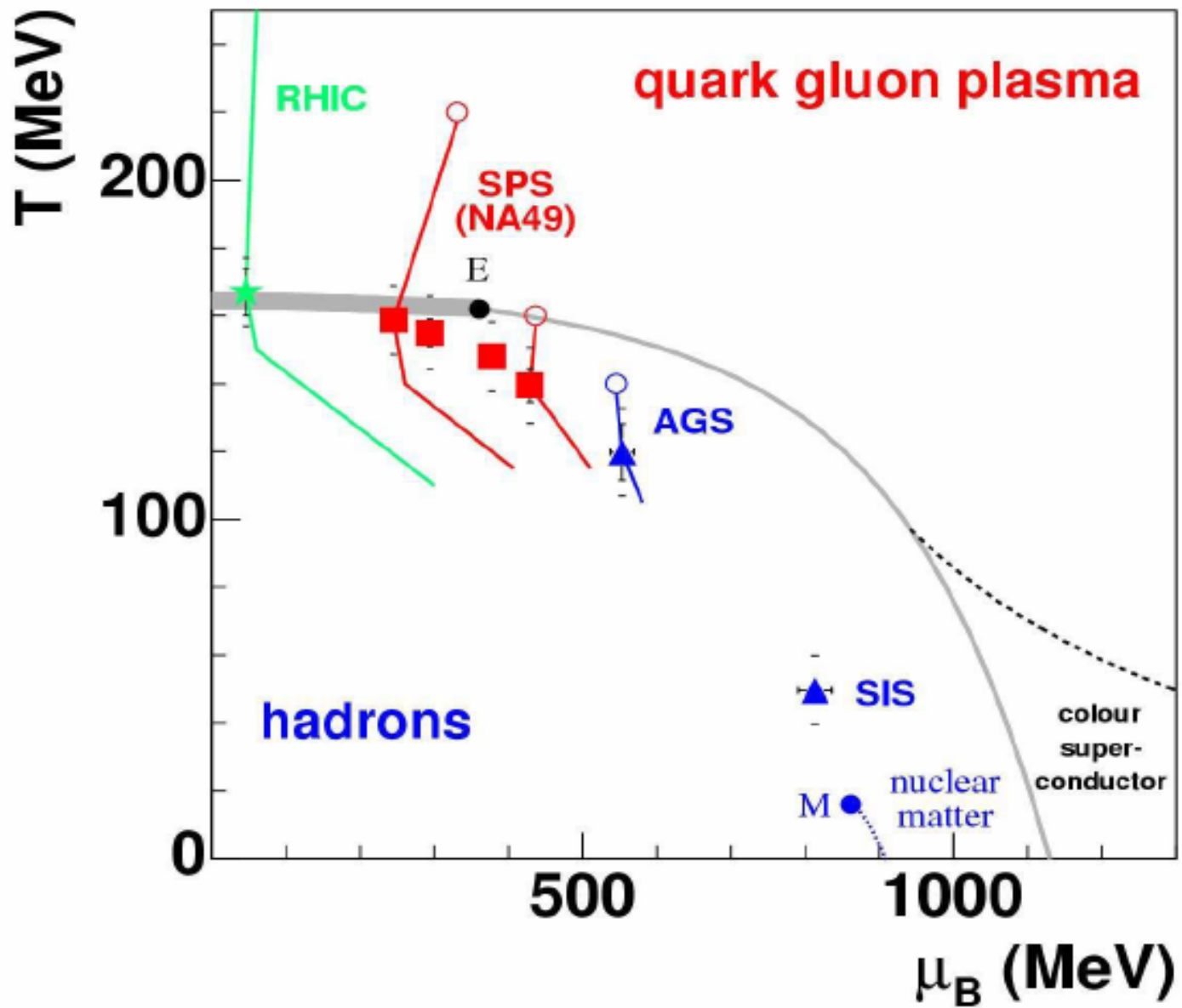


# Multiplicity Fluctuations in Nucleus-Nucleus Collisions: Statistical and Transport Models

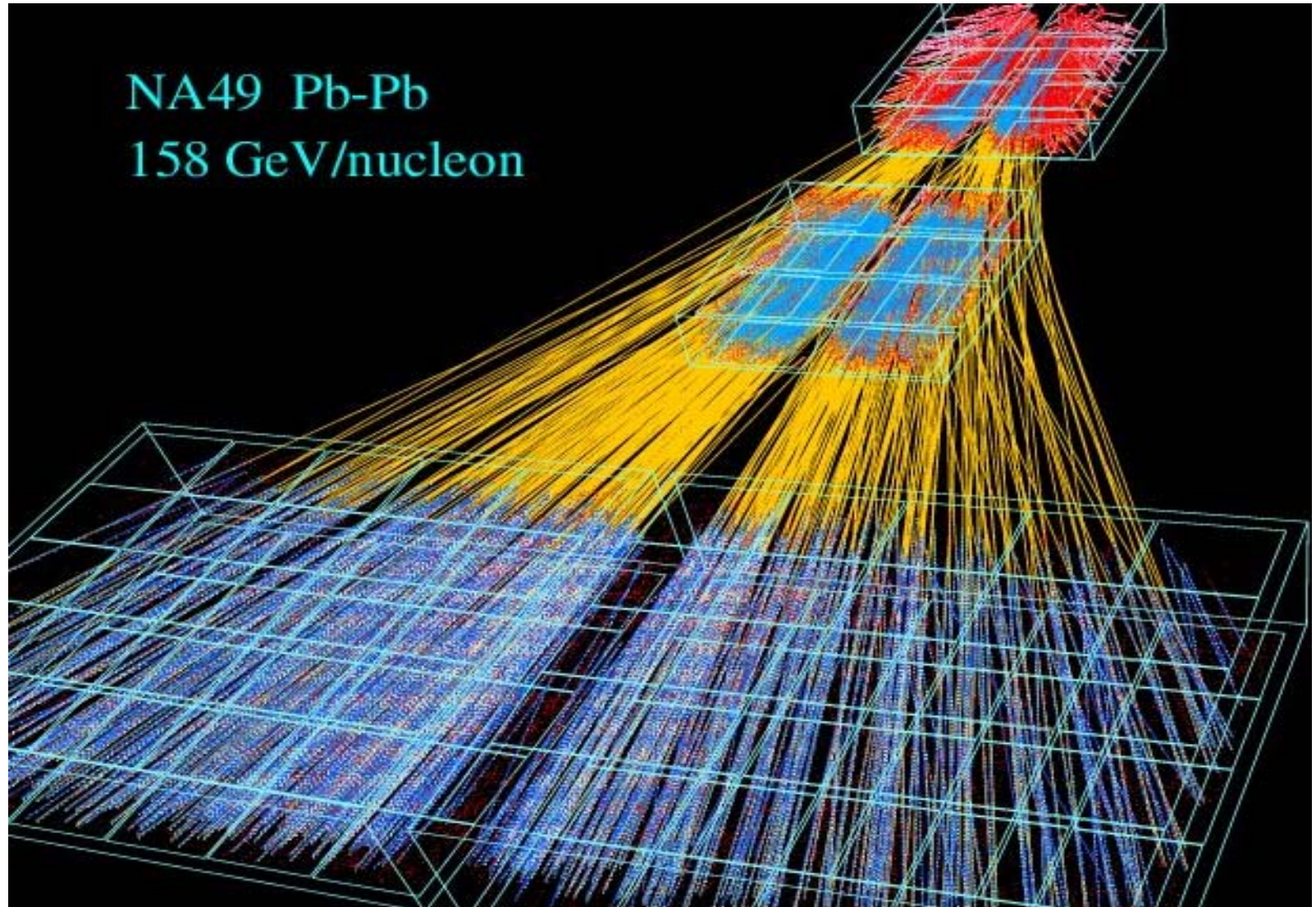
Mark Gorenstein

*Bogolyubov Institute for Theoretical Physics  
Kiev, Ukraine*

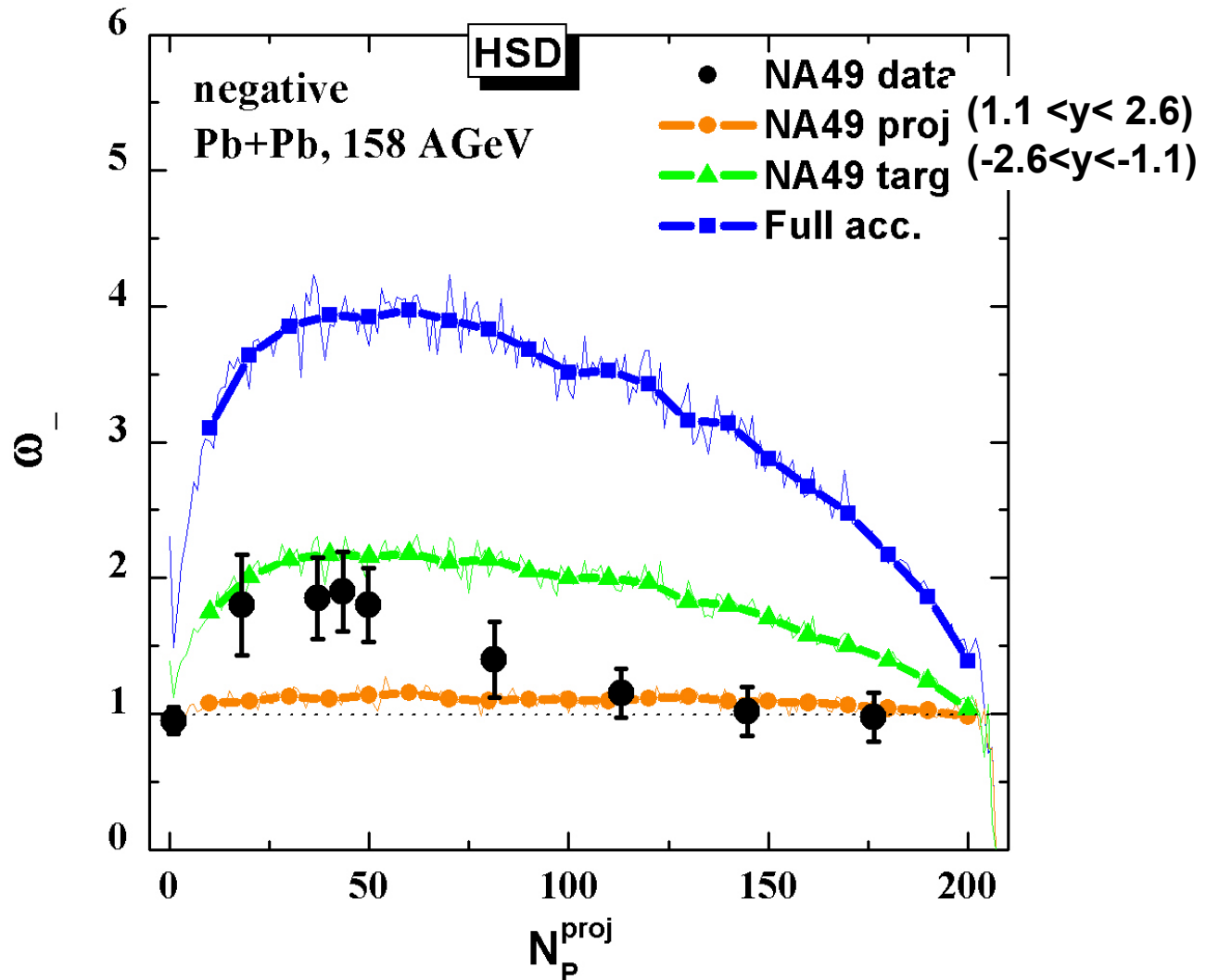
1. Motivation
2. Statistical Model: a) Exact Conservation Laws  
b) Resonance decays
3. Transport Models (HSD, UrQMD)
4. Transparency, mixing and reflection
5. Comparison with data, conclusions...



NA49 at SPS – a dedicated facility to study event-by-event fluctuations in nuclear collisions



# Transport Models (see details in the talk of Volodymyr Konchakovski)



Konchakovski, Hausler, M.I.G., Bratkovskaya, Bleicher, Stoecker  
Phys. Rev. C (2006)

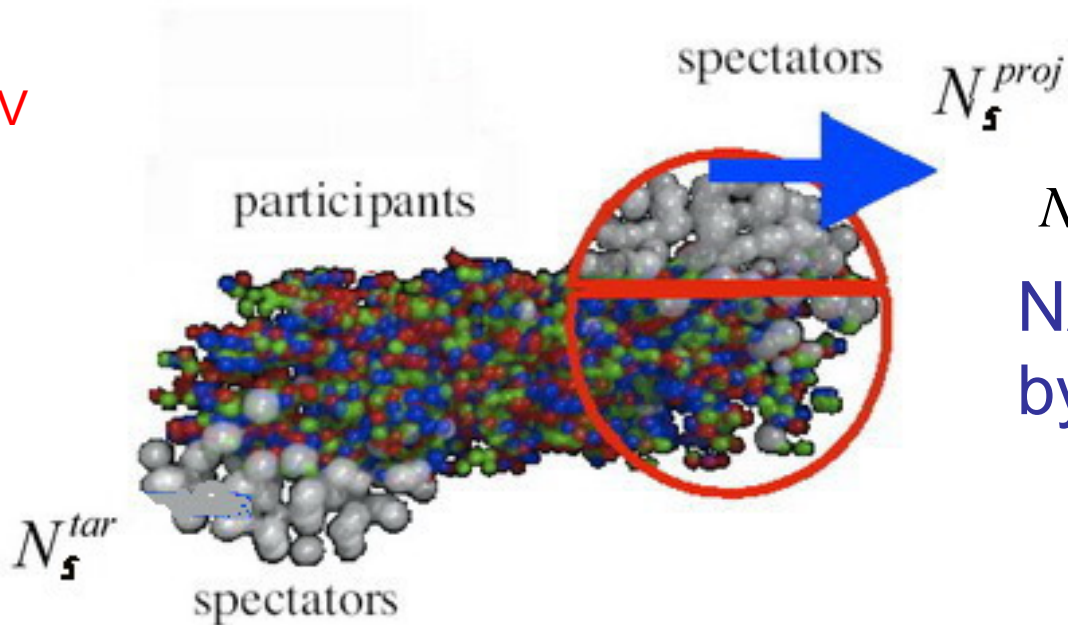
## Scaled variance

$$\omega_i = \frac{\langle N_i^2 \rangle - \langle N_i \rangle^2}{\langle N_i \rangle}, \quad \text{where } i = -, +, ch$$

$\omega = 1$  for Poisson distribution

## Nucleons: participants and spectators.

Pb-Pb  
158 AGeV



$$N_P^{proj} = A^{proj} - N_S^{proj}$$

NA49 Measured  
by ZDC

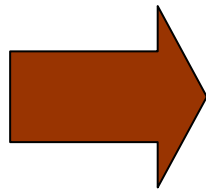
## Independent Sources

Number of Sources is Proportional to Number of Participants

$$n_i = \frac{\langle \bar{N}_i \rangle}{\langle N_P \rangle} \quad \text{- the particle number of i-th type per participant}$$

$$\omega_i = \omega_i^* + \frac{1}{2} \omega_P^{tar} n_i, \quad \text{where } i = -, +, ch$$

**HSD**  
**N+N**  
**158 GeV**



the fluctuation  
from a single  
source

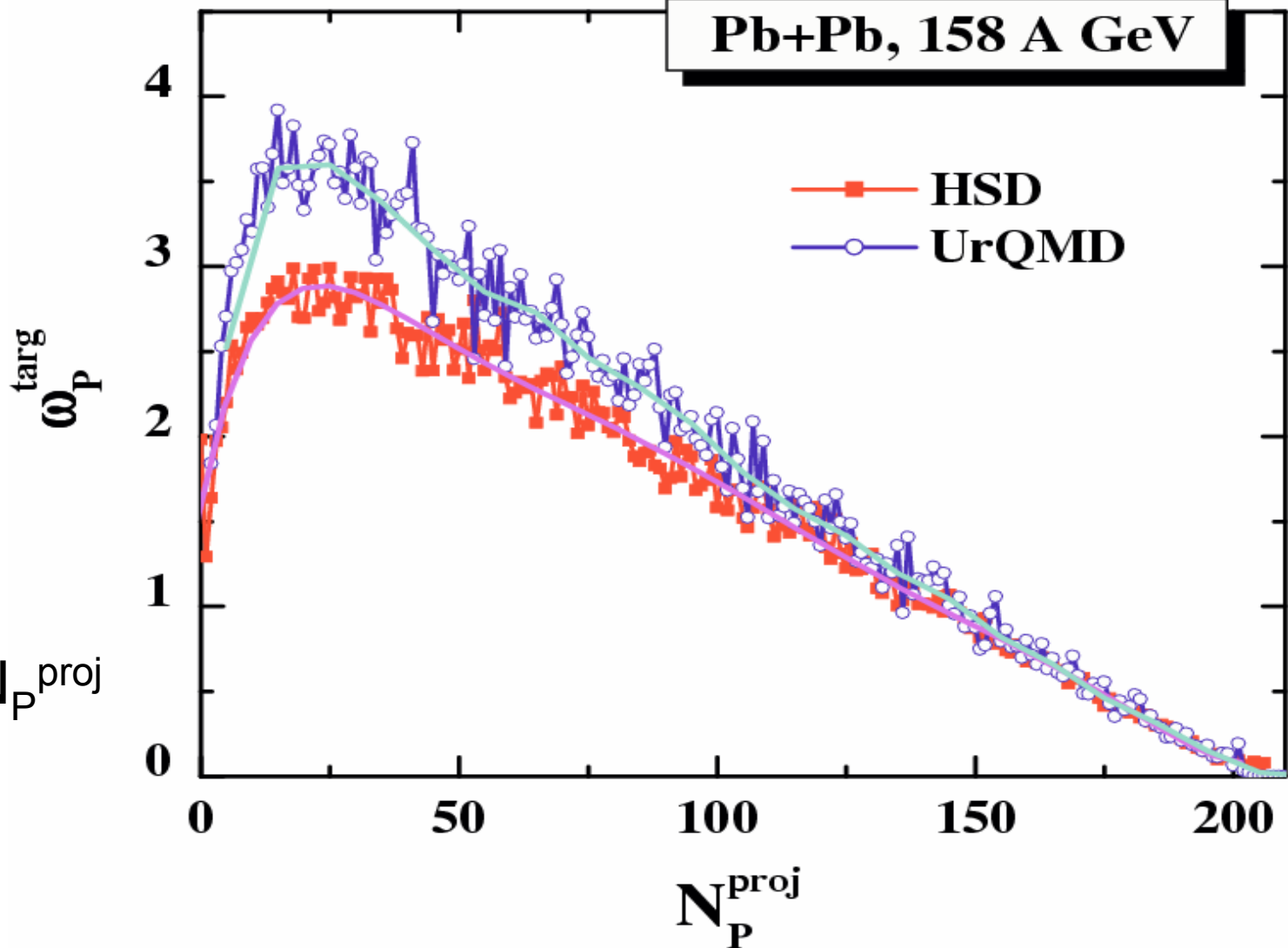
$$\omega_-^* = 1.5$$

$$\omega_+^* = 1.1$$

$$\omega_{ch}^* = 2.5$$



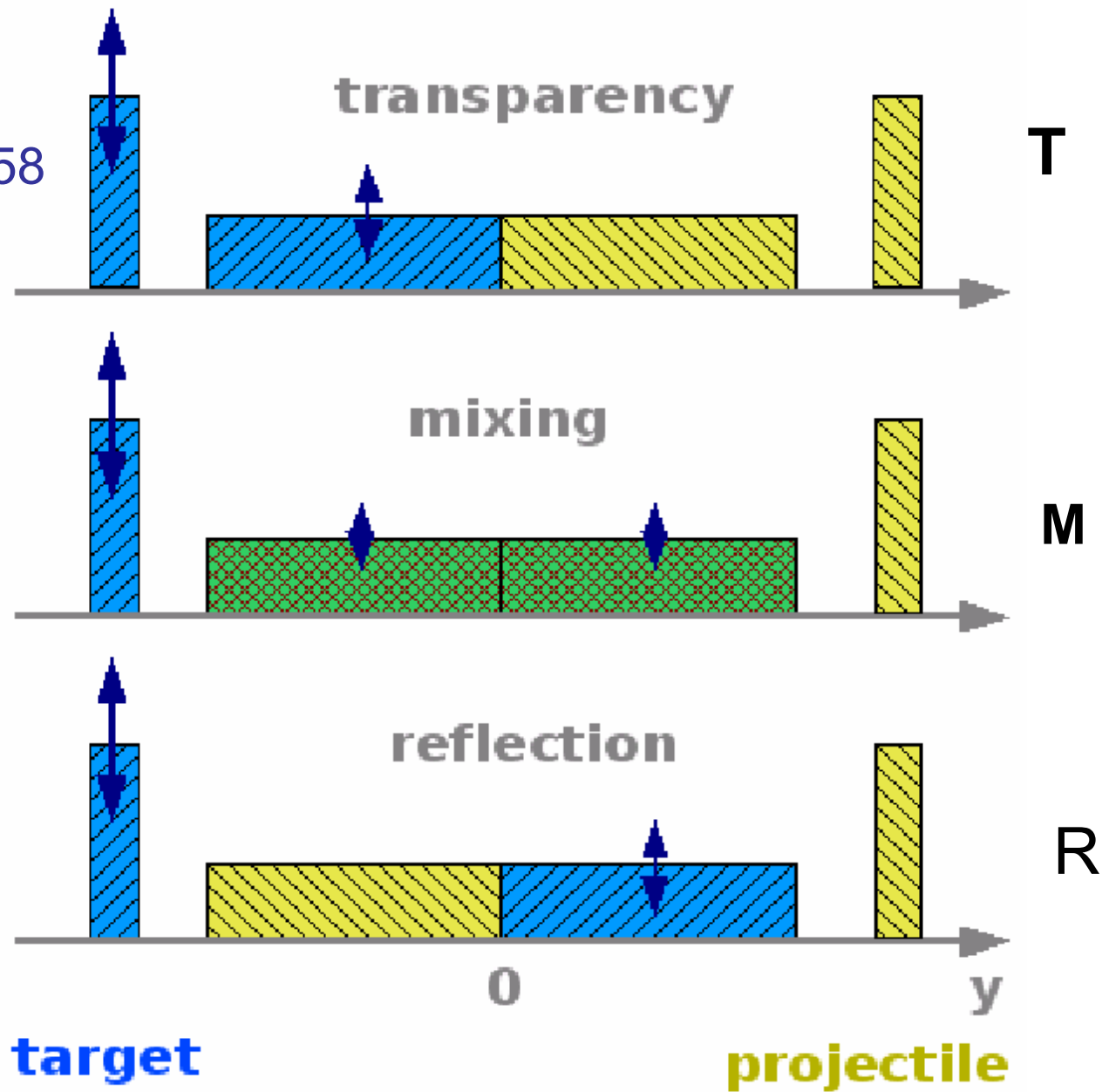
**Pb+Pb, 158 A GeV**



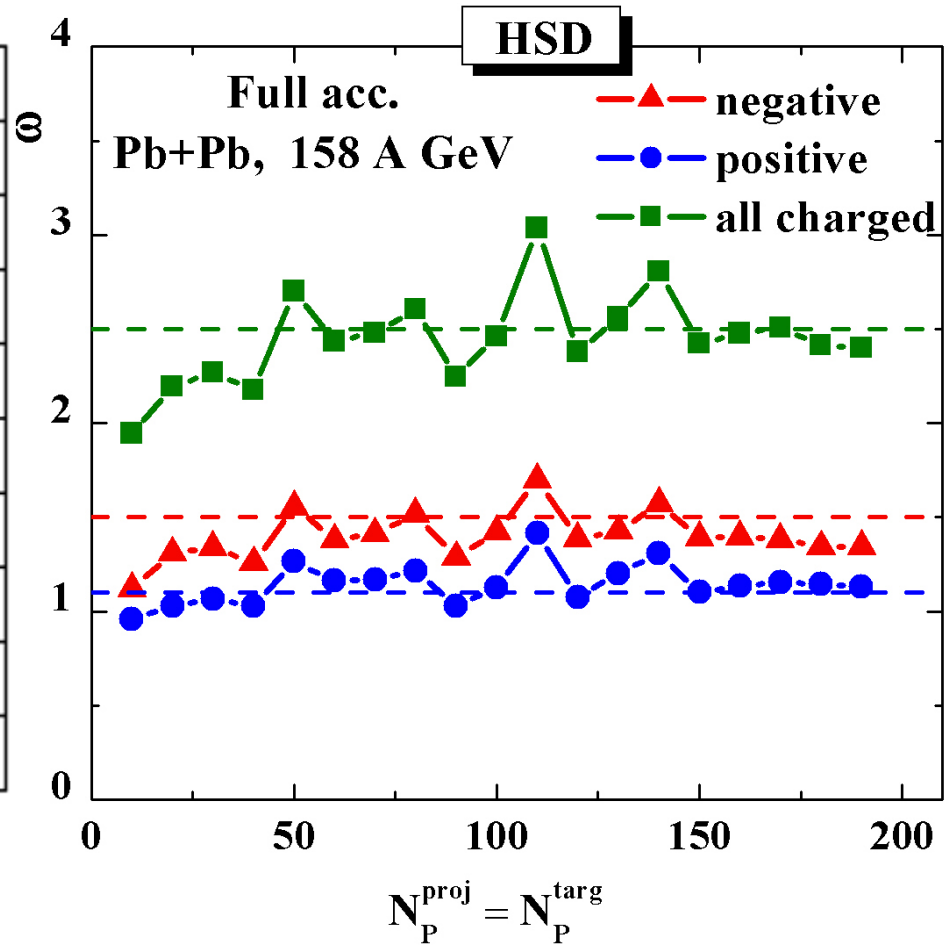
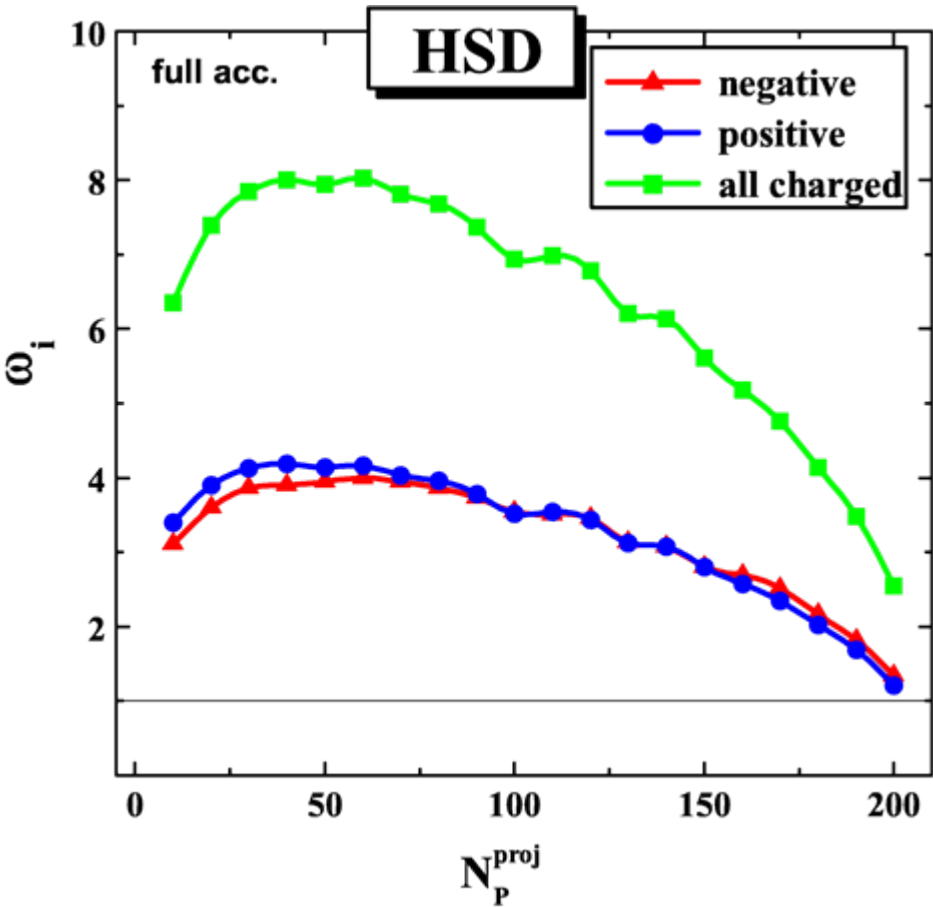
$$\langle N_P^{\text{targ}} \rangle = N_P^{\text{proj}}$$

Konchakovski, Hausler, M.I.G., Bratkovskaya, Bleicher, Stoecker, Phys. Rev. C (2006)

Gazdzicki  
and M.I.G.  
hep-ph/0511058  
Phys. Lett. B  
(2006)

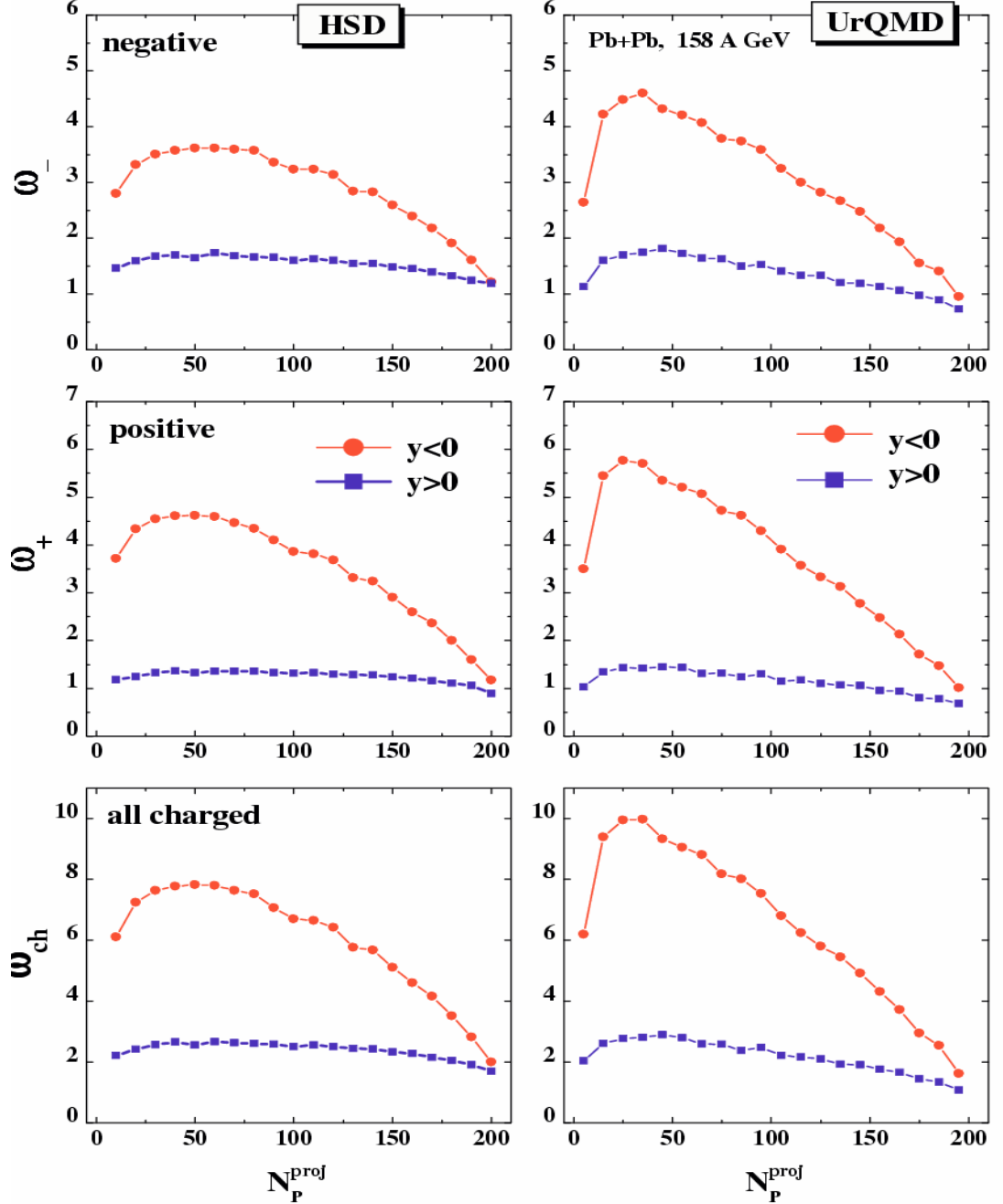


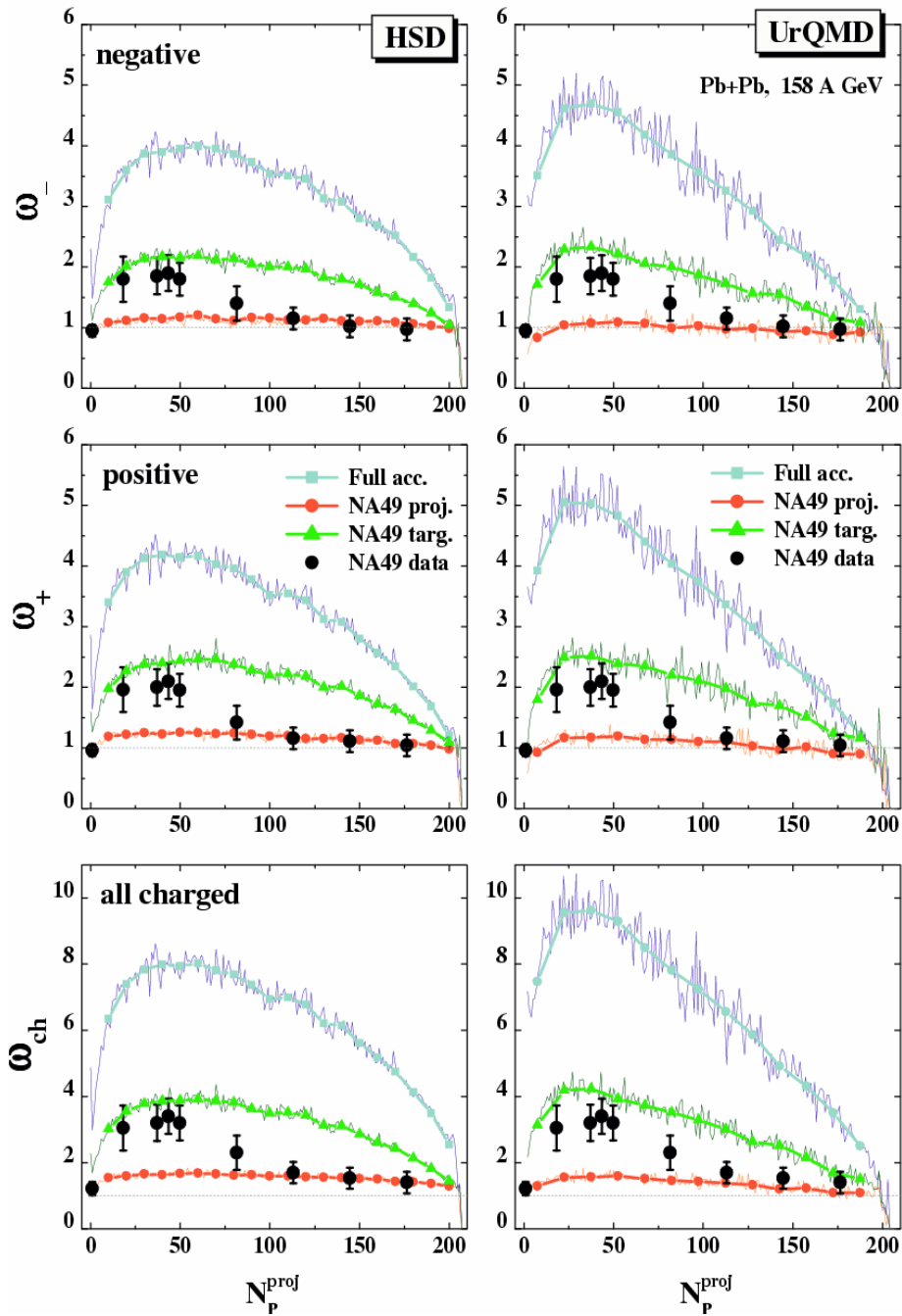


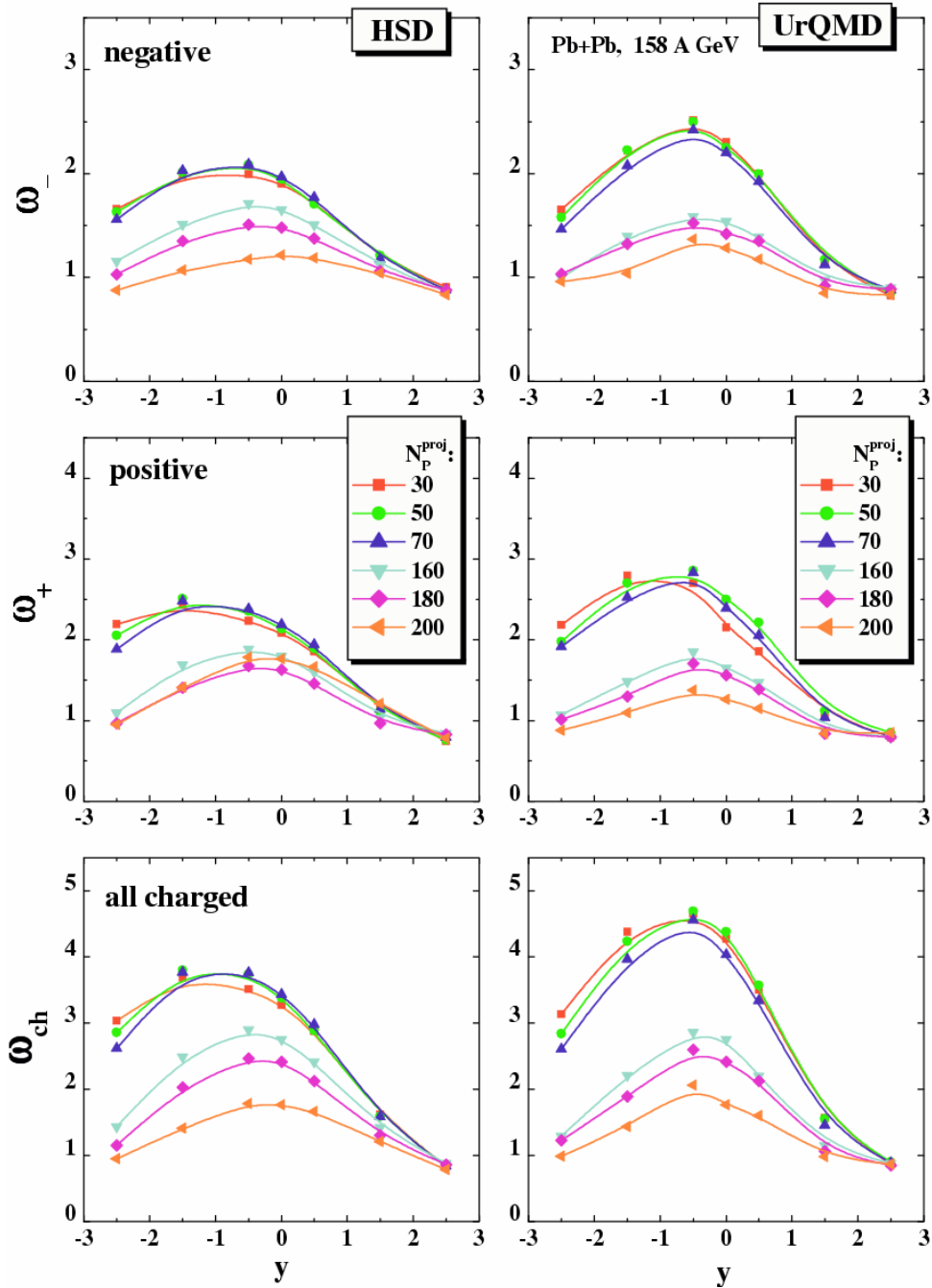


# HSD is close to T-models for pion sources

Konchakovski, Hausler  
M.I.G., Bratkovskaya,  
Bleicher, Stoecker,  
Phys. Rev. C (2006)

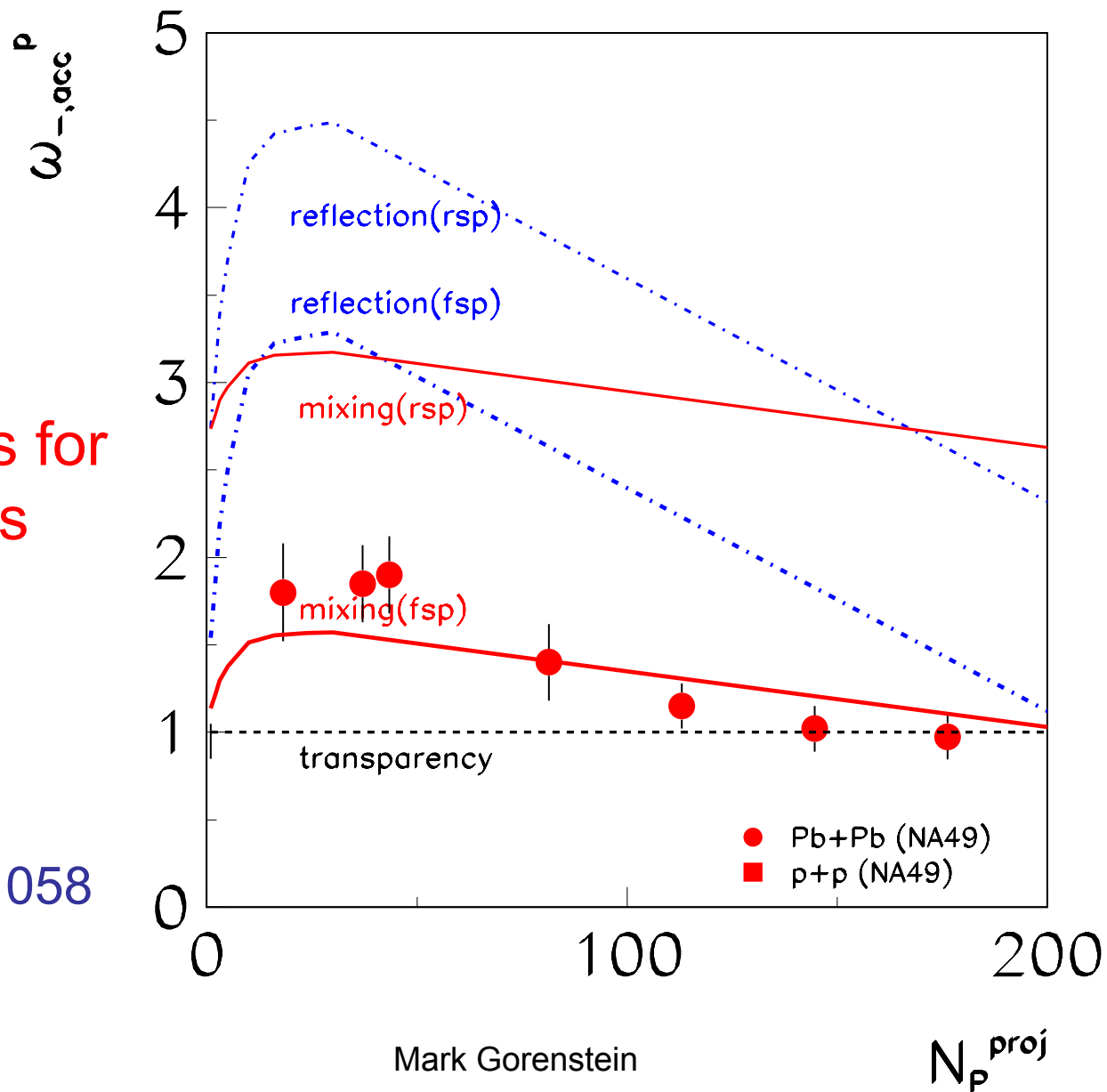




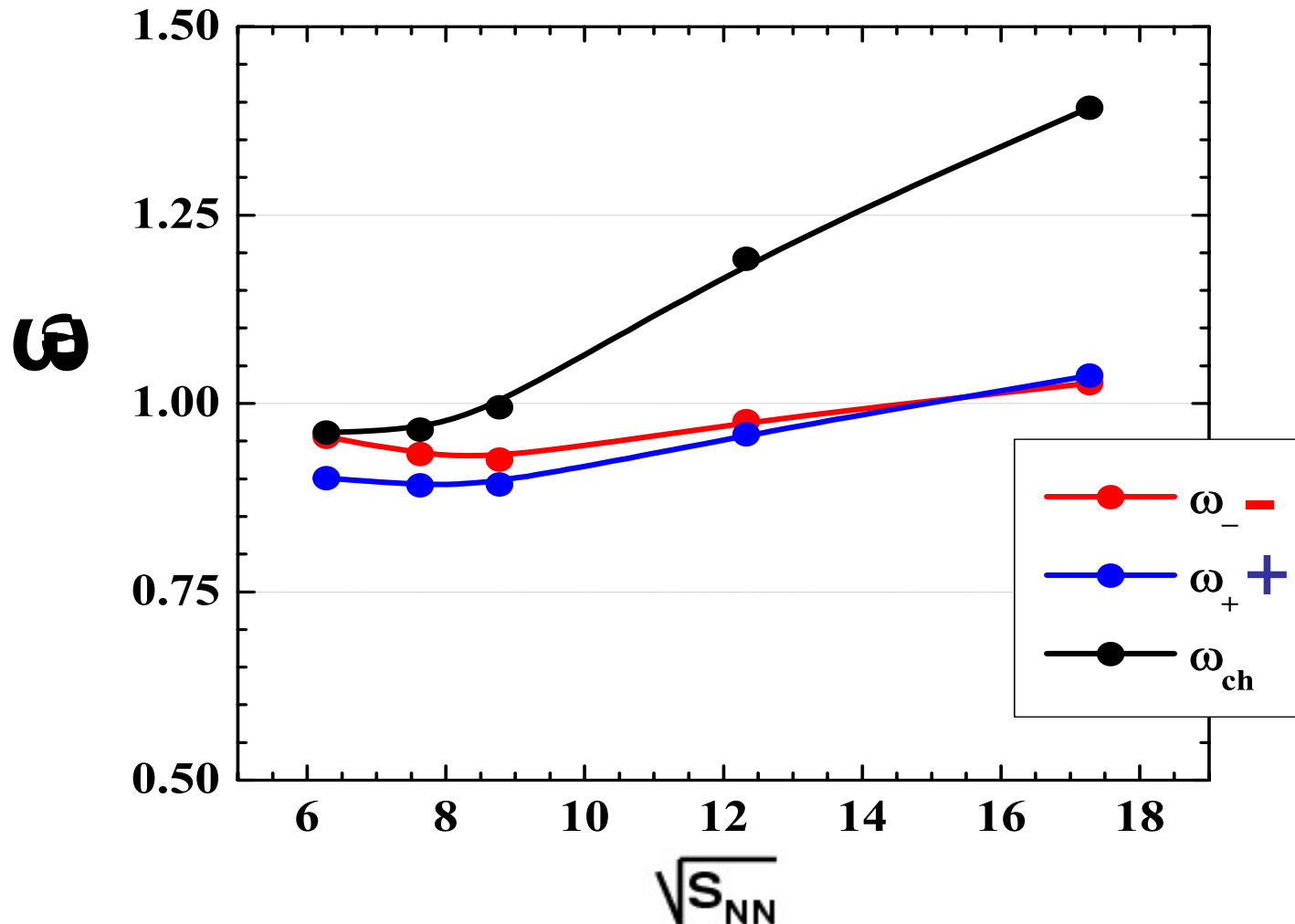


NA49 data  
are close  
to M-models for  
pion sources

Gazdzicki  
and M.I.G.  
hep-ph/0511058



# Scaled Variances in HSD for $N_p^{\text{proj}} = 180 - 200$





# Multiplicity Fluctuations in Hadron-Resonance Gas: CE and MCE

$$\omega^- = \frac{\langle (\Delta N_-)^2 \rangle}{\langle N_- \rangle}, \quad \omega^+ = \frac{\langle (\Delta N_+)^2 \rangle}{\langle N_+ \rangle},$$

Begun, Gazdzicki, M.I.G., Zozulya  
Phys. Rev. C (2004)

$$\langle N_i \rangle = \langle N_i^* \rangle + \sum_R \langle N_R \rangle \sum_r b_r^R n_{i,r}^R \equiv \langle N_i^* \rangle + \sum_R \langle N_R \rangle \langle n_i \rangle_R$$

$$\langle \Delta N_i \Delta N_j \rangle = \langle \Delta N_i^* \Delta N_j^* \rangle + \sum_R \left[ \langle \Delta N_R^2 \rangle \langle n_i \rangle_R \langle n_j \rangle_R + \langle N_R \rangle \langle \Delta n_i \Delta n_j \rangle_R \right],$$

$\langle \dots \rangle$

GCE

Stephanov, Rajagopal, Shuryak, Phys. Rev. B (1999)  
Jeon, Koch, Phys. Rev. Lett. (1999)

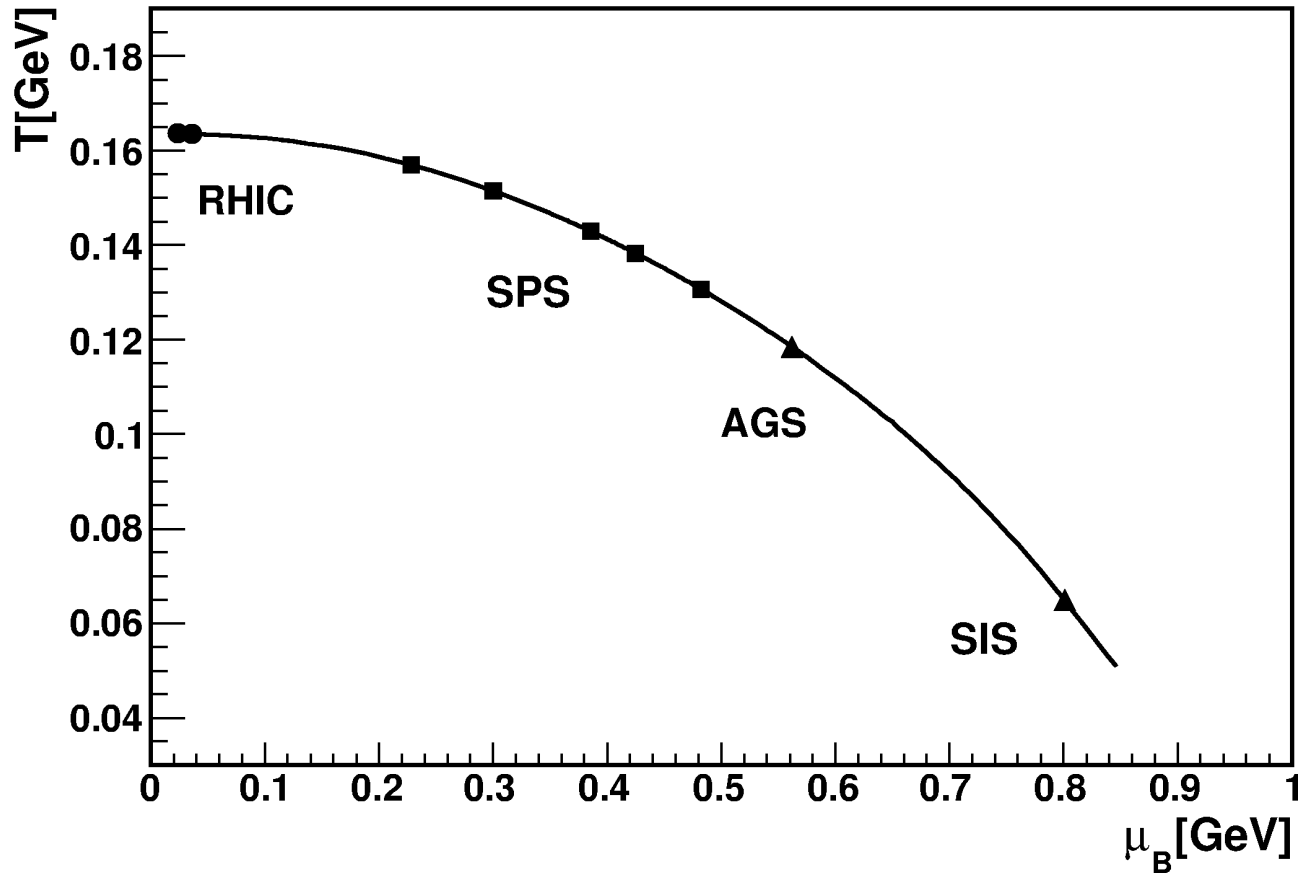
$$\begin{aligned} \langle \Delta N_i \Delta N_j \rangle_{c.e.} &= \langle \Delta N_i^* \Delta N_j^* \rangle_{c.e.} + \sum_R \langle N_R \rangle \langle \Delta n_i \Delta n_j \rangle_R + \sum_R \langle \Delta N_i^* \Delta N_R \rangle_{c.e.} \langle n_j \rangle_R \\ &+ \sum_R \langle \Delta N_j^* \Delta N_R \rangle_{c.e.} \langle n_i \rangle_R + \sum_{R,R'} \langle \Delta N_R \Delta N_{R'} \rangle_{c.e.} \langle n_i \rangle_R \langle n_j \rangle_{R'}. \end{aligned}$$

$\langle \dots \rangle_{c.e.}$   
22.08.2006

CE

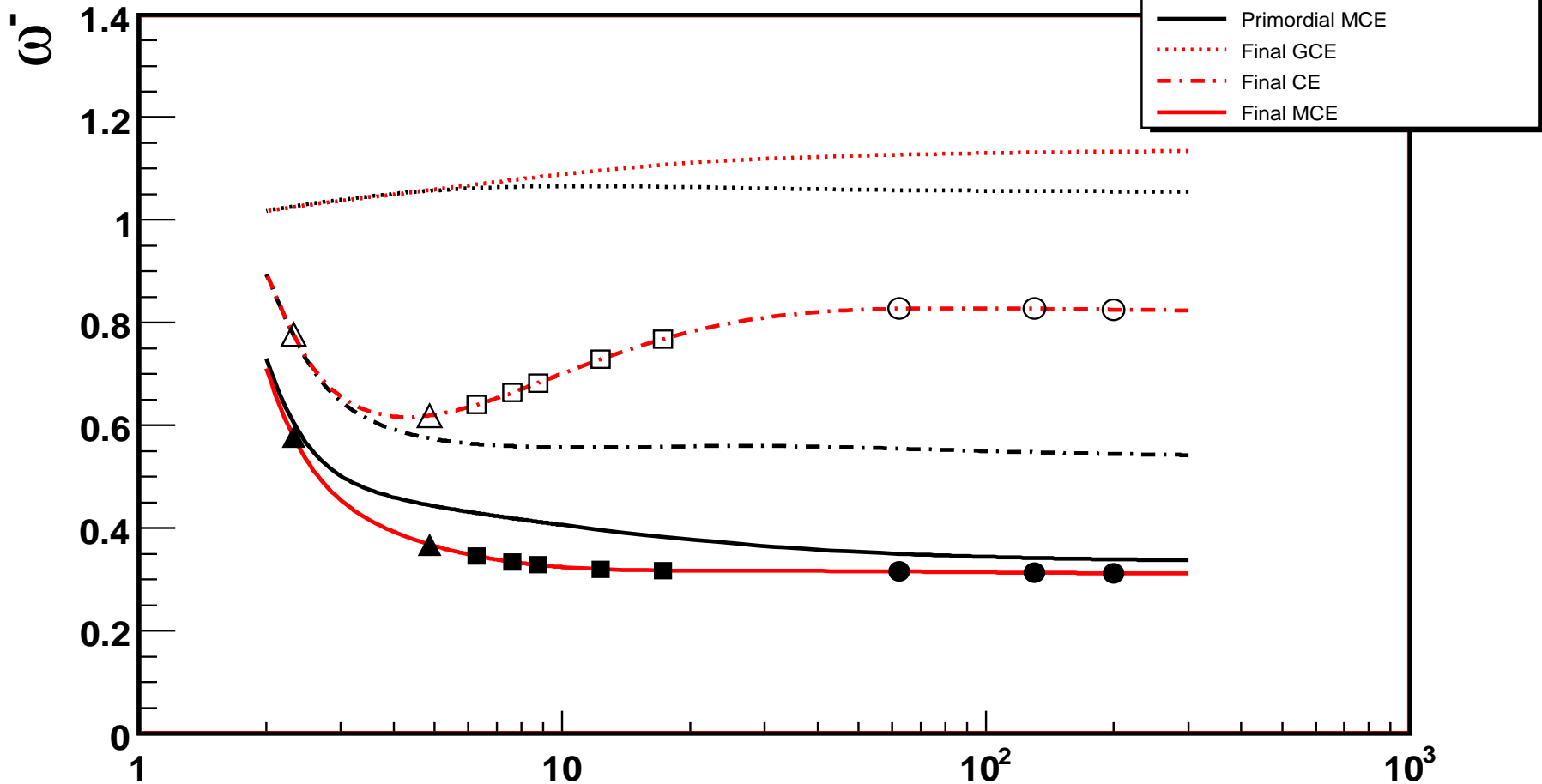
Begun, M.I.G., Hauer, Konchakovski, Zozulya, nucl-th/0606036

# Line of the chemical freeze-out



**E/N = 1 GeV** Cleymans and Redlich, PRL 81 (1998)

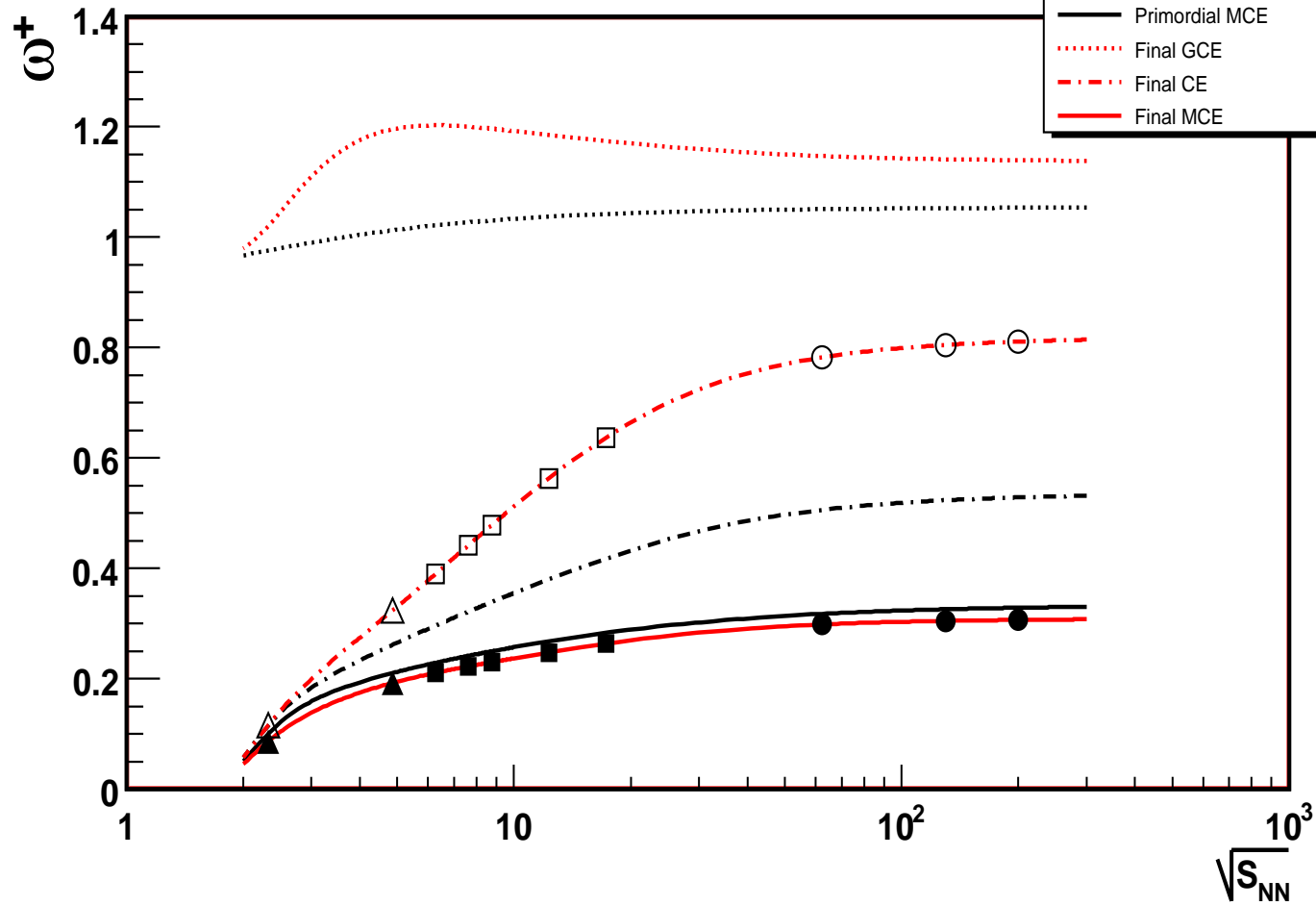
# Scaled Variance along $E/N = 1$ GeV freeze-out line



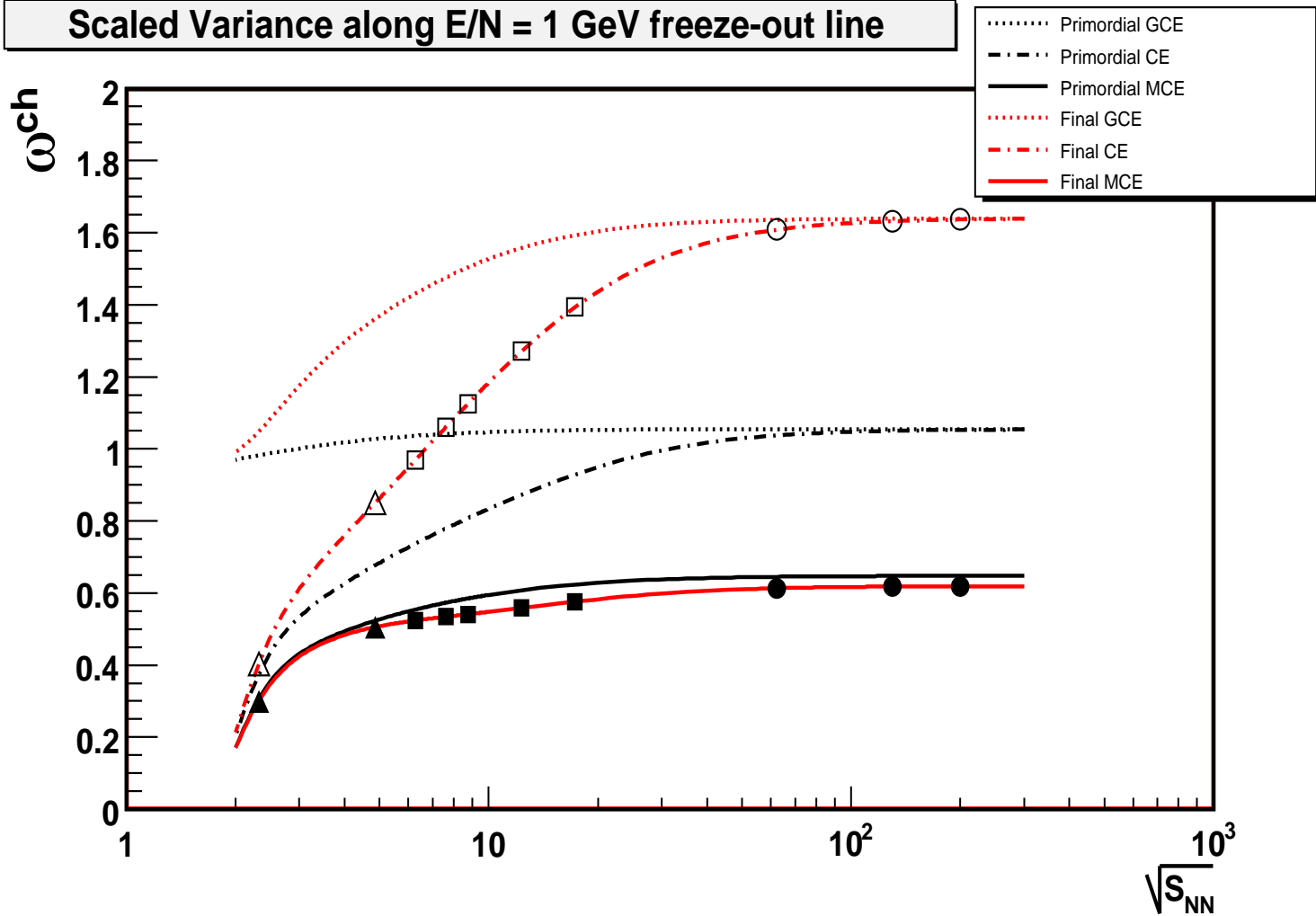
Begun, M.I.G. , Hauer, Konchakovski, Zozulya, nucl-th/0606036

Phys. Rev. C (2006)

### Scaled Variance along $E/N = 1$ GeV freeze-out line

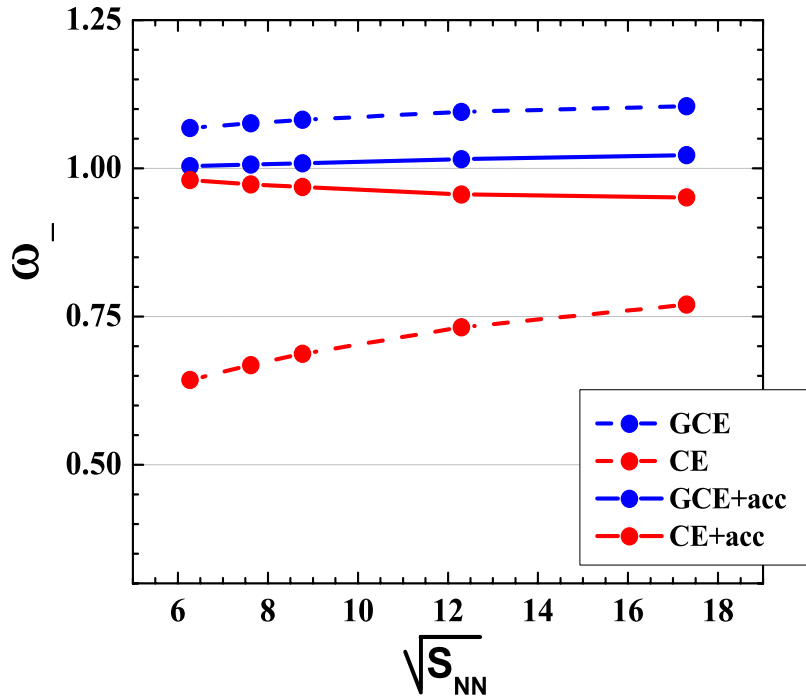


### Scaled Variance along $E/N = 1$ GeV freeze-out line

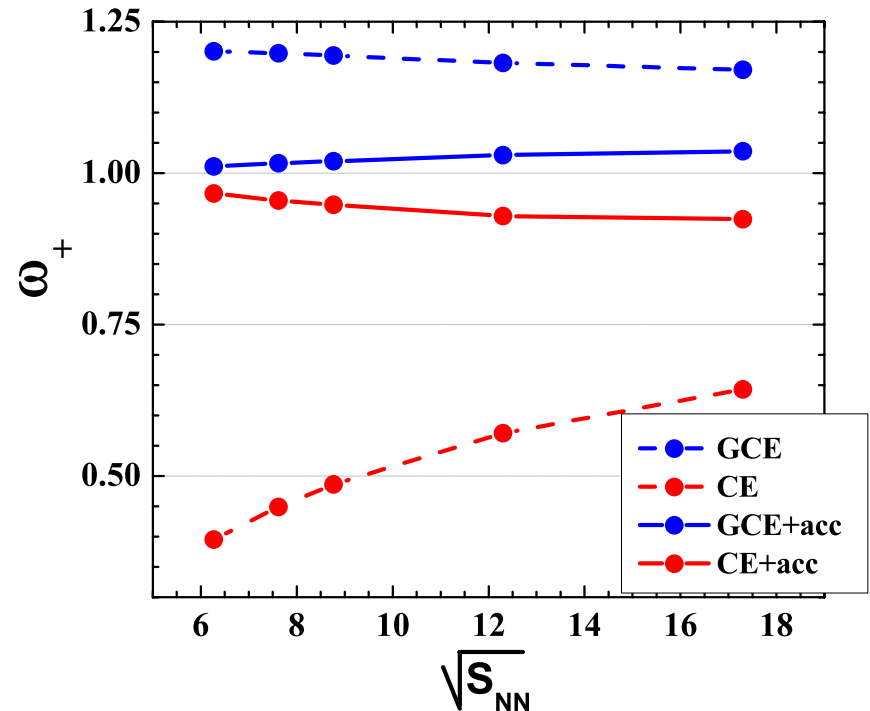


# Scaled Variances in Thermal Model

## Negative hadrons



## Positive hadrons

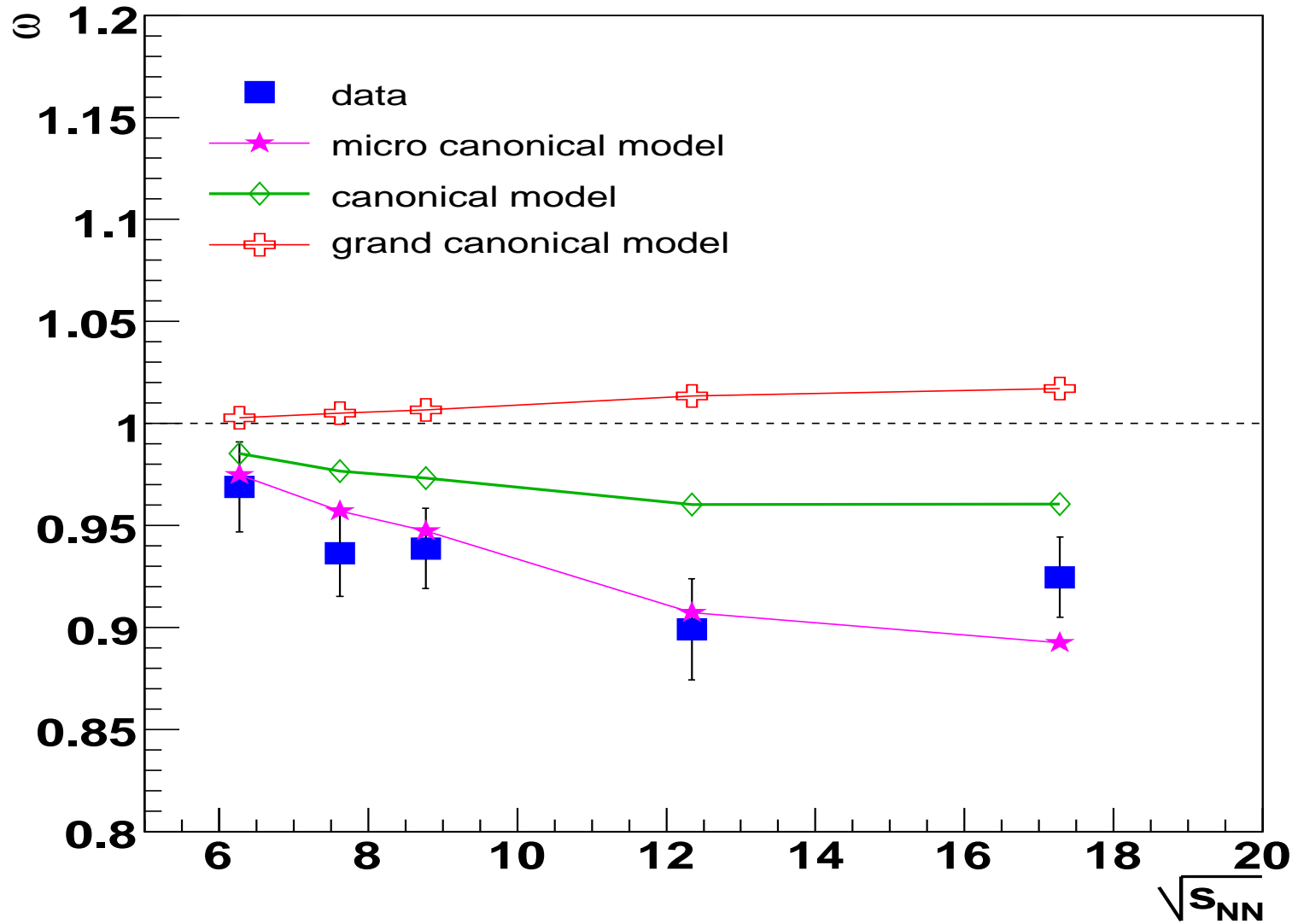


$$\omega_{4\pi}^{\pm} \equiv \frac{\langle (\Delta N_{\pm})^2 \rangle}{\langle N_{\pm} \rangle}$$

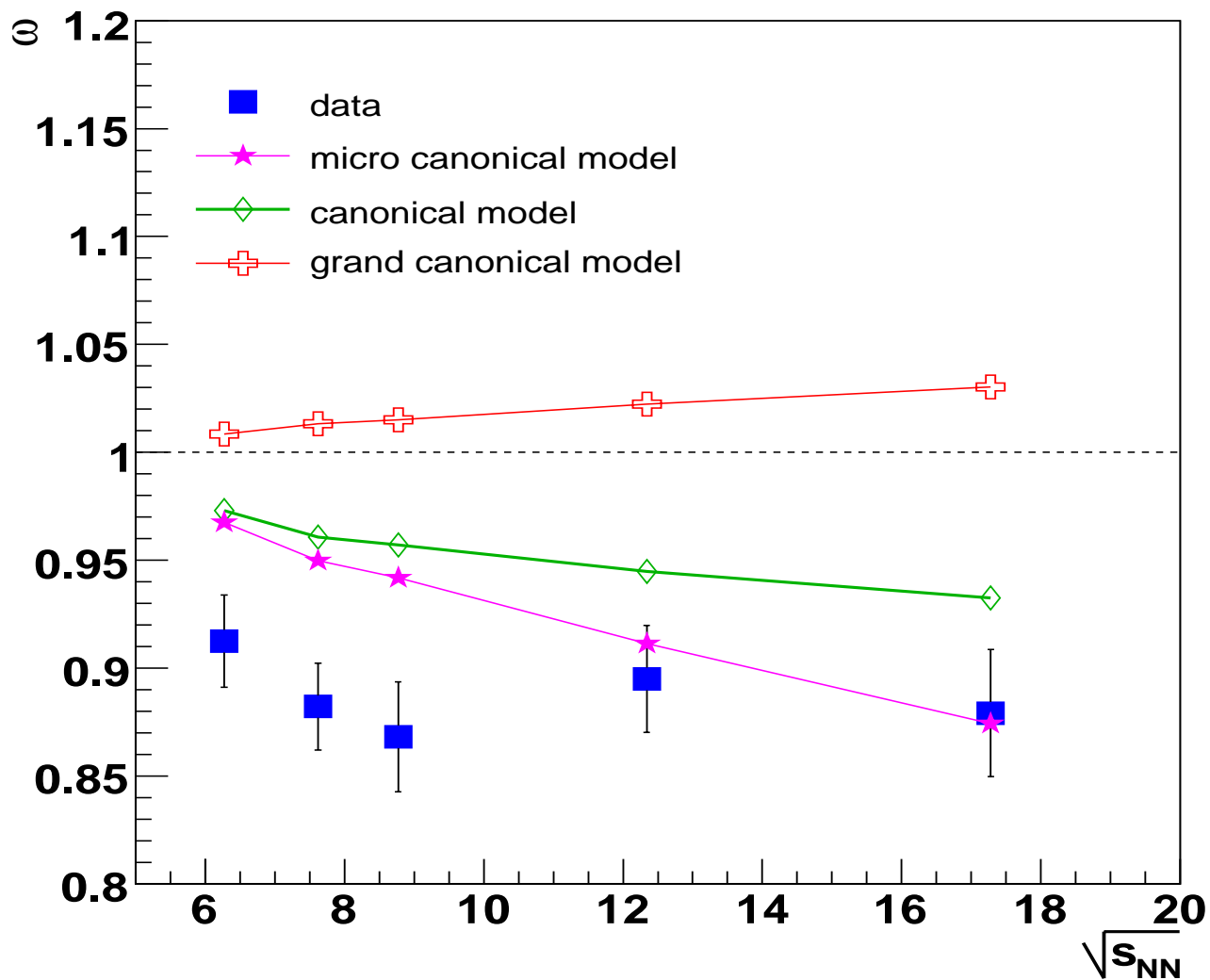
$$\omega_{acc}^{\pm} = 1 - q + q\omega_{4\pi}^{\pm}$$



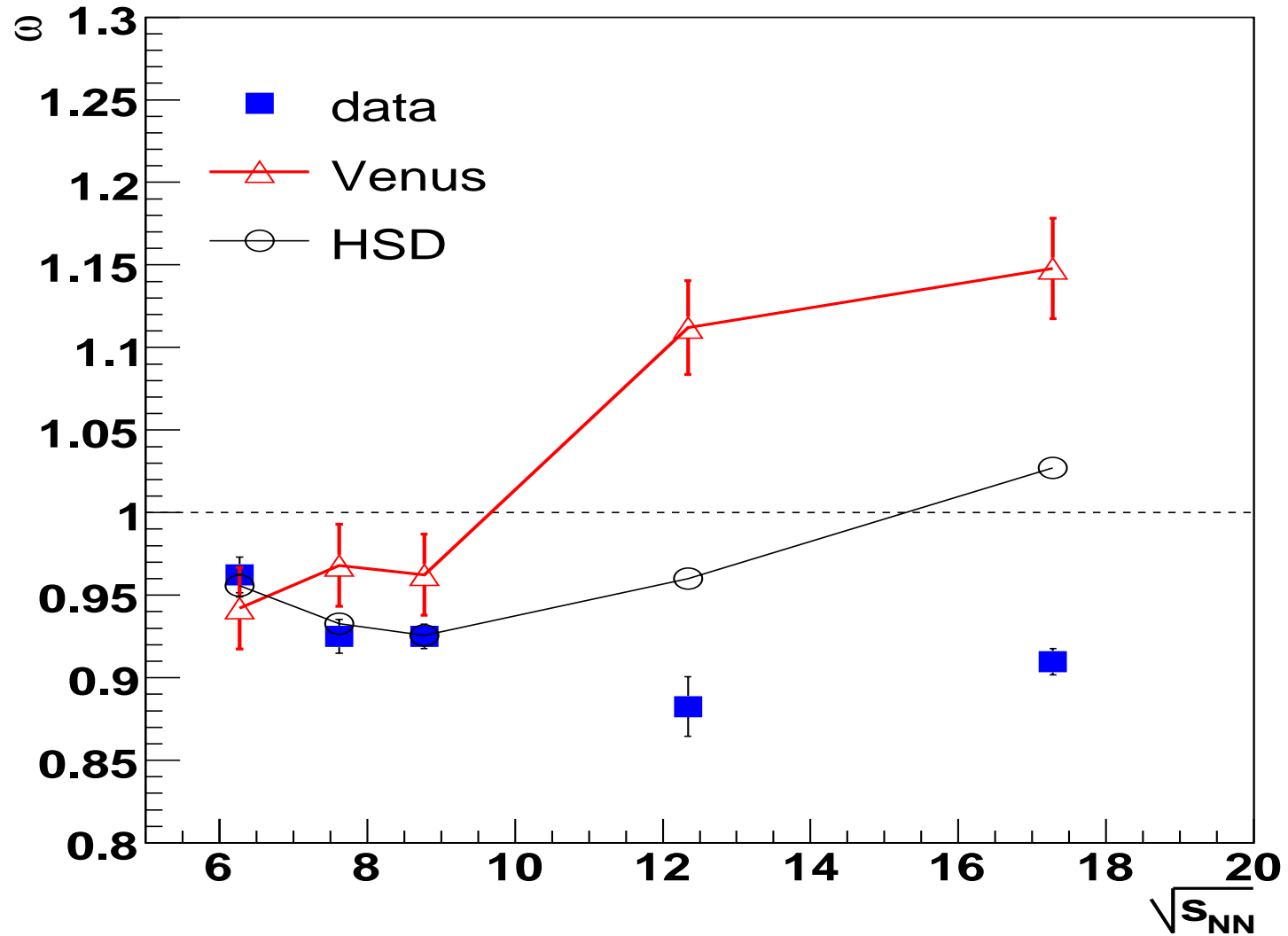
# Negative Hadrons



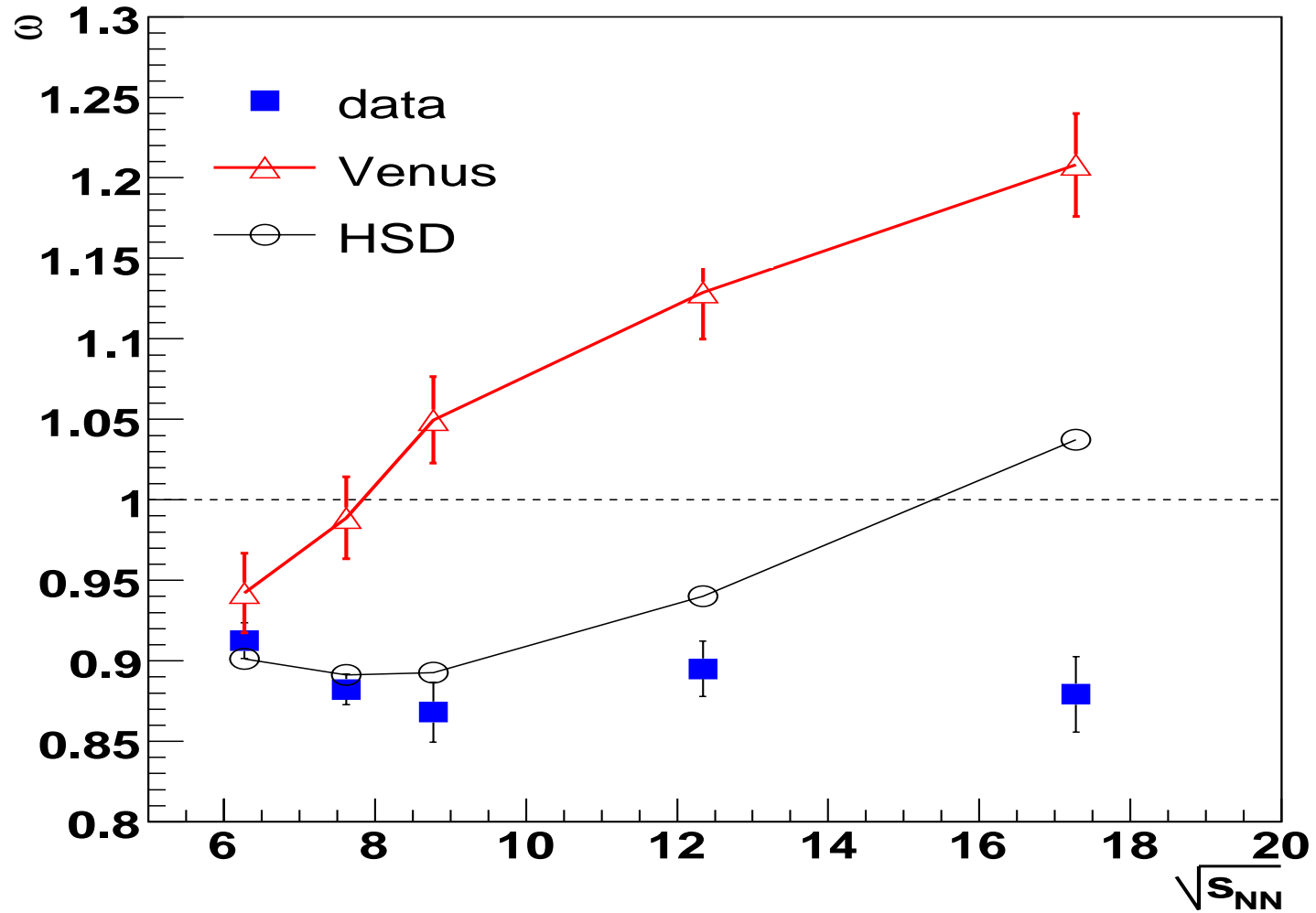
# Positive Hadrons



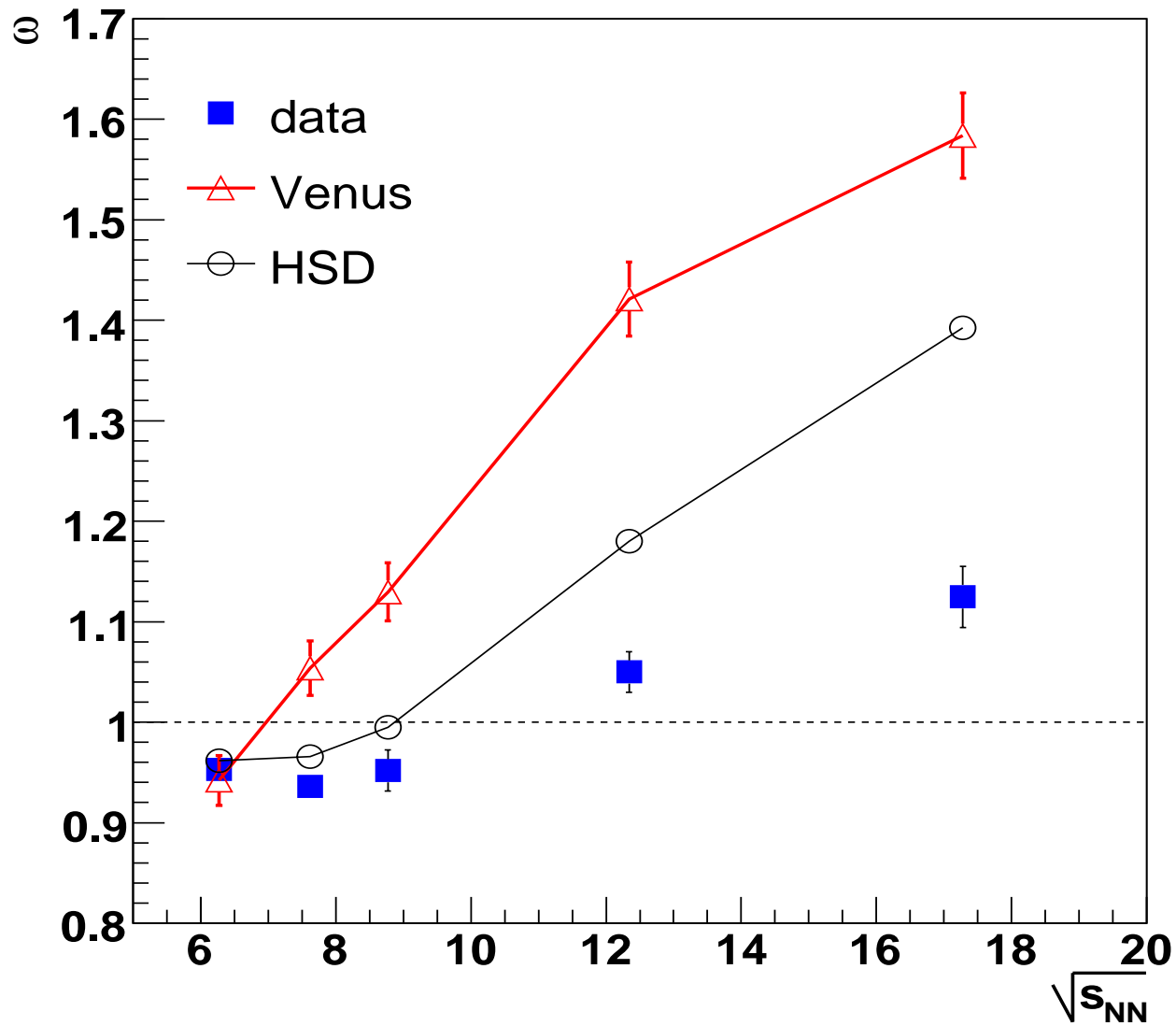
# Negative Hadrons



# Positive Hadrons



# Charged Hadrons

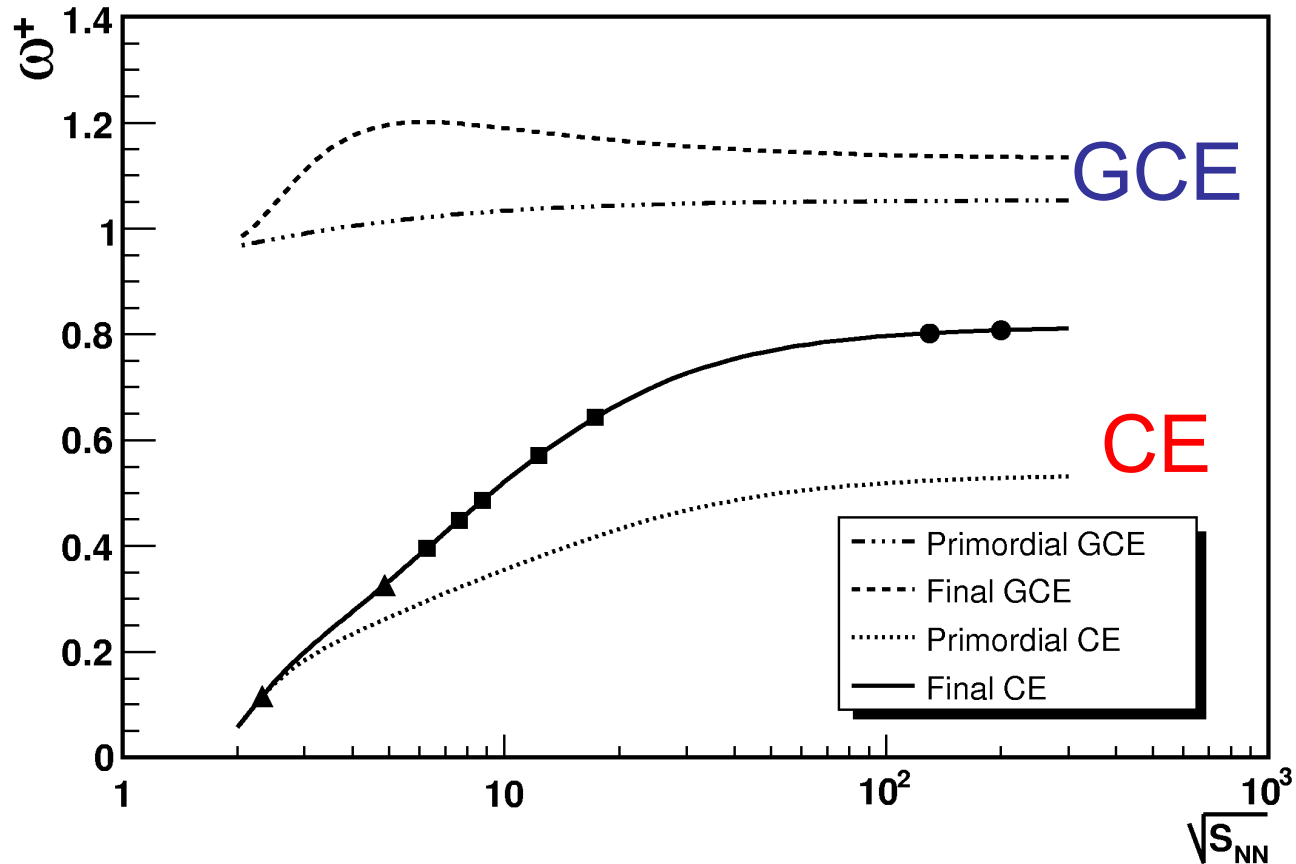


# Conclusions

1. Study of **fluctuations** gives a unique possibility to investigate the early stage **dynamics of initial flows**. The existing models are divided into 3 limiting groups: **T-, M-, R- models**.
3. **HSD** and **UrQMD** are close to **T-models**.
4. **NA49 data** for charged hadron multiplicity fluctuations are consistent with **M-models**.
5. Statistical fluctuations in A+A can be clearly seen in most central collisions,  **$N_p^{\text{proj}} = 180 - 200$  ( $< 1\%$ )**.
6. Scaled variances are very different in the GCE, **CE** and **MCE**.  
NA49 data in most central Pb+Pb collisions are close to the **MCE statistical results**.

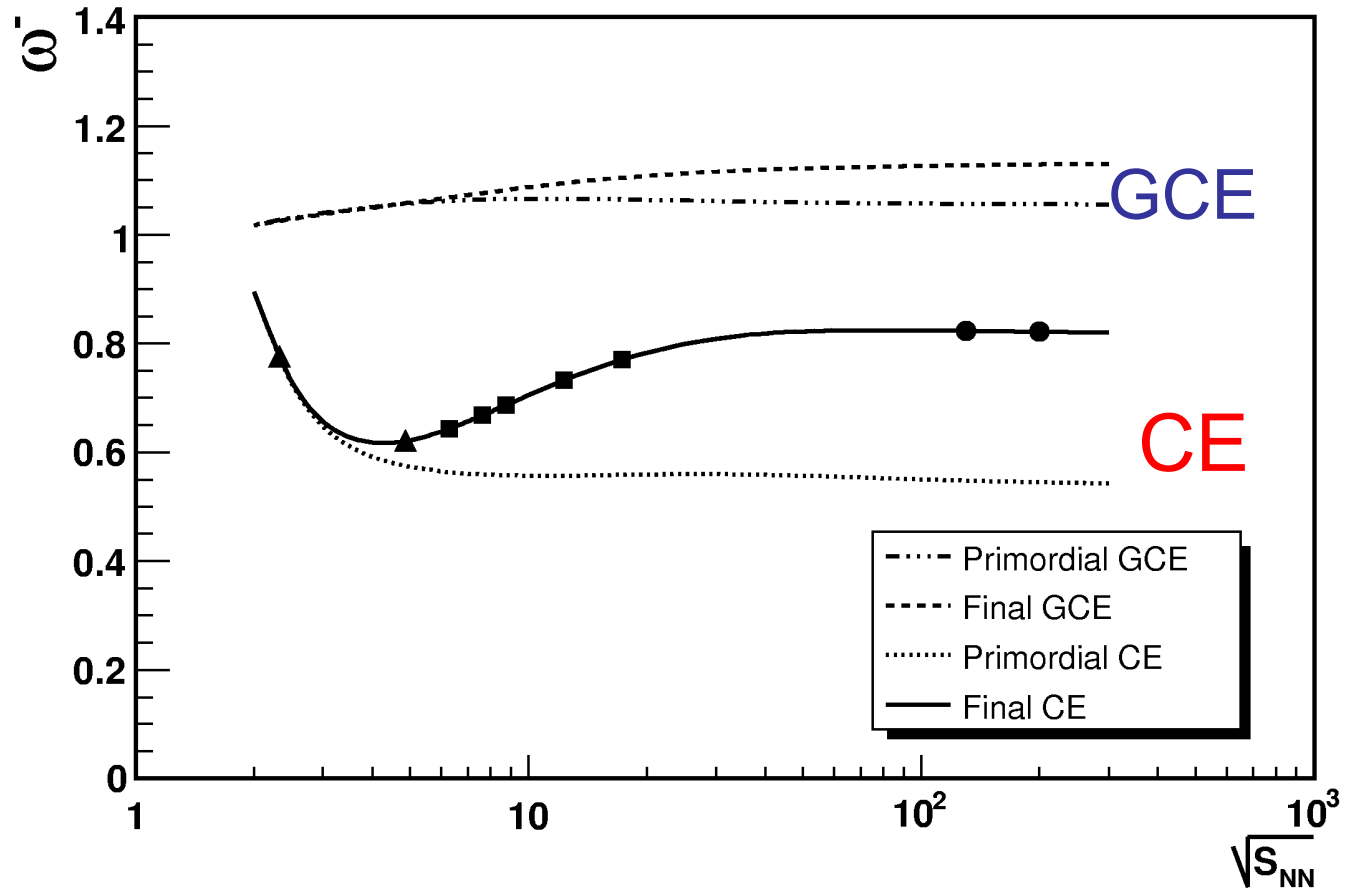


# Scaled Variance for positive hadrons



Begun, M.I.G. , Hauer, Konchakovski, Zozulya, nucl-th/0606036

# Scaled Variance for negative hadrons



Begun, M.I.G. , Hauer, Konchakovski, Zozulya, nucl-th/0606036