# YANG-MILLS THEORY AS A MASSLESS LIMIT OF A MASSIVE GAUGE MODEL.

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Yang-Mills theory is the basis of QCD.

Perturbation theory is not the proper instrument for study of QCD. Color confinement cannnot be explained in the framework of perturbation theory $\Rightarrow$ infrared problem.

Scattering theory for bound states does not exist.

Nevertheless we insist on the formal perturbative unitarity of the scattering matrix in the Yang-Mills theory.

Even the quantization procedure for the Yang-Mills theory is defined only in the framework of perturbation theory $\Rightarrow$ Gribov ambiguity.

Any differential gauge suffers from Gribov ambiguity (Singer). Any algebraic gauge is not manifestly Lorentz invariant.

Is it possible to construct the quantization procedure for nonabelian gauge fields which:

1. May be applied beyond perturbation theory.

2. Is manifestly Lorentz invariant.

3. Produces formally unitary scattering matrix.

4. Allows gauge invariant infrared regularization.

The first three questions have been answered in the papers: A.A.Slavnov, JHEP,08(2008)513; A.A.Slavnov, Theor.Math. Phys.161 (2009)1497; A.Quadri, A.A.Slavnov, JHEP,1007 (2010)087, were such quantization was performed, and renormalizability of this procedure was proven.

Perturbative infrared regularization was also constructed: A.A.Slavnov, 154 (2008) 178.

In this talk I will describe a gauge invariant infrared regularization of the Yang-Mills theory which may be used both in perturbation theory and beyond it.

The model is described by the Lagrangian:

$$L = -\frac{1}{4}F^a_{\mu\nu}F^a_{\mu\nu} - m^{-2}(D^2\tilde{\phi})^*(D^2\tilde{\phi}) + (D_\mu e)^*(D_\mu b) + (D_\mu b)^*(D_\mu e) + \alpha^2(D_\mu\tilde{\phi})^*(D_\mu\tilde{\phi}) - \alpha^2 m^2(b^*e + e^*b) \quad (1)$$

where $\phi, b, e$ are two component complex doublet.

$\alpha = 0$ corresponds to the nonregularized Yang-Mills theory. Integrating out the fields $\phi, b, e$, we get the usual Yang-Mills theory.

The Lagrangian (1) contains additional unphysical excitations, which however do not contribute to the physical amplitudes.

We shift the variable $\phi$

$$\tilde{\phi} = \phi - \hat{\mu}; \quad \hat{\mu} = (0, \mu\sqrt{2}g^{-1}) \tag{2}$$

and use the following parametrization of the classical fields in terms of Hermitean components:

$$\phi = (\frac{i\phi^1 + \phi^2}{\sqrt{2}}(1 + \frac{g}{2\mu}\phi^0), \frac{\phi^0 - i\phi^3(1 + g/(2\mu)\phi^0)}{\sqrt{2}}) \tag{3}$$

This parametrization makes possible to choose algebraic, completely relativistic gauge, free of Gribov ambiguity. Unitarity in the physical subspace should be proven.

The Lagrangian (1) is obviously invariant with respect to "shifted"gauge transformations

$$A_\mu^a \to A_\mu^a + \partial_\mu \eta^a - g\epsilon^{abc} A_\mu^b \eta^c$$
$$\phi^a \to \phi^a + \frac{g}{2}\epsilon^{abc}\phi^b\eta^c + \eta^a\mu + \frac{g^2}{4\mu}\phi^a\phi^b\eta^b$$
$$\ldots \tag{4}$$

For gauge transformations (4) the gauge $\phi^a = 0$ is admissible both in perturbation theory and beyond it. If $\phi^a = 0$, then under the gauge transformations (4) the variables $\phi^a$ become

$$\delta\phi^a = \mu\eta^a; \quad \phi^a = 0 \Rightarrow \eta^a = 0 \tag{5}$$

The Lagrangian (1)is also invariant with respect to the supersymmetry transformations

$$\phi \longrightarrow \phi - b\epsilon$$
$$e \rightarrow e - \frac{D^2(\phi - \widehat{\mu})}{m^2}\epsilon \tag{6}$$

This symmetry plays a crucial role in the proof of decoupling of unphysical excitations.

It holds for any $\alpha$, but for $\alpha = 0$ these transformations are also nilpotent. We need only the existence of the conserved charge $Q$ and nilpotency of the asymptotic charge $Q_0$, as the physical spectrum is determined by the asymptotic dynamics.

The nilpotency of the asymptotic charge requires $\alpha = 0$, and the massive theory with $\alpha \neq 0$ is gauge invariant but not unitary.

Usually the gauge invariance is a sufficient condition of unitarity, because one can pass freely from a renormalizable gauge to the unitary one, where the spectrum includes only physical excitations.

In the present case there is no "unitary"gauge. Even in the gauge $\phi^a = 0$, there are unphysical excitations.

The shift of the field $\phi$ produces the mass for the vector field.

$$\alpha^2 (D_\mu \widehat{\mu})^* (D_\mu \widehat{\mu}) = \frac{\alpha^2 \mu^2}{2} A_\mu^2 \tag{7}$$

Another term, produced by the shift

$$m^{-2} (D^2 \widehat{\mu})^* (D^2 \widehat{\mu}) = \frac{\mu^2}{2m^2} [(\partial_\mu A_\mu)^2 + \frac{g^2}{2} (A^2)^2] \tag{8}$$

makes the theory renormalizable for any $\alpha$.

The shifted Lagrangian is invariant with respect to simultaneous gauge transformations (4) and supersymmetry transformations.

Therefore the effective Lagrangian may be written in the form

$$L_{ef} = L + s_1 \bar{c}^a \phi^a = L(x) + \lambda^a \phi^a - \bar{c}^a (\mu c^a - b^a) \quad (9)$$

Here $L$ is the shifted Lagrangian (1), and $s_1$ is the nilpotent operator, corresponding to the simultaneous BRST transformation and supersymmetry transformation.

Integration over $\bar{c}, c$ leads to the substitution $c^a \to b^a \mu^{-1}$.

For asymptotic theory the symmetry transformations are

$$\begin{aligned} \delta A^a_\mu &= \partial_\mu b^a \mu^{-1} \epsilon \\ \delta \phi^a &= 0 \\ \delta \phi^0 &= -b^0 \epsilon \\ \delta e^a &= \partial_\mu A^a_\mu \mu^{-1} \\ \delta e^0 &= -\partial^2 \phi^0 \mu^{-2} \\ \delta b^a &= 0 \\ \delta b^0 &= 0. \end{aligned} \tag{10}$$

This invariance generates a conserved charge $Q$ and the asymptotic states may be chosen to satisfy the condition

$$\widehat{Q}_0|\psi>_{as}= 0 \tag{11}$$

We want to prove that the Lagrangian(9) really describes the infrared regularization of the Yang-Mills theory. That means for $\alpha \neq 0$ it corresponds to a massive gauge invariant theory and in the limit $\alpha = 0$ it describes the usual three dimensionally transversal excitations of the Yang-Mills field. For simplicity we put $\mu = m$.

In the limit $\alpha = 0$ all the terms proportional to $\alpha$ disappear . The remaining terms depend only on the fields $A_0, A_i$ and corresponding canonical momenta, which coincide with the usual Yang-Mills Hamiltonian in the diagonal Feynman gauge, and the fields $\phi_0, b_0, e_0$.

The fields $b_a, e_a$ play the role of the Faddeev-Popov ghosts.

By the usual arguments the longitudinal and temporal components of the Yang-Mills field as well as the fields $b_a, e_a$ decouple, and the physical states may include only transversal components of the Yang-Mills field and variables corresponding to the fields $\phi^0, b^0, e^0$.

The part of the Hamiltonian, depending on the fields $\phi^0, b^0, e^0$ is BRST exact. It can be presented as the anticommutator of the BRST operator with some operator $\hat{A}$.

$$\hat{H}_0 = [\hat{Q}_0, \hat{A}]_+ \tag{12}$$

Therefore the energy of any physical state, annihilated by $Q_0$ does not depend on these fields.

One may introduce the operator $\hat{K}$ whose anticommutator with $\hat{Q}_0$ is proportional to the number of unphysical modes, generated by the operators $\phi^0, b^0, e^0$.

$$[\hat{K}, \hat{Q}_0]_+ \sim \hat{N}$$

Therefore any vector $|\psi>_{phys}$, annihilated by $\hat{Q}_0$ may be presented in the form

$$|\psi>_{phys} = |\psi>_{tr} + Q_0|\chi> \tag{13}$$

and matrix element of any observable, calculated with the help of $|\psi>_{phys}$ coincides with the corresponding matrix element in the Yang-Mills theory.

# CONCLUSION

Infrared regularization of the Yang-Mills theory applicable beyond perturbation theory exists.

It may be used for the study of soliton mechanism of the color confinement.