Critical Line of the Deconfinement Phase Transitions

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1. Motivation

2. Phase Transitions in the System of Quark-Gluon Bags

Gorenstein, Petrov, Zinovjev (1981) ...

Gorenstein, Greiner, Yang (1998)

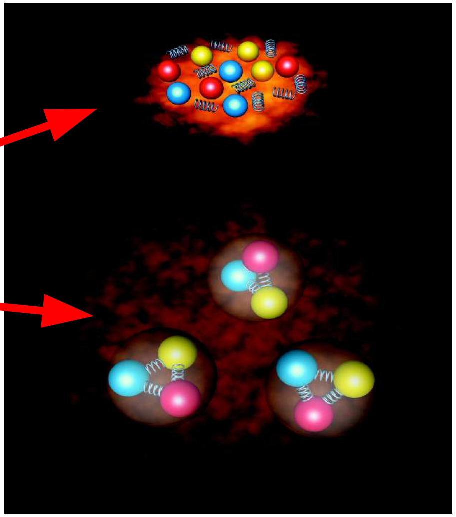
3. **1st** order PT, **2nd** order PT, **3rd** order PT, ...

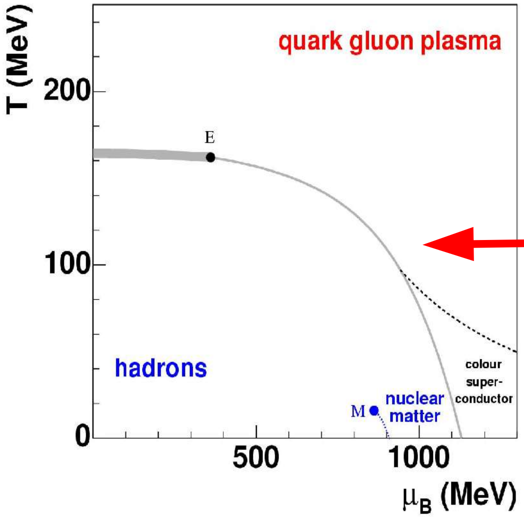
4. Phase Diagram in the Plane of Temperature -- Baryonic Chemical Potential

1. Motivation

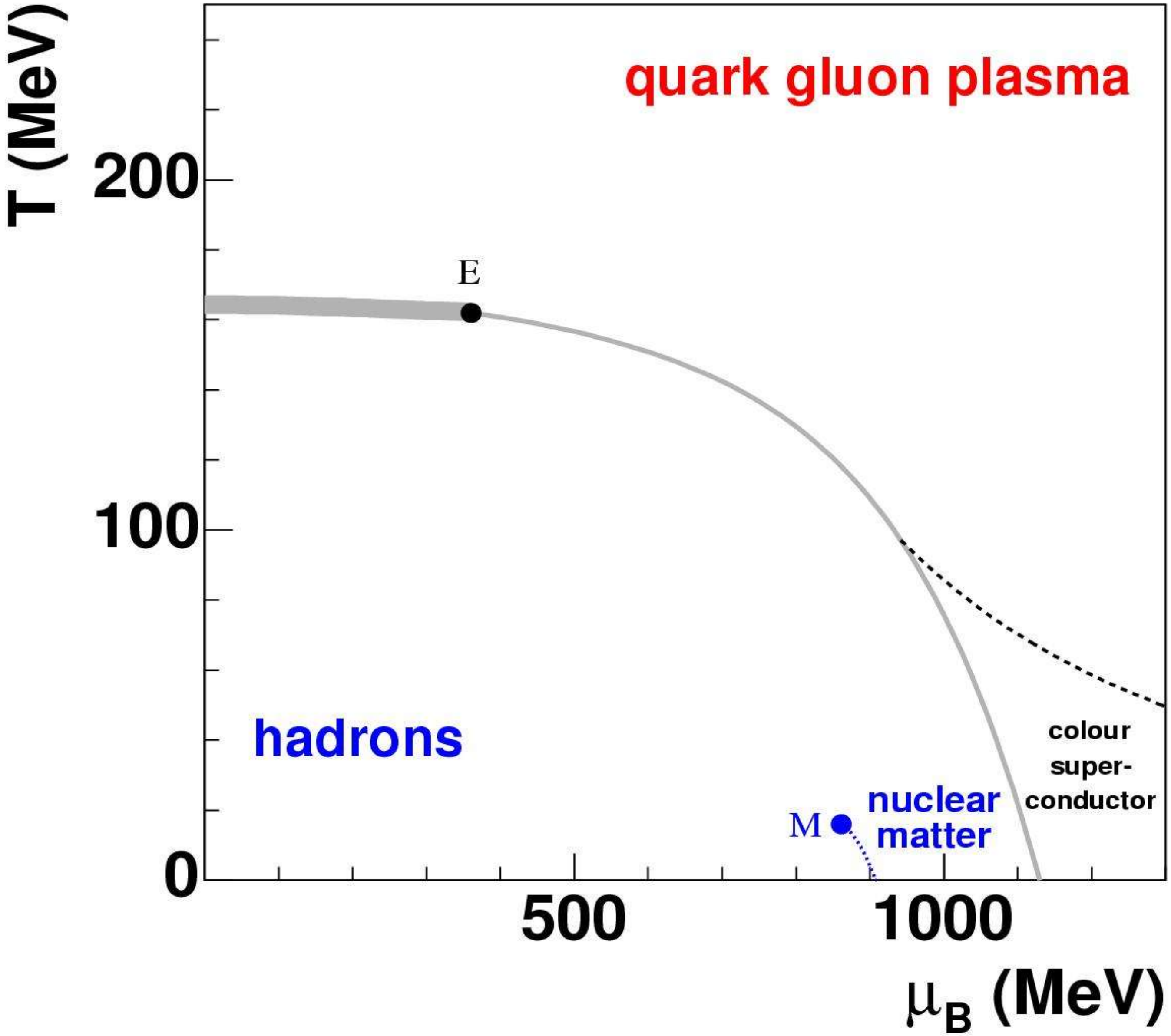
Two basic questions:

What are the phases of strongly interacting matter?

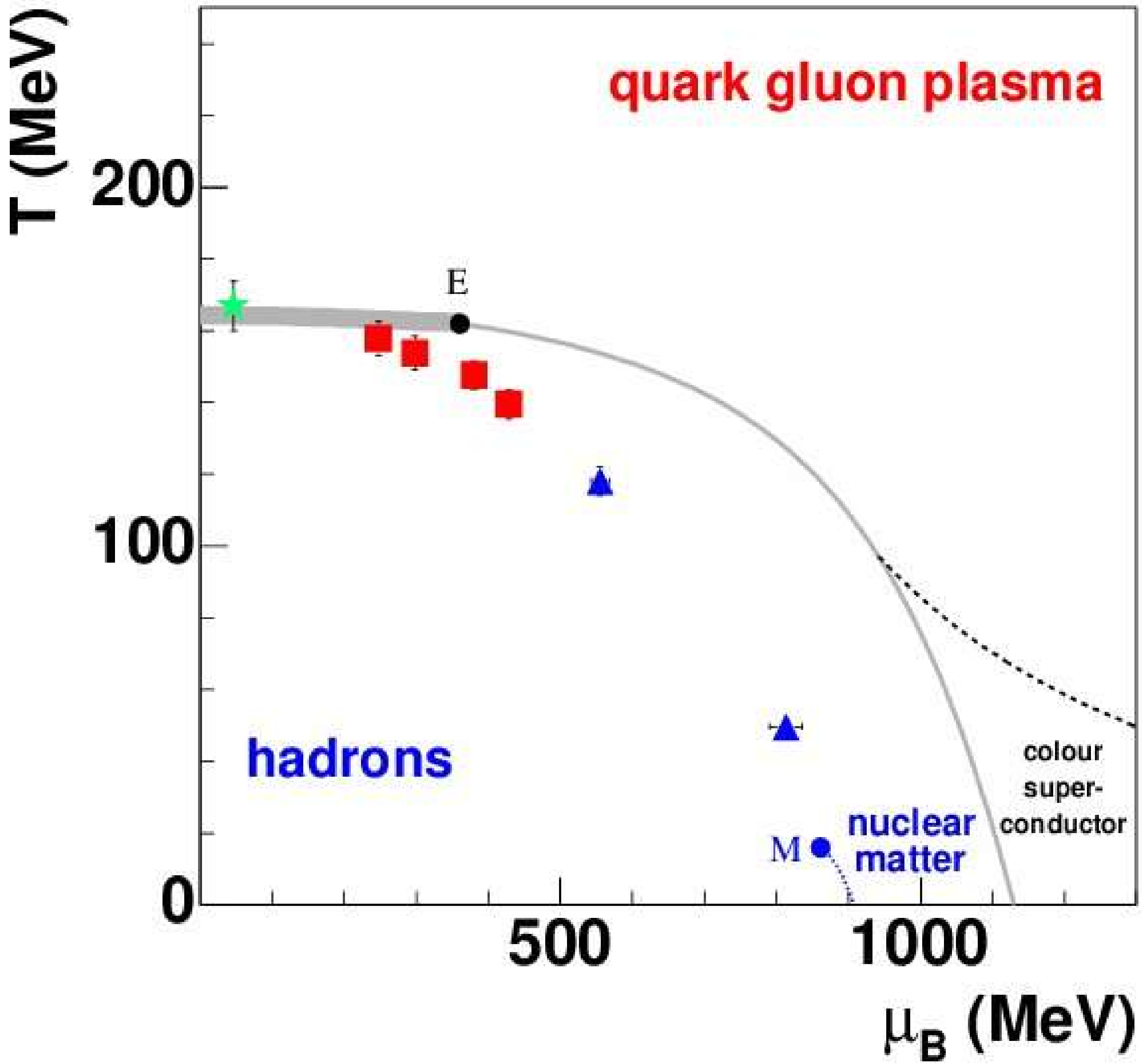




How do the transitions between them look like?

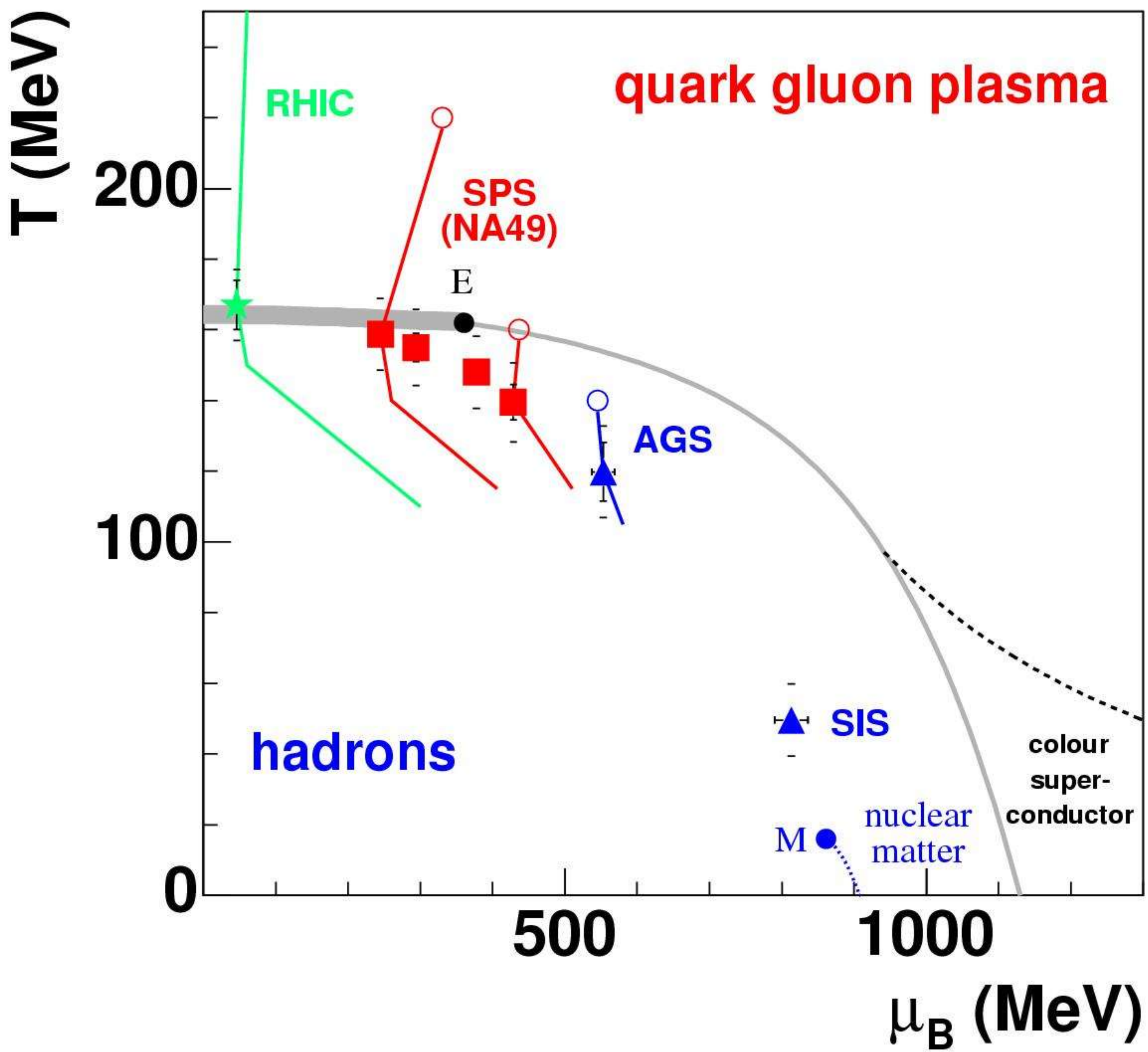


Rajagopal, Wilczek,
Stephanov, Shuryak, ...

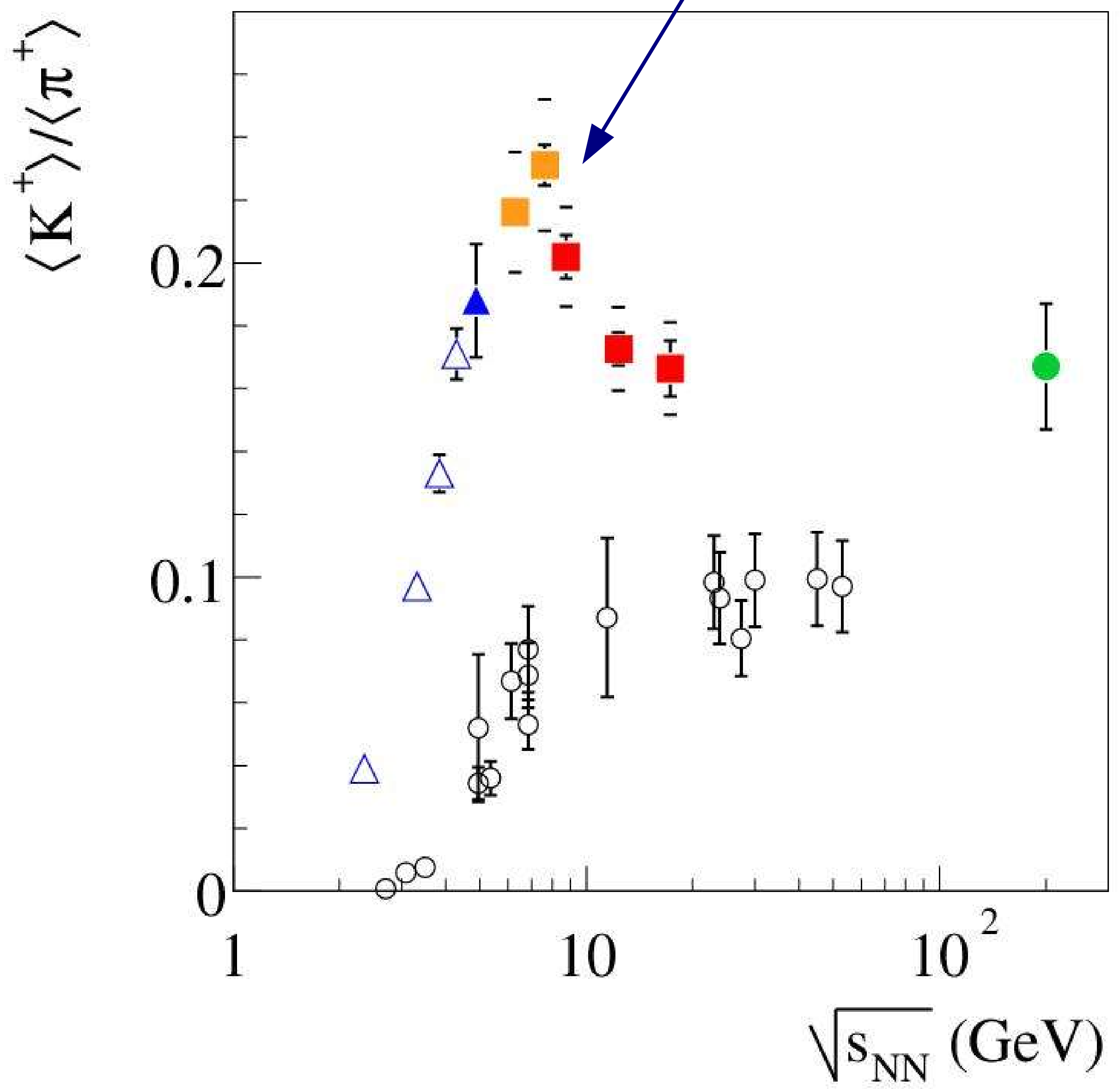


Data: NA49 (SPS), AGS, RHIC

Fit: Becattini et al.


T (MeV)
200
100
0
500
1000
μB (MeV)
RHIC
quark gluon plasma
SPS
(NA49)
E
AGS
SIS
hadrons
colour
super-
conductor
nuclear
matter
M

Onset of the Deconfinement



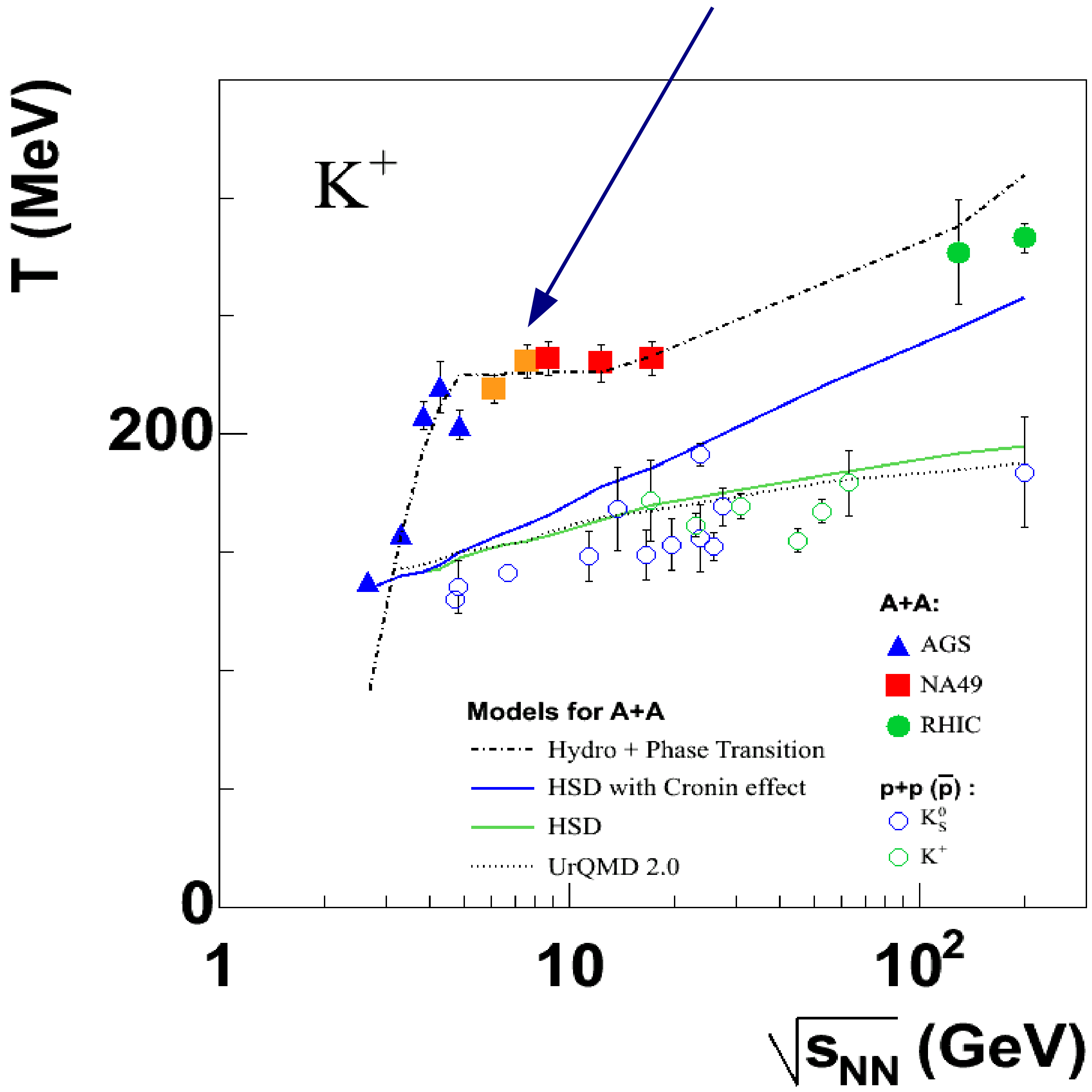
$$f \propto \exp(-m/T)$$

$$df \propto +m f\, dT/T^2$$

Data: NA49 (SPS), AGS, RHIC

Prediction: Gazdzicki, Gorenstein
Acta Phys. Pol. B30, 2705 (1999)

Onset of the Deconfinement



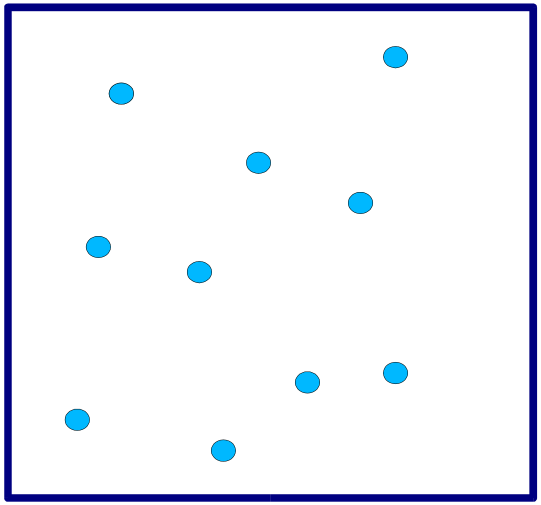
$p \approx$ const

$T \approx$ const

Data: NA49 (SPS), AGS, RHIC

Prediction: Gorenstein, Gazdzicki, Bugaev
Phys. Lett. B567, 175 (2003)

Partition Function of the Ideal Gas:



$$Z(V,T) = \sum_{N=0}^{\infty} \frac{1}{N!} \prod_{j=1}^{N} \int \frac{V\, d^3k_j}{(2\pi)^3} \times \exp\left[- \frac{(k_j^2 + m^2)^{1/2}}{T} \right]$$

$$= \sum_{N=0}^{\infty} \frac{[V\ \phi(T,m)]^N}{N!} = \exp[V\phi(T,m)]$$

Particle Number Density:

$$\phi(T,m) \equiv \frac{1}{2\pi^2} \int_0^{\infty} k^2 dk\ \exp\left[- \frac{(k^2 + m^2)^{1/2}}{T} \right]$$

$$= \frac{m^2 T}{2\pi^2} K_2\left(\frac{m}{T}\right)$$

$$\overline{N}(V,T) \;=\; V\;\phi(T,m)\;, \qquad n(T) \;\equiv\; \frac{\overline{N}}{V} \;=\; \phi(T,m)$$

Pressure:

$$p(T) \;\equiv\; T\,\frac{\ln Z(V,T)}{V} \;=\; T\,\phi(T,m)$$

Energy Density:

$$\varepsilon(T) \;\equiv\; T\,\frac{dp}{dT} \;-\; p(T) \;=\; T^2\,\frac{d\phi(T,m)}{dT}$$

Multi-Component Gas

$$Z(V,T) = \sum_{N_1=0}^{\infty} \dots \sum_{N_n=0}^{\infty} \frac{[V\phi(T,m_1)]^{N_1}}{N_1!} \dots$$

$$\dots \frac{[V\phi(T,m_n)]^{N_n}}{N_n!} = \exp\left[V \sum_{j=1}^{n} \phi(T,m_j) \right]$$

$$p(T) = T \sum_{j=1}^{n} \phi(T,m_j) \ , \quad \varepsilon(T) = T^2 \sum_{j=1}^{n} \frac{d\phi(T,m_j)}{dT}$$

Infinite Number of Components

$$\sum_{j=1}^{\infty} \dots = \int_0^{\infty} dm \dots \rho(m)$$

$$Z(V,T) = \exp\left[V \int_0^{\infty} dm \ \rho(m) \ \phi(T,m) \right]$$

$$p(T) \ = \ T \int_0^\infty dm \ \rho(m) \ \phi(T,m)$$

$$\varepsilon(T) \ = \ T^2 \int_0^\infty dm \ \rho(m) \ \frac{d\phi(T,m)}{dT}$$

Limiting Temperature

Hagedorn (1965), Frautschi (1971) SBM

$$\rho(m)_{m\to\infty} \simeq \ C \ m^{-a} \ \exp(bm) \ , \qquad b \ \equiv \ \frac{1}{T_H}$$

$$\phi(T,m) \ \simeq \ \left(\frac{mT}{2\pi}\right)^{3/2} \ \exp\left(- \ \frac{m}{T}\right)$$

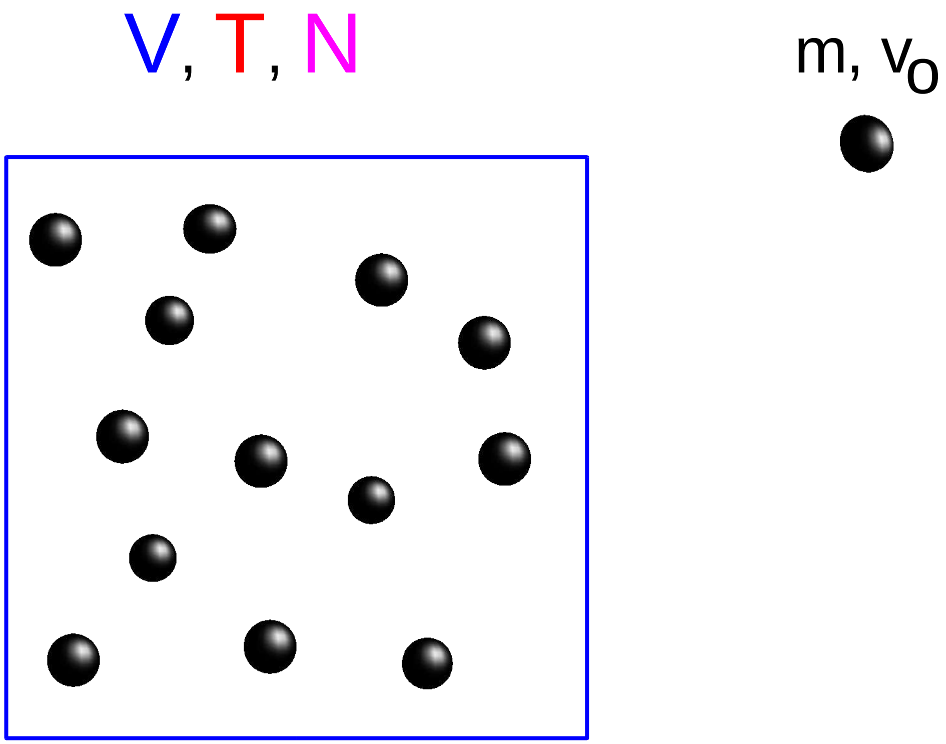
$$T \ < \ T_H \ , \qquad T \ \to \ T_H \ :$$

$$p \ , \ \varepsilon \ \to \ \infty \ , \quad a \ \le \ \frac{5}{2}$$

$$p \ \to const \ , \ \varepsilon \ \to \ \infty \ , \quad \frac{5}{2} \ \le \ a \ \le \ \frac{7}{2}$$

$$p \ , \ \varepsilon \ \to \ const \ , \quad a \ > \ \frac{7}{2}$$

van der Waals



$$V \longrightarrow V - N v_o$$

Van der Waals repulsion: $V \to V - v_o N$

$$Z(V,T) = \sum_{N=0}^{\infty} \frac{[(V - v_o N)\ \phi(T,m)]^N}{N!} \theta(V - v_o N)$$

$$\hat{Z}(s,T)\ \equiv \int_0^{\infty} dV \exp(-sV)\ Z(V,T)$$

$$= \sum_{N=0}^{\infty} \frac{[\phi(T,m)]^N}{N!} \int_{v_o N}^{\infty} dV \exp(-sV)(V\ -\ v_o N)^N$$

$$= \sum_{N=0}^{\infty} \frac{[\phi(T,m)]^N}{N!}\ \cdot\ \frac{\exp(-v_o s N)\ N!}{s^{N+1}}$$

$$= [\ s\ -\ \exp(-v_o s)\phi(T,m)]^{-1}$$

$$\hat{Z}(s,T) \equiv \int_0^\infty dV \exp(-sV)\ Z(V,T)$$

Farthest-Right Singularity of the Laplase Transform:

$$Z(V,T)_{V\to\infty} \simeq \exp\left[\frac{p(T)\ V}{T}\right] \quad \to \quad s^*(T) = \frac{p(T)}{T}$$

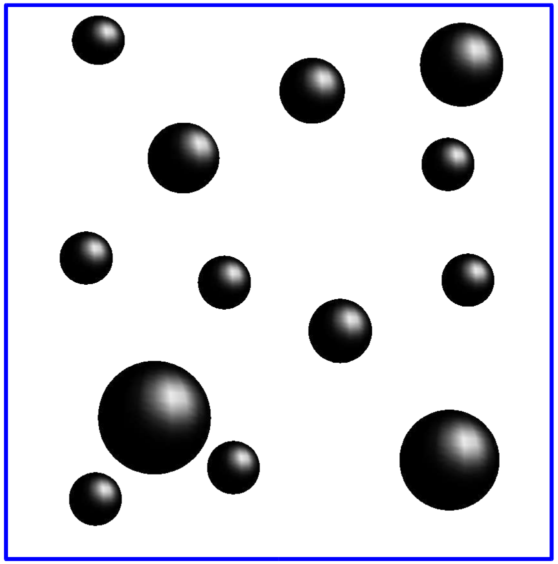
$$\hat{Z}(s,T) = [\, s - \exp(-v_o s)\phi(T,m)]^{-1}$$

Pole Singularity:

$$s^*(T) = \exp\left[-v_o s^*(T)\right]\ \phi(T,m)$$

Multi-Component VdW Gas

$m_1, v_1; \ldots; m_n, v_n$



V,T

$$Z(V,T) = \sum_{N_1=0}^{\infty} \ldots \sum_{N_n=0}^{\infty}$$

$$\times \frac{\left[(V - v_1 N_1 - \ldots - v_n N_n)\,\phi(T, m_1)\right]^{N_1}}{N_1!} \ldots \times$$

$$\ldots \times \frac{\left[(V - v_1 N_1 - \ldots - v_n N_n)\,\phi(T, m_n)\right]^{N_n}}{N_n!}$$

$$\times\ \theta\left(V - v_1 N_1 - \ldots - v_n N_n\right)$$

$$\sum_{j=1}^{n\to\infty} \ldots \to \int_0^{\infty} dm dv \ldots \rho(m,v)$$

Laplace Transform:

$$\hat{Z}(s,T) \equiv \int_0^\infty dV \exp(-sV)\, Z(V,T)$$

$$= [\, s \;-\; f(T,s)]^{-1}$$

$$f(T,s) \;=\; \int_0^\infty dmdv\; \rho(m,v)\; \exp(-vs)\; \phi(T,m)$$

Pressure: $$p(T) \;=\; T\, s^*(T)$$

Farthest-Right Singularity:

$$s^*(T) \;=\; max\{s_H(T), s_Q(T)\}$$

Pole Singularity:

$$s_H(T) \;=\; f\,(T, s_H(T))$$

Mass-Volume Spectrum of Quark-Gluon Bags

$$\rho(m,v) \simeq C\, v^{\gamma}(m-Bv)^{\delta} \times \exp\left[\frac{4}{3}\,\sigma_Q^{1/4}\, v^{1/4}\,(m-Bv)^{3/4}\right]$$

$$\sigma_Q = \frac{\pi^2}{30}\left(d_g + \frac{7}{8}d_{q\bar{q}}\right) = \frac{\pi^2}{30}\left(2\cdot 8 + \frac{7}{8}\cdot 2\cdot 2\cdot 3\cdot 3\right) = \frac{\pi^2}{30}\,\frac{95}{2}$$

$$f(T,s) \equiv f_H(T,s) + f_Q(T,s) = f_H(T,s)$$

$$+ \int_{V_o}^{\infty} dv \int_{M_o+Bv}^{\infty} dm\ \rho(m,v)\exp(-sv)\phi(T,m)$$

$$f_Q(T,s) \simeq C\, T^{4+4\delta} \left(\sigma_Q T^4 + B\right)^{3/2}$$

$$\times \int_{V_o}^{\infty} dv\, v^{2+\gamma+\delta} \exp\left[-\, v\, \left(s - s_Q(T)\right)\right]$$

$$s_Q(T) = \frac{\sigma_Q}{3} T^3 - \frac{B}{T}$$

$$f(T, s_H) = s_H \leftarrow\ ?\ \ s^*\ \ ?\ \rightarrow s_Q$$

The Farthest-Right Singularity?

$$f(T, s_H) = s_H, \quad \text{but } f(T,s) > 0, \text{ so that}$$

$s_H(T) > 0$ at all T.

$s_Q(T) < 0$ at small T, i.e. at $T < T_o \equiv \left(\frac{3B}{\sigma}\right)^{1/4}$.

Therefore, $s_H > s_Q$ at small T, and:

Hadron Gas

$$p(T) = T \cdot s_H \,, \qquad \varepsilon(T) = T^2 \cdot \frac{ds_H}{dT}$$

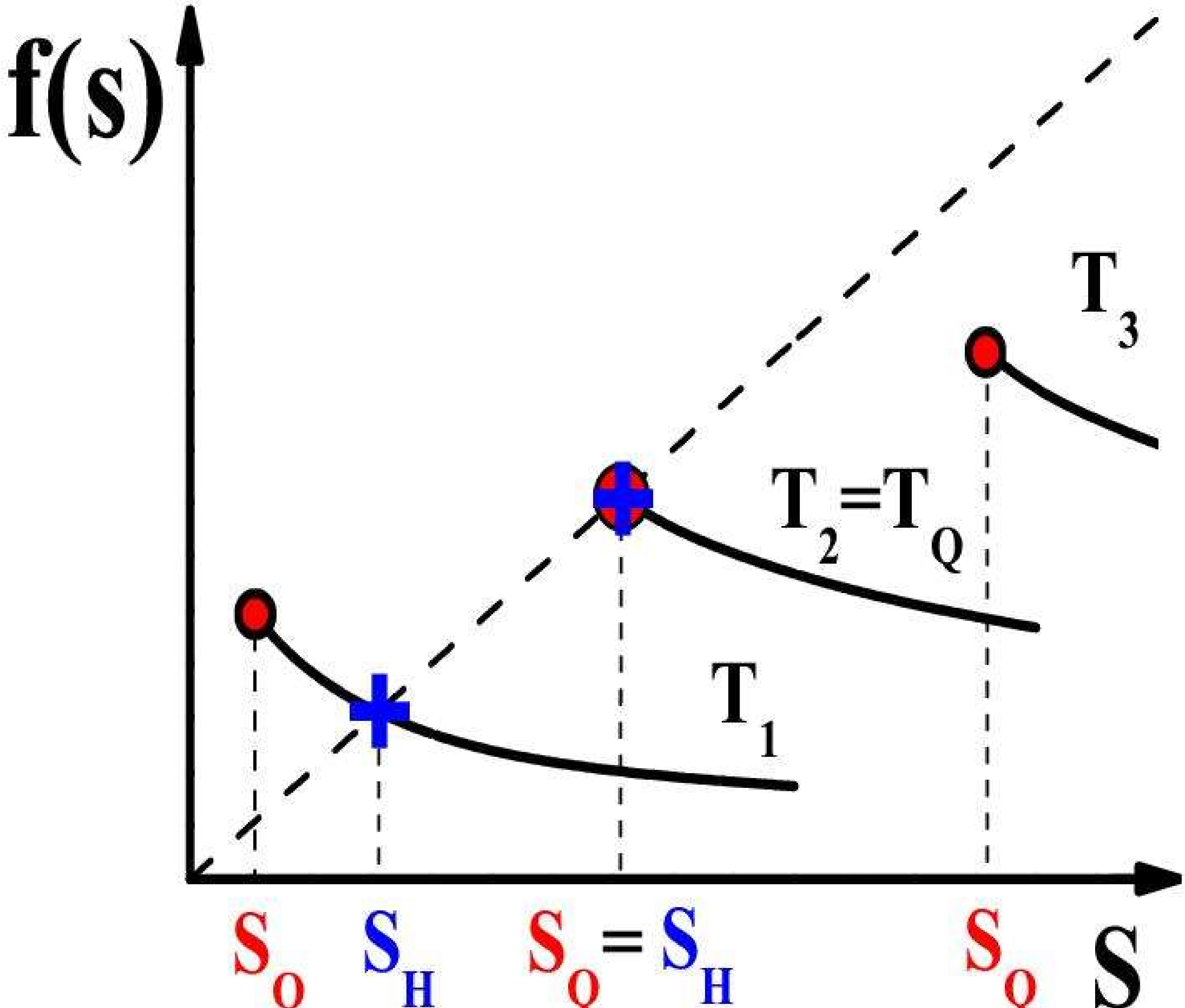
If $s_Q > s_H$ at $T > T_C$, then:

Quark-Gluon Plasma

$$p(T) = T \cdot s_Q = \frac{\sigma_Q}{3} T^4 - B,$$

$$\varepsilon(T) = T^2 \cdot \frac{ds_Q}{dT} = \sigma_Q T^4 + B$$

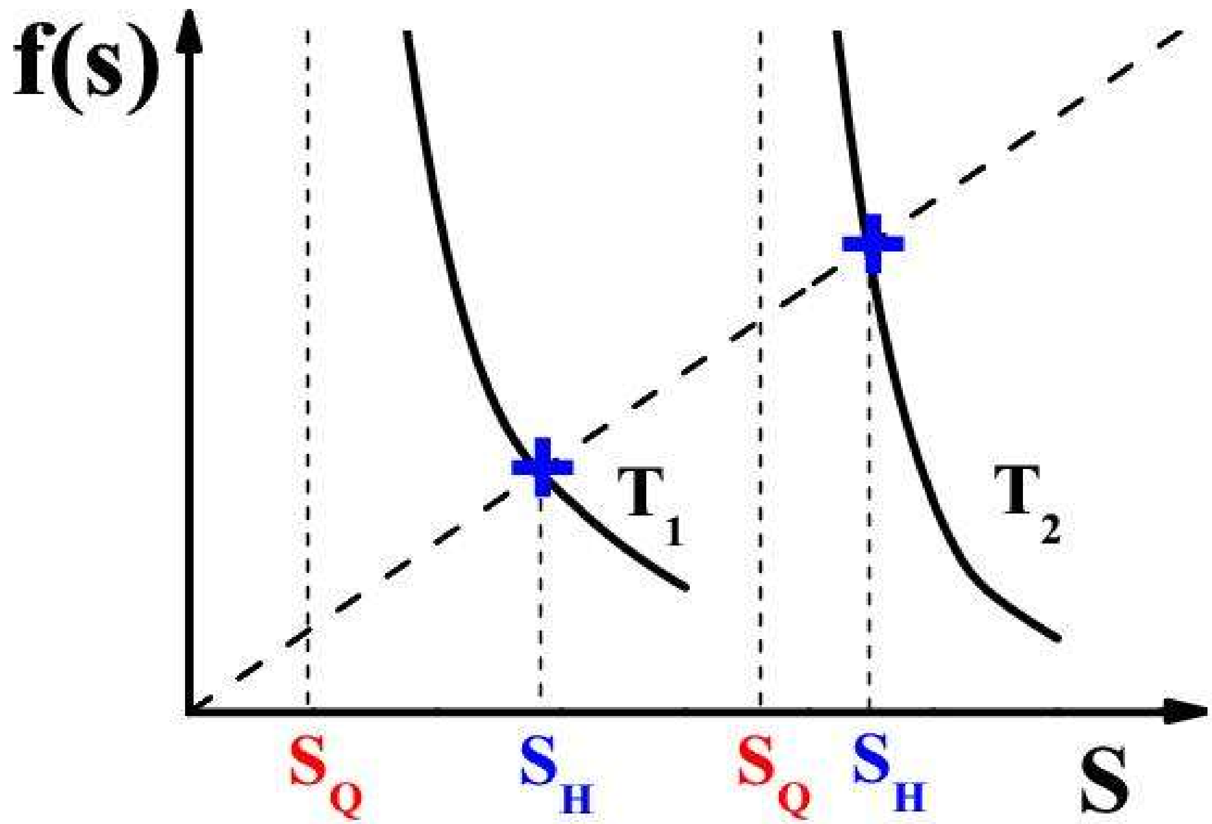
$$\gamma < -\frac{5}{4}, \qquad \delta < -\frac{7}{4}$$



To have $s^* = s_Q$ at high T one needs

$$\underline{\gamma + \delta < -3},$$

otherwise $f(T, s_Q) = \infty$, and $s_H > s_Q$ for all T:

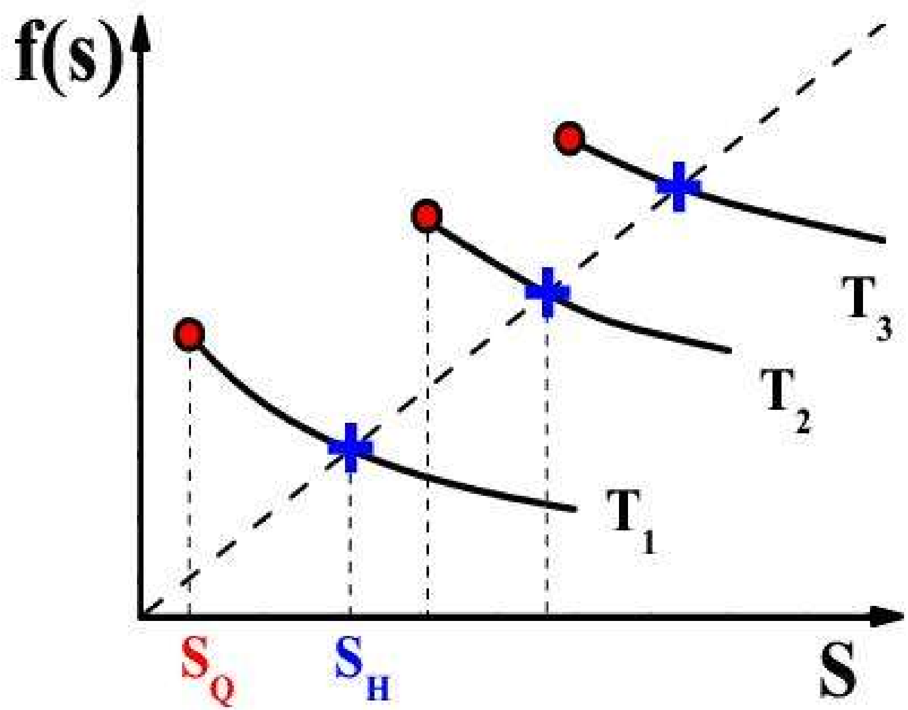


$$f_Q(T, s_Q)_{T\to\infty} \propto T^{10+4\delta} < s_Q(T)_{T\to\infty} \propto T^3$$

Therefore, $10 + 4\delta < 3$ or

$$\underline{\delta < -\frac{7}{4}},$$

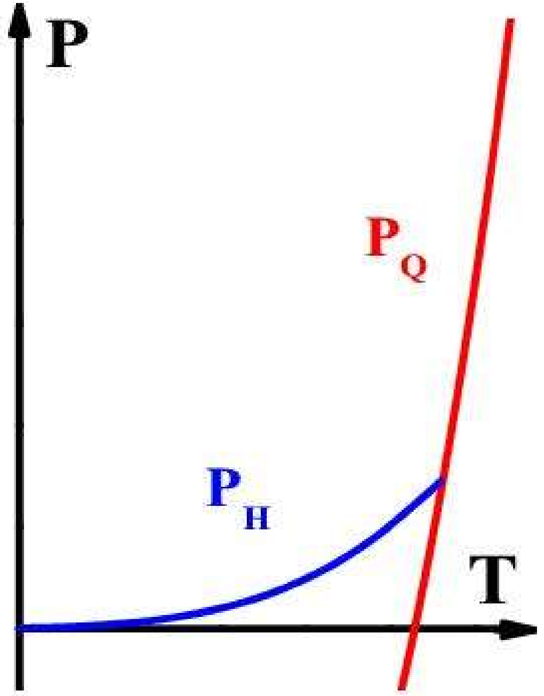
otherwise $f(T, s_Q) > s_Q$, and $s_H > s_Q$ for all T:

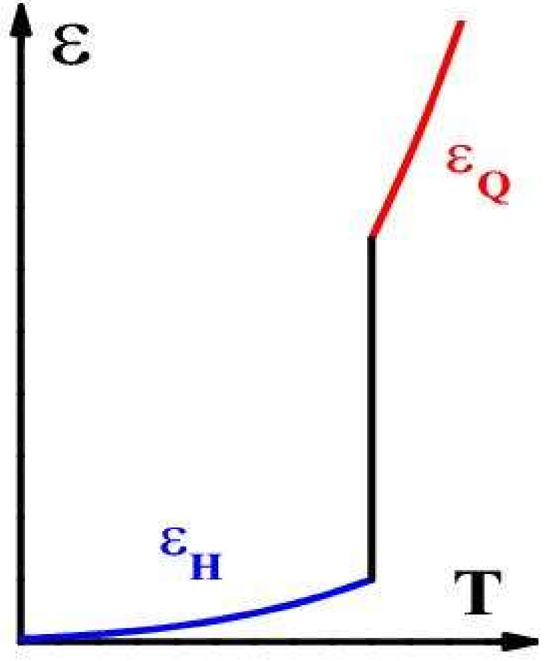


1^{st} Order PT

$$s_H(T_C) = s_Q(T_C)$$

$$s_H'(T_C) < s_Q'(T_C)$$



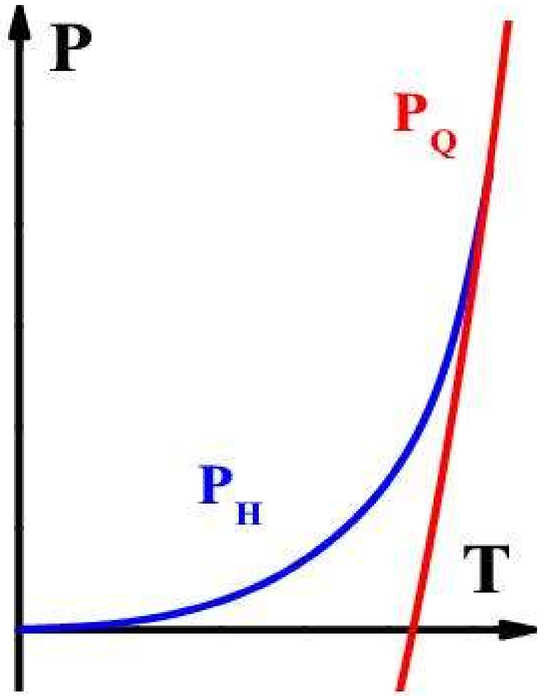


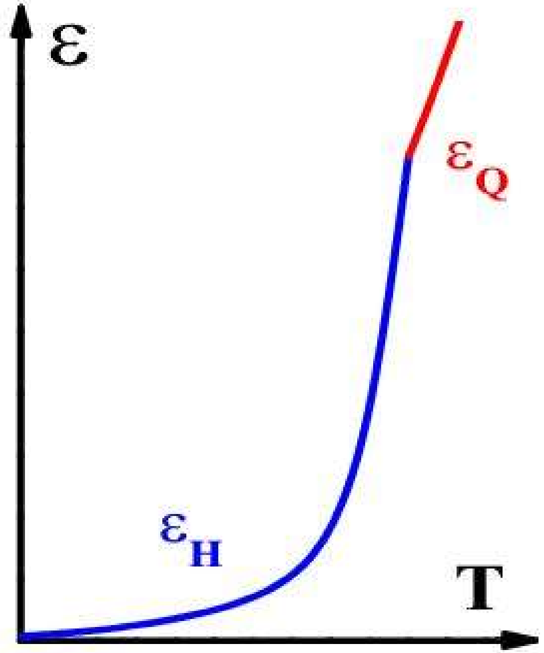
2^{nd} Order PT

$$s_H(T_C) = s_Q(T_C)$$

$$s_H'(T_C) = s_Q'(T_C)$$

$$s_H''(T_C) > s_Q''(T_C)$$





Formulas ...

$$s_H = f_H + u \int_{V_o}^{\infty} dv v^{-\alpha} \exp\left[-v\left(s_H - s_Q\right)\right]$$

$$\alpha = -(\gamma + \delta + 2) > 1\,,$$

$$u(T) = C\, T^{4+4\delta} \left(\sigma_Q T^4 + B\right)^{3/2}$$

$$s'_H = f'_H + u' \int_{V_o}^{\infty} dv v^{-\alpha} \exp\left[-v\left(s_H - s_Q\right)\right]$$

$$+ u \int_{V_o}^{\infty} dv v^{-\alpha+1} \exp\left[-v\left(s_H - s_Q\right)\right] \left(s'_Q - s'_H\right)$$

$$s'_H = \frac{G + F \cdot s'_Q}{1 + F}\,, \qquad s_H(T_C) = s_Q(T_C)$$

$$G \equiv f'_H + u' \int_{V_o}^{\infty} dv v^{-\alpha} \exp\left[-v\left(s_H - s_Q\right)\right]$$

$$F \equiv u \int_{V_o}^{\infty} dv v^{-\alpha+1} \exp\left[-v\left(s_H - s_Q\right)\right]$$

$$\int_{V_o}^{\infty} dv v^{-\alpha+1} \exp\left[-v\left(s_H - s_Q\right)\right]$$
$$= \left(s_H - s_Q\right)^{-2+\alpha} \Gamma\left[2-\alpha, (s_H - s_Q)V_o\right]$$

$$\propto \left(s_H - s_Q\right)^{-2+\alpha} , \quad \underline{\alpha < 2}$$

$$\propto - \ln\left(s_H - s_Q\right) , \quad \underline{\alpha = 2}$$

$$s'_Q - s'_H \propto \left(s_H - s_Q\right)^{2-\alpha} , \qquad \underline{\alpha < 2}$$
$$s'_Q - s'_H \propto - \ln^{-1}\left(s_H - s_Q\right) , \qquad \underline{\alpha = 2}$$

$\underline{\alpha \leq 1}$: No PTs

$\underline{\alpha > 2}$: 1st Order PT

$\underline{1 < \alpha \leq 2}$: 2nd and Higher Order PTs

$\underline{T < T_Q :}$

$$W(v) = C\, v^{-\alpha+1} \exp\left[-v\left(s_H - s_Q\right)\right] ,$$

$$C = \int_{V_o}^{\infty} dv v^{-\alpha+1} \exp\left[-v\left(s_H - s_Q\right)\right] ,$$

$$\overline{v} = \int_{V_o}^{\infty} dv\, v\, W(v)$$

$\underline{T \to T_Q :}$

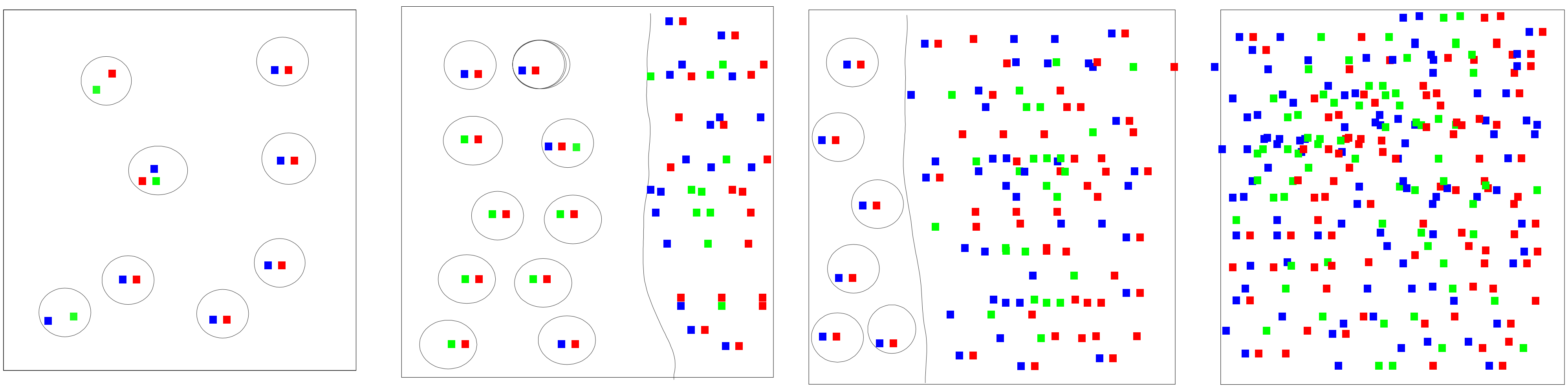
1^{st} Order PT

$$\overline{v} = const , \qquad \alpha > 2$$

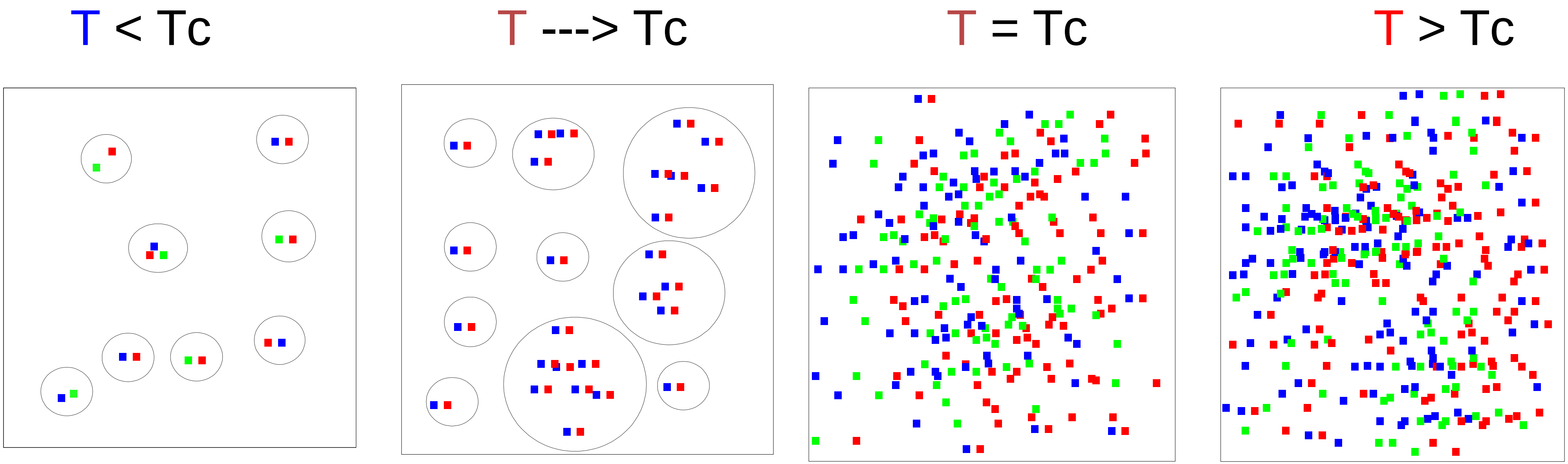
2^{nd} and Higher Order PTs

$$\overline{v} \to \infty , \qquad 1 < \alpha \leq 2$$

T < Tc   T = Tc   T=Tc   T > Tc



1^{st} Order Phase Transition

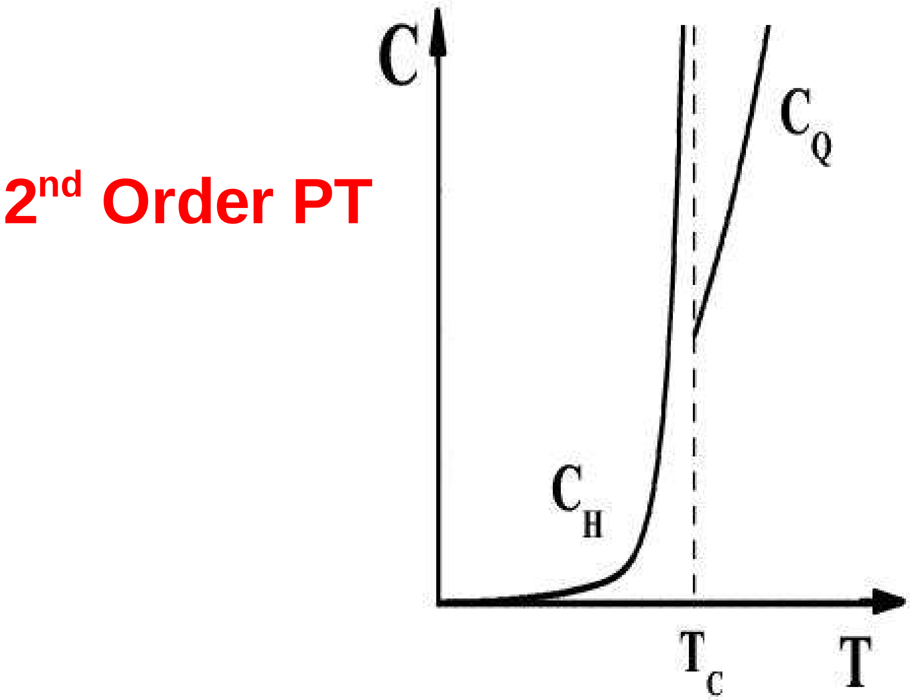


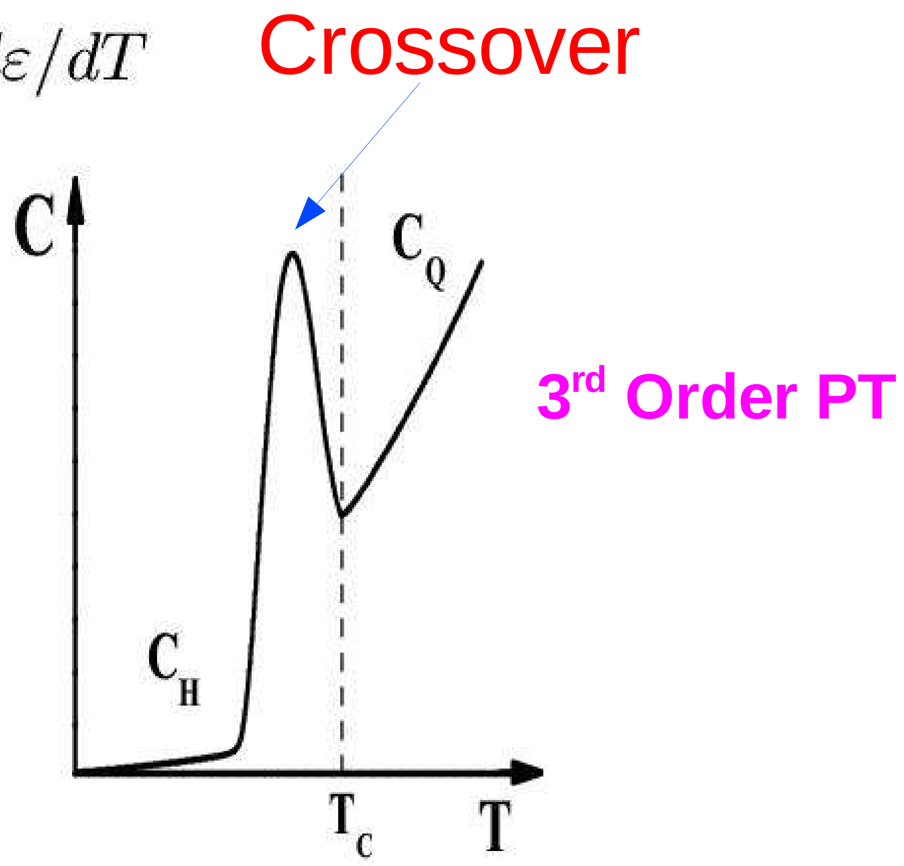
2nd Order and Higher Order Phase Transitions

For $\underline{3/2 < \alpha \leq 2}$ there is the 2^{nd} order PT

For $\underline{4/3 \leq \alpha < 3/2}$ there is the 3^{rd} order PT

The specific heat $C \equiv d\varepsilon/dT$





For $(n+1)/n \leq \alpha < n/(n-1)$ $(n = 4, 5, ...)$

there is the n^{th} order PT

Partition Function

$$Z(V,T,\mu_B) = \sum_{b=-\infty}^{\infty} \exp\left(\frac{b\,\mu_B}{T}\right) Z(V,T,B)$$

Laplace Transform

$$\hat{Z}(s,T,\mu_B) = \frac{1}{s - f(s,T,\mu_B)},$$

$$f(T,\mu_B,s) = f_H(T,\mu_B,s) + \int_{V_o}^{\infty} dv \int_{M_o+Bv}^{\infty} dm$$
$$\times\ \rho(m,v;\mu_B/T)\ \exp(-sv)\phi(T,m)\ ,$$

Hadrons:

$$f_H(T,\mu_B,s) = \sum_{j=1}^{n} g_j \exp\left(\frac{b_j\mu_B}{T} - v_j s\right) \phi(T,m_j)$$

QG Bags:

$$\rho(m,v;\mu_B/T) \equiv \sum_{b=-\infty}^{\infty} \exp\left(\frac{b\mu_B}{T}\right) \rho(m,v;b)$$

Farthest-Right Singularity

$$p(T,\mu_B) \;=\; T\, s^*(T,\mu_B) \;=\; T\cdot max\{s_H,\; s_Q\}$$

Pole Singularity

$$s_H(T,\mu_B) \;=\; f(s_H,T,\mu_B)$$

QGP Singularity

$$\begin{aligned} s_Q(T,\mu_B) \;&=\; \frac{\pi^2}{90}T^3 \cdot \frac{95}{2} \\ &+\; \frac{1}{9}T^3\left[\left(\frac{\mu_B}{T}\right)^2 + \frac{1}{162\pi^2}\left(\frac{\mu_B}{T}\right)^4\right] - \frac{B}{T} \\ &\equiv\; \frac{1}{3}\,\overline{\sigma}_Q(\mu_B)\,T^3 - \frac{B}{T} \end{aligned}$$

Energy Density and Baryonic Number Density

$$\varepsilon(T,\mu_B) \;=\; T^2\,\frac{\partial s^*(T,\mu_B)}{\partial T} + T\mu_B\,\frac{\partial s^*(T,\mu_B)}{\partial \mu_B}$$

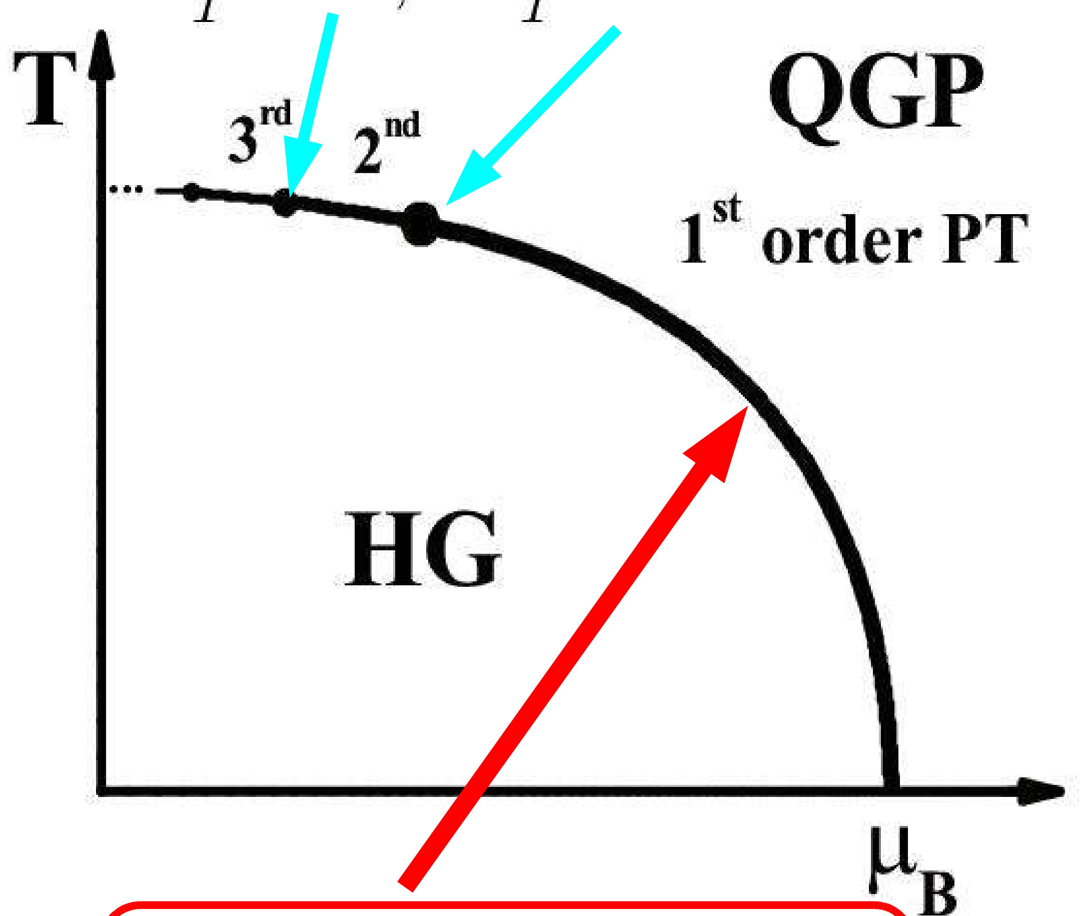
$$n_B(T,\mu_B) \;=\; T\,\frac{\partial s^*(T,\mu_B)}{\partial \mu_B}$$

$$\alpha > 2\,, \quad \tfrac{3}{2} \leq \alpha \leq 2\,, \quad 1 < \alpha \leq \tfrac{3}{2}$$

1st Order PT 2nd Order PT 3rd and Higher Order PTs

$$\alpha = \alpha_0 + \alpha_1 \frac{\mu_B}{T}\,, \quad \alpha_0 = 1 + \epsilon\,, \quad \alpha_1 \approx 0.5$$

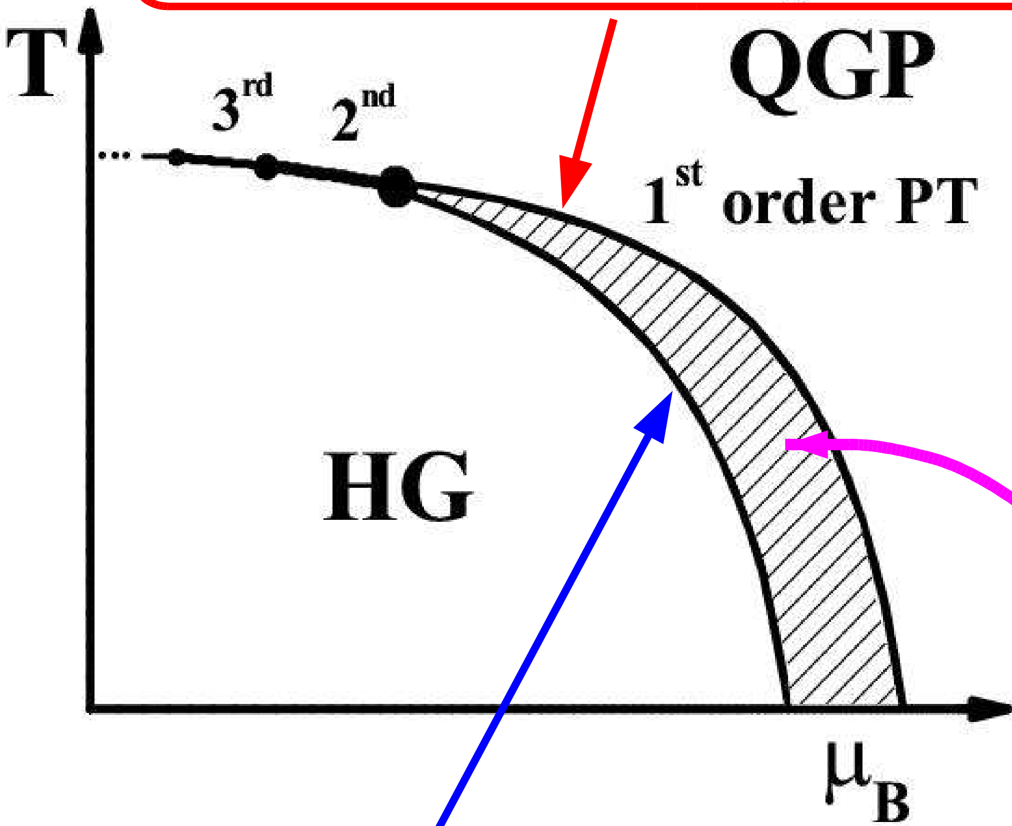
$$\frac{\mu_B}{T} \approx 1\,, \quad \frac{\mu_B}{T} \approx 2$$



$$s_H(T, \mu_B) = s_Q(T, \mu_B)$$

Strangeness

$$s_Q(T,\mu_B,\mu_S^Q) = s_H(T,\mu_B,\mu_S^Q)$$
$$\mu_S^Q = \mu_B/3$$



Strangeness Separation in the Mixed Phase

$$s_H(T,\mu_B,\mu_S^H) = s_Q(T,\mu_B,\mu_S^H)$$

$$s_H(T,\mu_B,\mu_S) = s_Q(T,\mu_B,\mu_S)$$
$$n_S^{mix} \equiv \delta \cdot n_S^Q + (1-\delta)\cdot n_S^H$$

Carsten Greiner et al. (1987)

SUMMARY

1st Order PT, 2nd Order PT, 3rd Order PT, …

Critical Line of the 2nd Order PT Instead of Critical Point

3rd Order and Higher Order PTs Instead of Crossover

1st Order PT Line is Transformed into the 'Strip'

Observables: Event-by-Event Fluctuations in A+A