# EXTENDED (SUPER)GRAVITY FROM NC

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DUBNA – DEC 17, 2012

1. NC FIELD THEORIES , $*$ PRODUCT

2. NC GRAVITY (VIELBEIN)
   $*$- DIFF. GEOMETRY : $\wedge_*$, $d$, $\int$, $\ell_A$

3. COUPLINGS :
   - FERMIONS
   - GAUGE FIELDS
   - SCALARS

4. GEOMETRICAL SEIBERG-WITTEN MAP

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6. OSp(1|4) - SUPERGRAVITY AND ITS NC EXTENSION $\longrightarrow$ EXTENDED $D=4$ $N=1$ SUPERGRAVITY

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20/09/12

P. Aschieri, LC

"Noncommutative D=4 gravity coupled to fermions", JHEP 0906(2009)086

"Noncommutative supergravity in D=3 and D=4", JHEP 0906(2009)087

"Noncommutative gravity coupled to fermions: second order expansion via the Seiberg-Witten map", JHEP 1207 (2012) 184

" Noncommutative gauge fields coupled to noncommutative gravity", hep-th 1205.1911 , GRG 2012

" Extended gravity theories from dynamical noncommutativity", hep-th 1206.4096 , GRG 2012

P.Aschieri, LC, M. Dimitrijevic

" Noncommutative gravity at second order via SW map" hep-th 1207.4346 , TO BE PUBL IN PHYS. REV. D

L.C., " OSp(1|4) SUPERGRAVITY AND ITS NONCOMMUTATIVE EXTENSION ", TO APPEAR

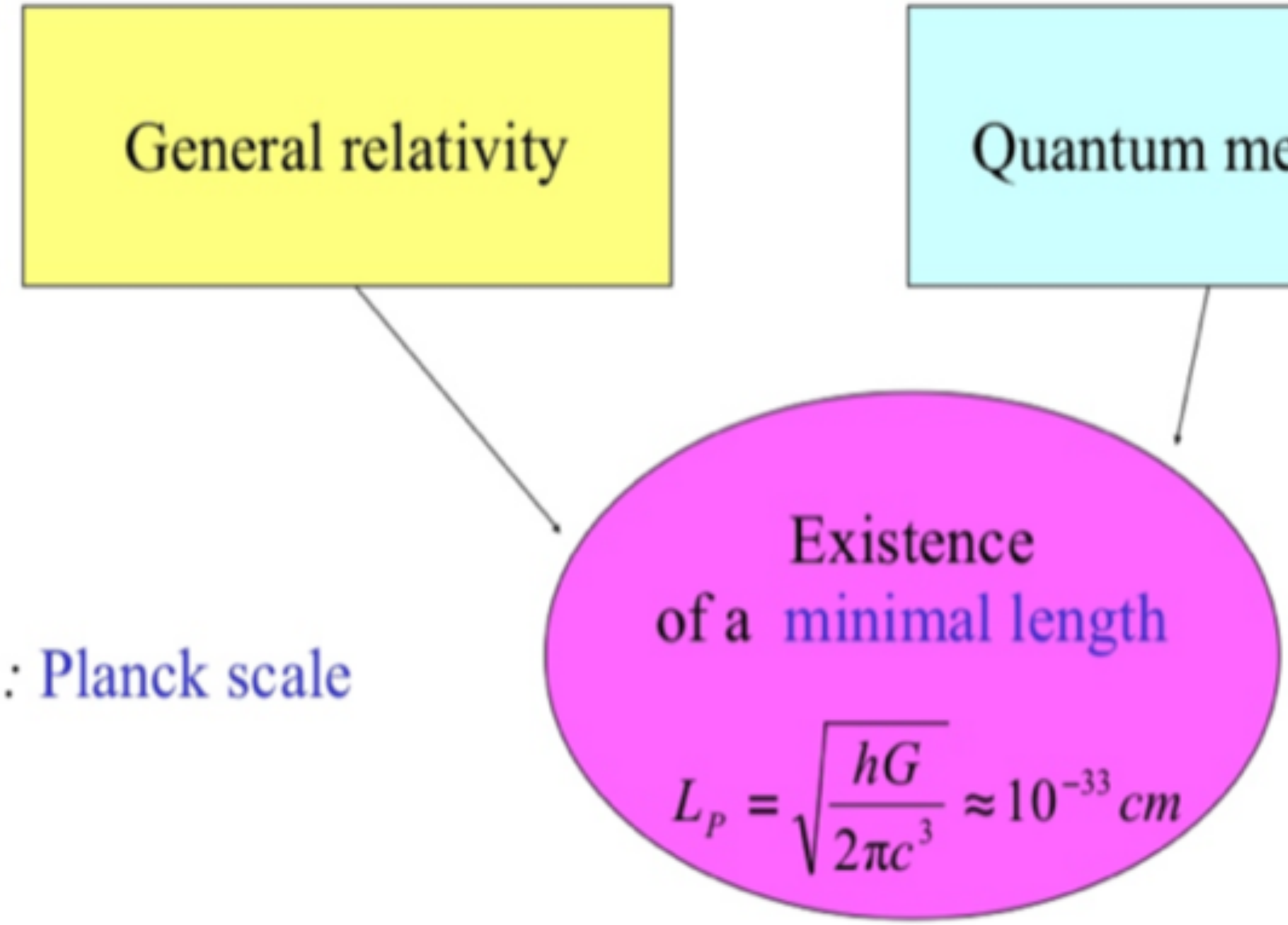
# MOTIVATIONS FOR NC SPACETIME

- TO ANALYSE SMALL SCALE STRUCTURE OF SPACETIME : HIGH ENERGY CONCENTRATION IN SMALL VOLUMES → CURVATURE → NO INFORMATION UNDER THE PLANCK LENGTH

- MINIMAL LENGTH → CURE OF UV DIV. IN QFT?

- $\star$-PRODUCTS (ENCODING NONCOMMUTATIVITY) → DEFORMED FIELD THEORIES → SW MAP → EXTENDED (HIGHER DER) FIELD TH.S → EXTENDED GRAVITY THEORIES

- Euristically:




$L_P$: Planck scale

- To "measure" geodetics in a gravitational field: use freely falling particles with mass $m$.

- How precisely can we measure geometry?

- Uncertainty in position of the particle:

$$\boxed{\Delta q \approx \frac{\hbar}{mc}}$$

corresponding to $\Delta p \sim mc$ (pair creation threshold)

- To decrease $\Delta q$, increase $m$ $\rightarrow$ then also the curvature of spacetime increases until …

$$\boxed{\text{curvature radius } \approx \Delta q}$$

• order of magnitude for $\Delta q$ ?

$$- R = \frac{8\pi G}{c^4} T \rightarrow 1/\Delta q^2 \approx \frac{8\pi G}{c^4} T^0{}_0 \approx \frac{8\pi G}{c^2} \frac{m}{\Delta q^3}$$

and substituting $m \sim h / (c\, \Delta q)$

$$\Delta q \approx ( hG / c^3)^{1/2}$$

$\approx$ Planck length

It is therefore impossible to observe phenomena (or spacetime structure) under the Planck scale $L_P$

- This indetermination emerges automatically if the coordinates are noncommutative :

$$x\,y - y\,x = \alpha\,(L_P)^2$$

or in general:

$$\boxed{[x^i, x^j] = i\,\vartheta^{ij}}$$

with $\vartheta$ = antisymmetric tensor (constant)

~ 1930 : MINIMAL LENGTH CUTOFF IN QFT
(BREAKS LORENTZ INVARIANCE ;
CUTOFF DEPENDS ON REF. SYSTEM)

FLINT, MARCH, MÖGLICH, GOUDSMIT, ...

$\sim 10^{-15}\,m$

~ 1936 : BRONSTEIN : GRAVITY DOES NOT
ALLOW ARBITRARILY LARGE MASS
CONCENTRATION IN SMALL REGION
OF SPACETIME ($\Rightarrow$ BLACK HOLE)

DIFFERENT FROM E.M.

1938 : HEISENBERG (ÜBER DIE IN DER THEORIE DER ELEMENTARTEILCHEN AUFTRETENDE UNIVERSELLE LÄNGE)

- UNIVERSAL LENGTH $r_0 \sim 100$ fm

(FROM CONSIDERATIONS OF VALIDITY LIMITS OF FERMI THEORY : "EXPLODES" AT SCALES $\sim r_0$, NOT RENORMALIZABLE)

- CONCLUDES THAT GRAVITY IS IRRELEVANT SINCE $\ell_p \ll r_0$

(I.E. MINIMAL LENGTH NOT DUE TO GRAVITY)

**1947:** SNYDER EXPLOITS AN IDEA OF HEISENBERG

$$[x_\mu, x_\nu] = \frac{J_{\mu\nu}}{m_{PL}}$$

LORENTZ COV.

**~1964:** MEAD. THOUGHT EXPERIMENTS TO SHOW THAT GRAVITY PREVENTS MEASURING DISTANCES UNDER $\ell_p$

**~1990:** DEFORMED COMM. RELATIONS $\rightarrow$ DFR

- QUANTUM GROUPS
- QUANTUM COSETS

$\Big]$ RFT

- K-POINCARÉ, q-MINKOWSKI




RFT = RESHETIKHIN, FADDEEV, TAKHTAJAN  DFR = DOPLICHER, FREDENHAGEN, ROBERTS

- FROM STRING THEORY :
  - CANNOT "RESOLVE" ARBITRARILY SMALL STRUCTURES WITH FINITE SIZE OBJECTS
    → GENERALIZED UNCERTAINTY PRINCIPLE
  - LOW EN. LIMIT OF OPEN STRINGS IN A BACKGROUND B-FIELD
- TODAY : NC AS A "GUIDE" TO

  EXTENDED GRAVITY THEORIES

# NC FIELD THEORIES , * PRODUCT

- FIELD THEORIES ON NC SPACES BECOME ESPECIALLY TRACTABLE WHEN NONCOMMUTATIVITY IS ENCODED IN A TWISTED *-PRODUCT BETWEEN ORDINARY FIELDS (NC, ASSOCIATIVE)

- EXAMPLE : MOYAL-GROENEWOLD *PRODUCT :

$$f(x) * g(x) \equiv \exp\left[\frac{i}{2}\theta^{\mu\nu}\frac{\partial}{\partial x^\mu}\frac{\partial}{\partial y^\nu}\right] f(x)g(y)\Big|_{y\to x}$$

$$= fg + \frac{i}{2}\theta^{\mu\nu}\partial_\mu f\,\partial_\nu g + \frac{1}{2!}\left(\frac{i}{2}\right)^2\theta^{\mu\nu}\theta^{\rho\sigma}(\partial_\mu\partial_\rho f)(\partial_\nu\partial_\sigma g)$$

$$+ \cdots$$

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- GENERALIZATION $\rightarrow$ $\boxed{\text{ABELIAN TWIST}}$

$$\partial_\mu \longrightarrow X_A^\mu(x)\,\partial_\mu \quad \text{WITH } [X_A, X_B] = 0$$

- EXTENSION TO p-FORMS : $\boxed{\wedge_\star\text{-PRODUCT}}$

$$X_A \longrightarrow \ell_{X_A} \quad \text{(LIE DERIVATIVE) :}$$

$$\tau \wedge_\star \tau' \equiv \tau\wedge\tau' + \frac{i}{2}\theta^{AB}\,\ell_{X_A}\tau \wedge \ell_{X_B}\tau' +$$

$$+\frac{1}{2!}\left(\frac{i}{2}\right)^2 \theta^{AB}\theta^{CD}\left(\ell_{X_A}\ell_{X_C}\tau\right)\wedge\left(\ell_{X_B}\ell_{X_D}\tau'\right) + \dots$$

- NC THEORIES ARE OBTAINED BY REPLACING PROD. BETWEEN FIELDS WITH * PROD IN CLASSICAL ACTIONS

→ NONLOCAL ACTIONS, HIGHER DER

→ INVARIANT UNDER NC * SYMMETRIES

- EXAMPLE : NC YANG-MILLS IN FLAT SPACE

- STUDY OF TWIST GEOMETRY, APPLIED TO *-METRIC GRAVITY (EQ.S OF MOTION) (MUNICH GROUP)

# * GAUGE THEORY

INGREDIENTS : $A_\mu(x) = A_\mu^I(x) T_I$

$\varepsilon(x) = \varepsilon^I(x) T_I$

$[T_I, T_J] = C_{IJ}^K T_K$ G-LIE A.

FIELD STRENGTH :

$$F_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu - A_\mu * A_\nu - A_\nu * A_\mu$$

GAUGE TRANSF : $\delta_\varepsilon A_\mu = \partial_\mu \varepsilon - A_\mu * \varepsilon + \varepsilon * A_\mu$

$$\Rightarrow \quad \delta_\varepsilon F_{\mu\nu} = - F_{\mu\nu} * \varepsilon + \varepsilon * F_{\mu\nu}$$

*-GAUGE ALGEBRA : $[\delta_{\varepsilon_1}, \delta_{\varepsilon_2}] = \delta_{\varepsilon_2 * \varepsilon_1 - \varepsilon_1 * \varepsilon_2}$

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- INVARIANT ACTION : $\int \mathrm{Tr}\left( F_{\mu\nu} * F^{\mu\nu} \right) d^4x$

(USE CYCLICITY OF $\mathrm{Tr}$ AND OF $\int$ )

- NOTE : $A_\mu * A_\nu - A_\nu * A_\mu =$

$$\left(A^I_\mu * A^J_\nu + A^J_\nu * A^I_\mu\right)[T_I, T_J] +$$
$$\left(A^I_\mu * A^J_\nu - A^J_\nu * A^I_\mu\right)\{T_I, T_J\}$$

$\Rightarrow$ $F^I_{\mu\nu}$ , $A^I_\mu$ BELONG TO THE U(G)

$\Rightarrow$ PROLIFERATION OF DEG. OF FREEDOM !

1) EXPLOIT PROPERTIES OF REPR. OF G

FOR EX. FOR $G = SU(2)$, IN THE 2-DIM REPR. (PAULI MATRICES):

$$[\tau_I, \tau_J] = \epsilon_{IJK} \tau_K$$
$$\{\tau_I, \tau_J\} = i \delta_{IJ} \mathbb{1}$$

- ONLY 1 ADDITIONAL GENERATOR ($\mathbb{1}$)
- $SU(2) \longrightarrow U(2)$

2) SEIBERG - WITTEN MAP $\hat{A} = \hat{A}(A)$, $\hat{\varepsilon} = \hat{\varepsilon}(A, \varepsilon)$

$\rightarrow$ ALL NC FIELDS IN TERMS OF CLASSICAL FIELDS

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# NONCOMMUTATIVE (VIELBEIN) GRAVITY

- CLASSICAL ACTION

$$S = \int R^{ab} \wedge V^c \wedge V^d \epsilon_{abcd} = -4 \int R \sqrt{-g}\, d^4x$$

$$R^{ab} \equiv d\omega^{ab} - \omega^{ac} \wedge \omega_c{}^b$$

$V^a = V^a_\mu dx^\mu$

$R^{ab} = R^{ab}{}_{\mu\nu} dx^\mu \wedge dx^\nu$

- INDEX-FREE:

$$S = \int \mathrm{Tr}\,(i\gamma_5 R \wedge V \wedge V)$$

$$V \equiv V^a \gamma_a \,,\quad \Omega \equiv \frac{1}{4}\omega^{ab}\gamma_{ab} \,,\quad R = d\Omega - \Omega \wedge \Omega$$

THEN $R = \frac{1}{4}\left(d\omega^{ab} - \omega^{ac} \wedge \omega_c{}^b\right)\gamma_{ab} = \frac{1}{4} R^{ab}\gamma_{ab}$

AND $\int \mathrm{Tr}\left(i\gamma_5 R \wedge V \wedge V\right) =$

$$\frac{1}{4}\int R^{ab} \wedge V^c \wedge V^d \underbrace{\mathrm{Tr}\left(i\gamma_5 \gamma_{ab}\gamma_c\gamma_d\right)}_{4\epsilon_{abcd}}$$

IS THE USUAL EINSTEIN-HILBERT ACTION

- INVARIANCES : G.C.T.

LOCAL LORENTZ

$\delta_\varepsilon \omega^{ab} = d\varepsilon^{ab} - \omega_c{}^a \varepsilon^{bc} + \varepsilon^{ac}\omega_c{}^b$

$\delta_\varepsilon V^a = \varepsilon^a{}_b V^b$

- LOCAL LORENTZ INVARIANCE IN INDEX-FREE NOTATION :

$$\delta_\varepsilon V = -V\varepsilon + \varepsilon V \ , \ \delta_\varepsilon \Omega = d\varepsilon - \Omega\varepsilon + \varepsilon\Omega$$

WITH $\varepsilon = \frac{1}{4}\varepsilon^{ab}\gamma_{ab}$

$$\Rightarrow \delta_\varepsilon R = -R\varepsilon + \varepsilon R$$

$$\Rightarrow \delta_\varepsilon \int Tr\left(i\gamma_5 R \wedge V \wedge V\right) = 0$$

BY CYCLICITY OF $Tr$ AND $[\gamma_5, \varepsilon] = 0$


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# * GRAVITY :

$$S = \int \mathrm{Tr}(i\gamma_5 R \wedge_* V \wedge_* V)$$

WITH $R \equiv d\Omega - \Omega \wedge_* \Omega$

NOTE: $\Omega \wedge_* \Omega$ CONTAINS $[\gamma^{ab}, \gamma^{cd}]$

AND $\{\gamma^{ab}, \gamma^{cd}\} \rightarrow 1\!\!1, \gamma_5$

$$\Rightarrow \left| \begin{array}{l} \Omega = \frac{1}{4}\omega^{ab}\gamma_{ab} + i\omega 1\!\!1 + \tilde{\omega}\gamma_5 \\ V = V^a\gamma_a + \tilde{V}^a\gamma_a\gamma_5 \\ R = \frac{1}{4}R^{ab}\gamma_{ab} + iR 1\!\!1 + \tilde{R}\gamma_5 \end{array} \right.$$

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- INVARIANCES :
  - GCT ( S IS AN INTEGRAL OF 4-FORM)
  - $\star$- GAUGE INVARIANCE UNDER

$$\delta_\varepsilon V = -V\star\varepsilon + \varepsilon\star V$$

$$\delta_\varepsilon \Omega = d\varepsilon - \Omega\star\varepsilon + \varepsilon\star\Omega$$

$$\Rightarrow \delta_\varepsilon R = -R\star\varepsilon + \varepsilon\star R$$

$$\delta_\varepsilon S = \delta_\varepsilon \int Tr(i\gamma_5 R\wedge_\star V\wedge_\star V) = 0$$

( CYCLICITY OF Tr AND OF $\int$ , AND $[\gamma_5, \varepsilon] = 0$ )

$$\varepsilon = \frac{1}{4}\varepsilon^{ab}\gamma_{ab} + i\varepsilon \mathbb{1} + \tilde{\varepsilon}\gamma_5$$



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- INDEX-FREE FORMALISM (→ CHAMSEDDINE 2003)

- $\star$ GRAVITY WITH COMPLEX VIELBEIN : CHAMSEDDINE 2003

- $\star$ GRAVITY WITH REAL VIELBEIN (C.C. CONDITIONS ON FIELDS, COMPATIBLE WITH $\star$-GAUGE TRANSF. P. ASCHIERI, L.C.) 2009

- $\theta^2$ - EXPANSION OF FIELDS AND ACTION VIA SW MAP P. ASCHIERI, L.C. 2011

- GAUGE-INV. $\theta^2$-EXPANSION P. ASCHIERI, L.C. M. DIMITRIJEVIC 2012

# GEOMETRICAL SW MAP FOR ABELIAN TWISTS

P. ASCHIERI, LC

- RELATES NC GAUGE FIELD $\hat{\Omega}$ TO ORDINARY (CLASSICAL) $\Omega$, AND $\hat{\varepsilon}$ TO $\varepsilon$ AND $\Omega$ SO AS TO SATISFY

$$\hat{\Omega}(\Omega) + \hat{\delta}_{\hat{\varepsilon}}\hat{\Omega}(\Omega) = \hat{\Omega}(\Omega + \delta_\varepsilon \Omega)$$

WHERE
$$\delta_\varepsilon \Omega = d\varepsilon - \Omega\varepsilon + \varepsilon\Omega$$
$$\hat{\delta}_{\hat{\varepsilon}}\hat{\Omega} = d\hat{\varepsilon} - \hat{\Omega} * \hat{\varepsilon} + \hat{\varepsilon} * \hat{\Omega}$$

- CAN BE SOLVED ORDER BY ORDER IN $\theta$

$$\hat{\Omega} = \Omega + \Omega^1(\Omega) + \Omega^2(\Omega) + \cdots$$

$$\hat{\mathcal{E}} = \mathcal{E} + \mathcal{E}^1(\mathcal{E},\Omega) + \mathcal{E}^2(\mathcal{E},\Omega) + \cdots$$

WITH $$\Omega^{n+1} = \frac{i}{4(n+1)}\theta^{AB}\{\hat{\Omega}_A, \ell_B\hat{\Omega} + \hat{R}_B\}^n_*$$

$$\mathcal{E}^{n+1} = \frac{i}{4(n+1)}\theta^{AB}\{\hat{\Omega}_A, \ell_B\hat{\mathcal{E}}\}^n_*$$

$$R^{n+1} = \frac{i}{4(n+1)}\theta^{AB}\left(\{\hat{\Omega}_A, (\ell_B + L_B)\hat{R}\}^n_* - [\hat{R}_A, \hat{R}_B]^n_*\right)$$

RECURSIVE REL.S: GENERALIZE ULKER (2008)

FOR EXAMPLE :

$$V^{1\,a} = 0$$

$$\tilde{V}^{1\,a} = \frac{1}{4}\,\vartheta^{AB}\,\chi^{\rho}_{A}\,\omega_{\rho}^{bc}\,\epsilon_{abcd}\left(\ell_{B}V^{d} - \frac{1}{2}\chi^{\sigma}_{B}\,\omega_{\sigma}^{de}V^{e}\right)$$

$$\omega^{1\,ab} = 0$$

$$\omega^{1} = -\frac{1}{16}\,\vartheta^{AB}\,\chi^{\rho}_{A}\,\omega_{\rho}^{ab}\left(\ell_{B}\,\omega^{ab} + i_{B}R^{ab}\right)$$

$$\tilde{\omega}^{1} = -\frac{1}{16}\,\vartheta^{AB}\,\chi^{\rho}_{A}\,\omega_{\rho}^{ab}\left(\ell_{B}\,\omega^{cd} + i_{B}R^{cd}\right)\epsilon_{abcd}$$

- APPLYING THE SW MAP TO THE FIELDS IN THE NC GRAVITY ACTION YIELDS A HIGHER DERIV. ACTION INVOLVING ONLY $V^a, \omega^{ab}$ (AND THE BACKGR. $X_A$ FIELDS DEFINING THE $\star$-PRODUCT)

$$S = S^0 + S^1 + S^2 + \cdots$$

WITH $S^0$ = CLASSICAL EINSTEIN-HILBERT ACTION

$$S^1 = 0$$

$$S^2 \neq 0$$

NOTE: THE EXPANDED ACTION, AFTER SW MAP, IS GAUGE INVARIANT UNDER <u>USUAL</u> GAUGE TRANSF. ORDER BY ORDER IN $\theta$

$$S = S^0 + S^1 + S^2 + \cdots$$

- BECAUSE USUAL GAUGE TRANSF. INDUCE $\star$-GAUGE TRANSF. ON THE NC FIELDS UNDER WHICH $S$ IS INVARIANT

- BECAUSE USUAL GAUGE TRANSF. DO NOT CONTAIN $\theta$

# EXTENDED GRAVITY ACTION AT $\theta^2$

$$S^2 = \frac{1}{32}\theta^{AB}\theta^{CD}\int -\frac{1}{2}\big((R^{ab}_C R^{cd}_D R^{ef})_{AB} - \frac{1}{2}R^{ef}_{CD}(R^{ab}R^{cd})_{AB}\big)V^g V^h(\varepsilon_{abcd}\delta^{gh}_{ef} + \varepsilon_{efgh}\delta^{cd}_{ab})$$

$$+ \big(2(L_C R^{ea} L_D R^{eb})_{AB} V^c V^d - (R^{ab}R^{cd})_{AB} L_C V^e L_D V^e$$

$$- 8R^{df} R^{ab}_{AC} L_B V^c L_D V^f - 4R^{ab}(L_A L_C V^c)(L_B L_D V^d)\big)\varepsilon_{abcd}\ .$$

- $L_A \equiv Di_A + i_A D$ (COVARIANT LIE DERIVATIVE)

  $L_A V = Di_A V + i_A(DV) = DV_A + T_A$

  $L_A R = Di_A R + i_A(DR) = DR_A$

- $T \equiv DV = dV - \Omega V - V\Omega$ (TORSION)

- $S^2$ IS MANIFESTLY INVARIANT UNDER LOCAL LORENZ ROTATIONS

→ AT THIS STAGE THE $X^\mu_A$ VECTOR FIELDS HAVE NO DYNAMICS

P. ASCHIERI, L.C. (2012)
P. ASCHIERI, L.C., M. DIMITRIJEVIC (2012)
E. DI GREZIA, G. ESPOSITO, M. FIGLIOLA, P. VITALE (2012)

# COUPLINGS OF NC $\star$-GRAVITY

## FERMIONS

- CLASSICAL ACTION

$$S = \int R^{ab} \wedge V^c \wedge V^d \epsilon_{abcd} +$$

$$+ \underbrace{i [\bar\psi \gamma^a D\psi - \overline{D\psi} \gamma^a \psi] \wedge V^b \wedge V^c \wedge V^d \epsilon_{abcd}}_{i \bar\psi \not{D} \psi \sqrt{-g}\, d^4x}$$

$$D\psi \equiv d\psi - \Omega\psi$$

- $\delta_\varepsilon \psi = \varepsilon \psi \qquad \left( \varepsilon = \frac{1}{4} \varepsilon^{ab} \gamma_{ab} \right)$

## * FERMIONS (SPIN 1/2)

$$S = \int \mathrm{Tr}\left[ i\gamma_5 \left( R \wedge_* V \wedge_* V - (D\psi * \bar{\psi} - \psi * D\bar{\psi}) \wedge_* V \wedge_* V \wedge_* V \right)\right]$$

$$D\psi \equiv d\psi - \Omega * \psi$$

- $*$-GAUGE INVARIANT UNDER SAME $*$-TRANSF AND

$$\delta_\varepsilon \psi \doteq \varepsilon * \psi$$

## * FERMIONS (SPIN 3/2)

→ NC SUPERGRAVITY

# OSp(1|4) SUPERGRAVITY AND ITS NONCOMMUTATIVE EXTENSION

LC, TO APPEAR

- OSp(1|4) CONNECTION :

$$\Omega = \begin{pmatrix} \Omega \equiv \frac{1}{4}\omega^{ab}\gamma_{ab} - \frac{i}{2}V^a\gamma_a & \psi \\ \bar\psi & 0 \end{pmatrix}$$

- OSp(1|4) CURVATURE : $\mathbb{R} = d\Omega - \Omega\Omega \rightarrow$

$$\mathbb{R} = \begin{pmatrix} R \equiv \frac{1}{4}R^{ab}\gamma_{ab} - \frac{i}{2}R^a\gamma_a & \Sigma \\ \bar\Sigma & 0 \end{pmatrix}$$

$$\Rightarrow R^{ab} = d\omega^{ab} - \omega^{ac}\omega^{cb} + V^aV^b + \frac{1}{2}\bar\psi\gamma^{ab}\psi$$

$$R^a = dV^a - \omega^{ab}V^b - \frac{i}{2}\bar\psi\gamma^a\psi$$

$$\Sigma = d\psi - \frac{1}{4}\omega^{ab}\gamma_{ab}\psi + \frac{i}{2}V^a\gamma_a\psi$$

$\mathcal{R}^{ab}$ (LORENTZ)

$\rho$ (GRAVITINO)

- ACTION (MAC DOWELL- MANSOURI)$_{1977}$

$$\boxed{S = i\int \mathrm{Tr}\left(R R \gamma_5 + \Sigma \bar{\Sigma} \gamma_5\right)}$$

- TAKING THE TRACE $\longrightarrow$

$$S = \int \frac{1}{4} R^{ab} R^{cd} \epsilon_{abcd} - 2i\, \bar{\Sigma} \gamma_5 \Sigma$$

- $R^{ab} = \mathcal{R}^{ab}(\omega) + V^a V^b + \frac{1}{2} \bar{\psi} \gamma^{ab} \psi$

  $\Sigma = \rho + \frac{i}{2} V^a \gamma_a \psi$

$$\boxed{S = \frac{1}{2}\int \mathcal{R}^{ab} V^c V^d \epsilon_{abcd} + 4 \bar{\rho} \gamma_a \gamma_5 \psi V^a + \frac{1}{2}\left(V^a V^b V^c V^d + 2 \bar{\psi} \gamma^{ab} \psi\, V^c V^d\right) \epsilon_{abcd}}$$

- ACTION (MAC DOWELL–MANSOURI)

$$\boxed{S = i\int \mathrm{Tr}\left(R R \gamma_5 + \Sigma \bar{\Sigma} \gamma_5\right)}$$

- TAKING THE TRACE $\longrightarrow$

$$S = \int \frac{1}{4} R^{ab} R^{cd} \epsilon_{abcd} - 2i\, \bar{\Sigma} \gamma_5 \Sigma$$

- $R^{ab} = \mathcal{R}^{ab}(\omega) + \lambda^2 V^a V^b + \frac{1}{2} \bar{\psi} \gamma^{ab} \psi$

$\Sigma = \rho + \lambda \frac{i}{2} V^a \gamma_a \psi$

$$\boxed{S = \frac{1}{2}\int \mathcal{R}^{ab} V^c V^d \epsilon_{abcd} + 4\bar{\rho}\gamma_a \gamma_5 \psi V^a + \lambda^2 \frac{1}{2}\left(V^a V^b V^c V^d + 2\bar{\psi}\gamma^{ab}\psi\, V^c V^d\right)\epsilon_{abcd}}$$

- USING $OSp(1|4)$ SUPERMATRICES :

$$\boxed{S = \int STr\left(R\left(1 - \frac{\Gamma^2}{2}\right) R\, \Gamma\right)}$$

PREITSCHOPF, VASILIEV
1998

- $\Gamma$ IS A CONSTANT 5x5 MATRIX $\quad \Gamma \equiv \begin{pmatrix} \gamma_5 & 0 \\ 0 & 0 \end{pmatrix}$

- $R$ IS THE $OSp(1|4)$ CURVATURE

- GAUGE VARIATIONS :

$$\delta\Omega = d\epsilon - \Omega\epsilon + \epsilon\Omega$$
$$\delta R = \quad - R\epsilon + \epsilon R$$

$$\epsilon = \begin{pmatrix} \frac{1}{4}\varepsilon^{ab}\gamma_{ab} - \frac{i}{2}\varepsilon^{a}\gamma_{a} & \epsilon \\ \bar{\epsilon} & 0 \end{pmatrix}$$

$\boxed{\delta S = 0 \text{ ONLY UNDER THE } SO(1,3) \text{ SUBGROUP OF } OSp(1|4)}$

CHAMSEDDINE, WEST, STELLE, TOWNSEND, FRÉ, D'ADDA, D'AURIA, REGGE, VAN NIEUWENHUIZEN...

- VARIATION UNDER SUPERSYMMETRY $\epsilon$

$$\delta S = 4 \int R^a \bar{\rho} \gamma_5 \gamma_a \epsilon$$

- VARIATION UNDER ARBITRARY $\delta\omega^{ab}$

$$\delta S = 16 \int R^a V^b \delta\omega^{cd} \epsilon_{abcd}$$

- THUS UNDER SUPERSYMMETRY WITH MODIFIED LAW FOR $\delta\omega^{ab}$

$$\delta_{MOD}\,\omega^{ab} = \delta_{\substack{GAUGE\\SUSY}}\,\omega^{ab} + \delta_{EXTRA}\,\omega^{ab} \Longrightarrow \delta S = 4 \int R^a \left( \bar{\rho} \gamma_5 \gamma_a \epsilon - 2\, \delta_{EXTRA}\,\omega^{bc} V^d \epsilon_{abcd} \right)$$

- TWO WAYS TO RESTORE SUPERSYMMETRY !

# $OSp(1|4)$ INVARIANT ACTION

PREITSCHOPF, VASILIEV

$$S = \int STr\left(\mathbb{R}\left(\mathbb{1} - \frac{\Phi^2}{2}\right)\mathbb{R}\Phi\right)$$

$$\Phi \equiv \begin{pmatrix} \frac{1}{4}\pi + \varphi\gamma_5 + i\varphi^a\gamma_a\gamma_5 & \chi \\ -\bar{\chi} & \pi \end{pmatrix}$$

SYMMETRIC TRACELESS REPR. OF $OSp(1|4)$

- $S$ IS INVARIANT UNDER THE $OSp(1|4)$ GAUGE TRANSF :

$$\begin{aligned} \delta\Phi &= -\Phi\epsilon + \epsilon\Phi \\ \delta\mathbb{R} &= -\mathbb{R}\epsilon + \epsilon\mathbb{R} \end{aligned} \Rightarrow \delta S = 0$$

- USE TRANSLATIONS AND SUPERSYMMETRY $\Rightarrow$ $\boxed{\varphi^a = 0,\ \chi = 0}$

- $OSp(1|4)$ - INVARIANT CONSTRAINT :

$$\boxed{\Phi^3 - \Phi = 0} \quad \Rightarrow \quad \Phi = \begin{pmatrix} \gamma_5 & 0 \\ 0 & 0 \end{pmatrix} = \Gamma$$

- GENERALIZES THE "SOLDERING GAUGE" $\Phi = \gamma_5$ OF $SO(2,3)$ - INVARIANT GRAVITY

- REDUCES THE $OSp(1|4)$ - INVARIANT THEORY TO MAC DOWELL - MANSOURI $D=4$ $N=1$ SUPERGRAVITY

- THE SAME GAME CAN BE PLAYED WITH THE $\star$-DEFORMED $OSp(1|4)$ - INV. THEORY. AFTER GAUGE FIXING $\longrightarrow$ HIGHER DER. MAC DOWELL - MANSOURI $D=4$ $N=1$ SUPERGRAVITY

# $\star$-OSp(1|4) INVARIANT ACTION

$$\boxed{S = \int \mathrm{STr}\left[\mathbb{R} \star \left(\mathbb{1} - \frac{\Phi \star \Phi}{2}\right) \wedge_\star \mathbb{R} \star \Phi\right]}$$

- INVARIANT UNDER

$$\delta \mathbb{R} = - \mathbb{R} \star \epsilon + \epsilon \star \mathbb{R}$$
$$\delta \Phi = - \Phi \star \epsilon + \epsilon \star \Phi$$

- EXTRA FIELDS IN $\Omega$, $\Phi$

- SW MAP FOR OSp(1|4) → EXTENDED (HIGHER DER) OSp(1|4) INVARIANT THEORY

- GAUGE FIXING $\Phi \rightarrow \Pi$

$\Rightarrow$ HIGHER DERIVATIVE $D=4$ $N=1$ SUPERGRAVITY

- IS SUPERSYMMETRY RESTORATION POSSIBLE IN THE $\ast$ DEFORMED CASE ?

- ANSWER IS : YES

  CAN FIND $\delta_{EXTRA}\,\omega^{ab}$ ORDER BY ORDER IN $\theta$ SUCH THAT THE EXTENDED $D=4$ $N=1$ SG IS SUPERSYMMETRIC

- NB : SUPERSYMM. TRANSF ON $\omega^{ab}$ INVOLVE $\theta$ !

# CONCLUSIONS

- USING NONCOMMUTATIVITY AS A BUILDING PRINCIPLE FOR EXTENDED (SUPER) GRAVITY THEORIES

- $\star$- $OSp(1|4)$ SUPERGRAVITY : FIRST INSTANCE OF $D=4$ LOCALLY SUPERSYMMETRIC NC - EXTENDED (HIGHER DER) THEORY.

- IN $D=3$ : NC AdS SUPERGRAVITY AS $U(1,1|1) \otimes U(1,1|1)$ $\star$ CHERN - SIMONS THEORY
  CACCIATORI, MARTUCCI 2002

# DYNAMICAL NONCOMMUTATIVITY

- RELATING $X_A$ TO SCALARS :

$$X^\mu_A = \left[\partial_\mu \phi^A\right]^{-1}$$

OR MORE GENERALLY

$$X^\mu_A = \left[\partial_\mu Z(\phi)^A\right]^{-1}$$

$$\Rightarrow \left[X_A, X_B\right] = 0$$

P. A., L. C., M. DIMITRIJEVIC (2008) ← FLAT SPACE
P. A., L. C. 2012 ← GRAVITY

# AdS D=3 SUPERGRAVITY

$$S = \int \mathrm{STr}\left(\Omega\, d\Omega - \frac{2}{3}\Omega\Omega\Omega\right) - \int \mathrm{STr}\left(\tilde{\Omega}\, d\tilde{\Omega} - \frac{2}{3}\tilde{\Omega}\tilde{\Omega}\tilde{\Omega}\right)$$

ACHUCARRO, TOWNSEND
1986

CACCIATORI, MARTUCCI
2002
(* EXTENSION)

$$\Omega = \begin{pmatrix} \Omega & \psi \\ -i\bar{\psi} & i a \end{pmatrix}$$

$$\Omega = \frac{1}{2}\omega^a \gamma_a + \frac{1}{2}\frac{V^a}{\ell}\gamma_a + \frac{i}{2}\omega \mathbb{1}$$

$\gamma_a, \mathbb{1} \longrightarrow U(1,1)$

$\rightarrow$

$$S = \int R^{ab} \wedge V^c \epsilon_{abc} + \frac{1}{3\ell^2} V^a V^b V^c \epsilon_{abc}$$
$$+ 2i\, \bar{\psi} D\psi - 2i\, \bar{\tilde{\psi}} D \tilde{\psi}$$
$$+ \cdots \quad (a \text{ and } b \text{ terms})$$

- LOCAL $SU(1,1|1) \times SU(1,1|1)$ SYMMETRY

$$\delta_{\epsilon} \Omega = d\epsilon - \Omega\epsilon + \epsilon\Omega \;, \quad \delta_{\epsilon} \tilde{\Omega} = 0$$
$$\delta_{\tilde{\epsilon}} \tilde{\Omega} = d\tilde{\epsilon} - \tilde{\Omega}\tilde{\epsilon} + \tilde{\epsilon}\tilde{\Omega} \;, \quad \delta_{\tilde{\epsilon}} \Omega = 0$$

$$\epsilon = \begin{pmatrix} \varepsilon^a \gamma_a + i\varepsilon \mathbb{1} & \epsilon \\ -i\bar{\epsilon} & i\alpha \end{pmatrix}$$

SUSY

$$\omega^{ab} = \epsilon^{abc} \omega_c$$


21/09/12

- SW MAP FOR $\Omega$
  ( $SU(1,1|1) \times SU(1,1|1)$ GAUGE FIELD)

- SUPERSYMM AND LORENTZ ARE PART OF SUPERGROUP $U(1,1|1) \times U(1,1|1)$

$\Rightarrow$ EXTENDED $D=3$ SUPERGRAVITY

(AFTER SW EXPANSION OF THE ACTION)