QCD at nonzero isospin asymmetry

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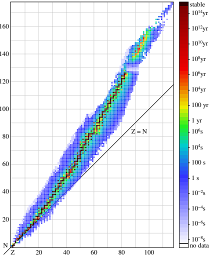
Isospin asymmetry: $n_I = n_u - n_d \neq 0$

- elements



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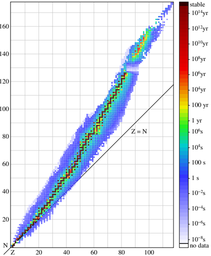

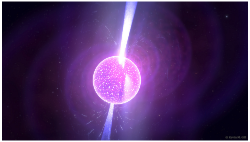

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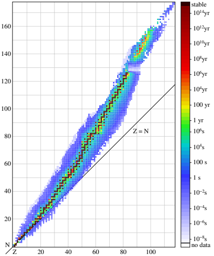

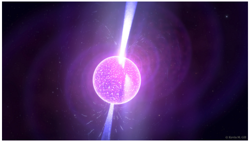

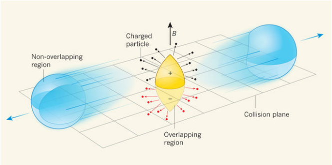

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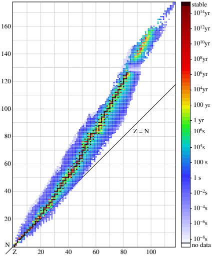
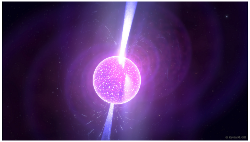
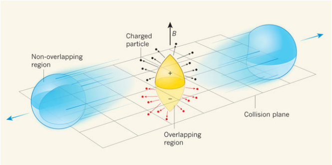
- heavy-ion collisions
- early universe







QCD at finite isospin chemical potential

Theoretical description in the grand canonical ensemble:

QCD at finite chemical potential ($N_f = 2$):
u quark: μ_u d quark: μ_d

- can be decomposed in baryon and isospin chemical potentials:

$$\mu_B = 3(\mu_u + \mu_d)/2 \text{ and } \mu_I = (\mu_u - \mu_d)/2$$

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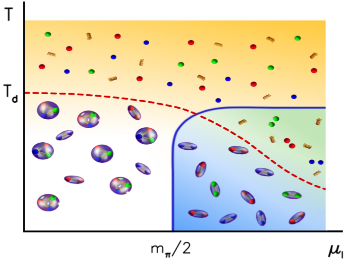
- symmetry breaking: finite μ_I breaks $SU_V(2)$ explicitly to $U_{\tau_3}(1)$

Phase structure at $\mu_B = 0$

[Son, Stephanov, PRL86 (2001); PAN64 (2001)]



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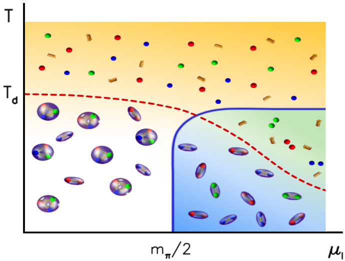

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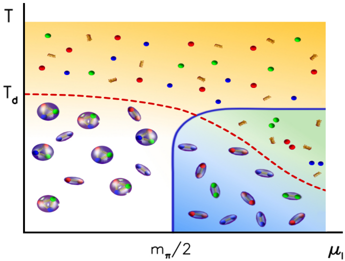

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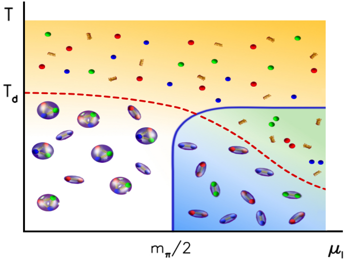

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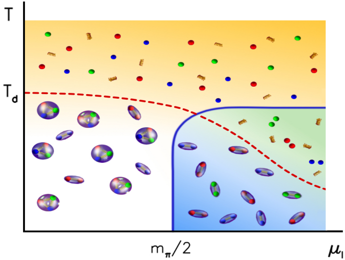
  main ingredients:

  pion condensation and deconfinement



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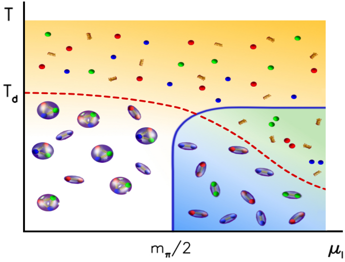

[BB, Endrődi, Schmalzbauer, PRD97 (2018)]

first results from lattice QCD: $N_t = 4$, unphysical masses, unimproved

$N_f = 2$ [Kogut, Sinclair, PRD66(2002); PRD70(2004)]

$N_f = 8$ [de Forcrand, *et al*, PoS LAT2007 (2007)]

Simulation setup $N_f = 2 + 1$

- use improved actions
- quark masses are tuned to their physical values.
- gauge action: Symanzik improved
- mass-degenerate u/d quarks: [Kogut, Sinclair, PRD66 (2002); PRD70 (2004)]

  fermion matrix: $M = \begin{pmatrix} D(\mu) & \lambda\gamma_5 \\ -\lambda\gamma_5 & D(-\mu) \end{pmatrix}$

  $D(\mu)$: staggered Dirac operator with 2×-stout smeared links

  λ: small explicit breaking of residual symmetry (unphysical)

  - necessary to observe spontaneous symmetry breaking at finite V
  - serves as a regulator in the pion condensation phase.

  $\Rightarrow$ need to extrapolate results to $\lambda = 0$
- strange quark: rooted staggered fermions (no chemical potential)

λ-extrapolations

main task for final analysis:

perform reliable extrapolation to $\lambda = 0$

problem: usual λ-dependence extremely steep

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extrapolation: $$\langle O \rangle_{\lambda=0} = \lim_{\lambda \to 0} \frac{1}{Z(\lambda)} \int [dU]\, O[U](\lambda)\, \det\big(M[U](\lambda)\big)\, e^{-S_G[U]}$$

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improvement program:

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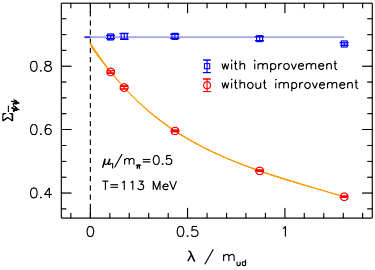

2. Phase diagram at finite μ_I

[PRD97 (2018), arXiv:1712.08190]

Pion condensation phase

main observable: renormalised pion condensate

$$\Sigma_\pi = \frac{m_{ud}}{m_\pi^2 f_\pi^2} \left\langle \pi^\pm \right\rangle_{T,\mu_I} \qquad \text{with} \qquad \left\langle \pi^\pm \right\rangle = \frac{T}{V} \frac{\partial \log Z}{\partial \lambda}$$

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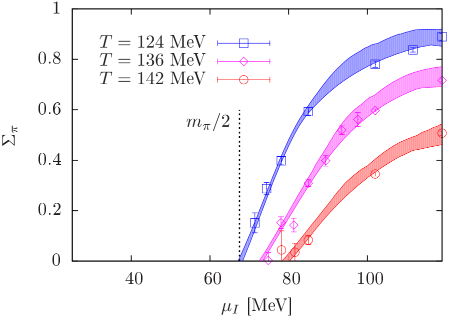


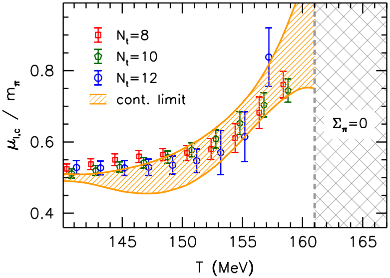
(use 2d cubic spline fit and MC generated nodepoints for interpolation)

Pion condensation phase: continuum etrapolation

parameterise phase boundary by: (include a^2 lattice artefacts)

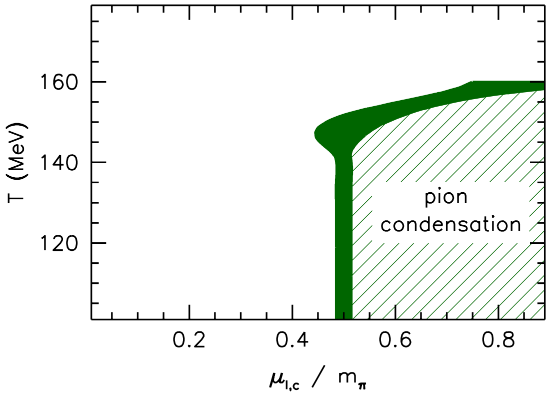
$$\mu_{I,c}(T,a) = \mu_{I,c}(T_0,a) + \sum_{n=2}^{4} b_n(a)(T-T_0)^n$$

with $\mu_{I,c}(T_0,0) = 67.5\,\text{MeV}$ and $T_0 = 140\,\text{MeV}$



(no pion condensation above $T = 161$ MeV)

Phase diagram



Chiral symmetry restoration

main observable: renormalised chiral condensate

$$\Sigma_{\bar\psi\psi} = \frac{m_{ud}}{m_\pi^2 f_\pi^2}\left[\left\langle\bar\psi\psi\right\rangle_{T,\mu_I} - \left\langle\bar\psi\psi\right\rangle_{0,0}\right] + 1$$

pseudocritical temperature T_{pc}: defined by inflection point of $\Sigma_{\bar\psi\psi}$

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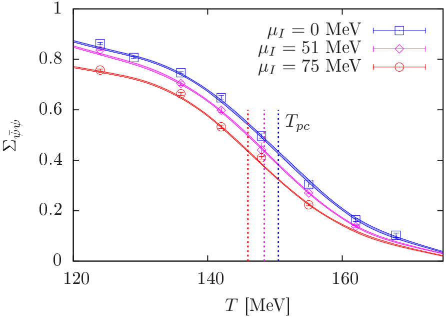
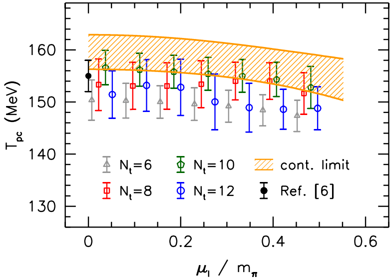


(use 2d cubic spline fit and MC generated nodepoints for interpolation)

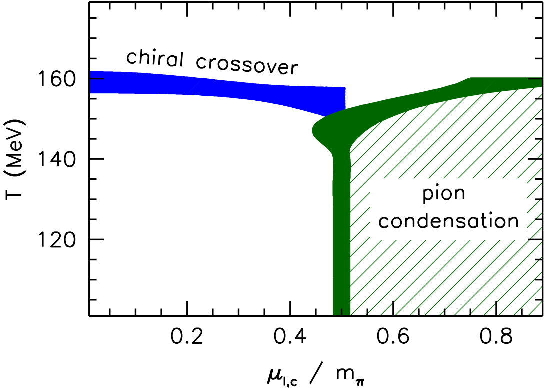
Chiral symmetry restoration

parameterise T_{pc} by: (include a^2 lattice artefacts)

$$T_{pc}(\mu_I, a) = T_{pc}(0, a) + d_2(a)\mu_I^2 \qquad \text{for} \quad \mu_I < 67.5\,\text{MeV}$$



Phase diagram

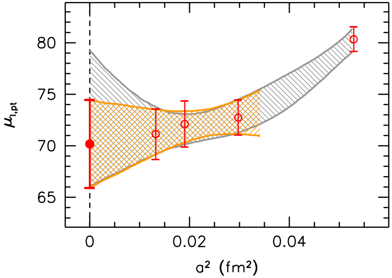


Pseudo-triple point

meeting point between T_{pc} and pion condensation phase boundary:

three phases coexist $\Rightarrow$ pseudo-triple point $(T_{pt}, \mu_{I,pt})$

here: defined by point where curves overlap within errors



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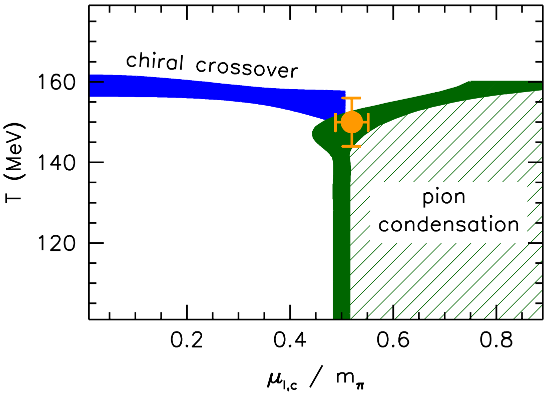
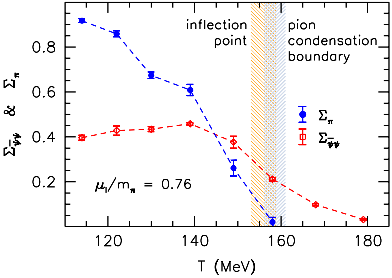


Chiral symmetry restoration for $\mu_I > m_\pi/2$

coincides with pion condensation phase boundary

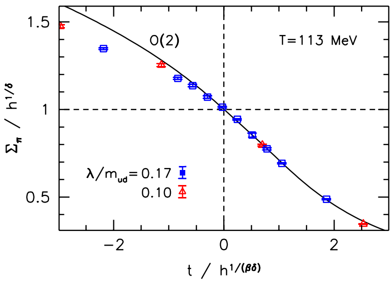


Order of the transition on the boundary

symmetry restoration pattern: 2nd order in $O(2)$ universality

check scaling: (including scaling violations)

$$\Sigma_\pi = h^{1/\delta} \cdot f_G\left(\frac{t}{h^{1/(\beta\delta)}}\right) + a_1 th + b_1 h + b_3 h^3 \qquad \text{with} \quad h = \frac{\lambda}{\lambda_0}, \quad t = \frac{\mu_{I,c} - \mu_I}{t_0}$$



$\Rightarrow$ data shows consistency with $O(2)$

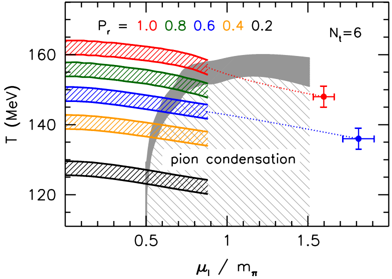
Polyakov loop and BCS phase

main ingredients for BCS superconducting phase:

pion condensation and deconfinement

measure for deconfinement: renormalised Polyakov loop

$$P_r(T,\mu_I) = Z \cdot P(T,\mu_I) \quad Z = \left(\frac{P_\star}{P(T_\star,\mu_I=0)}\right)^{T_\star/T}$$

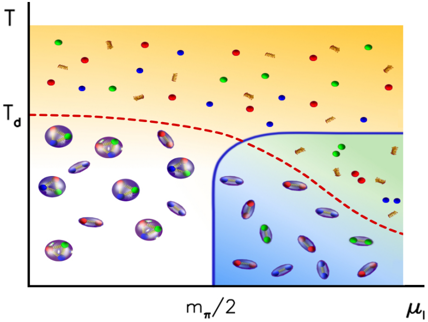


(possible definition for deconfinement transition: $P_r = 1$)

Polyakov loop and BCS phase

deconfinement transition:

smoothly penetrates into pion condensed phase



3. Comparison to Taylor expansion around $\mu_I = 0$

Taylor expansion around $\mu_I = 0$

simulations at finite μ_B: suffer from a sign problem!

one of the most important tools to obtain information at finite μ_B:
Taylor expansion around $\mu_B = 0$.

however: range of applicability at a given order is unknown

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here: test Taylor expansion method using our data for $\mu_I \neq 0$

- as an observable we use the isospin density (analogue to Baryon density):

$$\langle n_I \rangle = \frac{T}{V} \frac{\partial \log Z}{\partial \mu_I}$$

- associated Taylor expansion (follows from expansion of pressure p/T^4):

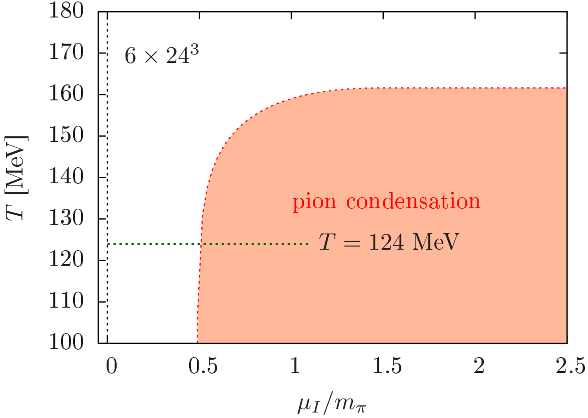
$$\frac{\langle n_I \rangle}{T^3} = c_2 \left(\frac{\mu_I}{T} \right) + \frac{c_4}{6} \left(\frac{\mu_I}{T} \right)^3$$

coefficients: take values from Budapest-Wuppertal

[BW: Borsanyi *et al*, JHEP1201 (2012)]

Comparison to data at finte μ_I

Compare data for 6×24^3 lattice:



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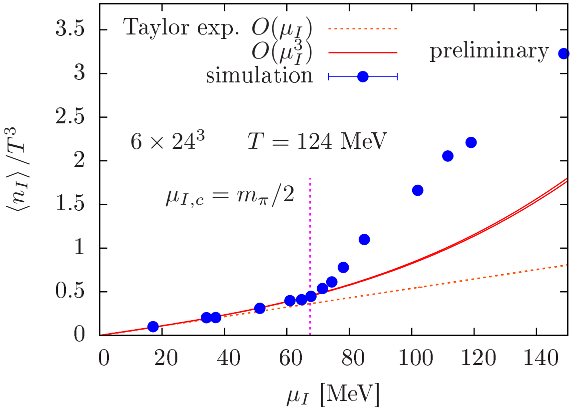

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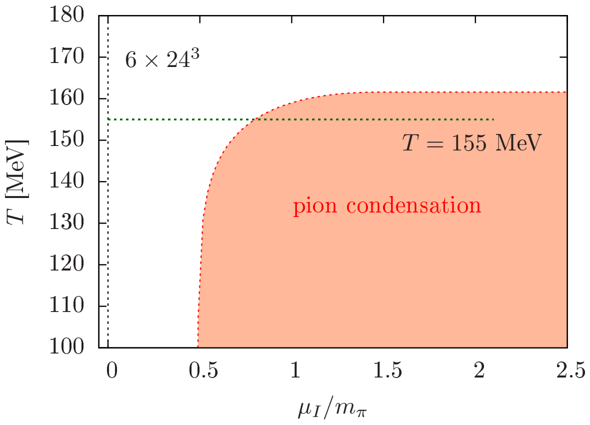

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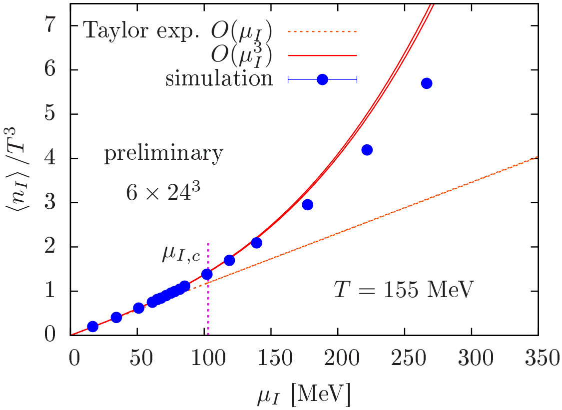

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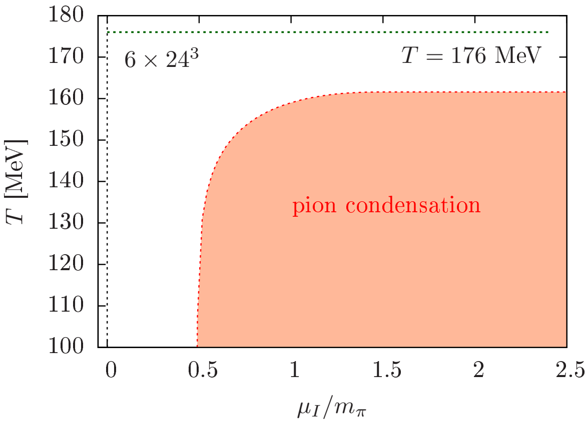

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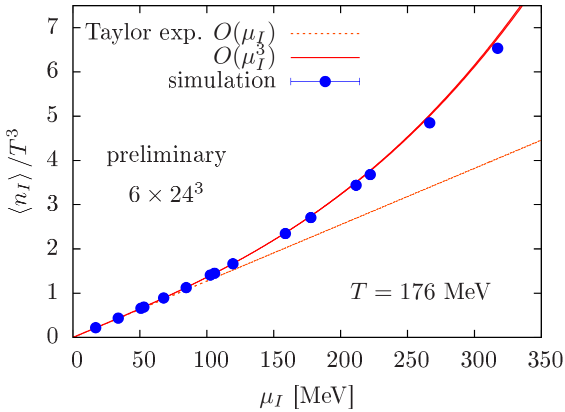
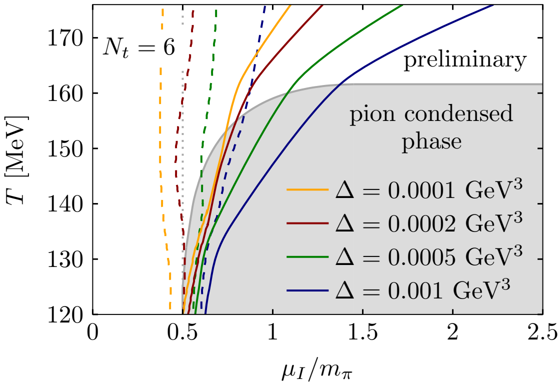


Comparison to data at finte μ_I

contour plot for $\Delta_{\mathrm{LO/NLO}} \equiv \left| \langle n_I \rangle - \langle n_I \rangle^{\mathrm{Taylor}}_{\mathrm{LO/NLO}} \right|$:



4. Equation of state at finite μ_I

Pressure and trace anomaly

Most important quantities to study equation of state (EOM):

- Pressure: $$\frac{p}{T^4} = -\frac{1}{T^3 V} \log \mathcal{Z}$$
- Trace anomaly: $$\frac{I}{T^4} = \frac{\epsilon - 3p}{T^4} = T \frac{\partial}{\partial T} \frac{p}{T^4} + \frac{\mu_I n_I}{T^4}$$

$\Rightarrow$ All other quantities derive from those and the number densities!

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Here: Consider these quantities at finite μ_I!

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Most important quantities to study equation of state (EOM):

- Pressure: $$\frac{p}{T^4} = -\frac{1}{T^3 V} \log \mathcal{Z}$$
- Trace anomaly: $$\frac{I}{T^4} = \frac{\epsilon - 3p}{T^4} = T \frac{\partial}{\partial T} \frac{p}{T^4} + \frac{\mu_I n_I}{T^4}$$

$\Rightarrow$ All other quantities derive from those and the number densities!

Here: Consider these quantities at finite μ_I!

First: Focus on the pressure!

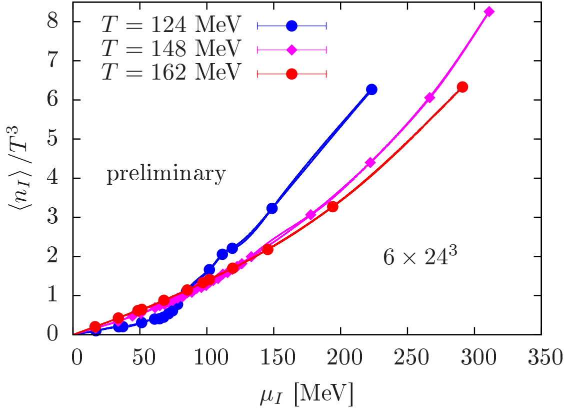
$$\Rightarrow \quad p(T, \mu_I) = p(T, 0) + \int_0^{\mu_I} d\mu_I' \, n_I(T, \mu_I') \equiv p(T, 0) + \Delta p(T, \mu_I)$$

(since $n_I = \frac{\partial p}{\partial \mu_I}$)

$p(T, 0)$ take results from [Borsanyi, *et al*, JHEP 1011 (2010)]

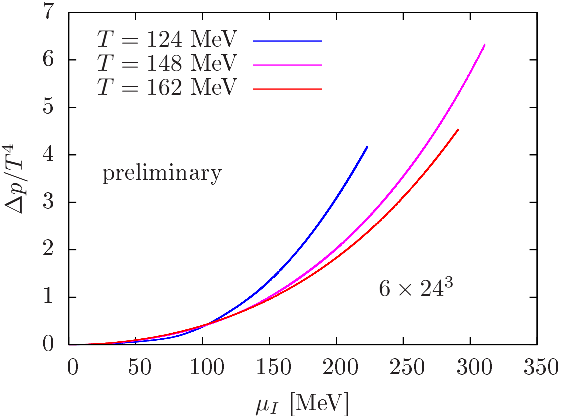
Pressure at finite μ_I

Interpolation of $\langle n_I \rangle$ for 6×24^3 lattice:



Pressure at finite μ_I

Pressure for 6×24^3 lattice:



5. An application for the EOS: pion stars

[arXiv:1802.06685]

Pion stars and EOS

pion condensed matter:

in principle allows for gravitationally stable objects

$\Rightarrow$ pion stars

mass-radius relation: can be obtained from solving TOV equation

[Glendenning, Compact stars: . . . (1997)]

Pion stars and EOS

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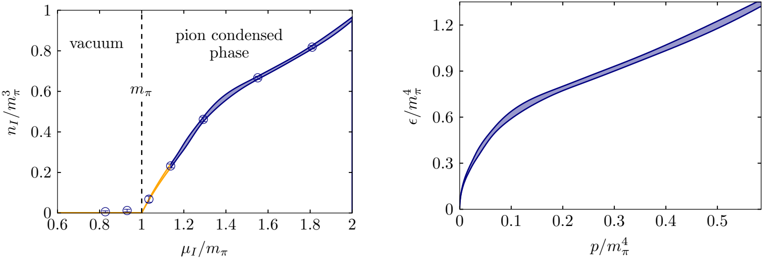
[Glendenning, Compact stars: ... (1997)]

input: EOS at $T = 0$ (for cold stars)



(results from 32×24^3 paper with $T \approx 0$; convention here: $\mu_I \to 2\mu_I$)

Pion stars: physical setup

condensing particles: charged pions $\Rightarrow$ Obtain a boson star!

- hypothetical objects [Kaup, PR172 (1968)]
- have been considered in the literature

[reviews: Jetzer, PR220 (1992); Liebling, Palenzuela, LRR15 (2012)]

Pion stars: physical setup

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condensate decay and charge of star:

$\Rightarrow$ need to include leptons

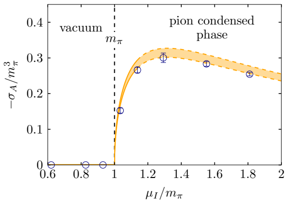
generic case:

include e, ν_e, μ, ν_e in chemical equilibrium

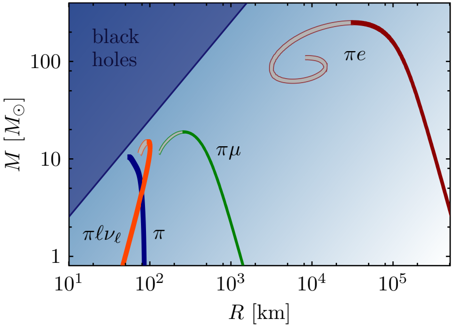
condensate decay:

no decaying exitation (Higgs effect for π and γ)

but: charged weak currents couple to
axial charge density $\sigma_A = \langle A_0^{\pm} \rangle \neq 0$



Mass-radius relation of pion stars



- stars fulfill stability criteria
  (robustness against density perturbations)
- generically they will decay (with which rate?)
  (however: neutrinos can be trapped in the condensate)
- in principle: could have been generated in the early universe?!

Summary and Perspectives

- we have investigated the phase structure of QCD at finite isospin chemical potential μ_I
- can use the theory to test Taylor expansion around $\mu_I = 0$
- started to measure the equation of state at finite μ_I
- an interesting application: pion stars
- relevance of pion condensation for early universe?
- reweighting to finite μ_B
  mapping out (μ_I, μ_B) phase diagram
- a lot of other interesting things to look at ...

Thank you for your attention!