \Phi)^2 + (\Phi R_\mu)^2] + 2 h_3 \mathrm{Tr}(L_\mu \Phi R^\mu \Phi^\dagger). \\
& + \bar{\Psi} i \partial\!\!\!/ \Psi - g_F \bar{\Psi} (\Phi_S + i \gamma_5 \Phi_{PS}) \Psi + g_V \bar{\Psi} \gamma^\mu \left( V_\mu + \frac{g_A}{g_V} \gamma_5 A_\mu \right) \Psi \\
& + \text{Polyakov loops}
\end{aligned}
$$

D. Parganlija, P. Kovacs, Gy. Wolf, F. Giacosa, D.H. Rischke, Phys. Rev. D87 (2013) 014011

# Lagrangian (2/2)

where

$$D^\mu\Phi = \partial^\mu\Phi - ig_1(L^\mu\Phi - \Phi R^\mu) - ieA_e^\mu[T_3, \Phi]$$

$$\Phi = \sum_{i=0}^{8}(\sigma_i + i\pi_i)T_i, \quad H = \sum_{i=0}^{8} h_i T_i \qquad T_i : U(3) \text{ generators}$$

$$R^\mu = \sum_{i=0}^{8}(\rho_i^\mu - b_i^\mu)T_i, \quad L^\mu = \sum_{i=0}^{8}(\rho_i^\mu + b_i^\mu)T_i$$

$$L^{\mu\nu} = \partial^\mu L^\nu - ieA_e^\mu[T_3, L^\nu] - \{\partial^\nu L^\mu - ieA_e^\nu[T_3, L^\mu]\}$$

$$R^{\mu\nu} = \partial^\mu R^\nu - ieA_e^\mu[T_3, R^\nu] - \{\partial^\nu R^\mu - ieA_e^\nu[T_3, R^\mu]\}$$

$$\bar{\Psi} = (\bar{u}, \bar{d}, \bar{s})$$

non strange – strange base:

$$\varphi_N = \sqrt{2/3}\varphi_0 + \sqrt{1/3}\varphi_8,$$

$$\varphi_S = \sqrt{1/3}\varphi_0 - \sqrt{2/3}\varphi_8, \qquad \varphi \in (\sigma_i, \pi_i, \rho_i^\mu, b_i^\mu, h_i)$$

broken symmetry: non-zero condensates $\langle\sigma_N\rangle, \langle\sigma_S\rangle \longleftrightarrow \phi_N, \phi_S$

# Symmetry properties of the model

Global $U(3)_L x U(3)_R$ transformation:

$$\Phi \rightarrow U_L \Phi U_R^\dagger$$
$$L^\mu \rightarrow U_L L^\mu U_L^\dagger \qquad R^\mu \rightarrow U_R R^\mu U_R^\dagger$$

Consequences (using the unitarity of U's):

$$D^\mu \Phi \rightarrow U_L D^\mu \Phi U_R^\dagger$$
$$L^{\mu\nu} \rightarrow U_L L^{\mu\nu} U_L^\dagger \qquad R^{\mu\nu} \rightarrow U_R R^{\mu\nu} U_R^\dagger$$

All terms are invariant except

the determinant: breaks $U_A(1)$

the explicit symmetry breaking term H: breaks $SU_A(3)$ and
- remains $U_V(1)xSU_V(3)$ if all 3 eigenvalues of H are equal
- remains $U_V(1)xSU_V(2)$ if 2 eigenvalues of H are equal
- remains $U_V(1)$ if all 3 eigenvalues of H are different.

# Determinant term

$$U_L = e^{-i\omega_L^a T^a} \qquad U_R = e^{-i\omega_R^a T^a}$$

$$\omega_V^a = 0.5(\omega_L^a + \omega_R^a) \qquad \omega_A^a = 0.5(\omega_L^a - \omega_R^a)$$

By $SU(3)_L$x$SU(3)_R$ transformation (if $\omega_L^0 = \omega_R^0 = 0 = \omega_V^0 = \omega_A^0$)

$$(\det\Phi)' = \det(U_L \Phi U_R^\dagger) = \det U_L \det\Phi \det U_R^\dagger = \det\Phi$$

Similarly $\det\Phi^\dagger$ is also invariant.

If $\omega_V^0 \neq 0$ and all the other $\omega$'s are 0 ($[T^a, T^0] = 0$)

$$(\det\Phi)' = \det(e^{-i\omega_V^0 T^0}\Phi e^{i\omega_V^0 T^0}) = \det(e^{-i\omega_V^0 T^0} e^{i\omega_V^0 T^0}\Phi) = \det\Phi$$

On the other hand, if $\omega_A^0 \neq 0$ and all the other $\omega$'s are 0

$$(\det\Phi)' = \det(e^{-i\omega_A^0 T^0}\Phi e^{-i\omega_A^0 T^0}) = \det(e^{-i\omega_A^0 T^0} e^{-i\omega_A^0 T^0}\Phi) = e^{-i2\omega_A^0} \det\Phi \mathrm{Tr}\, T^0$$

So the determinant term is invariant under $U(3)_V$x$SU(3)_A$ transformation and breaks explicitely the $U(1)_A$ symmetry.

# Explicit breaking term: $\mathrm{Tr}[\hat{\epsilon}(\Phi + \Phi^\dagger)]$

$$\hat{\epsilon} = \sum_{i=0}^{8} \epsilon_i T_i = \begin{pmatrix} \frac{\epsilon_N}{2} & 0 & 0 \\ 0 & \frac{\epsilon_N}{2} & 0 \\ 0 & 0 & \frac{\epsilon_S}{\sqrt{2}} \end{pmatrix} \qquad \text{only} \quad \epsilon^0, \epsilon^8 \neq 0$$

- axial transformation: if at least $\epsilon^0 \neq 0$ $U(3)_A$ is broken:

$$(\mathrm{Tr}[\hat{\epsilon}(\Phi)])' = \mathrm{Tr}(e^{-i2\omega_A^a T^a}\hat{\epsilon}\Phi)$$

- vector transformation

$$(\mathrm{Tr}[\hat{\epsilon}(\Phi)])' = \mathrm{Tr}(e^{-i\omega_V^a T^a}\hat{\epsilon}e^{i\omega_V^a T^a}\Phi)$$

Since $[\hat{\epsilon}, T^0] = 0$, $U(1)_V$ symmetry is preserved.
If all $\epsilon^a = 0$ except $\epsilon^0$, $U(3)_V$ is preserved.
If $\epsilon^8$ also non zero, then since $[T^K, T^8] = 0$ if $k = 1, 2, 3$, $U(1)_V$x$SU(2)_V$ survives (isospin symmetry)
(If $\epsilon^3 \neq 0$ too, then the isospin symmetry is broken, only $U(1)_V$.)

# Spontaneous symmetry breaking

Interaction is approximately chiral symmetric, spectra not, so SSB:

$$\sigma_{N/S} \to \sigma_{N/S} + \phi_{N/S} \qquad \phi_{N/S} \equiv <\sigma_{N/S}>$$

For tree level masses we have to select all terms quadratic in the new fields. Some of the terms include mixings arising from terms like $\mathrm{Tr}[(D_\mu\Phi)^\dagger(D_\mu\Phi)]$:

$$\begin{aligned}
\pi_N - a_{1N}^\mu &: \quad -g_1\phi_N a_{1N}^\mu \partial_\mu \pi_N, \\
\pi - a_1^\mu &: \quad -g_1\phi_N(a_1^{\mu+}\partial_\mu\pi^- + a_1^{\mu 0}\partial_\mu\pi^0) + \mathrm{h.c.}, \\
\pi_S - a_{1S}^\mu &: \quad -\sqrt{2}g_1\phi_S a_{1S}^\mu \partial_\mu \pi_S, \\
K_S - K_\mu^\star &: \quad \frac{ig_1}{2}(\sqrt{2}\phi_S - \phi_N)(\bar{K}_\mu^{\star 0}\partial^\mu K_S^0 + K_\mu^{\star -}\partial^\mu K_S^+) + \mathrm{h.c.}, \\
K - K_1^\mu &: \quad -\frac{g_1}{2}(\phi_N + \sqrt{2}\phi_S)(K_1^{\mu 0}\partial_\mu \bar{K}^0 + K_1^{\mu+}\partial_\mu K^-) + \mathrm{h.c.}.
\end{aligned}$$

Diagonalization$\to$ Wave function renormalization

# Determination of the parameters of the Lagrangian

16 unknown parameters $(m_0, \lambda_1, \lambda_2, c_1, m_1, g_1, g_2, h_1, h_2, h_3, \delta_S, \Phi_N, \Phi_S, g_F, g_V, g_A) \longrightarrow$ Determined by the min. of $\chi^2$:

$$\chi^2(x_1,\dots,x_N) = \sum_{i=1}^{M}\left[\frac{Q_i(x_1,\dots,x_N) - Q_i^{\exp}}{\delta Q_i}\right]^2,$$

where $(x_1,\dots,x_N) = (m_0, \lambda_1, \lambda_2, \dots)$, $Q_i(x_1,\dots,x_N)$ calculated from the model, while $Q_i^{\exp}$ taken from the PDG

multiparametric minimalization $\longrightarrow$ MINUIT

- PCAC $\rightarrow$ 2 physical quantities: $f_\pi, f_K$
- Tree-level masses $\rightarrow$ 15 physical quantities:

  $m_{u/d}, m_s, m_\pi, m_\eta, m_{\eta'}, m_K, m_\rho, m_\Phi, m_{K^\star}, m_{a_1}, m_{f_1^H}, m_{a_0}, m_{K_s}, m_{f_0^L}, m_{f_0^H}$
- Decay widths $\rightarrow$ 12 physical quantities:

  $\Gamma_{\rho\to\pi\pi}, \Gamma_{\Phi\to KK}, \Gamma_{K^\star\to K\pi}, \Gamma_{a_1\to\pi\gamma}, \Gamma_{a_1\to\rho\pi}, \Gamma_{f_1\to KK^\star}, \Gamma_{a_0}, \Gamma_{K_S\to K\pi}, \Gamma_{f_0^L\to\pi\pi}, \Gamma_{f_0^L\to KK}, \Gamma_{f_0^H\to\pi\pi}, \Gamma_{f_0^H\to KK}$
- $T_c = 155$ MeV from lattice

# Results

| | Cal(GeV) | Mass | | Cal(GeV) | Width |
|---|---|---|---|---|---|
| $m_\pi$ | 0.1405 | 0.1380 | $f_\pi$ | 0.0955 | 0.0922 |
| $m_K$ | 0.4995 | 0.4956 | $f_K$ | 0.1094 | 0.1100 |
| $m_\eta$ | 0.5421 | 0.5479 | $\Gamma_{f_0 L\rightarrow KK}$ | 0.0 | 0.0 |
| $m_{\eta'}$ | 0.9643 | 0.9578 | $\Gamma_{f_0 H\rightarrow KK}$ | 0.0 | 0.0 |
| $m_\rho$ | 0.8064 | 0.7755 | $\Gamma_\rho$ | 0.1515 | 0.149 |
| $m_\phi$ | 0.9901 | 1.0195 | $\Gamma_\phi$ | 0.003534 | 0.003545 |
| $m_{K*}$ | 0.9152 | 0.8938 | $\Gamma_{K*}$ | 0.04777 | 0.048 |
| $m_{f_1 H}$ | 1.4160 | 1.4264 | $\Gamma_{f_1\rightarrow KK}$ | 0.04451 | 0.0445 |
| $m_{a_1}$ | 1.0766 | 1.2300 | $\Gamma_{A_1\rightarrow\rho\pi}$ | 0.1994 | 0.425 |
| $m_{K_1}$ | 1.2999 | 1.2720 | $\Gamma_{A_1\rightarrow\gamma\pi}$ | 0.0003670 | 0.000640 |
| $m_{a_0}$ | 0.7208 | 0.980 | $\Gamma_{A_0}$ | 0.06834 | 0.075 |
| $m_{K_0^*}$ | 0.7529 | 0.682 | $\Gamma_{K_0^*}$ | 0.006001 | 0.00547 |
| $m_{f_0 L}$ | 0.2823 | 0.475 | $\Gamma_{f_0 L\rightarrow\pi\pi}$ | 0.5542 | 0.550 |
| $m_{f_0 H}$ | 0.7376 | 0.990 | $\Gamma_{f_0 H\rightarrow\pi\pi}$ | 0.08166 | 0.07 |
| $m_{ud}$ | 0.3224 | 0.308 | $m_s$ | 0.4577 | 0.483 |

# Parameters

| Parameter | Value |
|---|---|
| $\phi_N$ [GeV] | 0.1411 |
| $\phi_S$ [GeV] | 0.1416 |
| $m_0^2$ [GeV$^2$] | $2.3925E-4$ |
| $m_1^2$ [GeV$^2$] | $6.3298E-8$ |
| $\lambda_1$ | $-1.6738$ |
| $\lambda_2$ | 23.5078 |
| $\delta_S$ [GeV$^2$] | 0.1133 |
| $c_1$ [GeV] | 1.3086 |
| $g_1$ | 5.6156 |
| $g_2$ | 3.0467 |
| $h_1$ | 27.4617 |
| $h_2$ | 4.2281 |
| $h_3$ | 5.9839 |
| $g_F$ | 4.5708 |
| $M_0$ [GeV] | 0.3511 |

- with this set $f_0^l = 0.2837$ GeV

# Polyakov loops in Polyakov gauge

Polyakov loop variables: $\Phi(\vec{x}) = \frac{\mathrm{Tr}_c L(\vec{x})}{N_c}$ and $\bar{\Phi}(\vec{x}) = \frac{\mathrm{Tr}_c \bar{L}(\vec{x})}{N_c}$ with

$$L(x) = \mathcal{P}\exp\left[i\int_0^\beta d\tau A_4(\vec{x},\tau)\right]$$

$\longrightarrow$ signals center symmetry ($\mathbb{Z}_3$) breaking at the deconfinement transition

low $T$: confined phase, $\langle\Phi(\vec{x})\rangle, \langle\bar{\Phi}(\vec{x})\rangle = 0$
high $T$: deconfined phase, $\langle\Phi(\vec{x})\rangle, \langle\bar{\Phi}(\vec{x})\rangle \neq 0$

Polyakov gauge: the temporal component of the gauge field is time independent and can be gauge rotated to a diagonal form in the color space

$$G_{4,d}(\vec{x}) = \phi_3(\vec{x})\lambda_3 + \phi_8(\vec{x})\lambda_8; \quad \lambda_3, \lambda_8 \text{ : Gell-Mann matrices.}$$

In this gauge the Polyakov loop operator is

$$L(\vec{x}) = \mathrm{diag}(e^{i\beta\phi_+(\vec{x})}, e^{i\beta\phi_-(\vec{x})}, e^{-i\beta(\phi_+(\vec{x})+\phi_-(\vec{x}))})$$

where $\phi_\pm(\vec{x}) = \pm\phi_3(\vec{x}) + \phi_8(\vec{x})/\sqrt{3}$

# Improved Polyakov loops potential

Logarithmic potential K. Fukushima, Phys. Lett. **B591**, 277 (2004)

$$\frac{\mathcal{U}_{\log}(\Phi,\bar{\Phi})}{T^4} = -\frac{1}{2}a(T)\Phi\bar{\Phi} + b(T)\ln\left[1 - 6\Phi\bar{\Phi} + 4\left(\Phi^3+\bar{\Phi}^3\right) - 3\left(\Phi\bar{\Phi}\right)^2\right]$$

with $a(T) = a_0 + a_1\frac{T_0}{T} + a_2\frac{T_0^2}{T^2}$, $b(T) = b_3\frac{T_0^3}{T^3}$

$\mathcal{U}\left(\Phi,\bar{\Phi}\right)$ models the free energy of a pure gauge theory

Within FRG, the glue potential $U^{glue}(\Phi,\bar{\Phi})$ coming from the gauge dof propagating in the presence of dynamical quarks can be matched to the potential $U^{YM}(\Phi,\bar{\Phi})$ of the SU(3) YM theory by relating the reduced temperatures:

$$\frac{U^{glue}}{T^4}(\Phi,\bar{\Phi},t_{glue}) = \frac{U^{YM}}{(T^{YM})^4}(\Phi,\bar{\Phi},t_{YM}(t_{glue})), \quad t_{YM}(t_{glue}) \approx 0.57 t_{glue}$$

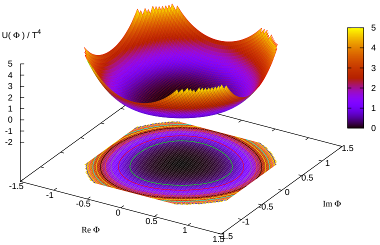
$$t_{glue} \equiv \frac{T - T_c^{glue}}{T_c^{glue}}, t_{YM} \equiv \frac{T^{YM} - T_0^{YM}}{T_0^{YM}}, \quad T_c^{glue} \in (180, 270) MeV$$

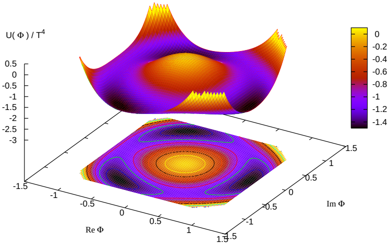
L.M.Haas et al., PRD 87, 076004 (2013)

# Polyakov loop potential

"Color confinement"
$\langle \Phi \rangle = 0 \longrightarrow$ no breaking of $\mathbb{Z}_3$
one minimum

"Color deconfinement"
$\langle \Phi \rangle \neq 0 \longrightarrow$ spontaneous breaking of $\mathbb{Z}_3$
minima at $0, 2\pi/3, -2\pi/3$
one of them spontaneously selected




from H. Hansen et al., PRD75, 065004 (2007)

# Effects of Polyakov loops on FD statistics

Inclusion of the Polyakov loop modifies the Fermi-Dirac distribution function

$$f(E_p-\mu_q) \longrightarrow f_\Phi^+(E_p) = \frac{\left(\bar\Phi + 2\Phi e^{-\beta\left(E_p-\mu_q\right)}\right) e^{-\beta\left(E_p-\mu_q\right)} + e^{-3\beta\left(E_p-\mu_q\right)}}{1+3\left(\bar\Phi + \Phi e^{-\beta\left(E_p-\mu_q\right)}\right) e^{-\beta\left(E_p-\mu_q\right)} + e^{-3\beta\left(E_p-\mu_q\right)}}$$

$$f(E_p+\mu_q) \longrightarrow f_\Phi^-(E_p) = \frac{\left(\Phi + 2\bar\Phi e^{-\beta\left(E_p+\mu_q\right)}\right) e^{-\beta\left(E_p+\mu_q\right)} + e^{-3\beta\left(E_p+\mu_q\right)}}{1+3\left(\Phi + \bar\Phi e^{-\beta\left(E_p+\mu_q\right)}\right) e^{-\beta\left(E_p+\mu_q\right)} + e^{-3\beta\left(E_p+\mu_q\right)}}$$

$\Phi, \bar\Phi \to 0 \Longrightarrow f_\Phi^\pm(E_p) \to f(3(E_p \pm \mu_q))$ $\quad \Phi, \bar\Phi \to 1 \Longrightarrow f_\Phi^\pm(E_p) \to f(E_p \pm \mu_q)$

three-particle state appears: mimics confinement of quarks within baryons

at $T = 0$ there is no difference between models with and without Polyakov loop

# $T/\mu_B$ dependence of the condensates

$\Omega$: grand canonical potential

$$\frac{\partial \Omega}{\partial \Phi} = \left.\frac{\partial \Omega}{\partial \bar{\Phi}}\right|_{\varphi_N=\phi_N, \varphi_S=\phi_S} = 0$$

$$\frac{\partial \Omega}{\partial \phi_N} = \left.\frac{\partial \Omega}{\partial \phi_S}\right|_{\Phi, \bar{\Phi}} = 0, \qquad \text{(after the SSB)}$$

Hybrid approach: fermions at one-loop, mesons at tree-level (their effects are much smaller)

# Masses

$$M^2_{i,ab} = \frac{\partial^2 \Omega(T,\mu_f)}{\partial\varphi_{i,a}\partial\varphi_{i,b}}\bigg|_{\min} = m^2_{i,ab} + \Delta_0 m^2_{i,ab} + \Delta_T m^2_{i,ab},$$

$m^2_{i,ab} \longrightarrow$ tree-level mass matrix,
$\Delta_{0/T} m^2_{i,ab} \longrightarrow$ fermion vacuum/thermal fluctuation,

$$\Delta_0 m^2_{i,ab} = \frac{\partial^2 \Omega^{\text{vac}}_{q\bar{q}}}{\partial\varphi_{i,a}\partial\varphi_{i,b}}\bigg|_{\min} = -\frac{3}{8\pi^2}\sum_{f=u,d,s}\left[\left(\frac{3}{2}+\log\frac{m_f^2}{M^2}\right)m^{2(i)}_{f,a}m^{2(i)}_{f,b} + m_f^2\left(\frac{1}{2}+\log\frac{m_f^2}{M^2}\right)m^{2(i)}_{f,ab}\right],$$

$$\Delta_T m^2_{i,ab} = \frac{\partial^2 \Omega^{\text{th}}_{q\bar{q}}}{\partial\varphi_{i,a}\partial\varphi_{i,b}}\bigg|_{\min} = 6\sum_{f=u,d,s}\int\frac{d^3p}{(2\pi)^3}\frac{1}{2E_f(p)}\Bigg[\left(f_f^+(p)+f_f^-(p)\right)\left(m^{2(i)}_{f,ab} - \frac{m^{2(i)}_{f,a}m^{2(i)}_{f,b}}{2E_f^2(p)}\right)$$
$$+ \left(B_f^+(p)+B_f^-(p)\right)\frac{m^{2(i)}_{f,a}m^{2(i)}_{f,b}}{2TE_f(p)}\Bigg],$$

where $m^{2(i)}_{f,a} \equiv \partial m_f^2/\partial\varphi_{i,a}$, $m^{2(i)}_{f,ab} \equiv \partial^2 m_f^2/\partial\varphi_{i,a}\partial\varphi_{i,b}$

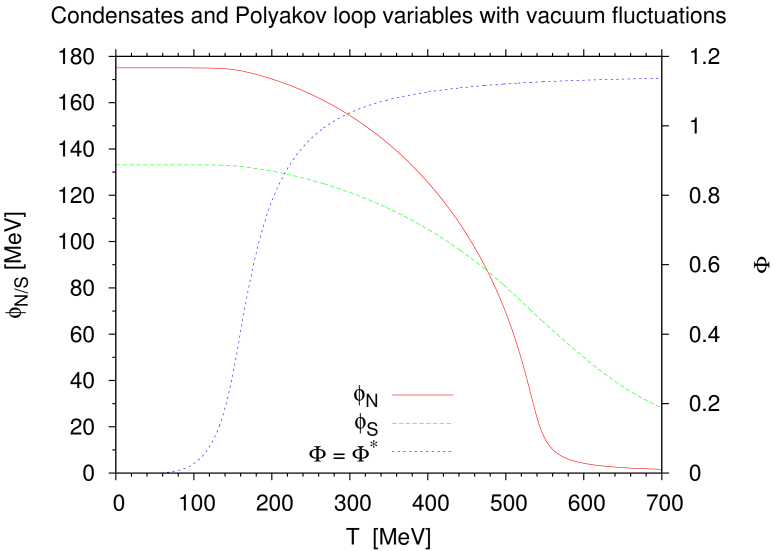
# Features of our approach

- D.O.F's: scalar, pseudoscalar, vector, axial vector nonets,
- Polyakov loop variables, $\Phi, \bar{\Phi}$ with $U^{YM}$ or $U^{glue}$
- u,d,s constituent quarks, $(m_u = m_d)$
- no mesonic fluctuations included in the grand canonical potential:

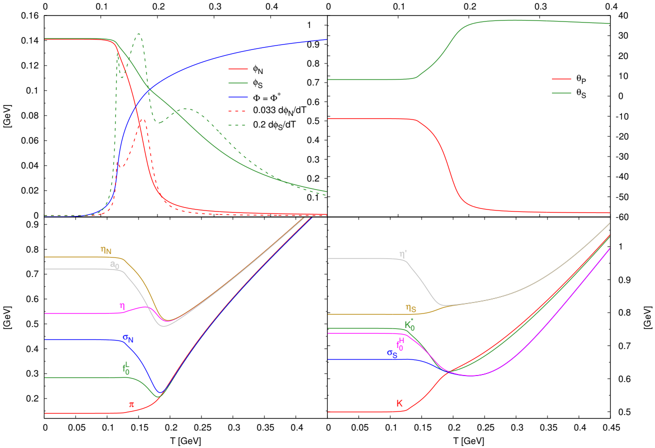
$$\Omega(T, \mu_q) = -\frac{1}{\beta V} ln(Z)$$

- Fermion vacuum and thermal fluctuations
- quarks do not couple to (axial) vector meson yet
- Four order parameters $(\phi_N, \phi_S, \Phi, \bar{\Phi}) \rightarrow$ four $T/\mu$-dependent equations
- thermal contribution of $\pi, K, f_0^L$ included in the pressure

# $\phi_{N/S}(T), \Phi/\bar{\Phi}(T)$ *with* Polyakov loop $m_{f_0^L} = 1326$ MeV



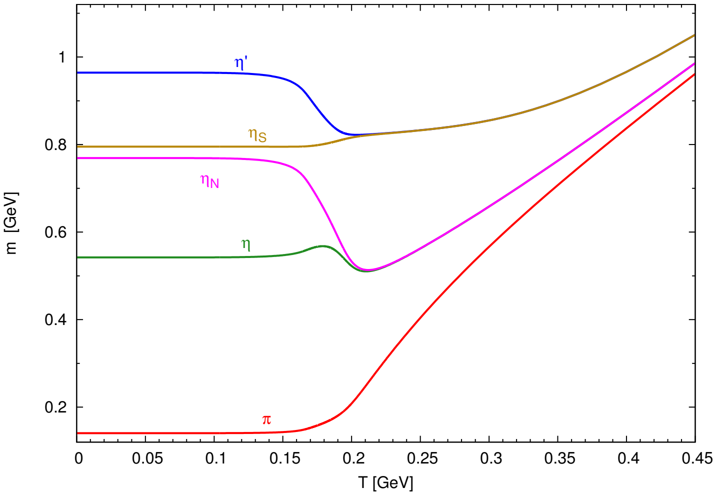
# With low mass scalars, $m_{f_0^L} = 300$ MeV




chiral symmetry is restored at high T as the chiral partners $(\pi, f_0^L)$, $(\eta, a_0)$ and $(K, K_0^\star)$, $(\eta', f_0^H)$ become degenerate

$U(1)_A$ symmetry is not restored, as the axial partners $(\pi, a_0)$ and $(\eta, f_0^L)$ do not become degenerate

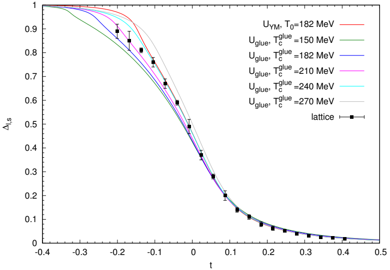
# Mass pattern in the $\eta$, $\eta'$ sector

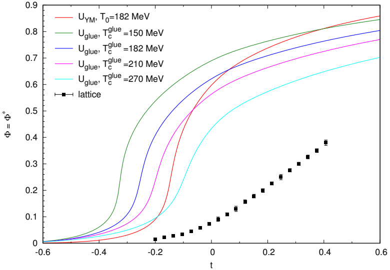



Our pattern: $m_\eta \leq m_{\eta_N} < m_{\eta_S} \leq m_{\eta'}$ in contrast to others

Schaefer, PRD79 014018, Tiwari PRD88, 074017

# T-dependence of condensates compared to lattice results




subtracted chiral condensate

$$\Delta = \frac{(\Phi_N - h_N/h_S \Phi_S)_T}{(\Phi_N - h_N/h_S \Phi_S)_{T=0}}$$

$U^{glue}$ with $T_c^{glue} \in (210-240)$MeV gives good agreement with the lattice result of Borsanyi et al.,JHEP 1009, 073 (2010)

- lattice shows smooth transition
- our result is completely off
- renormalization of the Polyakov loop may explain part of the discrepancy Andersen et al., PRD92, 114504

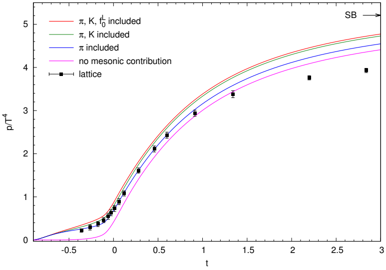
# Thermodynamical Observables

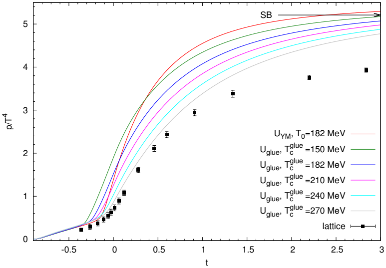
We include mesonic thermal contribution to $p$ for $(\pi, K, f_0^l)$

$$\Delta p(T) = -nT\int \frac{d^3q}{(2\pi)^3} ln(1-e^{-\beta E(q)}), \quad E(q) = \sqrt{q^2+m^2}$$

- pressure: $p(T,\mu_q) = \Omega_H(T=0,\mu_q) - \Omega_H(T,\mu_q)$
- entropy density: $s = \frac{\partial p}{\partial T}$
- quark number density: $\rho_q = \frac{\partial p}{\partial \mu_q}$
- energy density: $\epsilon = -p + Ts + \mu_q\rho_q$
- scaled interaction measure: $\frac{\Delta}{T^4} = \frac{\epsilon - 3p}{T^4}$
- speed of sound at $\mu_q = 0$: $c_s^2 = \frac{\partial p}{\partial \epsilon}$

# Normalized pressure



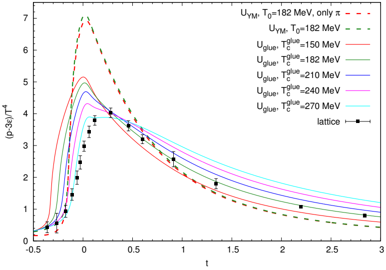




we use $U^{glue}$ with $T_c^{glue} = 270$ MeV
pion dominates at low $T$
at high $T$ pressure overshoots the lattice data

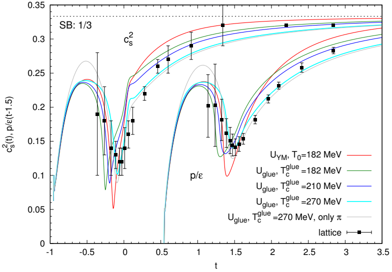
overshooting increases with decreasing $T_c^{glue}$
lattice: Borsányi et al., JHEP 1011, 077 (2010)

# Observables

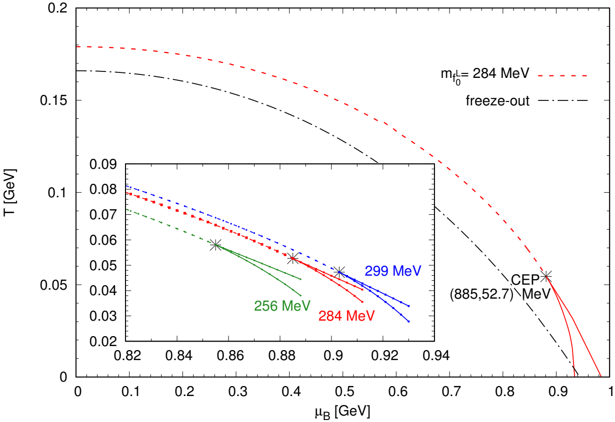
## interaction measure




## speed of sound



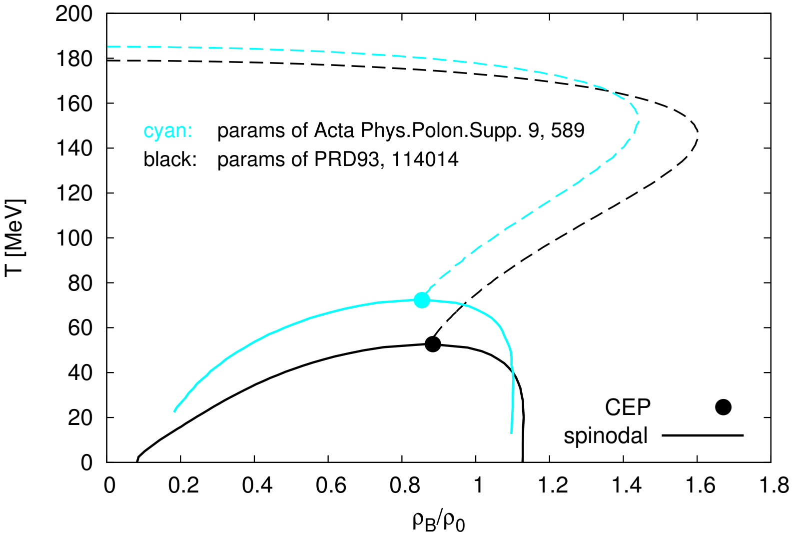
# $T - \mu_B$ phase diagram




- we use $U^{glue}$ with $T_c^{glue} = 210$ MeV
- freeze-out curve from Cleymans et al., J.Phys.G32, S165
- Curvature at $\mu_B = 0$ $\kappa = 0.0193$, close to the lattice value $\kappa = 0.020(4)$

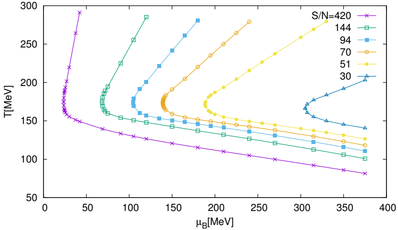
(Cea *et al.*,PRD93, 014507)

# $T - \rho_B$ phase diagram

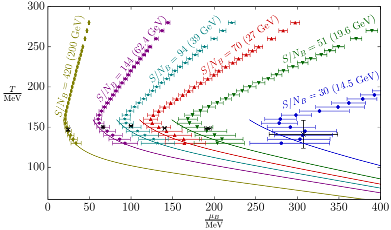


# Isentropic trajectories in the $\mathbf{T} - \mu_\mathrm{B}$ plane

our model, where $\mu_B^{\mathrm{CEP}} > 850\mathrm{MeV}$




lattice (analytic continuation)
Günther *et al.*, arXiv:1607.02493




same qualitative behavior of the isentropic trajectories for $\mu_B \leq 400$ MeV
$\Longrightarrow$ indication that in the lattice result there is no CEP in this region of $\mu_B$

# Summary and Conclusions

- The thermodynamics of the ePQM was studied after parametrizing of the model with a modification of the method used in Parganlija et al., PRD87, 014011
- 40 possible assignments of the scalars to the nonet states were investigated. Lowest $\chi^2$ for $a_0^{\bar{q}q} \to a_0(980)$, $K_0^{*,\bar{q}q} \to K_0^*(980)$, $f_0^{l,\bar{q}q} \to f_0(500)$, $f_0^{h,\bar{q}q} \to f_0(980)$
- The phase transition temperature requires low mass ($\leq 400$ MeV) $f_0$
- For the best set of parameters CEP was found in the $T - \mu_B$ plane
- The $T$-dependence of various thermodynamical observables measured on the lattice is reasonable well reproduced with an improved Polyakov loop potential. L.M. Hass et al., PRD87, 076004
- The model's predictions are unrealistic at large $\mu_B$

$\rightarrow$ To do . . .

$\rightarrow$ Improve the vacuum phenomenology by tetraquarks (and glueballs)

$\rightarrow$ coupling the quarks to the (axial)vectors

$\rightarrow$ including mesonic fluctuations

$\rightarrow$ find a way to improve the high density behaviour

Thank you for your attention!