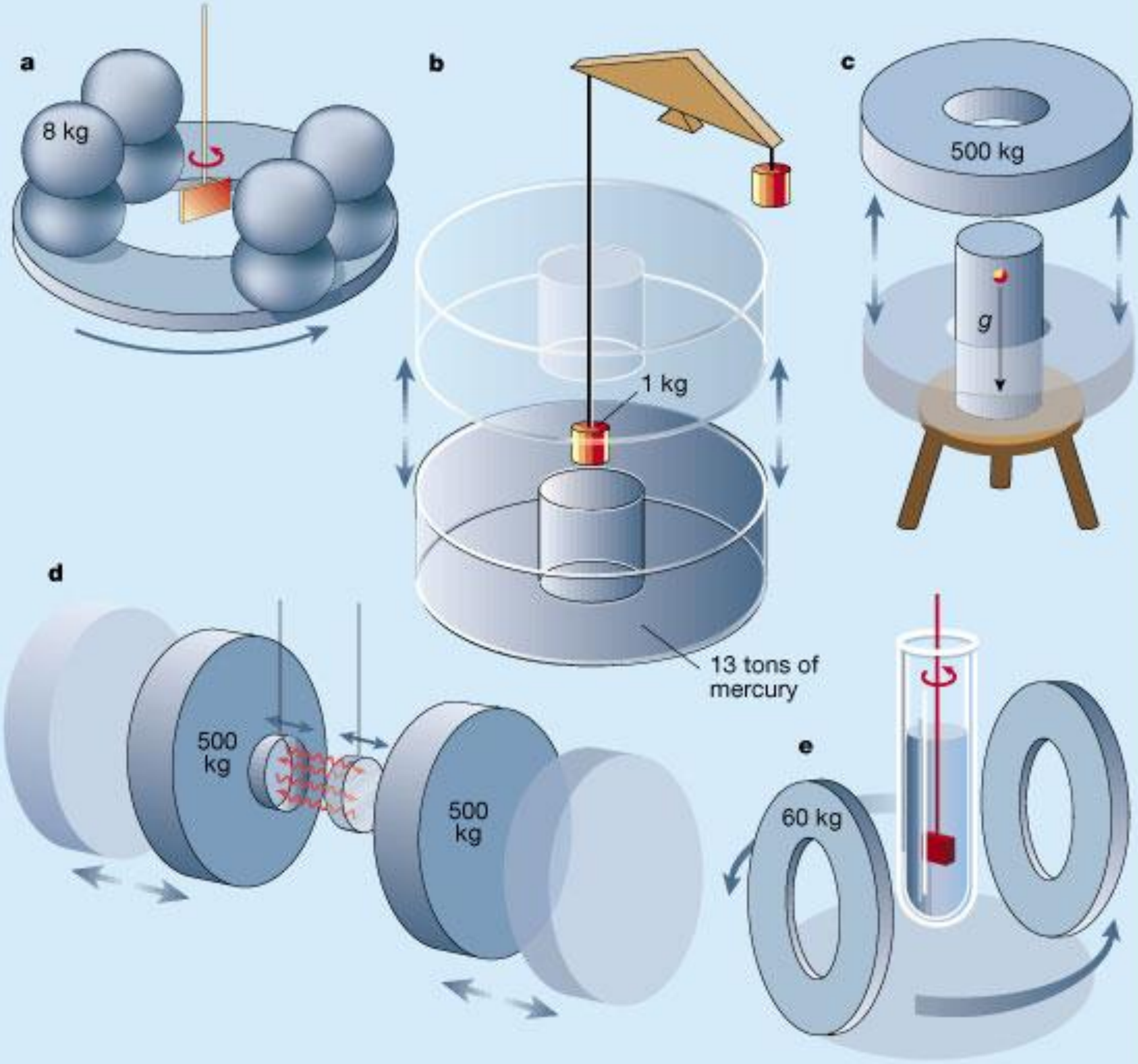


Terry Quinn NATURE|VOL 408 | 21/28 DECEMBER 2000

Variability of the Gravitational Constant – Forty-Three Years Later

Plamen Fiziev
Sofia University Foundation for
Theoretical and Computational Physics and Astrophysics
&
BLTF, JINR, Dubna
Tsveta Fizieva
Independent Researcher

FFK14,
Dubna, 1-5 December 2014

Plan of the talk:

1. **History and last results**
2. **The Problems with G measurements**
3. **Modern Space Results**

- **The Geoid Data**
- **Variations with time**
- **Correlation coefficients**

4. Fifth Force (1971-2014)

5. Experimental checks (1972-1992)

6. Sub-millimeter checks of gravity (2000-2009)

7. MDG Model, compact stars

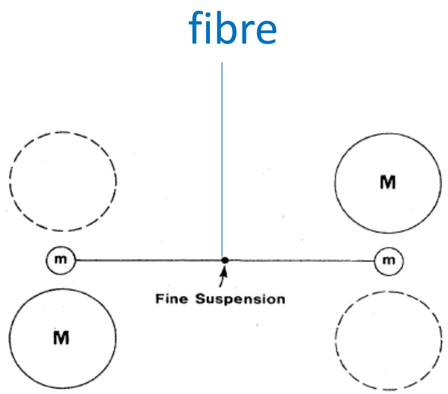
8. PREM

8. Novel MDG + PREM results

Newton consideration of **G×M** (1684): $F = GM_1M_2/r^2.$

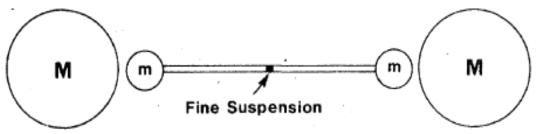
1-kg masses, **10 cm** apart ⇨ the acceleration of the other towards it is $\mathbf{6{\times}10^{-19}m\,s^{-2}}$.

Mitchel - Cavendish experiment of measurements of G torsion-balance method (1798):

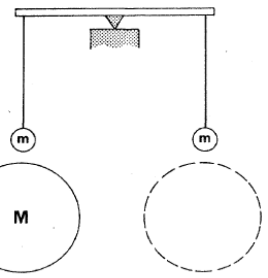


$$\mathbf{G = 6.754 \times 10^{-11}\ m^3\ kg^{-1}\ s^{-2}}$$

Torsional-pendulum method - **Von Eötvös** (1885):



Beam-balance method of von **Jolly and Poynting** (1892):

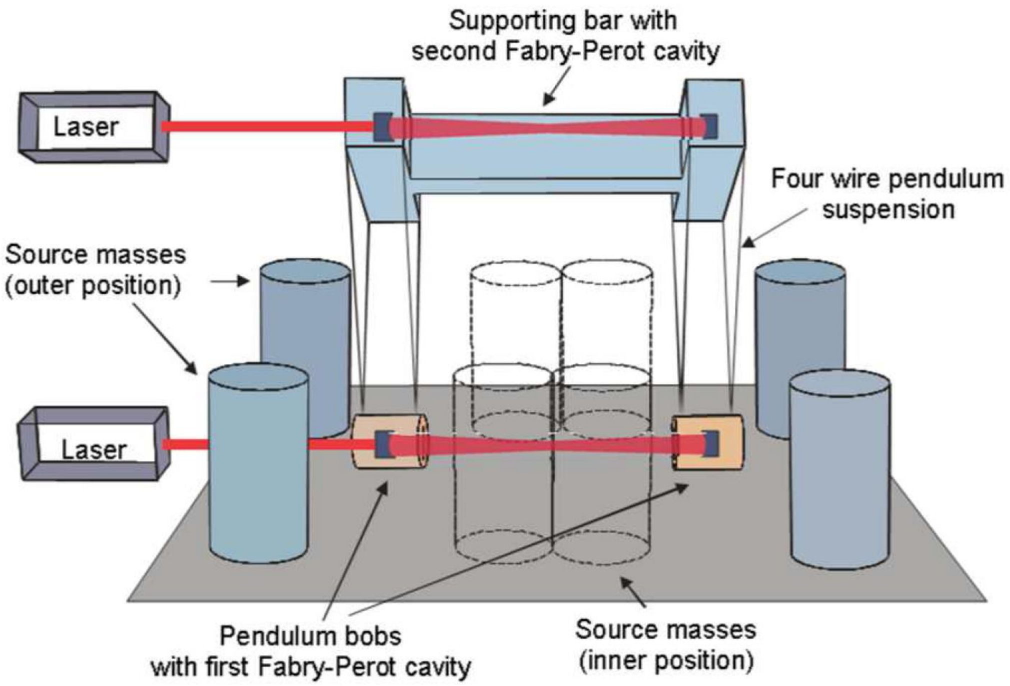


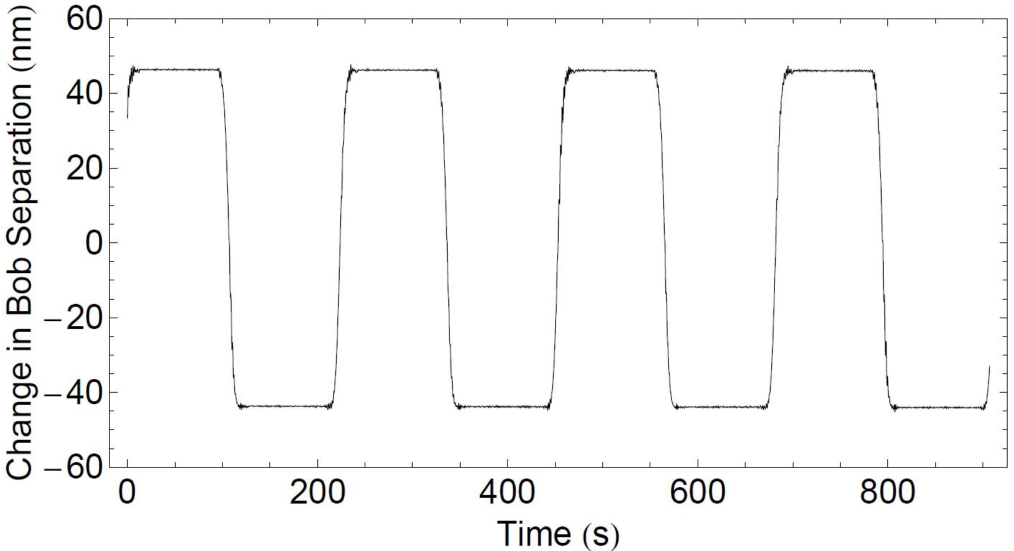
?

To what precision is **the Newton super position principle** validated ?
in nonlinear theories like GR and its modifications **?**
Additional forces ?
No definitive relationship between ***G* and the other fundamental constants ?**
Why gravity was alone and away from other physical interactions ?
…

Simple Pendulum Determination of the Gravitational Constant

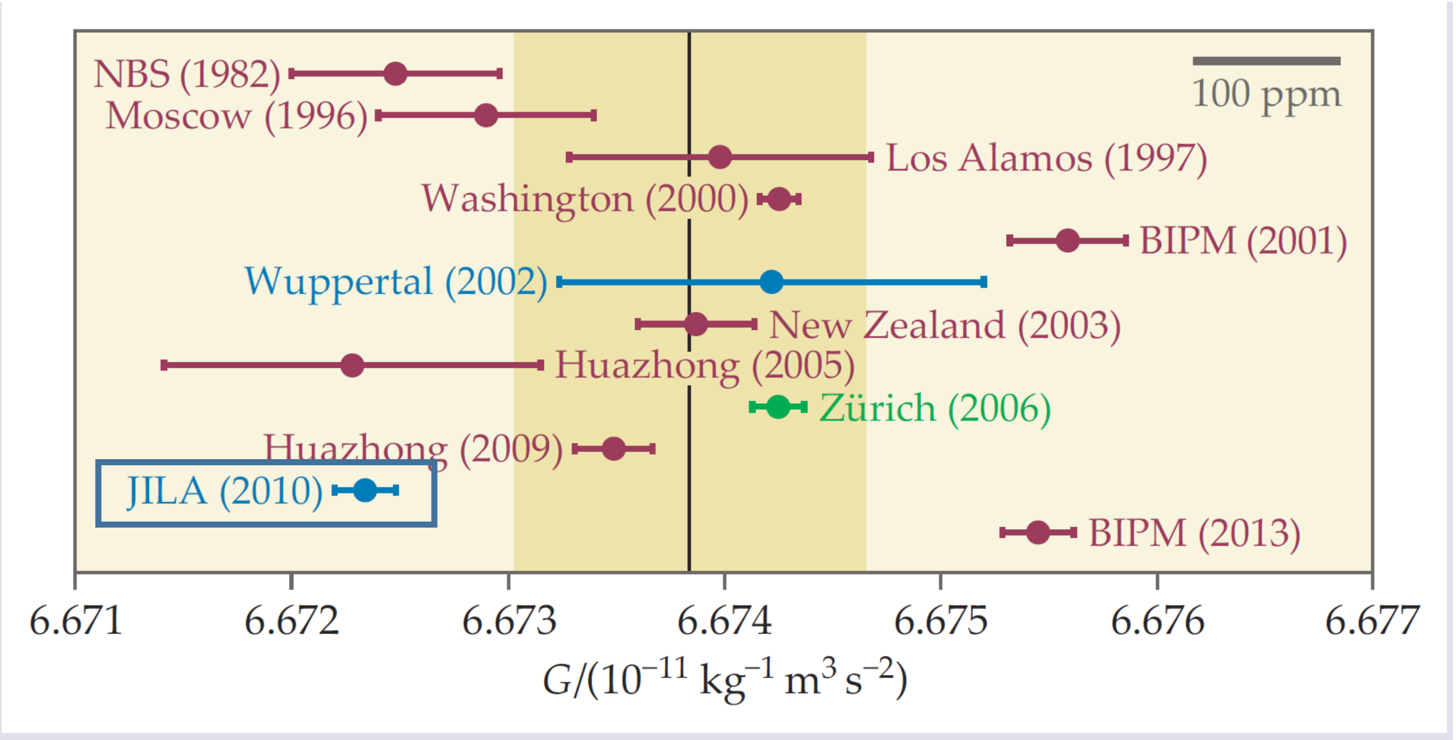
Harold V. Parks and James E. Faller **PRL 105, 110801 (2010)**





$$G = (6.672\,34 \pm 0.000\,14) \times 10^{-11}\ \mathrm{m^3\,kg^{-1}\,s^{-2}}$$

21 ppm

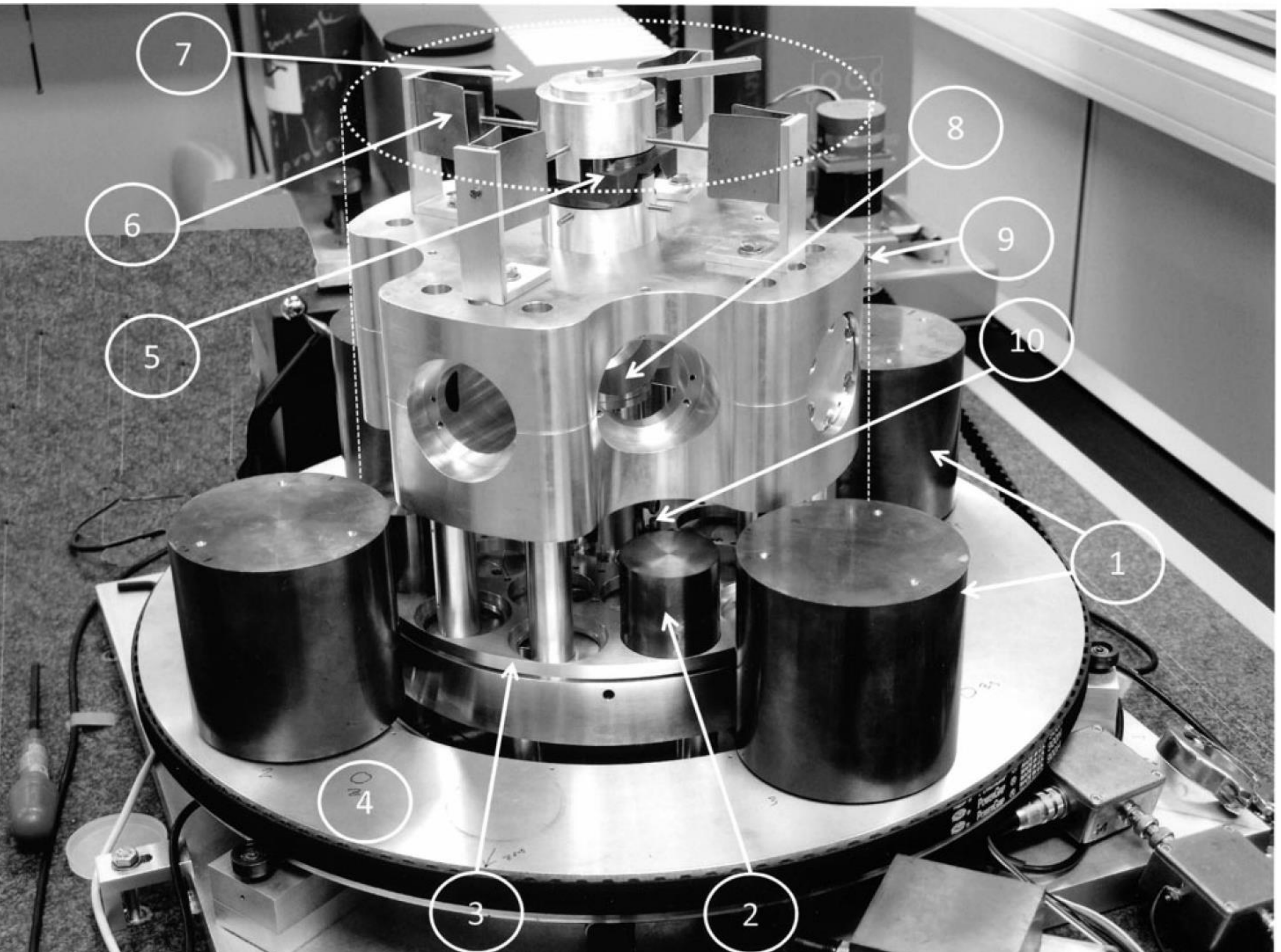


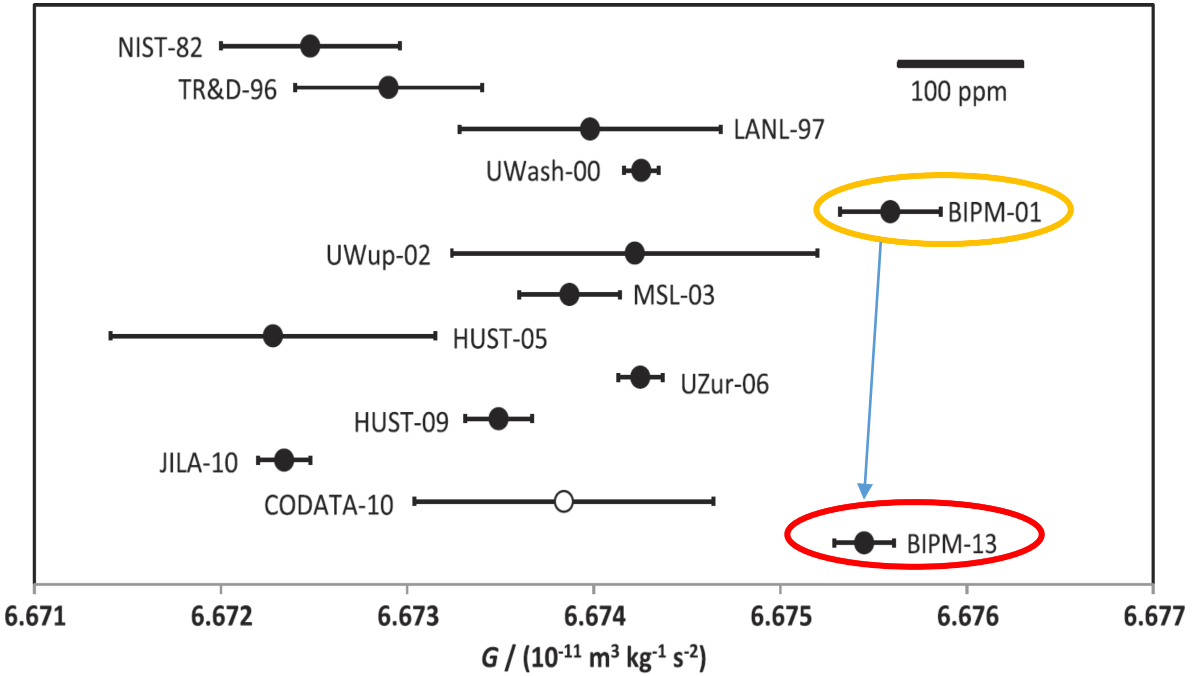
Improved Determination of *G* Using Two Methods

Terry Quinn, Harold Parks, **Clive Speake**, Richard Davis, **PRL 111, 101102 (2013)**

$$G = 6.67545(18) \times 10^{-11}\ \mathrm{m^3\,kg^{-1}\,s^{-2}}$$

27 ppm





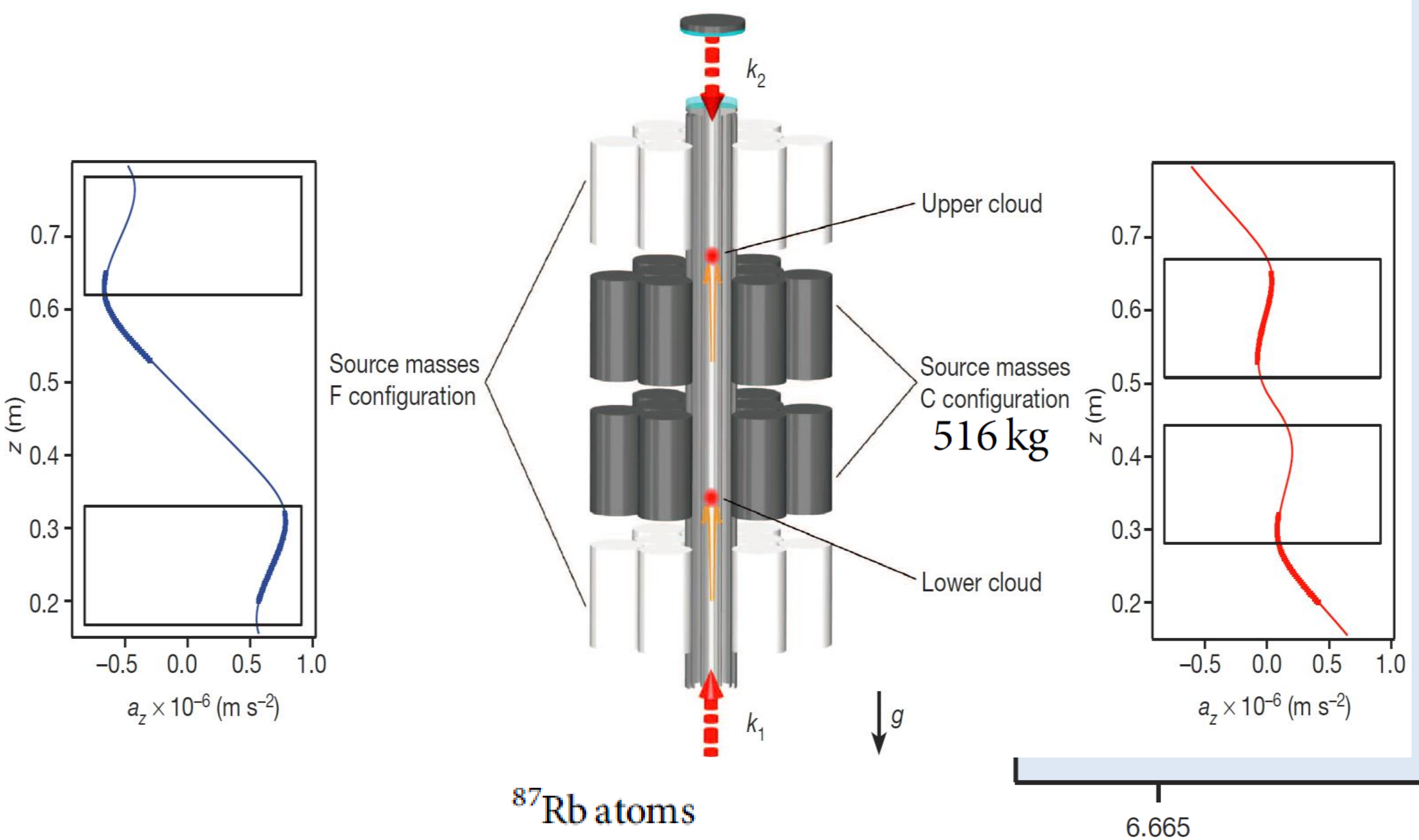
The present result (BIPM-13) compared with recent measurements of G

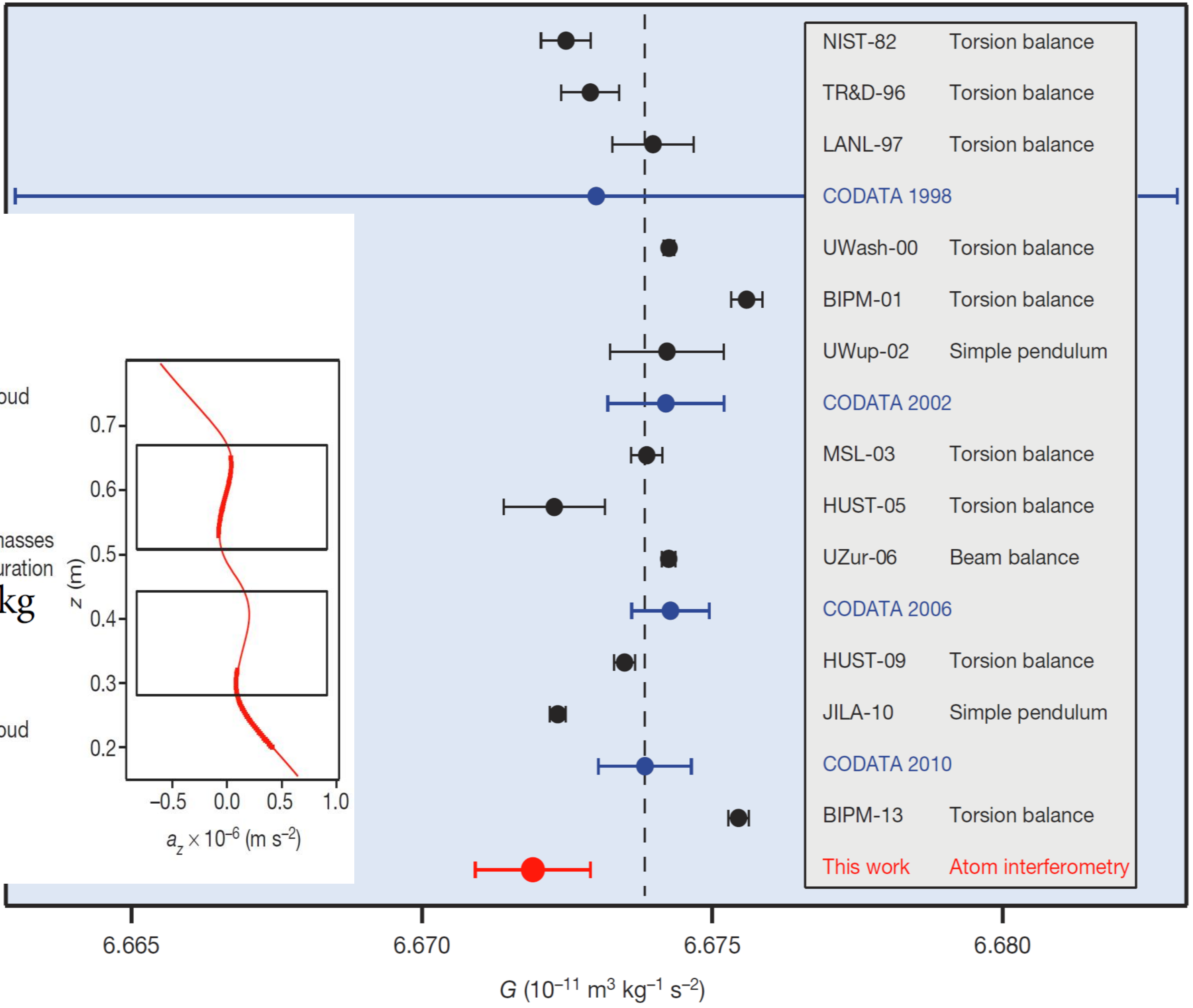
Precision measurement of the Newtonian gravitational constant using cold atoms

G. Rosi, F. Sorrentino, L. Cacciapuoti, M. Prevedelli & G. M. Tino, **NATURE, VOL 510, 26 (2014)**

$$G = 6.67191(99) \times 10^{-11}\ \mathrm{m^3\,kg^{-1}\,s^{-2}}$$

150 ppm





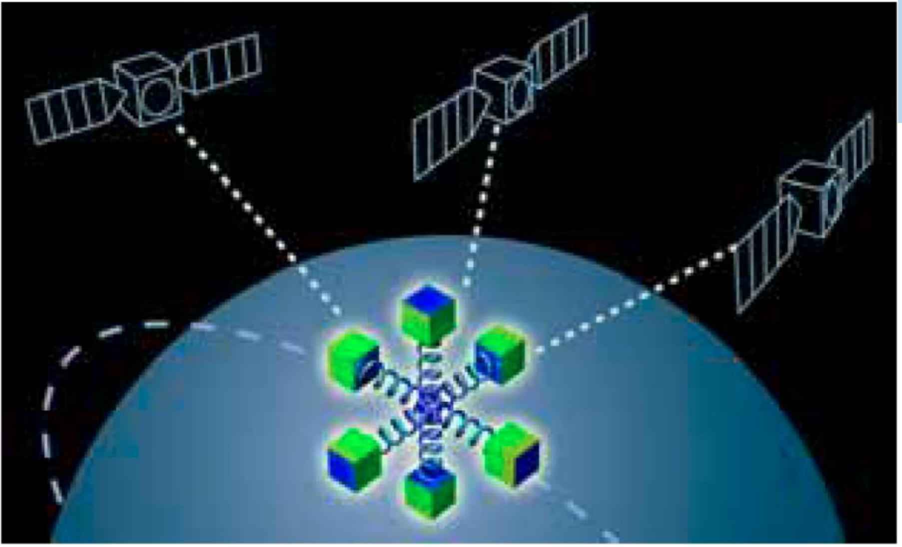
Values of G with altitudes, latitudes, longitudes of the places of the measurements [Recommended by CODATA-2010 G = 6.673 84(80)]

$2\pi/\Omega_{\oplus} \approx 7.2722\times10^{-5}$!?!

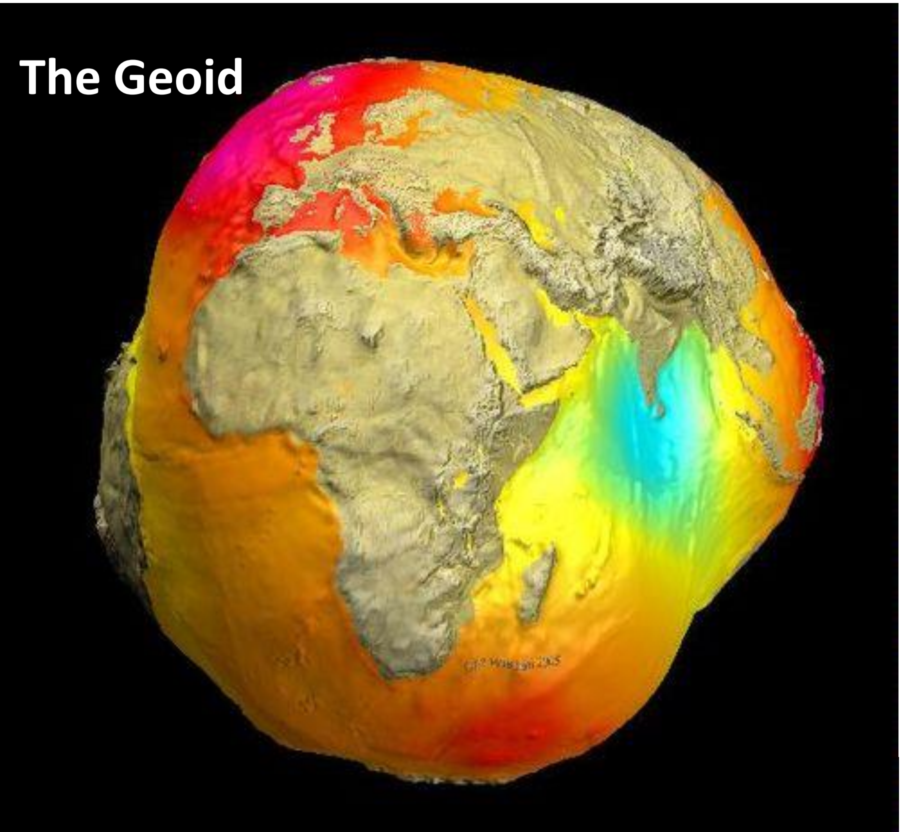
No	Value of G [10^{-11} m^3 kg^{-1} s^{-2}]	Uncertainty	Altitude [m]	Geoid height [m]	Latitude	Longitude
1	6.672 48(43)	6.4×10^{-5}	134.976	-30	39.13	-77.22
2	6.672 9(5)	7.5×10^{-5}	151.785	10	55.76	37.62
3	6.673 98(70)	1.0×10^{-4}	2160.594	-20	35.84	-106.29
4	6.674 255(92)	1.4×10^{-5}	28.618	-30	47.66	-122.30
5	6.675 59(27)	4.0×10^{-5}	89.050	50	48.82	2.21
6	6.674 22(98)	1.5×10^{-4}	259.822	50	51.25	7.15
7	6.673 87(27)	4.0×10^{-5}	0.000	20	-41.21	174.91
8	6.672 28(87)	1.3×10^{-4}	38.477	10	30.51	114.41
9	6.674 25(12)	1.9×10^{-5}	447.939	50	47.37	8.55
10	6.673 49(18)	2.7×10^{-5}	38.477	10	30.51	114.41
11	6.672 34(14)	2.1×10^{-5}	1640.384	-20	40.01	-105.27
12	6.67545(18)	2.7×10^{-5}	89.050	50	48.82	2.21
13	6.67191(99)	1.48×10^{-4}	50.875	50	43.78	11.26

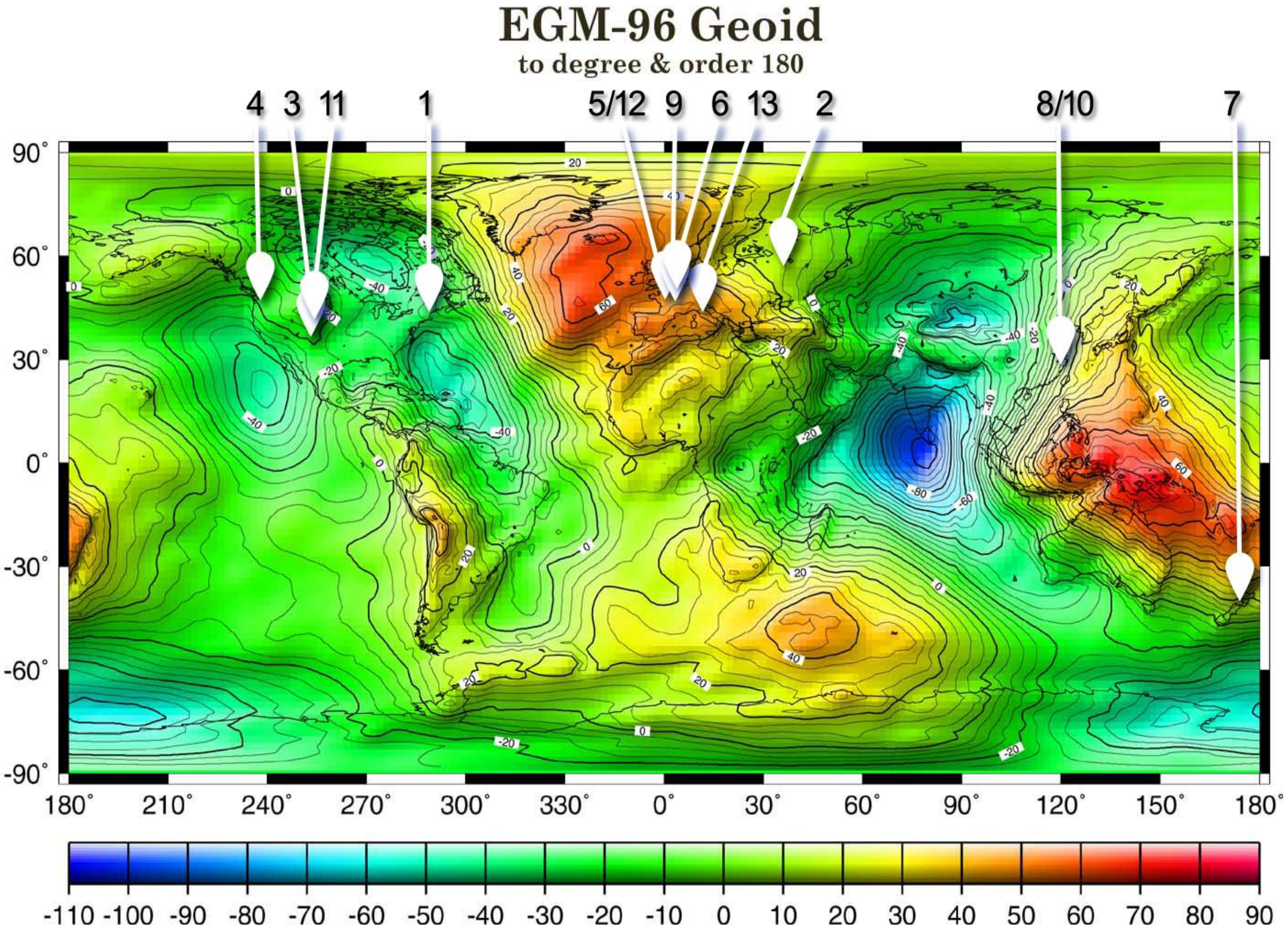
1. NIST(Gaithersburg, Maryland)-1982
2. TR&D(Moskow)-1996
3. LANL(Los Alamos)-1997
4. UWashington-2000
5. BIPM(Paris)-2001
6. UWuppertal(Deuchland)-2002
7. MSL(Wellington, NZ)-2003
8. HUST(Wenzhou, CHINA) -2005
9. UZurich-2006
10. HUST-2009
11. JILA(CU-Boulder, Colorado)-2010
12. BIPM(Paris)-2013
13. UFirenze, INFN-2014

The GOCE Satellite Data



Measurement principle of the GOCE Satellite





SCIENCE VOL 305 23 JULY 2004

GRACE Measurements of Mass Variability in the Earth System

Byron D. Tapley,[1] Srinivas Bettadpur,[1] John C. Ries,[1]* Paul F. Thompson,[1] Michael M. Watkins[2]

Monthly gravity field estimates made by the twin Gravity Recovery and Climate Experiment (GRACE) satellites have a geoid height accuracy of 2 to 3 millimeters at a spatial resolution as small as 400 kilometers. The annual cycle in the geoid variations, up to 10 millimeters in some regions, peaked predominantly in the spring and fall seasons. Geoid variations observed over South America that can be largely attributed to surface water and groundwater changes show a clear separation between the large Amazon watershed and the smaller watersheds to the north. Such observations will help hydrologists to connect processes at traditional length scales (tens of kilometers or less) to those at regional and global scales.

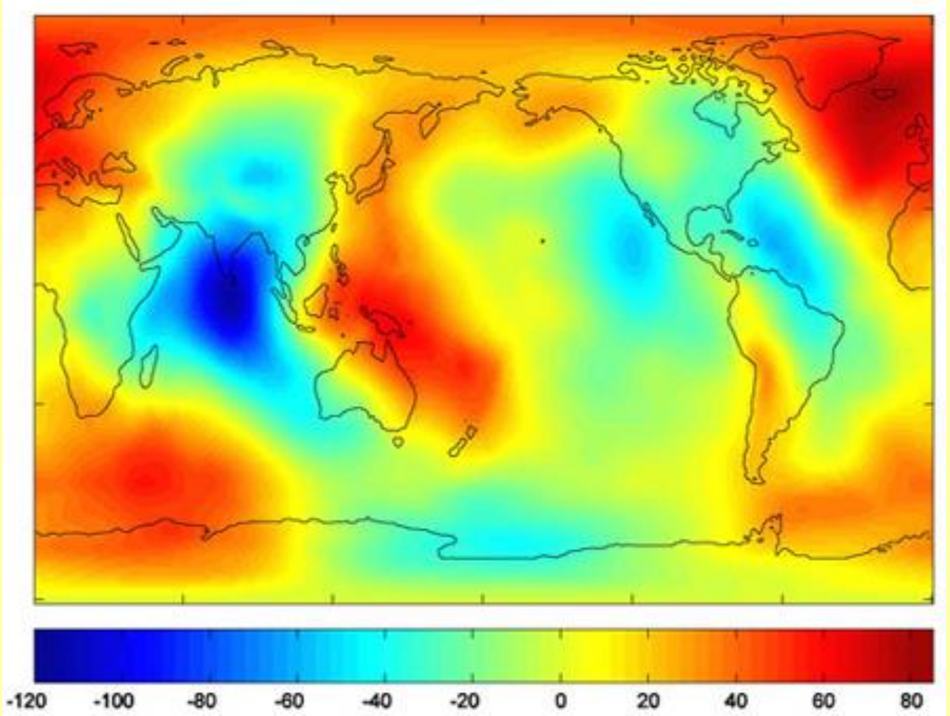


Image credit: University of Texas Center for Space Research and NASA

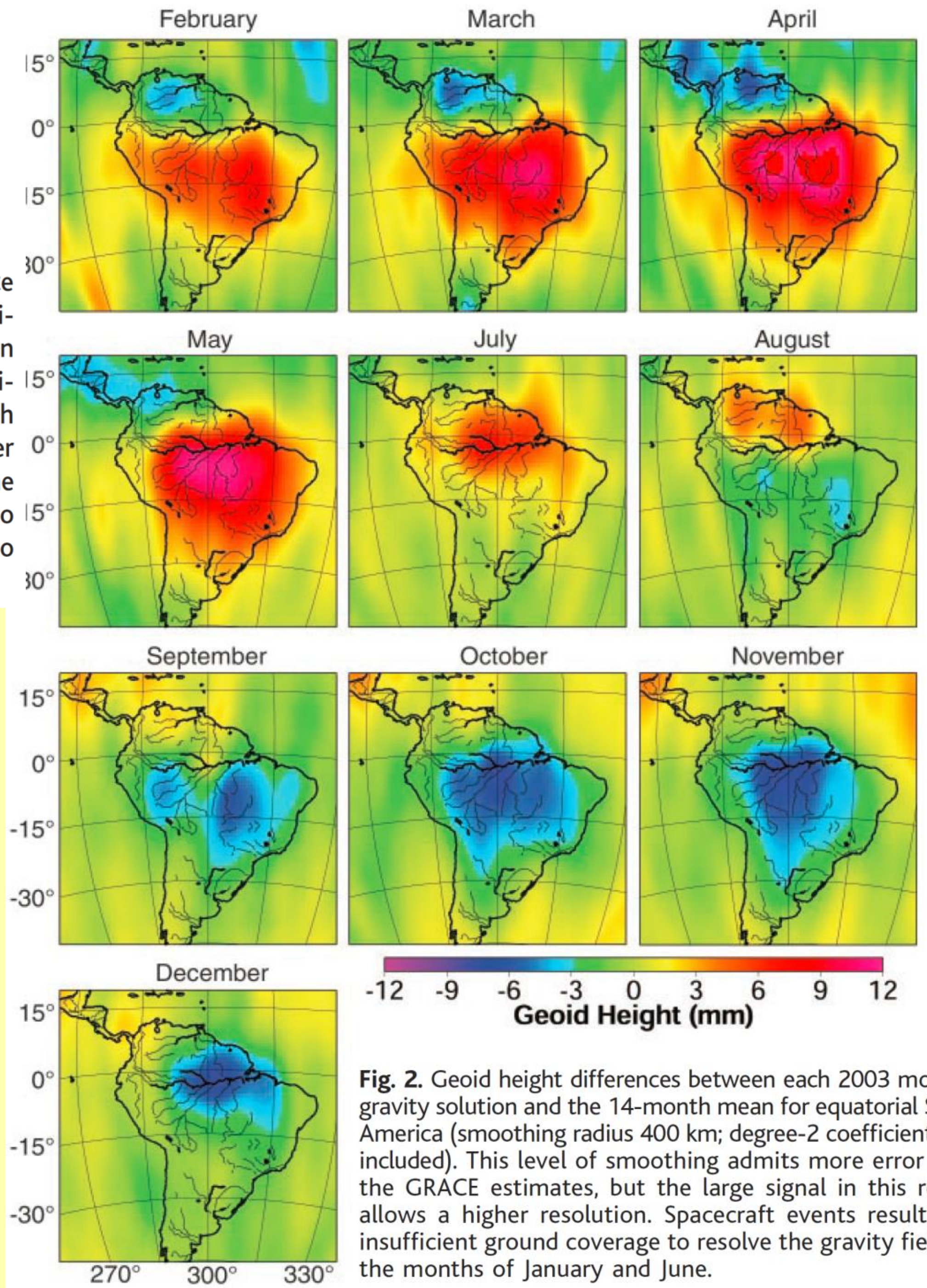


Fig. 2. Geoid height differences between each 2003 monthly gravity solution and the 14-month mean for equatorial South America (smoothing radius 400 km; degree-2 coefficients not included). This level of smoothing admits more error from the GRACE estimates, but the large signal in this region allows a higher resolution. Spacecraft events resulted in insufficient ground coverage to resolve the gravity field for the months of January and June.

Correlations:

[G, altitude = H, geoid height = gh, latitude = lat, longitude = long]

corr(G,H) = –.094	corr(gh,H) = –.412	corr(H,lat) = .085	corr(lat,long) = –.580
corr(G,gh) = .397	corr(gh,lat) = .090	corr(H,long) = –.553	
corr(G,lat) = .071	corr(gh,long) = .437		
corr(G,long) = –.035			

Weighted Correlations:

$corr(G,gh)_H$ = .453	$corr(gh,H)_G$ = –.412	$corr(H,long)_G$ = –.553
$corr(G,gh)_{lat}$ = .407	$corr(gh,H)_{lat}$ = –.396	$corr(H,long)_{gh}$ = .751
$corr(G,gh)_{long}$ = .492	$corr(gh,H)_{long}$ = .935	$corr(H,long)_{lat}$ = –.619

- $2\pi/\Omega_\oplus \approx 7.2722\times10^{-5}$!?!
- No significant correlations are seen in the available data.
- Much more amount of precise G-data are needed for a good statistics.
- Do the observed variations of G depend only on the equipment?
- Mobile precise measurements of G with the same equipment at different places may be useful.
- May the observed variations of G reflect some unknown laws of Nature?

The Fifth Force (The first time period: 1971-1992)

See F. D. Stacey, G. J. Tuck, G. I. Moore, S. C. Holding, B. D. Goodwin, R. Zhou, Rev. Mod. Phys. **59**, 157- 174 (1987)

The origin: Nambu-Goldstone massless dilaton Φ

Nambu, Y., *Phys. Rev. Lett.*, **4**, 380 (1960).
Goldstone, J., *Nuovo Cimento*, **19**, 154 (1961).

A model is proposed which allows a dilaton to show up in a possible non-Newtonian part of the gravitational force. By examining the available observational facts it can be shown that the force-range of the additional force, if it exists, will be either between 10 m and 1 km or smaller than ~1 cm.

$$m_\Phi > 0$$

1. Fujii, Y. *Nature phys. Sci.* **234**, 5–7 (1971).
2. Fujii, Y. *Ann. Phys.* **69**, 494–521 (1972).
3. Fujii, Y. *Phys. Rev.* **D9**, 874–876 (1974).
4. Fujii, Y. *Gen. Relat. Gravitat.* **6**, 29–34 (1975).

$$V(r) = -\frac{3}{4} G \frac{1}{r}\left(1 + \frac{1}{3} e^{-\mu r}\right)$$

$$V = -\frac{G_\infty m}{r}\left[1 - ae^{-r/v} + be^{-r/s}\right]$$

Frank D. Stacey, Gary J. Tuck, and G. Ian Moore, GEOPHYSICAL CONSIDERATIONS IN THE FIFTH FORCE CONTROVERSY, **JOURNAL OF GEOPHYSICAL RESEARCH, 93** ,575-10,587, (1988)

Many scalar and vector particles: G. W. Gibbons & B. F. Whiting *Nature Vol. 291 25 June 1981*

$$V = -G_\infty \frac{mm'}{r}\left(1 + \sum_{i=1}^{N} \alpha_i \exp(-r/\lambda_i)\right)$$

$$G_0 = G_\infty\left(1 + \sum_{i=1}^{i=N} \alpha_i\right)$$

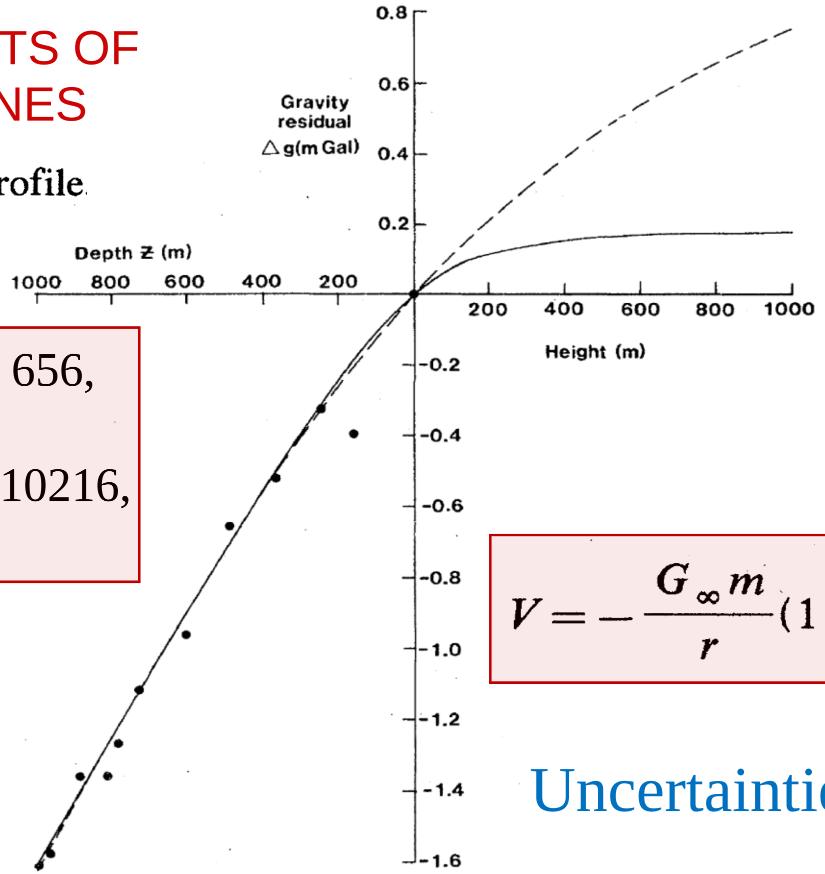
The effective gravitational constant $G(r)$ is given by

$$G(r) = G_\infty(1 + \alpha(1 + r/\lambda)\exp(-r/\lambda))$$

Some results for fifth force (1971-1992)

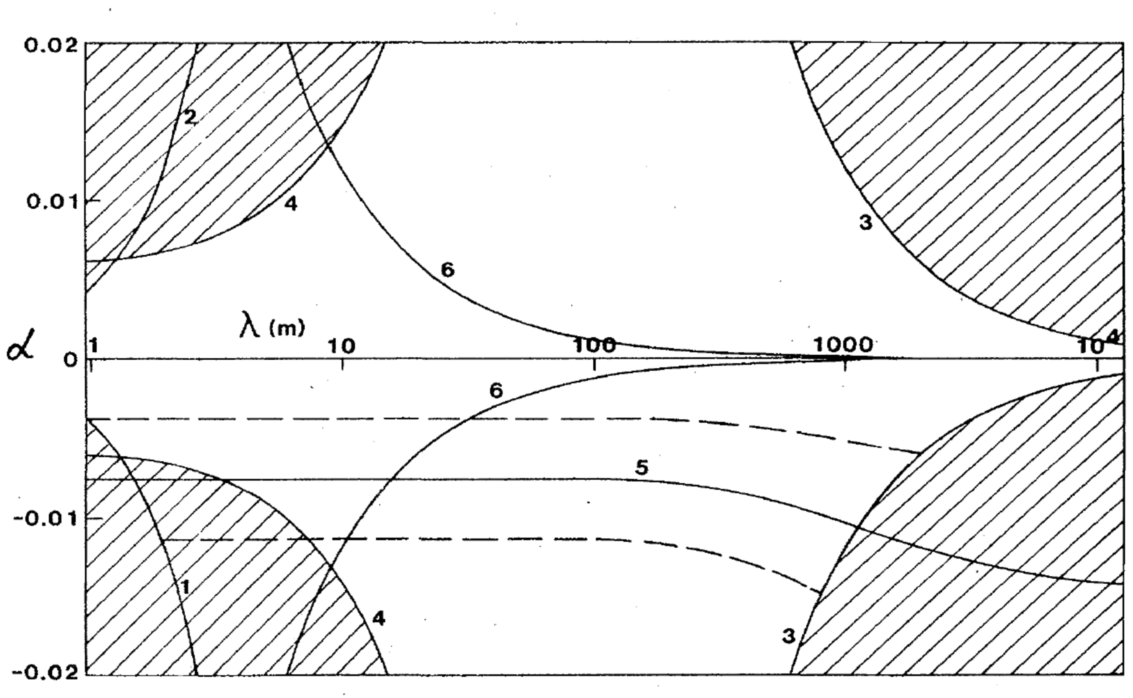
MEASUREMENTS OF GRAVITY IN MINES

the Hilton mine profile



Solid: α = -0.007 656, λ = 200 m;
Dashed: α = -0.010216, λ= 1000 m.

$$V = -\frac{G_\infty m}{r}(1+\alpha e^{-r/\lambda})$$

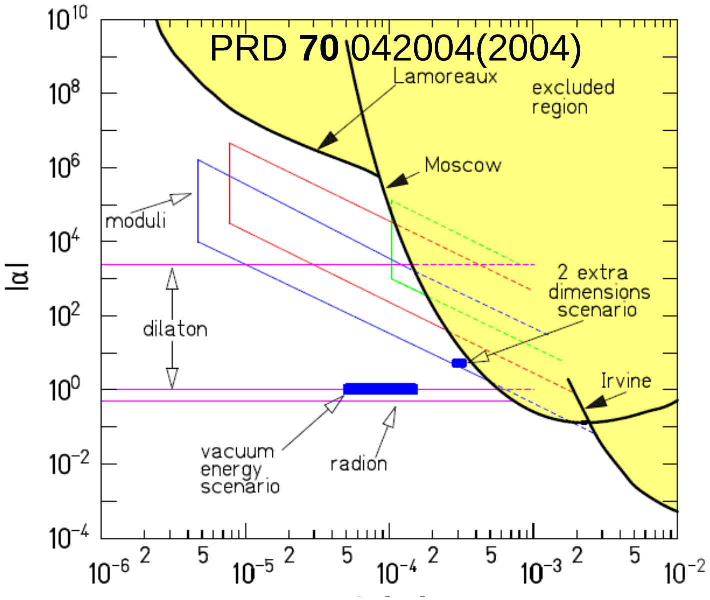


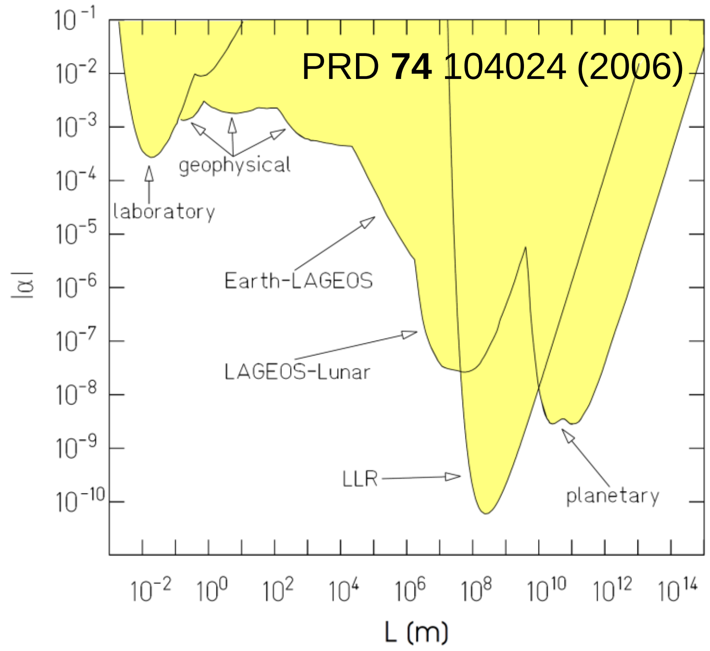
Uncertainties of the measurements in water and in air (towers).

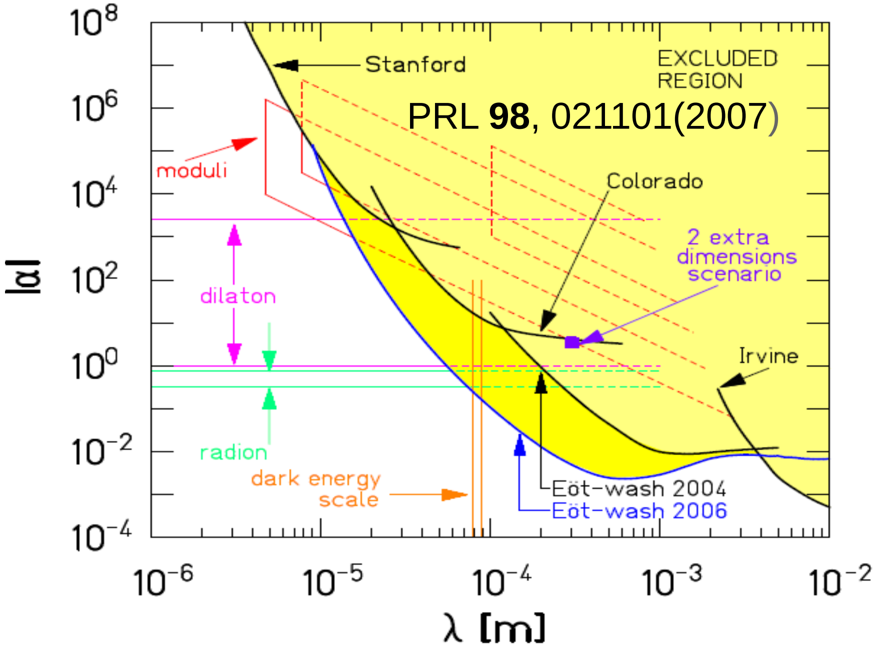
Yeong **E. KIM , David J. KLEPACKI ,** William J. HINZE, PHYSICS LETTERS B, 195 (1987) :
In summary, we find that the geophysical determination of the gravitational constant is extremely sensitive to the assumed global average mass density profile and that the mine data of Holding et al. are consistent with the laboratory value of G.

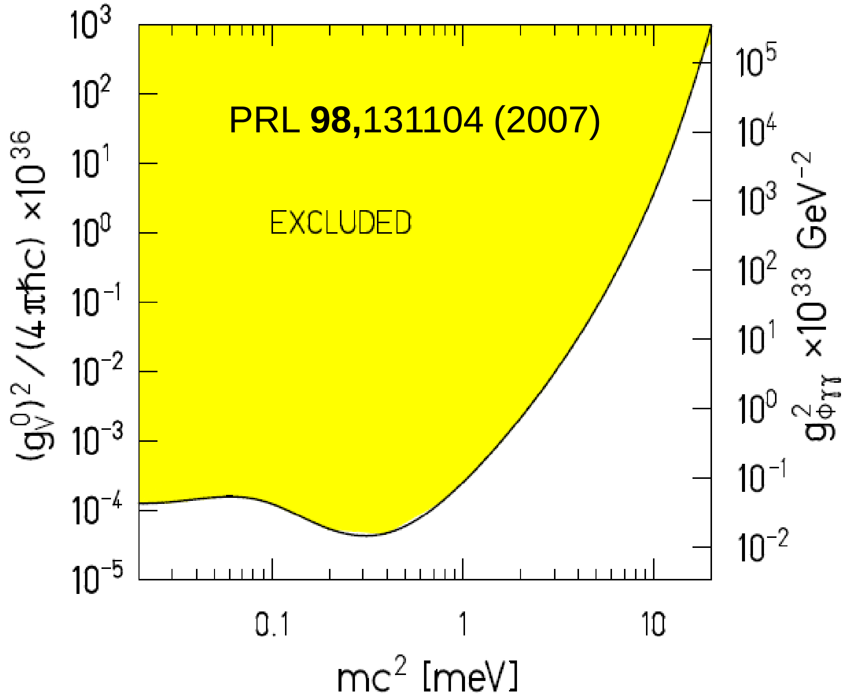
Yeong E. KIM, PHYSICS LETTERS B, **216,** 212 (1989): Apparent anomalies observed in borehole and seafloor gravity measurements are shown to be attributable to inaccurate implementation of Newton's gravitational law using **inadequate earth models**.

Sub-millimeter Tests of the Gravitational Inverse-square Law (2000-2009)

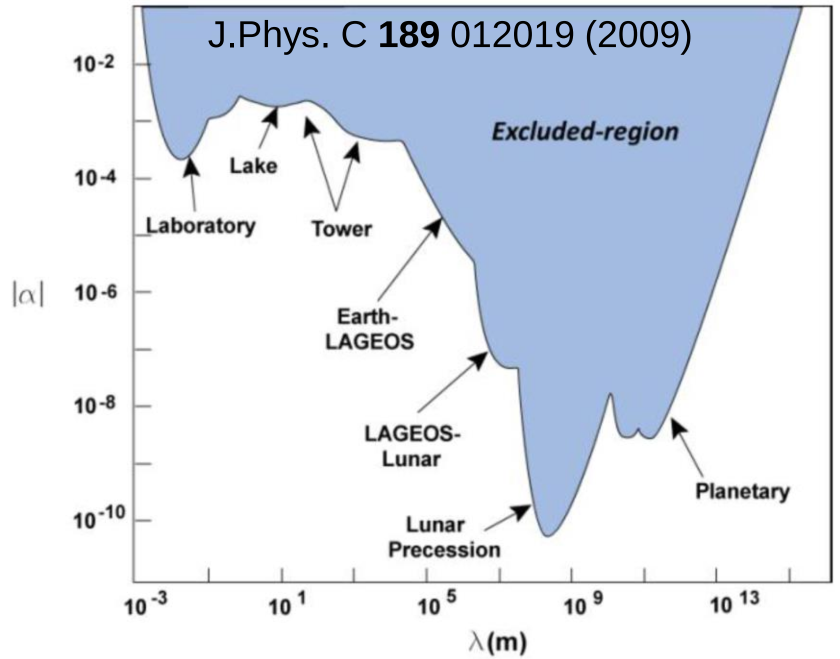








Only weak limitations on the two parameters α and λ were obtained so far



The Dark Energy (The *Universe expansion*) G

$$R_{\mu\nu} - \frac{1}{2} R\, g_{\mu\nu} + \Lambda\, g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}\,,$$ ← A. Einstein,1917

$\Lambda = 8\pi\ (G/c^2)\ \rho_\Lambda$

$\rho_\Lambda = \Omega_\Lambda\, \rho_{crit}$, $\rho_{crit} = \frac{3\,H^2}{8\pi G}$

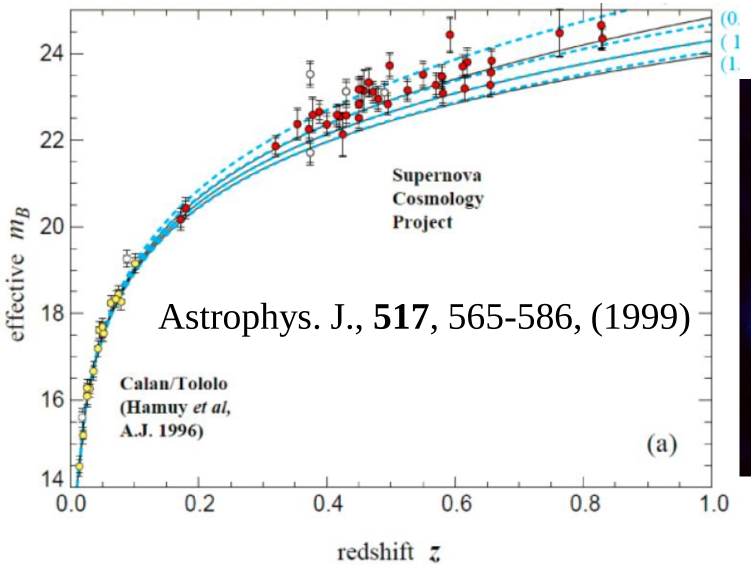
$\Lambda = \Omega_\Lambda \frac{3\,H^2}{c^2}$

$\Omega_\Lambda = 0.6825$
h = 0.6711

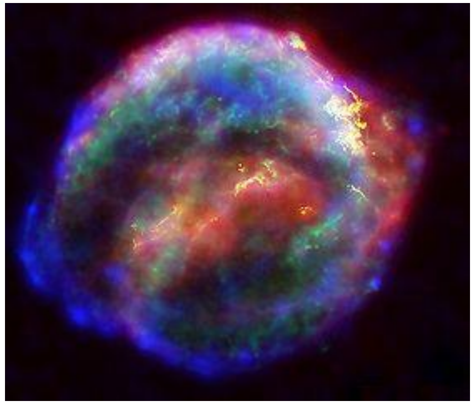
Planck 2013

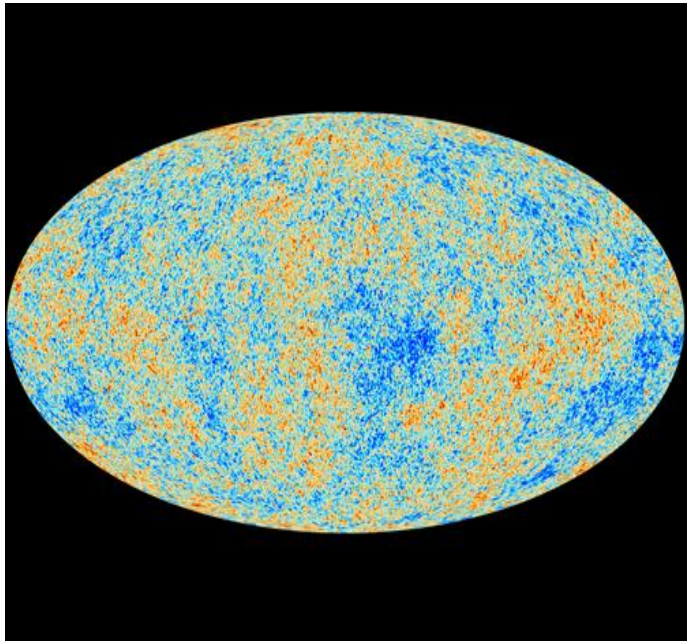
$\Lambda = 1.0776 \times 10^{-46}\ km^{-2}$

[Einstein and Hubble at Mt.Wilson in 1931, Caltech Archives]

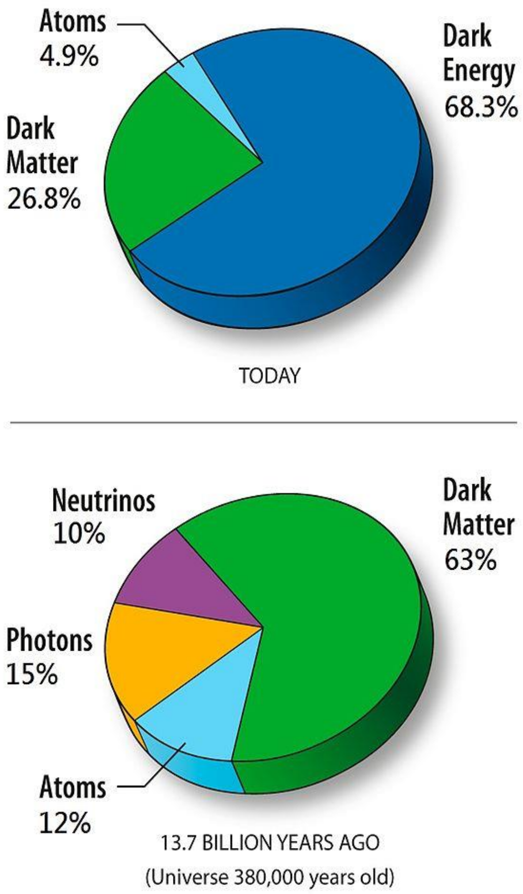


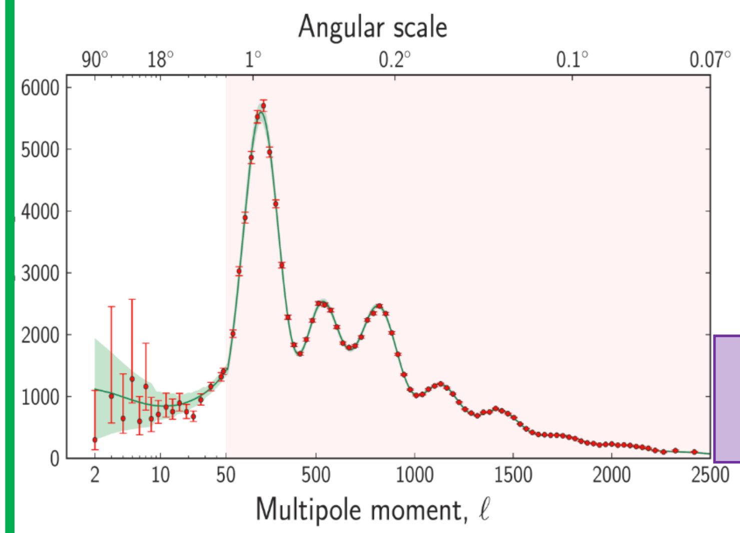
SN 1604HST/NASA/ESA





CMB data (2013):





$\Omega_\Lambda = 0.6825$ (best fit)

The Dark Matter (*The missing mass problem*)

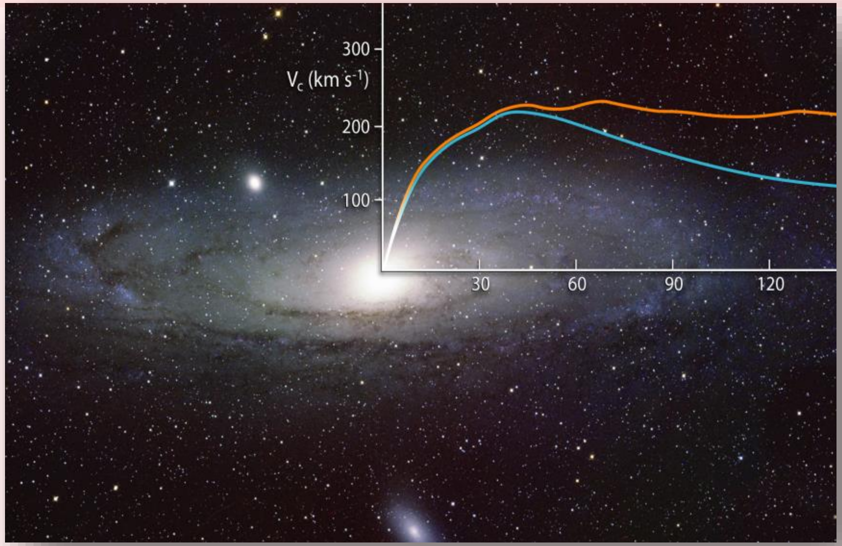
Mass distribution in Abell 1689, HST **2008**

First:

J. Oort (1932),

F. Zwicky (1932)

Rotation curve of a typical spiral galaxy

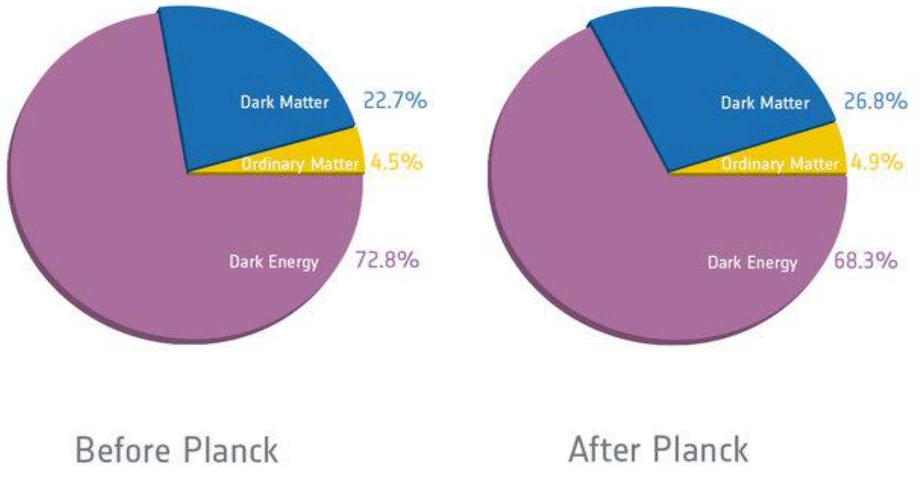


First:

H. W. Babcoc (1939)

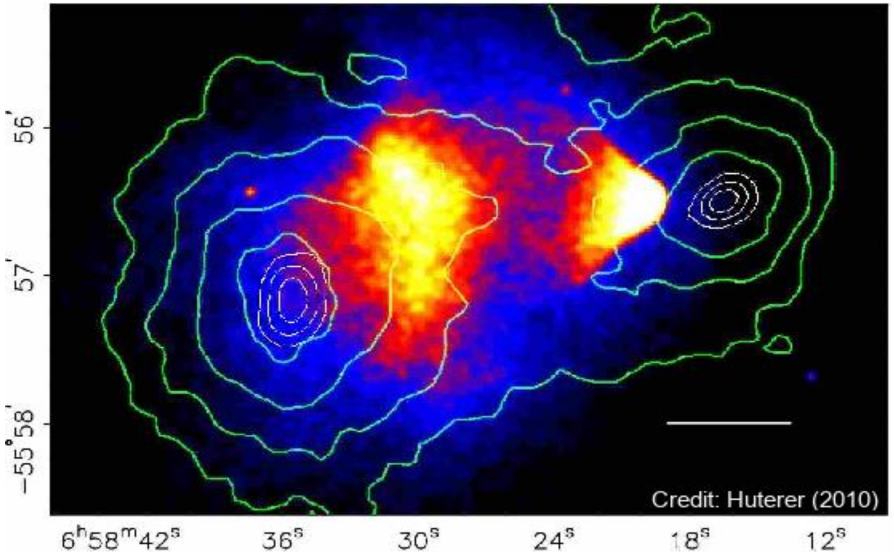
CMB data (2013):

$$\Omega_m = 0.315^{+0.016}_{-0.018}$$



A DIRECT EMPIRICAL PROOF OF THE EXISTENCE OF DARK MATTER

Merging cluster 1E0657–558



D. Clowe, et al., ApJ, 648, L109 (2006)

The most important lesson:

The clear understanding that
the Einstein general relativity (GR) and
Standard particle model (SPM)
are insufficient to explain
all observed phenomena in the Nature.

There exist three possible ways for further development:

1) To add some new content of the Universe beyond the SPM, like **dark matter** and **dark energy**.

2) To change the theory of gravity.

3) Some mixture of these two possibilities is not excluded by the current observational data.

The Minimal Dialtonic Gravity (MDG)

Proposed and studied in:

O'Hanlon, PRL **29** 137 (1972)
PPF, Mod.PL A, **15** 1077 (2000)
PPF, arXiv:gr-qc/0202074
PPF, Georgieva D., PRD **67** 064016 (2003)
PPF, PRD **87** 0044053 (2013)
PPF, arXiv:1402.2813; arXiv:1411.0242.

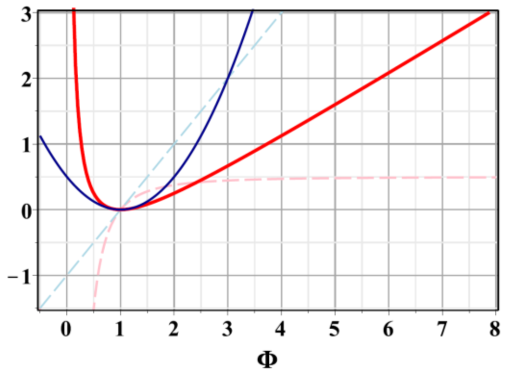
The action in gravi-dilaton sector: (GR: $\Phi \equiv 1,\ g(\Phi) \equiv 1$)

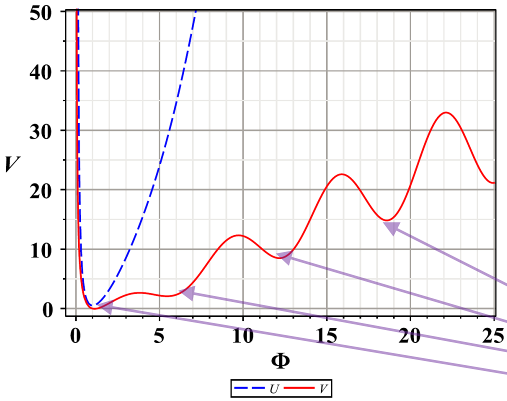
$$\mathscr{A}_{g,\Phi} = \frac{c^3}{16\pi G_N}\int d^4x\sqrt{|g|}\left(\Phi R - 2\Lambda U(\Phi)\right)$$

$$G_N \Rightarrow G(\Phi) = G_N/\Phi = G_N g(\Phi), \quad \Lambda \Rightarrow \Lambda U(\Phi)$$

Dilaton: $\Phi \in (0,\infty)$ ➡

Withholding potentials:





In contrast to **the fifth-force** models, in MDG we have **only one BASIC parameter** – the mass of the dilaton: m_Φ .
However, it is possible to have several of them!

Standard action of matter:

$$\mathcal{A}_{\rm matt} = \frac{1}{c}\int d^4x\sqrt{|g|}\,\mathcal{L}_{\rm matt}(\Psi, \nabla\Psi; g_{\alpha\beta})$$

WEP respected

Basic Equations:

$$\Phi\hat{R}_\alpha^\beta + \widehat{\nabla_\alpha\nabla^\beta}\Phi + 8\pi\hat{T}_\alpha^\beta = 0,$$

$$\Box\Phi + \Lambda V'(\Phi) = \frac{8\pi}{3}T.$$

$$\nabla_\nu T_\mu^\nu = 0.$$

$$\hat{X}_\alpha^\beta = X_\alpha^\beta - \tfrac{1}{4}X\delta_\alpha^\beta$$

$$V'(\Phi) = \tfrac{2}{3}\left(\Phi U'(\Phi) - 2U(\Phi)\right)$$

The basic equations of Static Spherically Symetric Solutions in MDG

***PPF:* arXiv:1402.281**
PoS (FFP14) 080

Generalized TOV equations:

$$
\begin{aligned}
m' &= 4\pi r^2 \epsilon_{eff}/\Phi, \\
\Phi' &= -4\pi r^2 p_{_\Phi}/\Delta, \\
p'_{_\Phi} &= -\frac{p_{_\Phi}}{r\Delta}\left(3r - 7m - \frac{2}{3}\Lambda r^3 + 4\pi r^3 \frac{\epsilon_{eff}}{\Phi}\right) - \frac{2}{r}\epsilon_{_\Phi}, \\
p' &= -\frac{p+\epsilon}{r}\frac{m + 4\pi r^3 p_{eff}/\Phi}{\Delta - 2\pi r^3 p_{_\Phi}/\Phi},
\end{aligned}
$$

+ boundary conditions

$$
\begin{aligned}
\epsilon_{eff} &= \epsilon + \epsilon_{_\Lambda} + \epsilon_{_\Phi}, \\
p_{eff} &= p + p_{_\Lambda} + p_{_\Phi}
\end{aligned}
$$

$$
\begin{aligned}
\mathrm{A} &= 4\pi r^2 \\
\Delta(r) &= r - 2m(r) - \frac{1}{3}\Lambda r^3
\end{aligned}
$$

NOVEL Quantities and EOS:

Cosmological energy-density and pressure:

$$\epsilon_\Lambda = \frac{\Lambda}{8\pi}\left(U(\Phi) - \Phi\right)$$
$$p_\Lambda = -\frac{\Lambda}{8\pi}\left(U(\Phi) - \frac{1}{3}\Phi\right)$$

$$-\Lambda U = \left(\epsilon_\Lambda + 3p_\Lambda\right)/2 < 0$$

Dilatonic energy-density and pressure:

$$\epsilon_\Phi = \frac{1}{8\pi}\frac{1}{A}\frac{d}{dl}\left(A\frac{d\Phi}{dl}\right)$$
$$p_\Phi = -\frac{1}{8\pi}\frac{1}{A}\frac{dA}{dl}\frac{d\Phi}{dl}$$

$$ds^2 = \alpha(l)^2 dt^2 - dl^2 - A(l)d^2\Omega$$
$$d^2\Omega = \left(d\theta^2 + \sin^2(\theta)d\phi^2\right)/4\pi$$

Three equations of state:

$$\epsilon_\Lambda = -p_\Lambda - \frac{\Lambda}{12\pi}\Phi\,; \qquad \longleftarrow \text{CEOS}$$

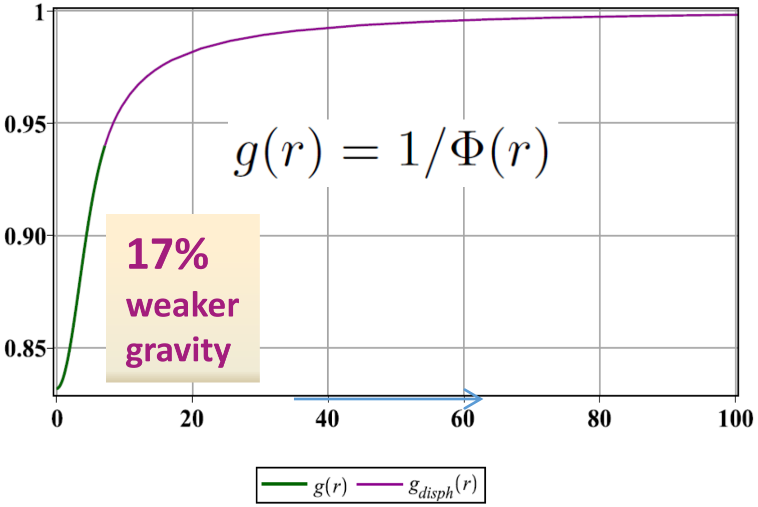
$$\epsilon_\Phi = p - \frac{1}{3}\epsilon + \frac{\Lambda}{8\pi}V'(\Phi) + \frac{p_\Phi}{2}\frac{m + 4\pi r^3 p_{eff}/\Phi}{\Delta - 2\pi r^3 p_\Phi/\Phi}\,; \qquad \longleftarrow \text{DEOS}$$

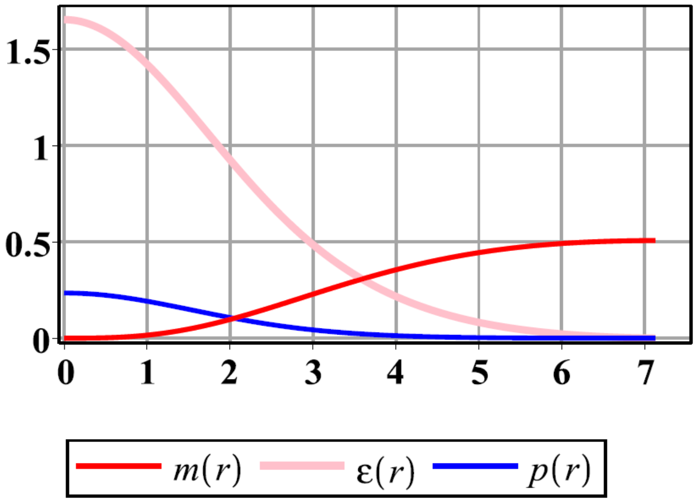
$$\epsilon = \epsilon(p) \qquad \longleftarrow \text{MEOS}$$

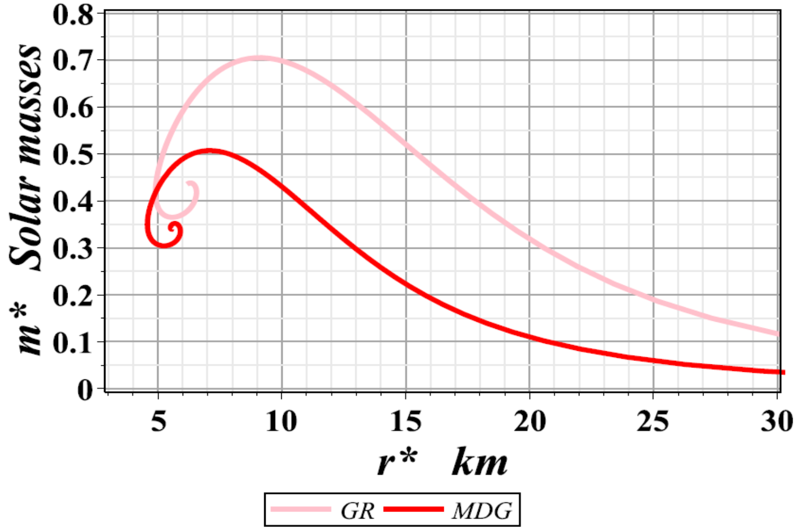
SSSS with Chandrasechkar (1935) & TOV (1939) MEOS (ideal Fermi gas T = 0) for NS in MDG

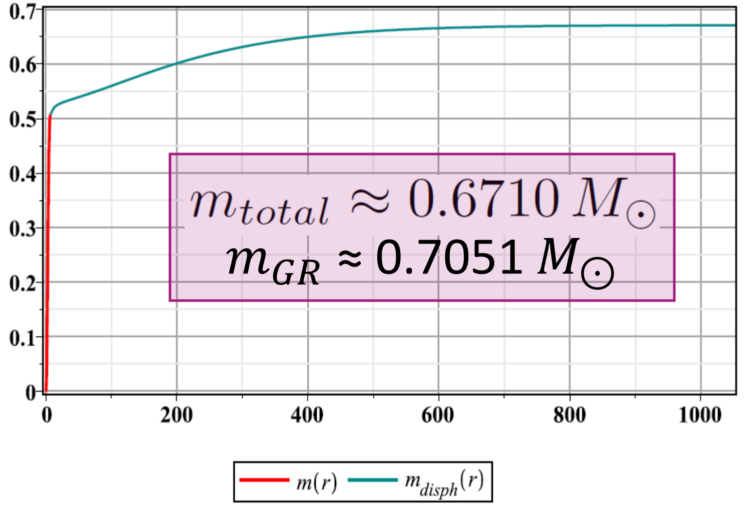
$$\epsilon = \frac{1}{4\pi}(\sinh t - t), \;\; p = \frac{1}{12\pi}(\sinh t - 8\sinh(t/2) + 3t).$$

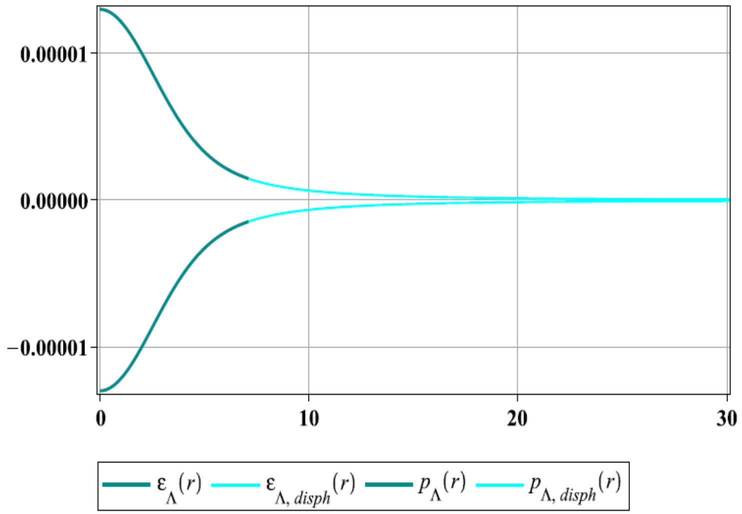
Similar results are obtained for NS with **polytropic** MEOS and $m_{total} \geq 2\, M_{\odot}$

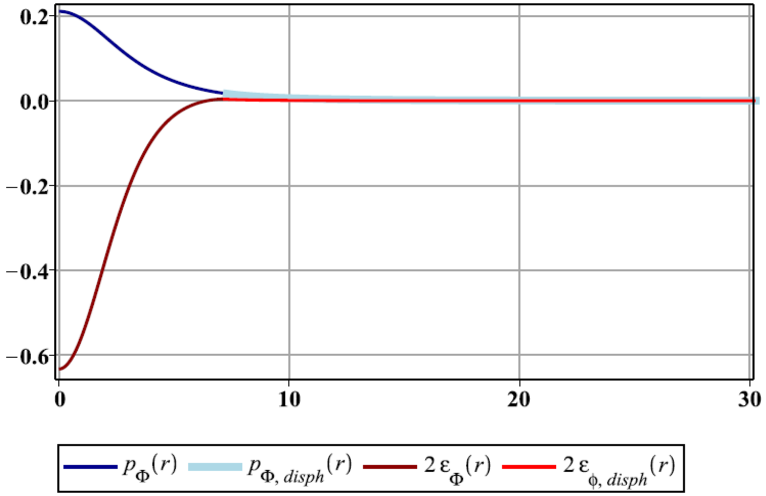












Preliminary Earth Model (PREM)

A.M. Dziewonski and D.L. Anderson, Phys. Earth Planet. Inter. **25**, 297 (1981).

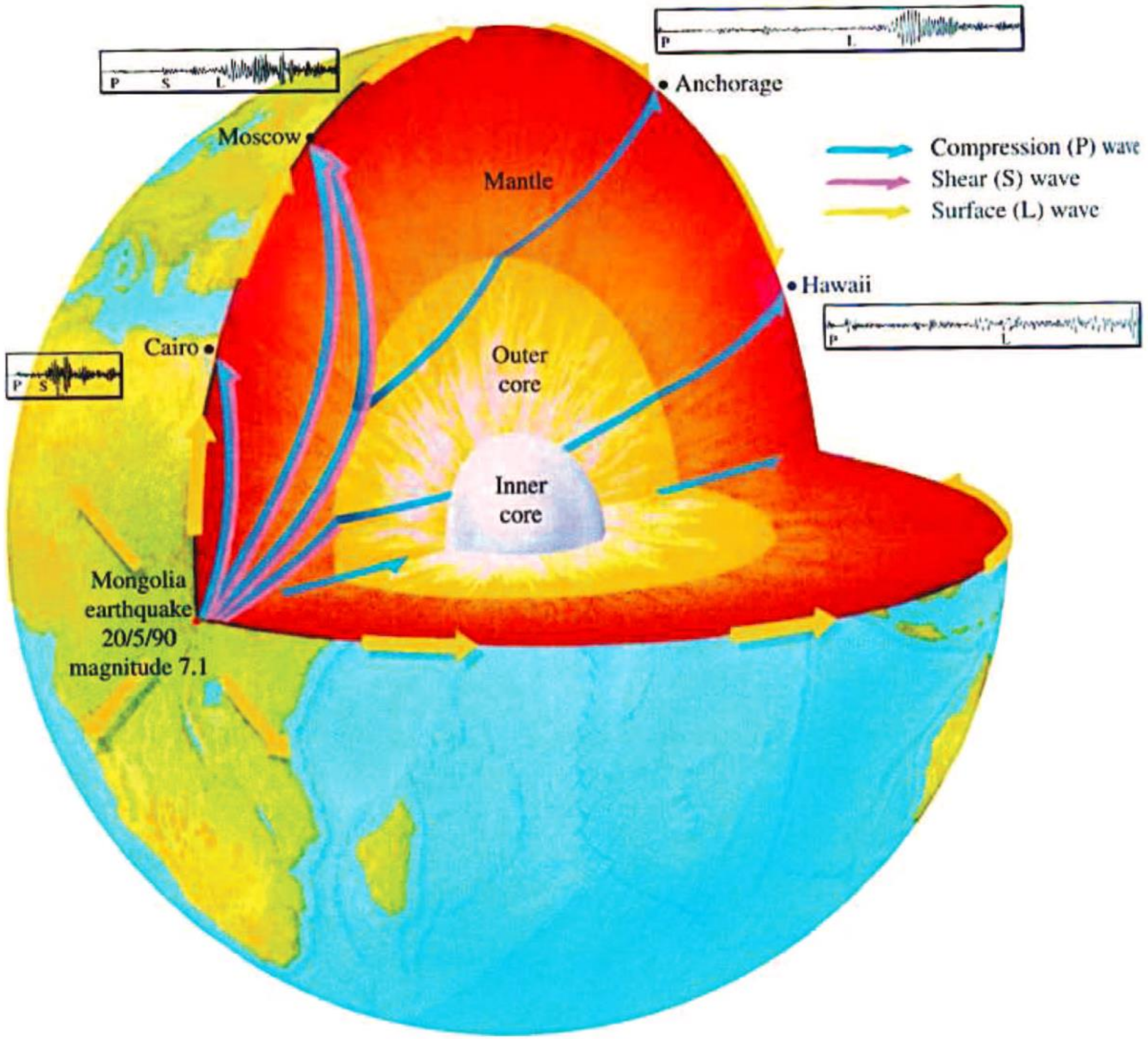
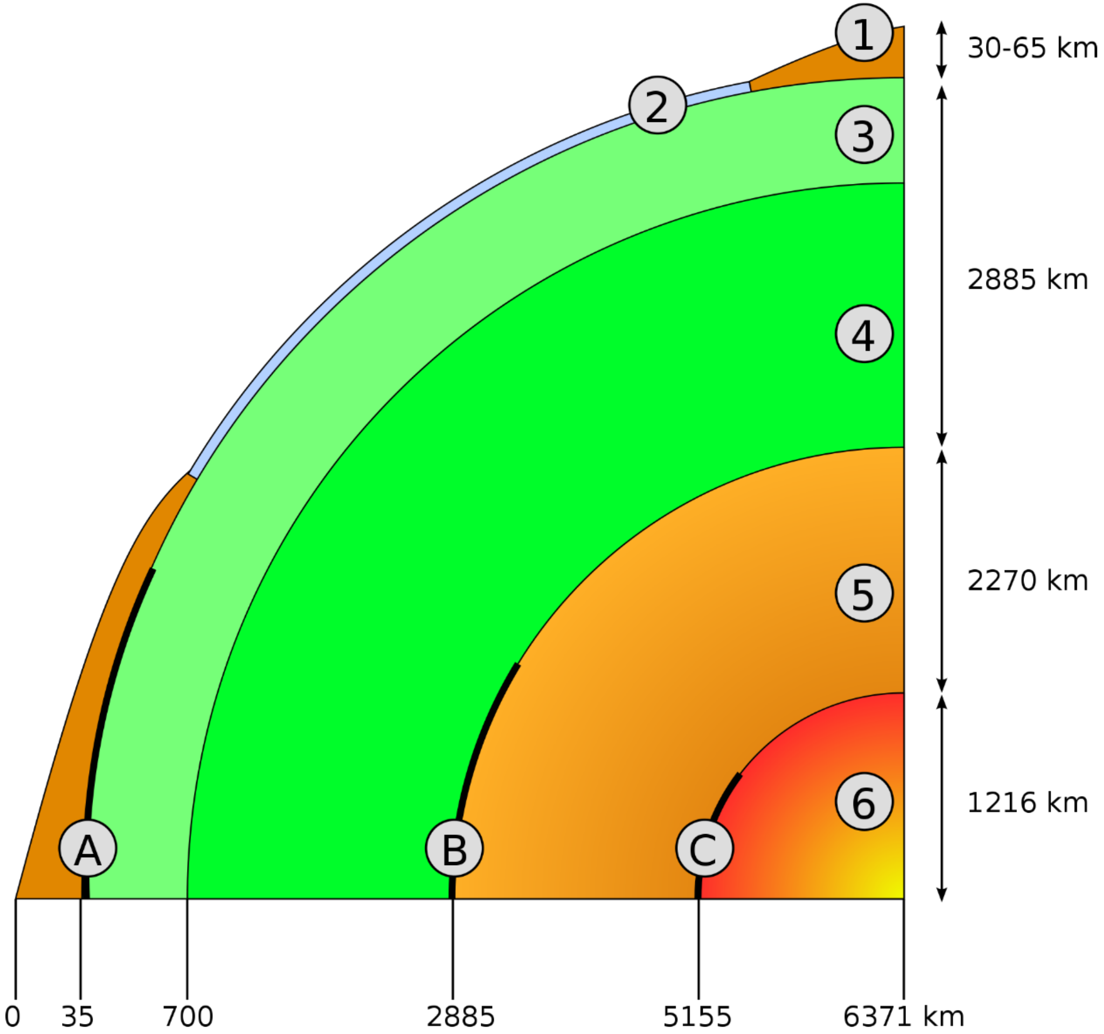


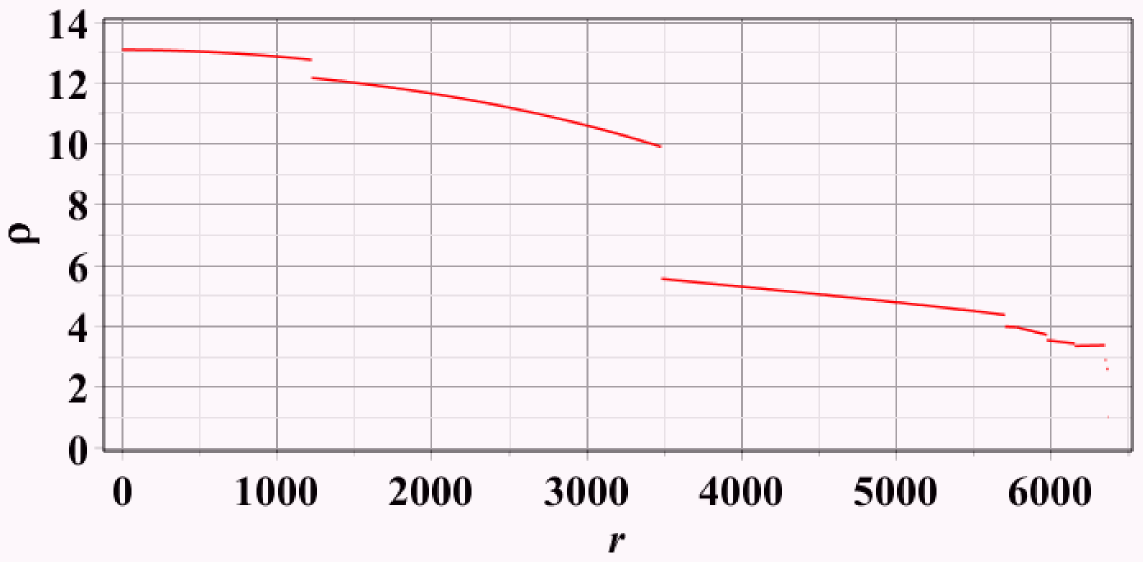
Figure 1. Earth's structure and schematic picture of travelling seismic waves inside the Earth.



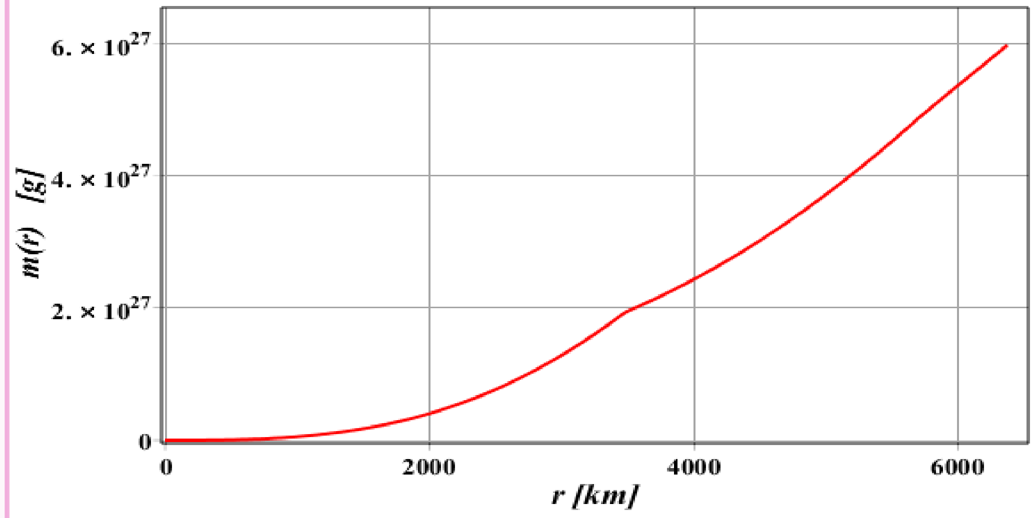
Preliminary Earth Model (PREM)

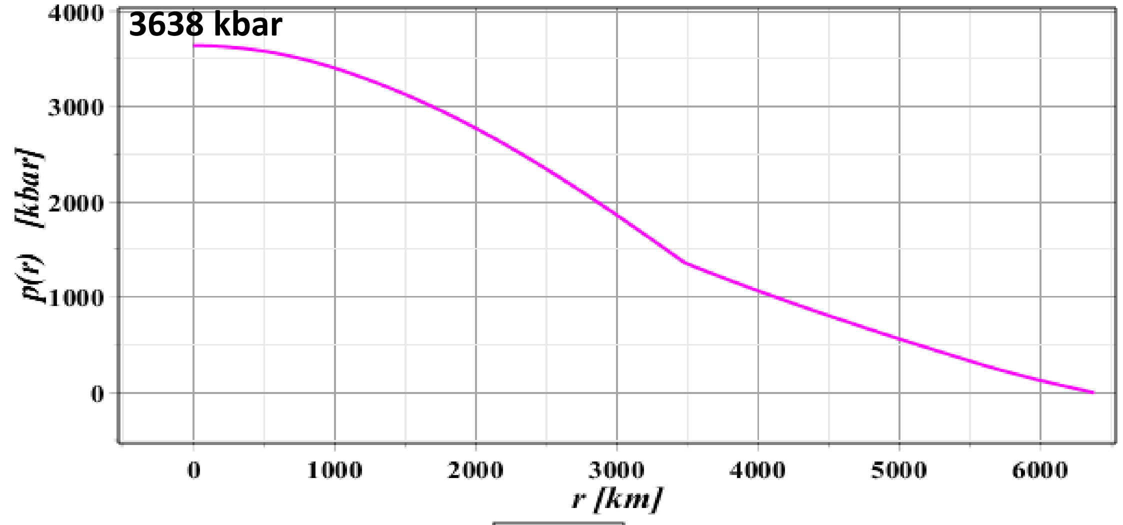
A.M. Dziewonski and D.L. Anderson, Phys. Earth Planet. Inter. **25**, 297 (1981).

The dependence of the mass density $\rho(r)$ $[g/cm^3]$ on the radius r [km] *in PREM*

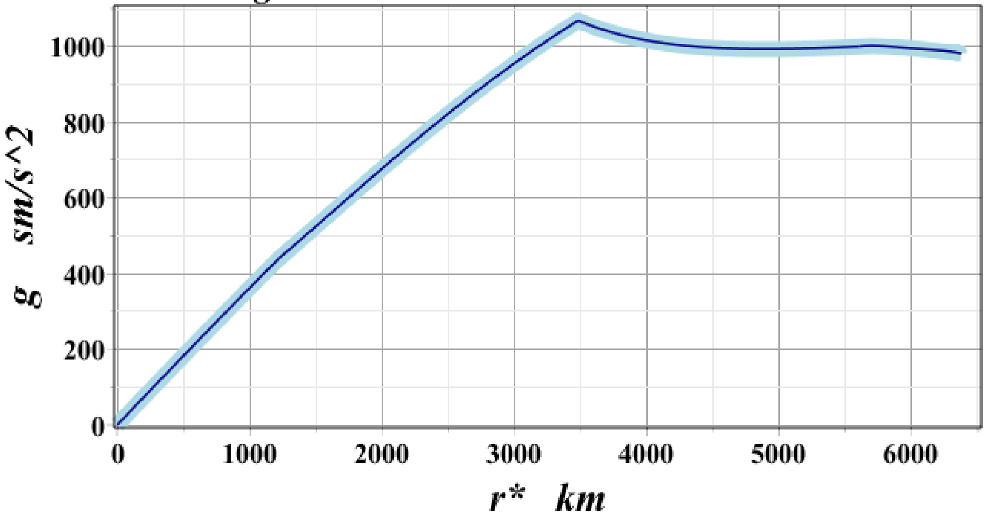


R = 6371 km, M = 5.9726 ×10 ^{27} g





The gravitational acceleration at radius r



GR corrections:

~ 2×10^{-9} for pressure

~ 4×10^{-15} for mass

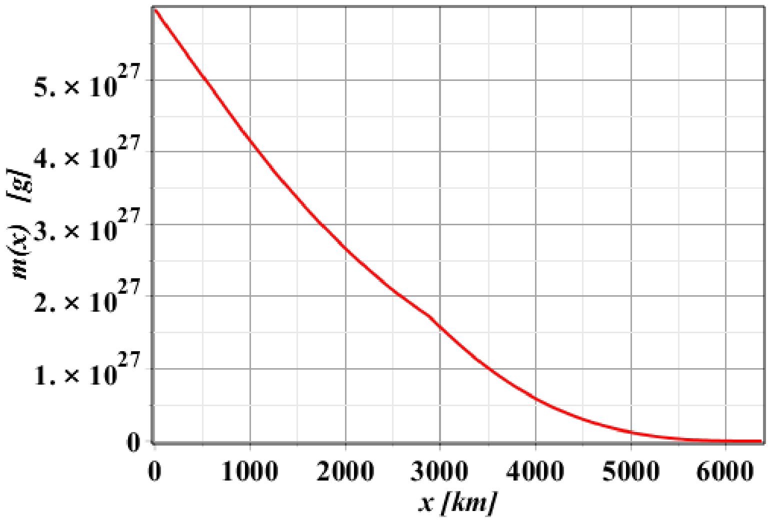
Completely ignorable

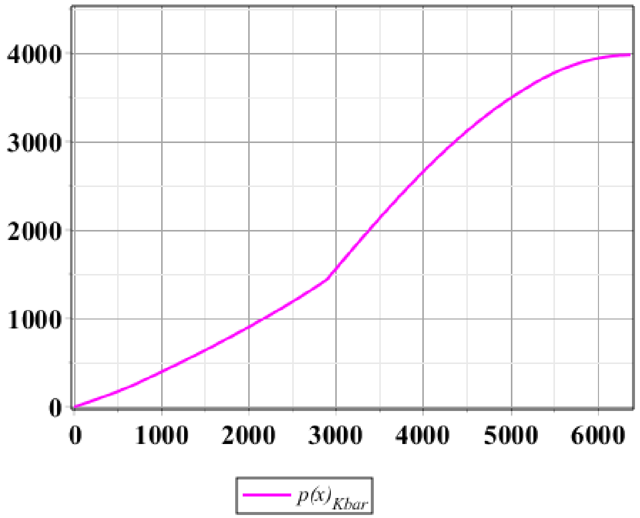
Earth structure in MDG

Instead of MEOS we know much more:

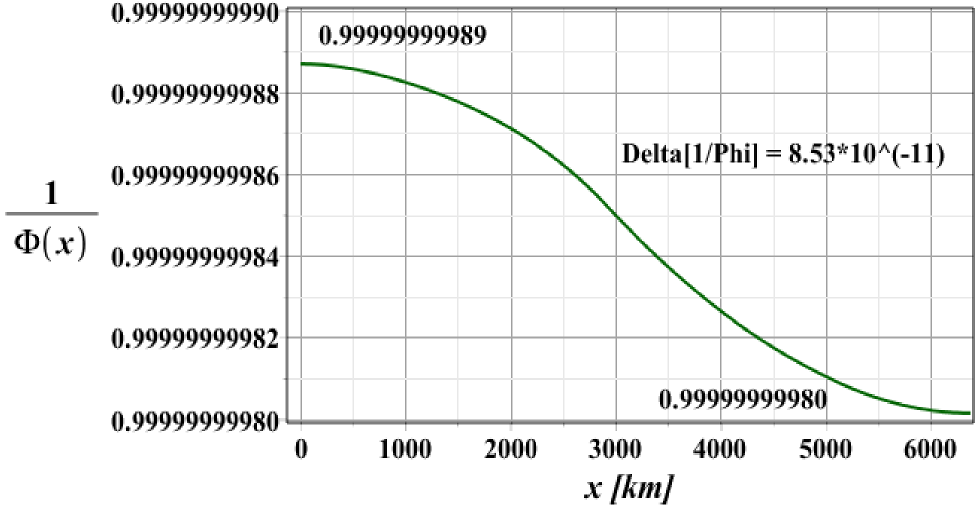
The mass density ρ_{Earth} (r) ; The total mass M_{Earth} ; The radius R_{Earth} (within 2% precision)

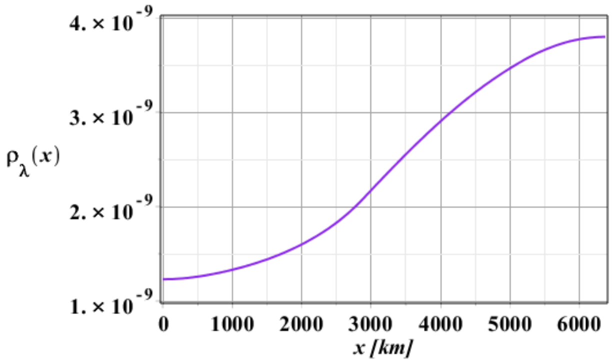
$\lambda_{\Phi} \approx$ **2890 km**, $m_{\Phi}c^2 \approx 4.297 \times 10^{-13}$ eV – **a basic novel result**





Variation of G inside the Earth in MDG





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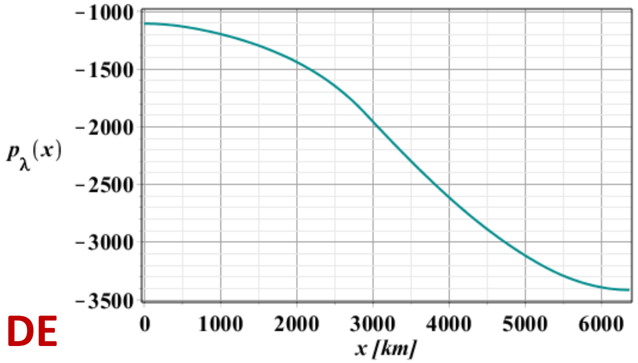
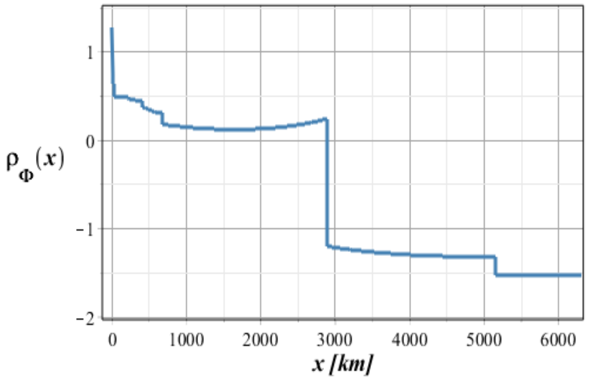
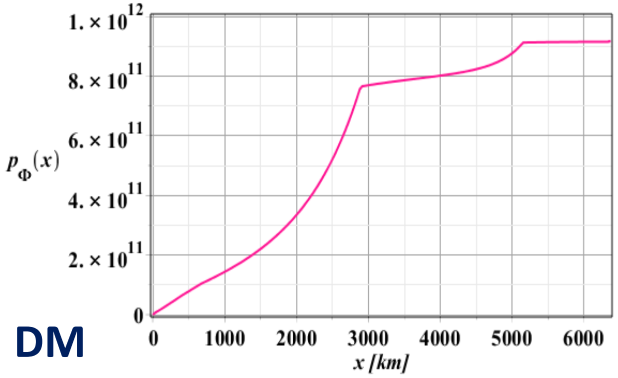




DM



Starobinsky 1980 potential

during INFLATION

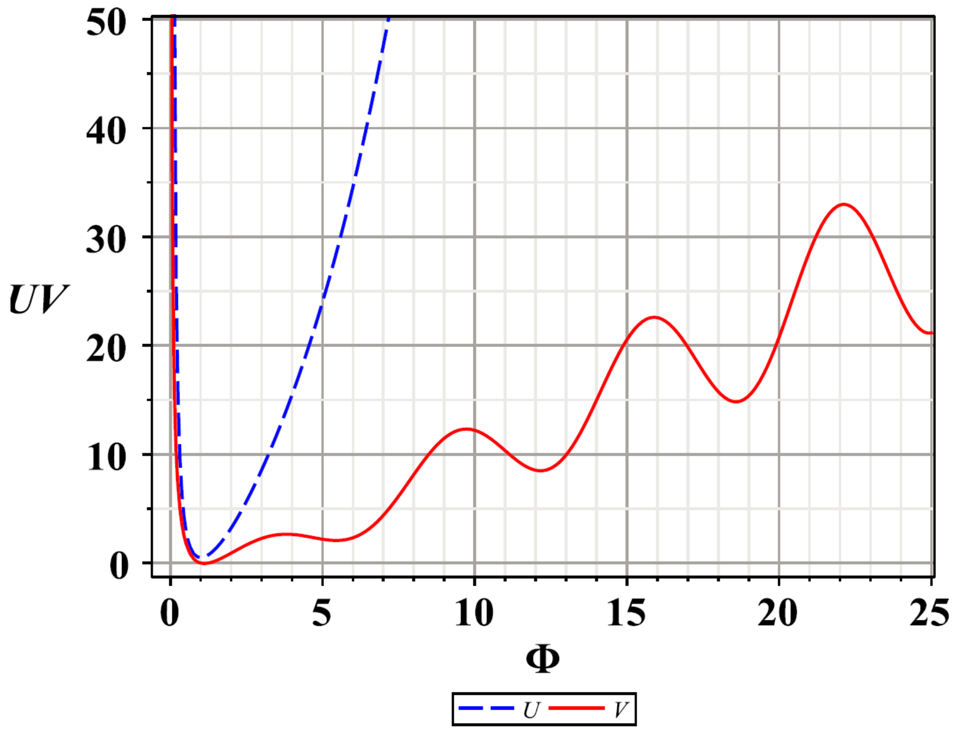
V**$_{st}$* and dilatonic potential ***V with identical masses of the scalaron and MDG-dilaton:

$$m_{scalaron} = m_{\Phi} \approx 3 \times 10^{-6} M_{Planck}$$

Todays Earth structure in MDG:

$$m_{\Phi\approx} \approx 4.297 \times 10^{-13}\ \mathrm{eV}/\ c^2$$

?



CONCLUSION:

We need models of gravity, which permit unified treatment of the physical problems at very different scales: from laboratory scales, planet and compact star scales to the scale of the visible Universe.

Such a unified approach may give much more definite justification of our models using all available information for the physical phenomena at all reachable scales.

Thank YOU

The boundary conditions for SSSS in MDG

Assuming: $r_c = 0$

$$m(0) = m_c = 0, \quad \Phi(0) = \Phi_c, \quad p(0) = p_c,$$
$$p_\Phi(0) = p_{\Phi c} = \frac{2}{3}\left(\frac{\epsilon(p_c)}{3} - p_c\right) - \frac{\Lambda}{12\pi}V'(\Phi_c).$$

SSSS edge: $P = 0 \Rightarrow r^*$

$$m^* = m(r^*; p_c, \Phi_c), \quad \Phi^* = \Phi(r^*; p_c, \Phi_c),$$
$$p_\Phi^* = p_\Phi(r^*; p_c, \Phi_c).$$

Cosmological horizon: r_U

$$r \in [r^*, r_U] \quad p \equiv 0 \text{ and } \epsilon \equiv 0$$
$$r_U: \ \Delta(r_U; p_c, \Phi_c) = 0, r_U \sim 1/\sqrt{\Lambda} \sim 10^{23}\,\text{km}$$

$$\Phi(r_U; p_c, \Phi_c) = 1 \quad \longleftarrow \quad \text{De Sitter vacuum}$$

$$F_\Phi(p_{\Phi c}, p_c, \Phi_c) = 0, \quad F_\Lambda(p_c, \Phi_c) = 0. \quad \longleftarrow \quad \text{Two specific MDG relations}$$

One parametric (p_c) family of SSSS – as in GR and the Newton gravity !

Matvey Petrovich Bronstein

The first attempt for quantization of gravity

Sow. Phys., 3, 73 (1933),
"Quantization of gravitational waves";

Phys. Zeitschr. der Sowjetunion 9, 140 (1936),
"Quantum theory of weak gravitational fields".

Proposed canonical quantization of week gravitational wave on flat background using relativistic invariant commutation relations and introducing for the first time gravitational quanta – gravitons, which meditate gravitational interaction between matter bodies.

1. **The Newton gravitational law is derived by calculating the exchange of gravitational quanta od spin 2.**

2. **The energy release by radiation of gravitational waves from matter bodies are calculated for the first time.**