Neutrino radiation from dense matter

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Outline

- The phenomenology
- Classifications of the reactions
- Self-energies and loop-expansions
- The role of pairing correlations
- Neutrino rates from color superconducting matter

Basic facts

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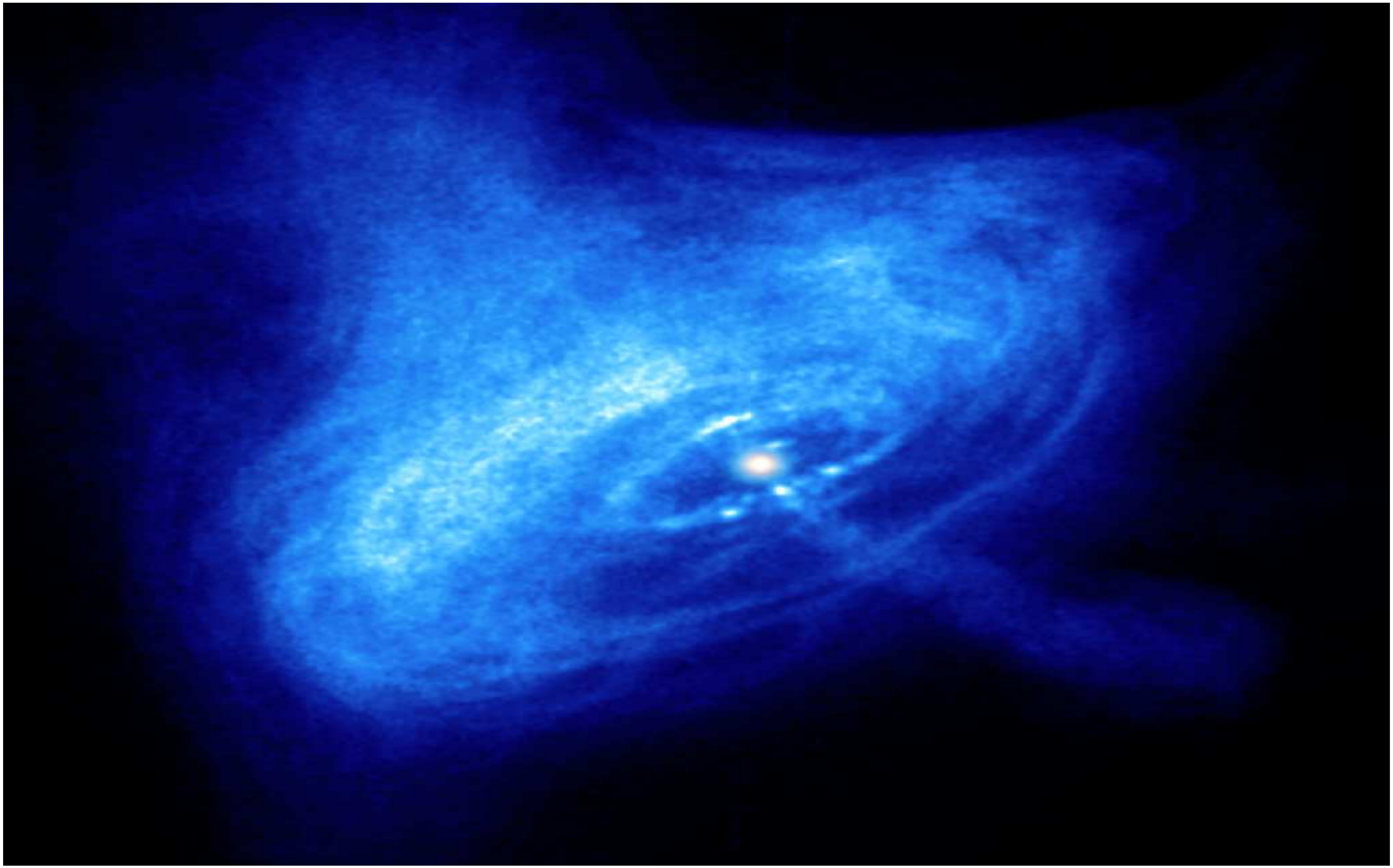
- Complex, multi-scale problem, which depends on many unknown parameters
- But... Continuing X-ray missions; True challenge to the many-body theory with potential to constraint the properties of dense matter.

Chandra image of Crab nebula in X-rays



Cooling simulations

Structure from Tolman-Opennheimer-Volkov equations

Evolution equations

$$\frac{\partial (Le^{2\phi})}{\partial r} = 4\pi r^2 e^{\Lambda} \left(-\epsilon_\nu e^{2\phi} + h e^{2\phi} - c_{\mathrm{v}} \frac{\partial (Te^{\phi})}{\partial t} \right) ,$$

$$\frac{\partial (Te^{\phi})}{\partial r} = -\frac{(Le^{2\phi}) e^{\Lambda - \phi}}{4\pi r^2 \kappa} .$$

Boundary conditions

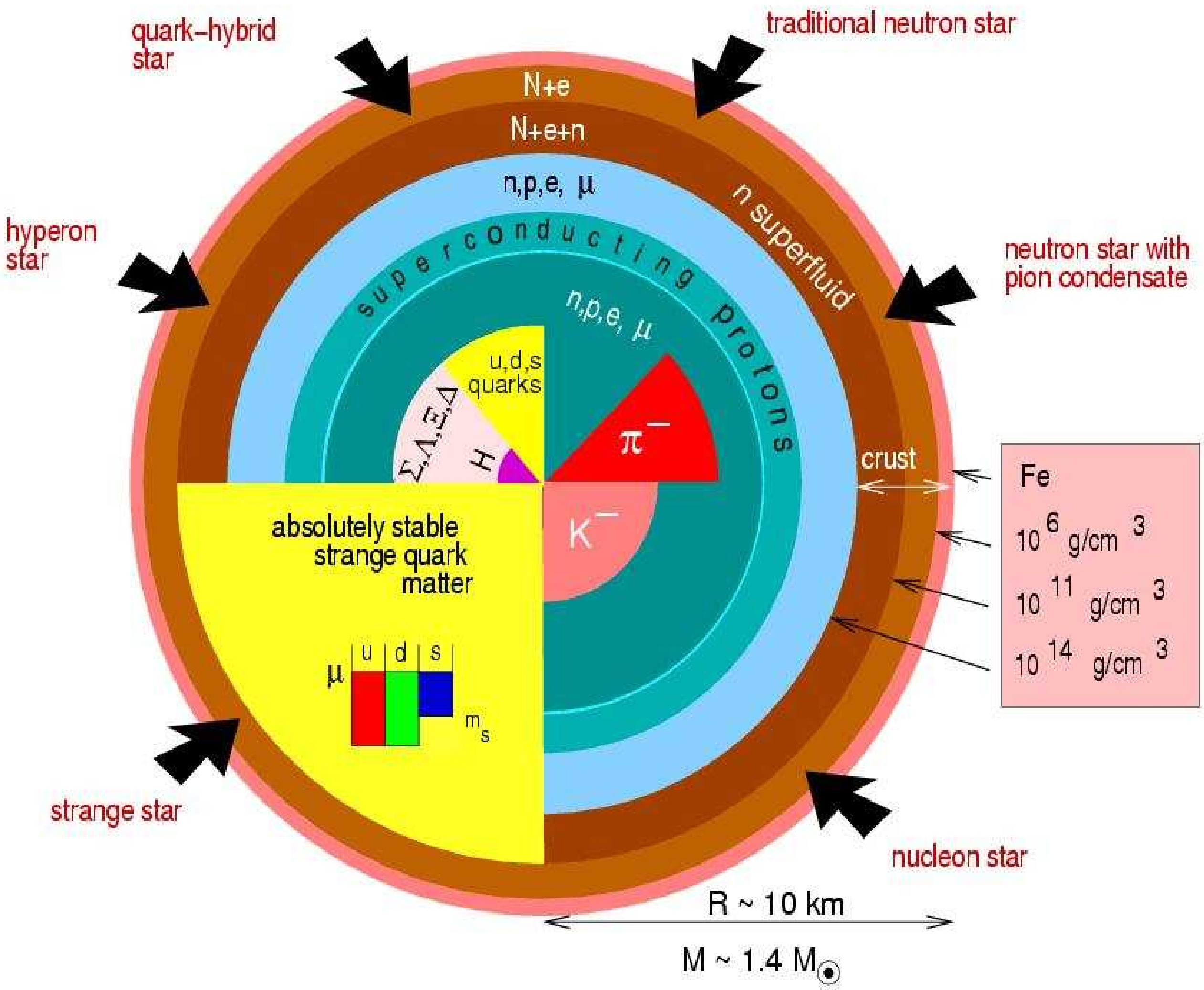
$$L(r = 0) = 0 ,$$

$$T(r = r_{\mathrm{m}}) = T_{\mathrm{m}}(r_{\mathrm{m}}, L_{\mathrm{m}}, M_{\mathrm{m}}) ,$$

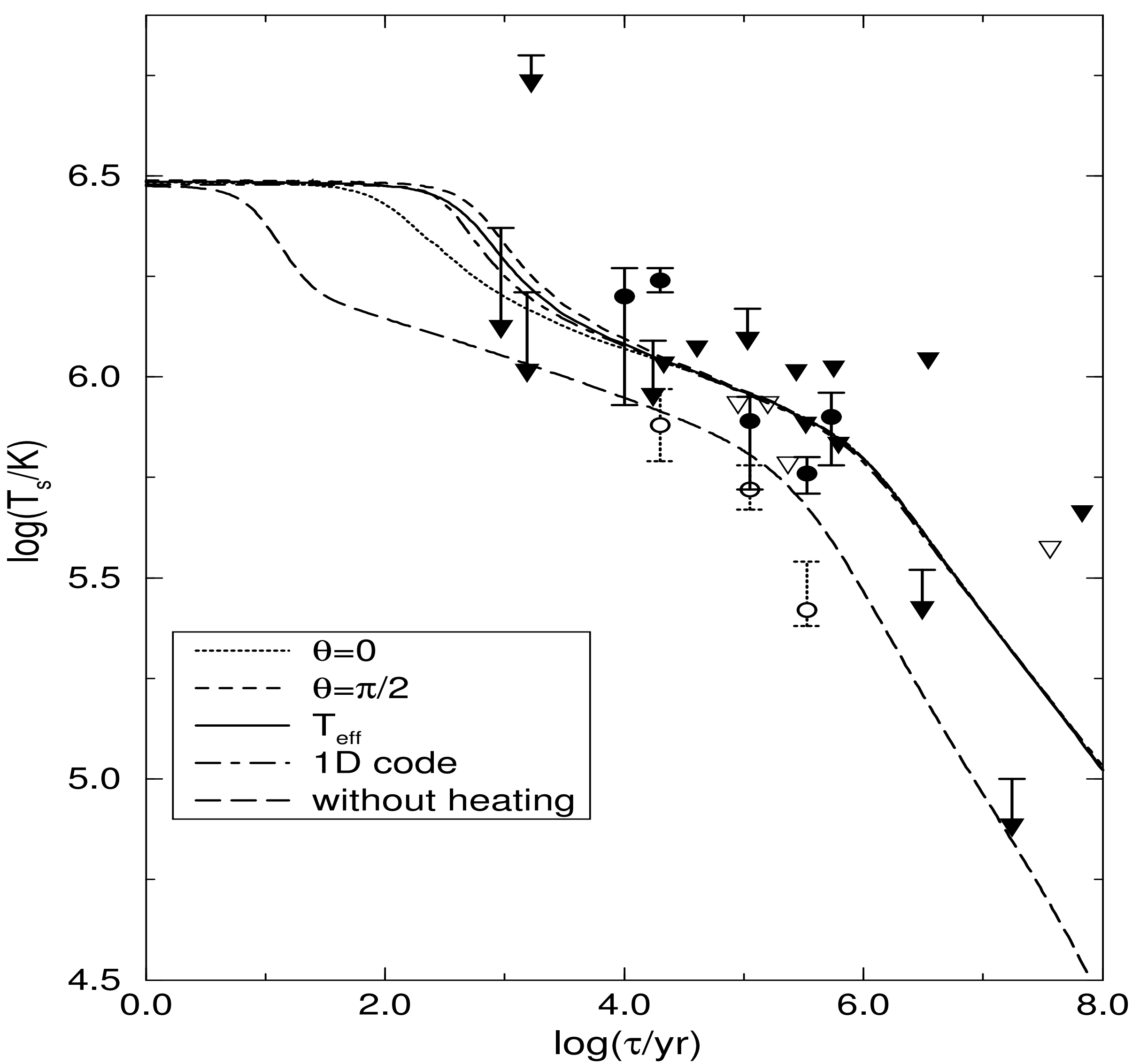
Input quantities

- Equations of state: crust, core, multiple phases
- Superfluidity of fermions
- Heat capacity
- Thermal conductivities: crust, core
- Neutrino emissivities: pair-, photon-, plasma-processes, bremsstrahlung, Urca processes
- Photosphere
- Surface composition

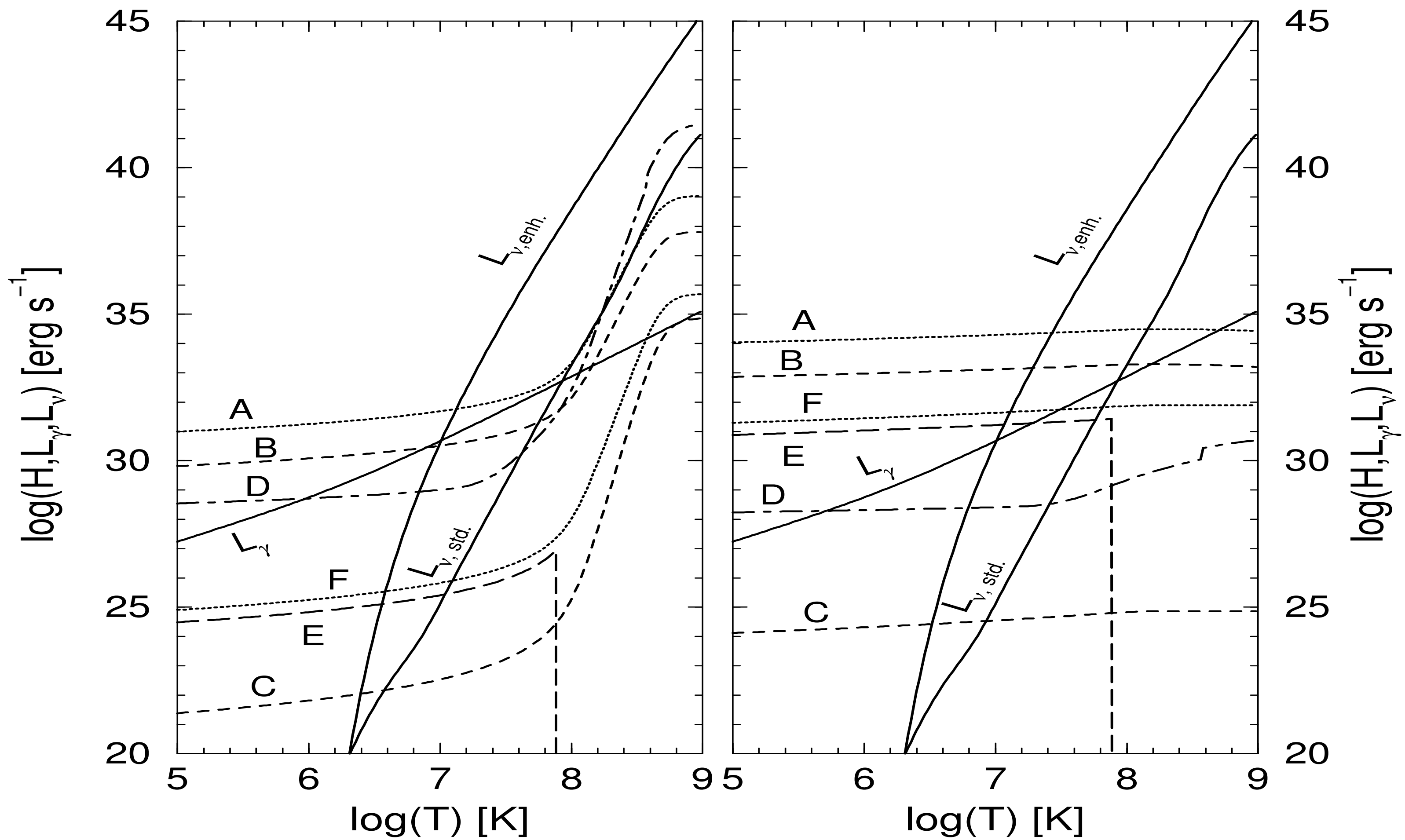
Structure


quark-hybrid star
traditional neutron star
N+e
N+e+n
n,p,e, μ
n superfluid
hyperon star
superconducting protons
neutron star with pion condensate
n,p,e, μ
u,d,s quarks
Σ,Λ,Ξ,Δ
H
π⁻
K⁻
crust
Fe
10⁶ g/cm³
10¹¹ g/cm³
10¹⁴ g/cm³
absolutely stable strange quark matter
μ
u
d
s
m_s
strange star
nucleon star
R ~ 10 km
M ~ 1.4 M☉

Cooling tracks



Luminosities



Classifying reactions

Processes on fermions

- Neutral current processes (Z_0 exchange)

$$\begin{cases} f_1 \longrightarrow f_2 + \nu_f + \bar{\nu}_f & \text{(brems)} \\ f_1 + f_1' \longrightarrow f_2 + f_2' + \nu_f + \bar{\nu}_f \end{cases} \qquad \text{(-2)}$$

- Charged current processes ($W^\pm$ exchange)

$$\begin{cases} f_1 \longrightarrow f_2 + e + \bar{\nu}_e & \text{(Urca)} \\ f_1 + f_1' \longrightarrow f_2 + f_2' + e + \bar{\nu}_e \end{cases} \qquad \text{(-3)}$$

Classifying reactions - 2

Processes on bosons

- Pion decay

$$\pi^- + n \rightarrow n + e^- + \bar{\nu}_e$$

- Condensation of pions leads to

$$\pi^- \rightarrow e^- + \bar{\nu}_e$$

- analogous processes in K condensed phases

Transport equations

- ν and $\bar{\nu}$ - Boltzmann equations

$$\left[\partial_t + \vec{\partial}_q\, \omega_\nu(\vec{q})\vec{\partial}_x\right] f_\nu(\vec{q}, x)$$
$$= \int_0^\infty \frac{dq_0}{2\pi} \mathrm{Tr}\left[\Omega^<(q,x)S_0^>(q,x) - \Omega^>(q,x)S_0^<(q,x)\right],$$

- ν-quasiparticle propagators:

$$S_0^<(q,x) = \frac{i\pi \not{q}}{\omega_\nu(\vec{q})}\Big[\delta\left(q_0 - \omega_\nu(\vec{q})\right) f_\nu(q,x)$$
$$-\delta\left(q_0 + \omega_\nu(\vec{q})\right)\left(1 - f_{\bar{\nu}}(-q,x)\right)\Big]. \quad \text{(-7)}$$

- definition of the Poisson bracket

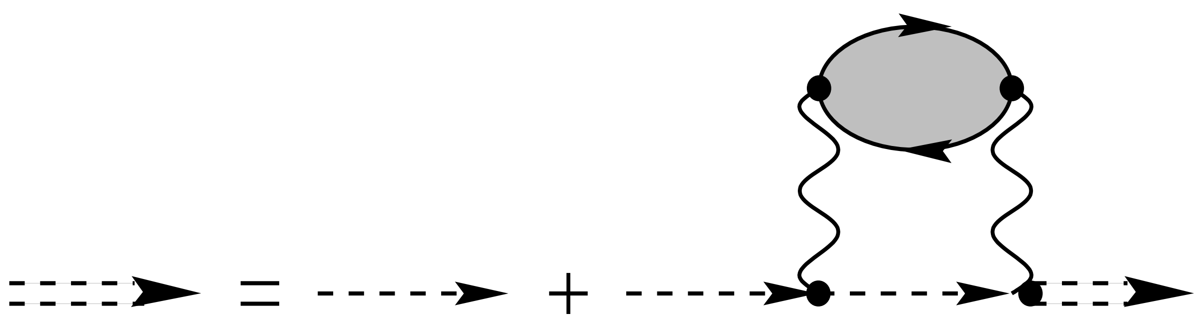
$$\{f,g\}_{P.B.} = \partial_\omega f\ \partial_t g - \partial_t f\ \partial_\omega g - \partial_{\vec{p}} f\ \partial_{\vec{r}} g + \partial_{\vec{r}} f\ \partial_{\vec{p}} g. \quad \text{(-8)}$$

Self-energies

- ν and $\bar{\nu}$-self-energies (second order in weak force)

$$-i\Omega^{>,<}(q_1,x) = \int \frac{d^4q}{(2\pi)^4}\frac{d^4q_2}{(2\pi)^4}(2\pi)^4\delta^4(q_1-q_2-q)$$

$$i\Gamma^{\mu}_{L\,q}\, iS_0^{<}(q_2,x) i\Gamma^{\dagger\,\lambda}_{L\,q} i\Pi^{>,<}_{\mu\lambda}(q,x), \quad \text{(-9)}$$

- 

- the problem is to compute the polarization tensor!

Bremsstrahlung emissivity

- energy loss per unit time and volume

$$\epsilon_{\nu\bar{\nu}} = \frac{d}{dt}\int \frac{d^3q}{(2\pi)^3}\left[f_\nu(\vec{q}) + f_{\bar{\nu}}(\vec{q})\right]\omega_\nu(\vec{q}) \qquad \text{(-10)}$$

- expressed through the collision integrals

$$\epsilon_{\nu\bar{\nu}} = -2\left(\frac{G}{2\sqrt{2}}\right)^2 \sum_f \int \frac{d^3q_2}{(2\pi)^3 2\omega_\nu(\vec{q}_2)} \int \frac{d^3q_1}{(2\pi)^3 2\omega_\nu(\vec{q}_1)} \int \frac{d^4q}{(2\pi)^4}$$
$$(2\pi)^4\delta^3(\vec{q}_1 + \vec{q}_2 - \vec{q})\delta(\omega_\nu(\vec{q}_1) + \omega_\nu(\vec{q}_2) - q_0)\ \left[\omega_\nu(\vec{q}_1) + \omega_\nu(\vec{q}_2)\right]$$
$$g_B(q_0)\left[1 - f_\nu(\omega_\nu(\vec{q}_1))\right]\left[1 - f_{\bar{\nu}}(\omega_\nu(\vec{q}_2))\right]\Lambda^{\mu\lambda}(q_1, q_2)\Im\mathrm{m}\,\Pi^R_{\mu\lambda}(q).$$

Urca emissivity

- energy loss per unit time and volume

$$\epsilon_{Urca} = \frac{d}{dt}\int\frac{d^3q}{(2\pi)^3}\left[f_{\bar\nu}(\vec{q})\right]\omega_\nu(\vec{q}) \tag{-12}$$

- expressed through the collision integrals

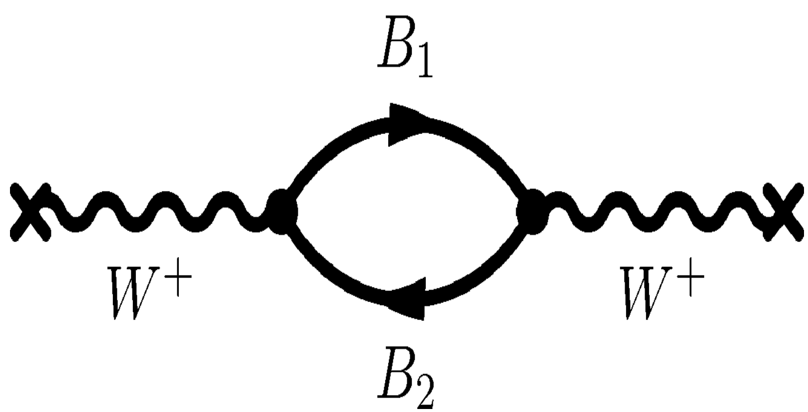
$$\begin{aligned}\epsilon_{\bar\nu} = {} & -2\left(\frac{\tilde{G}}{\sqrt{2}}\right)^2\int\frac{d^3q_1}{(2\pi)^3 2\omega_e(\boldsymbol{q}_1)}\int\frac{d^3q_2}{(2\pi)^3 2\omega_\nu(\boldsymbol{q}_2)} \\ & \int d^4q\,\delta(\boldsymbol{q}_1+\boldsymbol{q}_2-\boldsymbol{q})\delta(\omega_e+\omega_\nu-q_0)\,\omega_\nu(\boldsymbol{q}_2) \\ & g_B(q_0)\left[1-f_e(\omega_e)\right]\Lambda^{\mu\zeta}(q_1,q_2)\Im\mathrm{m}\,\Pi^R_{\mu\zeta}(q), \end{aligned} \tag{-13}$$

The direct Urca process

Simplest charge-current process is the β-decay

$$n \to p + e^- + \bar{\nu} \qquad e^- + p \to n + \nu \tag{-14}$$

The one-loop polarization tensor for charge current process. The wavy lines correspond to the W^+ propagators, the solid line to the baryonic propagators.



Urca process continued

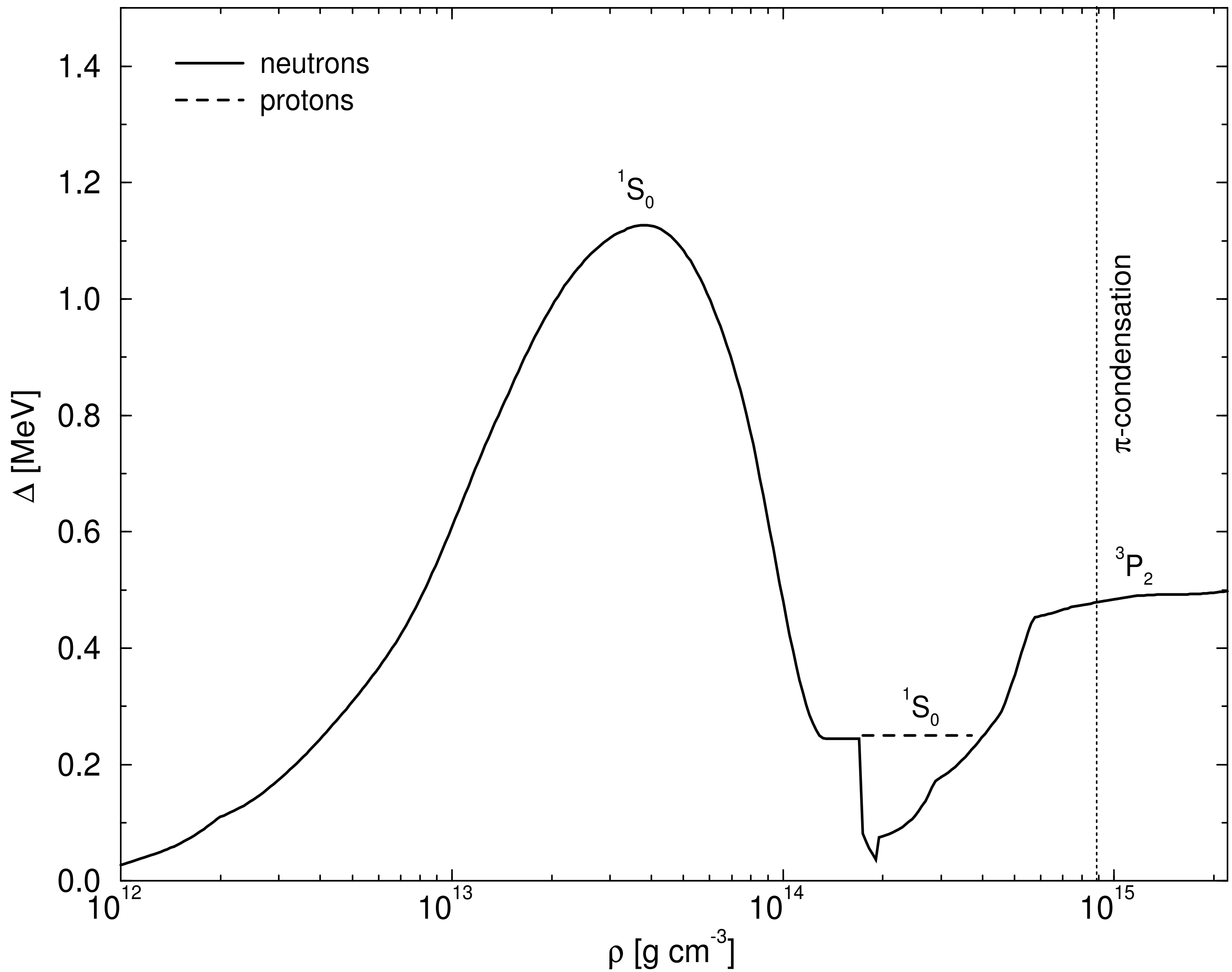
Direct Urca is forbidden by kinematics if the matter is strongly asymmetric, for proton fractions $x_p \geq 11 - 13\%$

$$\epsilon_{\bar{\nu}} = (1 + 3g_A^2)\frac{3\tilde{G}^2 m_n^* m_p^* p_{Fe}}{2\pi^5\beta^6}\int dy\, g_B(y)\,\ln\frac{1+e^{-x_{\min}}}{1+e^{-(x_{\min}+y)}}$$
$$\times \int dz z^3 f_e(z-y) \simeq 10^{26} \times T^6 \text{ erg cm}^{-3}\text{ s}^{-1}, \qquad \text{(-15)}$$

Temperature dependence $\epsilon_{\bar{\nu}} \propto T^6$.

- each degenerate fermion, i.e., e, p, n factor T/ϵ_F
- anti-neutrino T^3
- energy conservation T^{-1} and energy rate T

Effects of pairing on direct Urca process

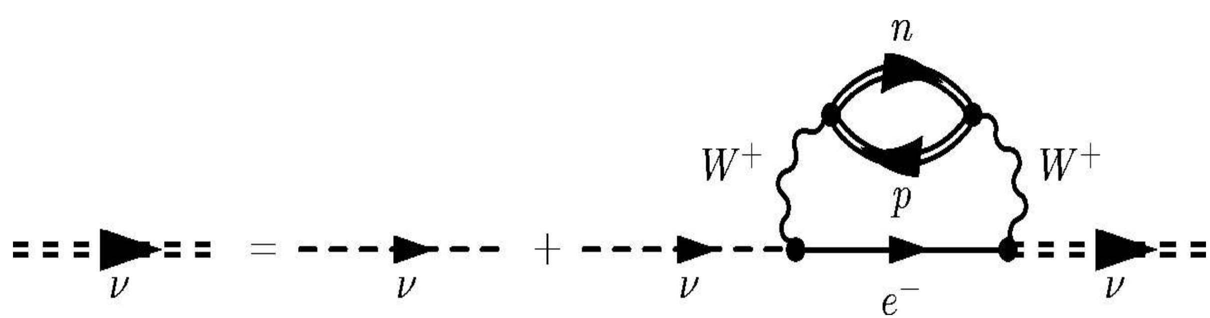


Effects of pairing on direct Urca process

Naive picture prescribes a suppression of the Urca process by the pairing gap [$\Delta_{\rm max} = \Delta_n,\ \Delta_p$]

$$\epsilon_{\bar\nu} \to \epsilon_{\bar\nu} \times \exp\left(-\frac{\Delta_{\rm max}}{T}\right). \qquad \text{(-16)}$$

A more systematic way ...



Polarization tensor at one loop

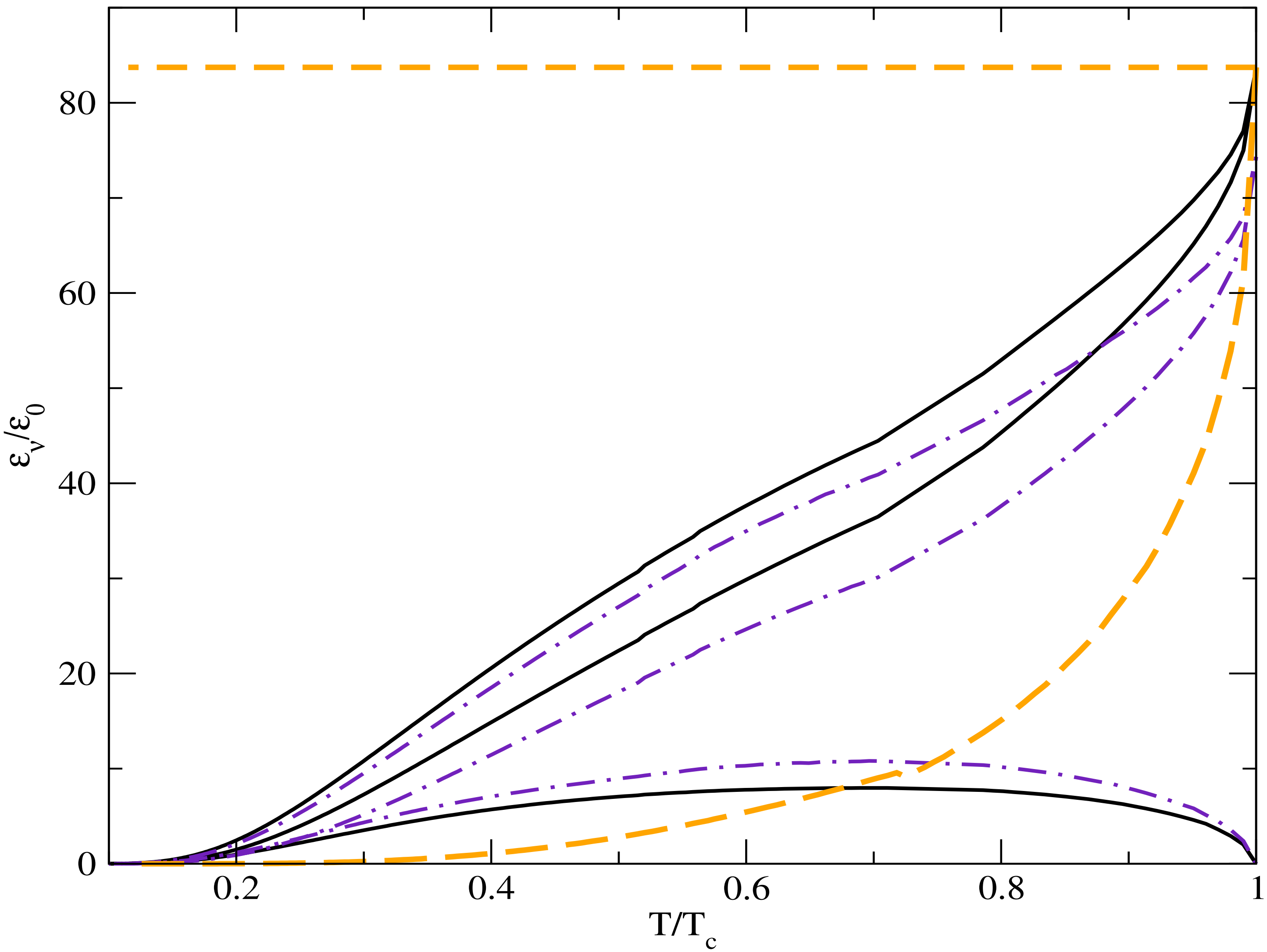
$$\Pi^R_{V/A}(\boldsymbol{q},\omega) = \sum_{\sigma,\vec{p}}\left\{\left(\frac{u_p^2u_k^2}{\omega+\varepsilon_p-\varepsilon_k+i\delta}-\frac{v_p^2v_k^2}{\omega-\varepsilon_p+\varepsilon_k+i\delta}\right)[f(\varepsilon_p)-f(\varepsilon_k)] + \left(\frac{u_p^2v_k^2}{\omega-\varepsilon_p-\varepsilon_k+i\delta}\right)[1-f(\varepsilon_p)-f(\varepsilon_k)]\right\}, \quad \text{(-17)}$$

with coherence factors

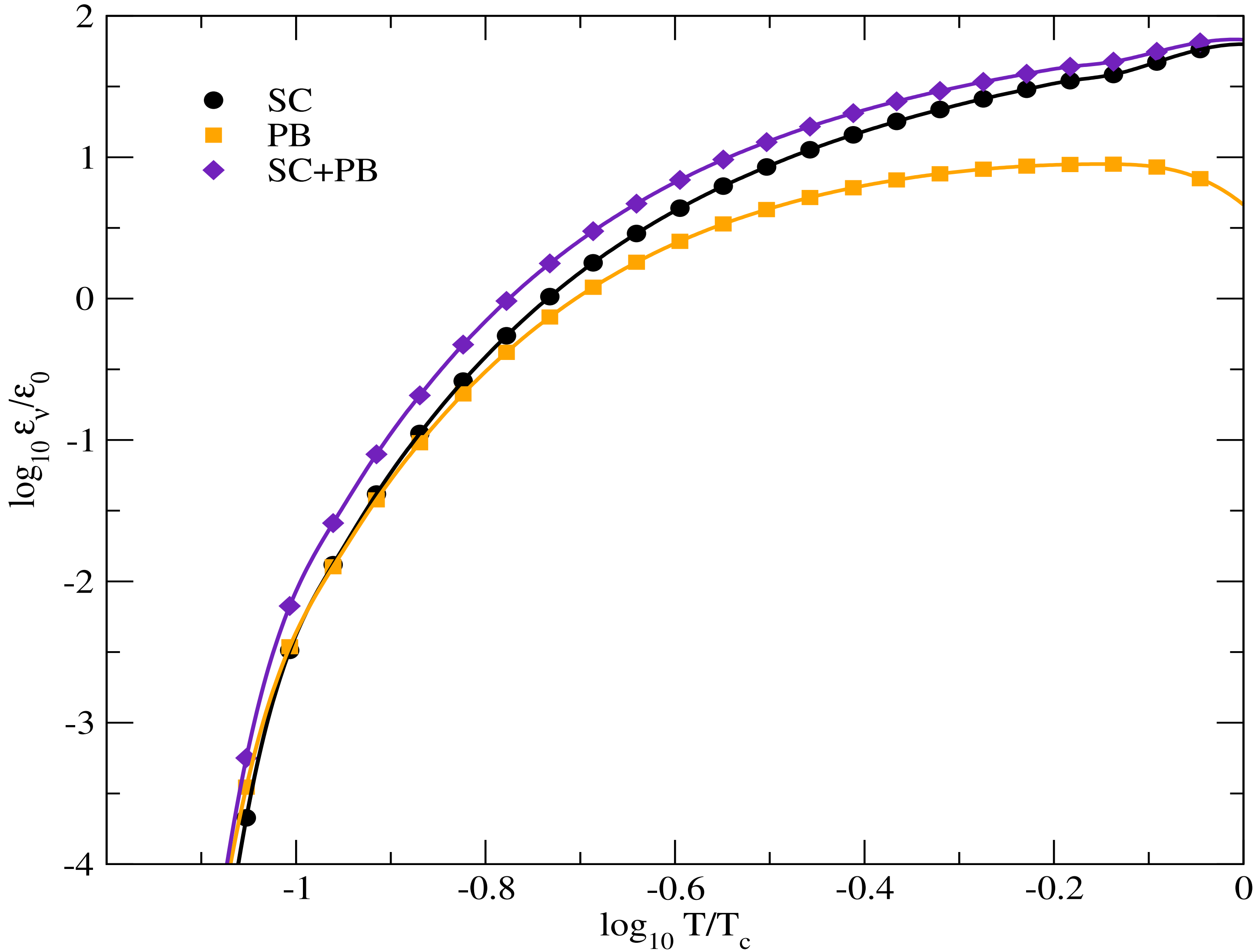
$$u_p^2 = \frac{1}{2}\left(1+\frac{\xi_p}{\varepsilon_p}\right) \qquad u_p^2+v_p^2 = 1. \quad \text{(-18)}$$

Scattering and pair-braking contributions

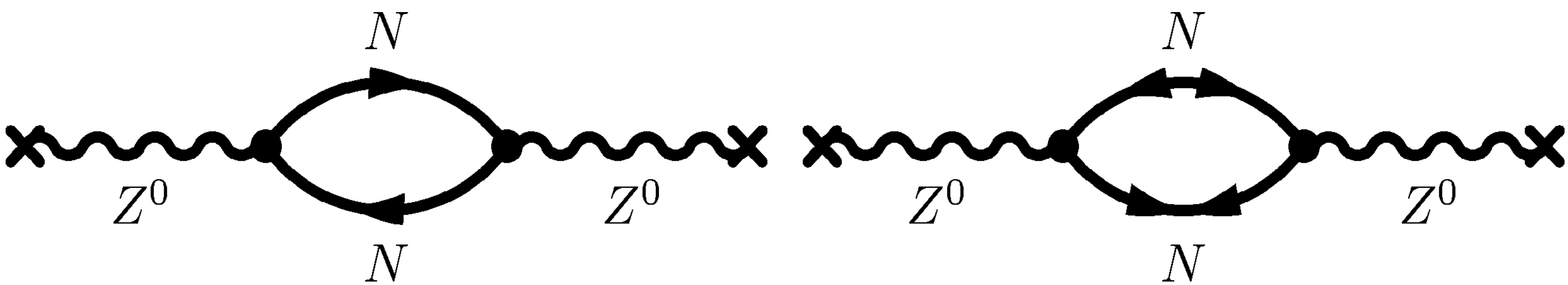
One-loop vs naive suppression



Pair-breaking contribution



Neutral current pair-breaking processes

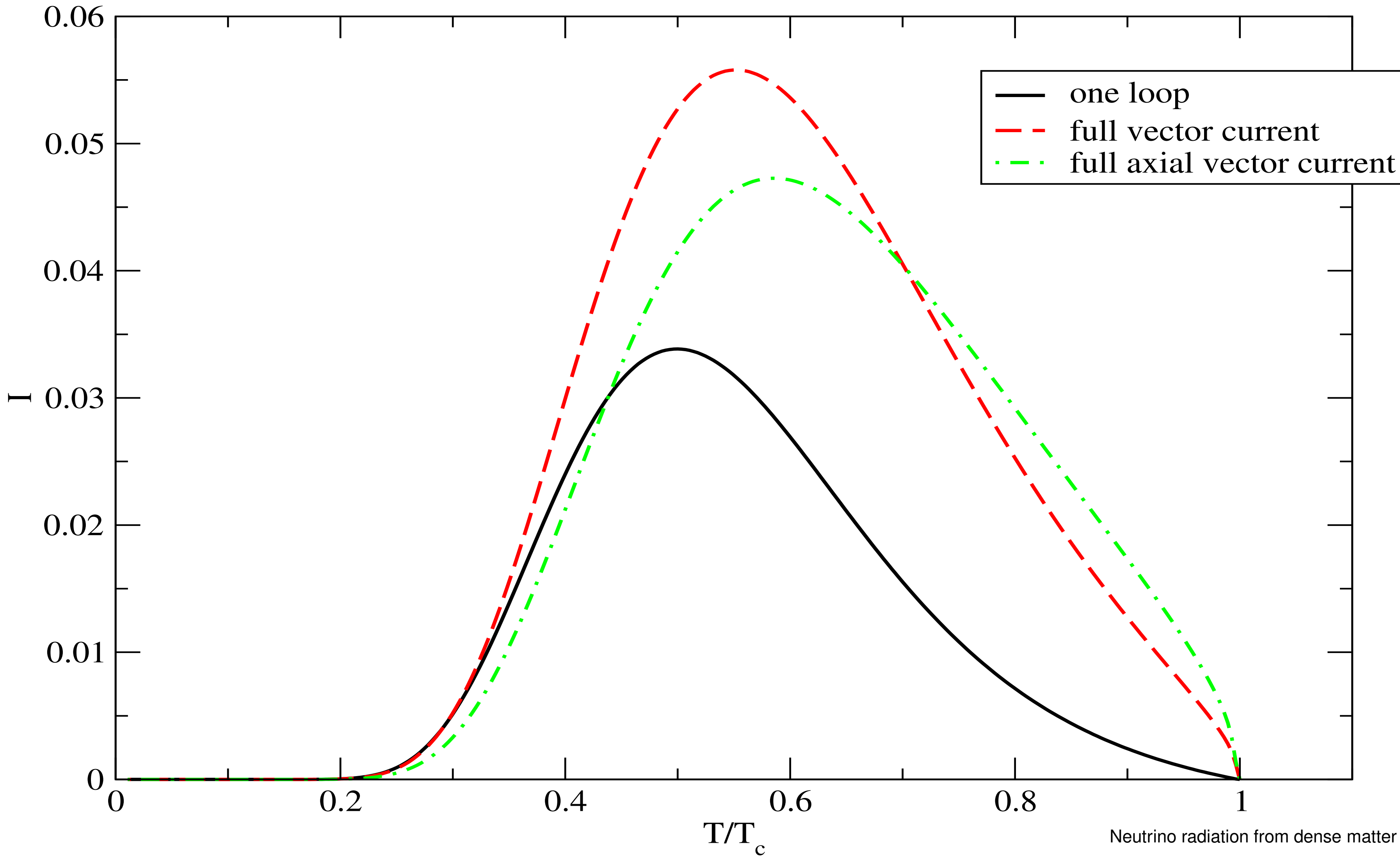


The one-loop contribution to the polarization tensor in the superfluid matter; solid lines refer to the baryon propagators, wavy lines to the (amputated) Z^0 propagator.

$$\epsilon_{\nu\bar{\nu}} = \frac{G^2\, c_V^2}{240\pi^3}\; \nu(p_F)\, T^7\; I(\zeta) \equiv \epsilon_0\; I(\zeta),$$

$$I(\zeta) = \zeta^7 \int_0^\infty d\phi\; (\cosh\phi)^5\; f(\zeta\cosh\phi)^2, \quad \zeta = 2\Delta(T)/T$$

Multi-loop processes continued



Neutrinos in superconducting quark matter

At moderate densities u, d and e plasma

Lagrangian:

$$\mathcal{L}_{\text{eff}} = \bar{\psi}(x)(i\gamma^{\mu}\partial_{\mu})\psi(x) + G_1(\psi^T C\gamma_5\tau_2\lambda_A\psi(x))^{\dagger}(\psi^T C\gamma_5\tau_2\lambda_A\psi(x)),$$

Pairing ansatz:

$$\Delta \propto \langle\psi^T(x)C\gamma_5\tau_2\lambda_2\psi(x)\rangle,$$

Stationary points of the thermodynamical potential

$$\frac{\partial\Omega}{\partial\Delta} = 0, \quad -\frac{\partial\Omega}{\partial\mu_f} = \rho_f;$$

Thermodynamic potential

Two-flavor systems with isospin asymmetry

$$\Omega = -\frac{1}{\beta}\sum_{\omega_n}\int \frac{d^3p}{(2\pi)^3}\frac{1}{2}\mathrm{Tr}\ \ln\left[\beta\begin{pmatrix} S_{11}^{-1}(i\omega_n,\vec{p}) & S_{12}^{-1}(i\omega_n,\vec{p}) \\ S_{21}^{-1}(i\omega_n,\vec{p}) & S_{22}^{-1}(i\omega_n,\vec{p}) \end{pmatrix}\right]$$

$$+\frac{\Delta^2}{4G_1},$$

In terms of quasiparticle spectra where $\xi_{\pm\pm} = (p \pm \mu) \pm \delta\mu$and
$E_{\pm\pm} = \sqrt{(p\pm\mu)^2 + |\Delta|^2} \pm \delta\mu,$

$$\Omega = -2\int \frac{d^3p}{(2\pi)^3}\left\{2p + \sum_{ij}\left[\frac{1}{\beta}\log\left(1+e^{-\beta\xi_{ij}}\right) + E_{ij}\right.\right.$$

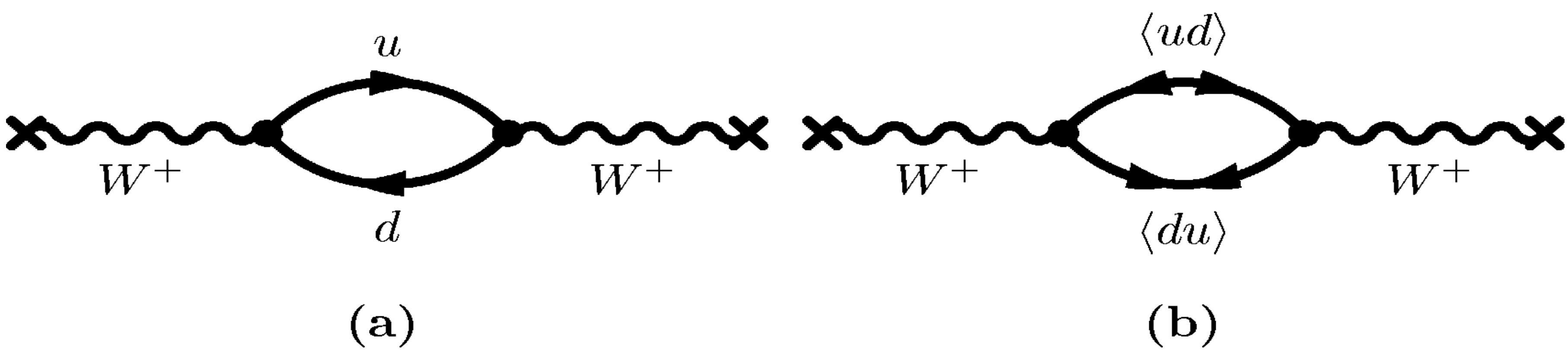
$$\left.\left.+\frac{2}{\beta}\log\left(1+e^{-\beta s_{ij}E_{ij}}\right)\right]\right\} + \frac{\Delta^2}{4G_1}, \qquad (-24)$$

One loop results

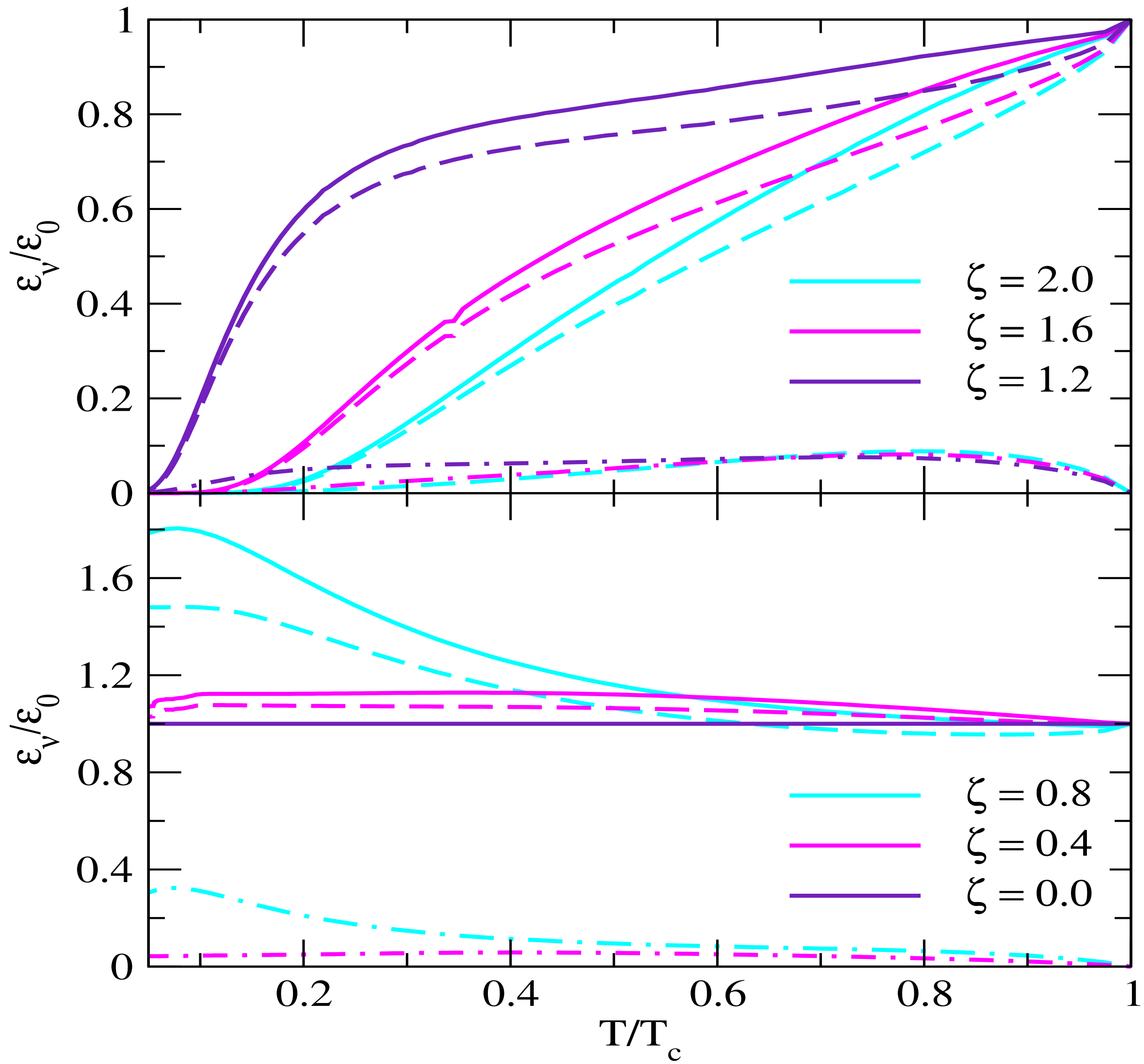
Polarization tensors

$$\Pi_{\mu\lambda}(q) = -i \int \frac{d^4p}{(2\pi)^4} \mathrm{Tr}\left[(\Gamma_-)_\mu S(p)(\Gamma_+)_\lambda S(p+q)\right]$$

$$\Gamma_\pm(q) = \gamma_\mu(1-\gamma_5) \otimes \tau_\pm$$



$$S_{f=u,d} = i\delta_{ab}\frac{\Lambda^+(p)}{p_0^2-\epsilon_p^2}(\not p - \mu_f\gamma_0), \qquad F(p) = -i\epsilon_{ab3}\epsilon_{fg}\Delta\frac{\Lambda^+(p)}{p_0^2-\epsilon_p^2}\gamma_5 C$$



$\zeta = \Delta/\delta\mu$, where $\delta\mu = \mu_d - \mu_u = \mu_e$.

- The neutrino radiation reactions can be computed systematically in the superfluid hadronic and quark phases within Green's function approach
- Accurate rates are need for cooling simulations of baryonic and quark stars.
- This program has the potential to constrain the properties of dense matter in compact stars.

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