Dynamics of relativistic heavy ion collisions II

V. Toneev

The Bogoliubov Laboratory of Theoretical Physics,
Joint Institute for Nuclear Research, Dubna

August 22, 2006

Observable

- Double differential cross sections

Invariant cross section (integrated with respect to azimuthal angle)

$$\int d\phi \; E \; \frac{d\sigma}{d^3p} = \frac{d\sigma}{2\pi \; p_\perp dp_\perp \; dy} = \frac{d\sigma}{2\pi \; m_\perp dm_\perp \; dy}$$

with the transverse mass : $m_\perp = \sqrt{m^2 + p_\perp^2}$ and longitudinal rapidity $y = \frac{1}{2} \; \ln \frac{E+p_z}{E-p_z}$

- ⋆ y is additive at the Lorentz transformation
- ⋆ if $E \to p \; , \quad y \to \ln \tan \theta/2$
- ⋆ mean particle multiplicity

$$n = \frac{1}{2\pi\sigma} \int \frac{d^3p}{E} \left(\frac{d\sigma}{m_\perp dm_\perp \; dy} \right)$$

Global analysis

- Collective variables

kinetic flow tensor (event-by-event)

$S_{ij} = \sum_{\nu=1}^{M} w_\nu \; p_i(\nu) \; p_j(\nu)$

with $w_\nu = (2m_\nu)^{-1}; \quad p_i(i = 1, 2, 3)$

ellipsoid defined by 3 eigenvalues

$f_1 \leq f_2 \leq f_3$ and 3 Euler angles

reaction plane !

- Collective variables

kinetic flow tensor (event-by-event)

$S_{ij} = \sum_{\nu=1}^{M} w_\nu \; p_i(\nu) \; p_j(\nu)$

with $w_\nu = (2m_\nu)^{-1}$; $p_i(i = 1, 2, 3)$

ellipsoid defined by 3 eigenvalues

$f_1 \leq f_2 \leq f_3$ and 3 Euler angles

reaction plane !

Θ_F - polar angle with largest eigenvector

$f_3/f_1, \; f_3/f_2$ - kinetic flow ratios

$f_3 = f_2 = f_1$ - sphere in the momentum space

$f_3 > f_2 = f_1$ - cigar pattern

$f_3 < f_2 = f_1$ - pancake pattern

Global analysis

- Collective variables

kinetic flow tensor (event-by-event)

$S_{ij} = \sum_{\nu=1}^{M} w_\nu \; p_i(\nu) \; p_j(\nu)$

with $w_\nu = (2m_\nu)^{-1}$; $p_i(i = 1, 2, 3)$

ellipsoid defined by 3 eigenvalues

$f_1 \leq f_2 \leq f_3$ and 3 Euler angles

reaction plane !

Θ_F - polar angle with largest eigenvector

f_3/f_1, f_3/f_2 - kinetic flow ratios

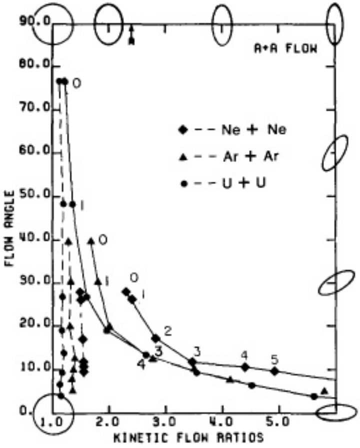
$f_3 = f_2 = f_1$ - sphere in the momentum space

$f_3 > f_2 = f_1$ - cigar pattern

$f_3 < f_2 = f_1$ - pancake pattern

finite flow angle

M.Gyulassi et al. Phys.Lett. **110B** (1982) 185



numbers besides symbols ⇒ b/b_{max} in steps of 0.1

Flow analysis

6 parameters are reduced to 3 relevant ones;

In the ultrarelativistic limit $S^1 = \begin{bmatrix} f_1 + f_3\theta_F^2 & 0 \\ 0 & f_2 \end{bmatrix}$

with $\quad f_1 \approx S_{11} = \sum_{\nu=1}^{M} w(\nu) p_x^2(\nu), \qquad f_2 \approx S_{22} = \sum_{\nu=1}^{M} w(\nu) p_y^2(\nu)$

Anisotropy measure $\alpha = \frac{f_1 - f_2}{f_1 + f_2} = \frac{\sum_{\nu=1}^{M} w(\nu)[p_x^2(\nu) - p_y^2(\nu)]}{\sum_{\nu=1}^{M} w(\nu)[p_x^2(\nu) + p_y^2(\nu)]}$

Flow analysis

6 parameters are reduced to 3 relevant ones;

In the ultrarelativistic limit $S^1 = \begin{bmatrix} f_1 + f_3\theta_F^2 & 0 \\ 0 & f_2 \end{bmatrix}$

with $f_1 \approx S_{11} = \sum_{\nu=1}^{M} w(\nu) p_x^2(\nu), \qquad f_2 \approx S_{22} = \sum_{\nu=1}^{M} w(\nu) p_y^2(\nu)$

Anisotropy measure $\alpha = \frac{f_1 - f_2}{f_1 + f_2} = \frac{\sum_{\nu=1}^{M} w(\nu)[p_x^2(\nu) - p_y^2(\nu)]}{\sum_{\nu=1}^{M} w(\nu)[p_x^2(\nu) + p_y^2(\nu)]}$

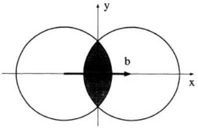
- Anisotropy in transverse momentum plane

Fourier series

$$\frac{dN}{d\phi} = \frac{1}{2\pi}(1 + 2\sum_{n\geq 1} v_n \cos n\phi)$$

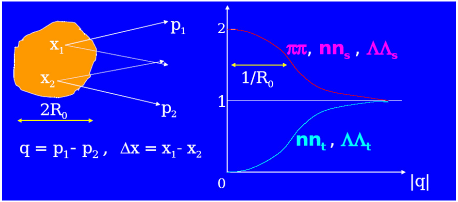
directed flow $v_1 =< \cos\phi >=< p_x/p_\perp >$,
elliptic flow $v_2 =< \cos 2\phi >=< (p_x^2 - p_y^2)/p_\perp^2 >$

(p_x, p_z) - reaction plane
non-flow effects $\Rightarrow$
n-particle correlations



HBT analysis

- Correlations of identical particles



$$\mathcal{A}_{12} = \frac{1}{\sqrt{2}} \left[e^{ip_1x_1 + ip_2x_2} + (-1)^s e^{ip_1x_2 + ip_2x_1} \right], \quad s \text{ - spin of a pair}$$

$$|\mathcal{A}_{12}|^2 = 1 + (-1)^s \cos[q(x_1 - x_2)] \quad \text{with} \quad q = p_1 - p_2$$

$$C_2(q) = \int d^4x_1 \; d^4x_2 \; |\mathcal{A}_{12}|^2 \; \rho(x,y)$$

For the Gaussian (pion) source ρ

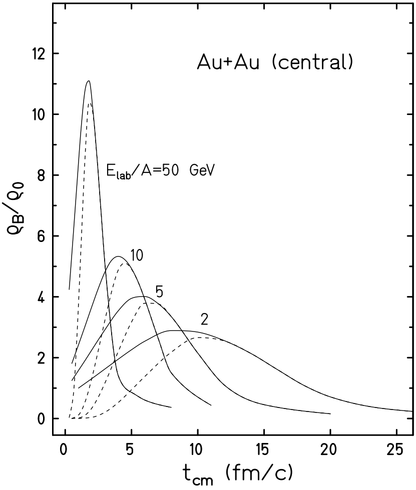
$$C_G(q) \sim 1 + \lambda \; \exp\{-[\vec{R}_G \vec{q} + \omega^2\tau^2]/2\}$$

space-time structure!

with $\omega = \sqrt{p_{\perp,1}^2 + m_\pi^2} - \sqrt{p_{\perp,2}^2 + m_\pi^2}$, τ- time difference

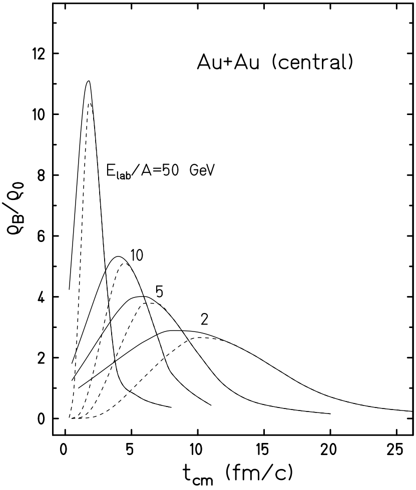
correlations in the initial state, source anisotropy, flow effects, final state interaction ...

Kinetic evolution (QGSM model)

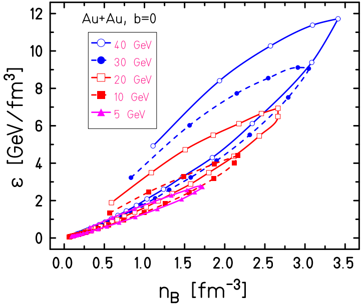


Transport model results

Kinetic evolution (QGSM model)



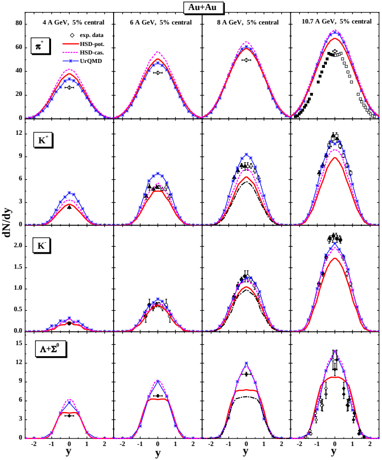
central Au+Au collisions



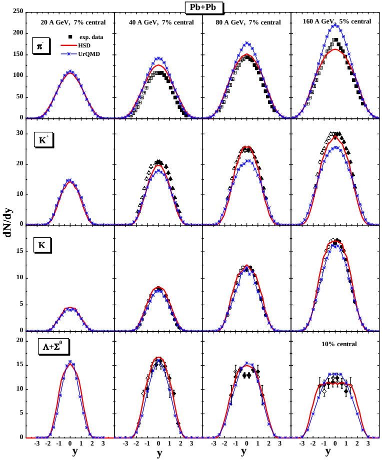
central cell $4 \times 4 \times 4/\gamma$ fm^3

Rapidity distributions

AGS energy range



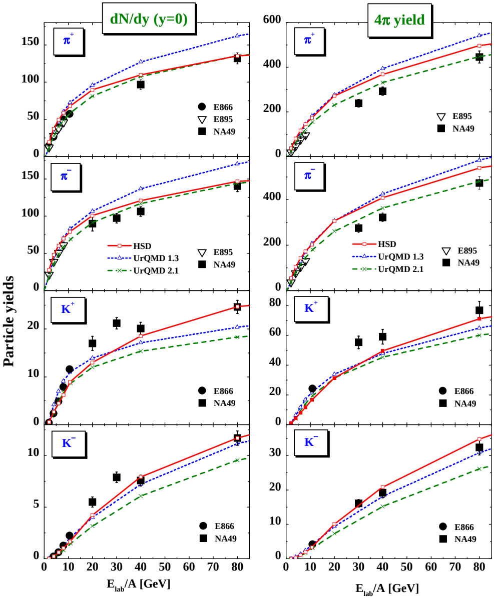
SPS energy range



H.Weber et al., Phys.Rev. **C67** (2003) 014904

Particle yield

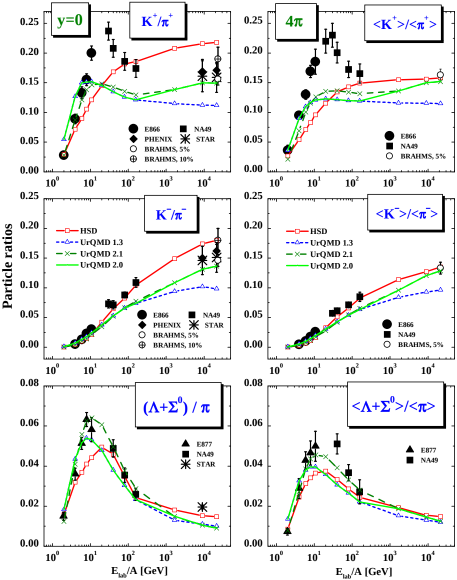
central Au+Au collisions



E.Bratkovskaya et al., Phys.Rev. **C69** (2004) 054907

Particle ratio

central Au+Au/Pb+Pb collisions

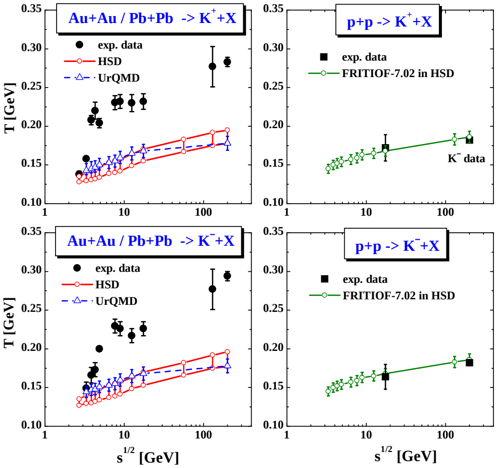


E.Bratkovskaya et al., Phys.Rev. **C69** (2004) 054907

"Transverse temperature"

$$\frac{dN}{m_\perp dm_\perp} \sim \exp(-\frac{m_\perp}{T})$$

central Au+Au/Pb+Pb collisions



E.Bratkovskaya et al., Phys.Rev.Lett. **92** (2004) 032302

Hydrodynamic approach

- Conservation laws (Gauss theorem) $\Rightarrow$ Fluid dynamics

$$\partial_\mu \; J_i^\mu = 0 \quad \text{net charge } i \text{ conservation} \quad \boxed{4}$$

$$\partial_\mu \; T^{\mu\nu} = 0 \quad \text{energy momentum conservation} \quad \boxed{10}$$

- Tensor decomposition of the charge current J^μ and energy-momentum tensor $T^{\mu\nu}$ with respect to 4-velocity u^μ

$$J_i^\mu = n_i u^\mu + (g_\nu^\mu - u^\mu u_\nu) \; J_i^\nu + \ldots$$

$$T^{\mu\nu} = \varepsilon \; u^\mu u^\nu - P \; (g^{\mu\nu} - u^\mu u^\nu) + \; q^\mu u^\nu + q^\nu u^\mu + \pi^{\mu\nu} + \ldots$$

$$\text{with} \quad J_i^\mu = \int \frac{d^3p}{p_0} \; p^\mu \; \left[f_i(x,p) - \bar{f}_i(x,p)\right]$$

$$T^{\mu\nu} = \int \frac{d^3p}{p_0} \; p^\mu p^\nu \left[f(x,p) + \bar{f}(x,p)\right]$$

$$f_i(x,p) = \frac{g_i}{(2\pi)^3} \; \exp\left[((u_\mu p^\mu(x) - \mu_i(x))/T(x)) \pm 1\right]^{-1}$$

Gradient expansion

⋆ Perfect hydro in local thermodynamical equilibrium

$$
\begin{aligned}
J_i^\mu &= n_i u^\mu \\
T^{\mu\nu} &= \underbrace{\varepsilon\, u^\mu u^\nu - P\,(g^{\mu\nu} - u^\mu u^\nu)}_{\text{perfect hydro}}
\end{aligned}
$$

6

+ EoS $\varepsilon(p)$

Gradient expansion

⋆ Perfect hydro in local thermodynamical equilibrium

$$\begin{aligned} J_i^\mu &= n_i u^\mu \\ T^{\mu\nu} &= \underbrace{\varepsilon\ u^\mu u^\nu - P\,(g^{\mu\nu} - u^\mu u^\nu)}_{\text{perfect hydro} \quad \boxed{6}} \end{aligned}$$

$$\boxed{+\ \text{EoS} \quad \varepsilon(p)}$$

⋆ First order dissipative corrections (viscosity, heat capacity)

$\Rightarrow$ acausality

Gradient expansion

⋆ Perfect hydro in local thermodynamical equilibrium

$$J_i^\mu = n_i u^\mu$$
$$T^{\mu\nu} = \underbrace{\varepsilon\ u^\mu u^\nu - P\left(g^{\mu\nu} - u^\mu u^\nu\right)}_{\text{perfect hydro} \quad \boxed{6}}$$

$$\boxed{+\ \text{EoS} \quad \varepsilon(p)}$$

⋆ First order dissipative corrections (viscosity, heat capacity)

$\Rightarrow$ acausality

⋆ Second order corrections $\Rightarrow$ + 14 Grad equations

Gradient expansion

⋆ Perfect hydro in local thermodynamical equilibrium

$$J_i^\mu = n_i u^\mu$$
$$T^{\mu\nu} = \underbrace{\varepsilon\ u^\mu u^\nu - P\ (g^{\mu\nu} - u^\mu u^\nu)}_{\text{perfect hydro} \quad \boxed{6}}$$

Yu.Ivanov

$\boxed{+\ \text{EoS} \quad \varepsilon(p)}$

⋆ First order dissipative corrections (viscosity, heat capacity)

$\Rightarrow$ acausality

⋆ Second order corrections $\Rightarrow$ + 14 Grad equations

Spatial-temporal variation of the macro fields has to be SMALL

Relation between kinetics and hydro

- The non-relativistic case (for nucleons)

$$\int d^3p \begin{bmatrix}\rho\\ \vec{v}\\ \epsilon\end{bmatrix} \frac{d}{dt} f(\vec{p},\vec{x},t) = \int d^3p \begin{bmatrix}1\\ \vec{p}/m_N\\ p^2/2m_N\end{bmatrix} \; C(\vec{p},\vec{x},t)$$

Boltzmann equation + local equilibrium hypothesis

$$\vec{v} \;=\; \vec{u}+\vec{c}\;,\; \text{hydro}\; \vec{u} = <\vec{v}>\;,\; \text{thermo}\; <\vec{c}> = 0$$

$$\rho < c_i c_k > \;=\; P\,\delta_{ik} + \Pi_{ik}\;,\quad \rho < c^2 c_k > = Q_k$$

$$\frac{\partial\rho}{\partial t} - \frac{\partial}{\partial x_k}\rho u_k = 0$$

$$\frac{\partial\rho u_i}{\partial t} - \frac{\partial}{\partial x_k}\rho u_i u_k = \{\frac{\partial}{\partial x_k}\Pi_{ik} - \frac{\partial}{\partial x_i}P\}$$

$$\frac{\partial\epsilon}{\partial t} - \frac{\partial}{\partial x_k}\epsilon u_k = \{\frac{\partial}{\partial x_k}\Pi_{ik}u_i - \frac{\partial}{\partial x_i}Pu_k - \frac{\partial}{\partial x_k}Q_k\}$$

$\rho, \vec{u}, \epsilon, P$

EOS

hydro: $\Lambda \ll L$; $Re = \frac{inertial}{viscous} \simeq \frac{M}{\Lambda/L} \simeq 4 - 10$ with $M = v/c_s$ and $c_s = \sqrt{\partial P/\partial\rho|_s} \approx 0.2$

turbulent regime $Re \simeq 10^2 - 10^3$

(near a phase transition ?)

Bjorken expansion

- Analytical solution

Longitudinal component of 4-velocity

$$u^\mu \partial^\nu T_{\nu\mu} \equiv u^\mu \ \partial_\mu \varepsilon + (p+\varepsilon)\ \partial_\mu u^\mu = 0$$
$$u^\mu \ \partial_\mu n_B + n_B \ \partial_\mu u^\mu = 0$$

Assuming Lorentz-invariant solution $\varepsilon = \varepsilon(\tau)$, $p = p(\tau)$ with $u^\mu = \frac{x^\mu}{\tau}$, $\tau = \sqrt{x_\mu x^\mu}$ (for longitudinal expansion $x^\mu = (t, 0, 0, z)$)

$$\partial_\tau \varepsilon + (\partial_\mu u^\mu)(\varepsilon + p) = 0 \qquad \Rightarrow \qquad \partial_\tau \varepsilon + \frac{\partial_\mu x^\mu - 1}{\tau}(\varepsilon + p) = 0$$
$$\partial_\tau n_B + (\partial_\mu u^\mu)\ n_B = 0 \qquad \qquad \partial_\tau n_B + \frac{\partial_\mu x^\mu - 1}{\tau}\ n_B = 0$$

After integration (EoS: $p = c_s^2 \varepsilon$)

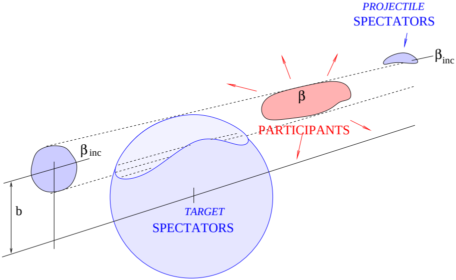
$$n_B(\tau) = n_B(\tau_0)\left(\frac{\tau_0}{\tau}\right)^{(\partial_\mu x^\mu - 1)} \qquad \varepsilon(\tau) = \varepsilon(\tau_0)\left(\frac{\tau_0}{\tau}\right)^{(1+c_s^2)(\partial_\mu x^\mu - 1)}$$

For Bjorken longitudinal expansion ($\partial_\mu x^\mu - 1 = 1$, $c_s^2 = 1/3$)

$$n_B(\tau) = n_B(\tau_0)\frac{\tau_0}{\tau}, \quad \varepsilon(\tau) = \varepsilon(\tau_0)\left(\frac{\tau_0}{\tau}\right)^{4/3}, \quad T(\tau) = T(\tau_0)\left(\frac{\tau_0}{\tau}\right)^{1/3}$$

Nonequilibrium and initial state

- "Frozen" hydrodynamics



equilibrium statistical description

- Landau and Bjorken initial sate $(u^\mu,\ \varepsilon,\ \rho_i$ at the moment $\tau_0)$ $+$ hydro expansion

Nonequilibrium and initial state

- Many fluid hydrodynamics

$$f(x,p) = \sum_{j}^{M} f_j^{eq}(x,p)$$

A single fluid may consist of several particle species. Different fluids may be of the same particle species

- Many fluid hydrodynamics

$$f(x,p) = \sum_{j}^{M} f_j^{eq}(x,p)$$

A single fluid may consist of several particle species. Different fluids may be of the same particle species

3-FLUIDS:

Yu.B.Ivanov, V.N.Russkikh, V.D.Toneev, Phys. Rev. **C 73** 044904 (2006)   Yu.Ivanov

⋆ Hadronic equation of state

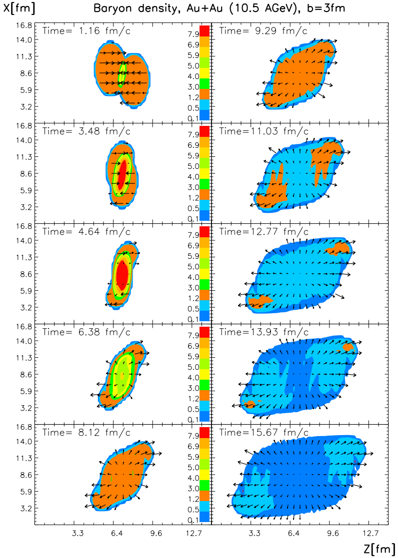
$$\begin{aligned} \varepsilon(n_B,T) &= \varepsilon_{gas}(n_B,T) + W(n_B), \\ P(n_B,T) &= P_{gas}(n_B,T) + n_B \frac{dW(n_B)}{dn_B} - W(n_B) \end{aligned}$$

nuclear potential $W(n_B) = n_B m_N \left[-b\left(\frac{n_B}{n_0}\right) + c\left(\frac{n_B}{n_0}\right)^{\gamma+1} \right]$

⋆ Friction force (parametrization)

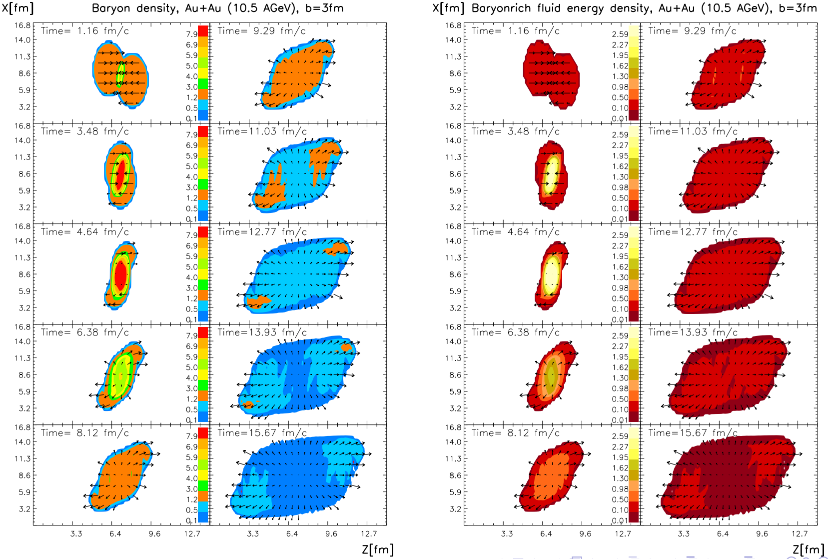
3-fluid model

Yu.B.Ivanov at al., Phys. Rev. **C 73** 044904 (2006)



3-fluid model

Yu.B.Ivanov at al., Phys. Rev. **C 73** 044904 (2006)



3-fluid model

Dynamics of relativistic HI collisions

V. Toneev

Observable

Transport model results

Hydrodynamics

General remarks

Hydrodynamic models

Evolution in 3F hydro

3F hydro - comparison

Fireball evolution

Hybrid model

EoS

Fireball evolution

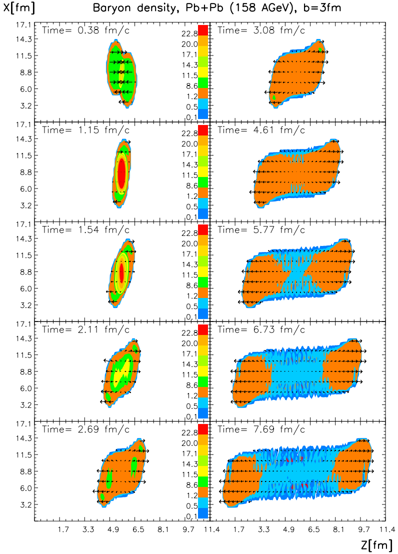
Evolution of averages

Hybrid - comparison

Dilepton production

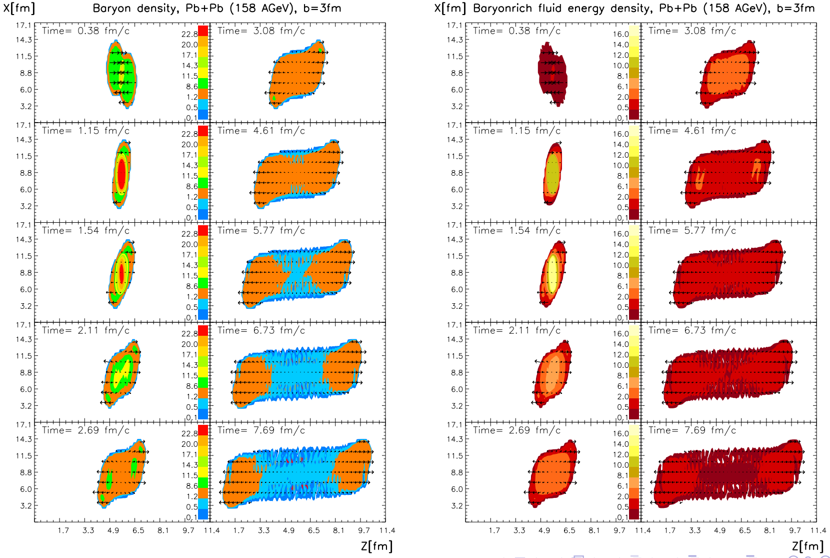
Hydro - concluding remarks

Yu.B.Ivanov at al., Phys. Rev. **C 73** 044904 (2006)



3-fluid model

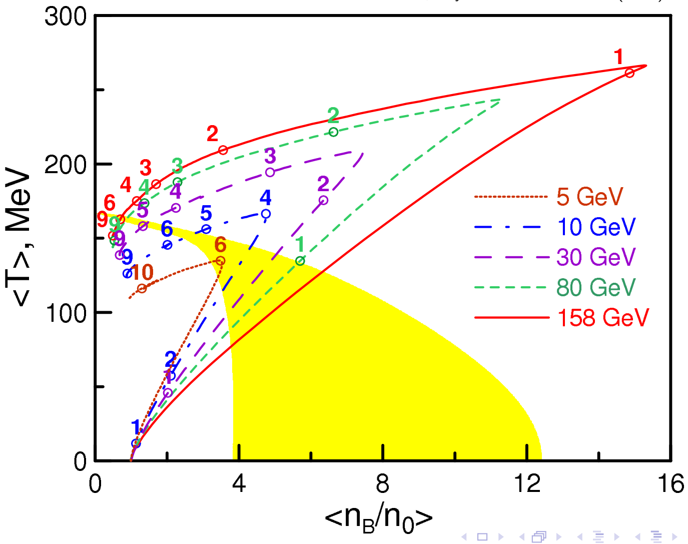
Yu.B.Ivanov at al., Phys. Rev. **C 73** 044904 (2006)



3-fluid model

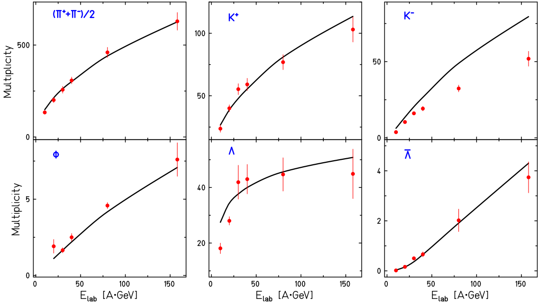
⋆ Phase diagram

Yu.B.Ivanov at al., Phys. Rev. **C 73** 044904 (2006)



3-fluid model

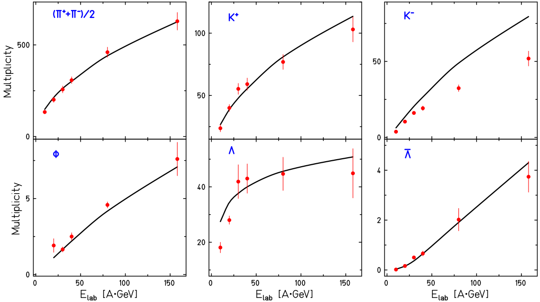
- ⋆ Freeze out (local $\varepsilon < \varepsilon_f$) $\Rightarrow$ observable



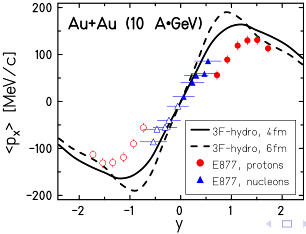
Yu.B.Ivanov at al.,
Phys. Rev. **C 73**
044904 (2006)

3-fluid model

★ Freeze out (local $\varepsilon < \varepsilon_f$) $\Rightarrow$ observable

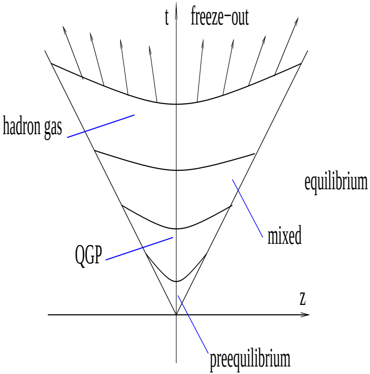


Yu.B.Ivanov at al., Phys. Rev. **C 73** 044904 (2006)



Space-time dynamics

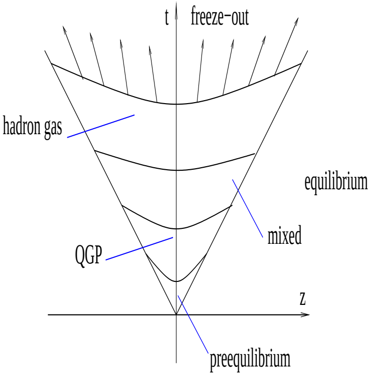
- Evolution of HIC in space-time



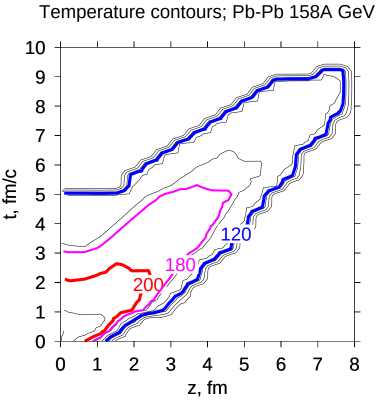
$$\tau^2 = t^2 - z^2$$

Space-time dynamics

- Evolution of HIC in space-time



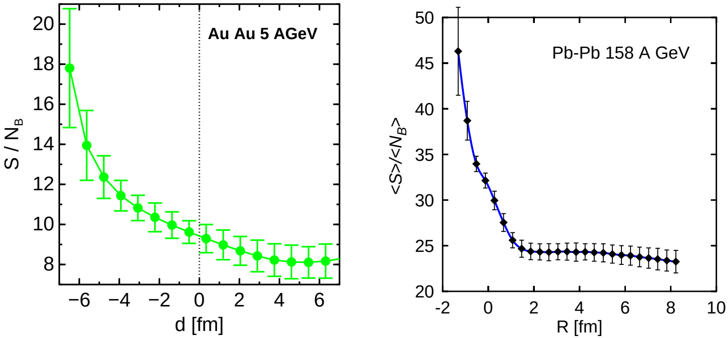
$$\tau^2 = t^2 - z^2$$



- Hybrid model : QGSM + relativistic 3D hydro expansion

Entropy creation and fireball formation are calculated within the **QGSM** transport code that defines an initial state for subsequent hydro stage

⋆ Marriage condition



Hybrid model

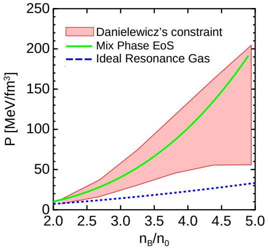
⋆ Equation of state (Statistical mixed phase model)

Hadronic sector:

interacting *h* gas in a mean field.

Saturation properties

(Binding energy, pressure, incompressibility at normal density)



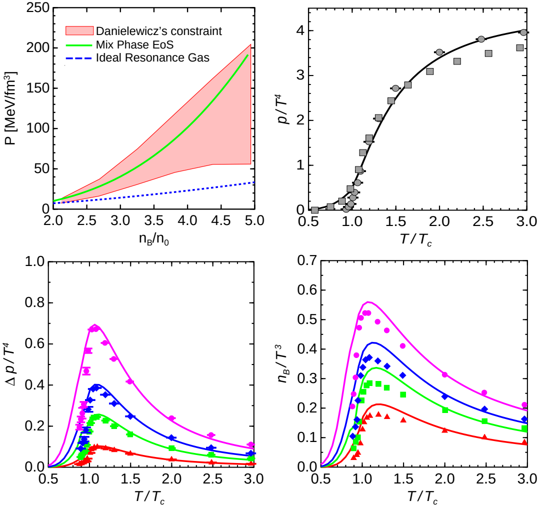
⋆ Equation of state (Statistical mixed phase model)

Hadronic sector:
interacting h gas in a mean field.
Saturation properties
(Binding energy, pressure, incompressibility at normal density)

Quark sector:
interacting q/g gas.
There is h-q/g interaction

crossover

$\Delta p = p(T, \mu_B) - p(T, \mu_B = 0)$

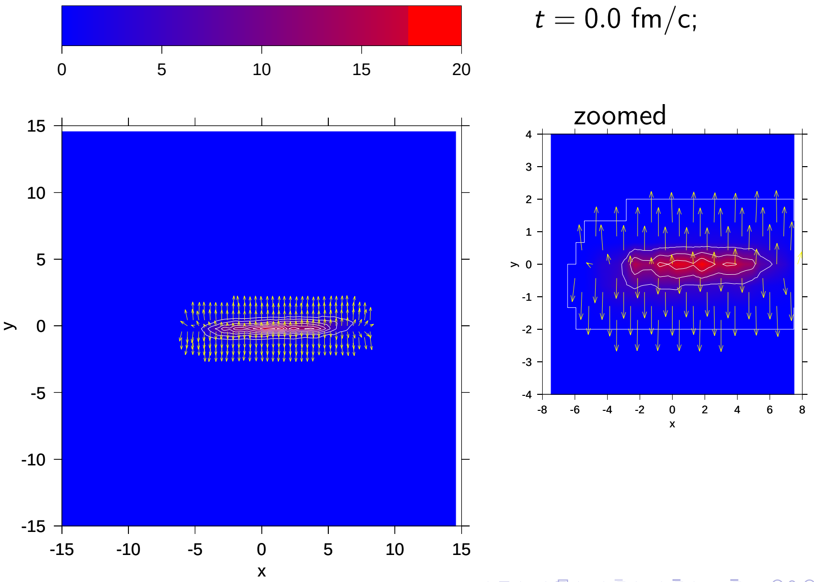


(2+1) QCD lattice results Z.Fodor and S.D.Katz, JHEP **203**, 14 (2002); JHEP **404**, 50 (2004)

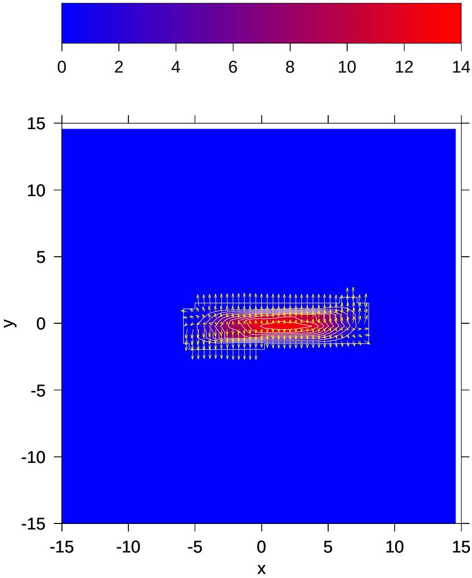
$\mu_B =$ 210, 330, 410, 530 MeV

Energy density evolution $Pb + Pb$ (158 AGeV)

$t = 0.0$ fm/c;



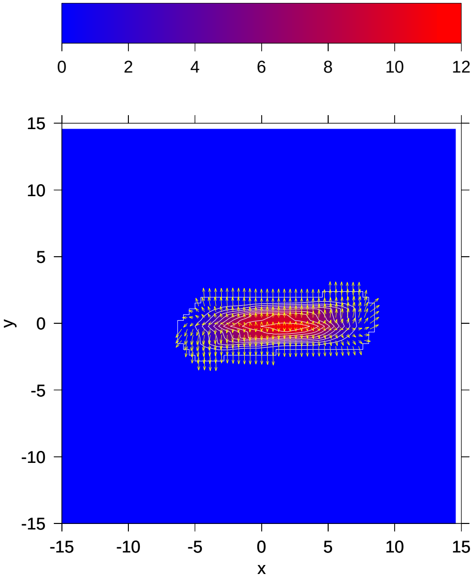
Dynamics of relativistic HI collisions
V. Toneev
Observable
Transport model results
Hydrodynamics
General remarks
Hydrodynamic models
Evolution in 3F hydro
3F hydro - comparison
Fireball evolution
Hybrid model
EoS
Fireball evolution
Evolution of averages
Hybrid - comparison
Dilepton production
Hydro - concluding remarks



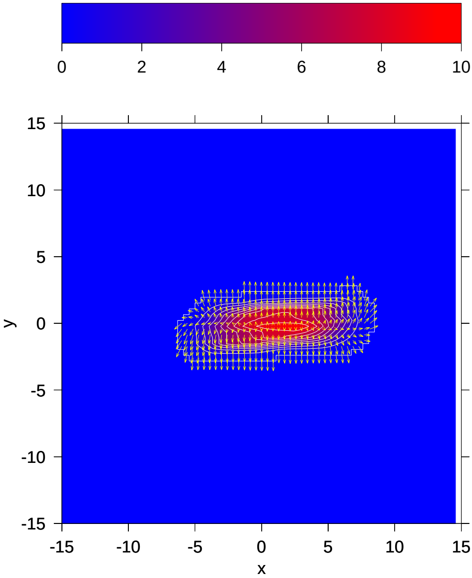
$t = 0.3$ fm/c

Energy density evolution $Pb + Pb$ (158 AGeV)

$t = 0.6$ fm/c

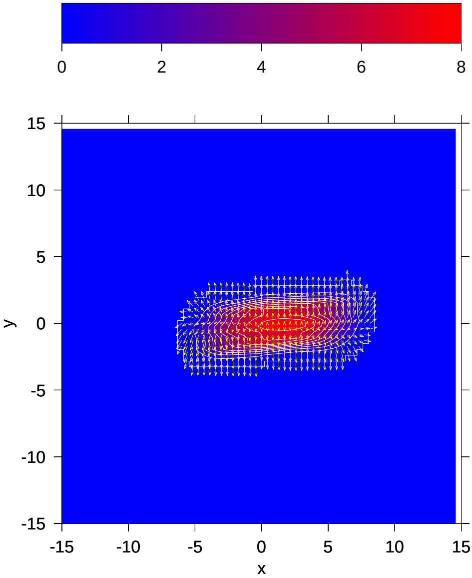


Energy density evolution $Pb + Pb$ (158 AGeV)



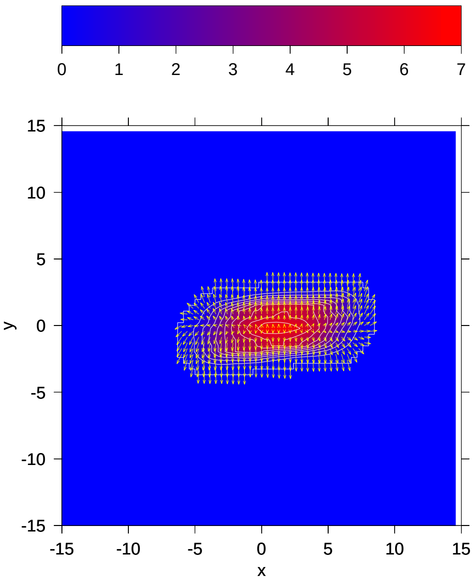
$t = 0.9$ fm/c

Energy density evolution $Pb + Pb$ (158 AGeV)



$t = 1.2$ fm/c

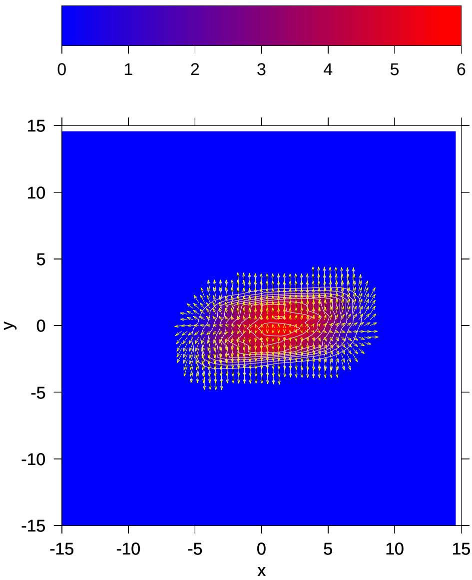
Energy density evolution $Pb + Pb$ (158 AGeV)



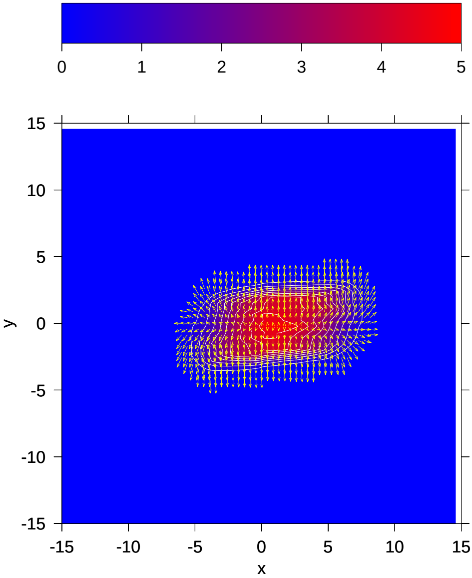
$t = 1.5$ fm/c

Energy density evolution $Pb + Pb$ (158 AGeV)

$t = 1.8$ fm/c

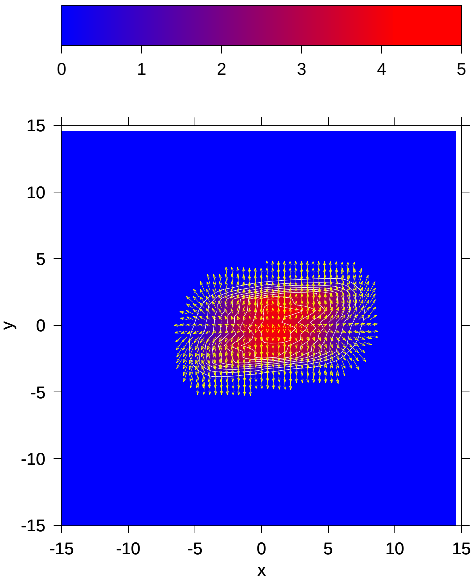


Energy density evolution $Pb + Pb$ (158 AGeV)



$t = 2.1$ fm/c

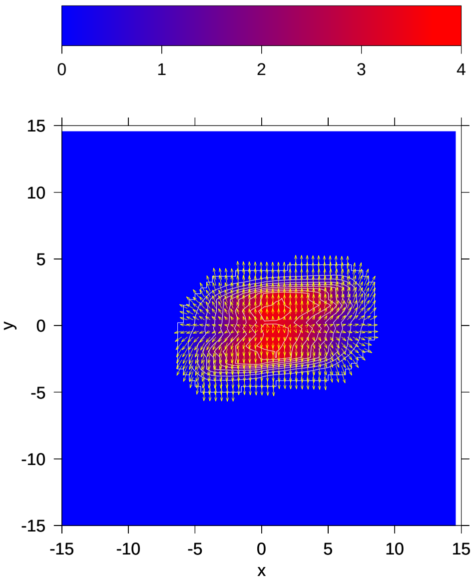
Energy density evolution $Pb + Pb$ (158 AGeV)



$t = 2.4$ fm/c

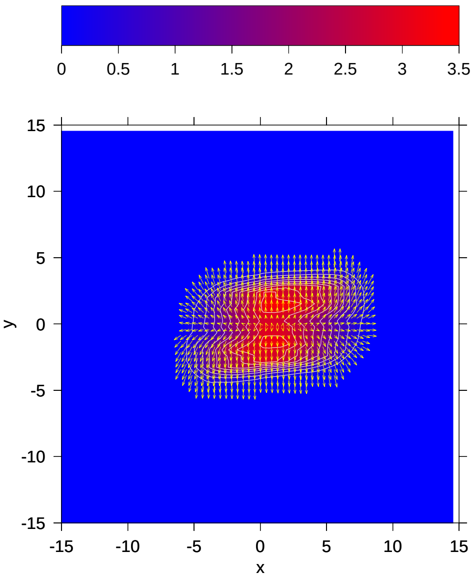
Energy density evolution $Pb + Pb$ (158 AGeV)

$t = 2.7$ fm/c



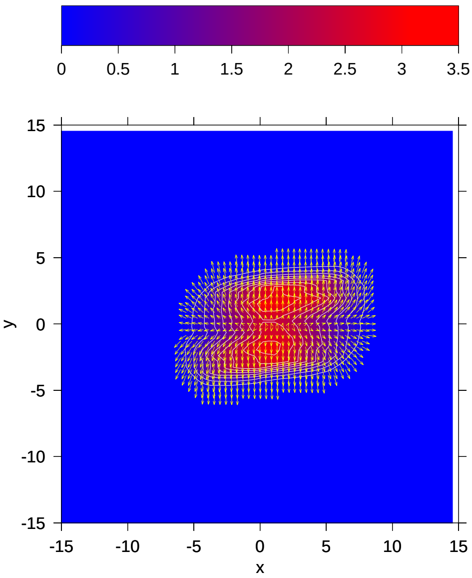
Energy density evolution $Pb + Pb$ (158 AGeV)

$t = 3.0$ fm/c

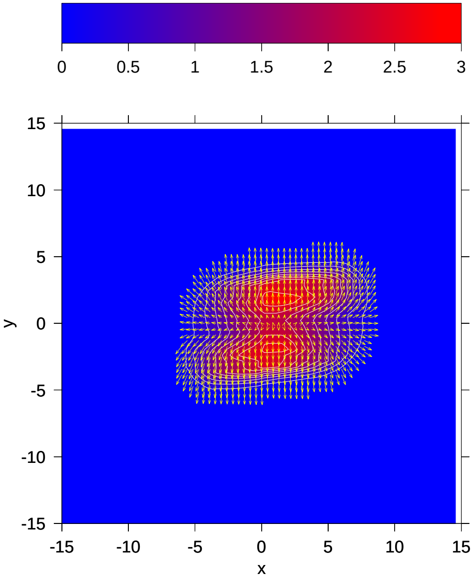


Energy density evolution $Pb + Pb$ (158 AGeV)

$t = 3.3$ fm/c



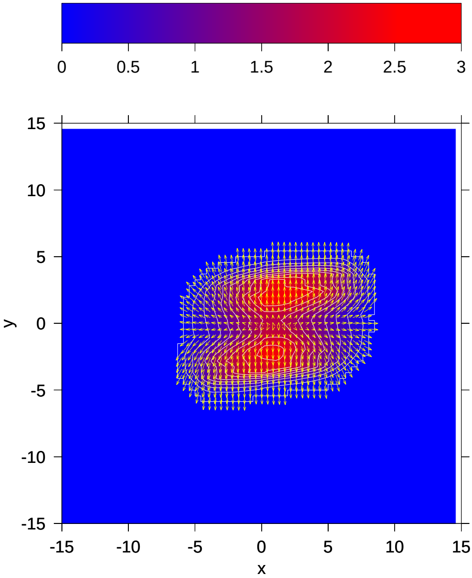
Energy density evolution $Pb + Pb$ (158 AGeV)



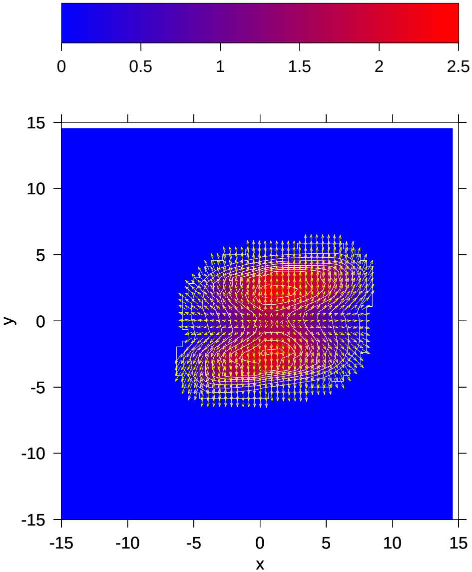
$t = 3.6$ fm/c

Energy density evolution $Pb + Pb$ (158 AGeV)

$t = 3.9$ fm/c

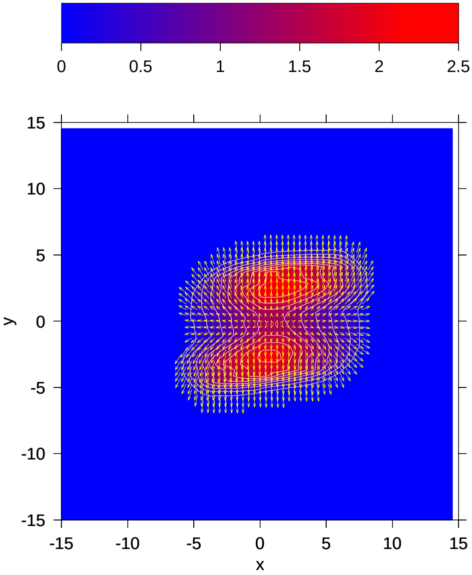


Energy density evolution $Pb + Pb$ (158 AGeV)



$t = 4.2$ fm/c

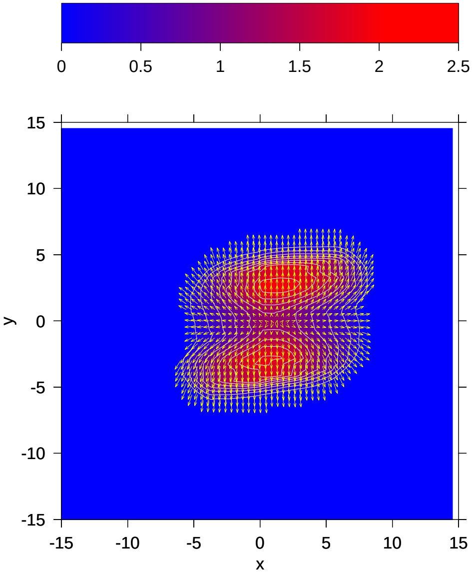
Energy density evolution $Pb + Pb$ (158 AGeV)

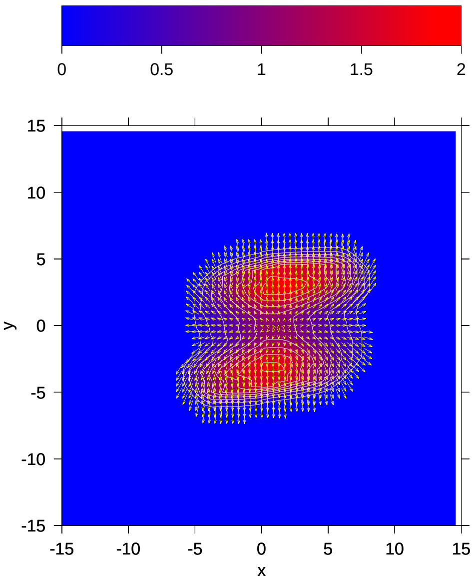


$t = 4.5$ fm/c

Energy density evolution $Pb + Pb$ (158 AGeV)

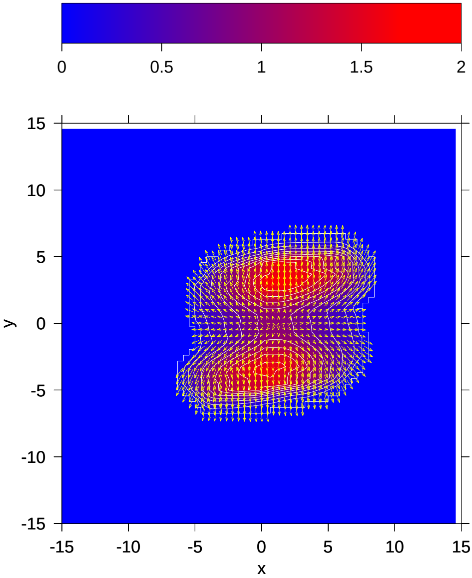
$t = 4.8$ fm/c





$t = 5.1$ fm/c

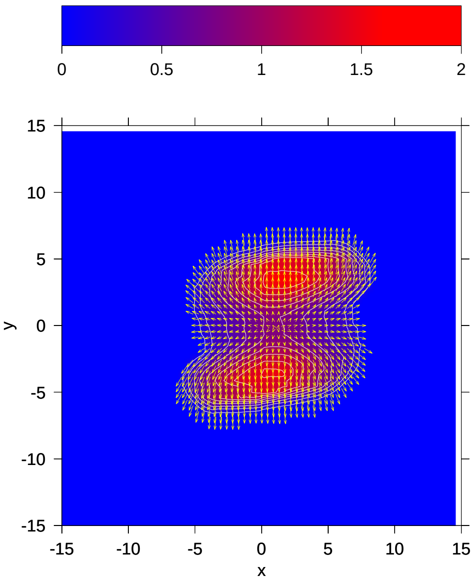
Energy density evolution $Pb + Pb$ (158 AGeV)



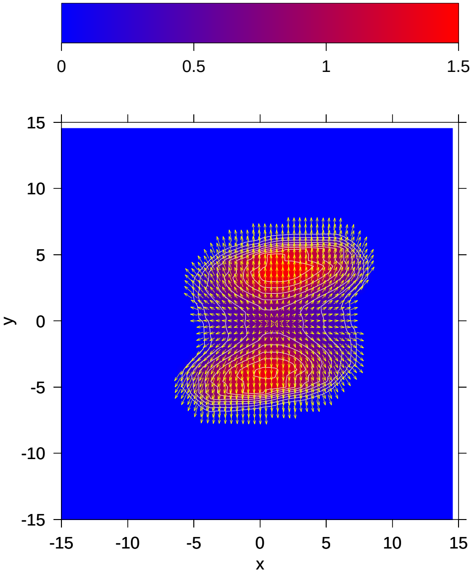
$t = 5.4$ fm/c

Energy density evolution $Pb + Pb$ (158 AGeV)

$t = 5.7$ fm/c



Energy density evolution $Pb + Pb$ (158 AGeV)



$t = 6.0$ fm/c

Energy density evolution $Pb + Pb$ (158 AGeV)

Dynamics of relativistic HI collisions

V. Toneev

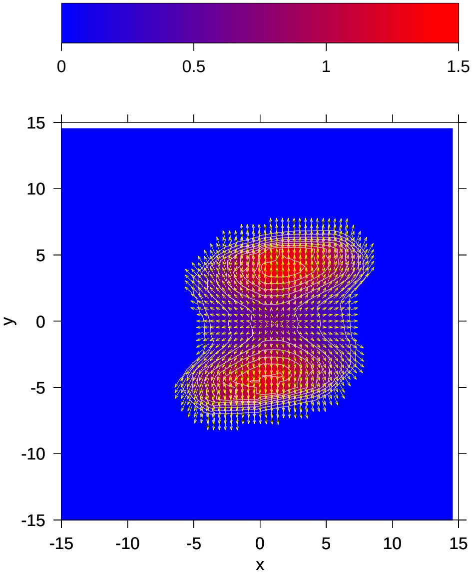
Observable

Transport model results

Hydrodynamics
General remarks
Hydrodynamic models
Evolution in 3F hydro
3F hydro - comparison
Fireball evolution

Hybrid model
EoS
Fireball evolution
Evolution of averages
Hybrid - comparison
Dilepton production

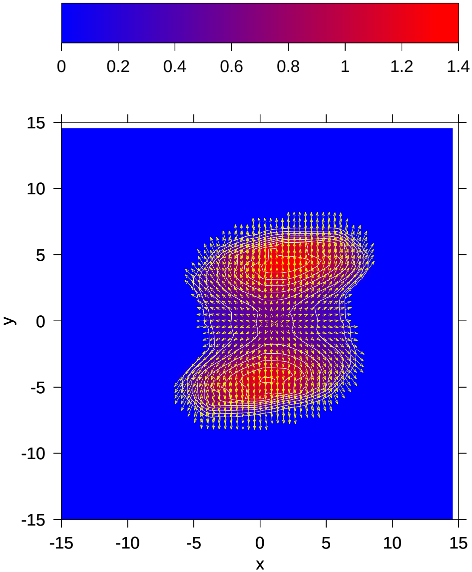
Hydro - concluding remarks



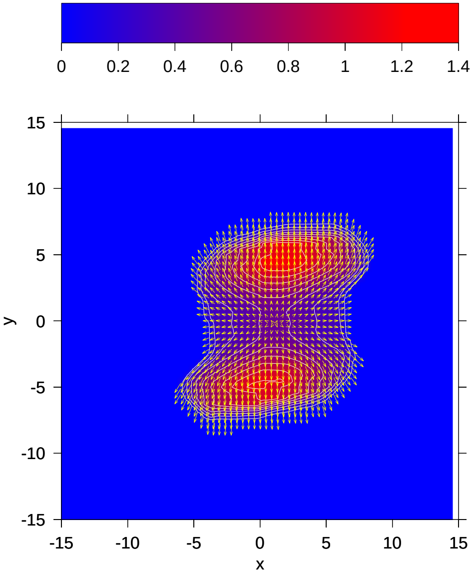
$t = 6.3$ fm/c

Energy density evolution $Pb + Pb$ (158 AGeV)

$t = 6.6$ fm/c

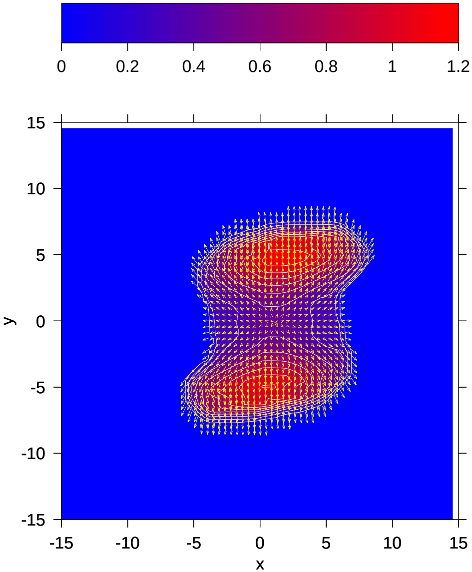


Energy density evolution $Pb + Pb$ (158 AGeV)



$t = 6.9$ fm/c

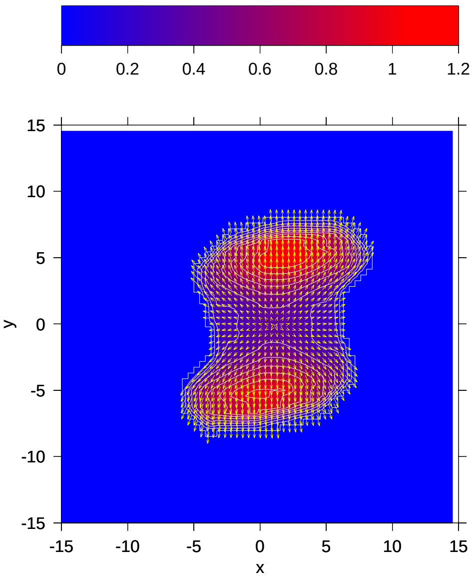
Energy density evolution $Pb + Pb$ (158 AGeV)



$t = 7.2$ fm/c

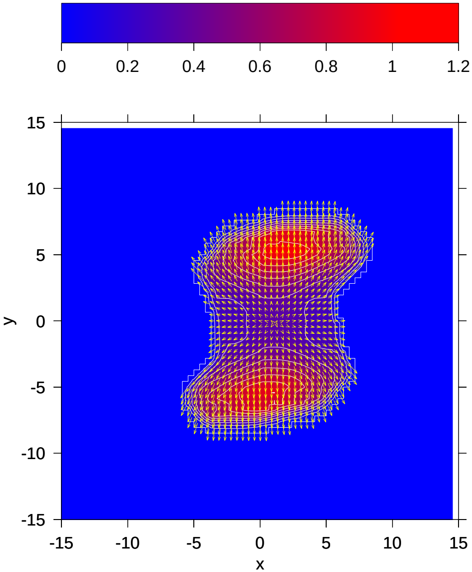
Energy density evolution $Pb + Pb$ (158 AGeV)

$t = 7.5$ fm/c

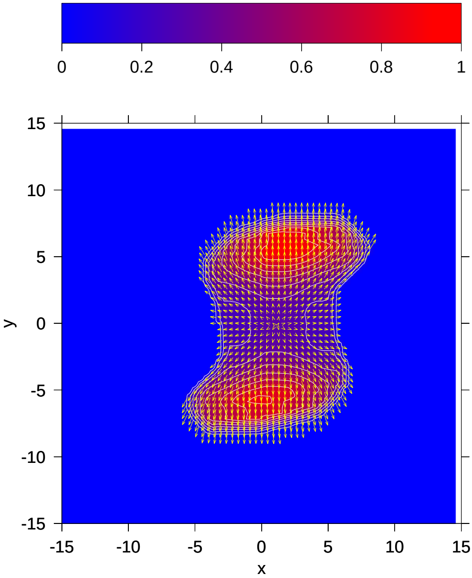


Energy density evolution $Pb + Pb$ (158 AGeV)

$t = 7.8$ fm/c



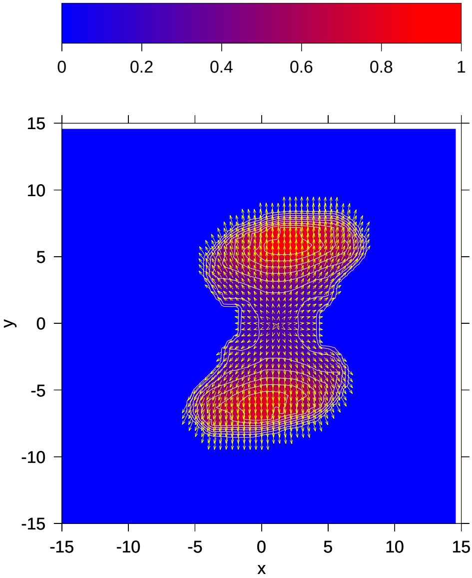
Energy density evolution $Pb + Pb$ (158 AGeV)



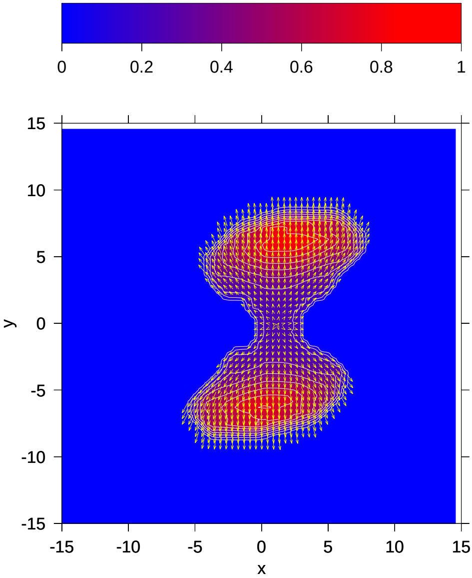
$t = 8.1$ fm/c

Energy density evolution $Pb + Pb$ (158 AGeV)

$t = 8.4$ fm/c

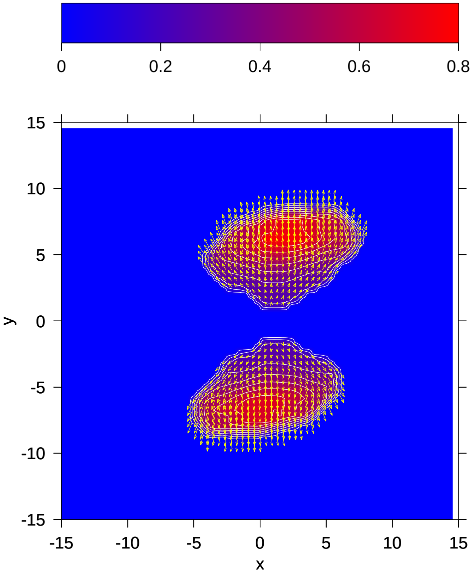


Energy density evolution $Pb + Pb$ (158 AGeV)



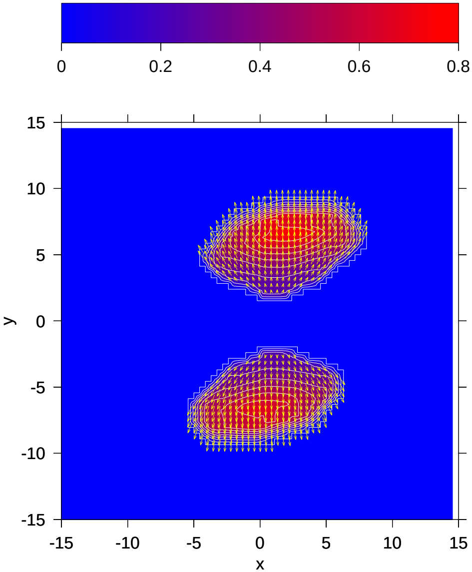
$t = 8.7$ fm/c

Energy density evolution $Pb + Pb$ (158 AGeV)



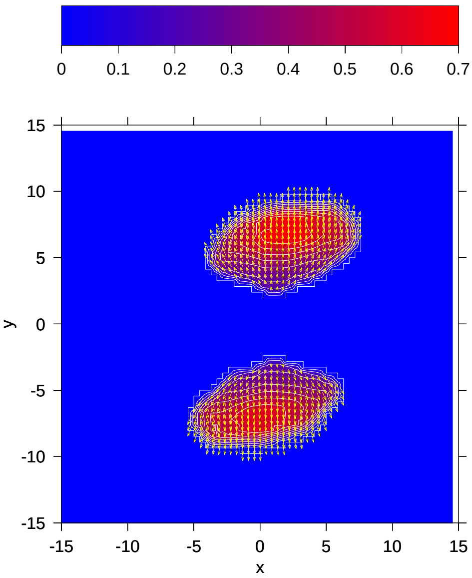
$t = 9.0$ fm/c

Energy density evolution $Pb + Pb$ (158 AGeV)



$t = 9.3$ fm/c

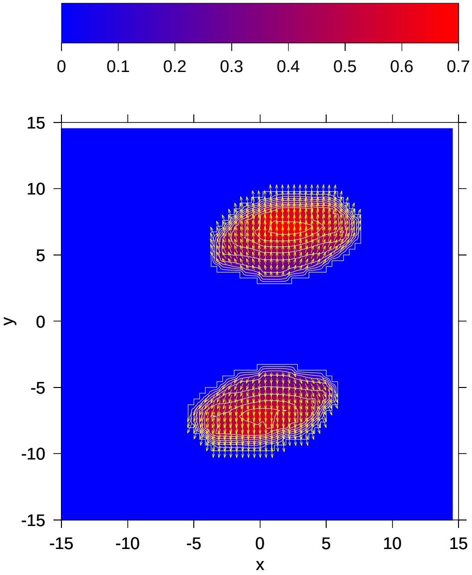
Energy density evolution $Pb + Pb$ (158 AGeV)



$t = 9.6$ fm/c

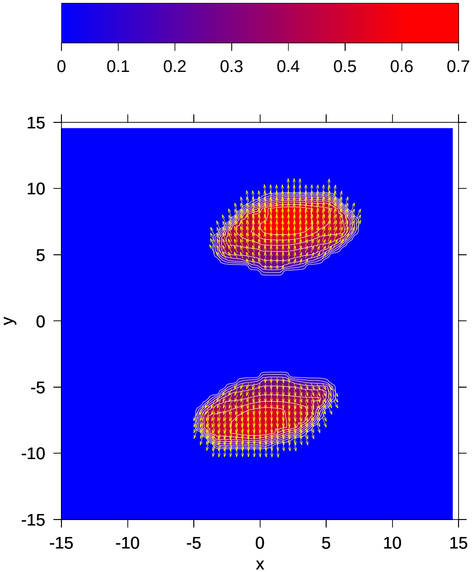
Energy density evolution $Pb + Pb$ (158 AGeV)

$t = 9.9$ fm/c

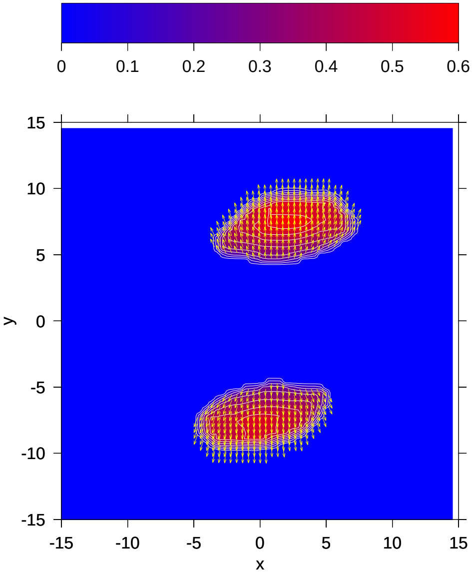


Energy density evolution $Pb + Pb$ (158 AGeV)

$t = 10.2$ fm/c

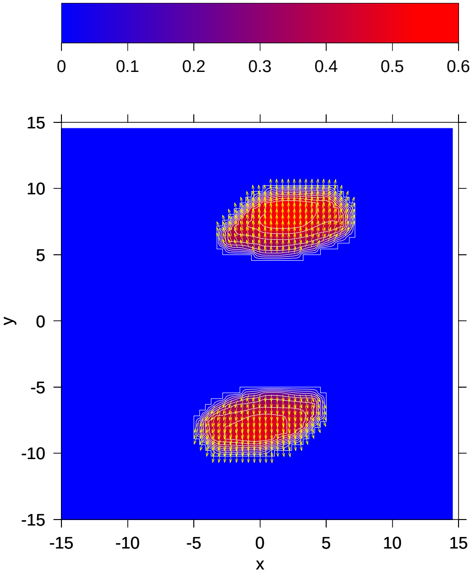


Energy density evolution $Pb + Pb$ (158 AGeV)



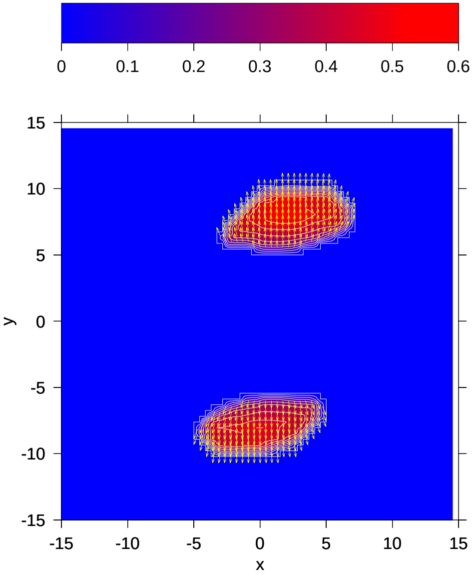
$t = 10.5$ fm/c

Energy density evolution $Pb + Pb$ (158 AGeV)



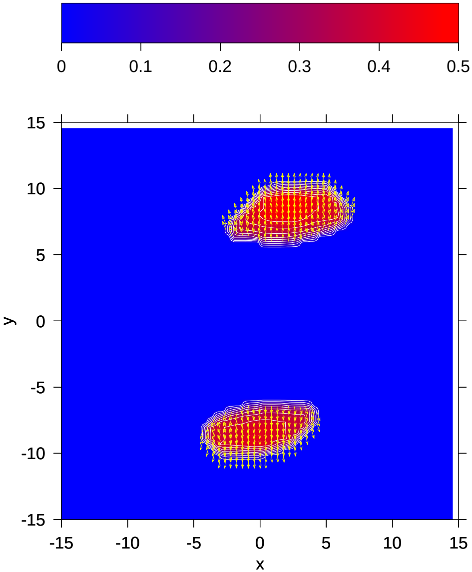
$t = 10.8$ fm/c

Energy density evolution $Pb + Pb$ (158 AGeV)



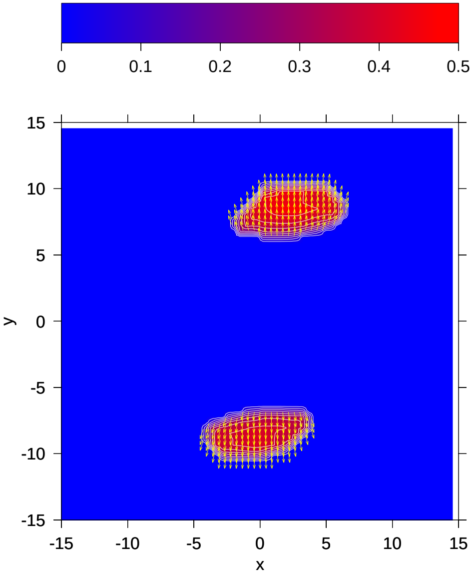
$t = 11.1$ fm/c

Energy density evolution $Pb + Pb$ (158 AGeV)



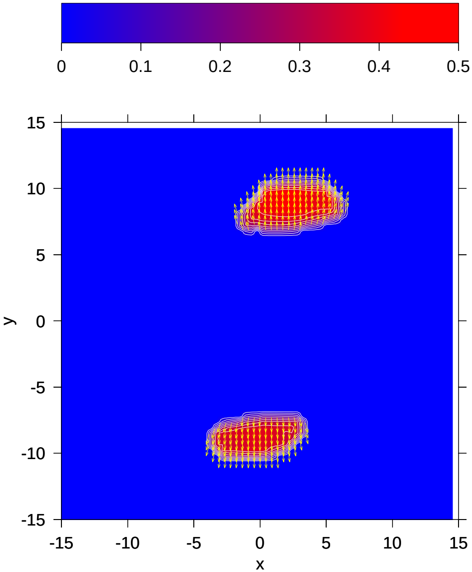
$t = 11.4$ fm/c

Energy density evolution $Pb + Pb$ (158 AGeV)



$t = 11.7$ fm/c

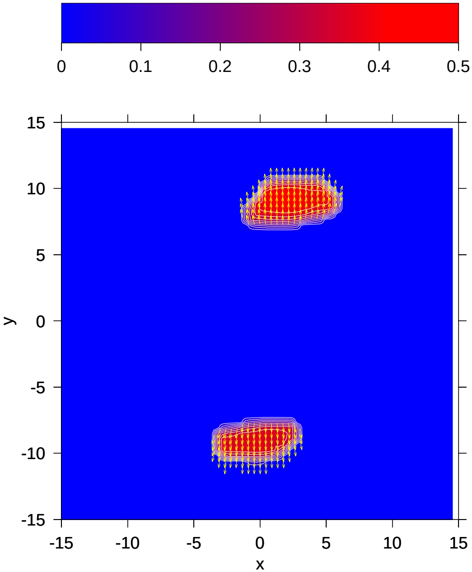
Energy density evolution $Pb + Pb$ (158 AGeV)



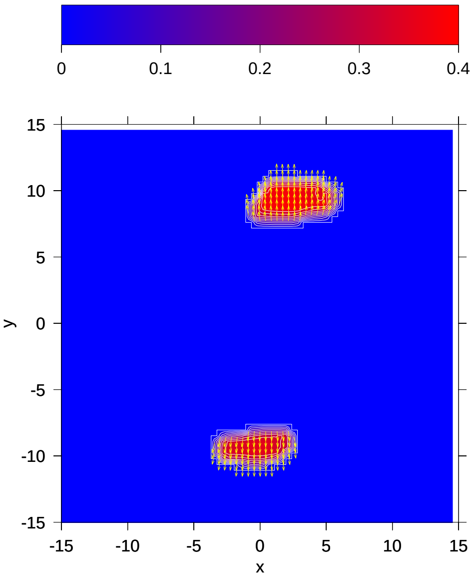
$t = 12.0$ fm/c

Energy density evolution $Pb + Pb$ (158 AGeV)

$t = 12.3$ fm/c

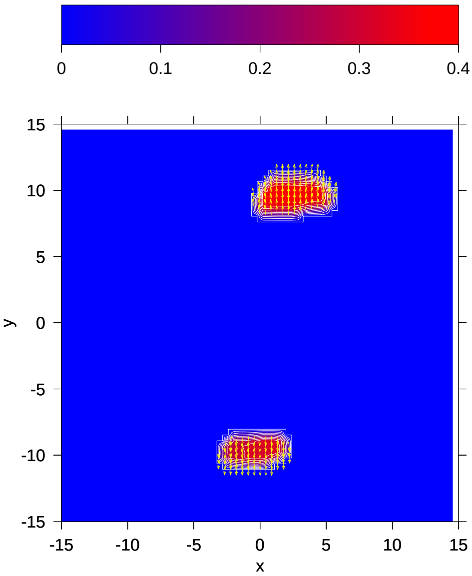


Energy density evolution $Pb + Pb$ (158 AGeV)



$t = 12.6$ fm/c

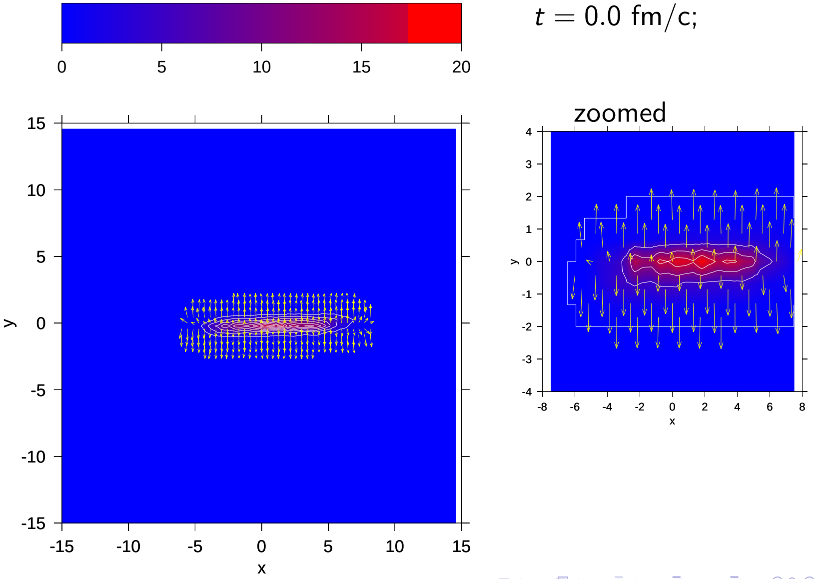
Energy density evolution $Pb + Pb$ (158 AGeV)



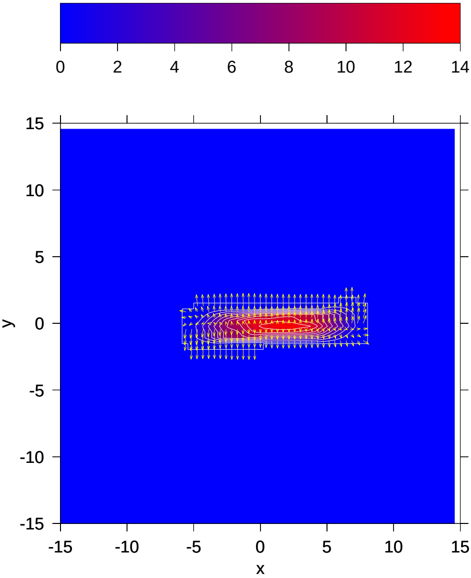
$t = 12.9$ fm/c

Time evolution of the fireball energy density

$t = 0.0$ fm/c;

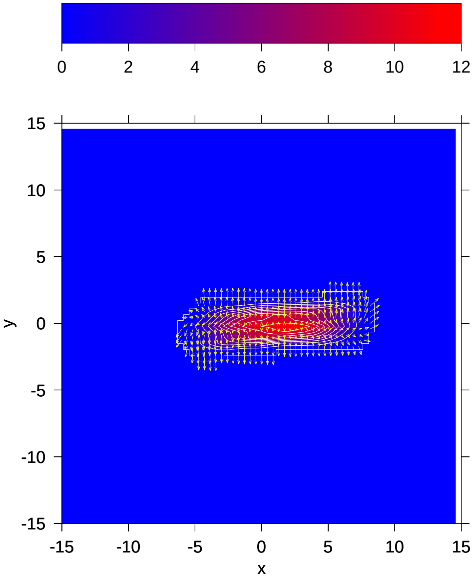


Time evolution of the fireball energy density



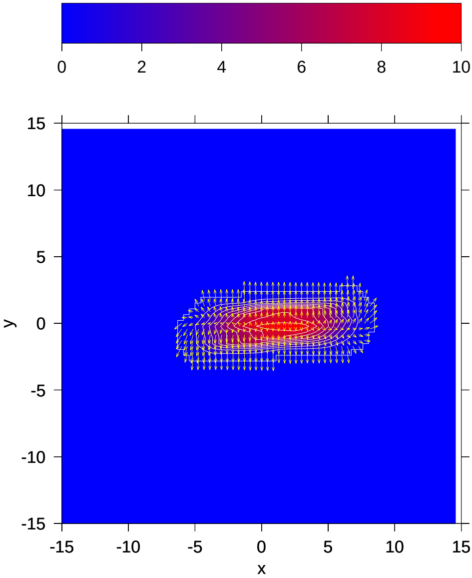
$t = 0.3$ fm/c

Time evolution of the fireball energy density



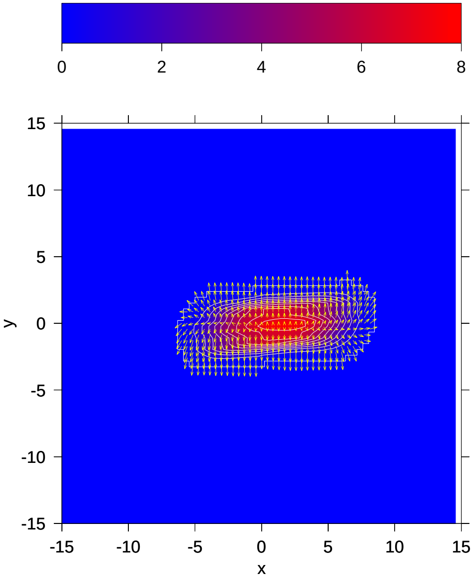
$t = 0.6$ fm/c

Time evolution of the fireball energy density



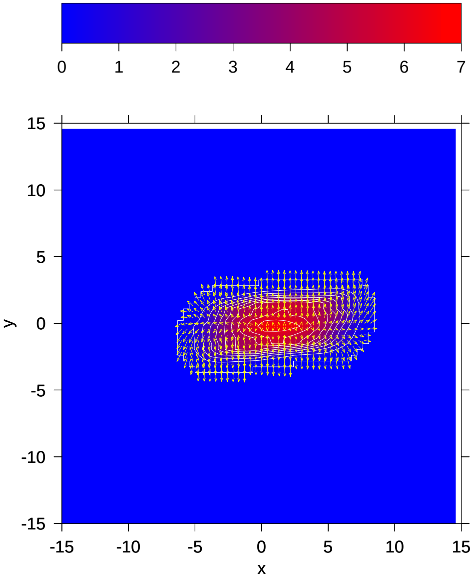
$t = 0.9$ fm/c

Time evolution of the fireball energy density



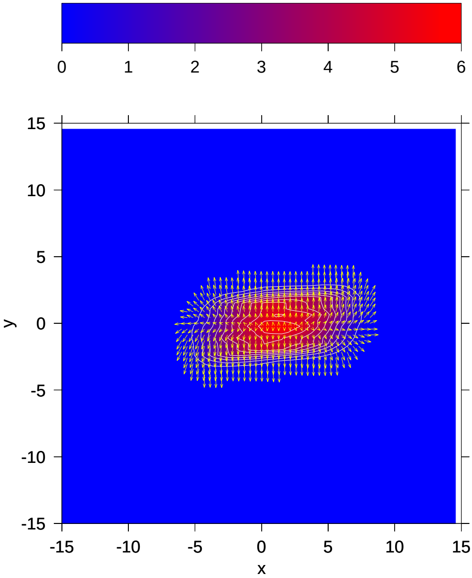
$t = 1.2$ fm/c

Time evolution of the fireball energy density



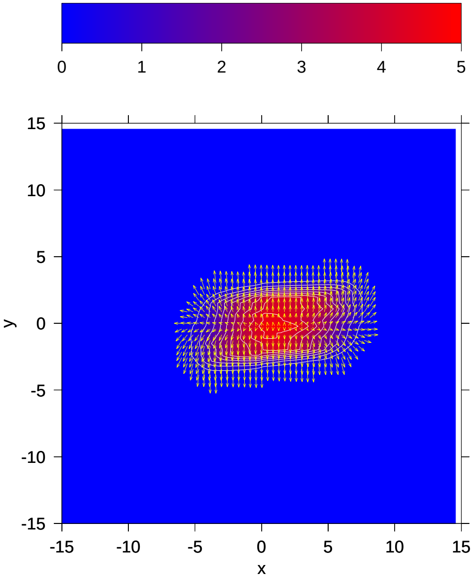
$t = 1.5$ fm/c

Time evolution of the fireball energy density



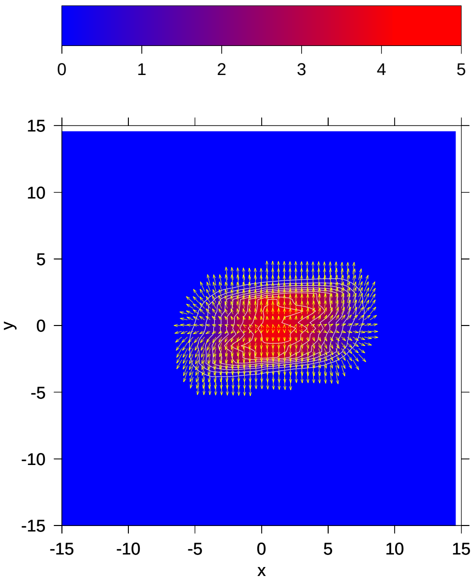
$t = 1.8$ fm/c

Time evolution of the fireball energy density



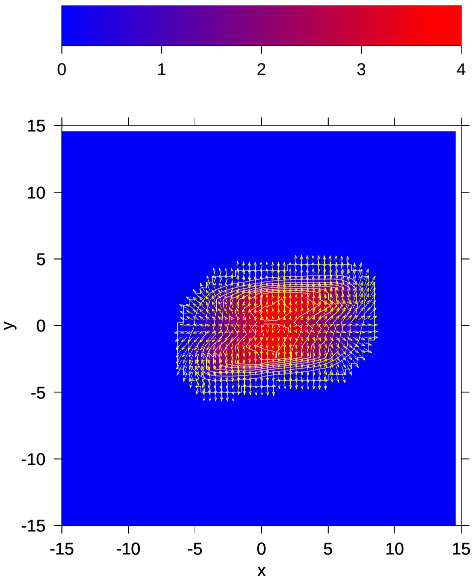
$t = 2.1$ fm/c

Time evolution of the fireball energy density



$t = 2.4$ fm/c

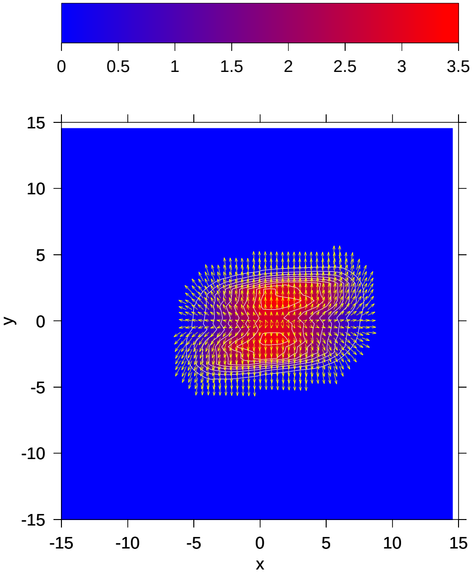
Time evolution of the fireball energy density



$t = 2.7$ fm/c

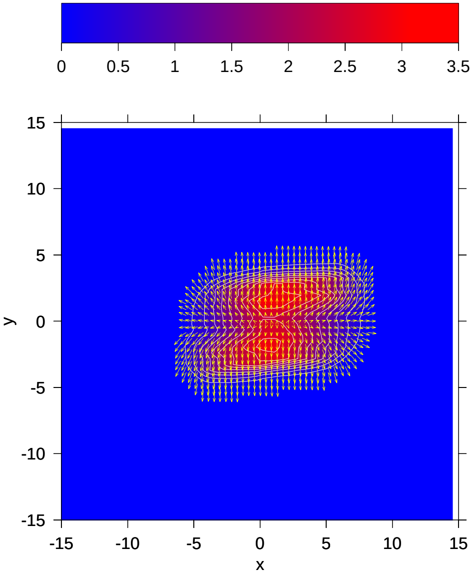
Time evolution of the fireball energy density

$t = 3.0$ fm/c

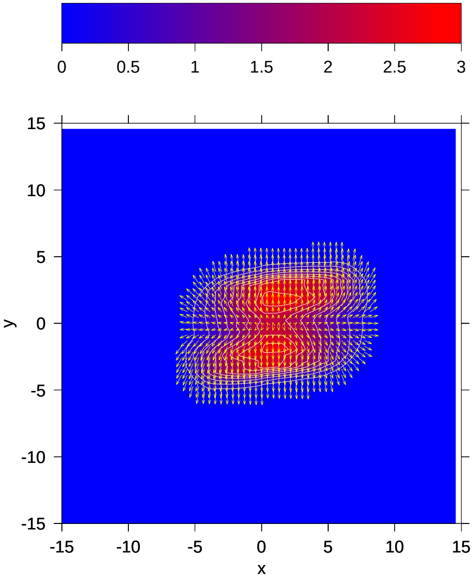


Time evolution of the fireball energy density

$t = 3.3$ fm/c

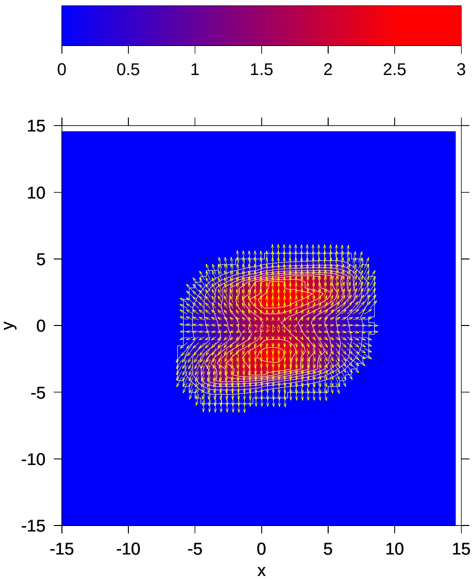


Time evolution of the fireball energy density



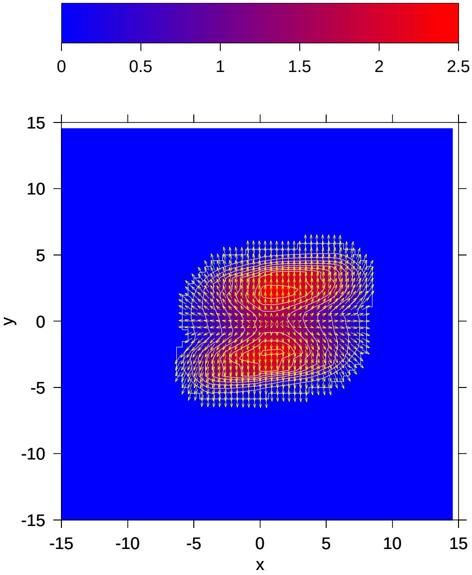
$t = 3.6$ fm/c

Time evolution of the fireball energy density



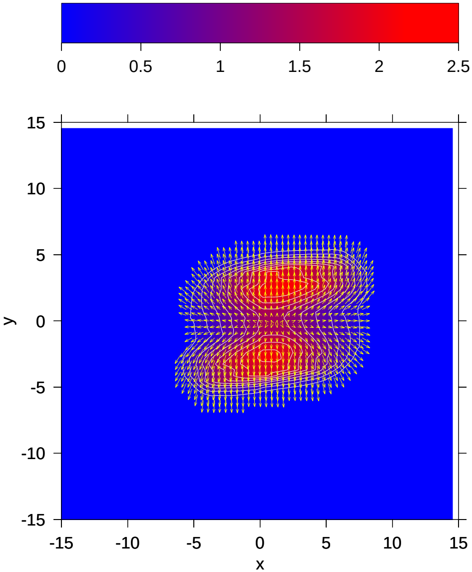
$t = 3.9$ fm/c

Time evolution of the fireball energy density



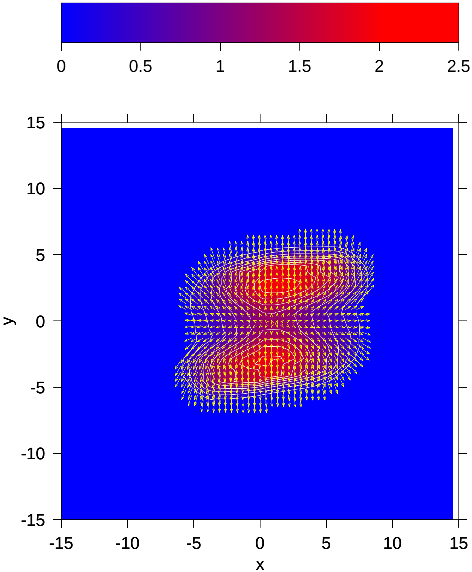
$t = 4.2\ \text{fm}/c$

Time evolution of the fireball energy density



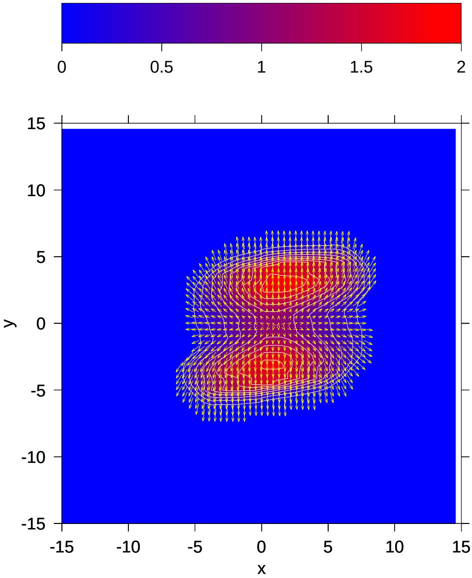
$t = 4.5$ fm/c

Time evolution of the fireball energy density



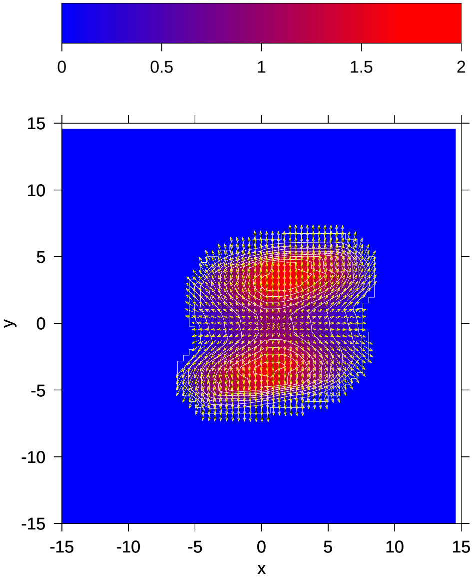
$t = 4.8$ fm/c

Time evolution of the fireball energy density



$t = 5.1$ fm/c

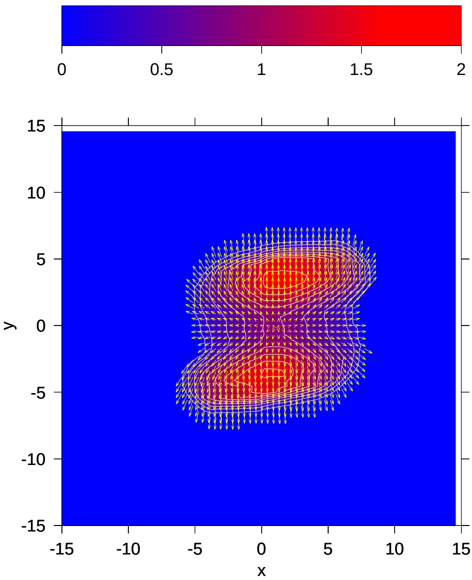
Time evolution of the fireball energy density



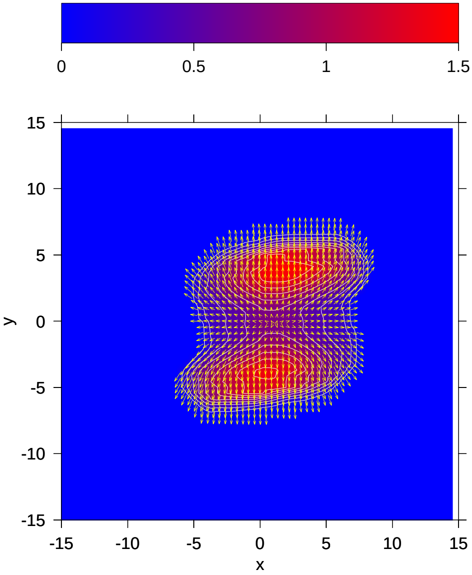
$t = 5.4$ fm/c

Time evolution of the fireball energy density

$t = 5.7$ fm/c



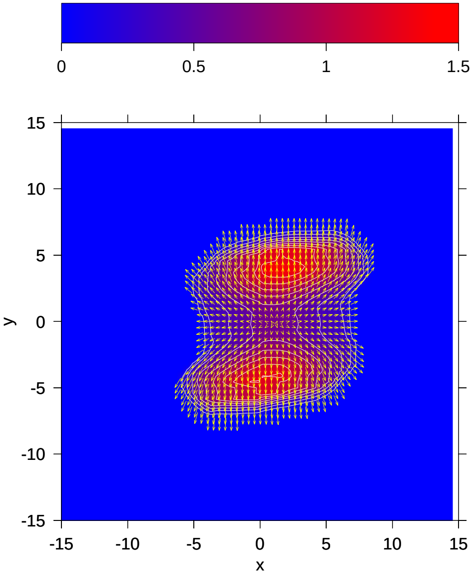
Time evolution of the fireball energy density



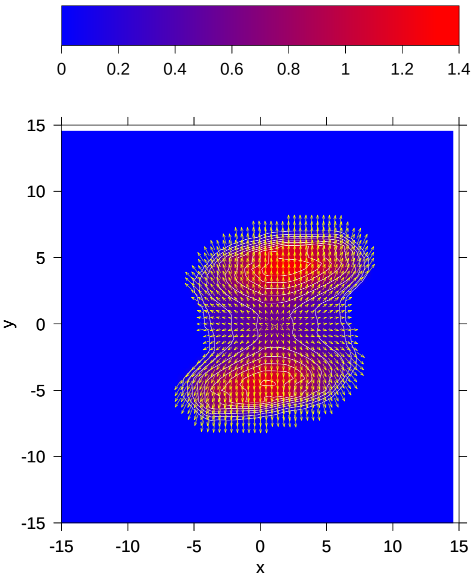
$t = 6.0$ fm/c

Time evolution of the fireball energy density

$t = 6.3$ fm/c

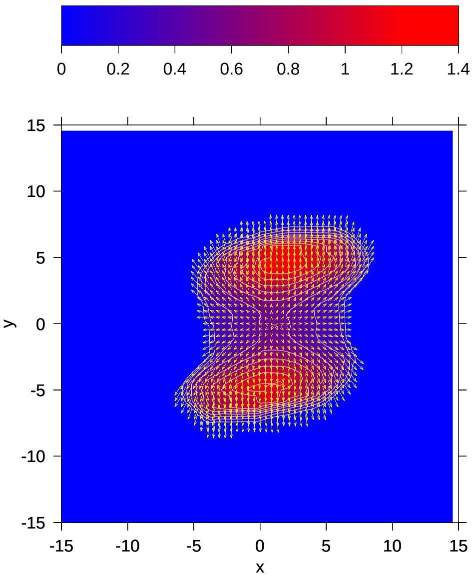


Time evolution of the fireball energy density



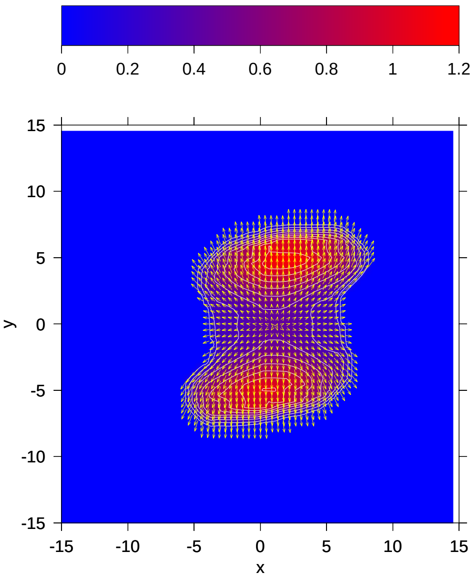
$t = 6.6$ fm/c

Time evolution of the fireball energy density



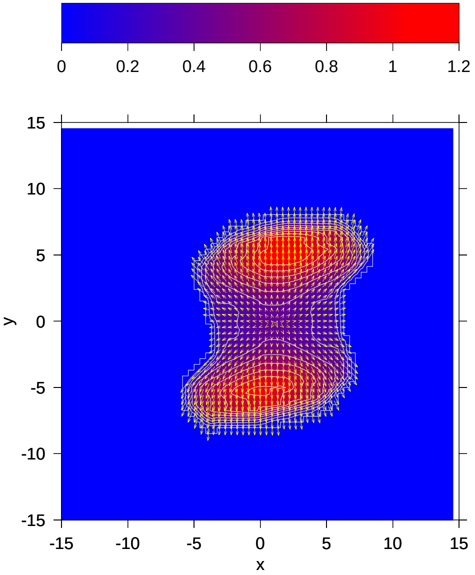
$t = 6.9$ fm/c

Time evolution of the fireball energy density



$t = 7.2$ fm/c

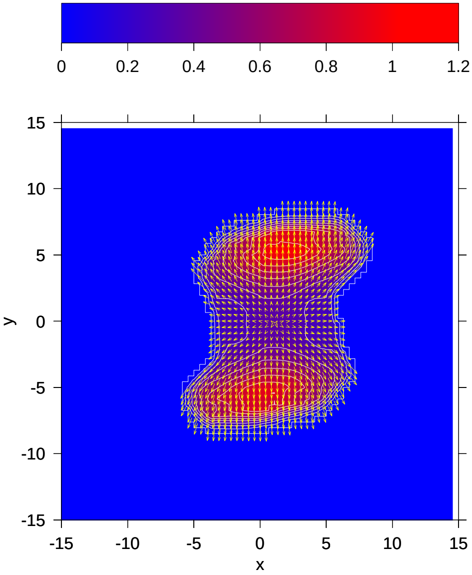
Time evolution of the fireball energy density



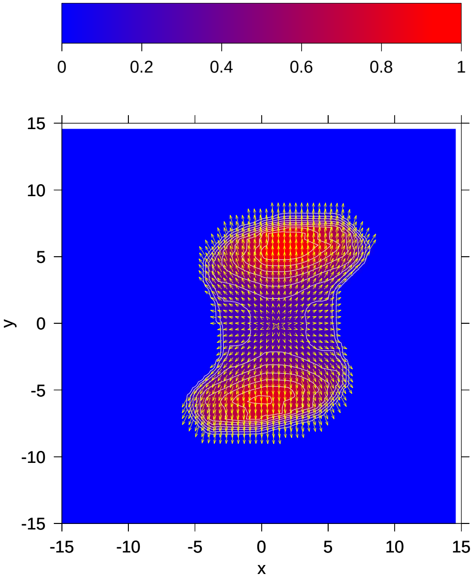
$t = 7.5$ fm/c

Time evolution of the fireball energy density

$t = 7.8$ fm/c

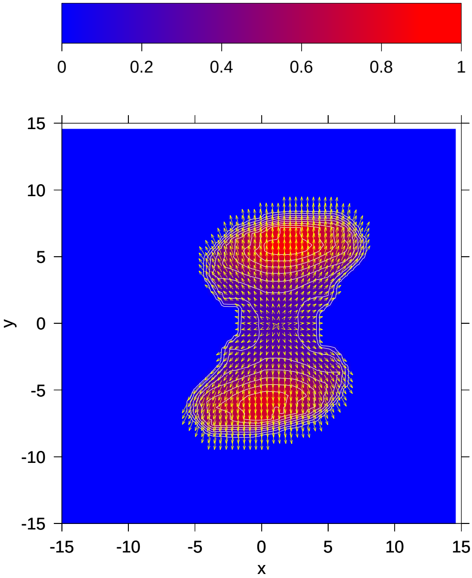


Time evolution of the fireball energy density



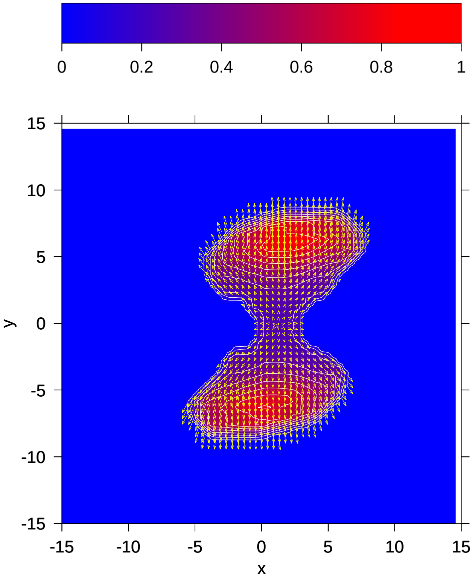
$t = 8.1$ fm/c

Time evolution of the fireball energy density

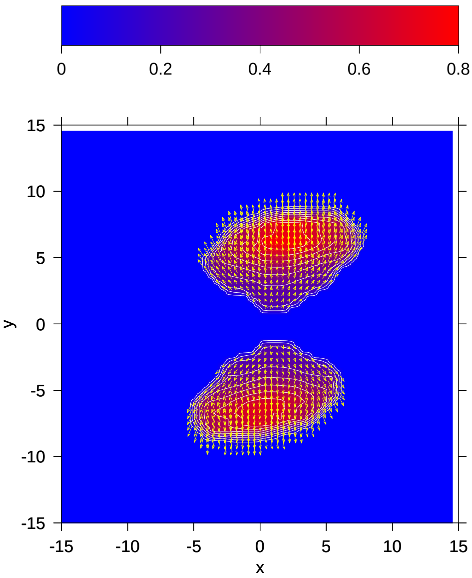


$t = 8.4$ fm/c

Time evolution of the fireball energy density

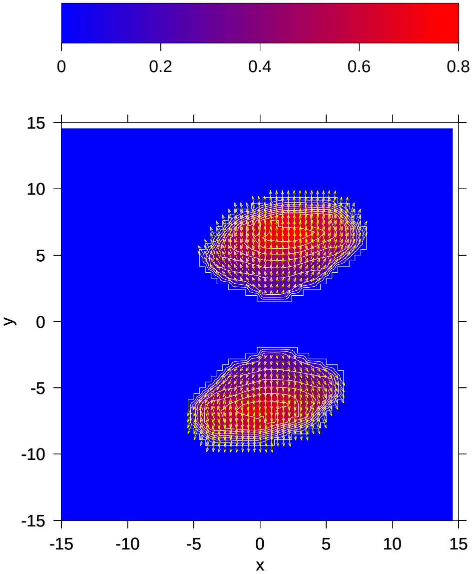


$t = 8.7$ fm/c



$t = 9.0$ fm/c

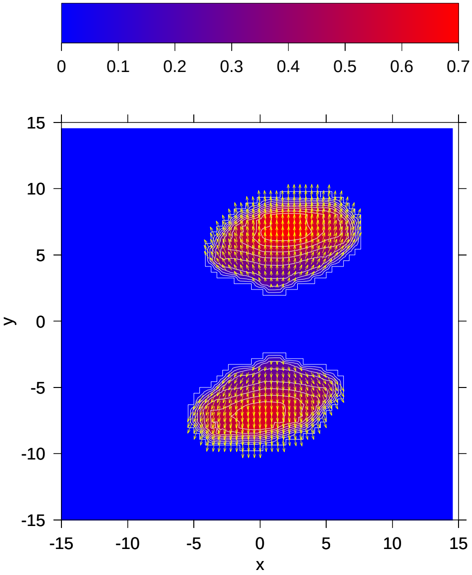
Time evolution of the fireball energy density



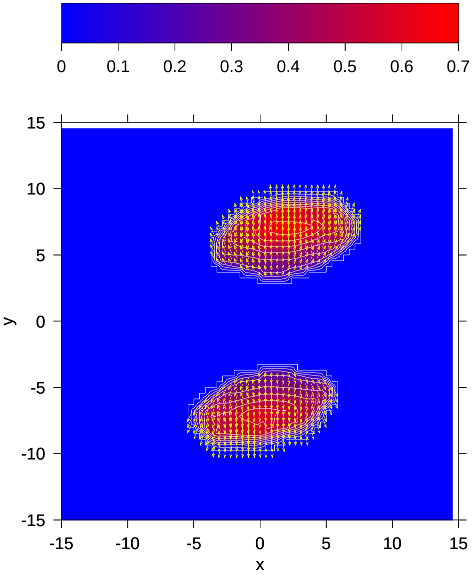
$t = 9.3$ fm/c

Time evolution of the fireball energy density

$t = 9.6$ fm/c



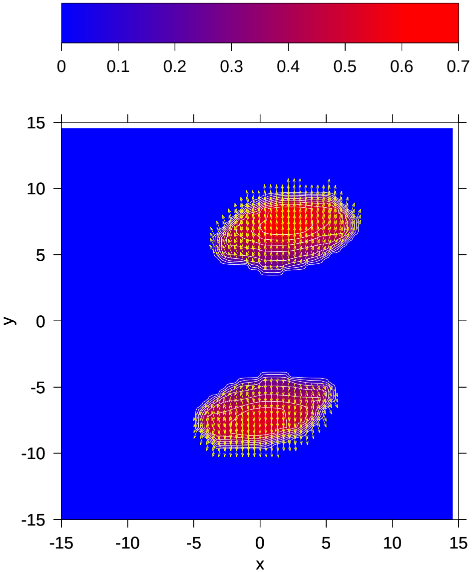
Time evolution of the fireball energy density



$t = 9.9$ fm/c

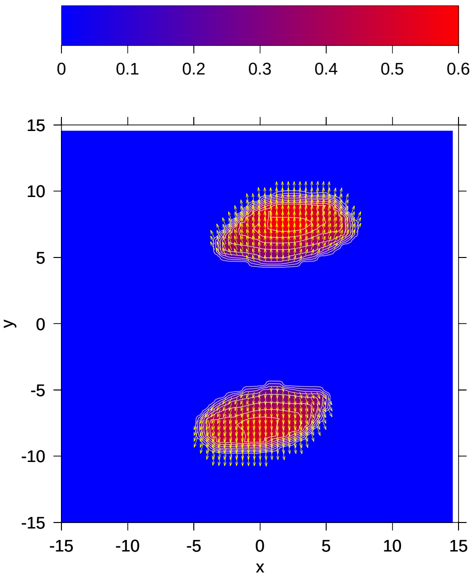
Time evolution of the fireball energy density

$t = 10.2$ fm/c

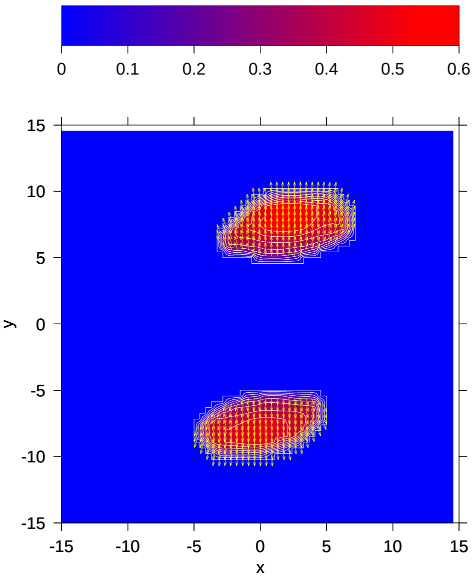


Time evolution of the fireball energy density

$t = 10.5$ fm/c

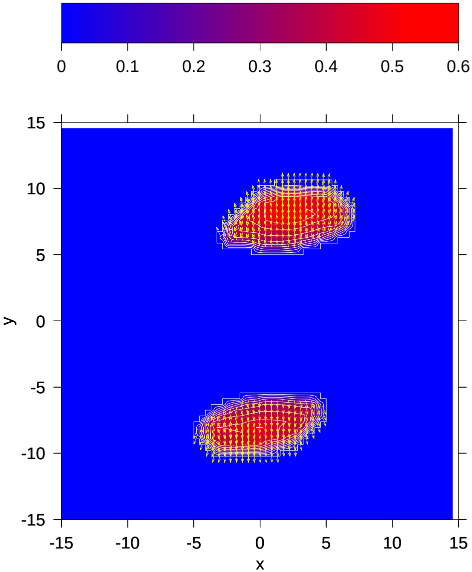


Time evolution of the fireball energy density



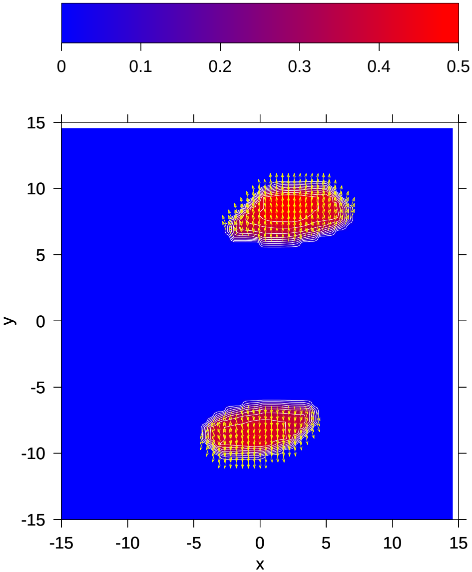
$t = 10.8$ fm/c

Time evolution of the fireball energy density



$t = 11.1$ fm/c

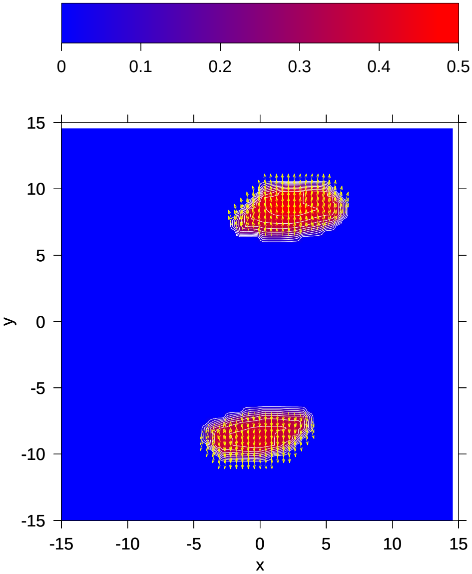
Time evolution of the fireball energy density



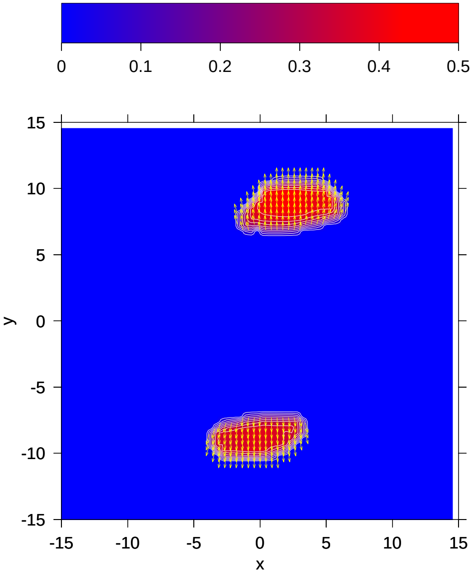
$t = 11.4$ fm/c

Time evolution of the fireball energy density

$t = 11.7$ fm/c

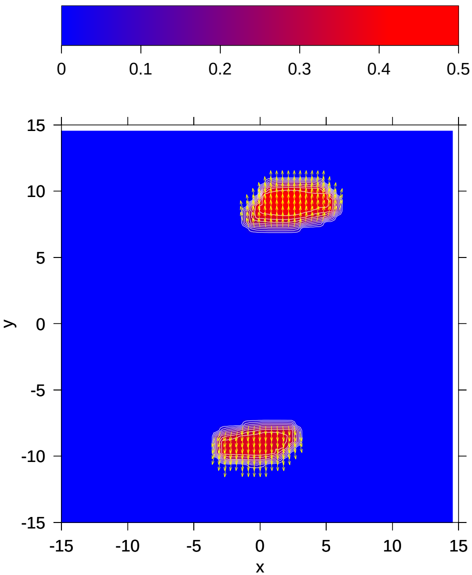


Time evolution of the fireball energy density



$t = 12.0$ fm/c

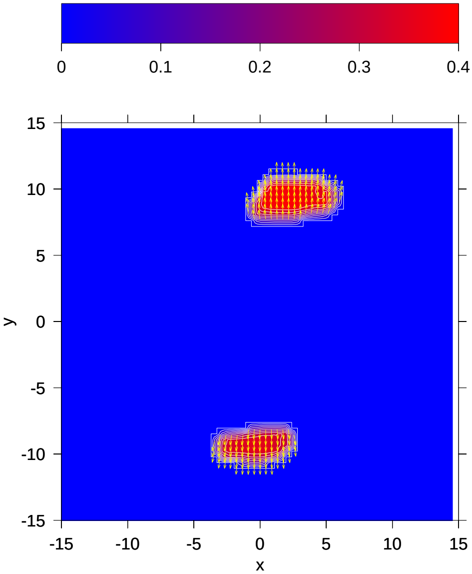
Time evolution of the fireball energy density



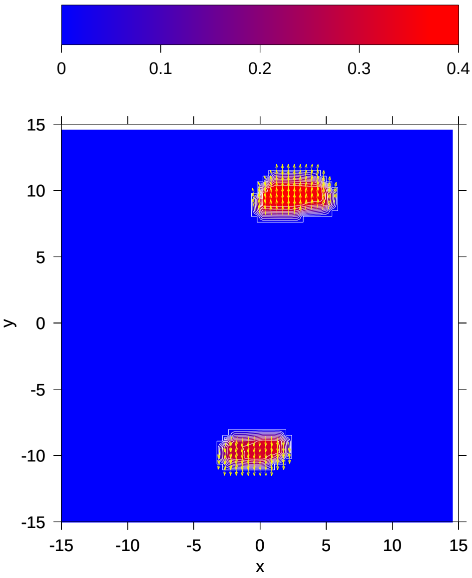
$t = 12.3$ fm/c

Time evolution of the fireball energy density

$t = 12.6$ fm/c



Time evolution of the fireball energy density

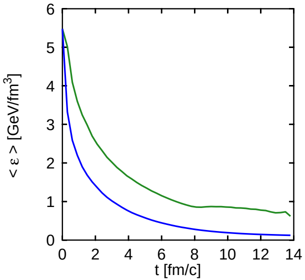


$t = 12.9$ fm/c

Average quantities

$Pb + Pb$ (158 $AGeV$)

- Average energy density (mixed phase EoS, mixed phase EoS without freeze-out)

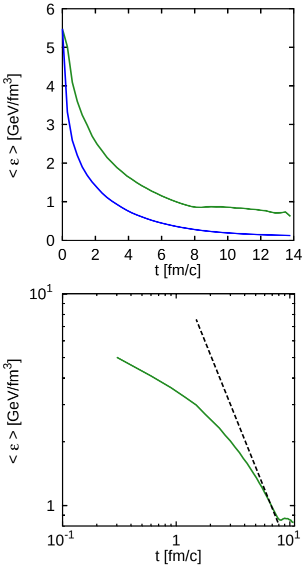


Average quantities

Pb + *Pb* (158 *AGeV*)

- Average energy density (mixed phase EoS, mixed phase EoS without freeze-out)

and comparison with the Bjorken scaling regime (ultrarelativistic ideal gas EoS: $\varepsilon = \frac{1}{3}P$, dashed line with the slope $-4/3$)



Average quantities

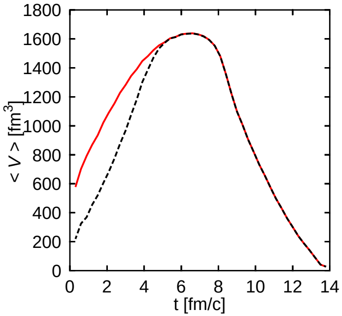
$Pb + Pb$ (158 $AGeV$)

- Evolution of the system volume

  There are two stages: "pure" expansion and freeze-out

  Hadron fraction (dashed) is defined by condition $N^{H}_{quarks} > N^{Q+G}_{quarks}$

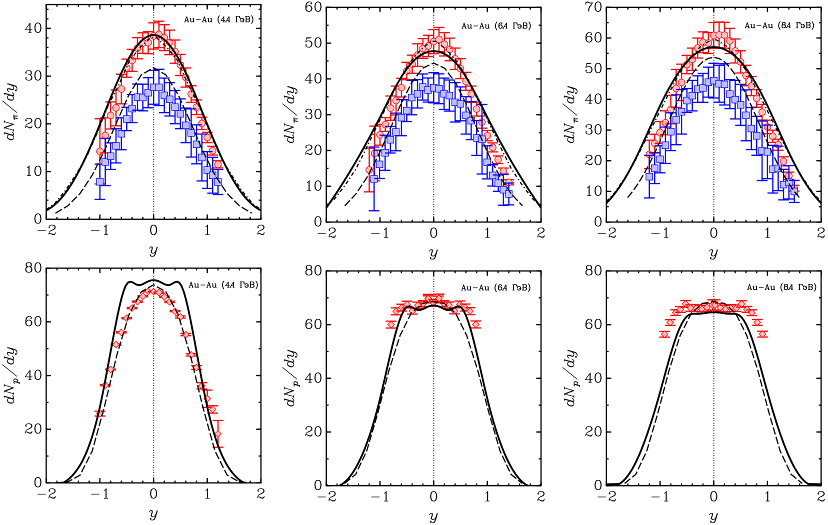
expansion⇓



⇑freeze-out

Comparison with experiment

- Rapidity spectra at AGS energies (pions &protons)

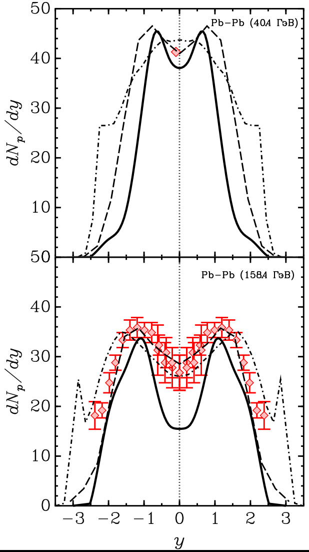


dashed lines ⇒ 3-fluid hydro

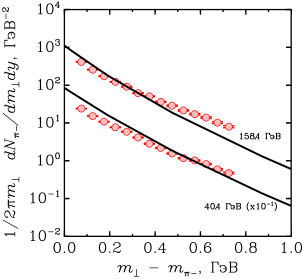
Comparison with experiment

- Spectra at SPS energies

protons



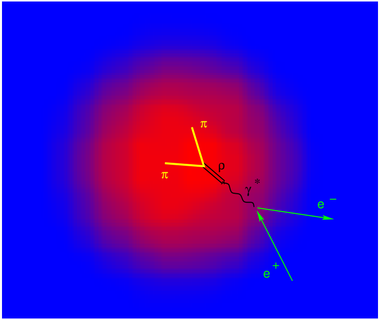
pions



dashed lines ⇒ 3-fluid hydro
dotted lines ⇒ HSD model

Dilepton production

- Introductional remarks

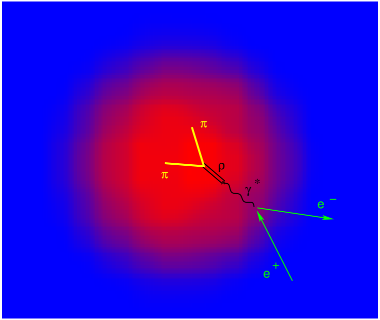


- in medium effects

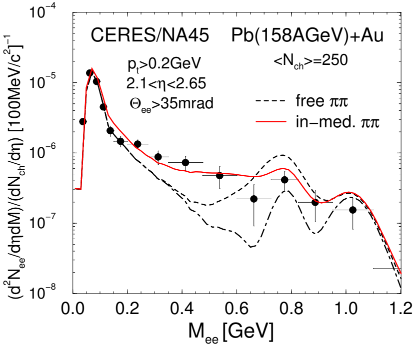
$$\frac{d^2N_{ee}}{dMd\eta} = \frac{M}{\Delta\eta_{e_\pm}} \int d\eta \int d^4x \int_0^{2\pi} d\phi \int_0^{\infty} p_T\ dp_T\ \frac{d^8N_{ee}(T(x), M, \eta, p_T)}{d^4x\ d^4p}\ Acc(M, \eta, p_T)$$

Dilepton production

- Introductional remarks



R. Rapp and J. Wambach, Eur. Phys. J. A **6** (1999) 415 [arXiv:hep-ph/9907502]



- in medium effects

$$\frac{d^2N_{ee}}{dMd\eta} = \frac{M}{\Delta\eta_{e_\pm}} \int d\eta \int d^4x \int_0^{2\pi} d\phi \int_0^\infty p_T\ dp_T\ \frac{d^8N_{ee}(T(x), M, \eta, p_T)}{d^4x\ d^4p}\ Acc(M, \eta, p_T)$$

Dileptons; time slices

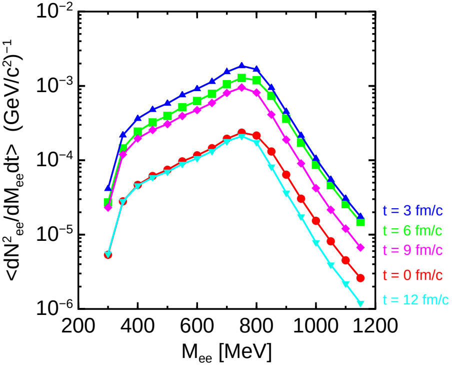
$$\frac{d^8 N_{ee}}{d^4 x d^4 q} = -\int d^4 x\, \mathcal{L}(M)\, \frac{\alpha^2}{\pi^3 q^2}\, \frac{\mathrm{Im}\Pi_{em}(q, T(x), \mu_b(x))}{e^{q_0/T} - 1}$$

$$q^2 = M^2 = q_0^2 - \vec{q}^2$$

(C. Gale and J. I. Kapusta,

Nucl. Phys. B **357** (1991) 65)

Pb + Au (158 AGeV)

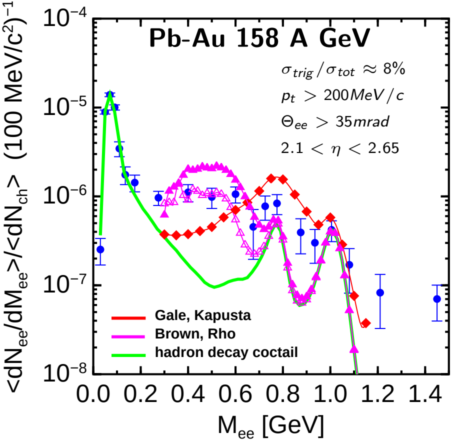


with CERES acceptance

Dielectros; comparison with CERES data

CERES/NA45 Collaboration, J. Phys. G **30** (2004) S1007, J. Phys. G **30** (2004) 2027,

G.E. Brown and M. Rho, Phys. Rep. **363** (2002) 85 ($k = 1/6$)



$$\frac{d^8 N_{ee}}{d^4 x d^4 q} = \frac{\alpha^2}{48\pi^4}\left(1+2\frac{m_l^2}{M^2}\right)\left(1-\frac{4m_\pi^2}{M^2}\right)^{3/2} \frac{\mathrm{Im}\Pi_{em}(M)}{e^{q_0/T}-1},$$

$$\mathrm{Im}\Pi_{em}(M) = \frac{m_\rho^4}{g^2}\frac{\mathrm{Im}\Pi}{(M^2-m_\rho^2)^2+(\mathrm{Im}\Pi)^2}, \quad \mathrm{Im}\Pi = -\frac{g_{\rho\pi\pi}^2}{48\pi}\frac{(M^2-4m_\pi^2)^{3/2}}{M}$$

$$m_\rho \sim \left(1-\frac{T^2}{T_c^2}\right)^k \left(1-0.18\frac{n_B}{n_0}\right) m_{\rho 0}$$

Muon pairs

- semi-central In+In (158 AGeV) collisions

NA60 Collaboration, Phys.Rev.Lett. **96** (2006) 163302

V.Skokov, V.Toneev, Phys. Rev. **C 73** (2006) 021902

$g_{\rho\pi\pi} = 6.05; \quad g = 5.03;$
$m_\rho = 770 MeV$

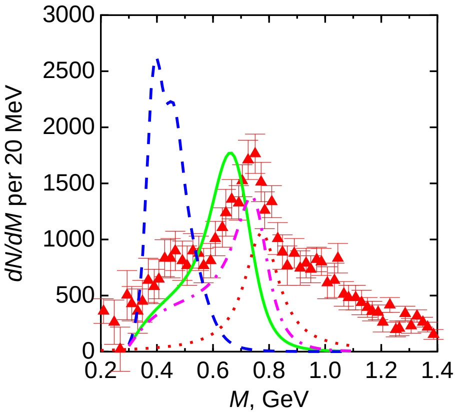
$m_\rho \to m^*_\rho(x) = m_\rho(1 - 0.15 \cdot n_B(x)/n_0)$

$m_\rho \to (1 - \frac{T^2}{T_c^2})^{0.3}(1 - 0.15\frac{n_B}{n_0})m_\rho$

$m^2_\rho/g \to m^{*2}_\rho/g^*$

dash-dotted - no in-medium effects

dotted - ρ decay at freeze-out



Concluding remarks

- Open question of hydrodynamics
  - ⋆ Initial state and non-equilibrium effect (memory effect)
  - ⋆ Equation of state (phase transition, critical end-point, dynamics of phase transition ...)
  - ⋆ Freeze-out procedure (Cooper-Fry prescription ?)
  - ⋆ . . .

THANK YOU

FOR ATTENTION

Special thanks to V.V. Skokov for help in preparing these lectures