On dynamics of the first order quark-hadron and liquid-gas phase transitions

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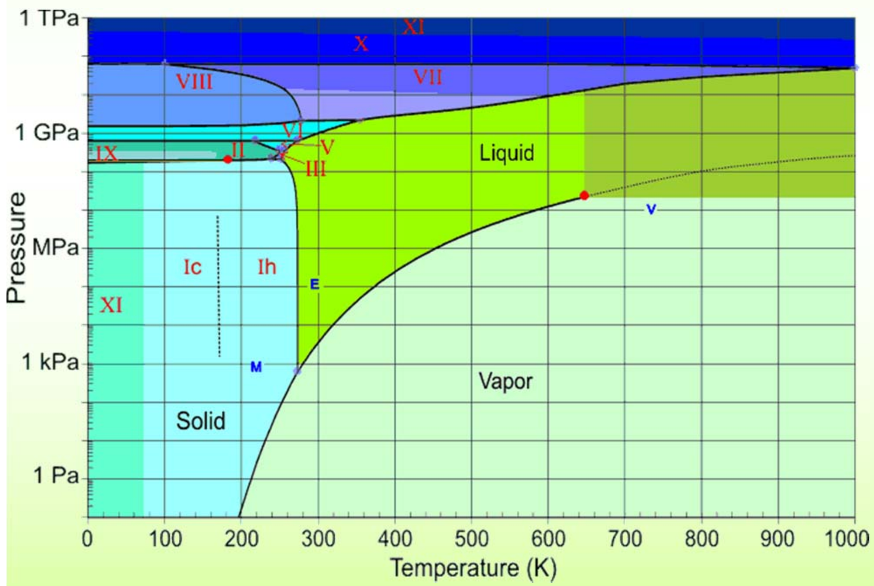
Plan:

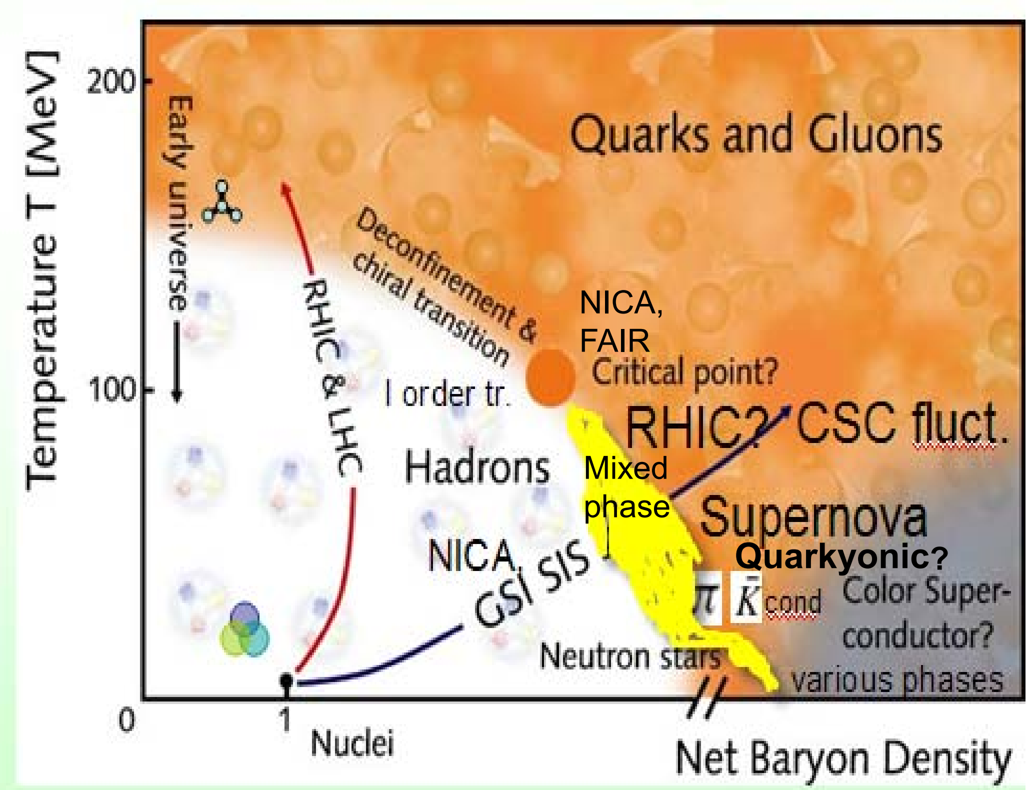
- Phenomenological description of phase transitions.
- Hydrodynamical description of I order phase transition (*on example of the hadron-quark and hadron liquid-gas I order phase transitions*). Demonstration of important role of non-zero viscosity and thermal conductivity.
- Liquid-gas or/and a scalar field condensation?

Phase Diagrams
Water and Nuclear Matter

Variety of phases: 12 crystalline, 3 glass, liquid, vapor, CEP



Chapline et al. (2007)

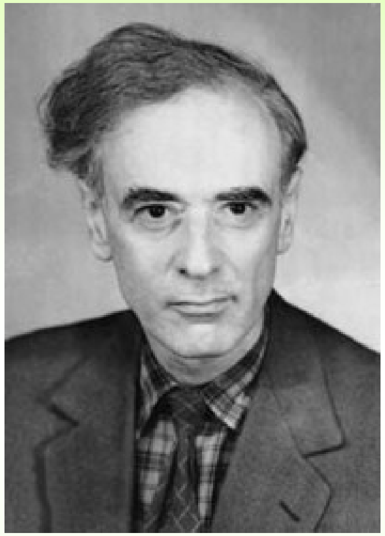


Low density,low T: *HIC* (liquid-gas); *excited nuclei* (high spin, pairing); high density, low T: *SN,NS*: (NN-pairing, π,K,ρ-condensates; CSC, quarkyonic); high T *HIC*: (chiral restoration, deconfinement), CEP

Landau Phenomenological Description of Phase Transitions

Simplest case: one order parameter.

Expand free energy in *Φ, grad. Φ,*
and then coefficients, in $T-T_{cr}$ near critical point:



$$F = Const + \int d^3x \left(\frac{m}{2}(\nabla\phi)^2 + \frac{a}{2}\phi^2 + \frac{b}{3}\phi^3 + \frac{c}{4}\phi^4 + h\phi \right)$$

Either cubic or linear term can be eliminated by the shift of the order parameter.

II order phase transition:

Specific heat C_v has finite value in crit. point ⇒ near critical point

$F = -\alpha^2(T-T_{cr})^2/4c$, $\Phi^2 \sim T-T_{cr}$, $a = \alpha\,(T-T_{cr})$, $b = h = 0$

I order phase transition: *Φ* has finite value, $h \neq 0$ (usually *b* is put zero)

Fluctuation region near T_{cr}

(thermodynamical treatment)

Ginzburg criterion: $W \sim \exp(-\delta F(T) V_{fl}/T)$,

energy loss $\delta F^{MF} \sim \alpha^2 (T-T_{cr})^2/c$,

in minimal volume $V_{fl} \sim l_0^3$, $l_0 \sim 1/(T_{cr}-T)^{1/2}$

is coherence length.

At $T \sim T_{fl}$ the fluctuation formed in a minimal volume $\sim l_0^3$ is probable ($W \sim 1$). Fluctuations are dominant for T near T_{cr} , $|T_{cr} - T_{fl}|/T_{cr} < 1$. Fl. region is estimated by Ginzburg number $Gi = |T_{cr} - T_{fl}|/T_{cr}$

in clean metalic supercond. $Gi \sim 10^{-8}$, fl. region is narrow , in He^4, quark-hadron and hadron liquid-gas phase tr. (strong interaction) $Gi \sim 1$, fl. region is broad

Energy variance $\overline{(\Delta E)^2} = T^2 C_V$ diverges in critical point.

Dynamical description

in condensed matter there always exist slowly dissipating modes:

deviation from equilibrium is proportional to a thermodynamical force

$$\Rightarrow \quad \frac{\partial \phi}{\partial t} = -\Gamma(\Delta)\frac{\delta F}{\delta \phi} \quad + \text{ white noise}$$

$$\Gamma(\Delta) = a_0 - a_1\Delta$$ is expanded in gradients

a_0=0 for conserved order parameter (like entropy)
a_1 =0 for non-conserved order parameter (like density)

Hydrodynamics of the first order phase transition:

V.Skokov, D.V. , arXiv 0811.3868, JETP Lett. 90 (2009) 223;
Nucl. Phys. A828 (2009) 401; A847 (2010) 253.

We solve the system of non-ideal hydro equations describing non-trivial fluctuations (droplets/bubbles, aerosol) in d=2 space +1 time dimensions numerically for van der Waals-like EoS, and for arbitrary d in the vicinity of the critical point analytically.

Non-ideal non-relativistic hydrodynamics

the less viscous the fluid is, the greater its ease of movement

$$mn\left[\partial_t u_i + (\mathbf{u}\nabla)u_i\right] = -\nabla_i P$$
$$+\nabla_k\left[\eta\left(\nabla_k u_i + \nabla_i u_k - \frac{2}{d}\delta_{ik}\mathrm{div}\mathbf{u}\right) + \zeta\delta_{ik}\mathrm{div}\mathbf{u}\right] \quad (8)$$
$$\partial_t n + \mathrm{div}(n\mathbf{u}) = 0, \quad (9)$$
$$T\left[\frac{\partial s}{\partial t} + \mathrm{div}(s\mathbf{u})\right] = \mathrm{div}(\kappa\nabla T)$$
$$+\eta\left(\nabla_k u_i + \nabla_i u_k - \frac{2}{d}\delta_{ik}\mathrm{div}\mathbf{u}\right)^2 + \zeta(\mathrm{div}\mathbf{u})^2. \quad (10)$$

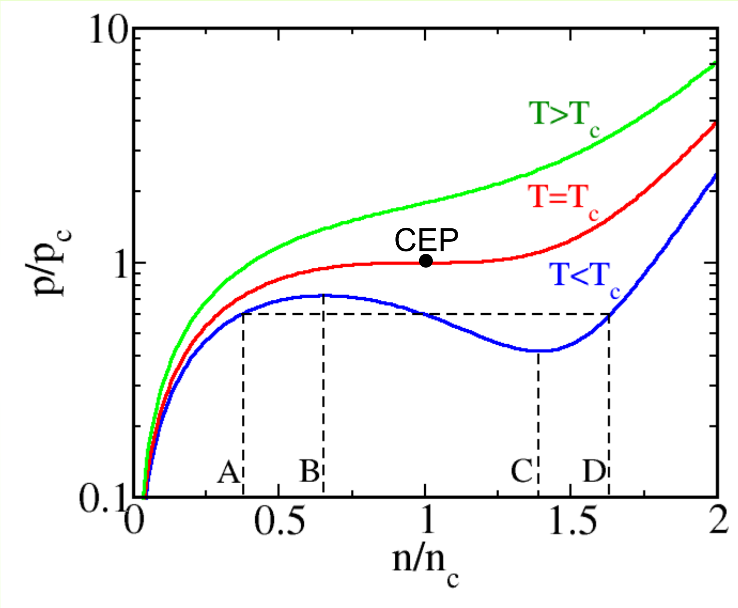
Here η and ζ are shear and bulk viscosities; $\mathbf{u}$ is the velocity of the element of the fluid; s is the entropy density; κ is the thermal conductivity; d is the dimensionality of space.

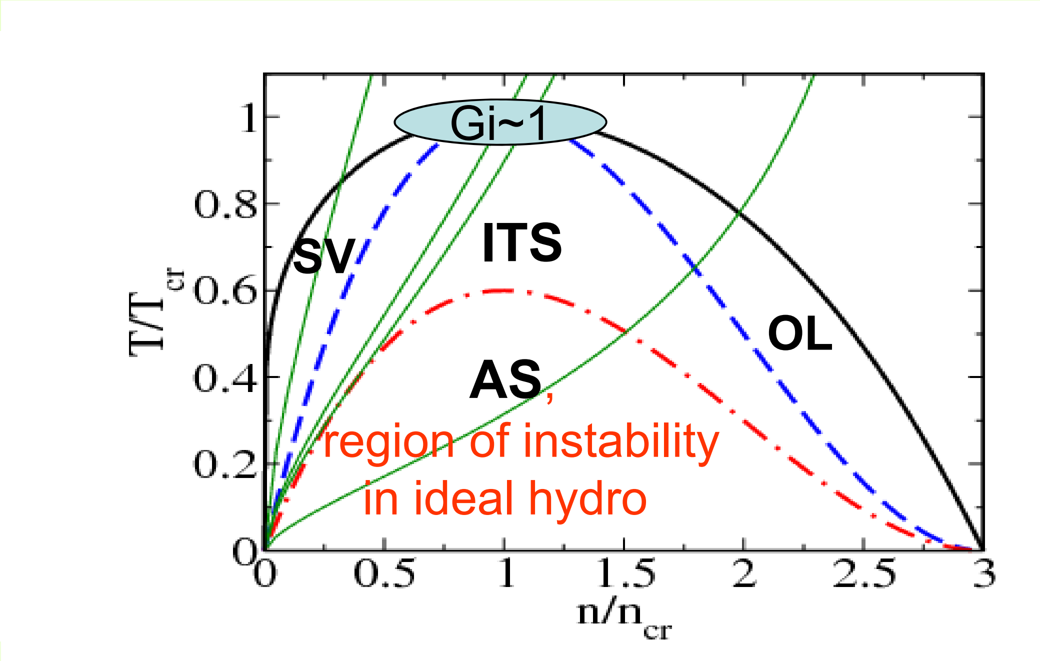
The reciprocal of thermal conductivity is *thermal resistivity*

In collective processes u is usually small,

therefore in analytical treatment we neglect u^2 terms

EoS and Constant entropy trajectories

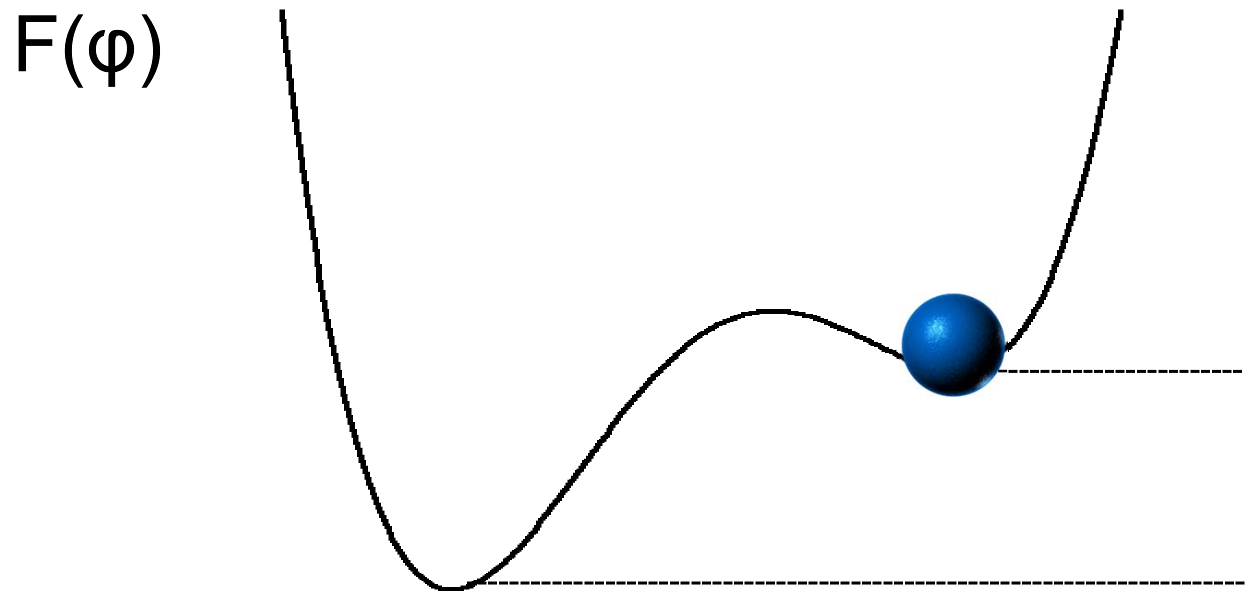




- - - isothermal spinodal (ITS), - . - . - adiabatic spinodal (AS),
▬ Maxwell construction

T_{max} =0.6 T_{cr} for van der Waals EoS

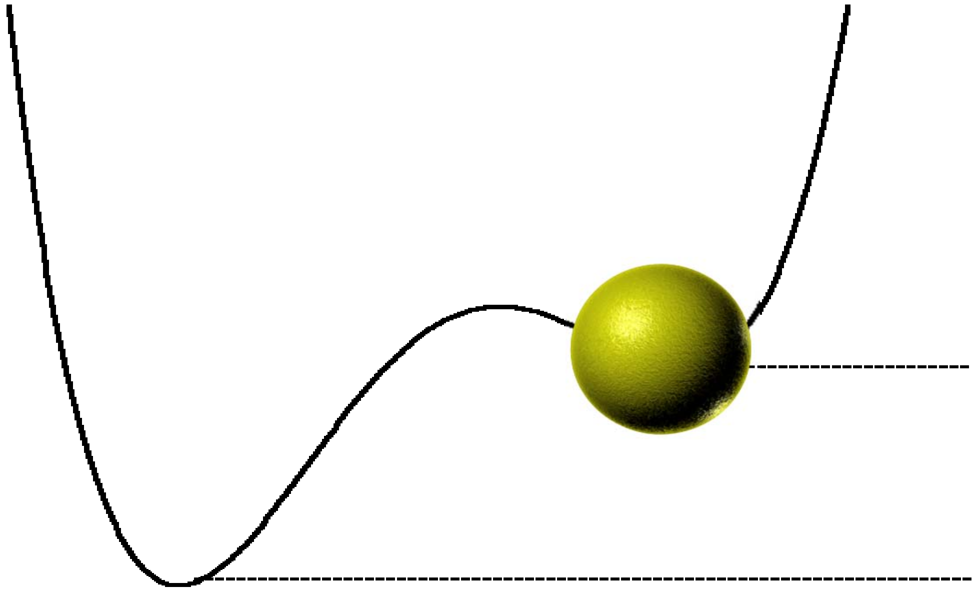
I order phase transition



$$a\phi + c\phi^3 + h = 0$$

,

I order phase transition



$$a\phi + c\phi^3 + h = 0$$

,

Near critical point:

$$\delta F_L = \int \frac{d^3x}{\rho_r}\left[\frac{c[\nabla(\delta\rho)]^2}{2} + \frac{\lambda(\delta\rho)^4}{4} - \frac{\lambda v^2(\delta\rho)^2}{2} - \epsilon\delta\rho\right],$$

From Navier-Stokes and continuity equations
neglecting u^2 terms:

viscosities

$\delta F / \delta \rho$

$$-\frac{\partial^2\delta\rho}{\partial t^2} = \Delta\left[c\Delta\delta\rho + \lambda v^2\delta\rho - \lambda(\delta\rho)^3 + \epsilon - \rho_r^{-1}\left(\frac{4}{3}\eta_r + \zeta_r\right)\frac{\partial\delta\rho}{\partial t}\right]$$ + white noise

See D.V. Phys.Scripta 47 (1993) 333 $\qquad \delta\rho = \rho - \rho_r$

cf. Landau Eq.

$\rho = mn$, m is baryon mass, ρ_r is near ρ_{cr}

neglecting u: $T\, \partial s / \partial t = \kappa \Delta T$, $\qquad$ s - is entropy density

$$t_T \sim R^2 c_V / \kappa$$

Peculiarities of hydro- description

Eq. is the 2-order in time derivatives -- beyond the standard Ginzburg-Landau description where:

$$-\ \rho_{\mathrm{r}}^{-2}\left(\tilde{d}\eta_{\mathrm{r}}+\zeta_{\mathrm{r}}\right)\frac{\partial\delta\rho}{\partial t}=\frac{\delta[F(T,\delta\rho)]}{\delta(\delta\rho)}|_{T}.$$

thermodynamical force

However for a produced fluctuation two initial conditions should be fulfilled

$$\delta\rho(t=0,\vec{r})=\delta\rho(0,\vec{r}), \qquad \frac{\partial\delta\rho(t,\vec{r})}{\partial t}|_{t=0}\simeq 0$$

initial stage of fluctuation dynamics is not described in Landau approximation; at large t one can use the Landau approximation.

Qualitative analysis and rough estimates

Dynamics is controlled by the slowest mode

typical time for density fluctuation: $t_\rho \sim R$ (constant velocity)

R (t) is the size of evolving seed

typical time for heat transport $t_T \sim R^2 c_V / \kappa$, c_V is specific heat density

We introduce R_{fog} -- typical seed size at which $t_\rho = t_T$

$t_\rho > t_T$ for $R(t) < R_{fog}$: **Density evolution stage (isothermal)**

$t_T > t_\rho$ for $R(t) > R_{fog}$: **Heat transport stage**

Seeds with $R \sim R_{fog}$ are accumulated with passage of time: ⇨ **fog stage**

for H-QGP phase transition $R_{fog} \sim 0.1$-1 fm, for liquid-gas ~1-10 fm,

Thermal conductivity effects should be incorporated in hydro simulations of HIC

Dimension-less equation of motion, typical scales

viscosities

$$-\frac{\partial^2 \delta\rho}{\partial t^2} = \Delta\left[c\Delta\delta\rho + \lambda v^2 \delta\rho - \lambda(\delta\rho)^3 + \epsilon - \rho_{\mathrm{r}}^{-1}\left(\frac{4}{3}\eta_{\mathrm{r}} + \zeta_{\mathrm{r}}\right)\frac{\partial\delta\rho}{\partial t}\right]$$

$$\delta\rho = \rho - \rho_{\mathrm{r}}$$

In dimensionless variables

$$\delta\rho = v\,\psi, \quad \xi_i = x_i/l, \quad \tau = t/t_0$$

$$-\beta\frac{\partial^2\psi}{\partial\tau^2} = \Delta_\xi\left(\Delta_\xi\psi + 2\psi(1-\psi^2) + \widetilde{\epsilon} - \frac{\partial\psi}{\partial\tau}\right)$$

$$l = \left(\frac{2c}{\lambda v^2}\right)^{1/2}, \quad t_0 = \frac{2(\frac{4}{3}\eta_{\mathrm{r}} + \zeta_{\mathrm{r}})}{\lambda v^2 \rho_{\mathrm{r}}}, \quad \widetilde{\epsilon} = \frac{2\epsilon}{\lambda v^3} \quad \beta = \frac{c\rho_{\mathrm{r}}^2}{(\frac{4}{3}\eta_{\mathrm{r}} + \zeta_{\mathrm{r}})^2}$$

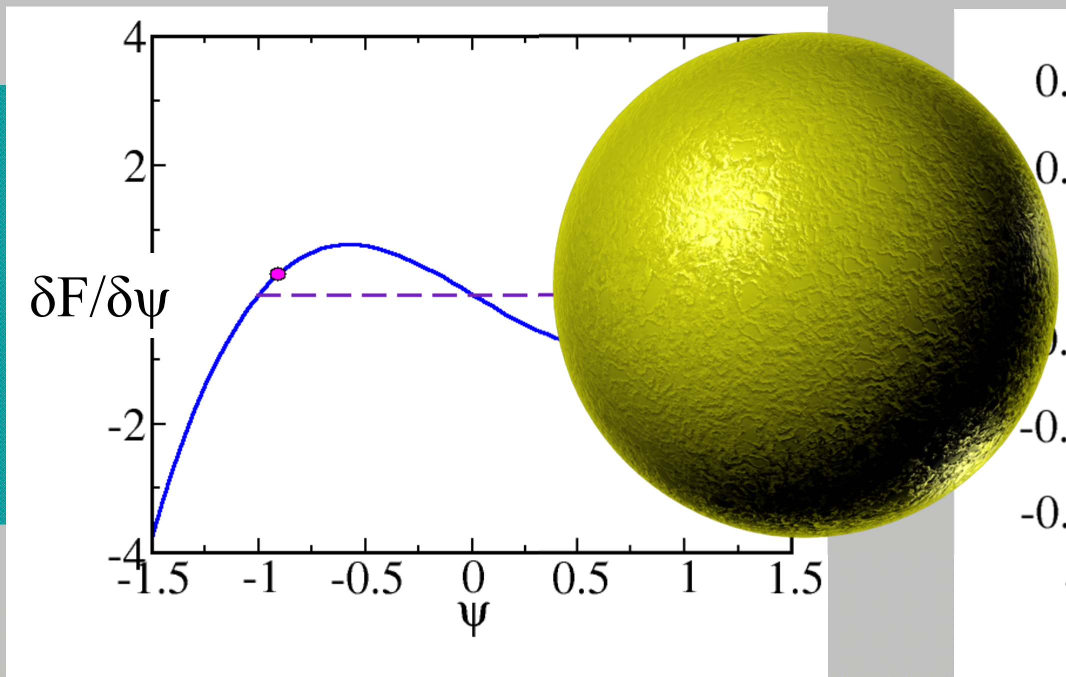
$$v \propto |T - T_{\mathrm{cr}}|^{1/2} \quad \longrightarrow \quad t_0 \propto |T - T_{\mathrm{cr}}|^{-1}$$

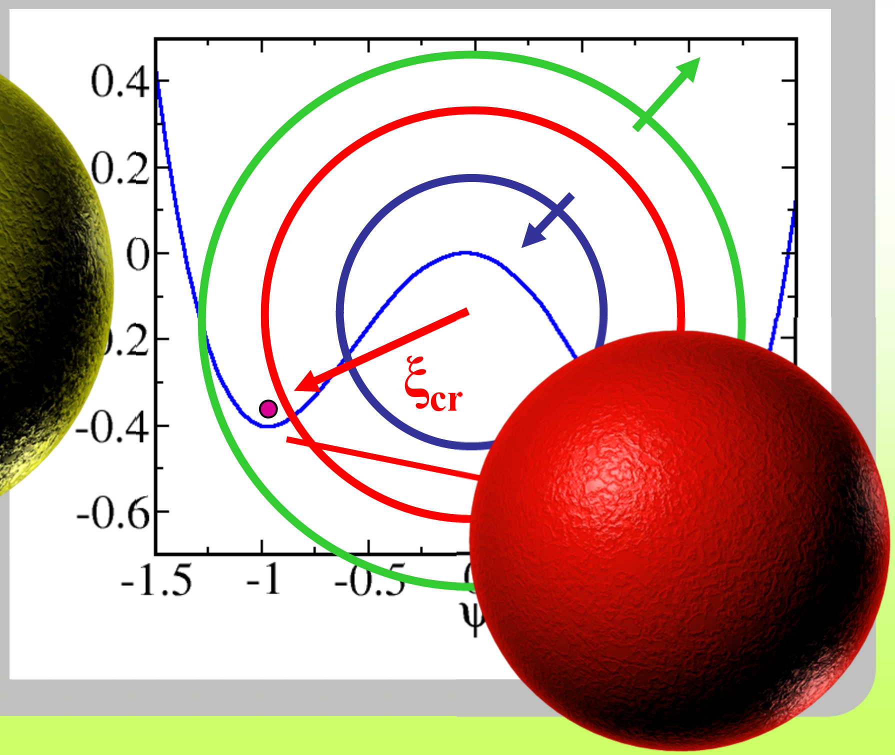
processes in the vicinity of the critical point prove to be very slow

Solution in the metastable region

d-dimensionality of space (d=1, or,2,or 3), ε«1, ε>0:

$$\psi = \mp \tanh(\xi - \xi_0) + \frac{\epsilon}{4}; \frac{\partial}{\partial \tau}\xi_0(\tau) = -\frac{d-1}{\xi_0(\tau)} \pm 3/2\epsilon$$





Baryon-less matter: Flow-experiments at RHIC indicate on very low viscosity

Conformal ADS/CFT theories show minimum $\eta/s \sim 1/4\pi$:

Baryon-rich matter: η/s did not appear in eqs of motion for fluctuations

Dynamics of the density mode is controlled by "inertial" parameter β, entering with the second derivative in time, expressed in terms of the **surface tension** and **viscosity**

$$\beta = \frac{\sigma_0^2\, m}{32\, T_{\rm cr}\, [\frac{4}{3}\,\eta_{\rm r} + \zeta_{\rm r}]^2}$$

σ_0 -surface tension for $T << T_c$

The **larger viscosity** and the **smaller surface tension**,
the effectively **more viscous (inertial)** is the fluidity of seeds.

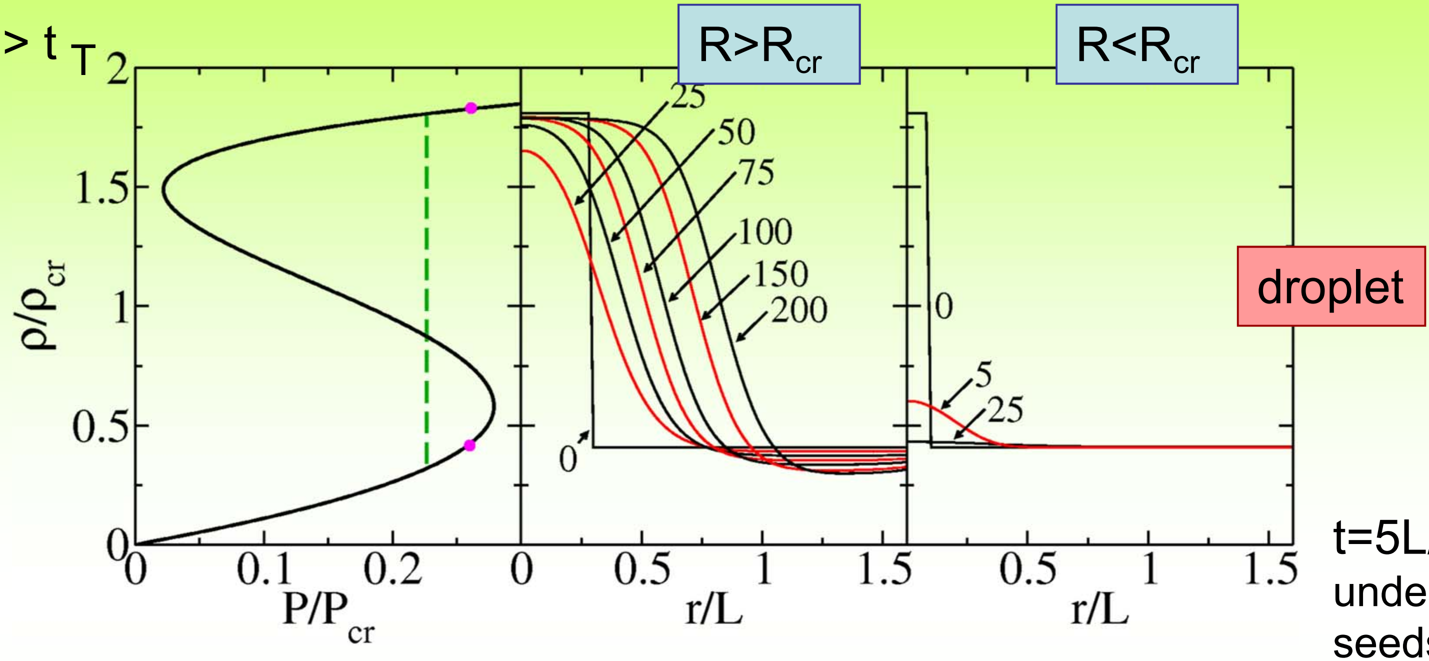
$\beta << 1$ effectively viscous fluidity; $\beta >> 1$ almost perfect fluidity

for liquid-gas phase transition $\beta \sim 0.01$;
for H-QGP phase transition: $\beta \sim 0.02$-0.2 , even for $\eta/s \sim 1/4\pi$:

⇨ **Effectively very viscous fluidity of density fluctuations in the course of the phase transitions in HIC!**

Hadron-QGP phase transition: droplet/bubble evolution from metastable phases

For $t_\rho \gg t_T$

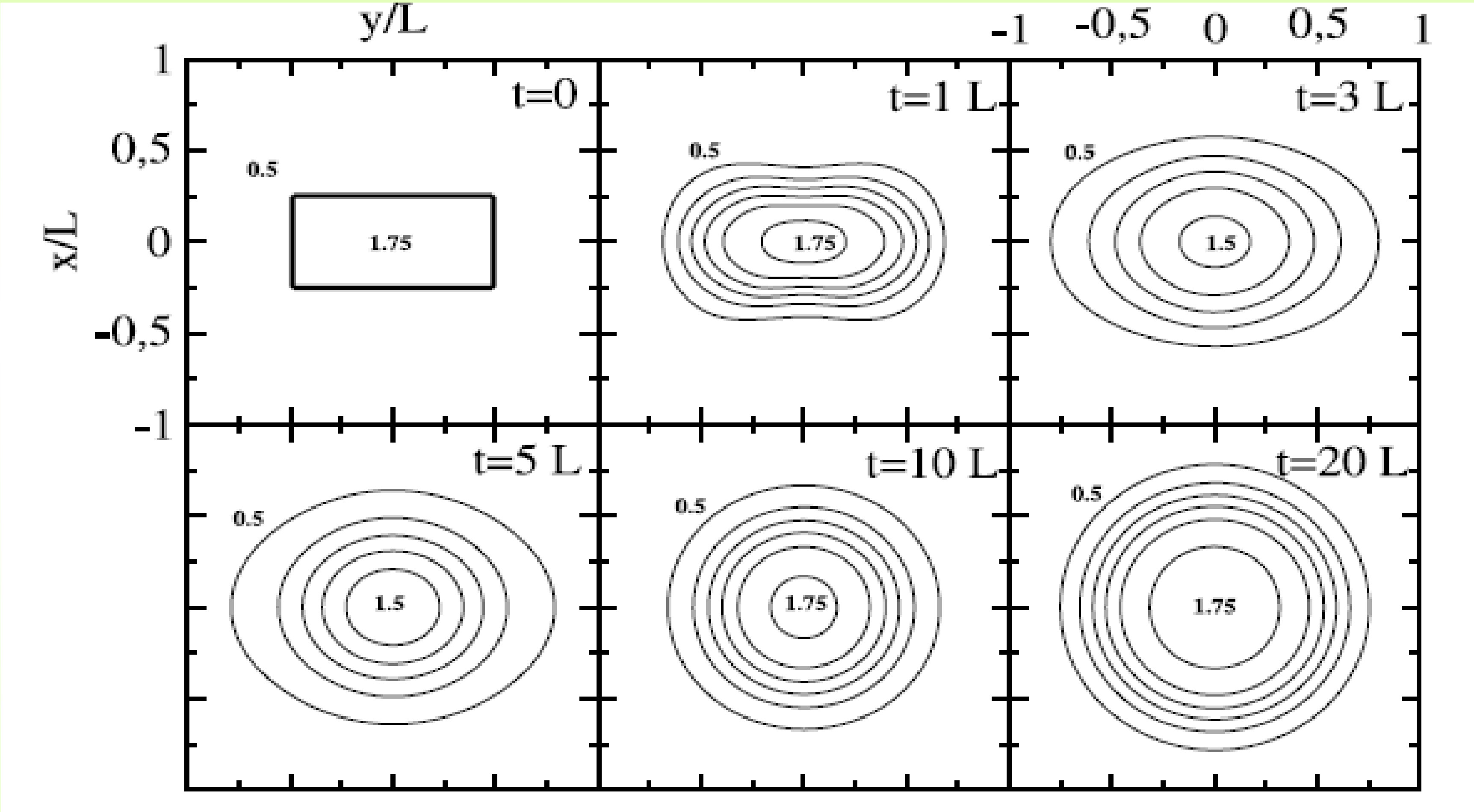


t=5L/c=25 fm/c, under-critical seeds dissolve more rapidly, overcritical-grow slowly

$(T_{cr} - T)/T_{cr} = 0.15$; $T_{cr} = 162$ MeV; $L = 5$ fm; $\beta = 0.2$

Change of the seed shape with time

Iso-lines of the density n/n_{cr} with increment 0.25



Initially anisotropic droplet slowly acquires spherical form $\beta = 0.1 \ll 1$

Thermodynamical fluctuations of conserved charges

scalar Landau parameter

$$\frac{\overline{(\Delta N)^2}}{N} = \frac{T}{n}\frac{\partial n}{\partial \mu}\Big|_T = \frac{\overline{(\Delta V)^2}}{V} = T/\frac{\partial P}{\partial n}\Big|_T, \quad u_T^2 = \frac{\partial P}{m_N \partial n}\Big|_T = p_F^2(1+f_0)/(3m_N m_N^*) .$$

At CEP and on isothermal spinodal (f_0=-1) normalized variance of particle number (volume) diverges (u_T^2 = 0),

Sasaki, Friman, Redlich PRL99 (2007):

enhanced fluctuations, as a signal of the spinodal decomposition. The spinodal phase separation can also lead to fluctuations in strangeness [19] and isospin densities [30].

Peculiarities of fluctuations in HIC

- **critical slowing down** (not enough time for critical thermal fluctuations to be developed)

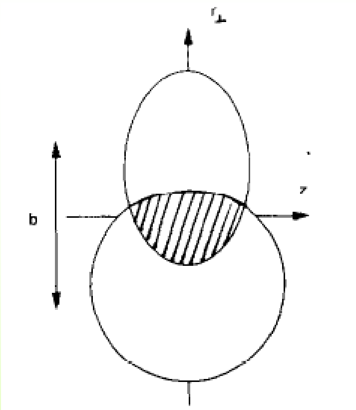
- **finite size effects:**

effectively reduced dimensionality: near CEP :

2d if coherence length $l \sim 1/|T_{cr}-T|^{1/2}$ fulfills inequalities $R_{\|} > l > R_{\perp}$,

1d -for $R_{\perp} > l > R_{\|}$,

0 d for l > R



In a narrow vicinity of T_c fluctuations behave as for d<3

Instabilities in spinodal region

aerosol of bubbles and droplets (**dynamical mixed phase**)

$$\delta n = \delta n_0 \exp[\gamma t + i\,\mathbf{pr}],$$

$$\delta s = \delta s_0 \exp[\gamma t + i\,\mathbf{pr}],$$

$$T = T_> + \delta T_0 \exp[\gamma t + i\,\mathbf{pr}]$$

$T_>$ is the temperature of the uniform matter

From equations of non-ideal hydro:

$$\gamma^2 = -p^2\left[u_T^2 + \frac{(\tilde{d}\eta + \zeta)\gamma}{mn} + cp^2 + \frac{u_{\tilde{s}}^2 - u_T^2}{1 + \kappa p^2/(c_V \gamma)}\right]$$

$u_{\tilde{s}}^2 = m^{-1}(\partial P/\partial n)_{\tilde{s}}$ and $u_T^2 = m^{-1}(\partial P/\partial n)_T$ are speeds of sound

Three solutions

For small momenta:

$$\gamma_{1,2} = \pm i u_{\tilde{s}} p + \left[\frac{\kappa}{c_V} \left(\frac{u_T^2}{u_{\tilde{s}}^2} - 1 \right) - \frac{\tilde{d}\eta + \zeta}{mn} \right] \frac{p^2}{2},$$

Density mode

$$\gamma_3 = -\frac{\kappa u_T^2 p^2}{u_{\tilde{s}}^2 c_V} \left[1 - \frac{u_T^2 - u_{\tilde{s}}^2}{u_{\tilde{s}}^2 u_T^2} \left(c + \frac{\kappa u_T^4}{u_{\tilde{s}}^2 c_V^2} - \frac{(\tilde{d}\eta + \zeta)\kappa u_T^2}{mnc_V u_{\tilde{s}}^2} \right) p^2 \right]$$

Thermal mode

in isothermal spinodal region, $u_T^2 < 0$, $u_s^2 > 0$; ⇨ thermal mode γ_3 is unstable
in adiabatic spinodal region, $u_s^2 < 0$, $u_T^2 < 0$;
⇨ thermal and density modes γ_2 , γ_3 are unstable

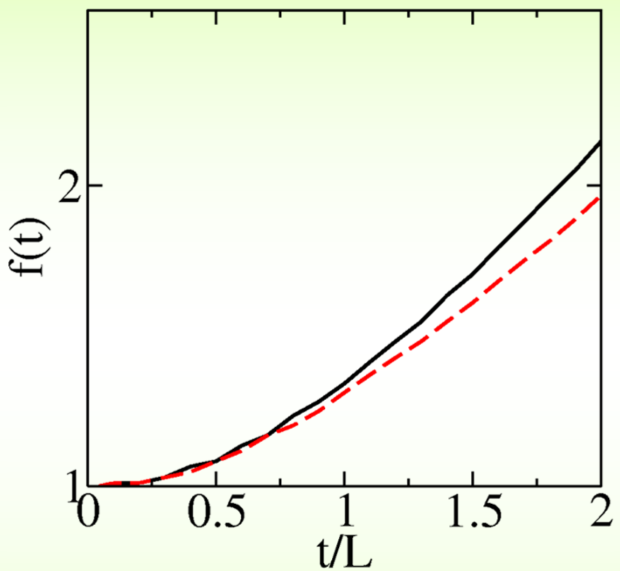
with an increase of momentum situation changes

Limit of large thermal conductivity

$\kappa \gg \nu c_V \sqrt{c}$, $\nu = (u_{\tilde{s}}^2 - u_T^2)/(-u_T^2)$

instability arises for the density mode (at finite momentum!) ,
when trajectory crosses isothermal spinodal line

amplitude of the growing modes



β=0.1 dash line, β=10 solid line

for most rapidly growing modes:

$$\gamma_m = \frac{(-u_T^2) m n_{cr}}{(2\sqrt{\beta}+1)(\tilde{d}\eta + \zeta)},$$

$$p_m^2 = \frac{(-u_T^2)\sqrt{\beta}}{(2\sqrt{\beta}+1)c}.$$

⇨ $R_m \sim 1/p_m$

Here $\delta \check{T} = (T_{cr} - T)/T_{cr} = 0.15$; $T_{cr} = 162$ MeV, $t = 2L = 10$ fm $(0.15 / \delta \check{T})$

Far from critical point fluctuations grow more rapidly
–effect of warm Champagne

Limit of small thermal conductivity $\quad \kappa \ll \nu c_V \sqrt{c}$.

Instability arises when trajectory crosses isothermal spinodal line, but now for the thermal mode

$$p_m^2 \simeq -u_T^2/(2c), \qquad \gamma_{3m} = \gamma_3(p_m) \simeq \frac{\kappa u_T^4}{4 c c_V u_{\tilde{s}}^2}.$$

Limit of $\kappa = 0$ (like in ideal hydro. calculations) **is special:**
no thermal mode

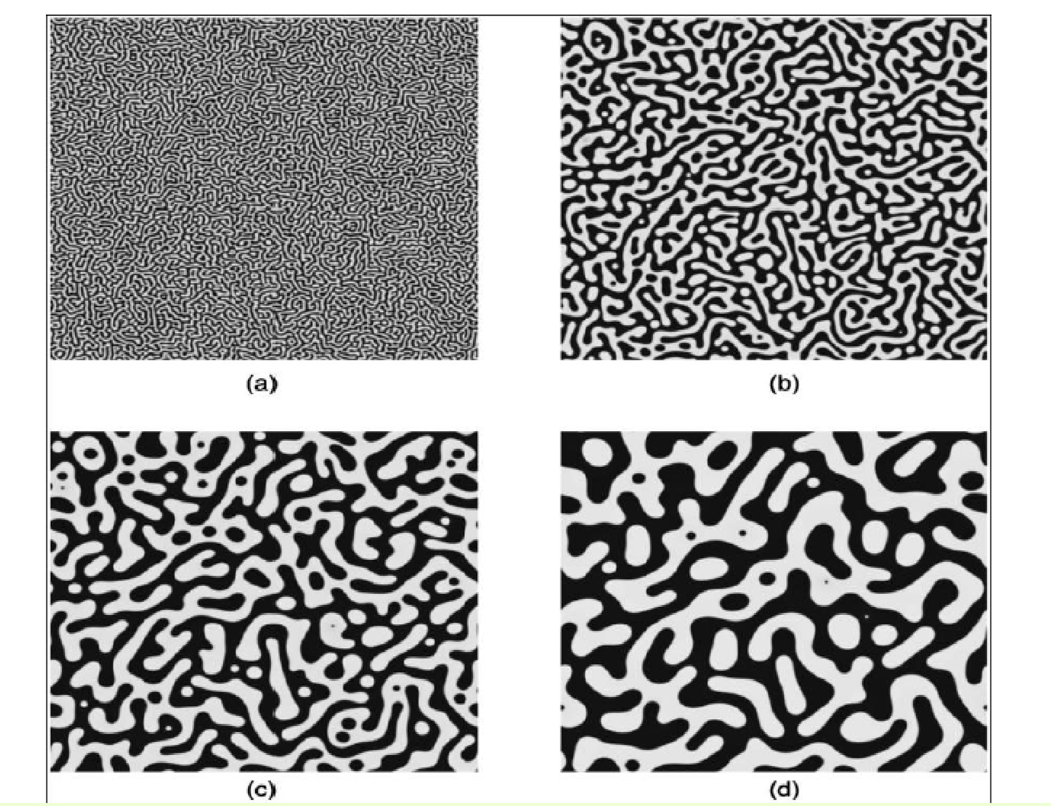
Instability arises for the density mode, but only when trajectory crosses adiabatic spinodal line

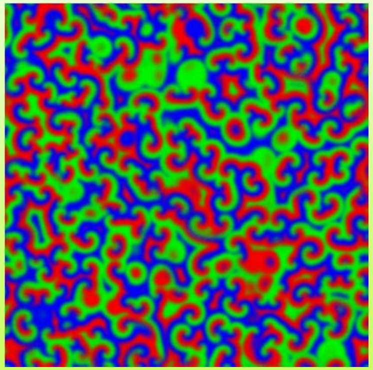
$$\gamma^2 = -p^2\left[u_{\tilde{s}}^2 + \cancel{\frac{(\tilde{d}\eta + \zeta)\gamma}{mn}} + cp^2\right].$$

Solution is similar to that for the density modes at large κ, but now the entropy per baryon is fixed rather than the temperature.

ideal hydro *(at least without taking of special care)* **cannot correctly describe dynamics of the first-order phase transition.**

Numerical simulations for conserved order parameter in Ginzburg-Landau approach J. Zhu, L. Q. Chen, J. Shen, V. Tikare, and A. Onuki, Phys. Rev. E, **60**, 3564 (1999).





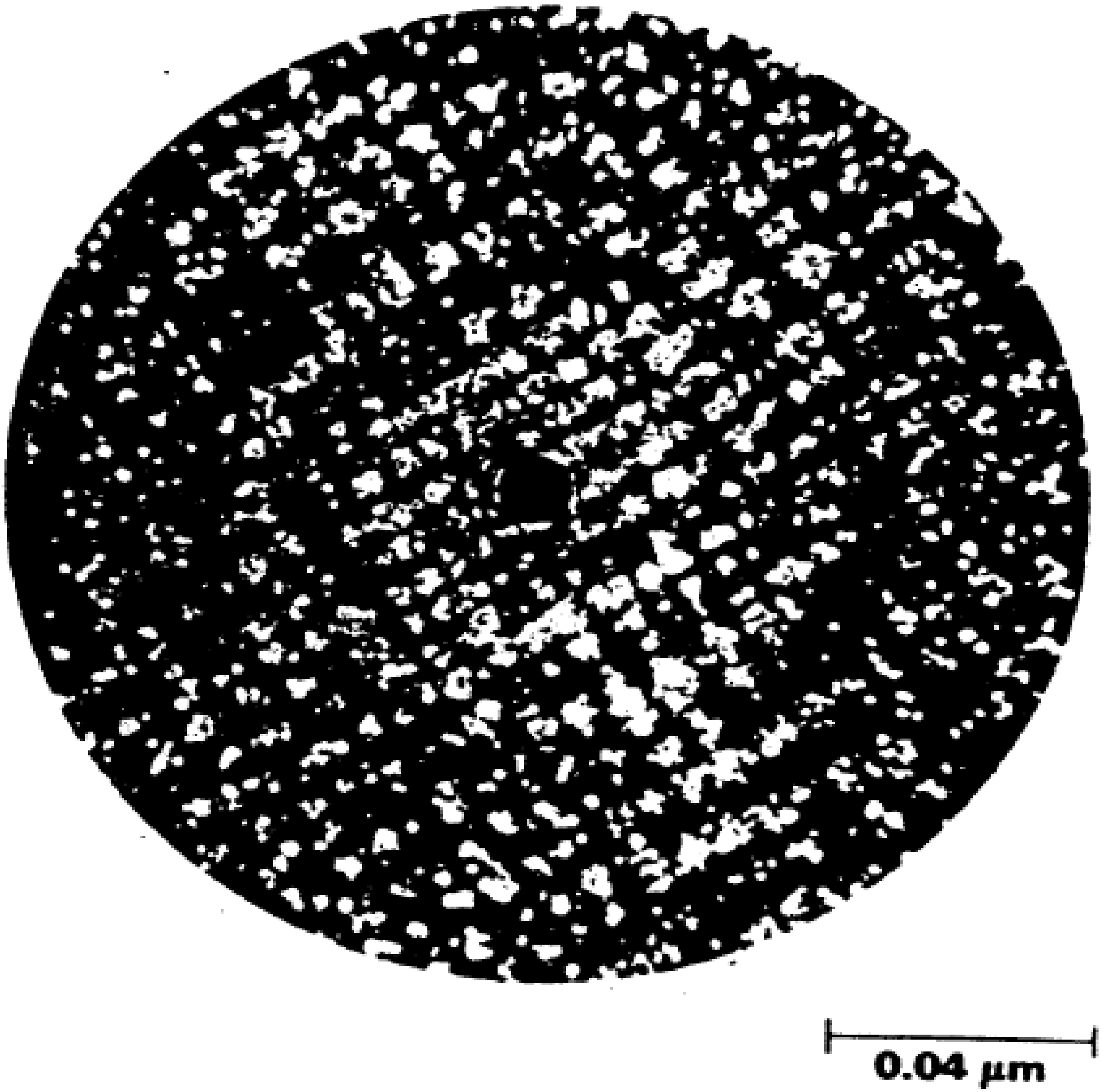
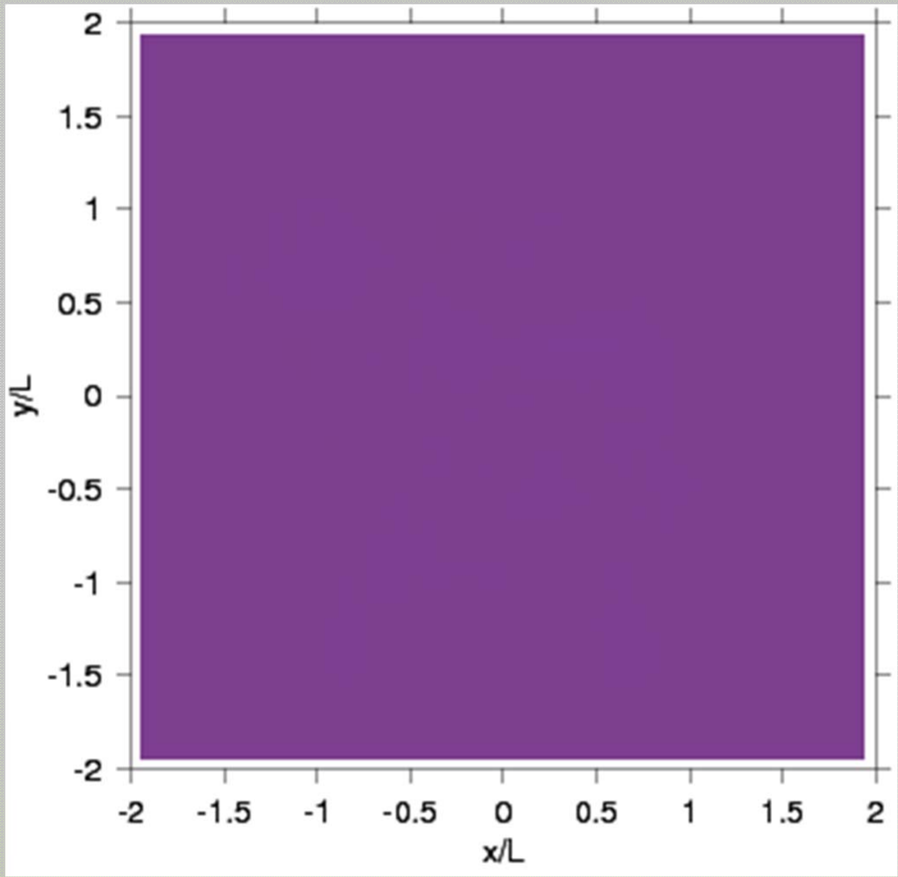


Fig. 9 Field-ion micrograph of spinodally decomposed Fe-25Be (at.%) alloy aged 20 min at 400 °C (750 °F). The axis of the needle-like specimen is [001]. The iron-rich phase images brightly because of the different contrast mechanism operating in the field-ion microscope. 375 000×. (M.K. Miller)

Spinodal instability

Dynamics in spinodal region.
Blue – hadrons,
Red – quarks.



high degree of connectivity

lack of periodicity

- **the larger viscosity and the smaller surface tension the effectively more viscous is the fluidity**

- **Anomalies in thermal fluctuations near CEP (*which are under extensive discussion*) may have not sufficient time to develop**
- **Effective system dimensionality for description of fluctuations near CEP might be <3**

- T_{cr} calculated in thermal models might be significantly higher than the value which may manifest in fluctuations in actual HIC
- **Heat transport effects play important role**

Effects of spinodal decomposition can be easier observed via fluctuation effects since they require a shorter time to develop

- **Since in reality κ is not zero, spinodal instabilities start to develop when the trajectory crosses the isothermal spinodal line rather than the adiabatic one as it were in ideal hydro, i.e. at essentially higher T. This favors observation of manifestation of spinodal decomposition in the Q-H transition in HIC**

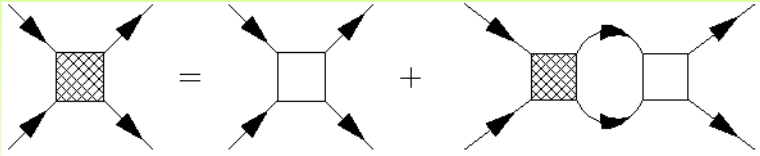
Liquid-gas spinodal instability vs. a scalar field condensation (?)

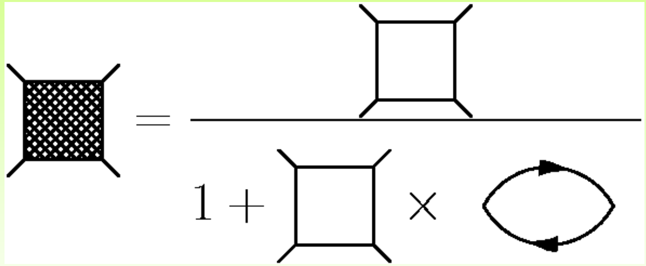
Kolomeitsev, Voskresensky., Eur. Phys. J. A (2016) 52: 362

Particle-hole interaction in Fermi liquid

interaction amplitude (low T)

$$N\,\Gamma_0(\theta) = f(\theta) = \sum_l (2l+1) f_l P_l(\cos\theta)$$





Lindhard function

$\Phi(\omega, k)$

scalar channel zeroth harmonics

$$T_{\mathrm{ph},0} = \frac{1}{[\Gamma_{00}]^{-1} + N\,\Phi(\omega, \boldsymbol{q})} = \frac{N^{-1}}{f_0^{-1} + \Phi(\omega, \boldsymbol{q})}$$

spectrum of low-lying excitations in the scalar channel $\omega(k)$- zero sound modes

$$f_0^{-1} + \Phi(\omega, \boldsymbol{q}) = 0$$

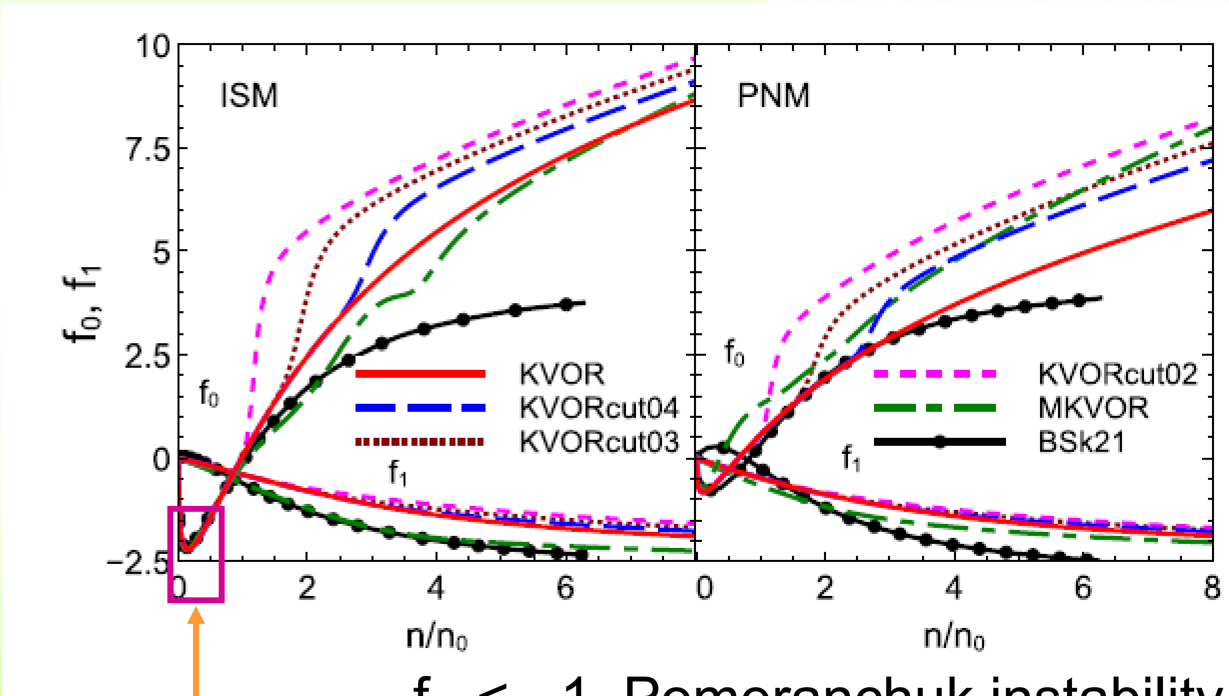
$$T_{\mathrm{ph},0} \approx \frac{V^2(k)}{\omega - \omega(k)}$$

$$V^{-2} = N \frac{\partial \mathrm{Re}\,\Phi}{\partial \omega}\Big|_{\omega(k)}$$

f_0 may be slightly momentum dependent quantity

Scalar Landau parameters f_0 (k=0) , f_1 (k=0) in nuclear matter

K.A. Maslov, E.E. Kolomeitsev, D.N. Voskresensky, Nuclear Physics A 950 (2016)



$f_0 < -1$, Pomeranchuk instability region

$f_0 < -1$ remains for $T << E_F$

Stability of Fermi liquid

I.Ia. Pomeranchuk, Soviet Phys. JETP 35 (1958) 524.

Pomeranchuck conditions for stability of the Fermi liquid with respect of small perturbations

For the scalar channel $-1 < f_l$

0th harmonics of the scalar Landau parameter are related to the incompressibility

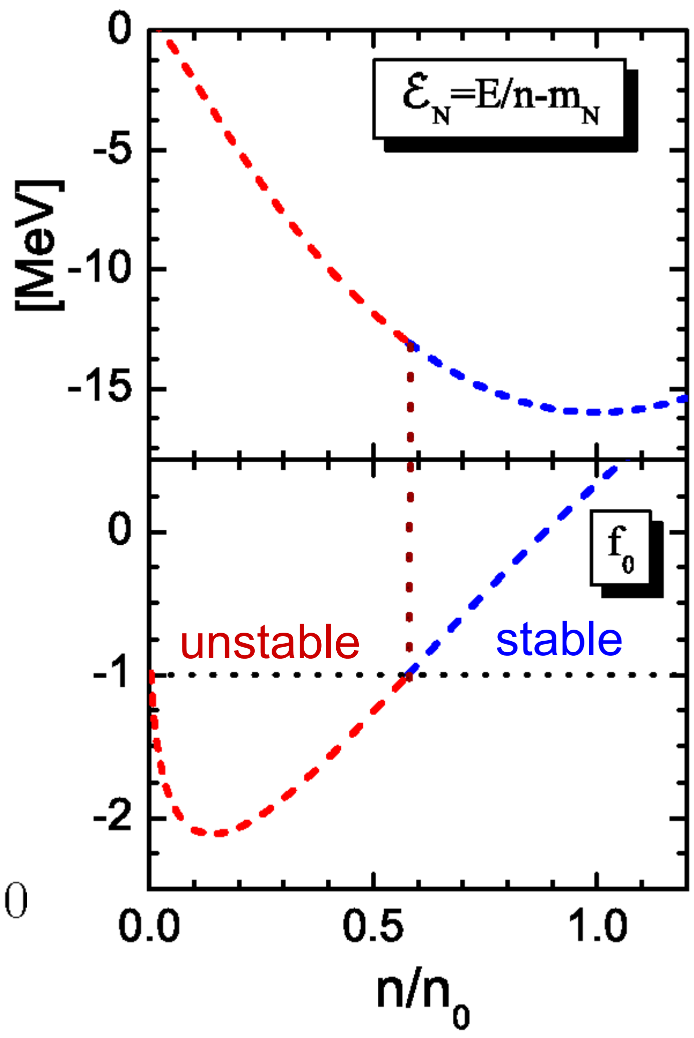
$$K = n\frac{\mathrm{d}^2 E}{\mathrm{d}n^2} = \frac{2}{3}\epsilon_{\mathrm{F}}(1+f_0)$$

Isospin symmetric nuclear matter is unstable for $n_1 < n < n_2 < n_0$

The zero-sound mode is unstable: $1 + f_0 < 0$

The first sound is unstable

speed of sound $u^2 = \frac{\partial P}{\partial \rho} = \frac{p_{\mathrm{F}}^2}{3mm^*}(1+f_0) < 0$



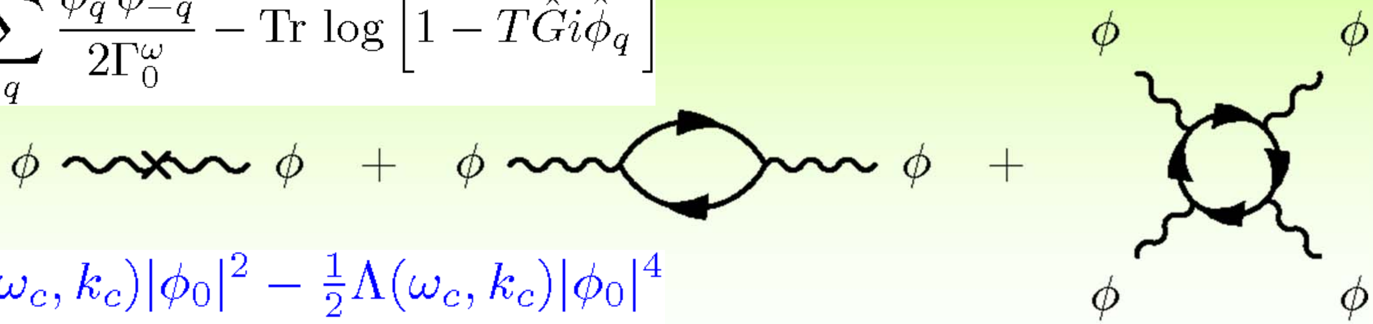
Bosonisation of the interaction

$$T^R_{\mathrm{ph},0}(\omega,k)=\frac{-(a^2N)^{-1}f_0m^2_{\mathrm{B}}}{(D^R_{\mathrm{B},0})^{-1}(\omega,k)-\Sigma^R_{\mathrm{B}}(\omega,k)}\equiv V^2_{\mathrm{B}}D^{\mathrm{R}}_{\mathrm{B}}(\omega,k).$$

Massive boson or we may use Hubbard-Stratonovich transformation: $\phi_q=\sum_p\psi^\dagger_p\psi_{p+q}$ $\quad q=(\omega,\boldsymbol{k})$ $\quad p=(\epsilon,\boldsymbol{p})$

$$S_{\mathrm{int}}[\phi]=\sum_q\frac{\phi_q\,\phi_{-q}}{2\Gamma^\omega_0}-\mathrm{Tr}\,\log\left[1-T\hat{G}i\hat{\phi}_q\right]$$

$S_{\mathrm{int}}[\phi]=$



$$L=\Re D^{-1}_\phi(\omega_c,k_c)|\phi_0|^2-\tfrac{1}{2}\Lambda(\omega_c,k_c)|\phi_0|^4$$

$$\phi=\phi_0e^{-i\omega_ct+i\boldsymbol{k}_c\boldsymbol{r}}$$

simplest form of the condensate field

$$D^{-1}_\phi(\omega,k)=-\mathrm{sgn}(f_0)[(\Gamma^\omega_0)^{-1}+a^2N\Phi(\omega,k)]$$

$$\Lambda=-2i\int GGGG\frac{\mathrm{d}^4p}{(2\pi)^4}$$

For f_0 <-1 minimum energy is realized for a static condensate $\quad \omega_c=0$

equation of motion for the static field

$$-a^2N\tilde{\omega}^2(k_c)\phi_0-\Lambda(0,k_c)\,|\phi_0|^2\phi_0=0$$

$$|\phi_0|^2=-N\frac{\tilde{\omega}^2(k_c)}{a^2\lambda\left(1+\frac{k_c^2}{2p_{\mathrm{F}}^2}\right)}$$

$$\tilde{\omega}^2(k_c)=-\frac{\Re D_\phi(0,k_c)}{a^2N}=\frac{1}{|f_0(k_c)|}-\Re\Phi(0,k_c)$$

$$E_{\mathrm{B}}=-N^2\frac{\tilde{\omega}^4(k_c)}{2\lambda\left(1+\frac{k_c^2}{2p_{\mathrm{F}}^2}\right)}$$

$$\Lambda(0,k_c)\approx a^4\lambda\left(1+\frac{k_c^2}{2p_{\mathrm{F}}^2}\right),\qquad \lambda=\frac{\nu}{\pi^2v_{\mathrm{F}}^3}$$

Application to isospin-symmetric nuclear matter

1. For a given energy per particle $\mathcal{E}_N(n) \longrightarrow f_{00} = \frac{3n}{2\epsilon_F}\frac{d^2(n\,\mathcal{E}_N)}{dn^2} - 1$

2. Mean-field energy with condensate (k_c=0)

$$\mathcal{E}_{\rm tot}^{(\rm MF)} = \mathcal{E}_N + \mathcal{E}_{\rm B}(n; f_{00}^{\rm tot}(n))$$

$$\mathcal{E}_{\rm B}(n,f) = -6\epsilon_{\rm F}(n)\frac{(f+1)^2}{f^2}$$

3. self-consistent Landau parameter

$$f_{00}^{\rm tot}(n) = f_{00}(n) + \frac{3n}{2\epsilon_{\rm F}}\frac{{\rm d}^2}{{\rm d}n^2}[n\mathcal{E}_{\rm B}(n, f_{00}^{\rm tot}(n))]$$

4. renormalized Landau parameter

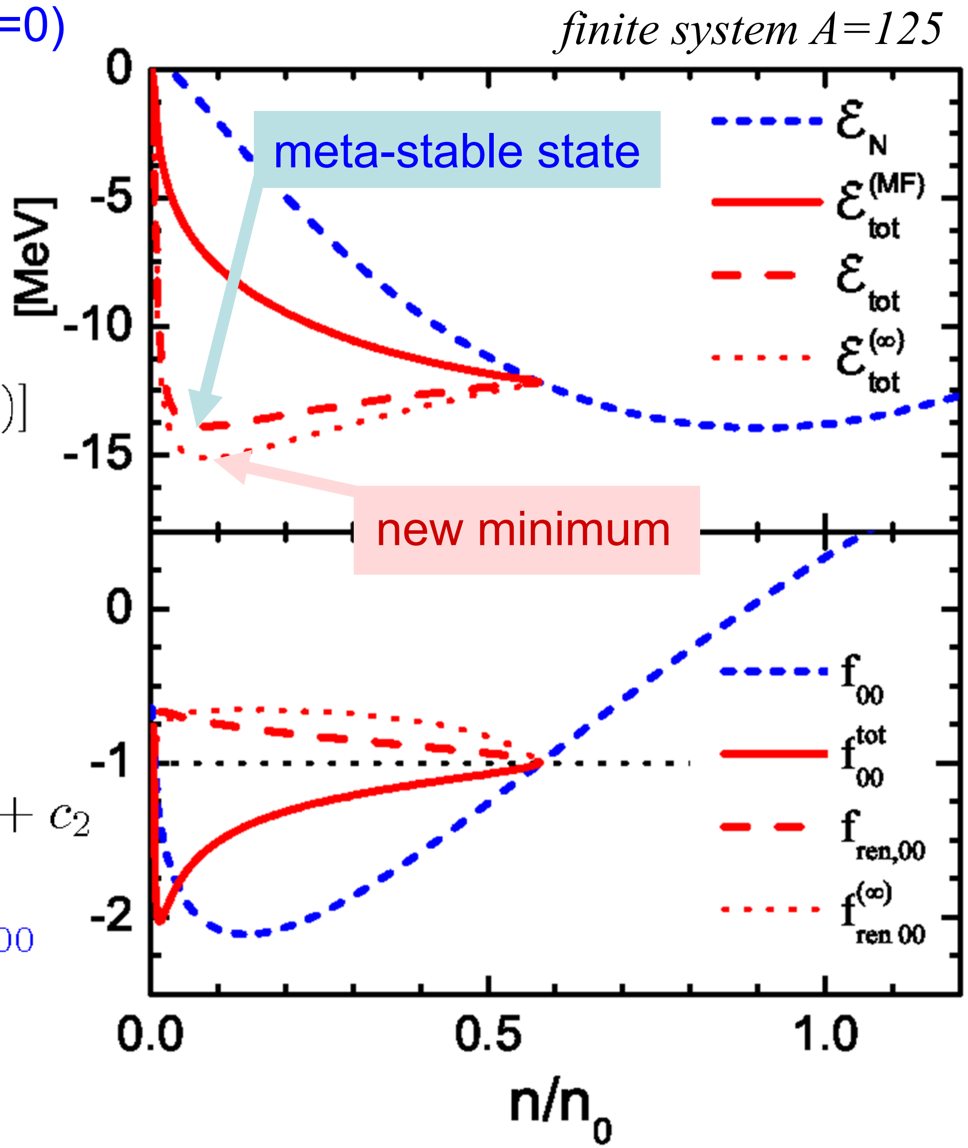
$$f_{{\rm ren},00} = -f_{00}^{\rm tot}/(2f_{00}^{\rm tot}+1)$$

5. reconstructed full energy per particle

$$\mathcal{E}_{\rm tot} = \frac{1}{n}\int {\rm d}n \int {\rm d}n \frac{2\epsilon_{\rm F}}{3n}(1+f_{{\rm ren},0}) + \frac{c_1}{n} + c_2$$

6. limit $\Lambda \to \infty$ then $\mathcal{E}_{\rm B} = 0,\ f_{00}^{\rm tot} = f_{00}$

$$f_{{\rm ren},00}^{(\infty)} = -f_{00}/(2\,f_{00}+1) \longrightarrow \mathcal{E}_{\rm tot}^{(\infty)}$$



Peripheral collisions

consider a peripheral nucleus-nucleus (A+A) collision in the frame associated with one of the nuclei (the target frame).

All the results obtained above do hold after the replacement

$$f_0(n)\Phi(\omega,k,n) \to f_0(n/2)[\Phi(\omega,k,n/2)+\Phi(\omega-\boldsymbol{k}\boldsymbol{u},k,n/2)].$$

the instability would occur for $f_0(n) < -1/2$ rather than for $f_0(n) < -1$. The instability would provoke a growth of the scalar condensate field ϕ with $\boldsymbol{k}_0 \perp \boldsymbol{p}_{\rm lab}$ in the course of peripheral heavy-ion collisions.

In peripheral collisions in a certain region of impact parameter first sound modes might be stable (no liquid-gas transition) but zero sound modes might be unstable

→ ***condensate of scalar modes***

- ✔ **Spectra** of scalar excitations in Fermi liquid are considered
- ✔ Local 4-fermion interaction is **bosonized** and the **effective Lagrangian** for the scalar excitations is constructed
- ✔ It is shown that the Pomeranchuck instability may lead to a condensation of scalar quanta.
- ✔ In the presence of condensate instabilities are **removed**
- ✔ **Reconstruction** of the equation of state for the **isospin-symmetric nuclear matter** is analysed.
- ✔ **New (meta-)stable state** is shown to be possible at small densities
- ✔ In peripheral collisions instability may appear already for $f_0 < -1/2$

Conclusion:

At NICA energies in HICs one may hope to observe non-monotonous behavior of different observables due to manifestation of non-trivial fluctuation effects especially of **spinodal decomposition** at I order hadron-quark phase transition in some collision energy interval and might be of **scalar field condensation,** e.g. in peripheral collisions.