Thermodynamic properties of a correlated nuclear system

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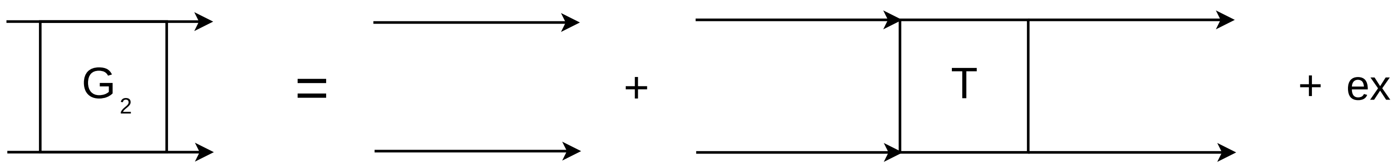
Introduction

- infinite system of strongly interacting nucleons
- equation of state of nuclear matter (heavy ion collisions, neutron stars)
- self-consistent finite temperature Green's function approach
- **energy per particle, pressure, entropy** at zero and finite temperature, different densities, symmetric matter
- free NN interaction (CD Bonn), comparison with an effective potential ($V_{\text{low k}}$), no three body forces
- as a reference *V. S. and P. Bożek, nucl-th/0604030*

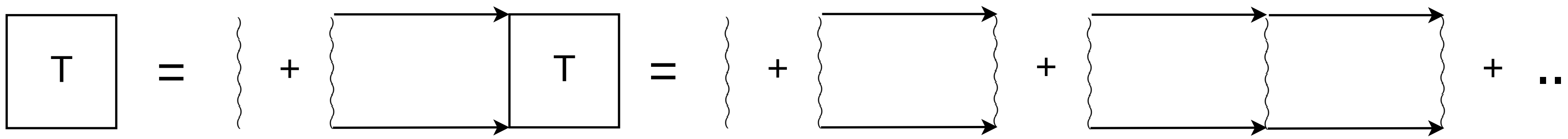
The T-matrix or ladder approximation

The two-particle propagator is expressed as

$$\mathcal{G}_2 = G_2^{nc} + G_2^{nc}\, T\, G_2^{nc} \; + \; exchange$$



where $\quad G_2^{nc} = G\, G \quad$ and $\quad T = V + V\, G_2^{nc}\, T \quad .$



All the ingredients are calculated iteratively:

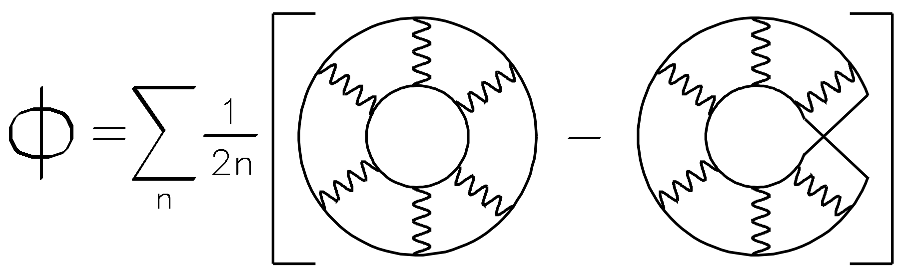
$$T = T\,[V, G] \;\; , \;\; \Sigma = \Sigma\,[T, G] \;\; , \;\; G = G\,[\Sigma]\; .$$

The generating functional Φ

It belongs to a class of approximations derivable from a suitably chosen generating functional (Baym and Kadanoff, 1961). Formally

$$\Sigma(x,y) = \frac{\delta \Phi}{\delta G(x,y)} \,.$$

In the case of the ladder approximation $\Phi = \sum_n \frac{1}{2n} \mathrm{Tr}\{(V\, G_2^{nc})^n\} \,.$



Thermodynamic consistency is ensured, thermodynamic relations are fulfilled. It is related to the thermodynamic potential Ω by

$$\Omega = -\mathrm{Tr}\{\ln[G^{-1}]\} - \mathrm{Tr}\{\Sigma G\} + \Phi \,.$$

Energy of the interacting system

The (total) internal energy is obtained as the expectation value of the Hamiltonian

$$\frac{E}{N} = \frac{1}{\rho} \left[\frac{\langle H_{kin} \rangle}{\mathscr{V}} + \frac{\langle H_{pot} \rangle}{\mathscr{V}} \right] .$$

In particular

$$\langle H_{pot} \rangle = \frac{\mathscr{V}}{2} \int \frac{d^3 P}{(2\pi)^3} \frac{d^3 k}{(2\pi)^3} \frac{d^3 k'}{(2\pi)^3} \frac{d\Omega}{2\pi} V(\mathbf{k}, \mathbf{k}') \, \langle \mathbf{k}' | G_2^<(\mathbf{P}, \Omega) | \mathbf{k} \rangle \ .$$

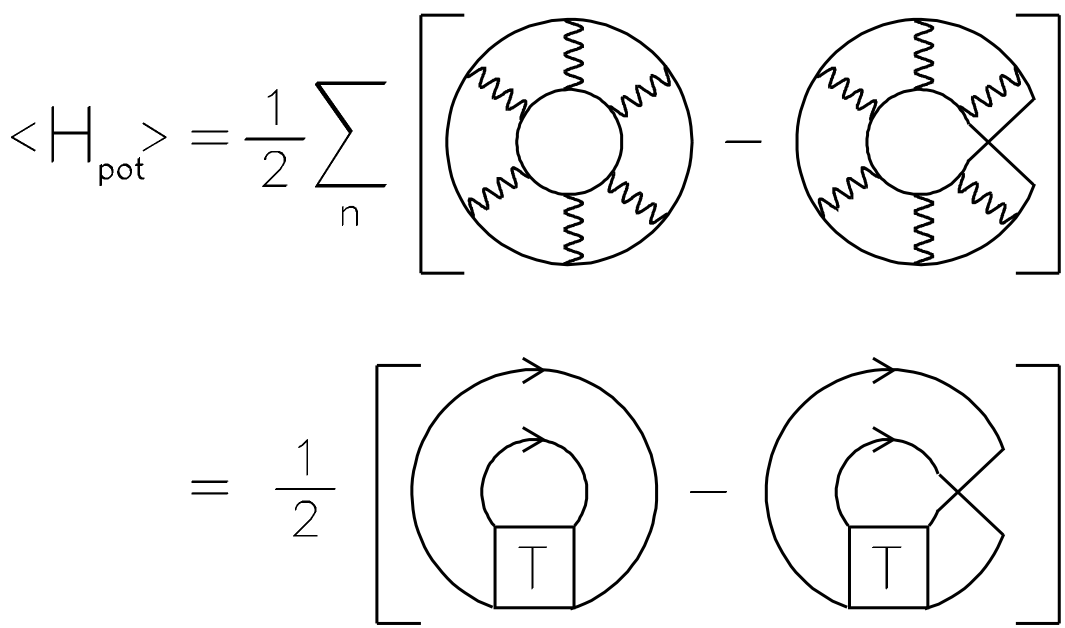
Alternatively, it is possible to estimate the internal energy through the Galitsky-Koltun's sum rule

$$\frac{E}{N} = \frac{1}{\rho} \int \frac{d^3 p}{(2\pi)^3} \frac{d\omega}{2\pi} \left[\frac{\mathbf{p}^2}{2m} + \omega \right] A(\mathbf{p}, \omega) f(\omega) \, .$$

Within the T-matrix approach, the interaction energy can be expressed as a function of T and G_2^{nc}.

$$
\begin{aligned}
V\mathcal{G}_2 &= V\,G_2^{nc} + V\,G_2^{nc}\,T\,G_2^{nc} \\
&= [V + V\,G_2^{nc}\,T]\,G_2^{nc} = TG_2^{nc}\,,
\end{aligned}
$$

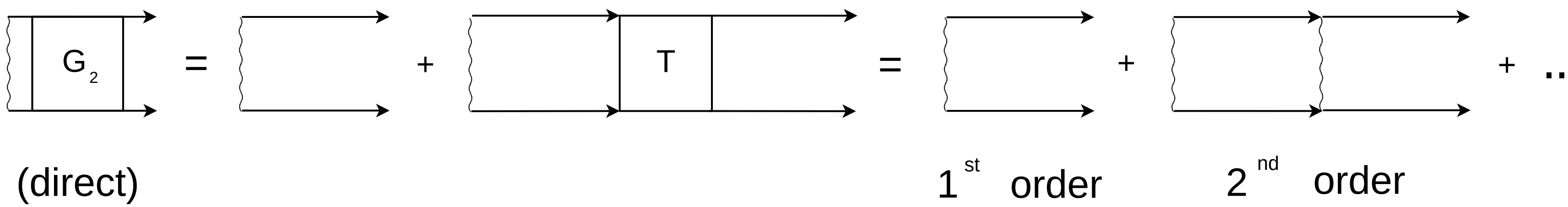
which, in terms of diagrams, looks like this

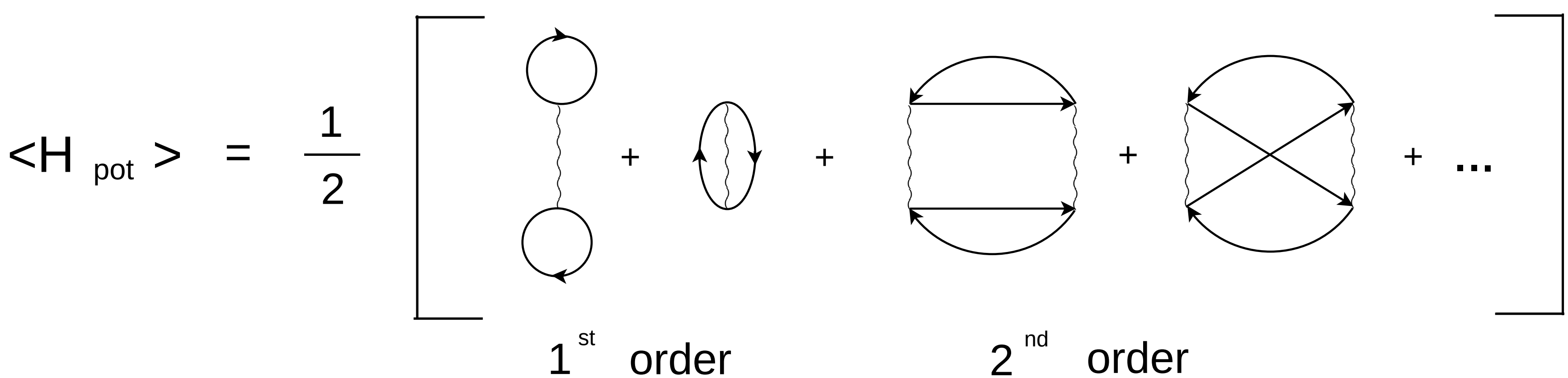


2^{nd} order approximation

Expand the interaction energy

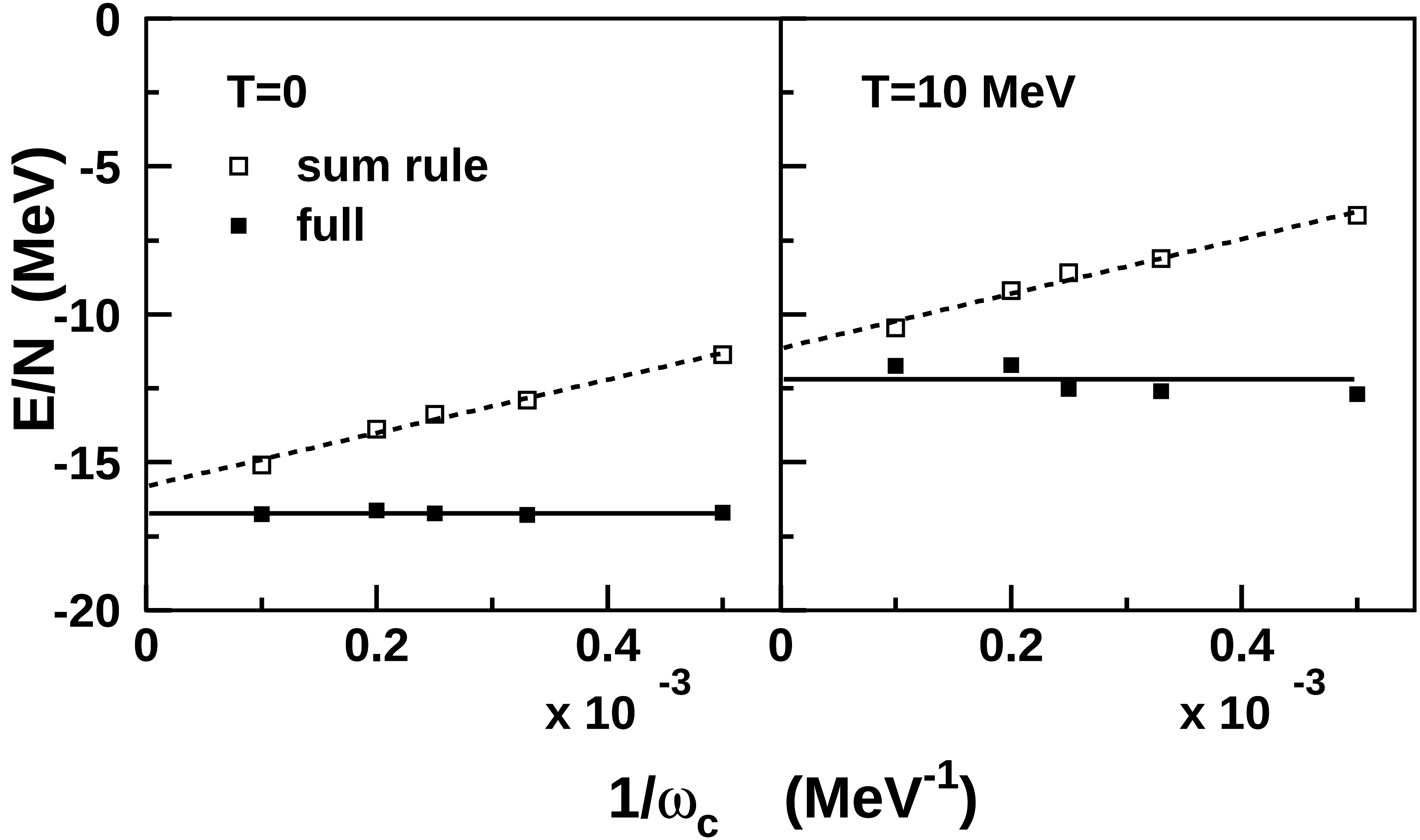
$$\langle H_{pot} \rangle = \frac{1}{2}\mathrm{Tr}\{(V\,\mathcal{G}_2)\} = \sum_n \frac{1}{2}\mathrm{Tr}\{(V\,G_2^{nc})^n\}\;.$$



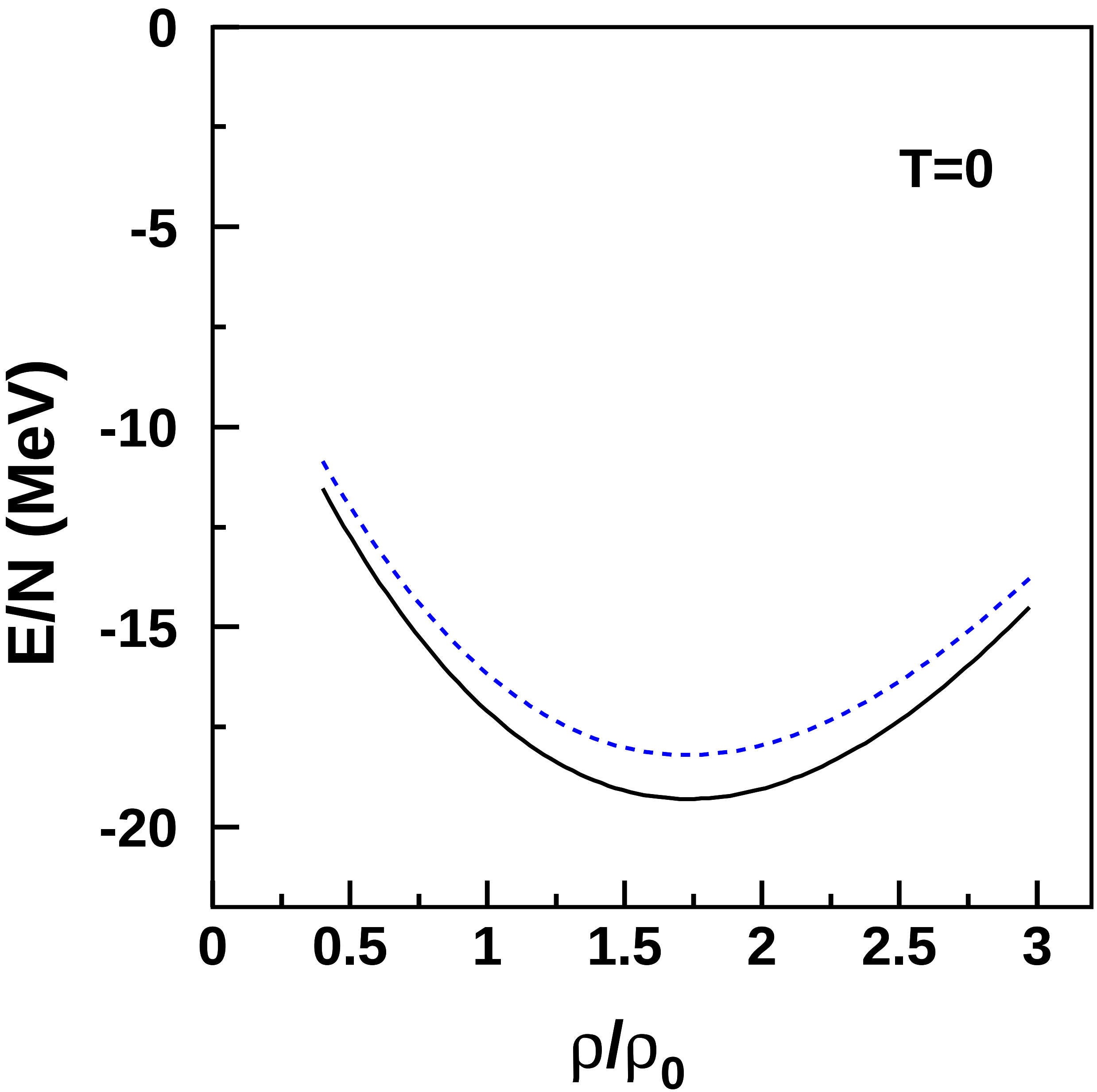


Up to 2^{nd} order, using an effective interaction $V_{\text{low k}}$ instead of V_{NN}.

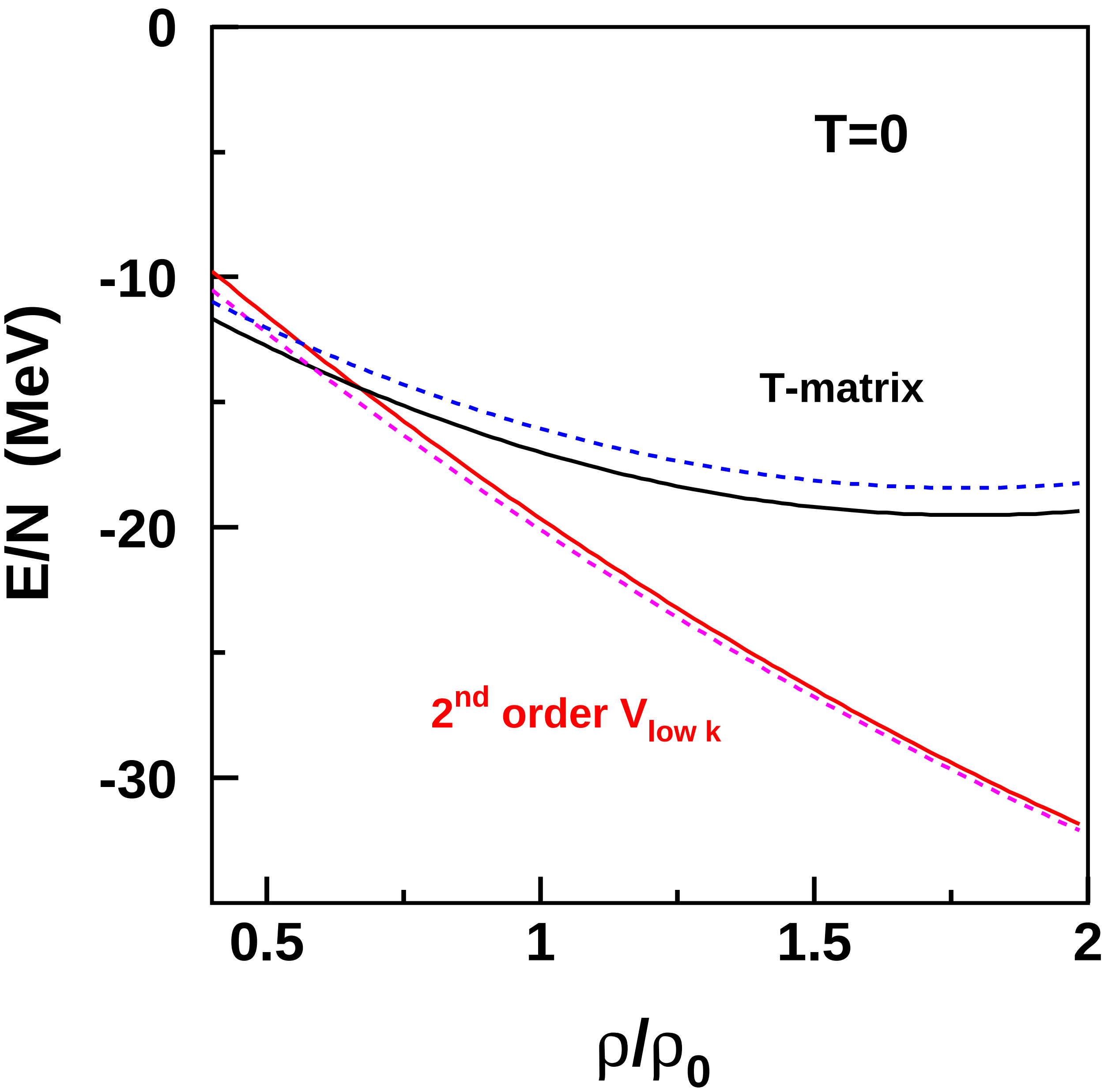
Cutoff dependence



Energy vs. density



Energy vs. density (comparison)



Pressure

The pressure is related to the thermodynamical potential

$$\Omega(T, \mu, V) = -PV \ .$$

Recall $\quad \Omega = -\mathrm{Tr}\{\ln[G^{-1}]\} - \mathrm{Tr}\{\Sigma G\} + \Phi \ .$

The first contribution to the pressure is

$$P_I = \frac{1}{V}\left[\mathrm{Tr}\{\ln[G^{-1}]\} + \mathrm{Tr}\{\Sigma G\}\right]$$

$$= T \int \frac{d^3p}{(2\pi)^3} \frac{d\omega}{2\pi} \ln(1 + e^{-\beta\omega}) \left[\ A(\mathbf{p}, \omega) \right.$$

$$\left. + \frac{\partial A(\mathbf{p}, \omega)}{\partial \omega} \mathrm{Re}\Sigma^R(\mathbf{p}, \omega) - 2\mathrm{Im}\Sigma^R(\mathbf{p}, \omega) \frac{\partial \mathrm{Re}G^R(\mathbf{p}, \omega)}{\partial \omega} \ \right] \ .$$

The contribution from Φ

The second contribution to the pressure comes from the functional Φ. It is calculated by introducing an integration over the (artificial) parameter λ:

$$\begin{aligned}\Phi &= \sum_n \frac{1}{2n}\mathsf{Tr}\{(V\,G_2)^n\} \\ &= \int_0^1 \frac{d\lambda}{\lambda}\sum_n \frac{1}{2}\mathsf{Tr}\{(\lambda V\,G_2)^n\} \\ &= \int_0^1 \frac{d\lambda}{\lambda} < H_{pot}(\lambda V, G_{\lambda=1}) > \quad .\end{aligned}$$

Since $< H_{pot}(V,G) > \sim \frac{1}{2}\mathsf{Tr}\{T\,G_2^{nc}\}$ and $T = \dfrac{V}{1 - V\,G_2^{nc}}$, finally

$$\Phi = \int_0^1 \frac{d\lambda}{\lambda} < H_{pot}(\lambda V, G_{\lambda=1}) > \sim \frac{1}{2}\int_0^1 d\lambda \frac{\mathsf{Tr}\{V\,G_2^{nc}\}}{1 - \mathsf{Tr}\{\lambda V\,G_2^{nc}\}} .$$

Entropy

The entropy is estimated through the thermodynamic relation

$$\frac{S}{N} = \frac{1}{T}\left[\frac{E}{N} + \frac{P}{\rho} - \mu\right]$$

and compared with two analytic expressions:

1. the dynamical quasiparticle formula

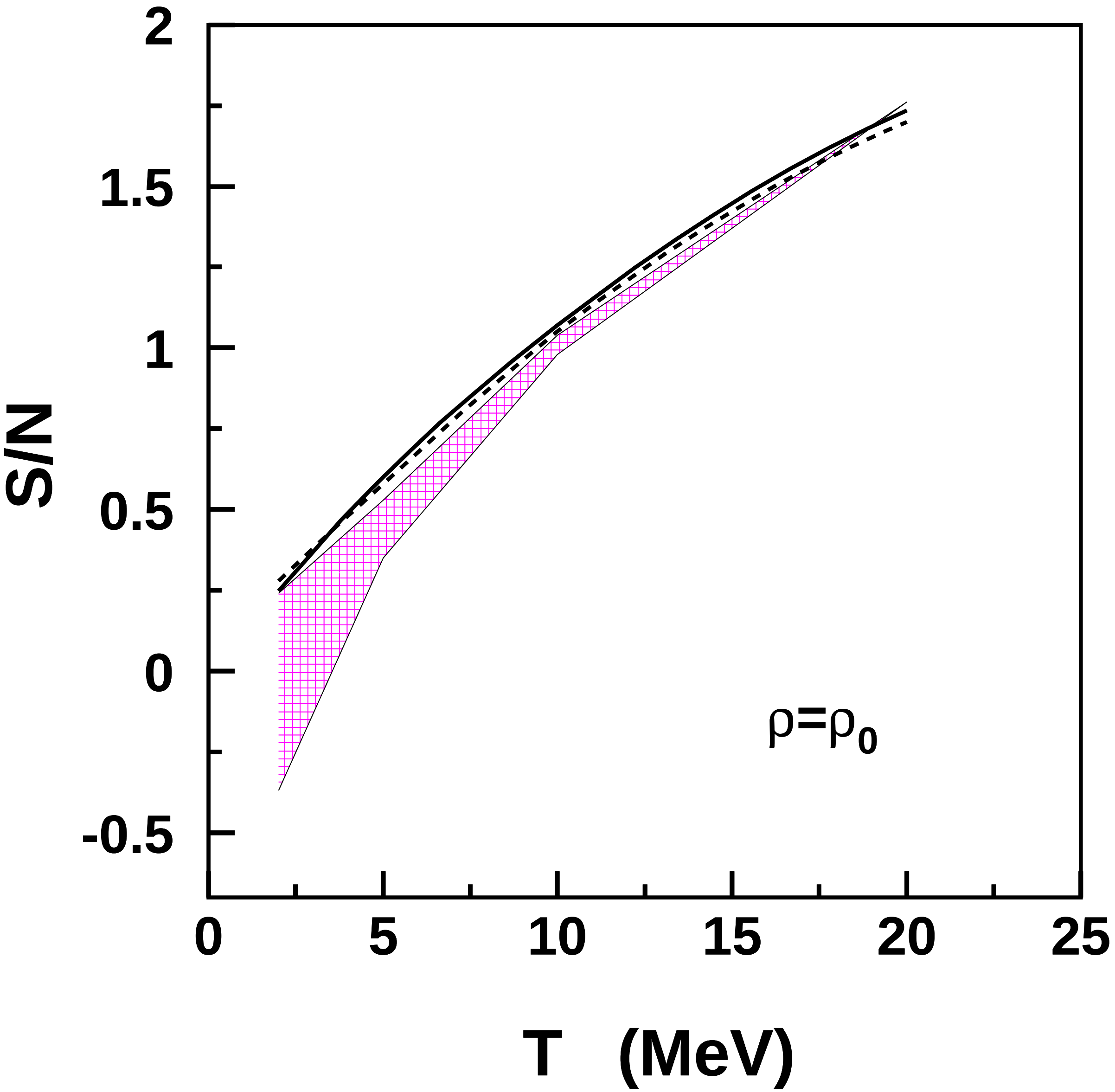
$$\frac{S_{DQ}}{N} = \frac{1}{\rho}\int \frac{d^3p}{(2\pi)^3}\frac{d\omega}{2\pi}\sigma(\omega)\left[A(\mathbf{p},\omega)\left(1 - \frac{\partial \mathrm{Re}\Sigma^R(\mathbf{p},\omega)}{\partial\omega}\right) + \frac{\partial \mathrm{Re}G^R(\mathbf{p},\omega)}{\partial\omega}\Gamma(\mathbf{p},\omega)\right]$$

where $\quad \sigma(\omega) = -f(\omega)\ln[f(\omega)] - [1 - f(\omega)]\ln[1 - f(\omega)]\,;$

2. the entropy for a free Fermi gas with effective masses

$$\frac{S_{free}}{N} = \frac{1}{\rho}\int \frac{d^3p}{(2\pi)^3}\,\sigma(\omega_p) \qquad \text{with} \quad \omega_p = \frac{p^2}{2m} - \mu - \Sigma(p,\omega_p) \quad .$$

Entropy vs. temperature



Summary

The presented scheme provides a way to calculate consistently thermodynamic properties of nuclear matter, at zero and finite temperatures, taking into account short range correlations.

The full (diagrammatic) calculation has been compared to others

- (energy) Galitsky-Koltun's sum rule, second order Born diagrams
- (entropy) dynamical quasiparticle formula, Fermi gas expression with effective masses

To be done:

- introduce three-body interactions
- deal with asymmetric matter
- ...