Simulations of HIC for NICA energies

Dubna, 12. 4. 2017

Light clusters in warm dense matter

Gerd Röpke, Rostock

Outline

- Light cluster production at NICA
- Nonequilibrium and equilibrium, Zubarev approach
- Equation of state: quantum statistical approach to nuclear systems at finite temperatures and subsaturation densities, bound states, spectral function, quasiparticle concept
- Light quasiparticles
- Advanced problems: Continuum correlations, cluster virial expansion, correlated matter, quantum condensates
- HIC: chemical constants, symmetry energy
- Heavy elements, thermodynamic instability, pasta structures
- Few-particle correlations in finite nuclear systems (nuclei),
- Transport codes, Mott effect and in-medium cross sections, relevance of the equilibrium EoS.

Light cluster production at NICA★

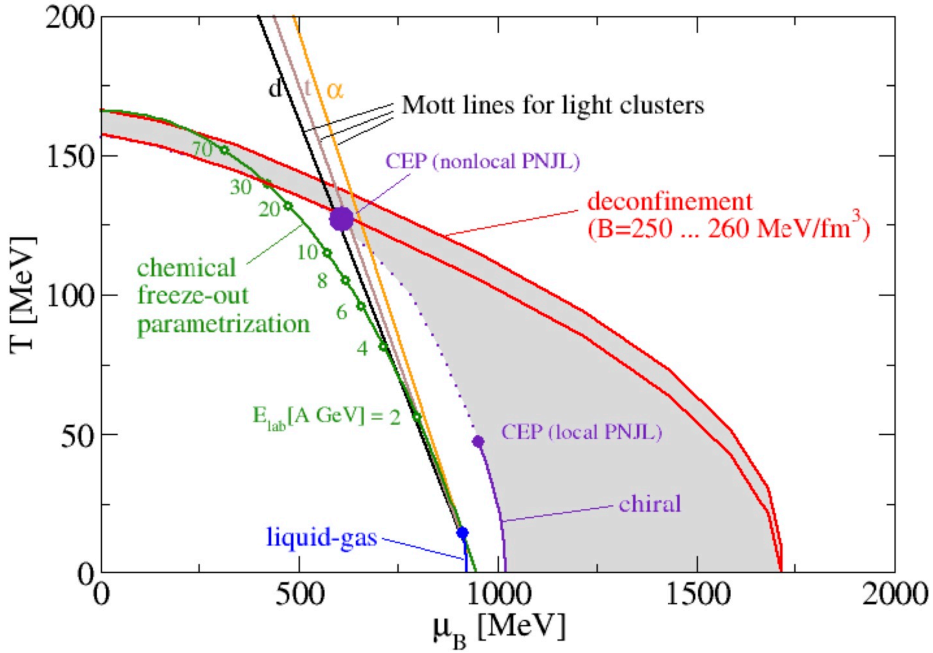
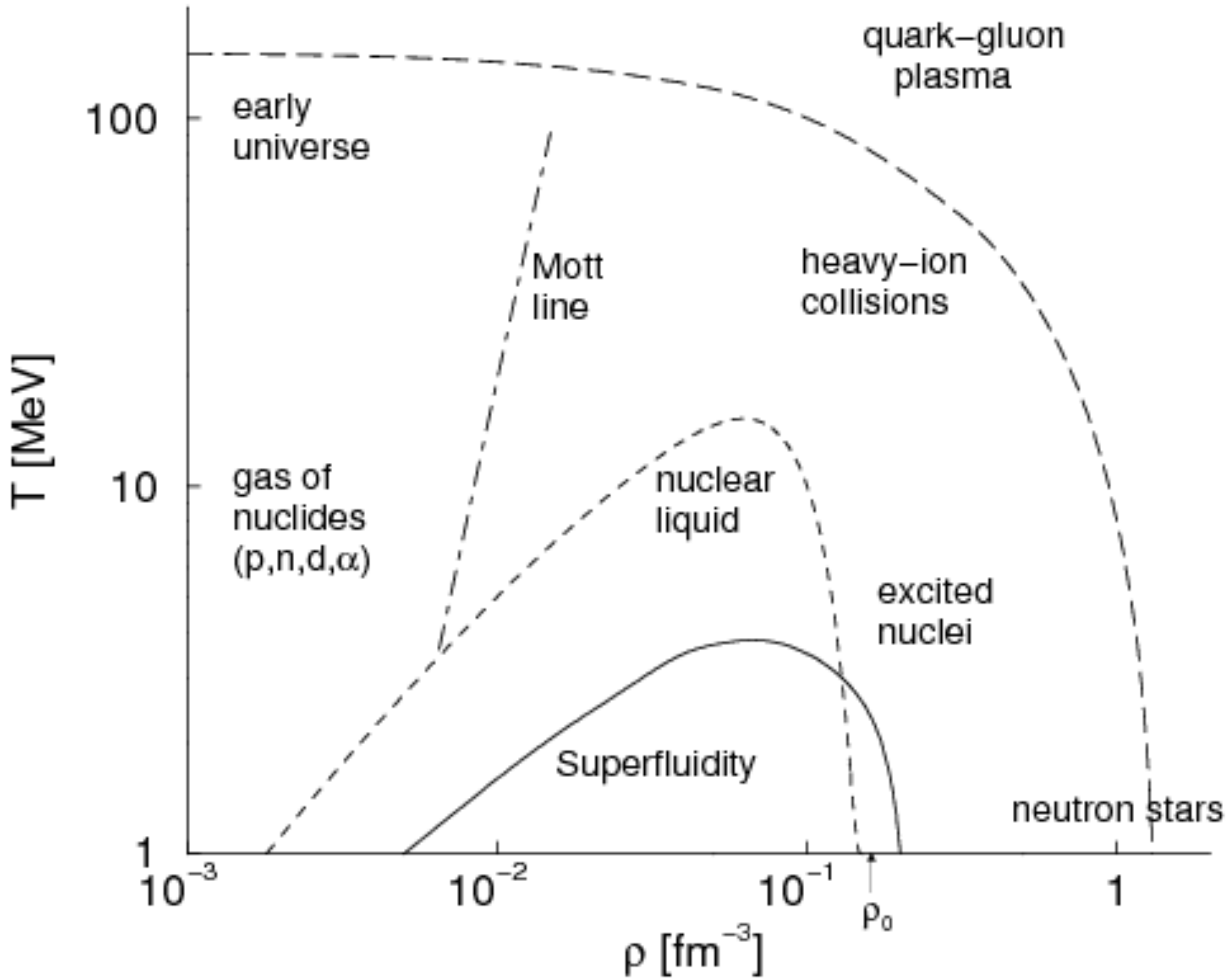


Fig. 1. Phase diagram of dense nuclear matter in the plane of temperature T and baryochemical potential μ_B. The diagram includes Mott lines for the dissociation of light nuclear clusters, extrapolated also to the deconfinement region. For details, see text.

N.-U. Bastian, P. Batyuk, D. Blaschke, P. Danielewicz, Yu.B. Ivanov, Iu. Karpenko, G. Ropke, O. Rogachevsky, and H.H. Wolter, Eur. Phys. J. A (2016) 52: 244

Symmetric nuclear matter: Phase diagram



Equilibrium and non-equilibrium

Statistical operator $\varrho(t)$

Extended von Neumann equation

$$\frac{\partial}{\partial t}\varrho_\varepsilon(t) + \frac{i}{\hbar}[H, \varrho_\varepsilon(t)] = -\varepsilon\left(\varrho_\varepsilon(t) - \varrho_{\rm rel}(t)\right)$$

The relevant statistical operator $\varrho_{\rm rel}(t)$ is obtained from the maximum of entropy reproducing the local, time dependent composition with parameter values $T(\mathbf{r}, t), \mu_n(\mathbf{r}, t), \mu_p(\mathbf{r}, t)$, but contains in addition the cluster distribution functions $f_{A\nu}^{\rm Wigner}(\mathbf{p}, \mathbf{r}, t)$ as relevant observables.[106,107]

$$\varrho(t) = \lim_{\varepsilon \to 0} \varrho_\varepsilon(t)$$

D.N. Zubarev, V.G. Morozov, and G. Ropke, Statistical Mechanics of Nonequilibrium Processes (1996)

D.N. Zubarev, V.G. Morozov, I.P. Omelyan, and M.V. Tokarchuk, Theoret. Math. Phys. **96**, 997 (1993)

G. Ropke and H. Schulz, Nucl. Phys. A **477**, 472 (1988)

Quantum statistics

- System in equilibrium: temperature T, volume Ω, particle numbers N_c (conserved)
- Nuclear systems, N_c: neutrons n_n, protons n_p, electrons n_e, …
- density $n_c(T, mu_{c'})$
  Thermodynamic potential: free energy $F(T, \Omega, N_c)$
  Internal energy $U(T, \Omega, N_c)$
- Nuclear structure $T=0$,
  astrophysics, heavy ion reactions (HIC): finite T
- Interaction: strong, Coulomb, weak
- Green function approach, Path integral, numerical simulatios

Equilibrium composition and Equation of State (EoS) of nuclear matter

Many-particle theory

$$n_\tau^{\rm tot}(T,\mu_n,\mu_p) = \frac{1}{\Omega}\sum_{p_1,\sigma_1}\int\frac{d\omega}{2\pi}\frac{1}{e^{(\omega-\mu_\tau)/T}+1}S_\tau(1,\omega)$$

Spectral function S (or A)

- Dyson equation and self energy (homogeneous system)

$$G(1,iz_\nu) = \frac{1}{iz_\nu - E(1) - \Sigma(1,iz_\nu)}$$

- Evaluation of $\Sigma(1,iz_\nu)$:
  perturbation expansion, diagram representation

$$A(1,\omega) = \frac{2{\rm Im}\,\Sigma(1,\omega+i0)}{[\omega - E(1) - {\rm Re}\,\Sigma(1,\omega)]^2 + [{\rm Im}\,\Sigma(1,\omega+i0)]^2}$$

approximation for self energy $\longrightarrow$ approximation for equilibrium correlation functions

alternatively: simulations, path integral methods

Different approximations

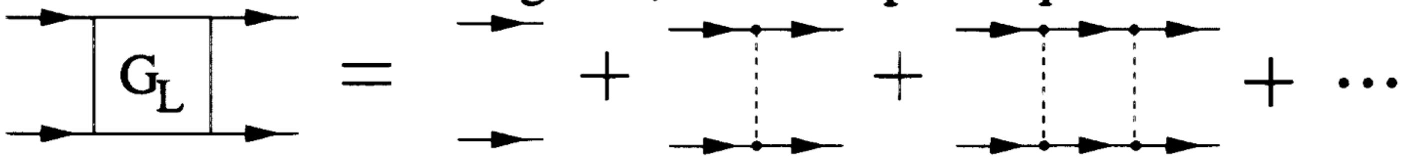
- Expansion for small Im $\Sigma(1,\omega+i\eta)$

$$A(1,\omega) \approx \frac{2\pi\delta(\omega - E^{\rm quasi}(1))}{1-\frac{\rm d}{{\rm d}z}{\rm Re}\ \Sigma(1,z)|_{z=E^{\rm quasi}-\mu_1}} - 2{\rm Im}\ \Sigma(1,\omega+i\eta)\frac{\rm d}{{\rm d}\omega}\frac{P}{\omega+\mu_1-E^{\rm quasi}(1)}$$

quasiparticle energy $E^{\rm quasi}(1) = E(1) + {\rm Re}\ \Sigma(1,\omega)|_{\omega=E^{\rm quasi}}$

- chemical picture: bound states $\hat{=}$ new species

summation of ladder diagrams, Bethe-Salpeter equation



Different approximations

Ideal Fermi gas:
protons, neutrons,
(electrons, neutrinos,…)

bound state formation

Nuclear statistical equilibrium:
ideal mixture of all bound states
(clusters:) chemical equilibrium

continuum contribution

Second virial coefficient:
account of continuum contribution,
scattering phase shifts, Beth-Uhl.E.

chemical & physical picture

Cluster virial approach:
all bound states (clusters)
scattering phase shifts of all pairs

medium effects

Quasiparticle quantum liquid:
mean-field approximation
Skyrme, Gogny, RMF

Chemical equilibrium with quasiparticle clusters:
self-energy and Pauli blocking

Generalized Beth-Uhlenbeck formula:
medium modified binding energies,
medium modified scattering phase shifts

Correlated medium
phase space occupation by all bound states
in-medium correlations,
quantum condensates

Different approximations

Ideal Fermi gas:

protons, neutrons,
(electrons, neutrinos,…)

Different approximations

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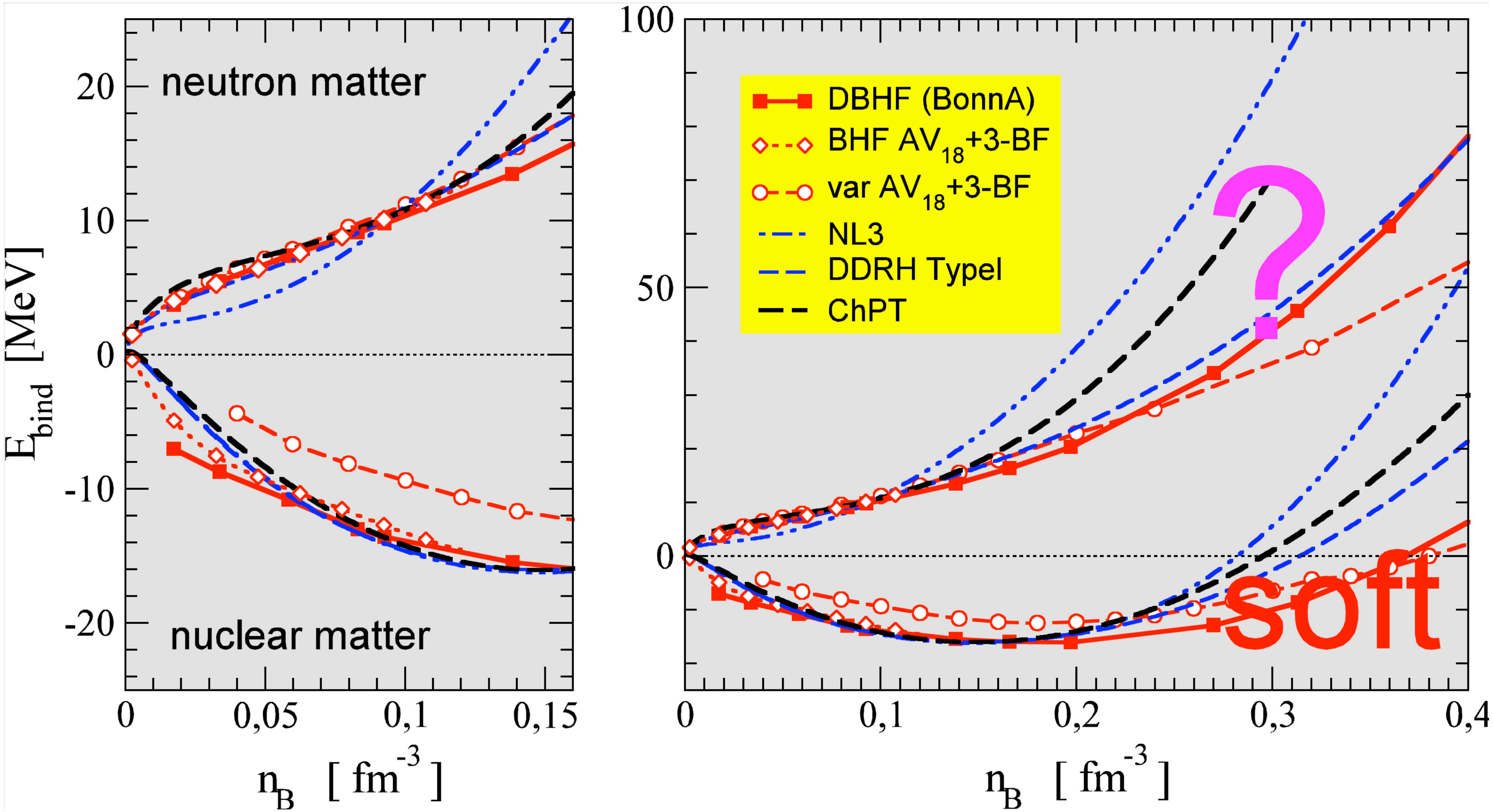
medium effects

Quasiparticle quantum liquid:
mean-field approximation
Skyrme, Gogny, RMF

Quasiparticle picture: RMF and DBHF

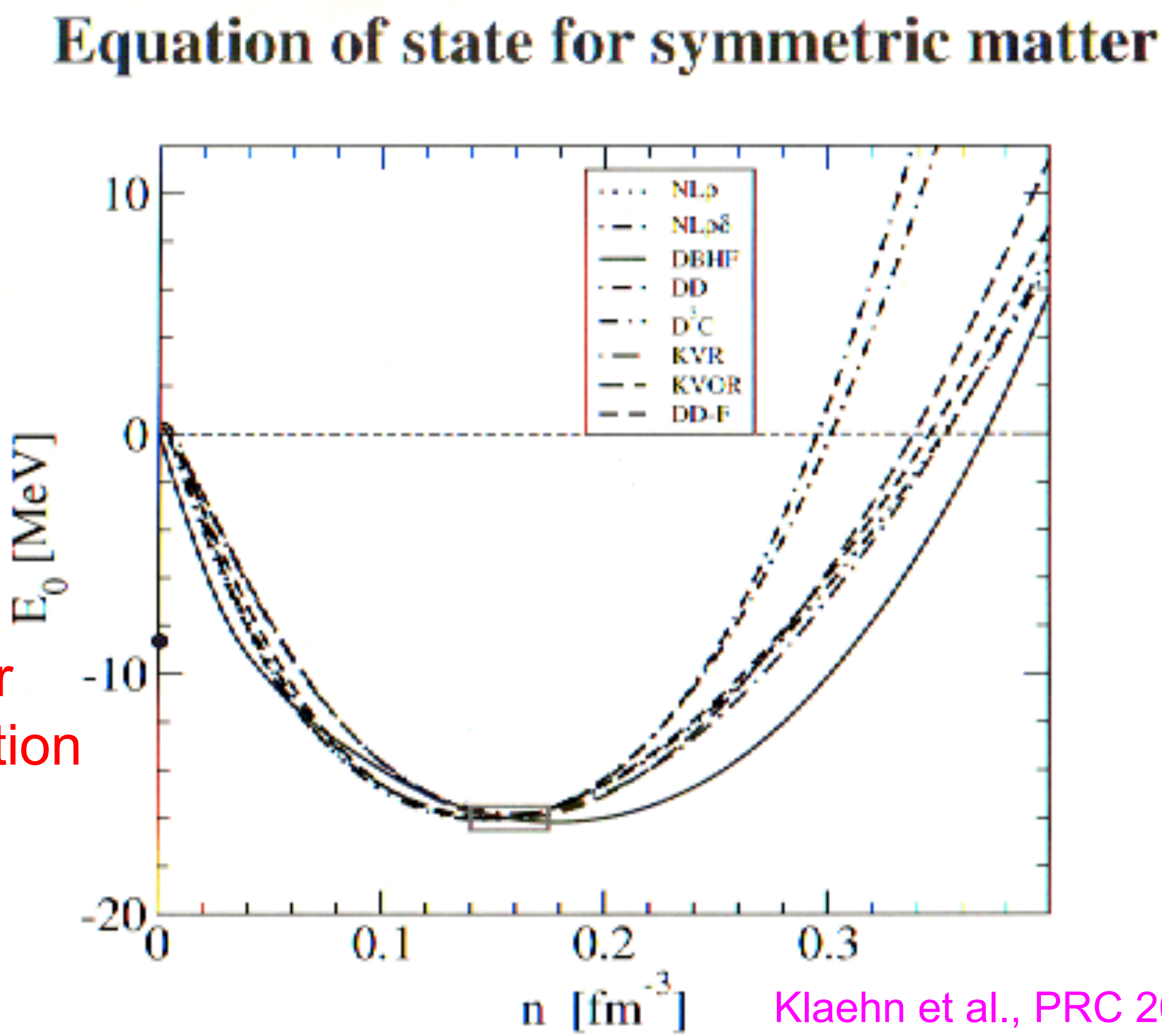


But: cluster formation

Incorrect low-density limit

C. Fuchs, H.H. Wolter, Eur. Phys. J. A **30**, 5 (2006)

Quasiparticle approximation for nuclear matter



But:
cluster
formation

Incorrect
low-density
limit

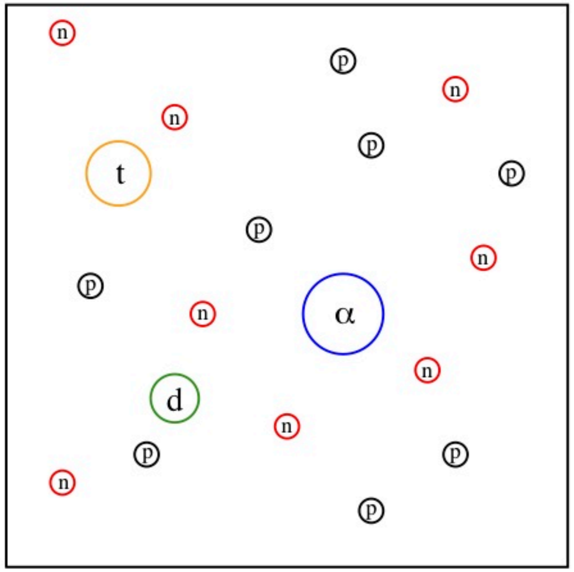
Klaehn et al., PRC 2006

Nuclear statistical equilibrium (NSE)

Chemical picture:

Ideal mixture of reacting components

Mass action law



Interaction between the components
internal structure: Pauli principle

Different approximations

medium effects

Ideal Fermi gas:

protons, neutrons,
(electrons, neutrinos,…)

bound state formation

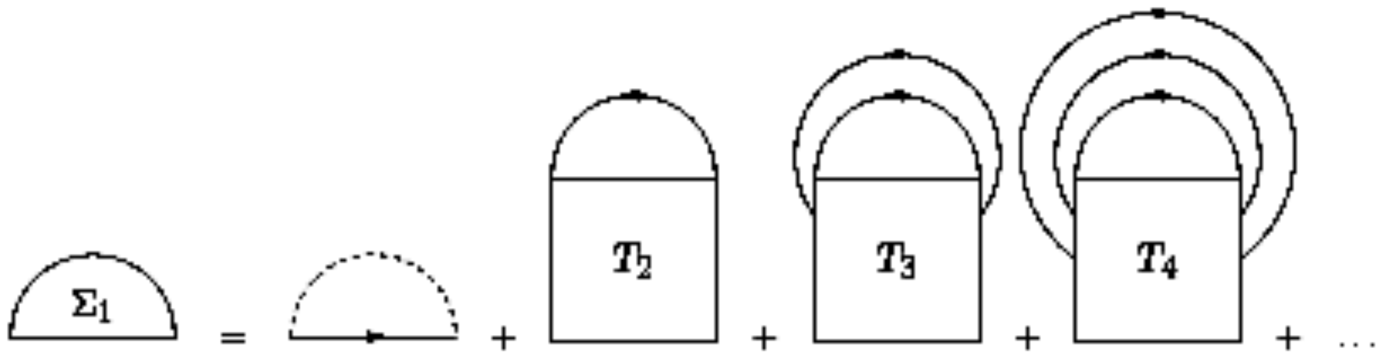
Nuclear statistical equilibrium:

ideal mixture of all bound states
(clusters:) chemical equilibrium

Quasiparticle quantum liquid:

mean-field approximation
Skyrme, Gogny, RMF

Cluster decomposition of the self-energy



T-matrices: bound states, scattering states
Including clusters like new components
chemical picture,
mass action law, nuclear statistical equilibrium (NSE)

Ideal mixture of reacting nuclides

$$n_p(T,\mu_p,\mu_n) = \frac{1}{V}\sum_{A,\nu,K} Z_A f_A\{E_{A,\nu K} - Z_A\mu_p - (A-Z_A)\mu_n\}$$

$$n_n(T,\mu_p,\mu_n) = \frac{1}{V}\sum_{A,\nu,K} (A-Z_A) f_A\{E_{A,\nu K} - Z_A\mu_p - (A-Z_A)\mu_n\}$$

mass number A,
charge Z_A,
energy $E_{A,\nu,K}$,
ν internal quantum number,
K center of mass momentum

$$f_{A(z)} = \frac{1}{\exp(z/T) - (-1)^A}$$

Chemical equilibrium, mass action law,
Nuclear Statistical Equilibrium (NSE)

Nuclear statistical equilibrium (NSE)

Chemical picture:

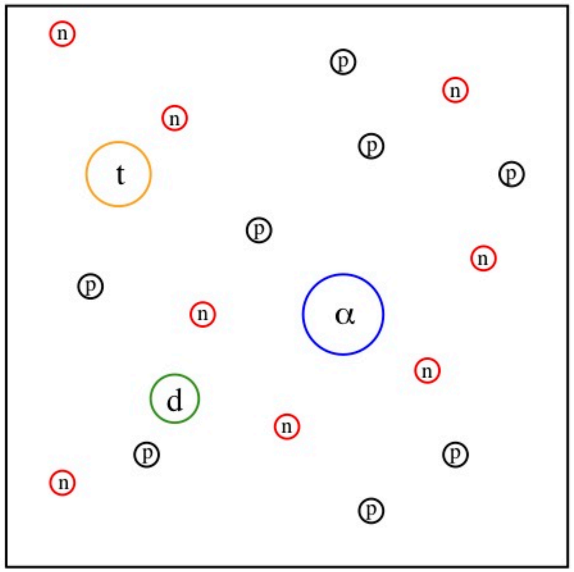
Ideal mixture of reacting components

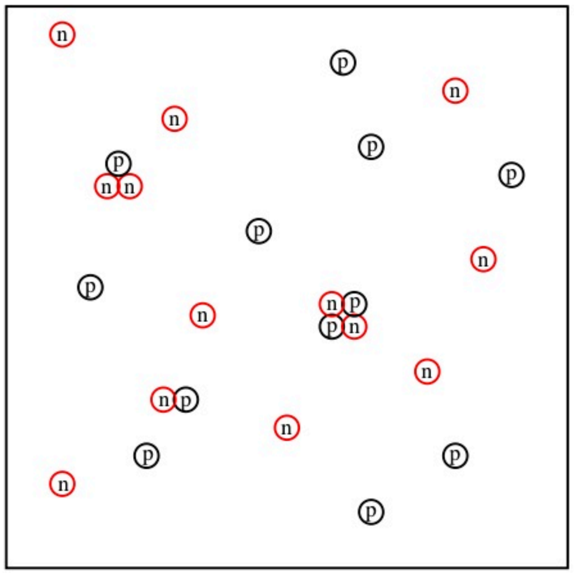
Mass action law

Physical picture:

"elementary" constituents

and their interaction





Interaction between the components

internal structure: Pauli principle

Quantum statistical (QS) approach,

quasiparticle concept, virial expansion

Composition of low-dense nuclear matter

$$
\begin{aligned}
n_p(T,\mu_p,\mu_n) &= \frac{1}{V}\sum_{A,\nu,K} Z_A f_A\{E_{A,\nu K} - Z_A\mu_p - (A-Z_A)\mu_n\} \\
n_n(T,\mu_p,\mu_n) &= \frac{1}{V}\sum_{A,\nu,K} (A-Z_A) f_A\{E_{A,\nu K} - Z_A\mu_p - (A-Z_A)\mu_n\}
\end{aligned}
$$

mass number A
charge Z_A
energy $E_{A,\nu,K}$
ν: internal quantum number

$$
f_{A(z)} = \frac{1}{\exp(z/T) - (-1)^A}
$$

Different approximations

medium effects

Ideal Fermi gas:

protons, neutrons,
(electrons, neutrinos,…)

Quasiparticle quantum liquid:

mean-field approximation
Skyrme, Gogny, RMF

bound state formation

Nuclear statistical equilibrium:

ideal mixture of all bound states
(clusters:) chemical equilibrium

Chemical equilibrium with quasiparticle clusters:

self-energy and Pauli blocking

Effective wave equation for the deuteron in matter

In-medium two-particle wave equation in mean-field approximation

$$\left(\frac{p_1^2}{2m_1}+\Delta_1+\frac{p_2^2}{2m_2}+\Delta_2\right)\Psi_{d,P}(p_1,p_2)+\sum_{p_1',p_2'}(1-f_{p_1}-f_{p_2})V(p_1,p_2;p_1',p_2')\Psi_{d,P}(p_1',p_2')$$

$$=E_{d,P}\Psi_{d,P}(p_1,p_2)$$

Add self-energy

Pauli-blocking

Thouless criterion

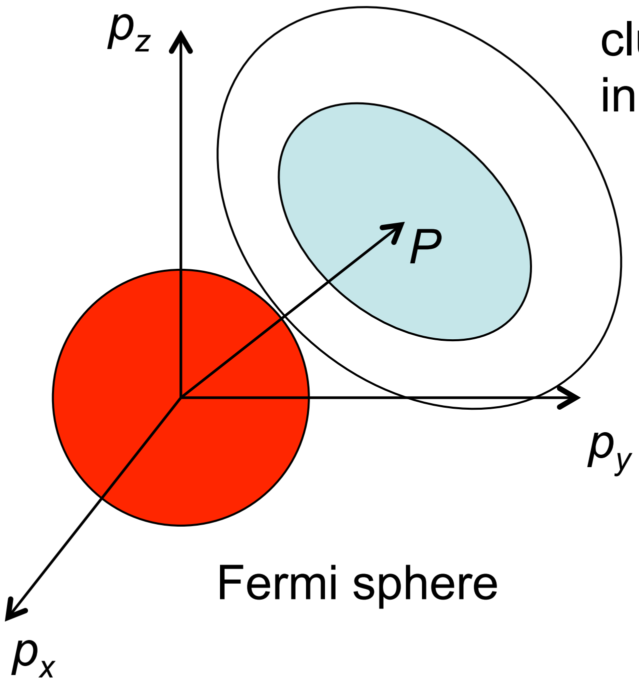
$$E_d(T,\mu)=2\mu$$

Fermi distribution function

$$f_p=\left[e^{(p^2/2m-\mu)/k_BT}+1\right]^{-1}$$

BEC-BCS crossover:
Alm et al.,1993

Pauli blocking – phase space occupation



cluster wave function (deuteron, alpha,…)
in momentum space

P - center of mass momentum

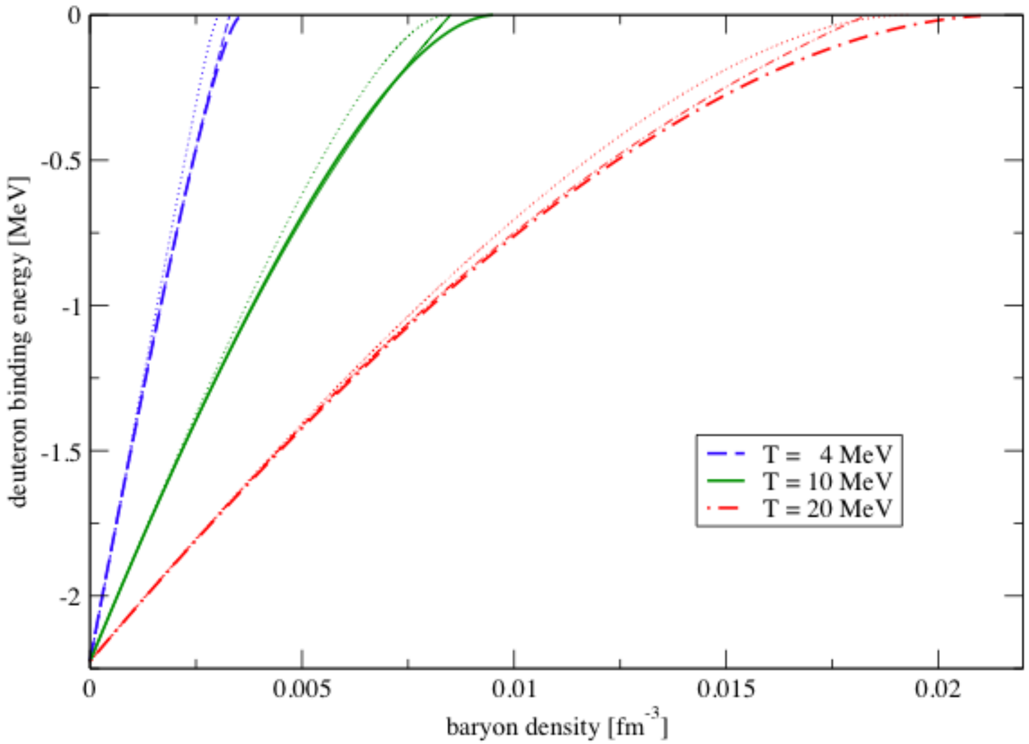
The Fermi sphere is forbidden,
deformation of the cluster wave function
in dependence on the c.o.m. momentum *P*

momentum space

The deformation is maximal at $P = 0$.
It leads to the weakening of the interaction
(disintegration of the bound state).

Shift of the deuteron binding energy

Dependence on nucleon density, various temperatures,
zero center of mass momentum



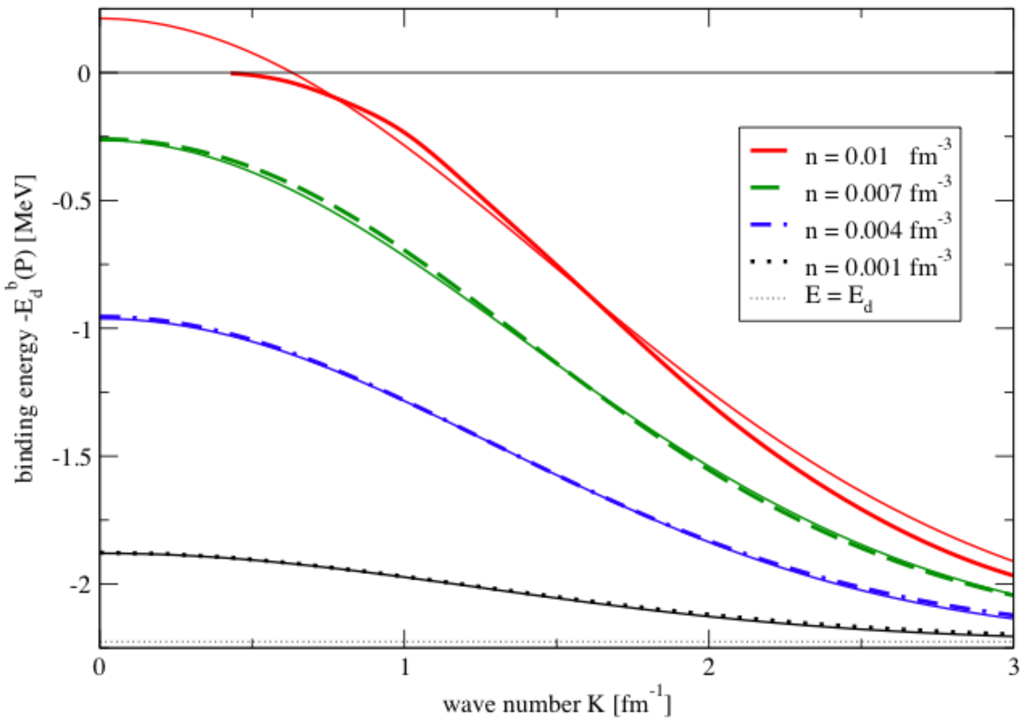
thin lines:

fit formula

G.R., NP A **867**, 66 (2011)

Shift of the deuteron bound state energy

Dependence on center of mass momentum,
various densities, T=10 MeV



thin lines:

fit formula

G.R., NP A **867**, 66 (2011)

Few-particle Schrödinger equation in a dense medium

4-particle Schrödinger equation with medium effects

$$\left(\left[E^{HF}(p_1)+E^{HF}(p_2)+E^{HF}(p_3)+E^{HF}(p_4)\right]\right)\Psi_{n,P}(p_1,p_2,p_3,p_4)$$

$$+\sum_{p_1',p_2'}(1-f_{p_1}-f_{p_2})V(p_1,p_2;p_1',p_2')\Psi_{n,P}(p_1',p_2',p_3,p_4)$$

$$+\{permutations\}$$

$$=E_{n,P}\Psi_{n,P}(p_1,p_2,p_3,p_4)$$

Medium modification of light clusters

- Single-particle, two-particle, etc. spectral function
  quasiparticle concept: Peak structures in the few-body spectral function

- Dispersion relation: quasiparticle energy is a function of total few-body momentum K, but also T, n_B, Y_e:
  $E_{A,nu,K}(T, n_B, Y_e)$

- Solution of a few-body equation. For practical use parametrization (like Skyrme or RMF, DFT)

- Alternative simple approaches to describe the medium effects:
  excluded volume

Composition of dense nuclear matter

$$n_p(T, \mu_p, \mu_n) = \frac{1}{V} \sum_{A,\nu,K} Z_A f_A\{E_{A,\nu K} - Z_A \mu_p - (A - Z_A)\mu_n\}$$

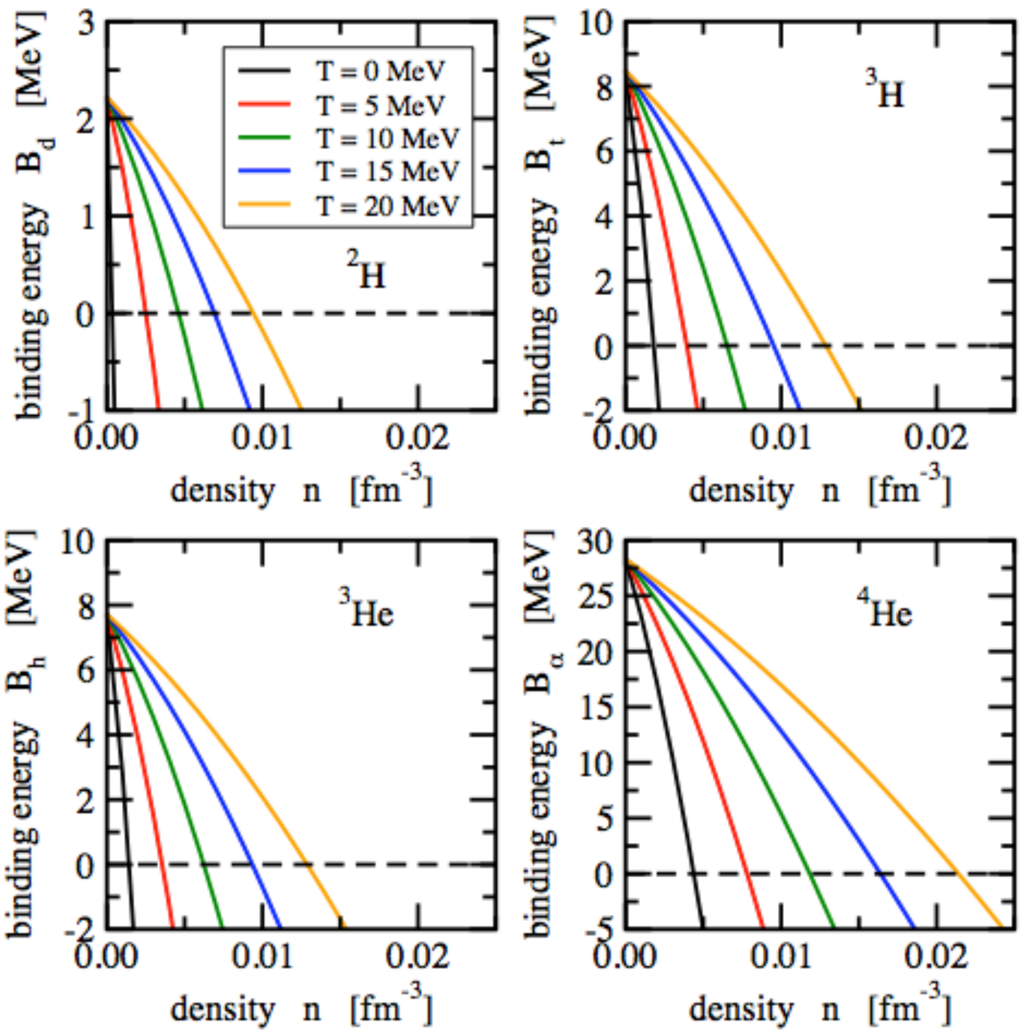
$$n_n(T, \mu_p, \mu_n) = \frac{1}{V} \sum_{A,\nu,K} (A - Z_A) f_A\{E_{A,\nu K} - Z_A \mu_p - (A - Z_A)\mu_n\}$$

mass number A
charge Z_A
energy $E_{A,\nu,K}$
ν: internal quantum number

$$f_{A(z)} = \frac{1}{\exp(z/T) - (-1)^A}$$

- Inclusion of excited states
- Medium effects:
  self-energy and Pauli blocking shifts of binding energies,
  Coulomb corrections due to screening (Wigner-Seitz,Debye)
- Bose-Einstein condensation

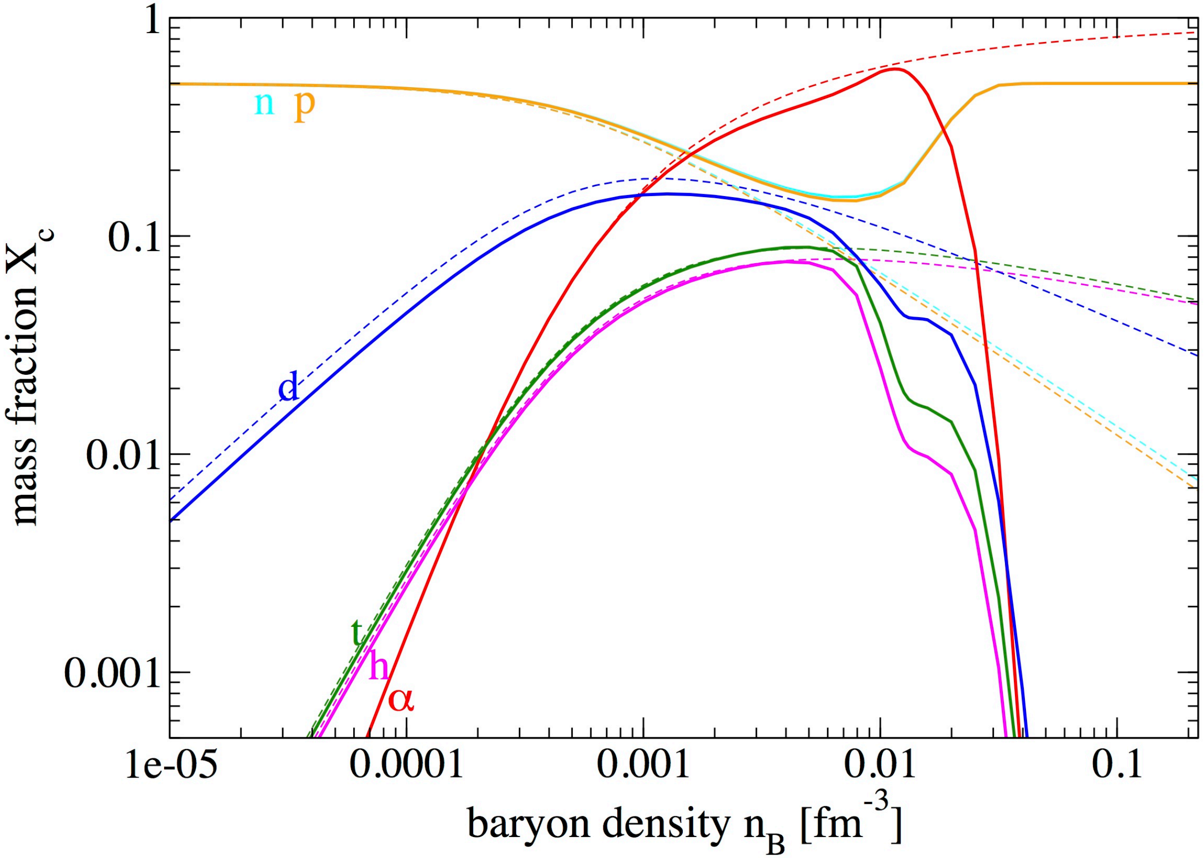
Shift of Binding Energies of Light Clusters



Symmetric matter

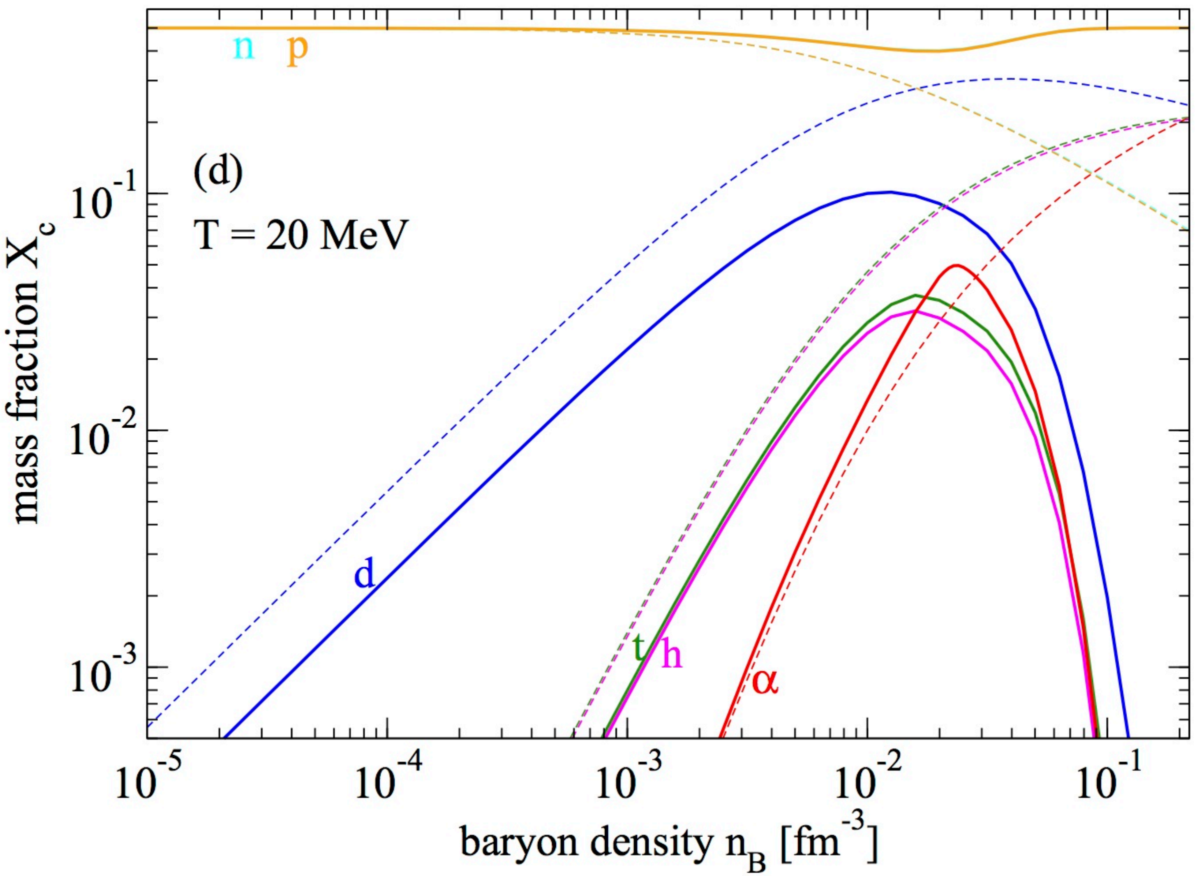
G.R., PRC **79**, 014002 (2009)
S. Typel et al.,
PRC **81**, 015803 (2010)

Light Cluster Abundances



Composition of symmetric matter in dependence on the baryon density n_B, T = 5 MeV.
Quantum statistical calculation (full) compared with NSE (dotted).

Light Cluster Abundances



Composition of symmetric matter in dependence on the baryon density n_B, T = 20 MeV.
Quantum statistical calculation (full) compared with NSE (dotted).

Intermediate nuclei

Quantum statistical calculation of cluster abundances in hot dense matter

For $4 < A < 12$ we have according Eq. (12)

$$\Delta E_{A0P}^{\rm Pauli} = \frac{\varrho_A^* \Lambda^3}{4} {\rm e}^{-\frac{x^2}{4\pi\delta}} \frac{\hbar\omega}{\delta^{3/2}} \left\{ \frac{3}{\delta} - 4\left(\frac{V_0}{\hbar\omega} + \frac{3}{2}\right) + \frac{x^2(\delta-1)\hbar\omega}{2\pi\delta^2} \right.$$
$$-2(A-4)\frac{x^2(\delta-1)}{12\pi\delta^2}\left[\left(\frac{V_0}{\hbar\omega}+\frac{5}{2}\right) - \frac{x^2(\delta-1)}{8\pi\delta^2}\right]$$
$$\left. -2(A-4)\frac{1}{3\delta}\left[-\frac{5x^2(\delta-1)}{8\pi\delta^2} + \frac{3}{2}\left(\frac{V_0}{\hbar\omega}+\frac{5}{2}\right)\right] + 2(A-4)\frac{5}{8\delta^2} \right\},$$
$$\delta = 1 + \frac{\hbar\omega}{2k_BT}.$$

G.R., J. Phys.: Conf. Series **436,** 012070 (2013)

Heavier clusters

In principle, clusters with arbitrary A should be considered.

Clusters with 4 < A <12 : weakly bound, no significant contributions

Heavy clusters: Thomas-Fermi model,

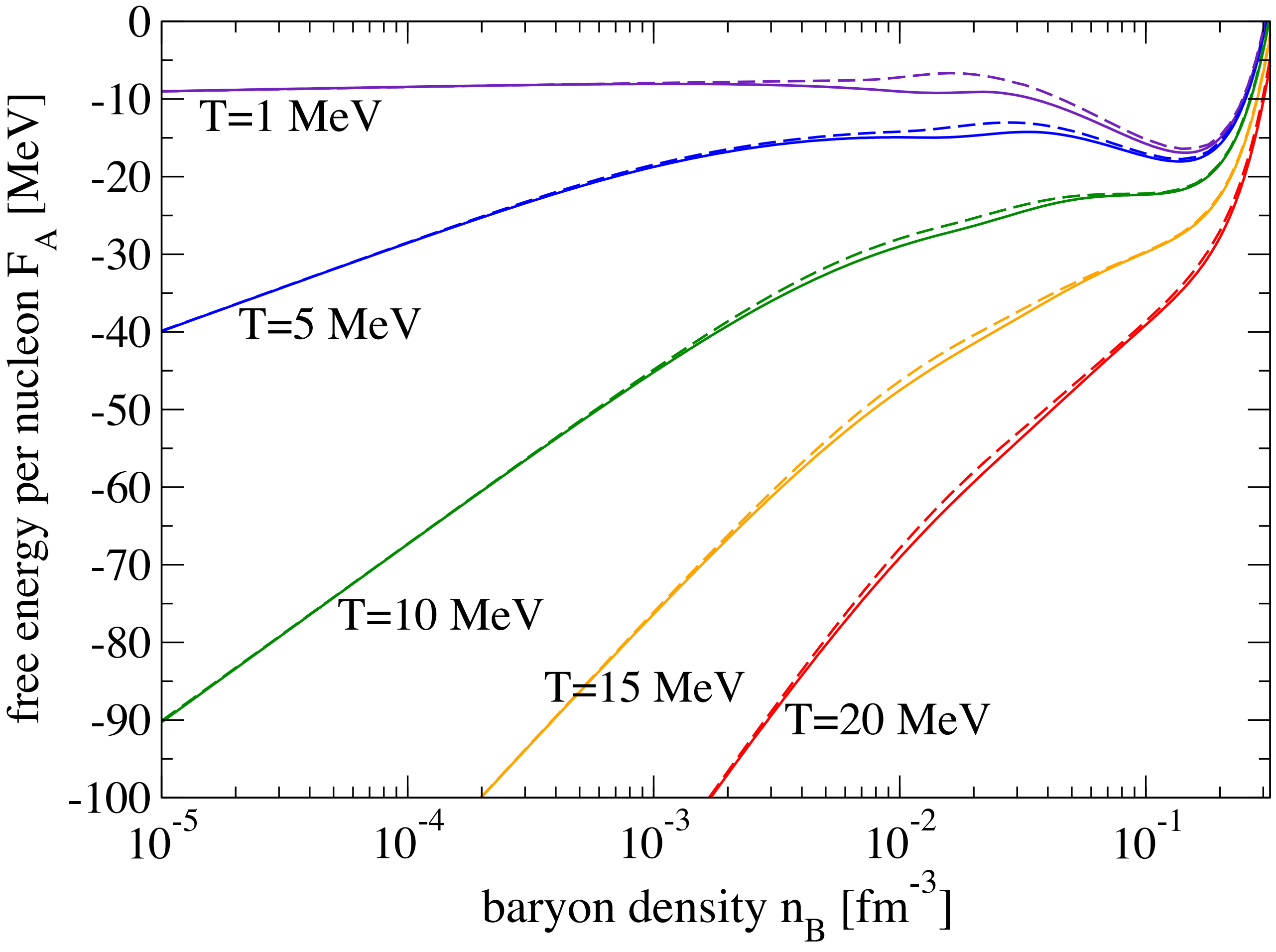
$$\Delta E_{A\nu P}(n_B) = \sum_{\tau=n,p} \int d^3 r \Delta E_\tau^{\rm SE}(n_n^A(r), n_p^A(r)) n_\tau$$
$$\times \int_{\Lambda_\tau p_F(n_\tau^A(r))}^{\infty} \frac{y dy}{2\pi x_\tau} \left[e^{-(y-x_\tau)^2/4\pi} - e^{-(y+x_\tau)^2/4\pi} \right]$$

$$n_B^A(r) = \frac{3A}{4\pi R^3} \frac{1}{1+(\pi b/R)^2} \left[\frac{1}{1+e^{(r-R)/b}} + \frac{1}{1+e^{(-r-R)/b}} \right]$$

V.V. Burov, Yu.N. Eldyshev, V.K. Lukyanov, and Yu.S. Pol, Dubna-preprint E4-8029, Joint Institute for Nuclear Research, Dubna 1974.

Open problems

Symmetric matter: phase instability



Dashed lines: no continuum correlations

Light Clusters and Pasta Phases in Warm and Dense Nuclear Matter

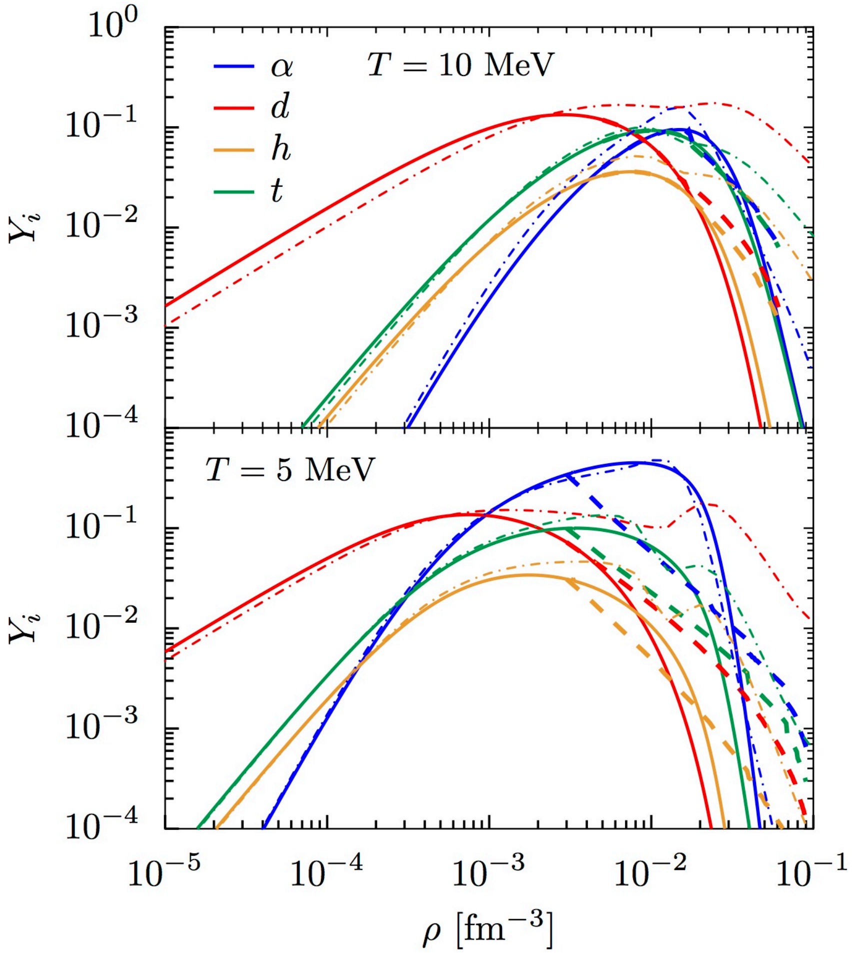


FIG. 7. Cluster fractions with $\eta = 0.70$ and $Y_p = 0.41$ as a function of density, for $T = 5$ MeV (bottom) and $T = 10$ MeV (top panels). Results for a TF calculation (dashed), homogeneous matter with clusters (solid), and the QS approach (dash-dotted lines) are shown. For $T = 5$ MeV, the TF calculation includes the five geometrical configurations, droplet, rod, slab, tube and bubble, for the heavy clusters.

Sidney S. Avancini et al., arXiv:1704.00054

Light Clusters and Pasta Phases in Warm and Dense Nuclear Matter

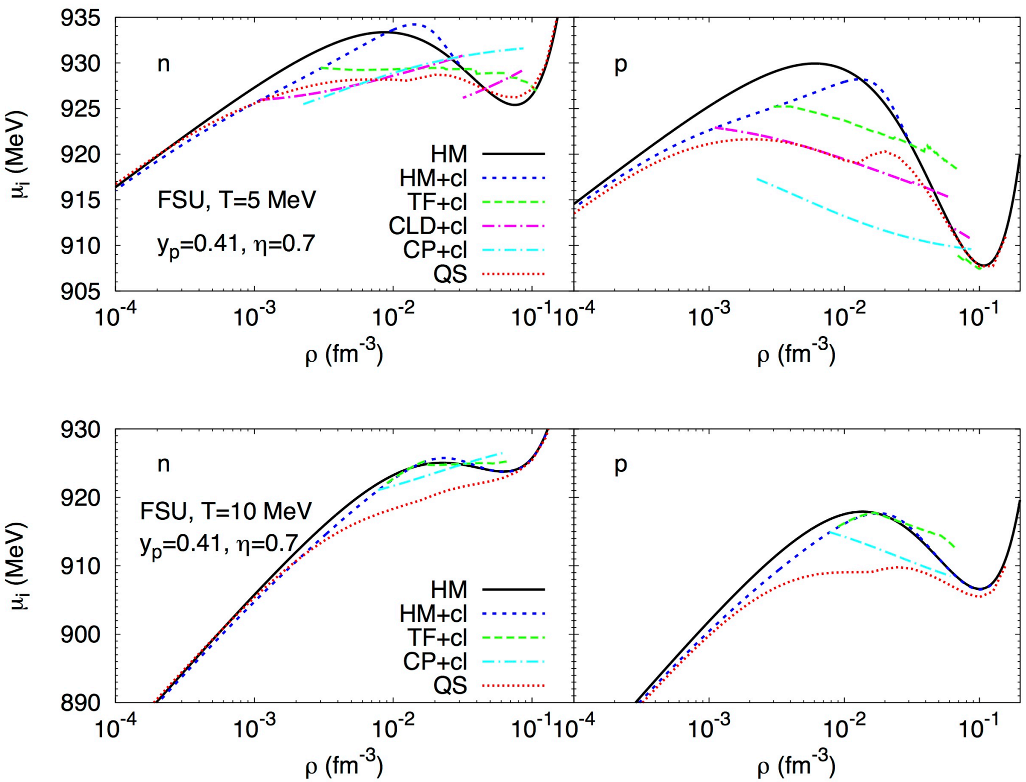


FIG. 8. Neutron (left panels) and proton (right panels) chemical potentials with $\eta = 0.7$ and $Y_p = 0.41$ as a function of density at $T = 5$ MeV (top) and $T = 10$ MeV (bottom), for homogeneous nuclear matter (HM) (solid), nuclear matter with light clusters (blue short-dashed), and mean-field pasta calculations with clusters [TF (green, dashed), CLD (pink, dash-dotted), CP (cyan, dash-dotted)]. QS results (red, dotted) are also shown.

[Sidney S. Avancini et al., arXiv:1704.00054]

Different approximations

medium effects

Ideal Fermi gas:

protons, neutrons,
(electrons, neutrinos,…)

Quasiparticle quantum liquid:

mean-field approximation
Skyrme, Gogny, RMF

bound state formation

Nuclear statistical equilibrium:

ideal mixture of all bound states
(clusters:) chemical equilibrium

Chemical equilibrium with quasiparticle clusters:

self-energy and Pauli blocking

continuum contribution

Second virial coefficient:

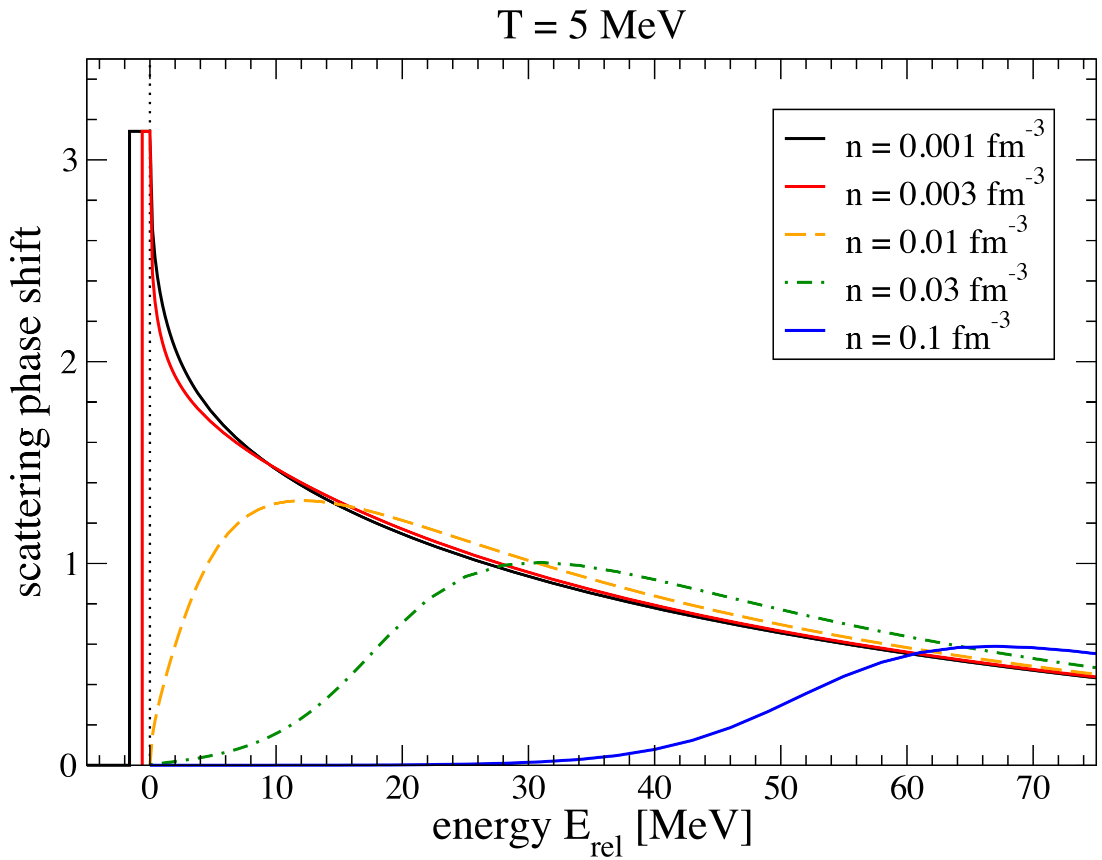
account of continuum contribution,
scattering phase shifts, Beth-Uhl.E.

Generalized Beth-Uhlenbeck formula:

medium modified binding energies,
medium modified scattering phase shifts

Deuteron-like scattering phase shifts

Virial coeff. $\propto e^{-E_d^0/T} - 1 + \frac{1}{\pi T} \int_0^\infty dE\; e^{-E/T} \left\{ \delta_c(E) - \frac{1}{2} \sin[2\delta_c(E)] \right\}$



deuteron bound state -2.2 MeV

G. Roepke, J. Phys.: Conf. Series 569, 012031 (2014).

EOS: continuum contributions

Partial density of channel A,c at P (for instance, $^3S_1$= d):

$$z_{A,c}^{\rm part}(\mathbf{P};T,\mu_n,\mu_p) = e^{(N\mu_n+Z\mu_p)/T}\left\{\sum_{\nu_c}^{\rm bound} g_{A,\nu_c}\, e^{-E_{A,\nu_c}(\mathbf{P})/T}\,\Theta\left[-E_{A,\nu_c}(\mathbf{P})+E_{A,c}^{\rm cont}(\mathbf{P})\right]+z_{A,c}^{\rm cont}(\mathbf{P})\right\}$$

separation: bound state part – continuum part ?

$$z_c^{\rm part}(\mathbf{P};T,n_B,Y_p) = e^{[N\mu_n+Z\mu_p-NE_n(\mathbf{P}/A;T,n_B,Y_p)-ZE_p(\mathbf{P}/A;T,n_B,Y_p)]/T}$$
$$\times g_c\left\{\left[e^{-E_c^{\rm intr}(\mathbf{P};T,n_B,Y_p)/T}-1\right]\Theta\left[-E_c^{\rm intr}(\mathbf{P};T,n_B,Y_p)\right]+v_c(\mathbf{P};T,n_B,Y_p)\right\}$$

parametrization (d – like):

$$v_c(\mathbf{P}=0;T,n_B,Y_p)\approx\left[1.24+\left(\frac{1}{v_{T_I=0}(T)}-1.24\right)e^{\gamma_c n_B/T}\right]^{-1}.$$

$$v_d^0(T)=v_{T_I=0}^0(T)\approx 0.30857+0.65327\;e^{-0.102424\;T/\mathrm{MeV}}$$

G. Roepke, PRC 92,054001 (2015)

Different approximations

bound state formation

continuum contribution

chemical & physical picture

medium effects

Ideal Fermi gas:

protons, neutrons,
(electrons, neutrinos,…)

Nuclear statistical equilibrium:

ideal mixture of all bound states
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Second virial coefficient:

account of continuum contribution,
scattering phase shifts, Beth-Uhl.E.

Cluster virial approach:

all bound states (clusters)
scattering phase shifts of all pairs

Quasiparticle quantum liquid:

mean-field approximation
Skyrme, Gogny, RMF

Chemical equilibrium with quasiparticle clusters:

self-energy and Pauli blocking

Generalized Beth-Uhlenbeck formula:

medium modified binding energies,
medium modified scattering phase shifts

Correlated medium

phase space occupation by all bound states
in-medium correlations,
quantum condensates

Composition of dense nuclear matter

$$n_p(T,\mu_p,\mu_n) = \frac{1}{V}\sum_{A,\nu,K} Z_A f_A\{E_{A,\nu K} - Z_A\mu_p - (A-Z_A)\mu_n\}$$

$$n_n(T,\mu_p,\mu_n) = \frac{1}{V}\sum_{A,\nu,K} (A-Z_A) f_A\{E_{A,\nu K} - Z_A\mu_p - (A-Z_A)\mu_n\}$$

mass number A
charge Z_A
energy $E_{A,\nu,K}$
ν: internal quantum number

$$f_{A(z)} = \frac{1}{\exp(z/T) - (-1)^A}$$

- Medium effects: correct behavior near saturation
  self-energy and Pauli blocking shifts of binding energies,
  Coulomb corrections due to screening (Wigner-Seitz,Debye)

- Inclusion of excited states and continuum correlations,
  correct virial expansions

- Bose-Einstein condensation, phase instabilities

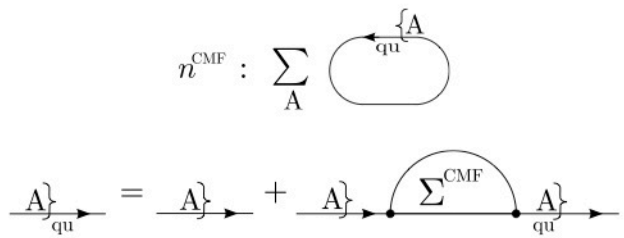
Cluster virial expansion for nuclear matter within a quasiparticle approach

Generalized Beth-Uhlenbeck approach

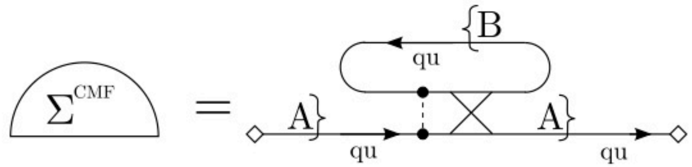
$$n_1^{\mathrm{qu}}(T,\mu_p,\mu_n) = \sum_{A,Z,\nu} \frac{A}{\Omega} \sum_{\substack{\vec{P} \\ P>P_{\mathrm{Mott}}}} f_A\big(E_{A,Z,\nu}(\vec{P};T,\mu_p,\mu_n),\mu_{A,Z,\nu}\big)$$

$$\begin{aligned} n_2^{\mathrm{qu}}(T,\mu_p,\mu_n) \;=\; & \sum_{A,Z,\nu} \sum_{A',Z',\nu'} \frac{A+A'}{\Omega} \sum_{\vec{P}} \sum_{c} g_c \frac{1+\delta_{A,Z,\nu;A',Z',\nu'}}{2\pi} \\ & \times \int_0^\infty dE\, f_{A+A'}\left(E_c(\vec{P};T,\mu_p,\mu_n)+E,\mu_{A,Z}+\mu_{A',Z'}\right) 2\sin^2(\delta_c)\,\frac{d\delta_c}{dE} \end{aligned}$$

Avoid double counting

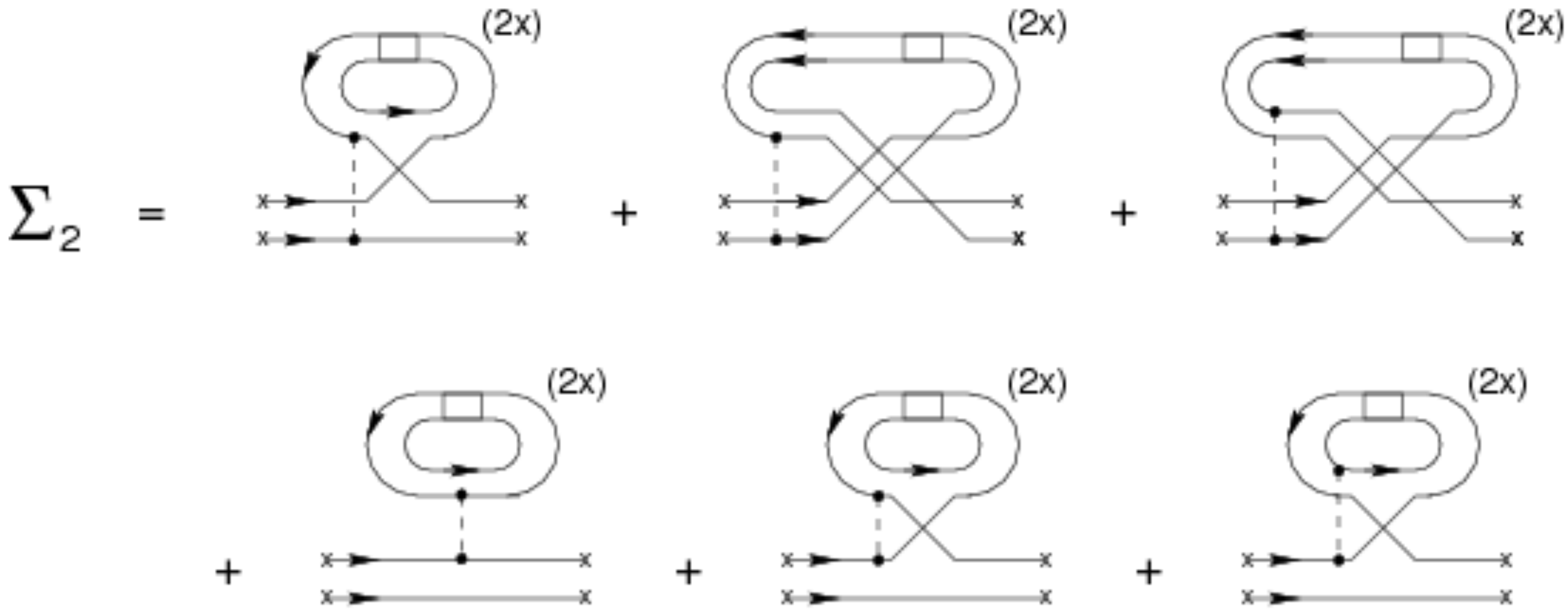


Generating functional



G.R., N. Bastian, D. Blaschke, T. Klaehn, S. Typel, H. Wolter, NPA **897**, 70 (2013)

Correlations in the medium



cluster mean-field approximation

Pauli blocking, correlated medium

In-medium Schroedinger equation

$$[E_{\tau_1}(\mathbf{p}_1;T,\mu_n,\mu_p)+\dots+E_{\tau_A}(\mathbf{p}_A;T,\mu_n,\mu_p)-E_{A\nu}(\mathbf{P};T,\mu_n,\mu_p)]\psi_{A\nu\mathbf{P}}(1\dots A)$$
$$+\sum_{1'\dots A'}\sum_{i<j}[1-n(i;T,\mu_n,\mu_p)-n(j;T,\mu_n,\mu_p)]V(ij,i'j')\prod_{k\neq i,j}\delta_{kk'}\psi_{A\nu\mathbf{P}}(1'\dots i'\dots j'\dots A')=0$$

effective occupation numbers

$$n(1)=f_{1,\tau_1}(1)+\sum_{B=2}^{\infty}\sum_{\bar\nu,\bar{\mathbf{P}}}\sum_{2\dots B}Bf_B\left(E_{B,\bar\nu}(\bar{\mathbf{P}};T,\mu_n,\mu_p)\right)|\psi_{B\bar\nu\bar{\mathbf{P}}}(1\dots B)|^2$$

effective Fermi distribution

$$n(1;T,\mu_n,\mu_p)\approx f_{1,\tau_1}(1;T_{\text{eff}},\mu_n^{\text{eff}},\mu_p^{\text{eff}})$$

blocking by all nucleons

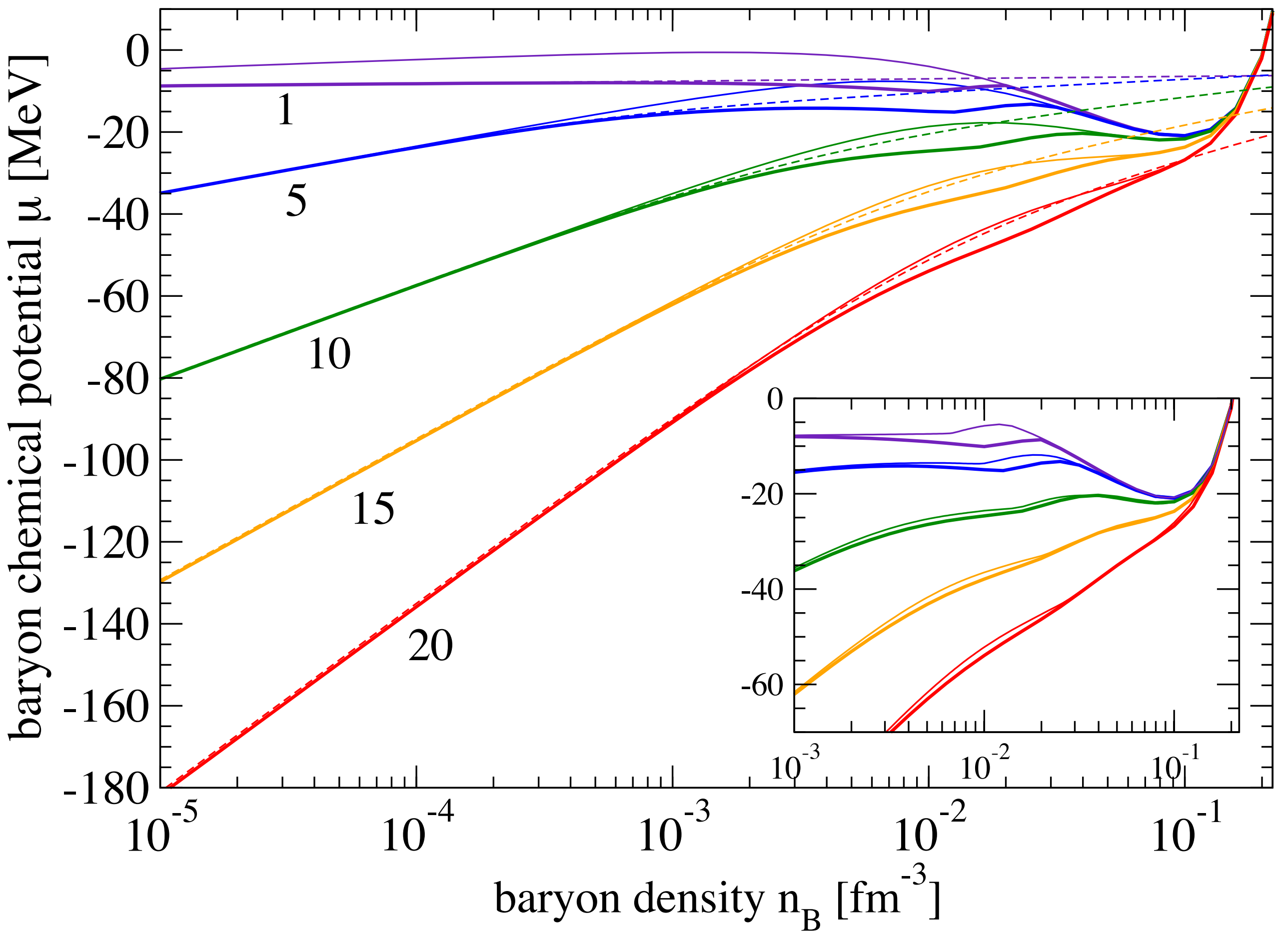
$$n(1;T,\mu_n,\mu_p)\approx \tilde f_{1,\tau_1}(1;T_{\text{eff}},n_B,Y_p)$$

effective temperature $\quad T_{\text{eff}}\approx 5.5\,\text{MeV}+0.5\,T+60\,n_B\,\text{MeV fm}^3$

G. Roepke, PRC 92,054001 (2015)

Symmetric matter: chemical potential

QS compared with RMF (thin) and NSE (dotted)



G. Roepke, arXiv: 1411.4593, submitted to PRC

Insert: no continuum correlations (thin)

Heavy ion collisions

EOS at low densities from HIC

PRL **108,** 172701 (2012) PHYSICAL REVIEW LETTERS week ending 27 APRIL 2012

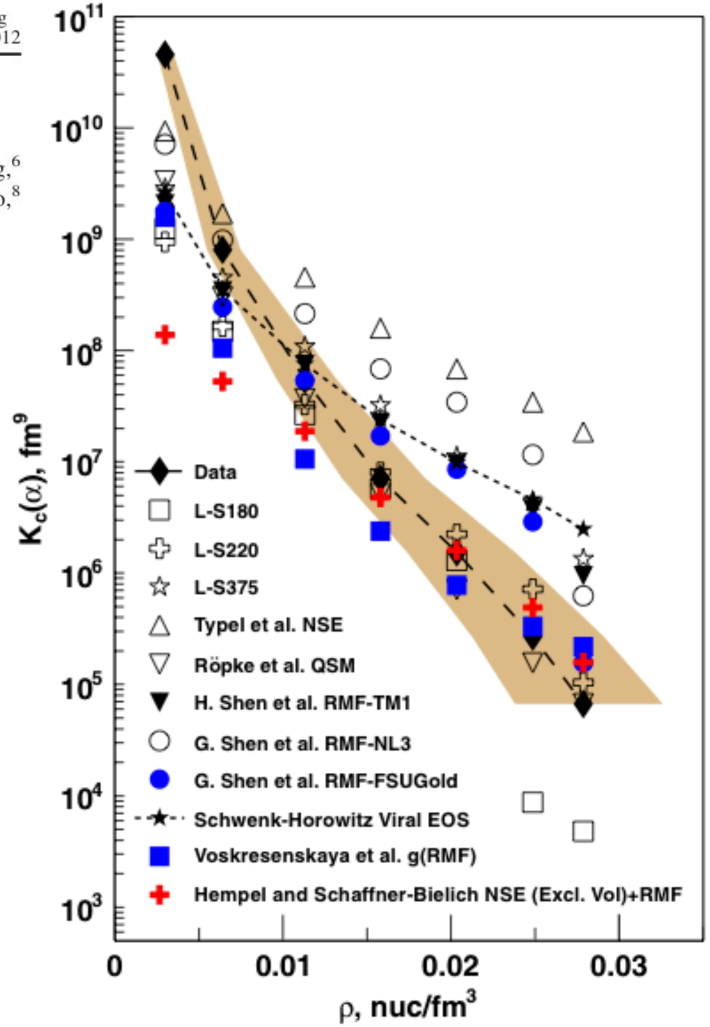
Laboratory Tests of Low Density Astrophysical Nuclear Equations of State

L. Qin,[1] K. Hagel,[1] R. Wada,[2,1] J. B. Natowitz,[1] S. Shlomo,[1] A. Bonasera,[1,3] G. Röpke,[4] S. Typel,[5] Z. Chen,[6] M. Huang,[6] J. Wang,[6] H. Zheng,[1] S. Kowalski,[7] M. Barbui,[1] M. R. D. Rodrigues,[1] K. Schmidt,[1] D. Fabris,[8] M. Lunardon,[8] S. Moretto,[8] G. Nebbia,[8] S. Pesente,[8] V. Rizzi,[8] G. Viesti,[8] M. Cinausero,[9] G. Prete,[9] T. Keutgen,[10] Y. El Masri,[10] Z. Majka,[11] and Y. G. Ma[12]

Yields of clusters from HIC: p, n, d, t, h, α

chemical constants

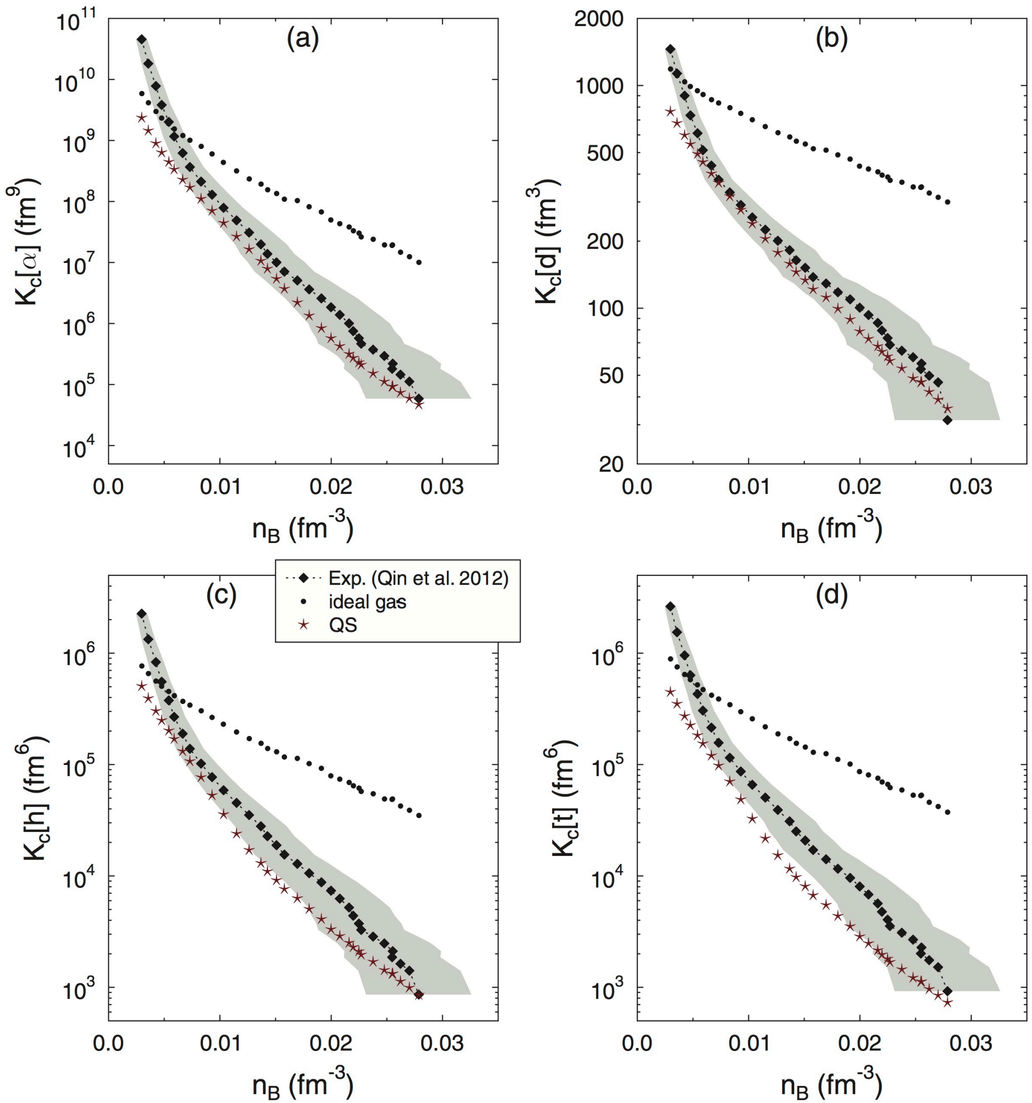
$$K_c(A,Z) = \rho_{(A,Z)}/[(\rho_p)^Z(\rho_n)^N]$$



inhomogeneous,
non-equilibrium

Chemical constants

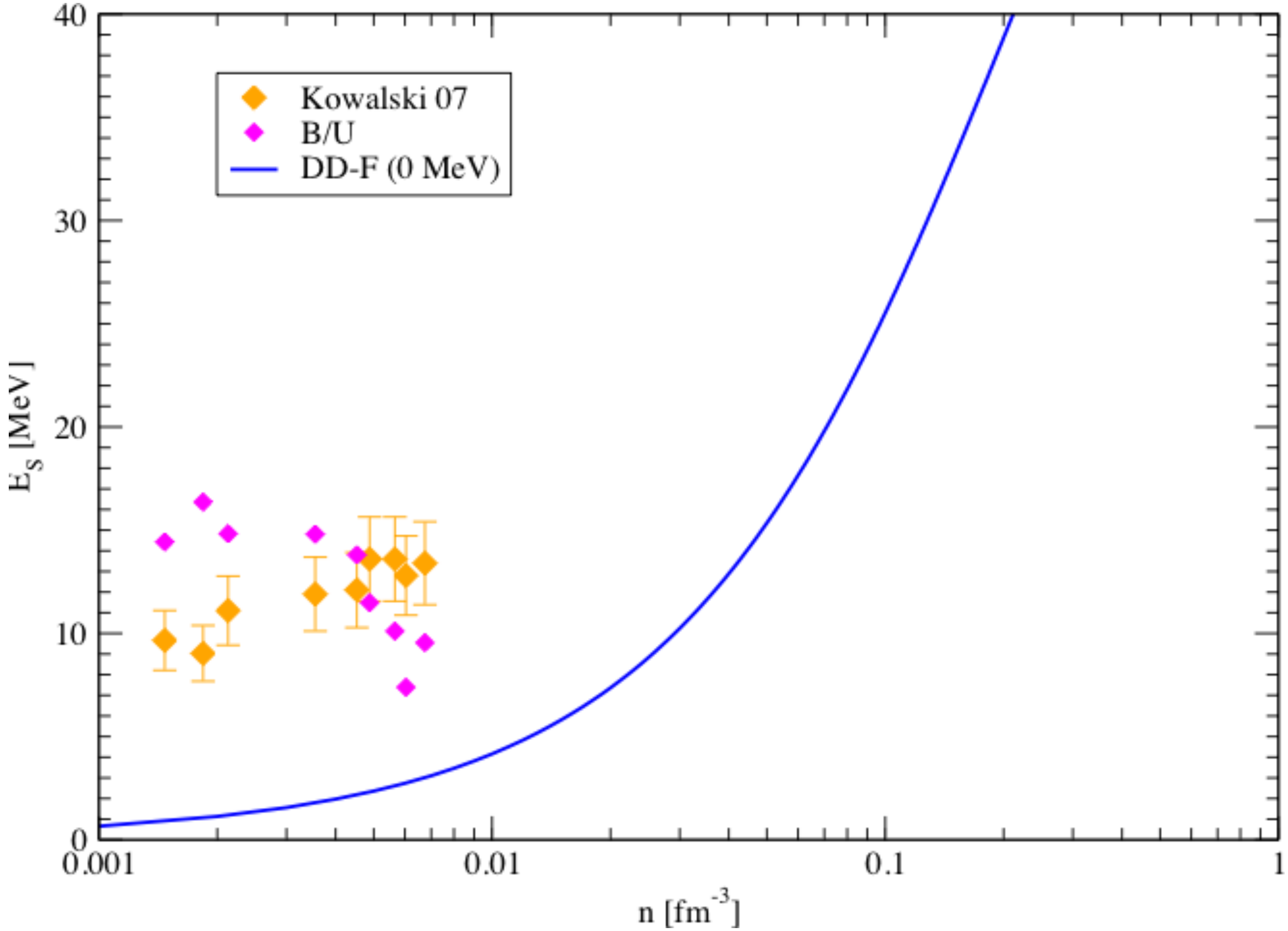
Comparison:
experiment
NSE (ideal mixture)
QS (quantum statistics)



Matthias Hempel,
Kris Hagel,
Joseph Natowitz,
Gerd Röpke, and
Stefan Typel
Phys. Rev. C **91**,
045805 (2015)

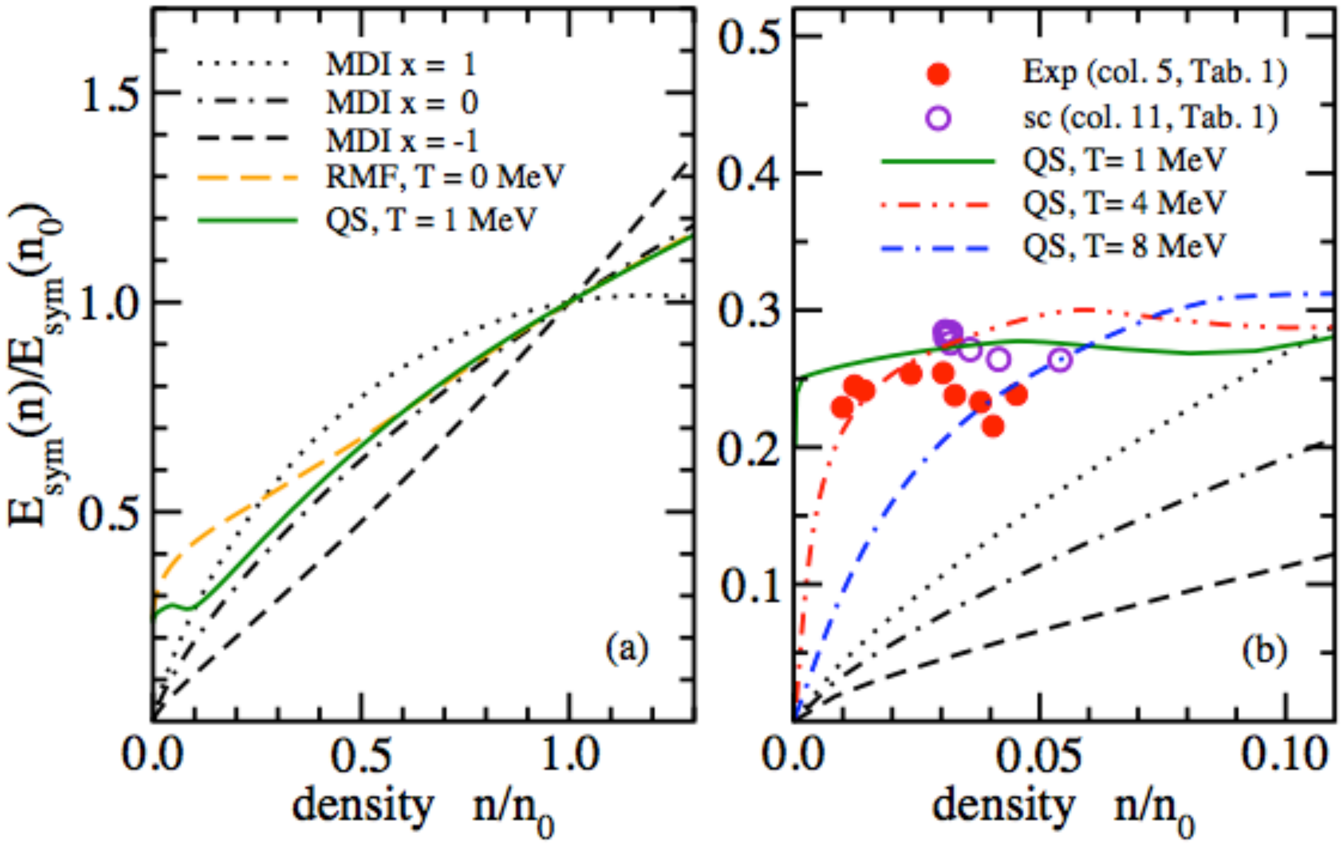
Symmetry energy

Heavy-ion collisions, spectra of emitted clusters,
temperature (3 - 10 MeV), free energy



S. Kowalski et al.,
PRC **75**, 014601
(2007)

Symmetry Energy

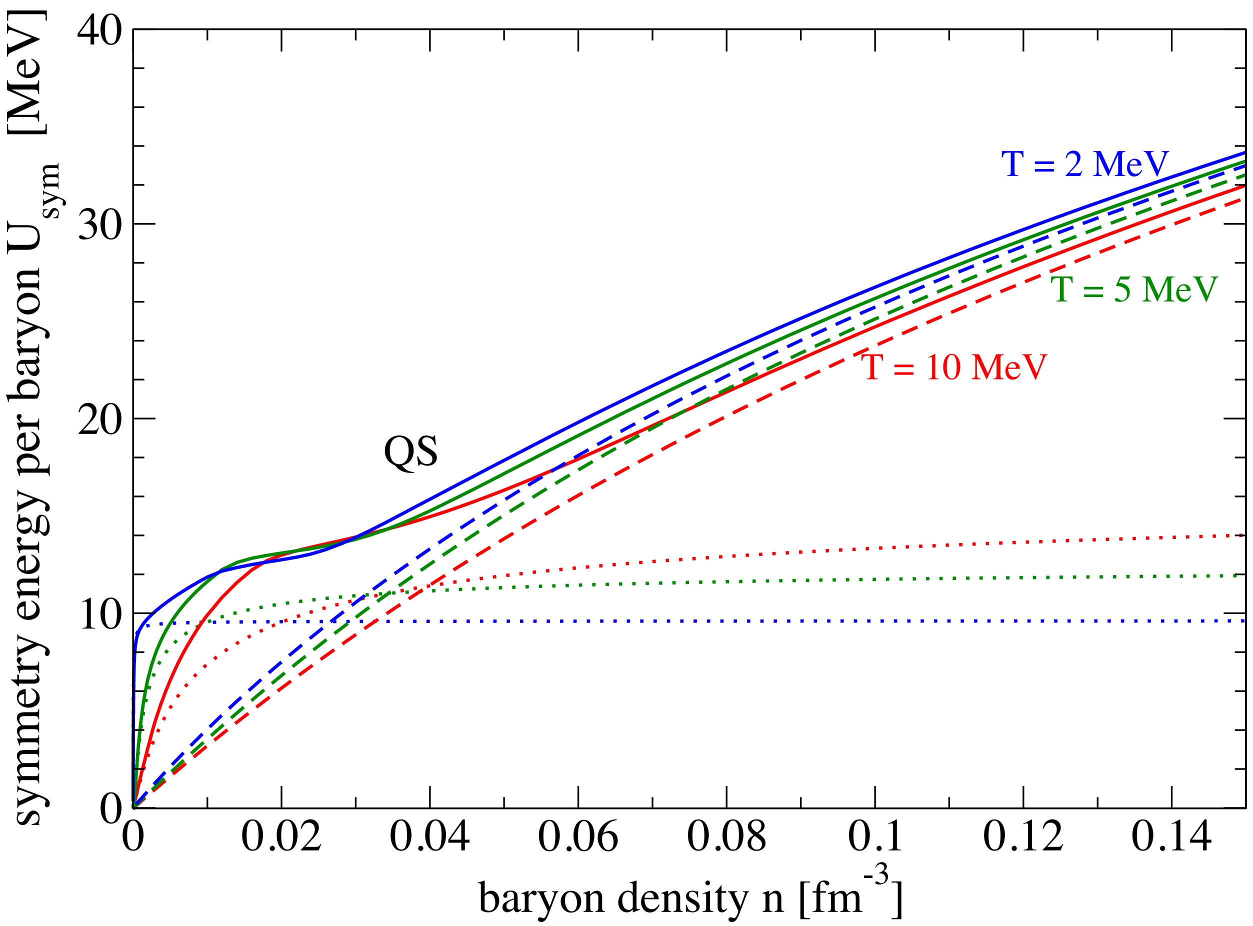


Scaled internal symmetry energy as a function of the scaled total density.
MDI: Chen et al., QS: quantum statistical, Exp: experiment at TAMU

J.Natowitz et al. PRL, May 2010

Light clusters and symmetry energy

dependent on T

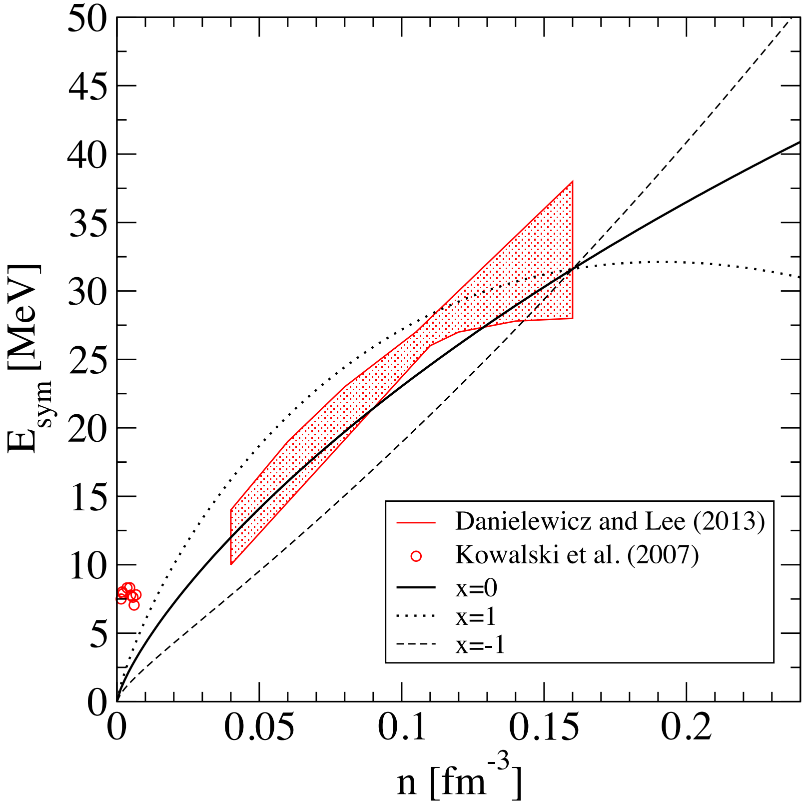


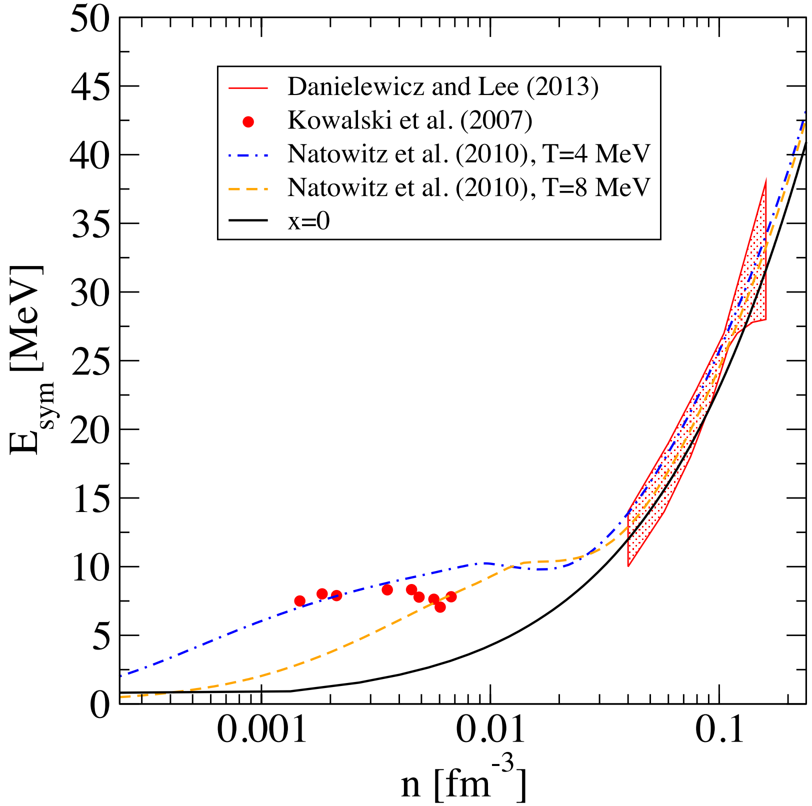
Full lines: QS
Dashed: RMF

Dotted: NSE

K. Hagel et al.Eur. Phys. J. A (2014) 50: 39

Symmetry energy: low density limit





K. Hagel et al., Eur. Phys. J. A (2014) 50: 39

Correlations in nuclei

Cluster formation in nuclei

- Clustering in low-density matter
- Alpha-like clustering and condensation in expanded N=Z nuclei: Hoyle state
- Clustering in the neck region
- Clustering at the surface of heavy nuclei,
  Preformation for alpha decay
- Pairing - quartetting

Self-conjugate 4n nuclei

^{12}C:

0^+ state at 0.39 MeV above the 3α threshold energy:
α cluster interact predominantly in relative S waves,
gaslike structure

α-particle condensation in low-density nuclear matter
$(\rho \leq \rho_0/5)$

$n\alpha$ cluster condensed states
– a general feature in $N = Z$ nuclei?

Self-conjugate 4n nuclei

$n\alpha$ nuclei: ^{8}Be, ^{12}C, ^{16}O, ^{20}Ne, ^{24}Mg, ...

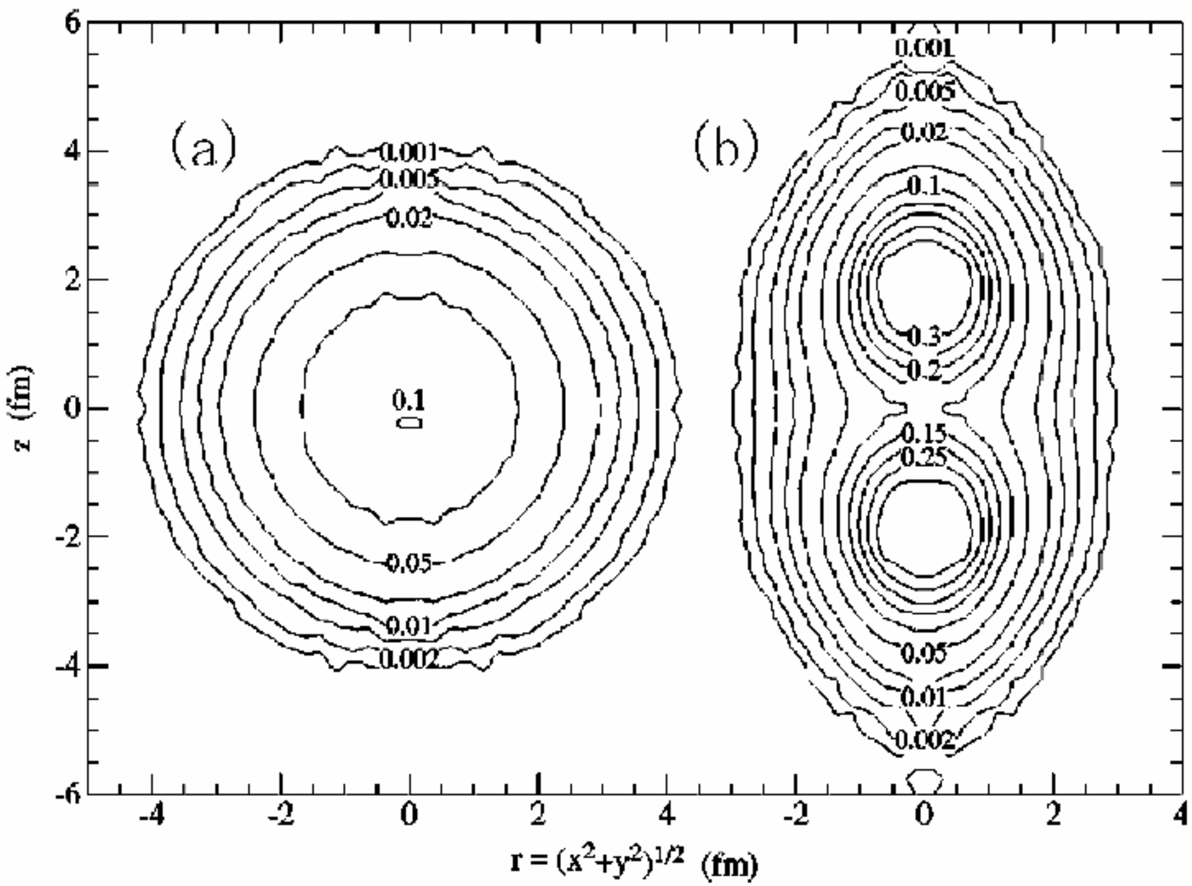
Single-particle shell model, or

Cluster type structures

ground state, excited states

$n\alpha$ break up at the threshold energy $E_{n\alpha}^{\mathrm{thr}} = nE_{\alpha}$

Alpha cluster structure of Be 8



R.B. Wiringa et al., PRC **63**, 034605 (01)

Contours of constant density, plotted in cylindrical coordinates, for 8Be(0+) .
The left side is in the laboratory frame while the right side is in the intrinsic frame.

Results

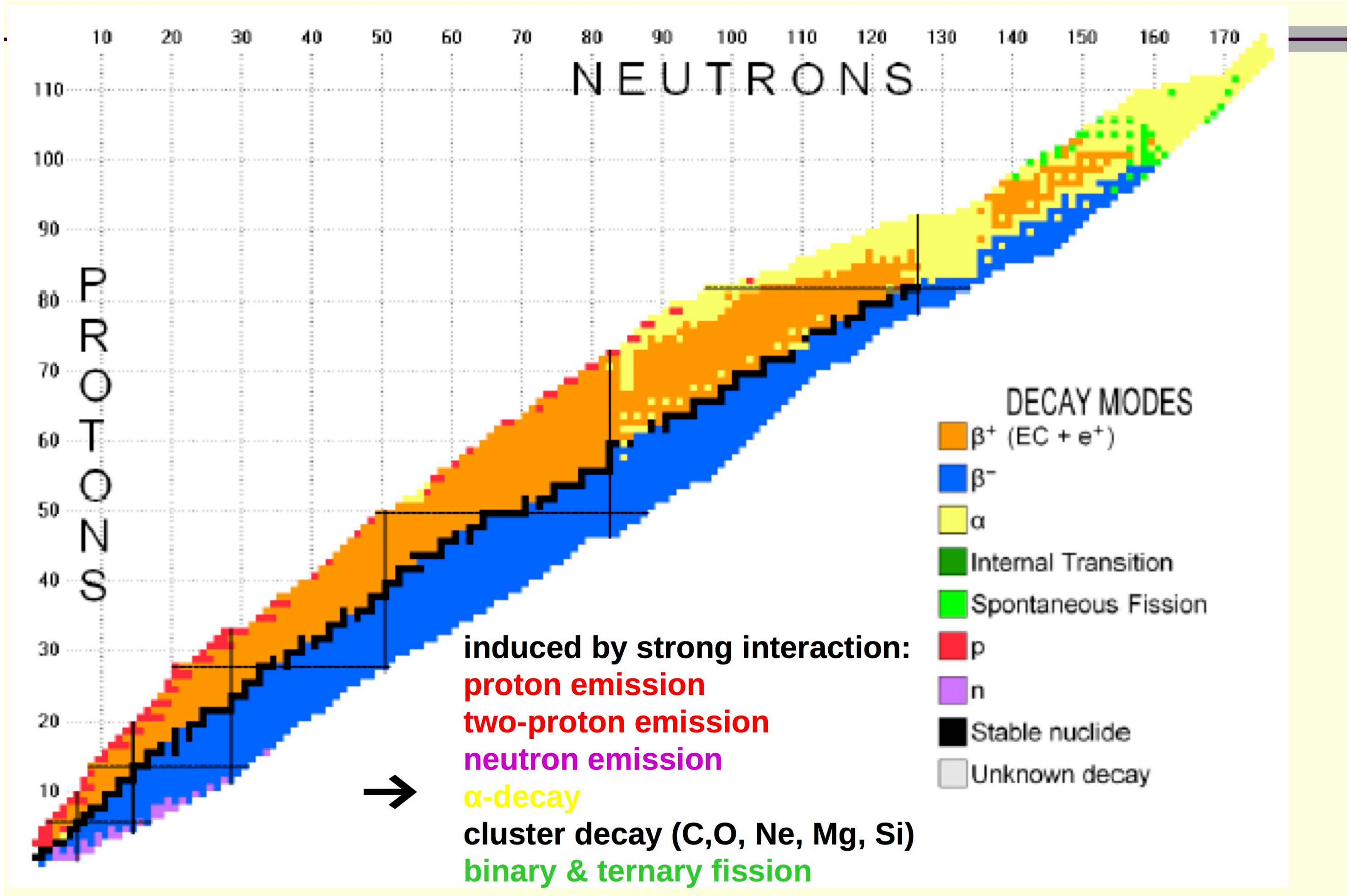
		E_k (MeV)	E_{exp} (MeV)	$E_k - E^{thr}_{n\alpha}$ (MeV)	$(E - E^{thr}_{n\alpha})_{exp}$ (MeV)	$\sqrt{\langle r^2 \rangle}$ (fm)	$\sqrt{\langle r^2 \rangle}_{exp}$ (fm)
^{12}C	$k = 1$	-85.9	-92.16 (0_1^+)	-3.4	-7.27	2.97	2.65
	$k = 2$	-82.0	-84.51 (0_2^+)	+0.5	0.38	4.29	
	$E^{thr}_{3\alpha}$	-82.5	-84.89				
^{16}O	$k = 1$	-124.8	-127.62 (0_1^+)	-14.8	-14.44	2.59	2.73
		(−128.0)*		(−18.0)*			
	$k = 2$	-116.0	-116.36 (0_3^+)	-6.0	-3.18	3.16	
	$k = 3$	-110.7	-113.62 (0_5^+)	-0.7	-0.44	3.97	
	$E^{thr}_{4\alpha}$	-110.0	-113.18				
^{8}Be				−0.17	+0.1		

Tabelle 1: Comparison of the generator coordinate method calculations with experimental values. $E^{thr}_{n\alpha} = nE_\alpha$ denotes the threshold energy for the decay into α-clusters, the values marked by * correspond to a refined mesh.

M. Chernykh et al., PRL **98**, 032501 (07); Y. Funaki et al., PRL **101**, 082502 (08)

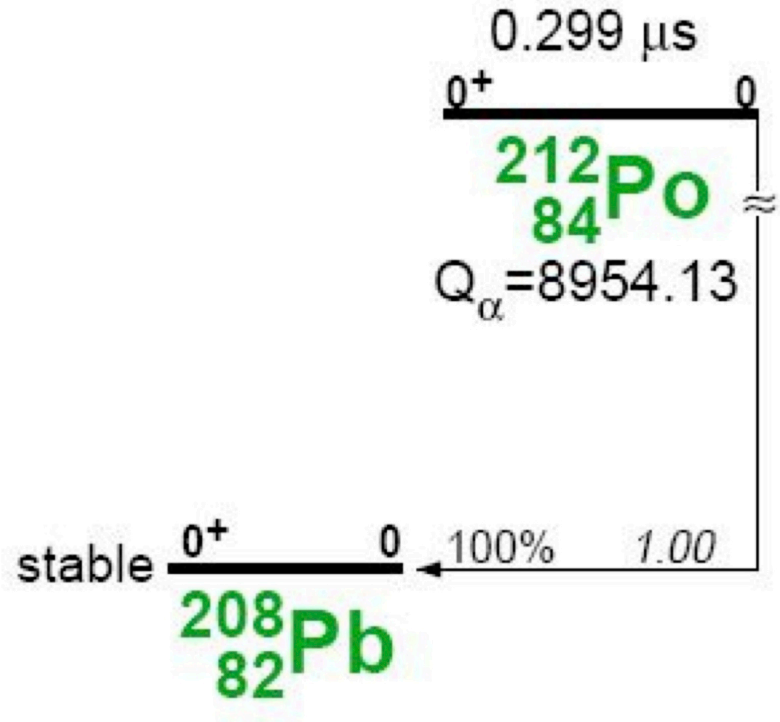
α decay of heavy nuclei

Decay modes of nuclei

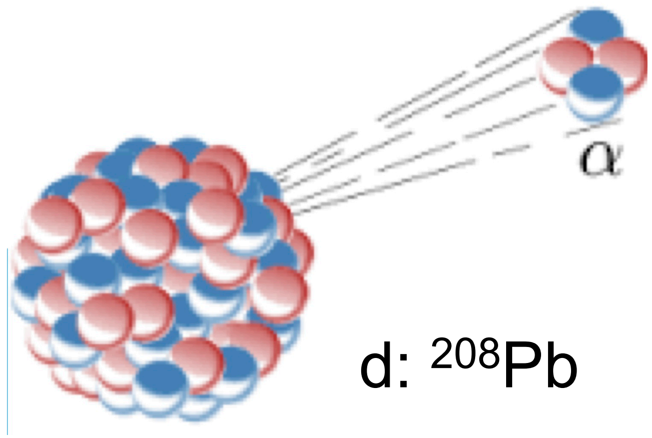


induced by strong interaction:
proton emission
two-proton emission
neutron emission
→ **α-decay**
cluster decay (C,O, Ne, Mg, Si)
binary & ternary fission

α decay of ^{212}Po



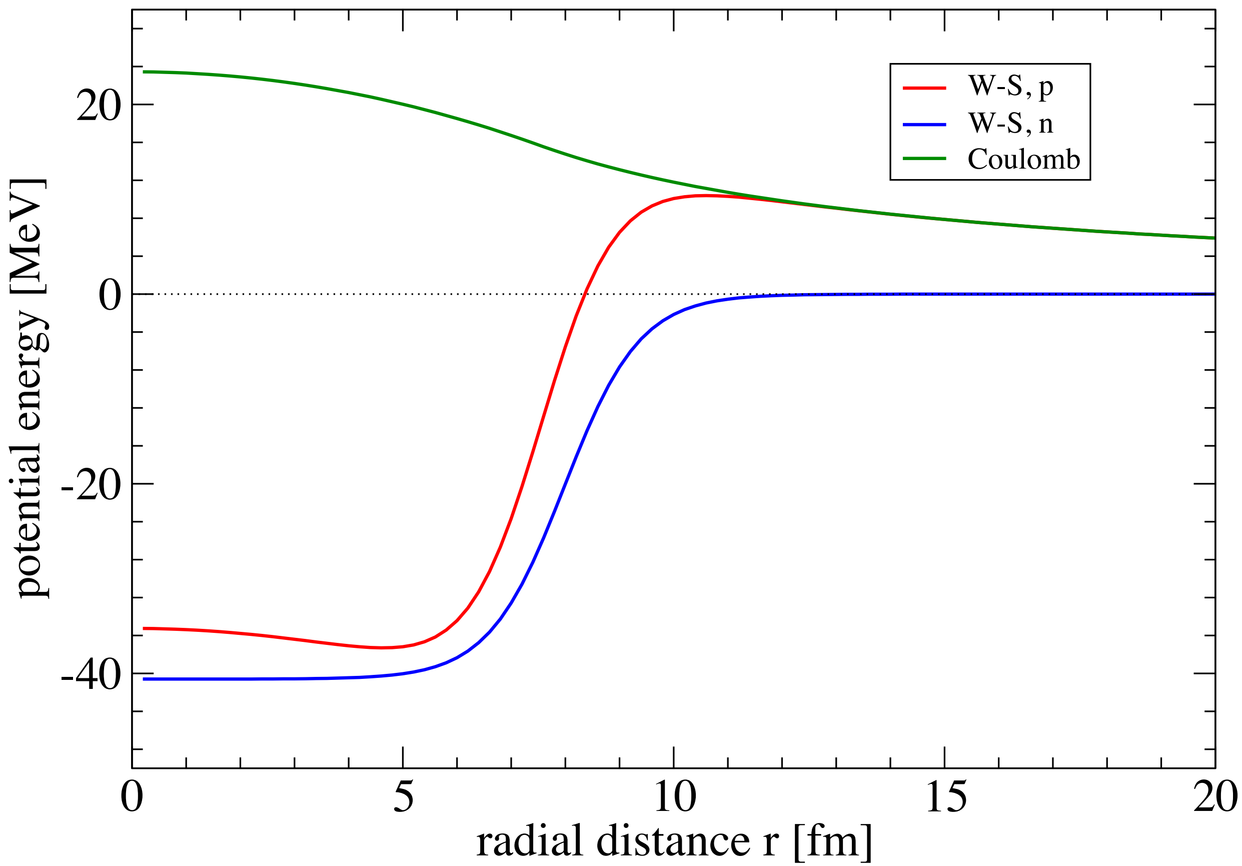
m: ^{212}Po



d: ^{208}Pb

Woods-Saxon potentials for the ^{208}Pb core

Woods-Saxon potential
for neutrons
and protons
(containing
Coulomb repulsion)



D.S. Delion and R. J. Liotta, Phys. Rev. C **87**, 041302(R) (2013)

Few-particle Schrödinger equation in a dense medium

4-particle Schrödinger equation with medium effects

$$\left(\left[E^{HF}(p_1) + E^{HF}(p_2) + E^{HF}(p_3) + E^{HF}(p_4)\right]\right)\Psi_{n,P}(p_1,p_2,p_3,p_4)$$

$$+ \sum_{p_1',p_2'} (1 - f_{p_1} - f_{p_2}) V(p_1,p_2;p_1',p_2')\Psi_{n,P}(p_1',p_2',p_3,p_4)$$

$$+\{permutations\}$$

$$= E_{n,P}\Psi_{n,P}(p_1,p_2,p_3,p_4)$$

Thouless criterion
for quantum condensate:

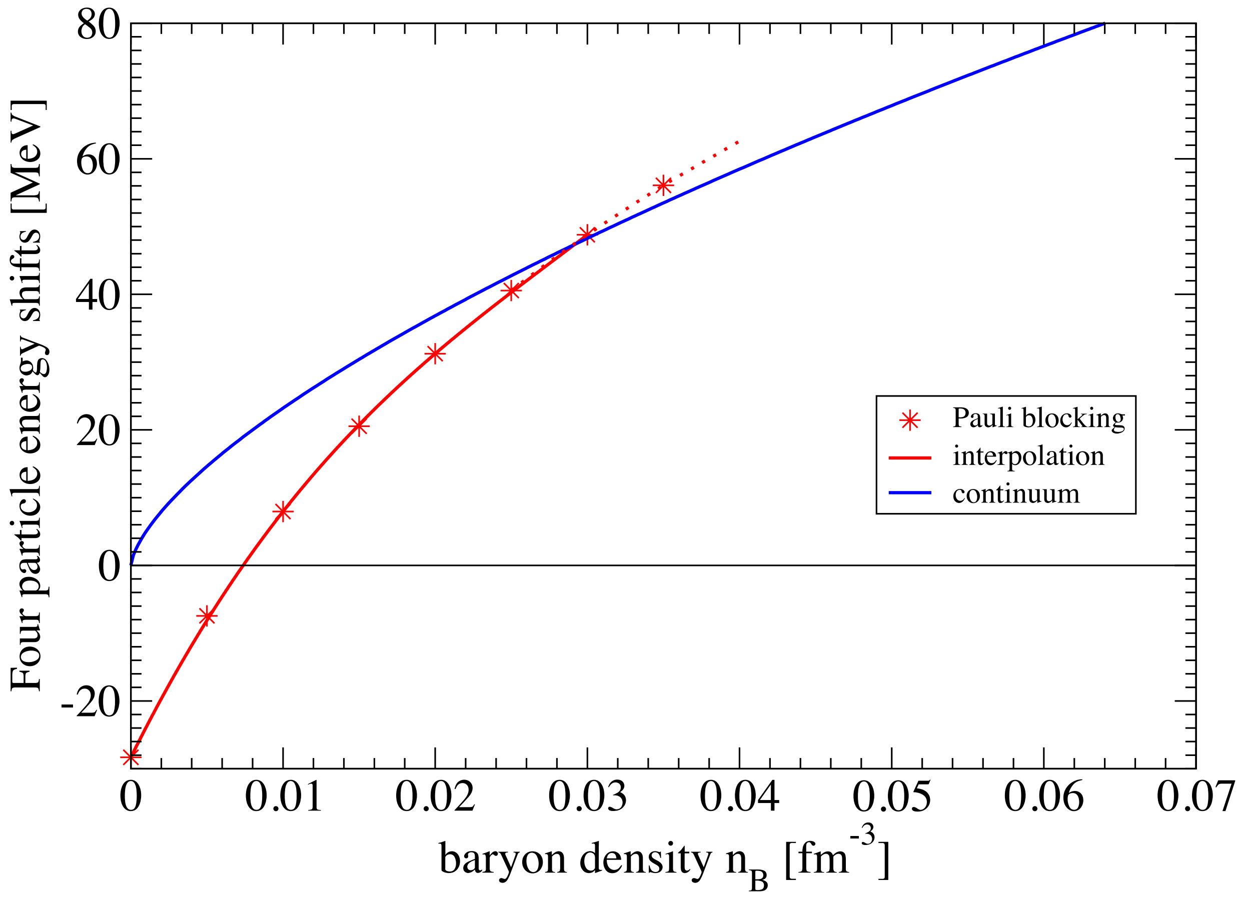
$E_{n,P=0}(T,\mu) = 4\mu$

Four-nucleon energies at finite density

Solution of the in-medium wave equation, T = 0

4 free nucleons
at the Fermi energy
(continuum)

bound state
(α particle)
with Pauli blocking



critical baryon density $n_{cr} = 0.03\ fm^{-3}$

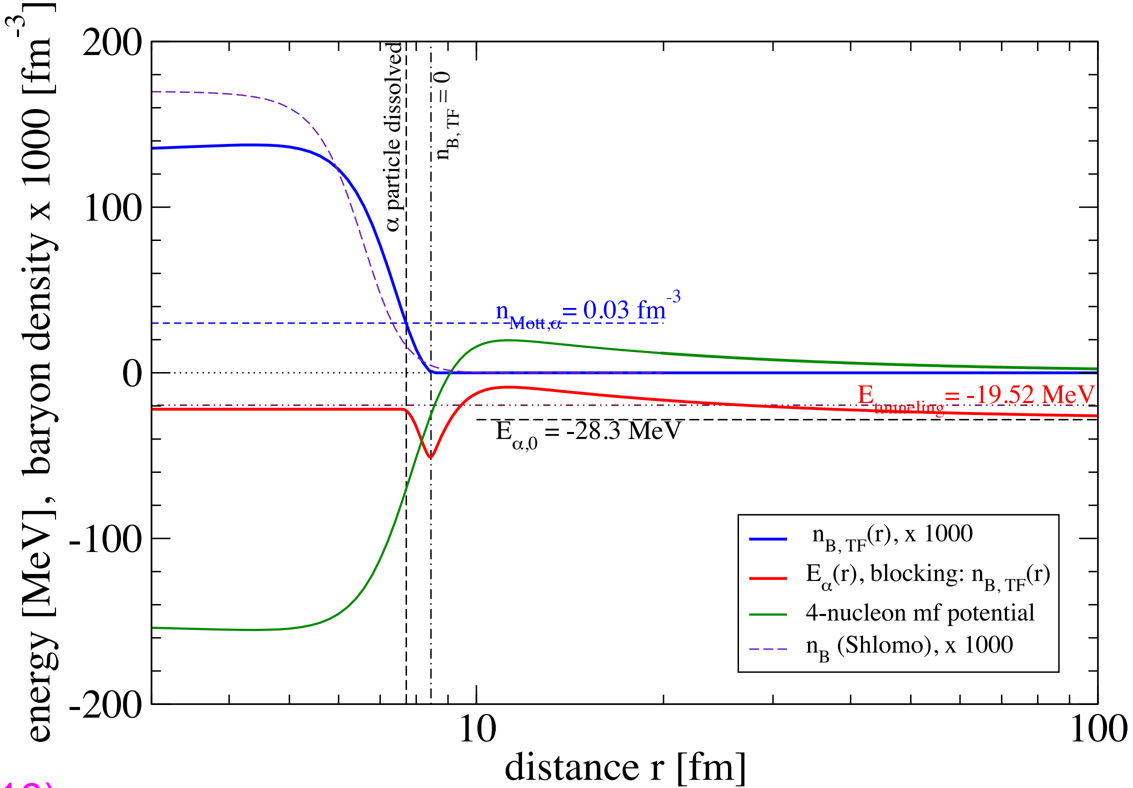
^{212}Po: α on top of ^{208}Pb

- Woods-Saxon potential (Delion 2013)
- Thomas-Fermi nucleon density
- Pauli-blocking of the α particle

Local effective potential W(**R**)
with respect to the ^{208}Pb core.

Woods-Saxon potential
of 2 neutrons and 2 protons
including Coulomb repulsion.

Density in Thomas-Fermi
approximation
with chemical potential fixed
by the total nucleon number



C. Xu et al., PRC **93**, 011306(R) (2016)

Cluster: center of mass motion as collective degree of freedom,
Separation of the c.o.m. motion from the internal motion. Exact wave equations?

Results for α decay of ^{212}Po

C. Xu et al., PHYSICAL REVIEW C **93**, 011306(R) (2016) RAPID COMMUNICATIONS

Potential	c (MeV fm)	d (MeV fm)	E_{tunnel} (MeV)	Fermi energy μ_4 (MeV)
A	13866.30	4090.51	−19.346	−19.346
B	11032.08	3415.56	−19.346	−19.771

$E_{tunnel} - \mu_4$ (MeV)	Preform. factor P_α	Decay half-life $T_{1/2}$ (s)
0	0.367	2.91×10^{-8}
0.425	0.142	2.99×10^{-7}

$$v(s) = c\exp(-4s)/(4s) - d\exp(-2.5s)/(2.5s)$$

Transport codes including light clusters

Equilibrium and non-equilibrium

Statistical operator $\varrho(t)$

Extended von Neumann equation

$$\frac{\partial}{\partial t}\varrho_\varepsilon(t) + \frac{i}{\hbar}\left[H, \varrho_\varepsilon(t)\right] = -\varepsilon\left(\varrho_\varepsilon(t) - \varrho_{\rm rel}(t)\right)$$

The relevant statistical operator $\varrho_{\rm rel}(t)$ is obtained from the maximum of entropy reproducing the local, time dependent composition with parameter values $T(\mathbf{r},t), \mu_n(\mathbf{r},t), \mu_p(\mathbf{r},t)$, but contains in addition the cluster distribution functions $f_{A\nu}^{\rm Wigner}(\mathbf{p},\mathbf{r},t)$ as relevant observables.[106,107]

$$\varrho(t) = \lim_{\varepsilon\to 0}\varrho_\varepsilon(t)$$

Future work is necessary to devise a transport theory for HIC which is compatible with the thermodynamic properties and the EoS as equilibrium solution.

D.N. Zubarev, V.G. Morozov, and G. Ropke, Statistical Mechanics of Nonequilibrium Processes (1996)
D.N. Zubarev, V.G. Morozov, I.P. Omelyan, and M.V. Tokarchuk, Theoret. Math. Phys. **96**, 997 (1993)
G. Ropke and H. Schulz, Nucl. Phys. A **477**, 472 (1988)

Boltzmann equation

- Relevant observables: single particle distribution function (classical, quantal)
- Mean-field and collisions
- Entropy and conservation of kinetic energy
- Equilibrium solution: ideal gases
- Time-dependent Green functions
- Quasiparticles, spectral function

Formation of light clusters in heavy ion reactions, transport codes

PHYSICAL REVIEW C, VOLUME 63, 034605

Medium corrections in the formation of light charged particles in heavy ion reactions

C. Kuhrts,[1] M. Beyer,[1,*] P. Danielewicz,[2] and G. Röpke[1]
[1]*FB Physik, Universität Rostock, Universitätsplatz 3, D-18051 Rostock, Germany*
[2]*NSCL, Michigan State University, East Lansing, Michigan 48824*
(Received 13 September 2000; published 12 February 2001)

Wigner distribution

cluster mean-field potential

$$\partial_t f_X + \{\mathcal{U}_X, f_X\} = \mathcal{K}_X^{\mathrm{gain}}\{f_N, f_d, f_t, \ldots\}(1 \pm f_X) - \mathcal{K}_X^{\mathrm{loss}}\{f_N, f_d, f_t, \ldots\} f_X,$$

$$X = N, d, t, \ldots.$$

loss rate

in-medium
breakup transition operator

$$\mathcal{K}_d^{\mathrm{loss}}(P,t) = \int d^3k \int d^3k_1\, d^3k_2\, d^3k_3 |\langle k_1 k_2 k_3 | U_0 | kP\rangle|^2_{dN\to pnN} \times f_N(k_1,t) f_N(k_2,t) f_N(k_3,t) f_N(k,t) + \cdots. \quad (3)$$

breakup cross section

$$\sigma_{\mathrm{bu}}^0(E) = \frac{1}{|v_d - v_N|} \frac{1}{3!} \int d^3k_1\, d^3k_2\, d^3k_3 |\langle kP | U_0 | k_1 k_2 k_3\rangle|^2 \times 2\pi\delta(E' - E)(2\pi)^3 \delta^{(3)}(k_1 + k_2 + k_3), \quad (4)$$

Mott effect, in-medium cross section

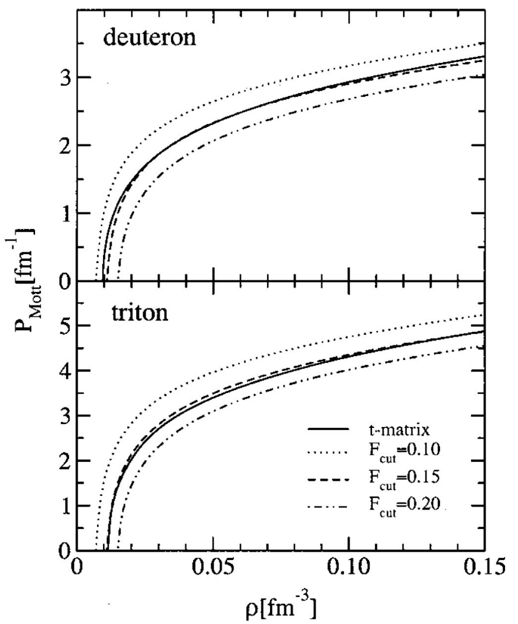


FIG. 1. Deuteron and triton Mott momenta P_{Mott} shown as a function of density ρ at fixed temperature of $T = 10$ MeV. The solid line represents results of the t matrix approach. The dashed, dotted, and dashed-dotted lines represent the deuteron Mott momenta from the parametrization given in Eq. (24) for three different cutoff values F_{cut}.

$$\int d^3 q \; f\left(\mathbf{q} + \frac{\mathbf{P}_{\mathrm{c.m.}}}{2}\right) \; |\phi(\mathbf{q})|^2 \leqslant F_{\mathrm{cut}}$$

C. Kuhrts, PRC **63**,034605 (2001)

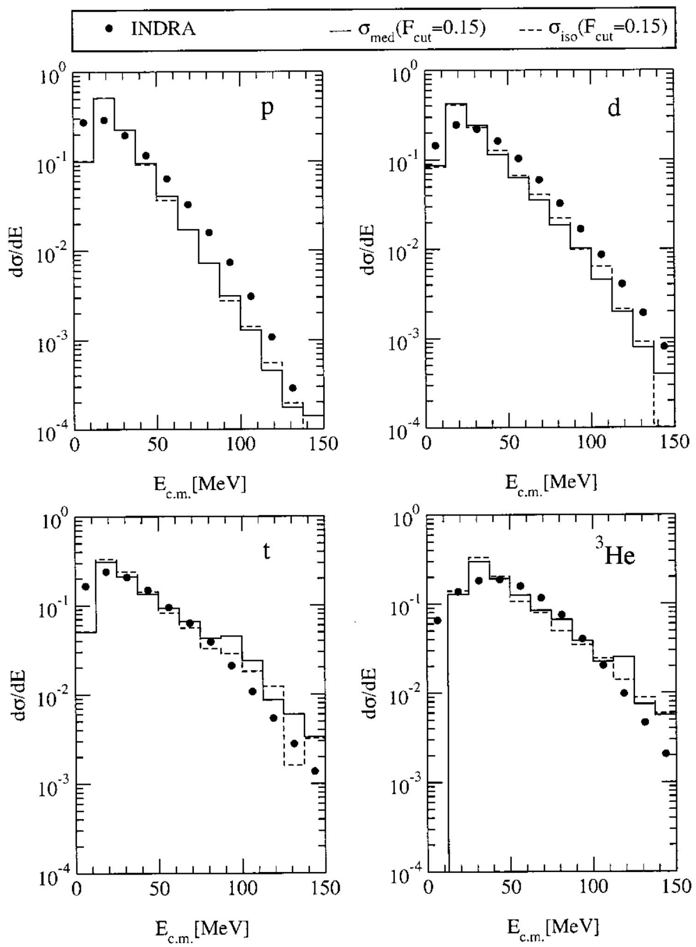


FIG. 5. Renormalized light charged light particle spectra in the center of mass system for the reaction ^{129}Xe+^{119}Sn at 50 MeV/nucleon. The filled circles represent the data of the INDRA Collaboration [21]. The solid line shows the calculations with the in-medium Nd reaction rates, while the dashed line shows a calculation using the isolated Nd breakup cross section; both with $F_{\mathrm{cut}} = 0.15$.

Equilibrium correlations and transport codes

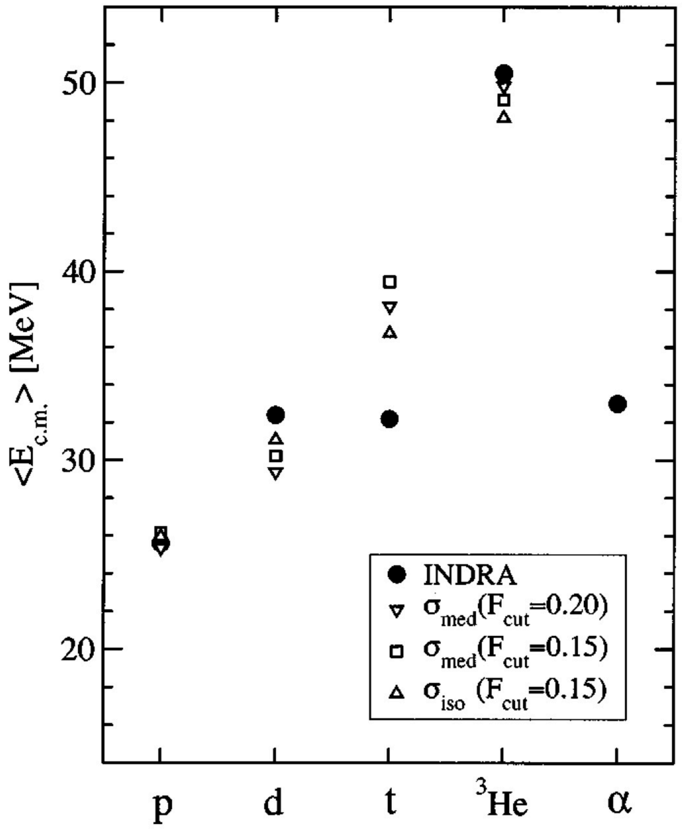


FIG. 6. Mean transverse energy of light charged fragments in the angular range of $-0.5 \leq \cos\theta_{c.m.} \leq 0.5$.

C. Kuhrts, PRC **63**,034605 (2001)

Important: Mott effect

Minor effects:
in medium cross sections

Missing: inclusion of alphas

Correlated continuum,
correlated medium

Freeze-out and local
thermodynamic equilibrium

single-particle quantum kinetic
equations and correlations

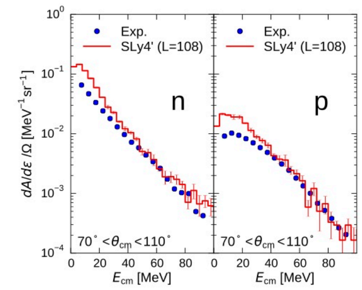
Equilibrium solution?

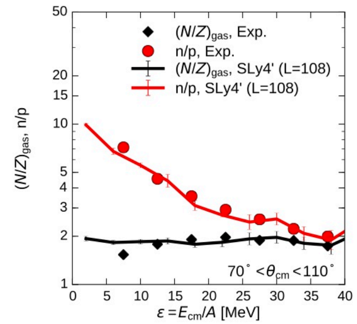
AMD (Akira Ono)

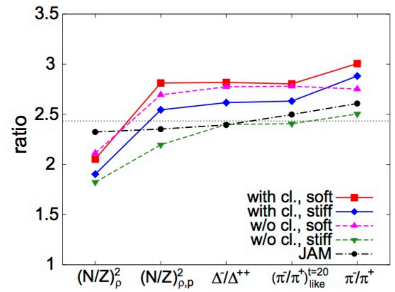
Summary

- AMD has been extended to include cluster correlations.
  - The correlation to bind several light clusters is also important.
  - Transition from a wave packet to a plane wave is taken into account to improve nucleon spectra.
- Clusters have strong impacts.
  - Good reproduction of cluster and fragment productions, in various reaction systems simultaneously.
  - The neutron/proton ratio is sensitive to the production of α particles (as well as to the density dependence of the symmetry energy).
  - If clusters start to appear at early times, they change the way how the symmetry energy is reflected in final observables such as the π^-/π^+ ratio.

One-body dynamics $\Leftarrow$ $\Rightarrow$ Clusters
Bulk properties (EOS)







Various transport theories

Based on the one-body distribution function $f(\mathbf{r},\mathbf{p},t)$ $\Leftrightarrow$ One-body density matrix $\rho(\mathbf{r},\mathbf{r}')$

$$\frac{\partial f(\mathbf{r},\mathbf{p},t)}{\partial t} + \left\{f,\ h_{\mathrm{MF}}[f]\right\} = I_{\mathrm{coll}} + \text{fluct.}$$

Mean Field Models (BUU, VUU, BNV, SMF, BLOB, . . .)

"Nucleon motions in the mean field should be solved without any limitation."

Molecular Dynamics Models (∗QMD, CoMD, AMD, . . .)

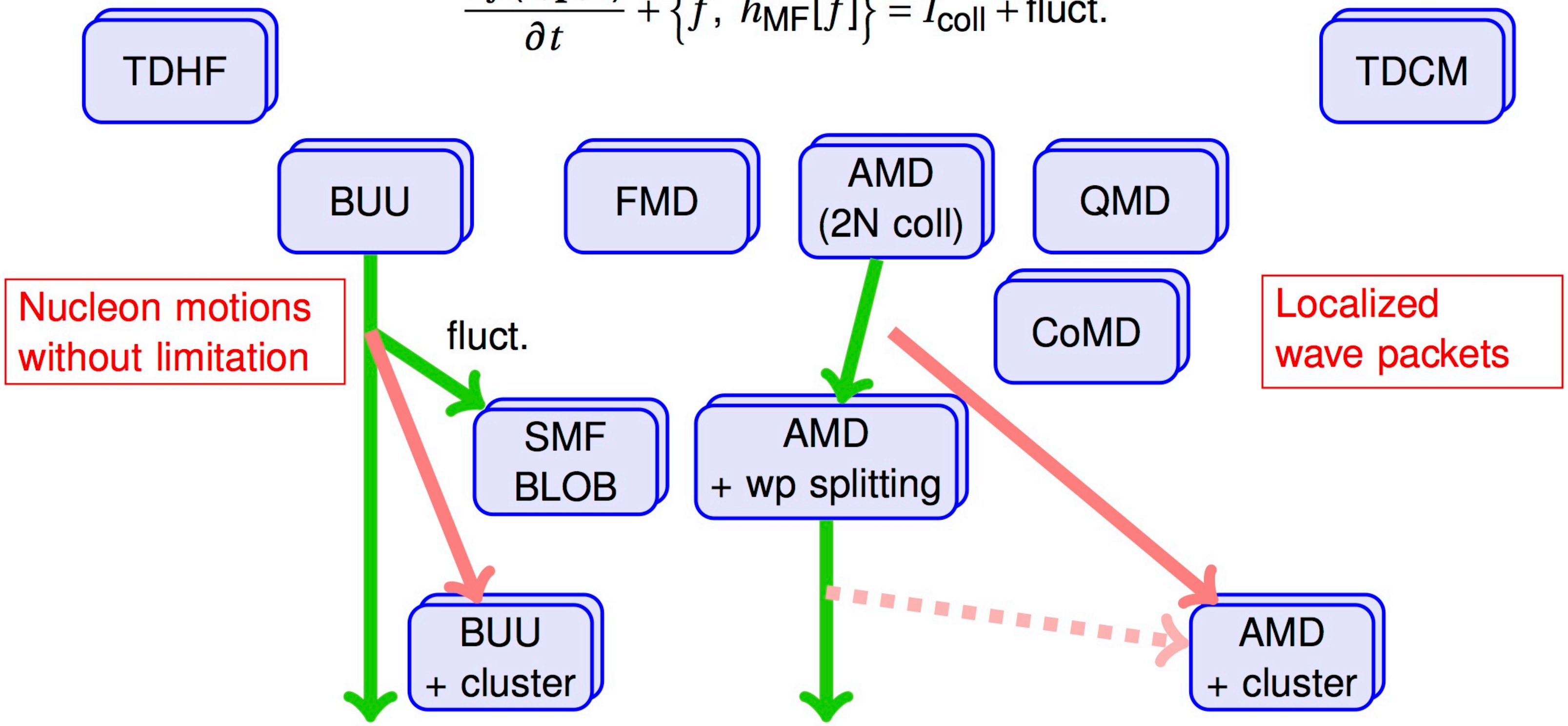
"Each nucleon should be localized because it has to be in a fragment at the end."

- Fluctuation/branching is a way to handle many-body correlations, even with the single-nucleon distribution function $f(\mathbf{r},\mathbf{p},t)$.
- Not many models treat cluster correlations explicitly.

Various transport theories

Based on the one-body distribution function $f(\mathbf{r},\mathbf{p},t)$ ⇔ One-body density matrix $\rho(\mathbf{r},\mathbf{r}')$

$$\frac{\partial f(\mathbf{r},\mathbf{p},t)}{\partial t} + \left\{ f,\ h_{\mathrm{MF}}[f] \right\} = I_{\mathrm{coll}} + \text{fluct.}$$



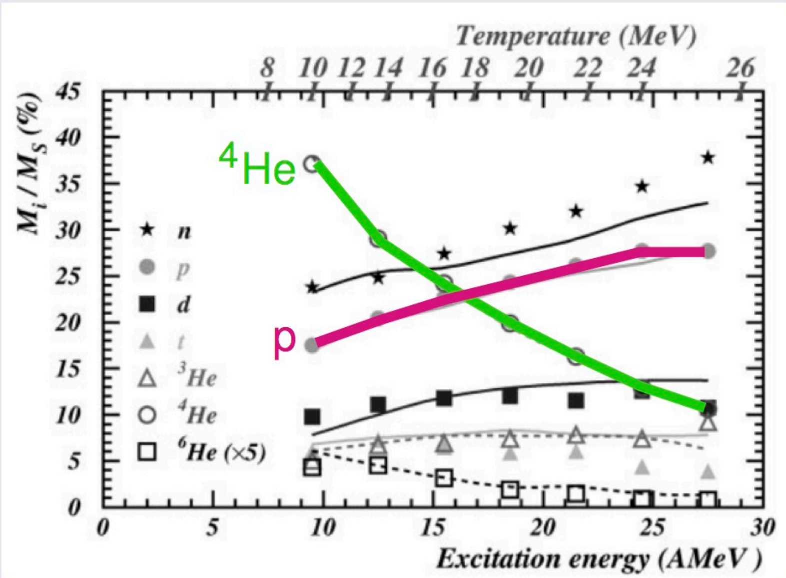
- Fluctuation/branching is a way to handle many-body correlations.
- Not many models treat cluster correlations explicitly.

Vaporized nuclei and nuclear matter

Heavy-Ion Collisions

Experimental data of cluster abundance in $^{36}Ar + ^{58}Ni$ for the events where the quasi-projectile is vaporized.

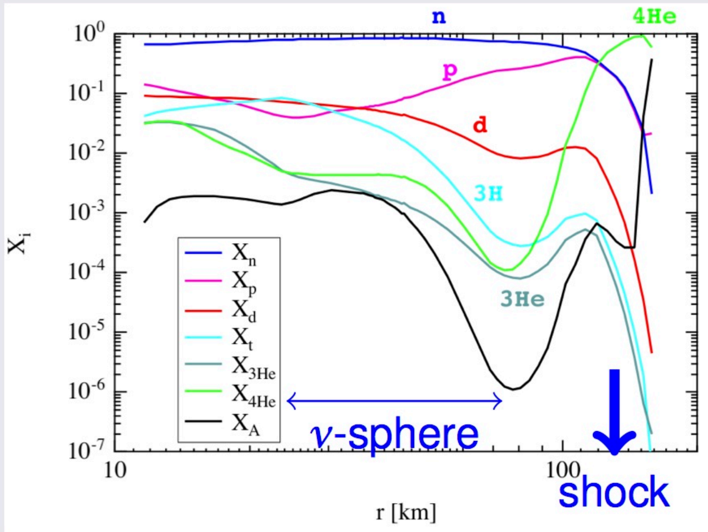
Borderie et al., EPJA6 (1999) 197, PLB388 (1996) 224.



Supernova

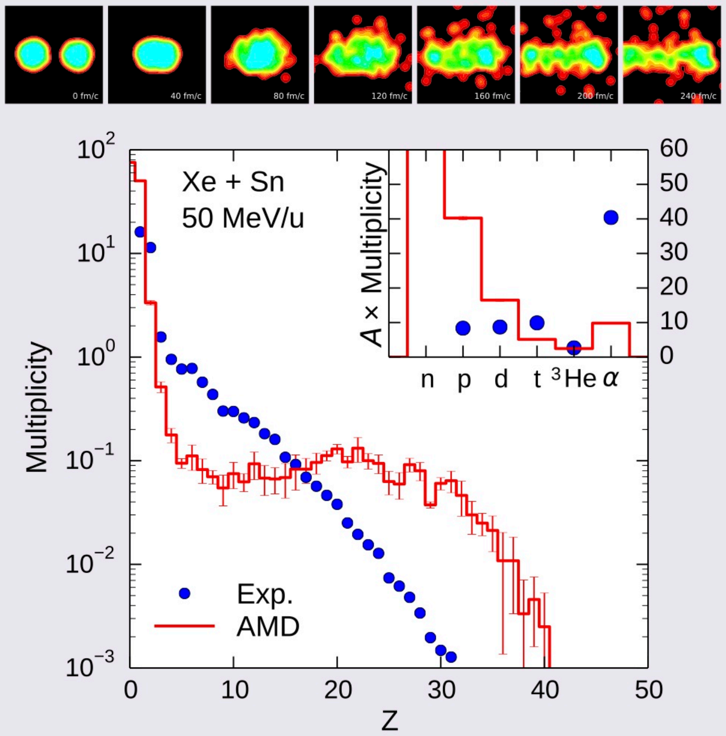
Mass fraction of light clusters in the post-bounce supernova core, based on nuclear statistical equilibrium.

Sumiyoshi and Röpke, PRC77 (2008) 055804.

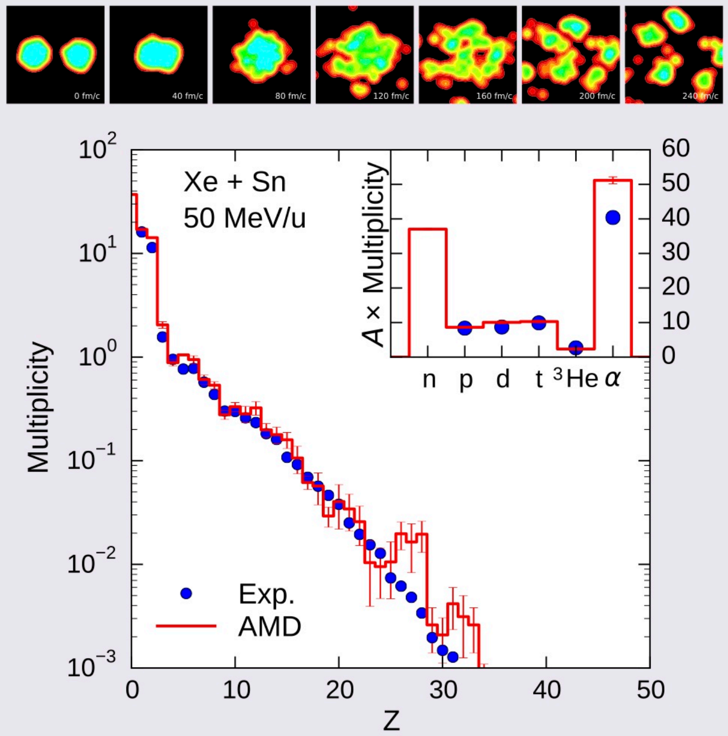


Effect of cluster correlations: central Xe + Sn at 50 MeV/u

Without clusters



With clusters



Applications

- Astrophysics
- Heavy ion collisions

Nuclear matter phase diagram

Core collapse supernovae

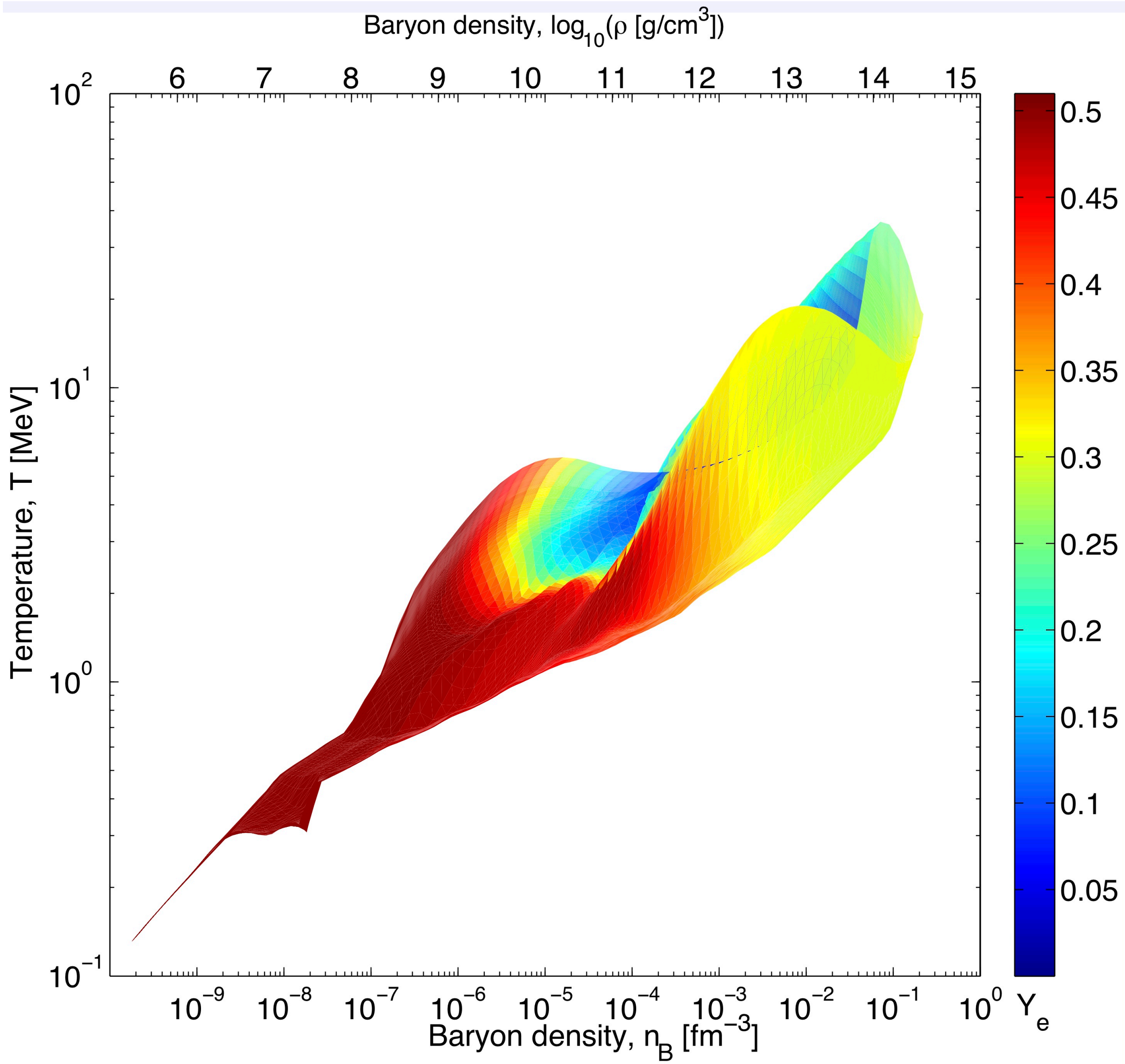
Thermodynamic parameters:

baryon density n_B

temperature T

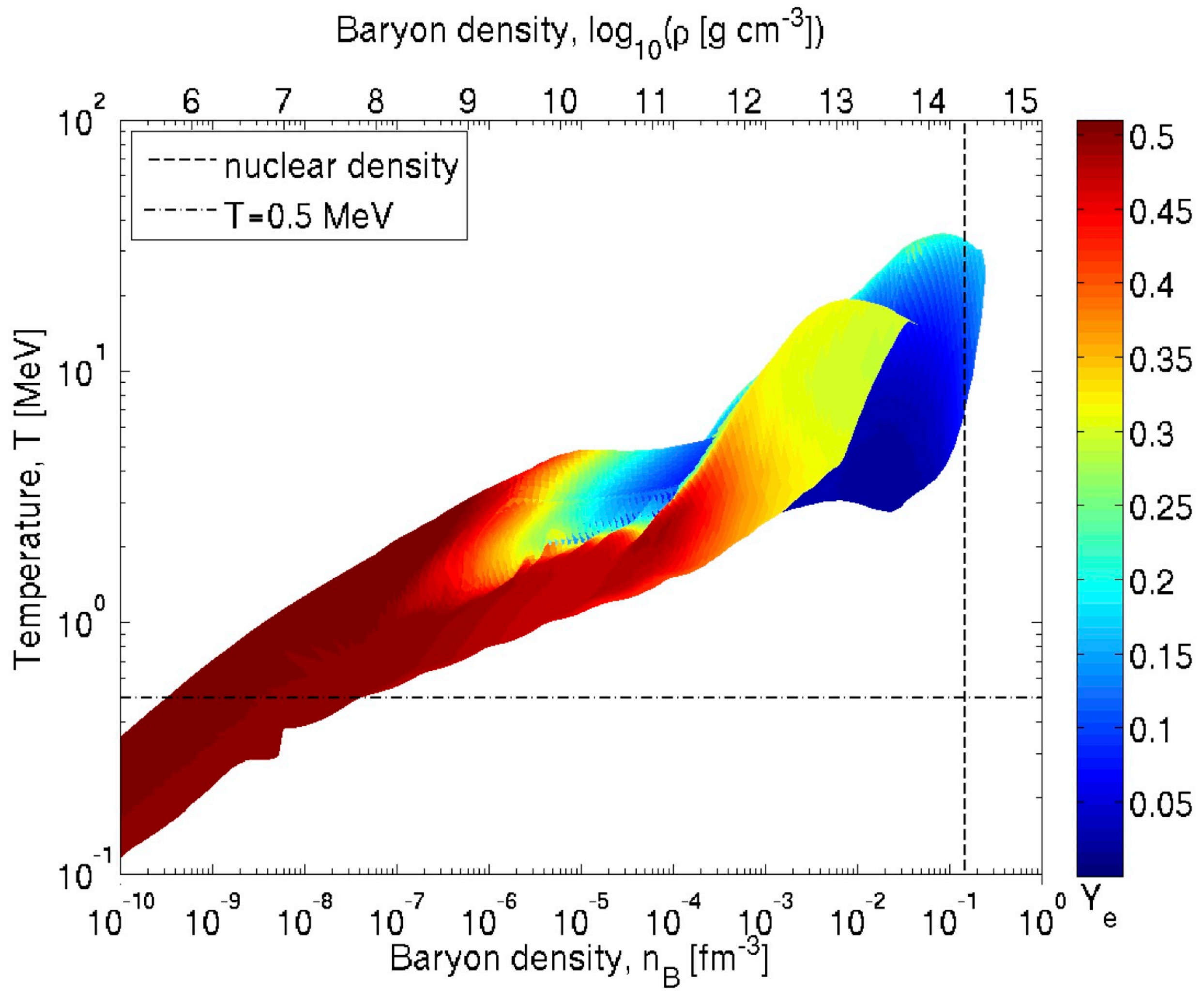
electron fraction Y_e

T. Fischer et al., ApJS 194, 39 (2011)



Nuclear matter phase diagram

Exploding supernova



T. Fischer et al., arXiv 1307.6190

Light cluster production at NICA★

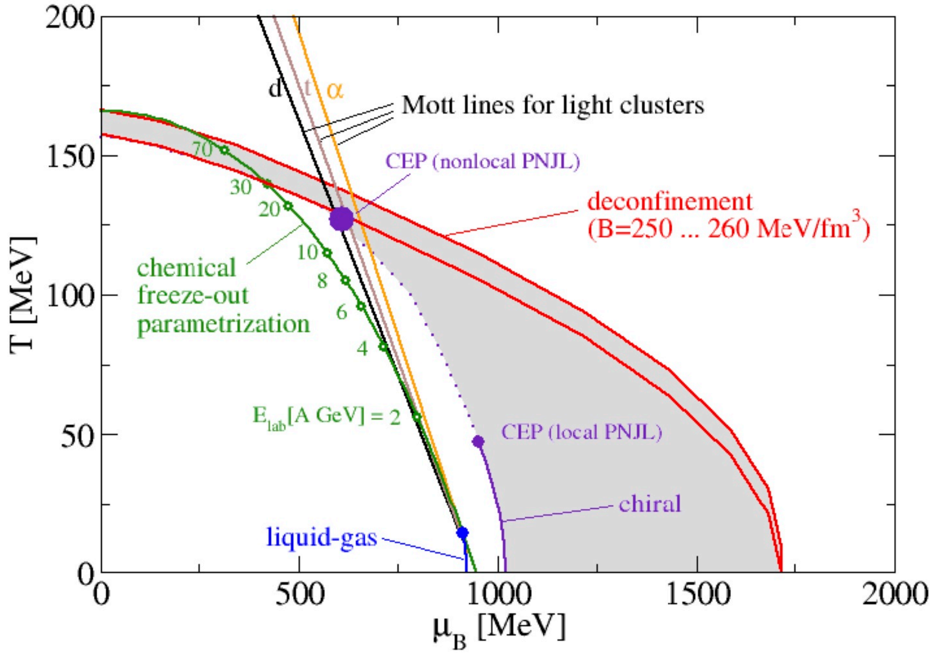


Fig. 1. Phase diagram of dense nuclear matter in the plane of temperature T and baryochemical potential μ_B. The diagram includes Mott lines for the dissociation of light nuclear clusters, extrapolated also to the deconfinement region. For details, see text.

N.-U. Bastian, P. Batyuk, D. Blaschke, P. Danielewicz, Yu.B. Ivanov, Iu. Karpenko, G. Ropke, O. Rogachevsky, and H.H. Wolter, Eur. Phys. J. A (2016) 52: 244

Light cluster production at NICA★

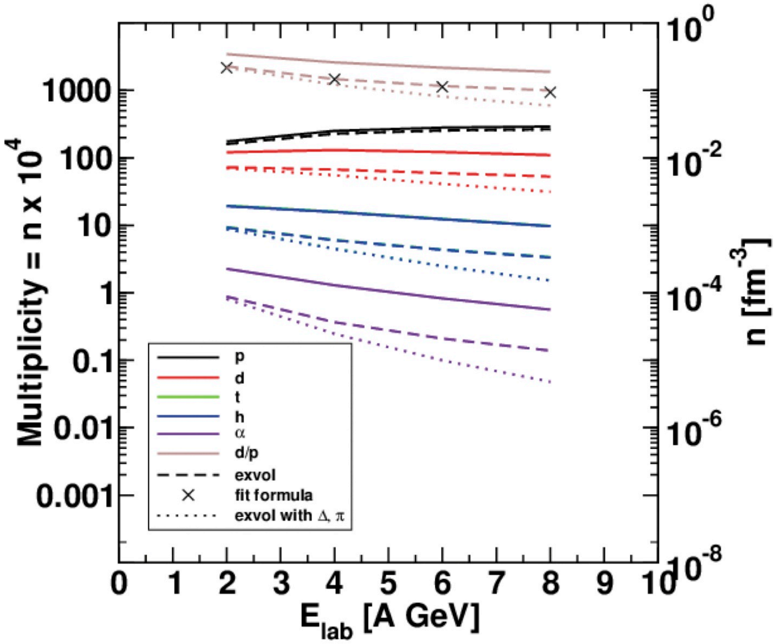


Fig. 2. Abundances of protons and of light clusters following LMA for temperature and chemical potential values along the freeze-out line eq. (1) in Au + Au collisions in the NICA energy range (anticipating an energy scan with $E_{\rm lab} = 2, 4, 6, 8\,A\,\text{GeV}$). In each case solid lines are for the pointlike particles, dashed for the excluded-volume correction for nucleons and clusters, and dotted ones including also the volume of pions and deltas.

N.-U. Bastian, P. Batyuk, D. Blaschke, P. Danielewicz, Yu.B. Ivanov, Iu. Karpenko, G. Ropke, O. Rogachevsky, and H.H. Wolter, Eur. Phys. J. A (2016) 52: 244

Light cluster production at NICA★

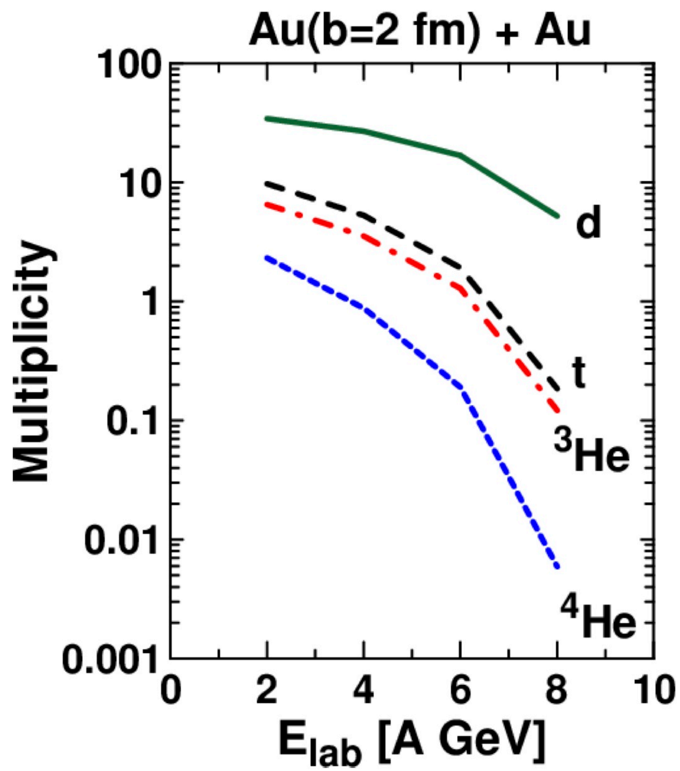


Fig. 5. Multiplicities of light clusters in central Au + Au collisions in the NICA energy range (calculated for an energy scan with $E_{\rm lab} = 2, 4, 6, 8\,A\,\text{GeV}$). Results from a 3-fluid hydrodynamics description with cluster coalescence [22].

N.-U. Bastian, P. Batyuk, D. Blaschke, P. Danielewicz, Yu.B. Ivanov, Iu. Karpenko, G. Ropke, O. Rogachevsky, and H.H. Wolter, Eur. Phys. J. A (2016) 52: 244

Light cluster production at NICA★

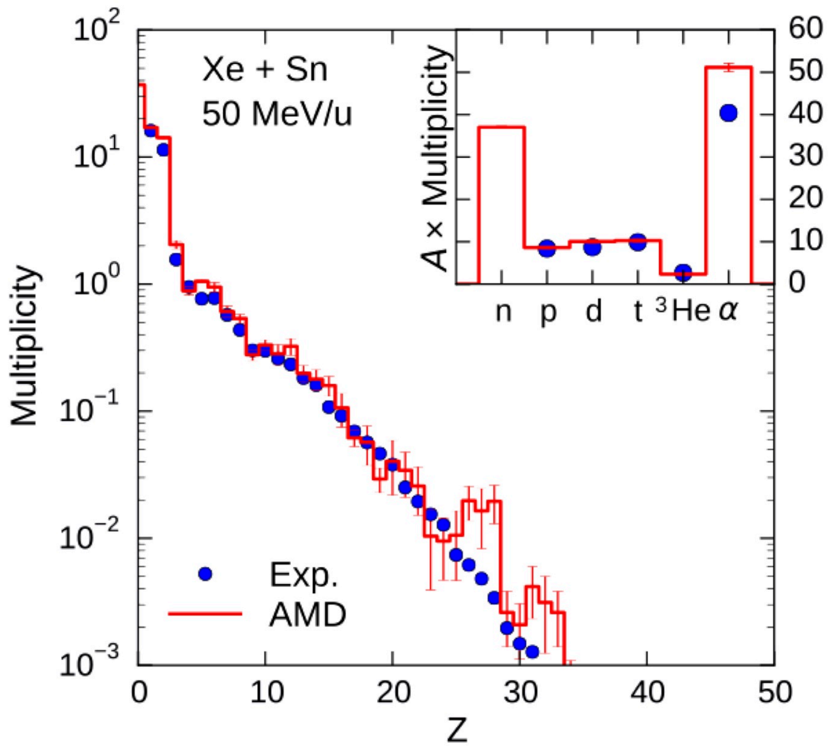


Fig. 4. Multiplicity of different charged fragments in Xe + Sn collisions at 50 MeV/nucleon. Results from the AMD model of ref. [20], including also heavier-cluster formation from cluster-cluster collisions.

N.-U. Bastian, P. Batyuk, D. Blaschke, P. Danielewicz, Yu.B. Ivanov, Iu. Karpenko, G. Ropke, O. Rogachevsky, and H.H. Wolter, Eur. Phys. J. A (2016) 52: 244

Light cluster production at NICA⋆

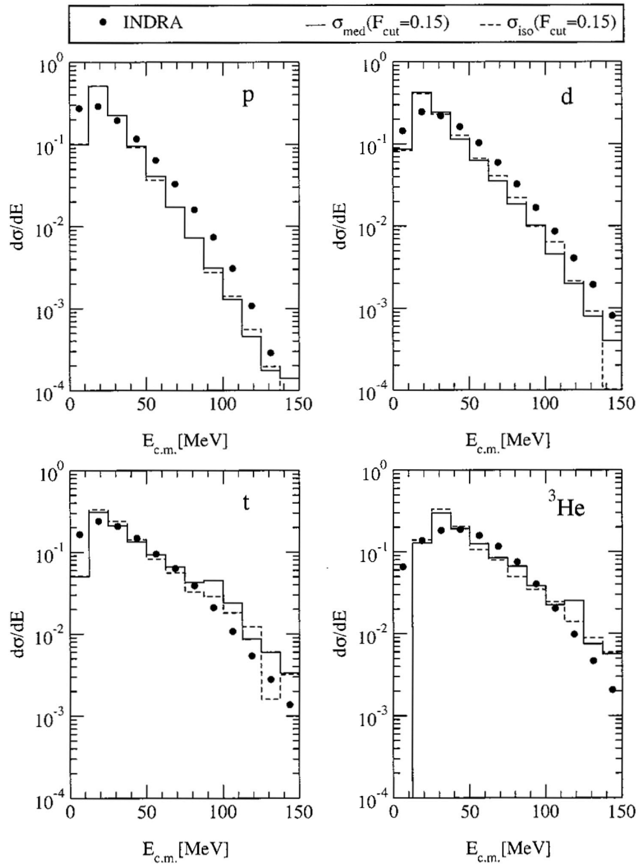


Fig. 3. Differential cross sections for production of charged $A \leq 3$ fragments in Xe + Se collisions at 50 MeV/nucleon. Results from the model of ref. [19] with cluster of mass $A \leq 3$ as explicit degrees of freedom.

N.-U. Bastian, P. Batyuk, D. Blaschke, P. Danielewicz, Yu.B. Ivanov, Iu. Karpenko, G. Ropke, O. Rogachevsky, and H.H. Wolter, Eur. Phys. J. A (2016) 52: 244

Summary

- Quantum statistical approach: light clusters with in-medium quasiparticle energies. The Pauli blockiing is strongly depending on temperature T and P. Mott effect.

- Clusters are observed. Which signatures can be obtained for the source?

- The influence of continuum correlations (clusters) at increasing densities requires detailed investigations.

- Continuum correlations contribute to the symmetry energy (density dependent virial coefficients).
- The blocking of bound states is modified because of correlations in the medium (α matter).

- Cluster formation is relevant for HIC (freeze-out, transport theory) and astrophysics (supernova explosions, pasta structures)

Thanks

to D. Blaschke, C. Fuchs, Y. Funaki, H. Horiuchi,
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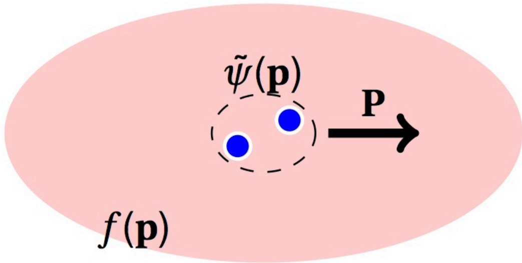
A. Sedrakian, K. Sumiyoshi, A. Tohsaki, S. Typel,
H. Wolter, Z. Xu, T. Yamada, B. Zhou
for collaboration

to you

for attention

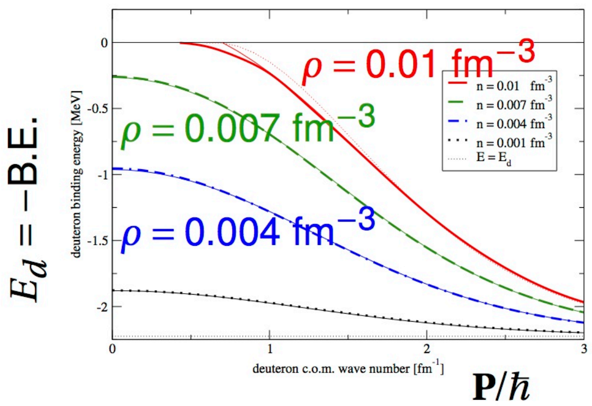
D.G.

A cluster in medium & Clusterized nuclear matter



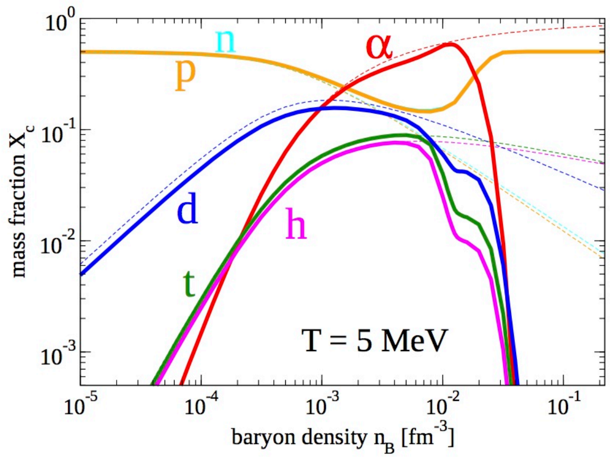
Equation for a deuteron in uncorrelated medium

$$\left[e(\tfrac{1}{2}\mathbf{P}+\mathbf{p})+e(\tfrac{1}{2}\mathbf{P}-\mathbf{p})\right]\tilde{\psi}(\mathbf{p}) + \left[1-f(\tfrac{1}{2}\mathbf{P}+\mathbf{p})-f(\tfrac{1}{2}\mathbf{P}-\mathbf{p})\right]\int\frac{d\mathbf{p}'}{(2\pi)^3}\langle\mathbf{p}|v|\mathbf{p}'\rangle\tilde{\psi}(\mathbf{p}') = E\tilde{\psi}(\mathbf{p})$$



Momentum (**P**) dependence of B.E.

Röpke, NPA867 (2011) 66.

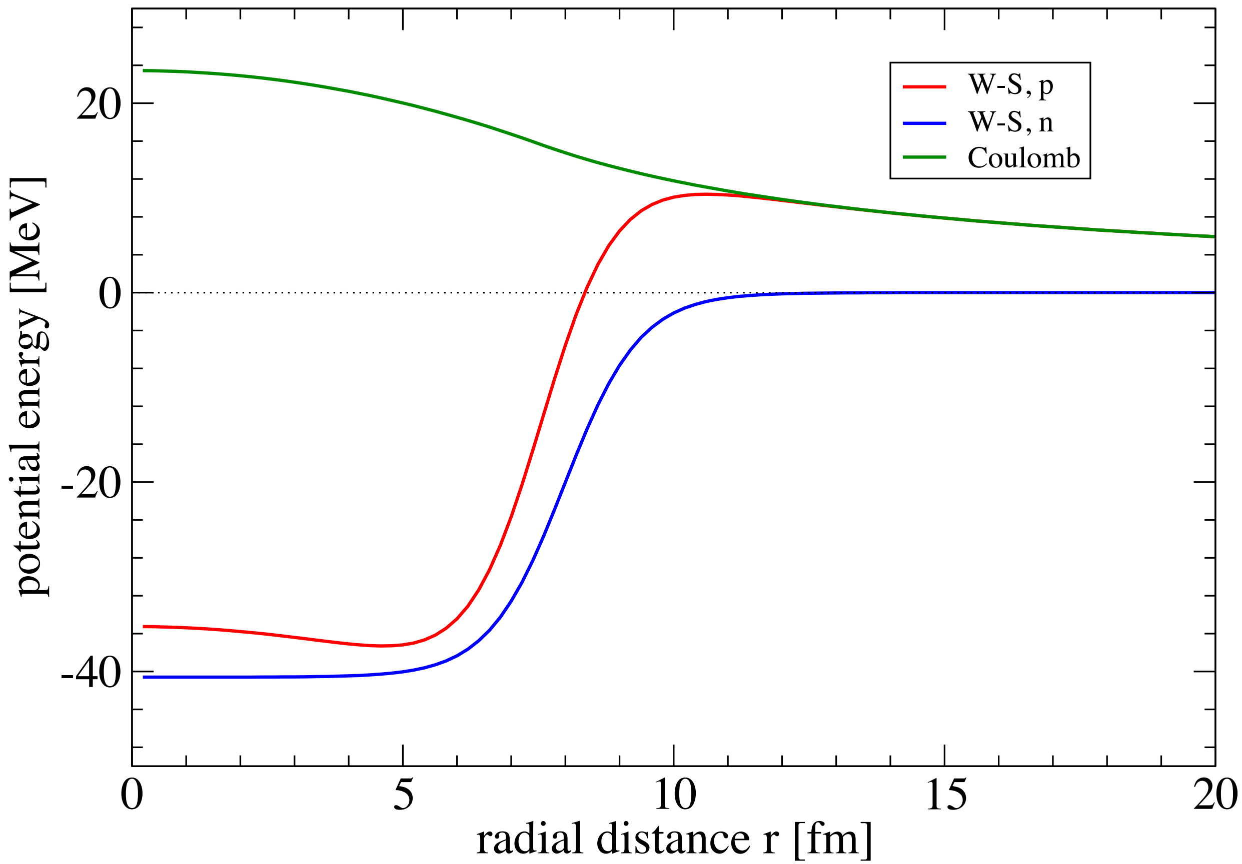


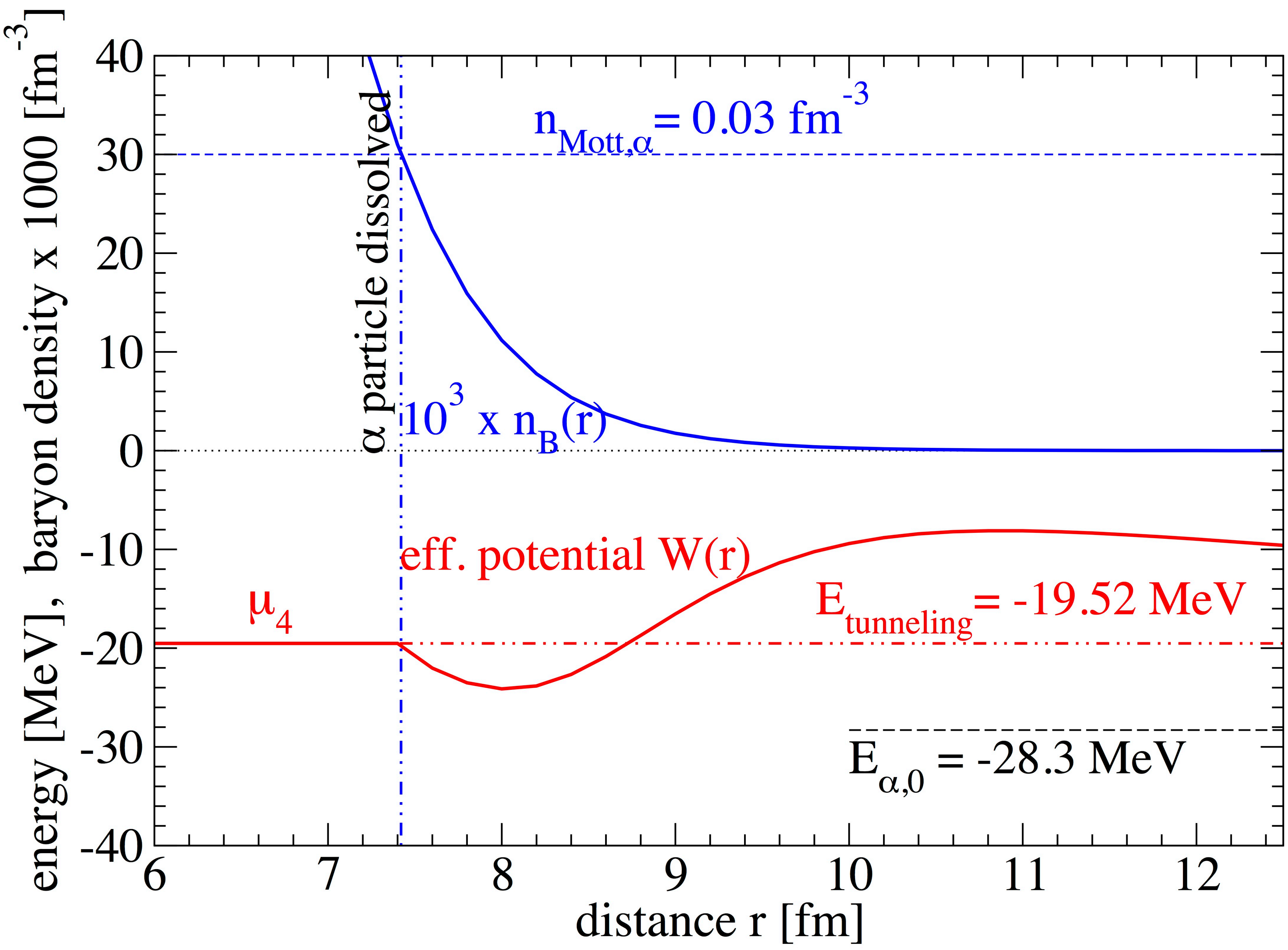
QS for symmetric nuclear matter

Röpke, PRC 92 (2015) 054001.

Woods-Saxon potentials for the ^{208}Pb core

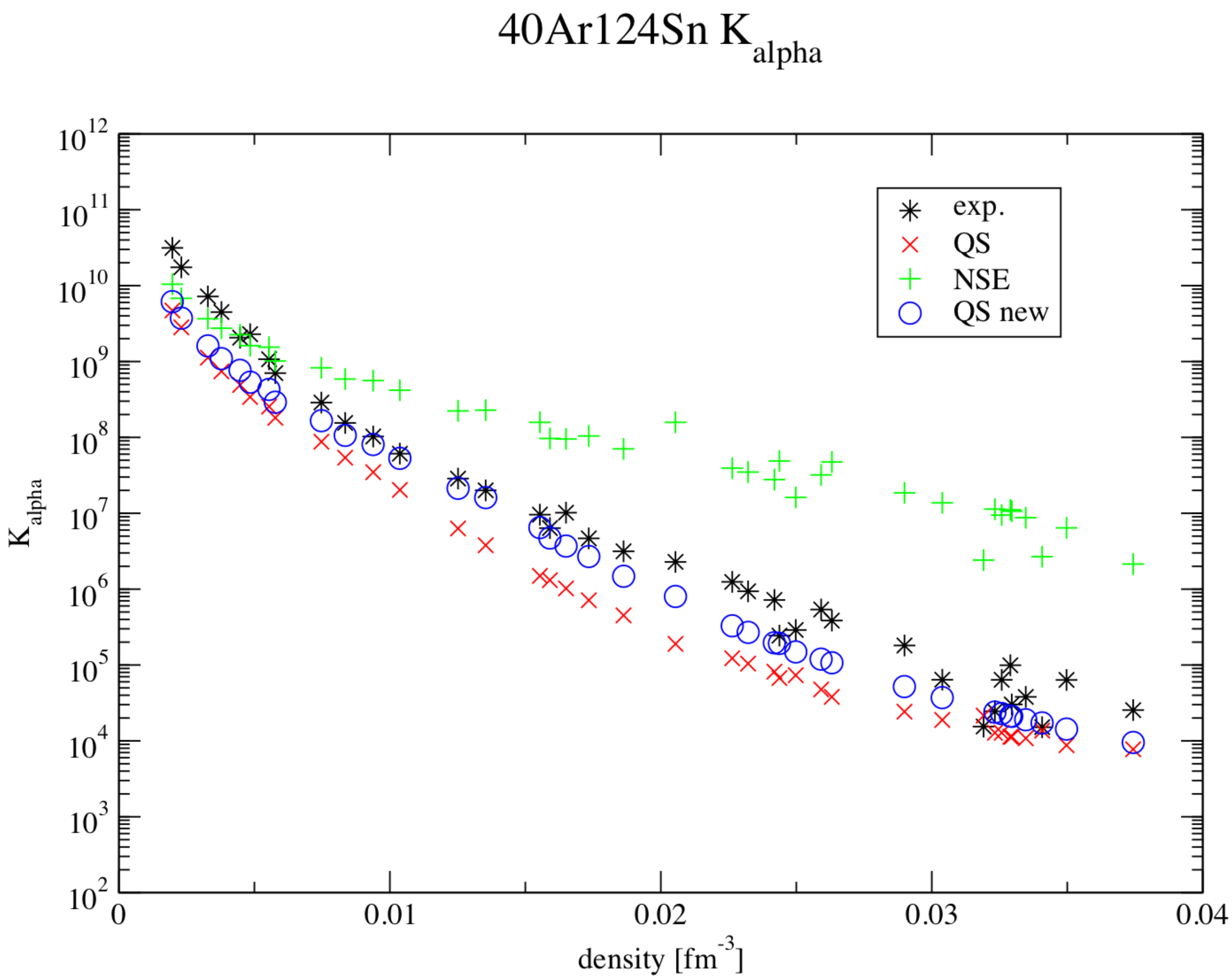
Woods-Saxon potential
(Delion 2013)
for neutrons
and protons
(containing
Coulomb repulsion)





energy [MeV], baryon density x 1000 [fm^{-3}]
distance r [fm]
$n_{Mott,\alpha}$ = 0.03 fm^{-3}
α particle dissolved
10^3 x $n_B(r)$
eff. potential W(r)
μ_4
$E_{tunneling}$ = -19.52 MeV
$E_{\alpha,0}$ = -28.3 MeV
40
30
20
10
0
-10
-20
-30
-40
6
7
8
9
10
11
12

QS versus NSE: comparison with data



Center of mass and intrinsic Schroedinger equation

c. o. m. coordinate R, relative coordinates s_j

$$\Psi(\mathbf{R}, \mathbf{s}_j) = \varphi^{\mathrm{intr}}(\mathbf{s}_j, \mathbf{R})\, \Phi(\mathbf{R})$$

normalization

$$\int dR\, |\Phi(\mathbf{R})|^2 = 1 \qquad \int ds_j |\varphi^{\mathrm{intr}}(\mathbf{s}_j, \mathbf{R})|^2 = 1$$

Wave equation for the c.o.m. motion

$$-\frac{\hbar^2}{2Am}\nabla_R^2\Phi(\mathbf{R}) - \frac{\hbar^2}{Am}\int ds_j \varphi^{\mathrm{intr},*}(\mathbf{s}_j, \mathbf{R})[\nabla_R \varphi^{\mathrm{intr}}(\mathbf{s}_j, \mathbf{R})][\nabla_R \Phi(\mathbf{R})]$$
$$-\frac{\hbar^2}{2Am}\int ds_j \varphi^{\mathrm{intr},*}(\mathbf{s}_j, \mathbf{R})[\nabla_R^2 \varphi^{\mathrm{intr}}(\mathbf{s}_j, \mathbf{R})]\Phi(\mathbf{R}) + \int dR'\, W(\mathbf{R}, \mathbf{R}')\Phi(\mathbf{R}') = E\,\Phi(\mathbf{R})$$

c.o.m. effective potential

$$W(\mathbf{R}, \mathbf{R}') = \int ds_j\, ds'_j\, \varphi^{\mathrm{intr},*}(\mathbf{s}_j, \mathbf{R})\, \left[T[\nabla_{s_j}]\delta(\mathbf{R} - \mathbf{R}')\delta(\mathbf{s}_j - \mathbf{s}'_j) + V(\mathbf{R}, \mathbf{s}_j; \mathbf{R}', \mathbf{s}'_j)\right]\, \varphi^{\mathrm{intr}}(\mathbf{s}'_j, \mathbf{R}')$$

Wave equation for the intrinsic motion

$$-\frac{\hbar^2}{Am}\Phi^*(\mathbf{R})[\nabla_R \Phi(\mathbf{R})][\nabla_R \varphi^{\mathrm{intr}}(\mathbf{s}_j, \mathbf{R})] - \frac{\hbar^2}{2Am}|\Phi(\mathbf{R})|^2 \nabla_R^2 \varphi^{\mathrm{intr}}(\mathbf{s}_j, \mathbf{R})$$
$$+\int dR'\, ds'_j\, \Phi^*(\mathbf{R})\, \left[T[\nabla_{s_j}]\delta(\mathbf{R} - \mathbf{R}')\delta(\mathbf{s}_j - \mathbf{s}'_j) + V(\mathbf{R}, \mathbf{s}_j; \mathbf{R}', \mathbf{s}'_j)\right]\, \Phi(\mathbf{R}')\varphi^{\mathrm{intr}}(\mathbf{s}'_j, \mathbf{R}') = F(\mathbf{R})\varphi^{\mathrm{intr}}(\mathbf{s}_j, \mathbf{R})$$

Four-particle correlations

Four-particle wave equation in position space representation

$$[E_4 - \hat{h}_1 - \hat{h}_2 - \hat{h}_3 - \hat{h}_4]\Psi_4(\mathbf{r}_1\mathbf{r}_2\mathbf{r}_3\mathbf{r}_4) = \int d^3\mathbf{r}'_1\, d^3\mathbf{r}'_2 \langle \mathbf{r}_1\mathbf{r}_2|B\ V_{N-N}|\mathbf{r}'_1\mathbf{r}'_2\rangle \Psi_4(\mathbf{r}'_1\mathbf{r}'_2\mathbf{r}_3\mathbf{r}_4)$$

$$+ \int d^3\mathbf{r}'_1\ d^3\mathbf{r}'_3 \langle \mathbf{r}_1\mathbf{r}_3|B\ V_{N-N}|\mathbf{r}'_1\mathbf{r}'_3\rangle \Psi_4(\mathbf{r}'_1\mathbf{r}_2\mathbf{r}'_3\mathbf{r}_4) + \text{four further permutations.}$$

Single-nucleon Hamiltonian h_i , Pauli blocking B: Tamm-Dancoff $[1 - f_1(\hat{h}_1)][1 - f_2(\hat{h}_2)]$

Homogeneous nuclear matter: momentum representation

$$\left[-\frac{\hbar^2}{8m}\mathbf{P}^2 + \tilde{W}(\mathbf{P})\right]\tilde{\Phi}(\mathbf{P}) = E_4(\mathbf{P})\,\tilde{\Phi}(\mathbf{P})\,.$$

Intrinsic motion

$$\frac{\hbar^2}{2m}[k^2 + 2k_{12}^2 + 2k_{34}^2]\tilde{\varphi}^{\text{intr}}(\mathbf{k},\mathbf{k}_{12},\mathbf{k}_{34},\mathbf{P}) + \int \frac{d^3k'}{(2\pi)^3}\frac{d^3k'_{12}}{(2\pi)^3}\frac{d^3k'_{34}}{(2\pi)^3}\tilde{V}^{(4)}(\mathbf{k},\mathbf{k}_{12},\mathbf{k}_{34};\mathbf{k}',\mathbf{k}'_{12},\mathbf{k}'_{34};\mathbf{P})\tilde{\varphi}^{\text{intr}}(\mathbf{k}',\mathbf{k}'_{12},\mathbf{k}'_{34},\mathbf{P})$$

$$= \tilde{W}(\mathbf{P})\tilde{\varphi}^{\text{intr}}(\mathbf{k},\mathbf{k}_{12},\mathbf{k}_{34},\mathbf{P})$$

Effective in-medium interaction

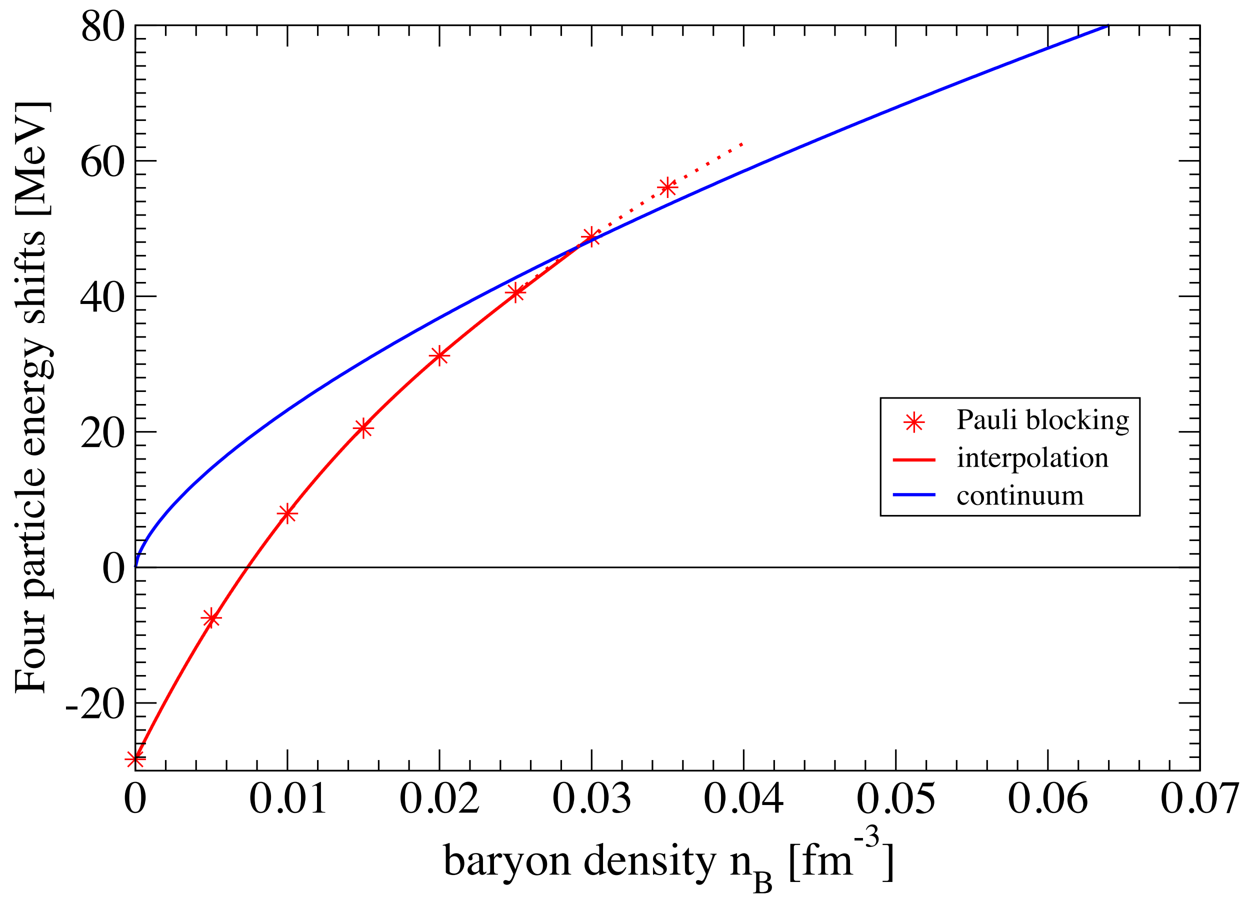
$$V^{(4)}_{N-N}(\mathbf{p}_1,\mathbf{p}_2,\mathbf{p}_3,\mathbf{p}_4;\mathbf{p}'_1,\mathbf{p}'_2,\mathbf{p}'_3,\mathbf{p}'_4) = (1 - f_{\tau_1,\mathbf{p}_1})(1 - f_{\tau_2,\mathbf{p}_2})V_{N-N}(\mathbf{p}_1,\mathbf{p}_2;\mathbf{p}'_1,\mathbf{p}'_2)\delta(\mathbf{p}'_3 - \mathbf{p}_3)\delta(\mathbf{p}'_4 - \mathbf{p}_4)$$
$$+\text{five permutations}$$

Four-nucleon energies at finite density

Solution of the in-medium wave equation, T = 0

4 free nucleons
at the Fermi energy
(continuum)

bound state
(α particle)
with Pauli blocking



α-like correlations in a nucleus

c. o. m. wave equation
$$-\frac{\hbar^2}{8m}\nabla_R^2\Phi(\mathbf{R})+\int d^3R'\,W(\mathbf{R},\mathbf{R}')\;\Phi(\mathbf{R}')=E\,\Phi(\mathbf{R})$$

Effective c. o. m. potential
$$W(\mathbf{R},\mathbf{R}')=E_4^{\rm intr}(\mathbf{R})\delta(\mathbf{R}-\mathbf{R}')$$

$$\int d^3R'\,d^9s'_j\;[T_4[\nabla_{s_j}]\delta(\mathbf{R}-\mathbf{R}')\delta(\mathbf{s}_j-\mathbf{s}'_j)+V_4(\mathbf{R},\mathbf{s}_j;\mathbf{R}',\mathbf{s}'_j)]\,\frac{\Phi(\mathbf{R}')}{|\Phi(\mathbf{R})|^2}\varphi_4^{\rm intr}(\mathbf{s}'_j,\mathbf{R}')=E_4^{\rm intr}(\mathbf{R})\varphi_4^{\rm intr}(\mathbf{s}_j,\mathbf{R})$$

Local approximation for the four nucleon effective potential

$$V_4(\mathbf{R},\mathbf{s}_j;\mathbf{R}',\mathbf{s}'_j)=V_4^{\rm ext}(\mathbf{R},\mathbf{s}_j;\mathbf{R}',\mathbf{s}'_j)+V_4^{\rm intr}(\mathbf{R},\mathbf{s}_j;\mathbf{R}',\mathbf{s}'_j)$$

External contribution together with mean-field contribution to the effective potential

$$V_4^{\rm ext}(\mathbf{R},\mathbf{s}_j;\mathbf{R}',\mathbf{s}'_j)=\Big[V_{\tau_1}^{\rm mf}(\mathbf{R}+\frac{1}{2}\mathbf{s}+\frac{1}{2}\mathbf{s}_{12})+V_{\tau_2}^{\rm mf}(\mathbf{R}+\frac{1}{2}\mathbf{s}-\frac{1}{2}\mathbf{s}_{12})$$
$$+V_{\tau_3}^{\rm mf}(\mathbf{R}-\frac{1}{2}\mathbf{s}+\frac{1}{2}\mathbf{s}_{34})+V_{\tau_4}^{\rm mf}(\mathbf{R}-\frac{1}{2}\mathbf{s}-\frac{1}{2}\mathbf{s}_{34})\Big]\,\delta(\mathbf{R}-\mathbf{R}')\delta(\mathbf{s}-\mathbf{s}')\delta(\mathbf{s}_{12}-\mathbf{s}'_{12})\delta(\mathbf{s}_{34}-\mathbf{s}'_{34})$$

Intrinsic contribution containing Pauli blocking

$$V_4^{\rm intr}(\mathbf{r}_i;\mathbf{r}'_i)=\int d^3r''_1\,d^3r''_2\;\langle\mathbf{r}_1\mathbf{r}_2|[1-f_1(\varepsilon_{n_1})][1-f_2(\varepsilon_{n_2})]|\mathbf{r}''_1\mathbf{r}''_2\rangle\langle\mathbf{r}''_1\mathbf{r}''_2|V_{N-N}|\mathbf{r}'_1\mathbf{r}'_2\rangle\delta(\mathbf{r}'_3-\mathbf{r}_3)\delta(\mathbf{r}'_4-\mathbf{r}_4)$$
$$+\ \text{five permutations}$$

Mixed representation
$$\langle\mathbf{r}_1|f_1(E_{n_1})|\mathbf{r}''_1\rangle=\int\frac{d^3p_1}{(2\pi)^3}\,\mathrm{e}^{i\mathbf{p}_1\cdot(\mathbf{r}_1-\mathbf{r}''_1)}f_1^{\rm Wigner}\left(\frac{\mathbf{r}_1+\mathbf{r}''_1}{2},\mathbf{p}_1\right)$$

Quantum condensate

Ideal Bose condensate : $|0\rangle = b_0^\dagger b_0^\dagger \cdots b_0^\dagger |vac\rangle$

α-particle condensate : $|\Phi_{\alpha C}\rangle = C_\alpha^\dagger C_\alpha^\dagger \cdots C_\alpha^\dagger |vac\rangle$

In r-space :

$$\langle \vec{r}_1, \vec{r}_2, \cdots, \vec{r}_{4n} | \Phi_{\alpha C} \rangle = \mathcal{A}\{\Phi(\vec{r}_1, \vec{r}_2, \vec{r}_3, \vec{r}_4)\Phi(\vec{r}_5, \vec{r}_6, \vec{r}_7, \vec{r}_8)\cdots\Phi(\vec{r}_{4n-3}, \vec{r}_{4n-2}, \vec{r}_{4n-1}, \vec{r}_{4n})\}$$

In comparison with pairing :

$$\langle \vec{r}_1, \vec{r}_2, \cdots | \mathrm{BCS} \rangle = \mathcal{A}\{\Phi(\vec{r}_1, \vec{r}_2)\Phi(\vec{r}_3, \vec{r}_4)\cdots\}$$

A. Tohsaki et al., PRL **87**, 192501 (2001)

Variational ansatz

Variational ansatz for $\Phi(\vec{r}_1,\vec{r}_2,\vec{r}_3,\vec{r}_4)$: $\Phi(\vec{r}_1,\vec{r}_2,\vec{r}_3,\vec{r}_4) = e^{-\frac{2}{B^2}\vec{R}^2}\phi_\alpha(\vec{r}_i - \vec{r}_j)$

Center of mass : $\vec{R} = \frac{1}{4}(\vec{r}_1 + \vec{r}_2 + \vec{r}_3 + \vec{r}_4)$

Intrinsic α-wave function :

$$\phi_\alpha(\vec{r}_i - \vec{r}_j) = e^{-\frac{1}{8b^2}\left\{(\vec{r}_4-\vec{r}_1)^2+(\vec{r}_4-\vec{r}_2)^2+(\vec{r}_4-\vec{r}_3)^2+\cdots\right\}}$$

Two variational parameters : $B,\ b$

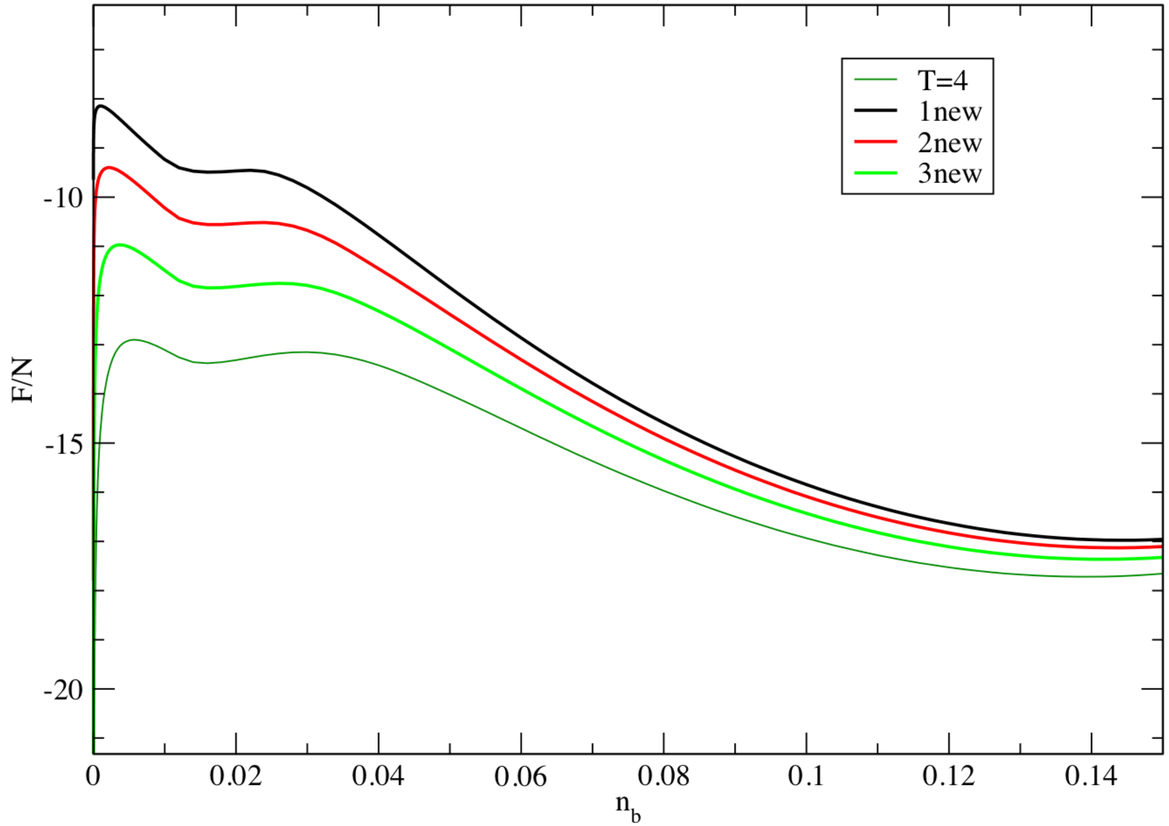
Two limits : $B = b$ $\quad |\Phi_{\alpha C}\rangle =$ Slater determinant

$B \gg b$ $\quad |\Phi_{\alpha C}\rangle =$ gas of independent α-particles

Two dimensional surface : $E(B,b) = \frac{\langle \Phi_{\alpha C} | H | \Phi_{\alpha C} \rangle}{\langle \Phi_{\alpha C} | \Phi_{\alpha C} \rangle}$

but deuteron / dineutron case: non-Gaussian wf

Free energy per nucleon



(preliminary)

correlated
medium

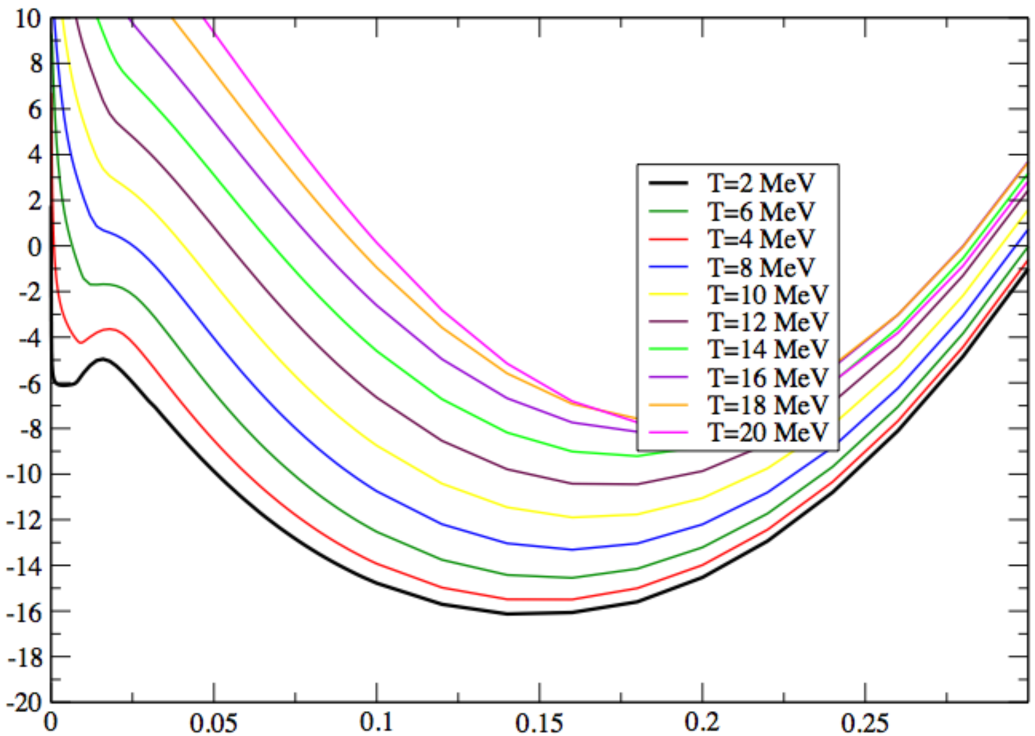
Constrained THSR calculations as function of the c.o.m. width B?

Quantum condensates in nuclei?

Lot of semantics – my position

- Pairing is well accepted.
- Quartetting is not very well-known and simple.
- The main point is the formation of clusters (correlations) in low-density matter.
- We are interested in an efficient description (optimal wave function) for the cluster state.
- The center of mass motion has to be considered as new (collective) degree of freedom.

Internal energy per nucleon



Quantum statistical approach:

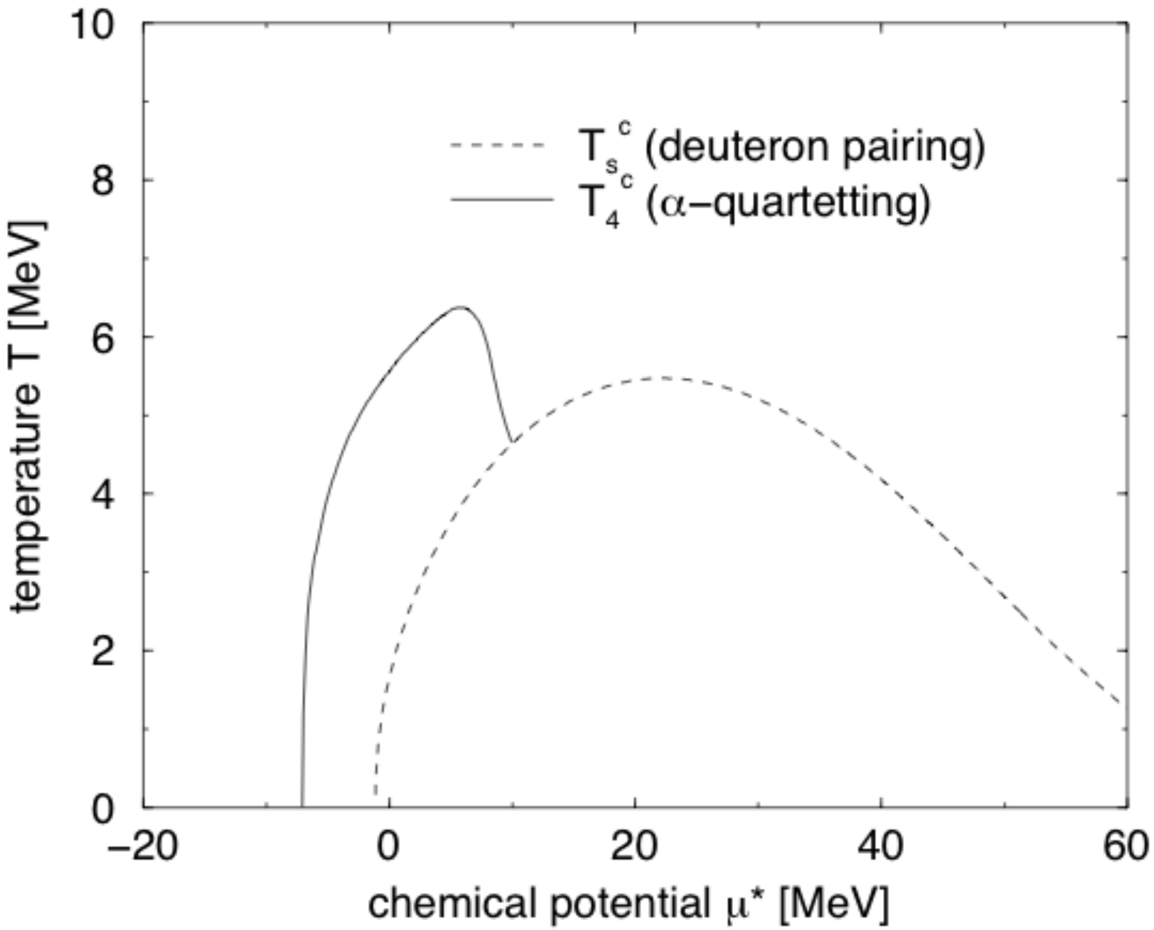
Cluster ?

Condensate?

EOS for symmetric matter - low density region?

α-cluster-condensation

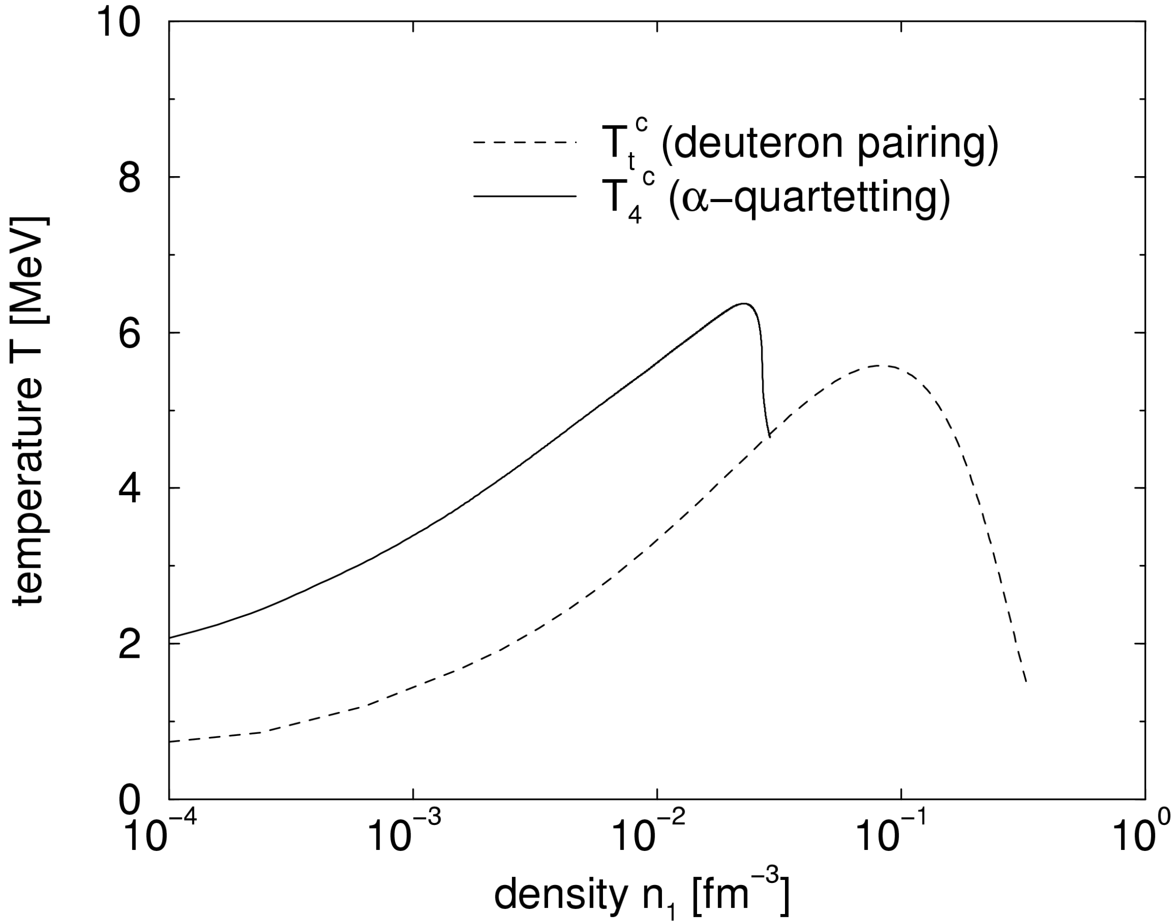
(quartetting)



G.Röpke, A.Schnell, P.Schuck, and P.Nozieres, PRL 80, 3177 (1998)

α-cluster-condensation

(quartetting)



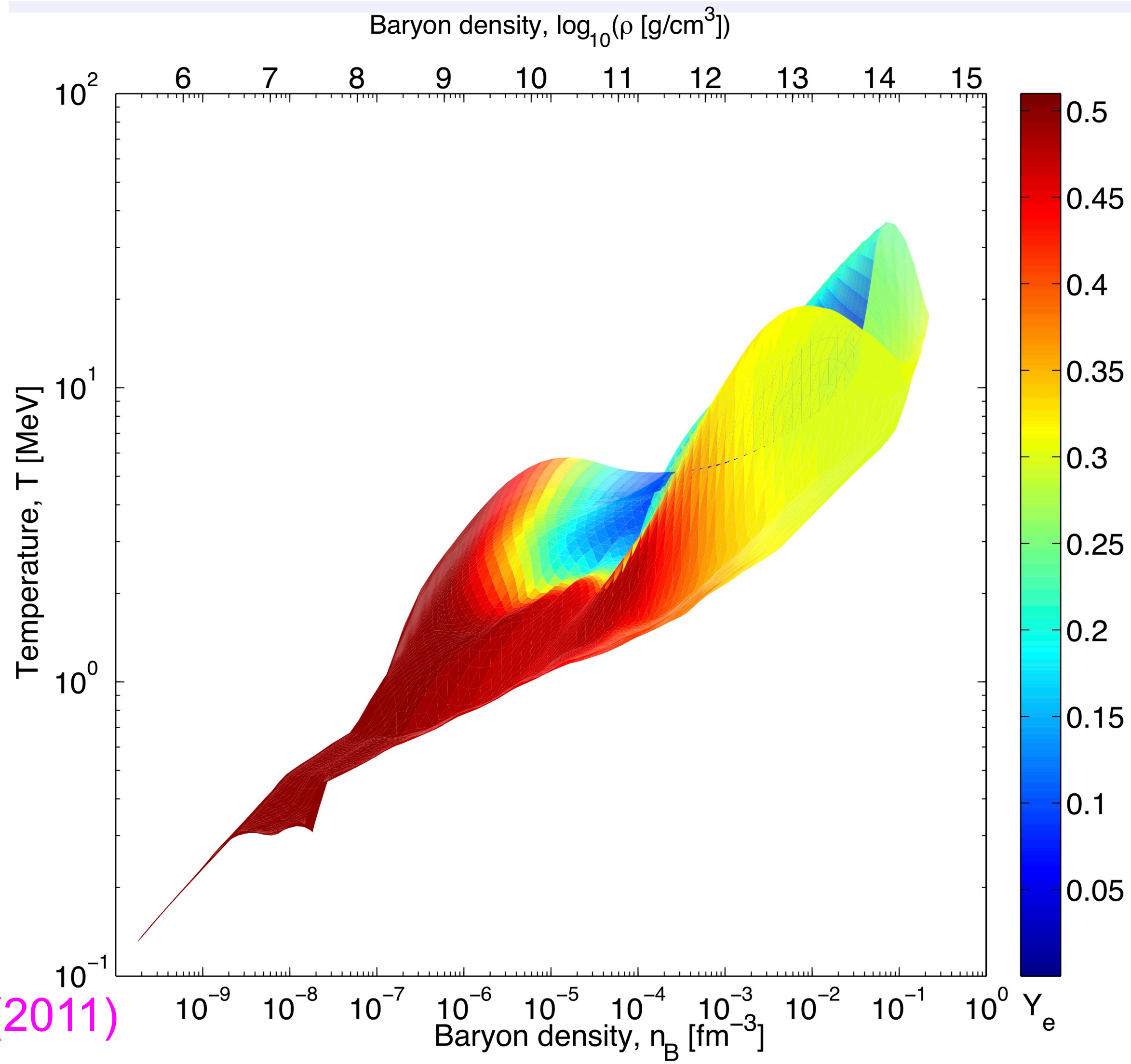
G.Röpke, A.Schnell, P.Schuck, and P.Nozieres, PRL 80, 3177 (1998)

Nuclear matter phase diagram

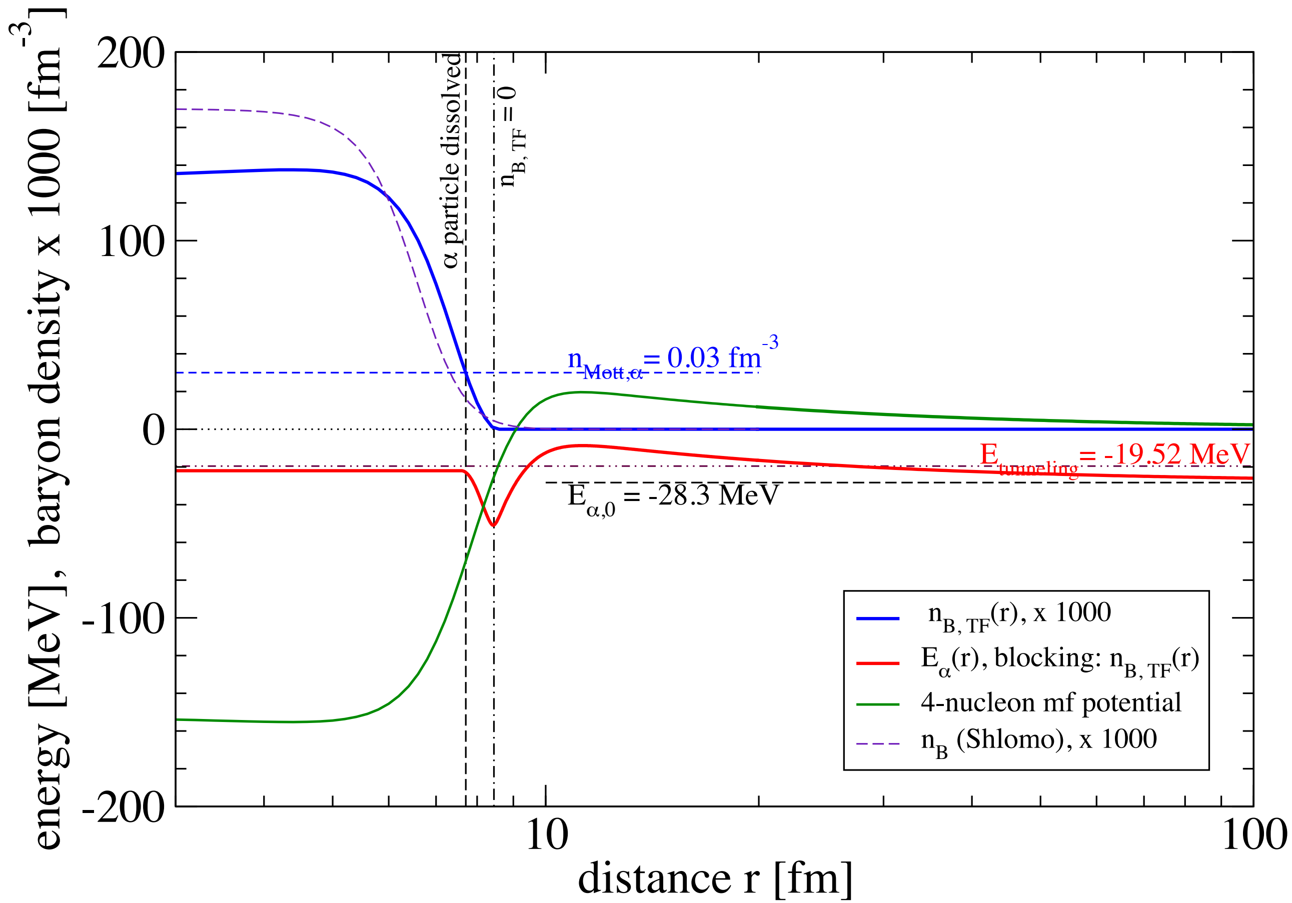
Core collapse supernovae

Relevant Parameters:

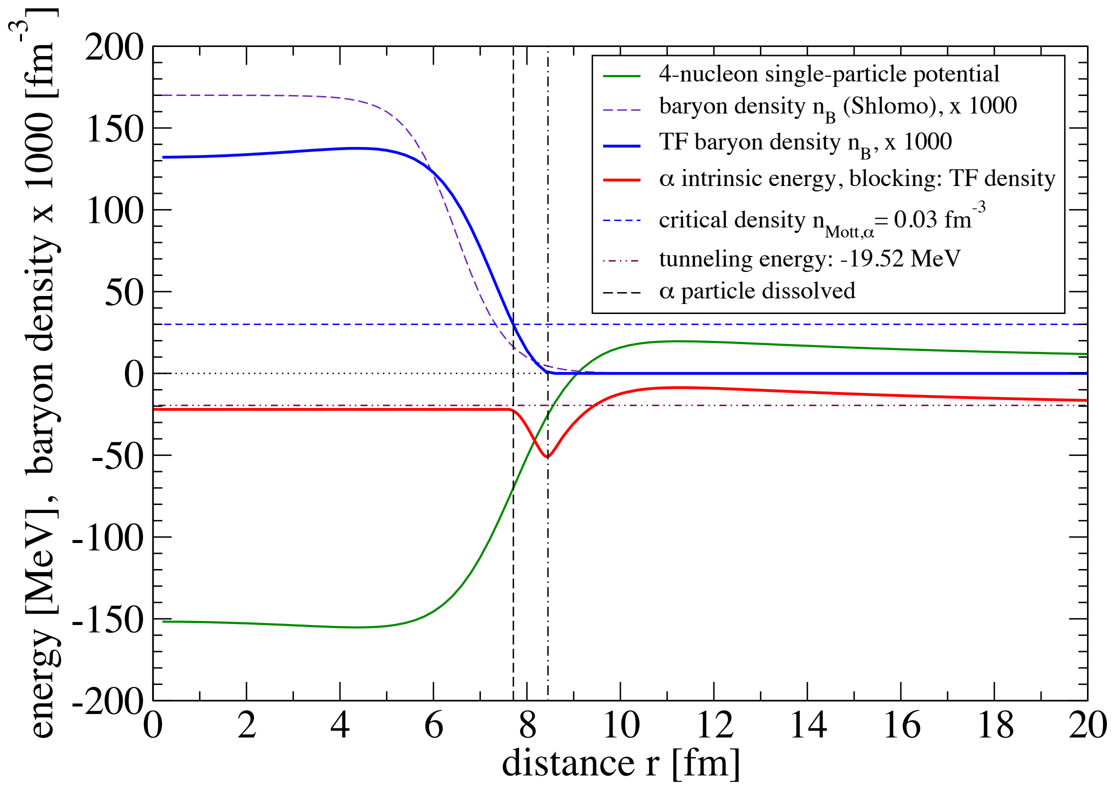
- **density**:
  $10^{-9} \lesssim \varrho/\varrho_{\rm sat} \lesssim 10$
  with nuclear saturation density
  $\varrho_{\rm sat} \approx 2.5 \cdot 10^{14}$ g/cm^3
  ($n_{\rm sat} = \varrho_{\rm sat}/m_n \approx 0.15$ fm^{-3})
- **temperature**:
  0 MeV $\leq k_B T \lesssim 50$ MeV
  ($\hat{=}\ 5.8 \cdot 10^{11}$ K)
- **electron fraction**:
  $0 \leq Y_e \lesssim 0.6$



T. Fischer et al., ApJS 194, 39 (2011)



energy [MeV], baryon density x 1000 [fm^-3]
200
100
0
-100
-200
10
100
distance r [fm]
α particle dissolved
n_B, TF = 0
n_Mott,α = 0.03 fm^-3
E_tunneling = -19.52 MeV
E_α,0 = -28.3 MeV
n_B, TF(r), x 1000
E_α(r), blocking: n_B, TF(r)
4-nucleon mf potential
n_B (Shlomo), x 1000



energy [MeV], baryon density x 1000 [fm^-3]
200
150
100
50
0
-50
-100
-150
-200
0
2
4
6
8
10
12
14
16
18
20
distance r [fm]
4-nucleon single-particle potential
baryon density n_B (Shlomo), x 1000
TF baryon density n_B, x 1000
α intrinsic energy, blocking: TF density
critical density n_Mott,α = 0.03 fm^-3
tunneling energy: -19.52 MeV
α particle dissolved

Excited light nuclei

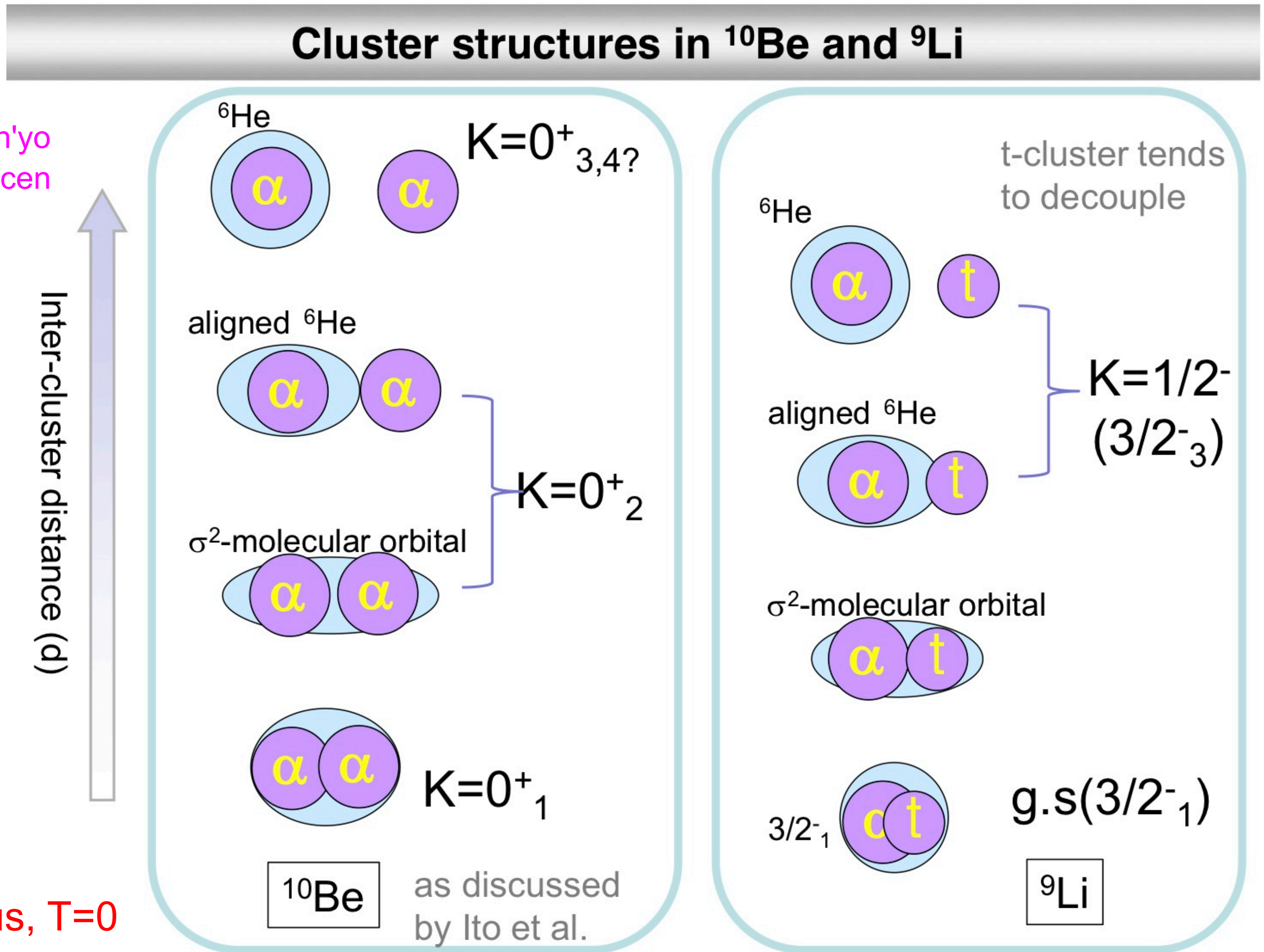
Yoshiko Kanada-En'yo
Cluster2012,Debrecen

decreasing
density

deuterons?

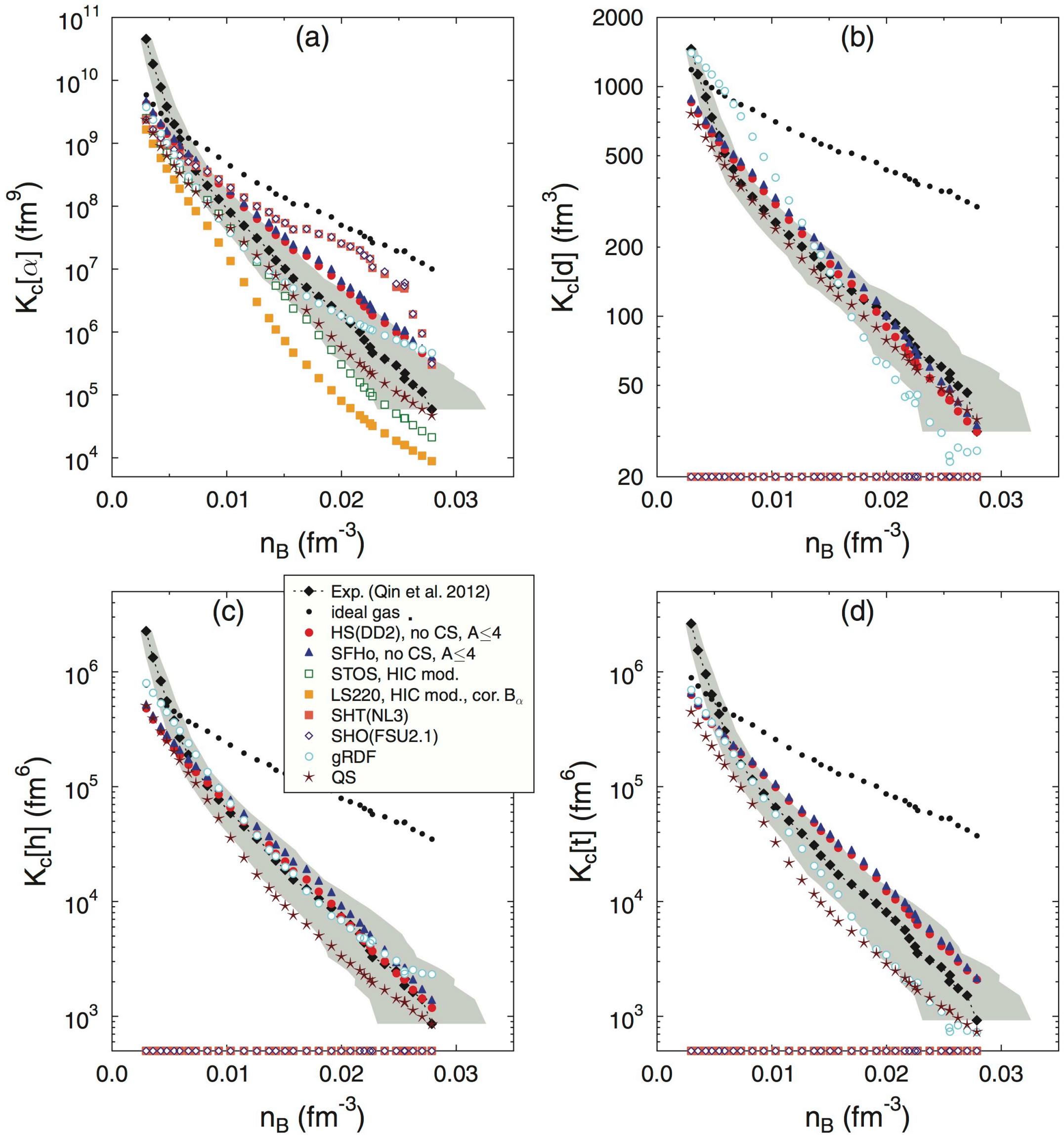
systematics
in
weakly bound
light
elements

light
clusters
in neutron
matter



inhomogeneous, T=0

Chemical constants



Matthias Hempel,
Kris Hagel,
Joseph Natowitz,
Gerd Röpke, and
Stefan Typel
Phys. Rev. C **91**,
045805 (2015)

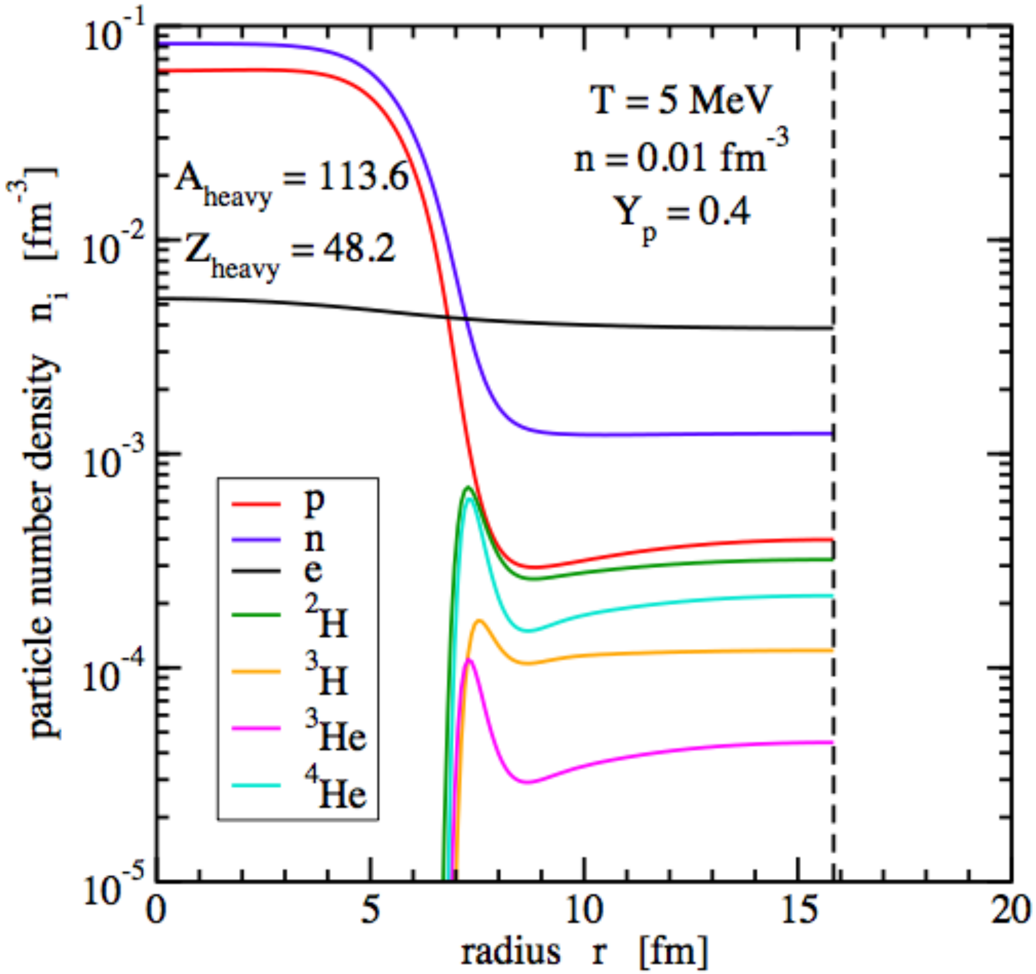
α cluster in astrophysics

Crust of neutron stars

Protons in droplets
(heavy nuclei)

$\alpha-$cluster outside,
at the surface,
condensate?

S. Typel, Proc. Int. Workshop XII Hadron Physics



Two-particle correlations

Generalized
Beth-Uhlenbeck Approach
for Hot Nuclear Matter

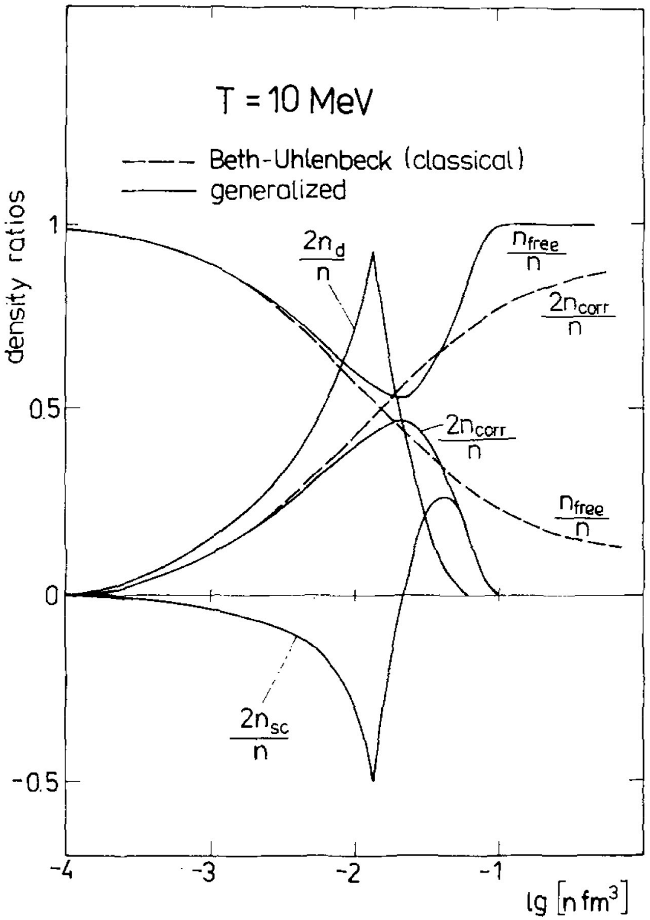


FIG. 7. The composition of nuclear matter as a function of the density n for given temperature $T = 10$ MeV. The solid and dashed lines show the results of the generalized and classical Beth–Uhlenbeck approach, respectively. Note the distinct behavior of n_{free} and n_{corr} predicted by the two approaches in the low and high density limit!

M. Schmidt, G.R., H. Schulz
Ann. Phys. **202**, 57 (1990)

^{212}Po: α on top of ^{208}Pb

pocket formation for the c.m. motion