BOUND STATES AND SUPERCONDUCTIVITY IN DENSE MATTER (II)
BCS/BEC CROSSOVER IN QUARK MATTER

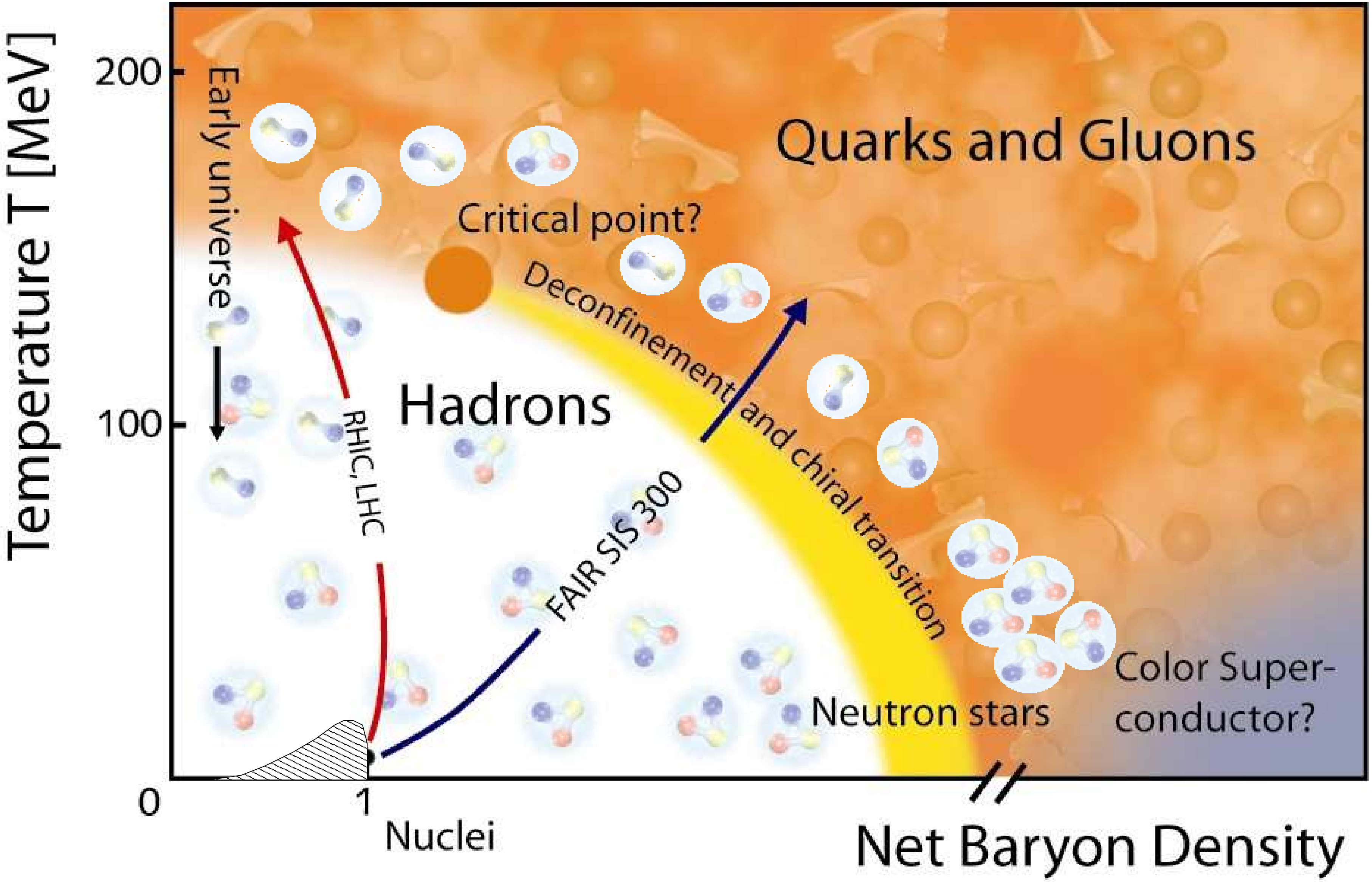
David Blaschke

Institut für Physik, Universität Rostock, Germany and BLTP @ JINR Dubna, Russia

- Path Integral Formulation
  - Nozières–Schmitt-Rink theory for relativistic fermion systems
  - Thermodynamic potential with correlations: spectral density vs. phase shift
  - Thouless criterion
  - Particle number conservation
- Thermodynamics of the BCS-to-BEC crossover
- Evolution of the soft mode dynamics
  - Ginzburg-Landau equation and (relativistic) Gross-Pitaevskii equation
  - Transport properties
- Summary / Outlook

Reference: H. Abuki, arxiv:hep-ph/0605081 (May 2006)

Hadronic Correlations in the Phasediagram of QCD



RELATIVISTIC NOZIERES–SCHMITT-RINK THEORY (I)

Lagrangian for $N_f = 2$, $N_c = 3$ quark matter, OGE inspired

$$\mathcal{L} = \bar{q}(i\partial\!\!\!/ + \mu\!\!\!/ - m)q - \frac{g^2}{2}\sum_{a=1}^{8} \bar{q}T^a\gamma_\mu q\, \bar{q}T^a\gamma^\mu q$$

Fierz transformation, select scalar diquark channel $(P_\eta)^{ij}_{ab} = iC\gamma_5\epsilon^{ij}\epsilon_{abc}$

$$\mathcal{L} = \bar{q}(i\partial\!\!\!/ + \mu\!\!\!/ - m)q + \frac{G}{4}\sum_{\eta=1}^{3}[iq^t P_\eta q][i\bar{q}\bar{P}_\eta \bar{q}^t] + \frac{G}{4}\sum_{\eta=1}^{3}[iq^t P_\eta\gamma_5 q][i\bar{q}\bar{P}_\eta\gamma_5\bar{q}^t]\cdots$$

Spinor doublet (bispinor) $Q = (q, \bar{q}^t)$ representation, $\bar{P}_\eta = \gamma_0(P_\eta)^\dagger\gamma_0$

$$\mathcal{L} = \frac{1}{2}\bar{Q}\begin{pmatrix} i\partial\!\!\!/ + \mu\!\!\!/ - m & 0 \\ 0 & i\partial\!\!\!/^t - \mu\!\!\!/^t + m \end{pmatrix} Q + \frac{G}{4}\bar{Q}\begin{pmatrix} 0 & 0 \\ P_\eta & 0 \end{pmatrix} Q\bar{Q}\begin{pmatrix} 0 & \bar{P}_\eta \\ 0 & 0 \end{pmatrix} Q$$

Partition function with $\mathcal{L}_E = -\mathcal{L}|_{t\to -i\tau}$

$$Z = \int \mathcal{D}Q \exp\big[-\int_0^\beta d\tau \int d\boldsymbol{x}\mathcal{L}_E(Q)\big]$$

Relativistic Nozieres–Schmitt-Rink theory (II)

Hubbard–Stratonovich transformation to fermion pair fields

$$\Delta_\eta(\tau,x) = \tfrac{G}{2}\langle \bar{Q} \begin{pmatrix} 0 & 0 \\ P_\eta & 0 \end{pmatrix} Q\rangle, \quad \Delta^*_\eta(\tau,x) = \tfrac{G}{2}\langle \bar{Q} \begin{pmatrix} 0 & \bar{P}_\eta \\ 0 & 0 \end{pmatrix} Q\rangle,$$

Integrating out the fermion fields

$$Z = \int \mathcal{D}\Delta\mathcal{D}\Delta^* \exp\left[-\int d\tau d\boldsymbol{x} \left(\frac{|\Delta(-i\tau,\boldsymbol{x})|^2}{G}\right)\right] \times \exp\left[\frac{1}{2}\log \mathrm{Det}_{(x,y)} \begin{pmatrix} S^{-1}_{Fx,y} & \bar{P}_\eta\Delta(x)\delta_{x,y} \\ P_\eta\Delta^*(x)\delta_{x,y} & \bar{S}^{-1}_{Fx,y} \end{pmatrix}\right]$$

Bare fermion propagator

$$S^{-1}_{Fx,y} = (i\partial\!\!\!/ + \mu\!\!\!/ - m)\delta_{x,y}, \quad \bar{S}^{-1}_{Fx,y} = \gamma_5 C S^{-1}_{Fy,x} C\gamma_5; \quad \delta_{x,y} = T\sum_n \int \frac{d\boldsymbol{p}}{(2\pi)^3} e^{-i\omega_n(\tau_x-\tau_y)+i\boldsymbol{p}\cdot(\boldsymbol{x}-\boldsymbol{y})}$$

Nambu-Gorkov propagator and self energy

$$\boldsymbol{S}^{-1}_{Fx,y} = \begin{pmatrix} S^{-1}_{Fx,y} & 0 \\ 0 & \bar{S}^{-1}_{Fx,y} \end{pmatrix}, \quad -\boldsymbol{\Sigma}_{x,y} = \begin{pmatrix} 0 & \bar{P}_\eta\Delta_\eta(x)\delta_{x,y} \\ P_\eta\Delta^*_\eta(x)\delta_{x,y} & 0 \end{pmatrix},$$

Factorize the fluctuation contribution to the partition function

$$Z(\mu,T) \equiv e^{-\beta\Omega(\mu,T)} = Z_0(\mu,T) Z_{\rm fluc}(\mu,T),$$

Relativistic Nozieres–Schmitt-Rink theory (III)

The thermodynamic potential for the free quark system is ($\epsilon_{p\pm} = \sqrt{m^2+p^2} \mp \mu$)

$$\frac{\Omega_0}{V} = -\frac{1}{\beta V}\ln Z_0 = -2N_f N_c T \sum_{\alpha=\pm}\int\frac{d\boldsymbol{p}}{(2\pi)^3}\ln\left(1+e^{-\epsilon_{p\alpha}/T}\right),$$

and the fluctuation contribution is determined from $\Omega_{\rm fluc} = -\ln Z_{\rm fluc}/\beta$ with

$$Z_{\rm fluc} = \int \mathcal{D}\Delta\mathcal{D}\Delta^* \exp\left[-\int d\tau d\boldsymbol{x}\left(\frac{|\Delta_\eta(-i\tau,\boldsymbol{x})|^2}{G}\right) + \frac{1}{2}\ln {\rm Det}_{(x,y)}\left(\delta_{x,y} - \sum_z \boldsymbol{S}_{Fx,z}\boldsymbol{\Sigma}_{z,y}\right)\right].$$

Expanding up to quadratic order in (Δ,Δ^*) (Gaussian approximation), we have

$$Z_{\rm fluc} \equiv e^{-\beta\Omega_{\rm fluc}} = \prod_{\eta,N,\boldsymbol{P}}\int d\Delta_\eta(i\Omega_N,\boldsymbol{P})d\Delta^*_\eta(i\Omega_N,\boldsymbol{P})\exp\left[-\frac{T}{V}\left(\frac{1}{G}-\chi_{\mu,T}(i\Omega_N,\boldsymbol{P})\right)|\Delta_\eta(i\Omega_N,\boldsymbol{P})|^2\right],$$

where Ω_N and $\boldsymbol{P}$ denote the *bosonic* Matsubara-frequency and momentum.
Integrating out the Gaussian fluctuation leads to (vacuum contr. $\chi_0 \equiv \chi_{\mu=0,T=0}$ subtracted)

$$\Omega_{\rm fluc} = d_B T\sum_{N:\text{even}}\int\frac{d\boldsymbol{P}}{(2\pi)^3}\log\left[\frac{1}{G}-\chi_{\mu,T}(i\Omega_N,\boldsymbol{P})\right] - d_B\int_{-\infty}^{\infty}d\Omega\int\frac{d\boldsymbol{P}}{(2\pi)^3}\log\left[\frac{1}{G}-\chi_0(i\Omega,\boldsymbol{P})\right],$$

Relativistic Nozieres–Schmitt-Rink theory (IV)

The *pair correlation function* $\chi_{\mu,T}(i\Omega_N, \boldsymbol{P})$ at one-loop level is defined by $\Longrightarrow$ Seminar

$$\begin{aligned}
\chi_{\mu,T}(i\Omega_N, \boldsymbol{P}) &= 2T\sum_n \int\frac{d\boldsymbol{q}}{(2\pi)^3}\mathrm{tr}[S_F(i\omega_n + i\Omega_N, \boldsymbol{q}+\boldsymbol{P})S_F(-i\omega_n, -\boldsymbol{q})] \\
&= -2\int\frac{d\boldsymbol{q}}{(2\pi)^3}(1+\frac{m^2+\boldsymbol{q}\cdot(\boldsymbol{q}+\boldsymbol{P})}{E_{\boldsymbol{q}}E_{\boldsymbol{q}+\boldsymbol{P}}})\frac{1-f_F(E_{\boldsymbol{q}+\boldsymbol{P}}-\mu)-f_F(E_{\boldsymbol{q}}-\mu)}{i\Omega_N+2\mu-E_{\boldsymbol{q}+\boldsymbol{P}}-E_{\boldsymbol{q}}} \\
&\quad -4\int\frac{d\boldsymbol{q}}{(2\pi)^3}(1-\frac{m^2+\boldsymbol{q}\cdot(\boldsymbol{q}+\boldsymbol{P})}{E_{\boldsymbol{q}}E_{\boldsymbol{q}+\boldsymbol{P}}})\frac{-f_F(E_{\boldsymbol{q}+\boldsymbol{P}}-\mu)+f_F(E_{\boldsymbol{q}}+\mu)}{i\Omega_N+2\mu-E_{\boldsymbol{q}+\boldsymbol{P}}+E_{\boldsymbol{q}}} \\
&\quad +2\int\frac{d\boldsymbol{q}}{(2\pi)^3}(1+\frac{m^2+\boldsymbol{q}\cdot(\boldsymbol{q}+\boldsymbol{P})}{E_{\boldsymbol{q}}E_{\boldsymbol{q}+\boldsymbol{P}}})\frac{1-f_F(E_{\boldsymbol{q}+\boldsymbol{P}}+\mu)-f_F(E_{\boldsymbol{q}}+\mu)}{i\Omega_N+2\mu+E_{\boldsymbol{q}+\boldsymbol{P}}+E_{\boldsymbol{q}}},
\end{aligned}$$

The (retarded) dynamic pair susceptibility $\Gamma(\omega, \boldsymbol{P})$ can be obtained by the analytic continuation of the pair correlation to the real ω-axis:

$$\Gamma^{-1}_{\mu,T}(\omega, \boldsymbol{P}) = \frac{1}{G} - \chi_{\mu,T}(\omega + i\delta, \boldsymbol{P}).$$

It corresponds to the quark pair propagator and its spectral density for $G = \mathcal{G}$

$$\rho^{\mathcal{G}}_{\mu,T}(\omega, \boldsymbol{P}) = \mathrm{Im}\,\Gamma^{\mathcal{G}}_{\mu,T}(\omega, \boldsymbol{P})\ ; \quad \Gamma^{\mathcal{G}}_{\mu,T}(\omega, \boldsymbol{P}) = \frac{1}{1/\mathcal{G} - \chi_{\mu,T}(i\Omega_N, \boldsymbol{P})} = -\int_{-\infty}^{\infty}\frac{d\omega}{\pi}\frac{\rho^{\mathcal{G}}_{\mu,T}(\omega, \boldsymbol{P})}{i\Omega_N - \omega}.$$

THERMODYNAMIC POTENTIAL IN TERMS OF SPECTRAL DENSITY

Differentiation and integration of Ω_{fluc} with respect to G gives

$$\Omega_{\text{fluc}} = d_B \int_0^G \frac{d\mathcal{G}}{\mathcal{G}^2} \int_{-\infty}^{\infty} \frac{d\omega}{\pi} \frac{d\boldsymbol{P}}{(2\pi)^3} T \sum_N \frac{\omega \rho^{\mathcal{G}}_{\mu,T}(\omega, \boldsymbol{P})}{\Omega_N^2 + \omega^2} - (T = \mu = 0 \text{ part})$$

Identity $\rho^{\mathcal{G}}_{-\mu,T}(\omega, \boldsymbol{P}) = -\rho^{\mathcal{G}}_{\mu,T}(-\omega, \boldsymbol{P})$ entails charge conjugation symmetry $\mu \leftrightarrow -\mu$.
Matsubara summation yields [$\tilde{f}_B(\omega) = f_B(\omega) + \theta(-\omega)$ with $f_B(\omega) = 1/(e^{\beta\omega} - 1)$]

$$\Omega_{\text{fluc}} = -d_B \int_0^G \frac{d\mathcal{G}}{\mathcal{G}^2} \int_{-\infty}^{\infty} \frac{d\omega}{\pi} \frac{d\boldsymbol{P}}{(2\pi)^3} \tilde{f}_B(\omega) \rho^{\mathcal{G}}_{\mu,T}(\omega, \boldsymbol{P}) - d_B \int_0^G \frac{d\mathcal{G}}{\mathcal{G}^2} \int_{-\infty}^{\infty} \frac{d\omega}{\pi} \frac{d\boldsymbol{P}}{(2\pi)^3} \frac{\epsilon(\omega)}{2} \delta\rho^{\mathcal{G}}_{\mu,T}(\omega, \boldsymbol{P})$$

In-medium spectral shift: $\delta\rho^{\mathcal{G}}_{\mu,T}(\omega, \boldsymbol{P}) = \rho^{\mathcal{G}}_{\mu,T}(\omega, \boldsymbol{P}) - \rho^{\mathcal{G}}_{0,0}(\omega, \boldsymbol{P})$. The result

$$\Omega_{\text{fluc}} = \Omega_{\text{NSR}} + \Omega_{\text{qfl}}$$

shows quantum fluctuation Ω_{qfl} besides the Nozieres–Schmitt-Rink contribution Ω_{NSR}.

Ω_{qfl} can be ignored for:

(i) high temperature $T > T_c$ and (ii) weak coupling $T_c/E_F \ll 1$

THERMODYNAMIC POTENTIAL IN TERMS OF PHASE SHIFT

Integrating the spectral density over the coupling constant leads to

$$\int_0^G \frac{d\mathcal{G}}{\mathcal{G}^2} \rho^{\mathcal{G}}_{\mu,T}(\omega, \boldsymbol{P}) = \frac{i}{2} \log \left(\frac{\frac{1}{G} - \chi_{\mu,T}(\omega + i\delta, \boldsymbol{P})}{\frac{1}{G} - \chi_{\mu,T}(\omega - i\delta, \boldsymbol{P})} \right) \equiv \delta_{\mu,T}(\omega, \boldsymbol{P}).$$

The in-medium phase shift $\delta_{\mu,T}(\omega, \boldsymbol{P})$ is the argument of the dynamic pair susceptibility

$$\frac{\frac{1}{G} - \chi_{\mu,T}(\omega \pm i\delta, \boldsymbol{P})}{|\frac{1}{G} - \chi_{\mu,T}(\omega, \boldsymbol{P})|} = e^{\mp i\delta_{\mu,T}(\omega,\boldsymbol{P})}.$$

The contributions to the thermodynamical potential can be expressed as

$$\Omega_{\rm NSR} = -d_B \int_{-\infty}^{\infty} \frac{d\omega}{\pi} \frac{d\boldsymbol{P}}{(2\pi)^3} \tilde{f}_B(\omega) \delta_{\mu,T}(\omega, \boldsymbol{P}),$$

$$\Omega_{\rm qfl} = -d_B \int_{-\infty}^{\infty} \frac{d\omega}{\pi} \frac{d\boldsymbol{P}}{(2\pi)^3} \frac{\epsilon(\omega)}{2} \left[\delta_{\mu,T}(\omega, \boldsymbol{P}) - \delta_{0,0}(\omega, \boldsymbol{P})\right],$$

where $\Omega_{\rm NSR}$ is exactly the Nozieres–Schmitt-Rink result in J. Low Temp. Phys. **59** (1985) 195

THOULESS CRITERION AND RENORMALIZATION OF COUPLING (I)

In the weak attractive coupling regime, the Thouless condition defines the critical temperature T_c by the divergence of the static and long wavelength limit of the dynamic pair susceptibility

$$\Gamma^{-1}_{\mu,T_c}(0,\mathbf{0}) = \frac{1}{G} - \chi_{\mu,T_c}(0,\mathbf{0}) = 0\ , \quad \chi_{\mu,T_c}(0,\mathbf{0}) = 2\int^{\Lambda}\frac{d\boldsymbol{q}}{(2\pi)^3}\frac{\tanh\frac{E_q-\mu}{2T_c}}{E_q-\mu} + (\mu\to-\mu).$$

Cutoff-dependence of $T_c = \Lambda f(m/\Lambda, \mu/\Lambda, G\Lambda^2)$ reduced by partial renormalization using low-energy scattering T-matrix (“1” $= (\boldsymbol{p}, a, i, h_1)$ labels quark momentum, color, flavor, spin)

$$T(12\to 34) = T(\boldsymbol{p},\boldsymbol{k})(\Gamma_{12}\Gamma_{34} - (3\leftrightarrow 4)), \quad \Gamma_{12} = \frac{\varepsilon_{ab}}{\sqrt{2}}\frac{\varepsilon_{ij}}{\sqrt{2}}\frac{\sigma^3_{h_1h_2}}{\sqrt{2}},$$

where $T(\boldsymbol{p},\boldsymbol{k})$ can be evaluated with the Lippman-Schwinger resummation

$$T(\boldsymbol{p},\boldsymbol{k}) = \frac{-G}{1-G\chi_{0,0}(2E_p+i\delta,\mathbf{0})} \equiv -\Gamma_{0,0}(2E_p,\mathbf{0}).$$

At sufficiently low energy, $2E_p = 2m + \frac{p^2}{m}$, the scattering amplitude $f(\boldsymbol{p},\boldsymbol{k}) = -\frac{m}{4\pi}T(\boldsymbol{p},\boldsymbol{k})$ is

$$f(\boldsymbol{p},\boldsymbol{k}) = \frac{e^{i\delta}\sin\delta}{p} = \frac{1}{p\cot\delta - ip} \sim \frac{1}{-\frac{1}{a_s}+\frac{1}{2}r_e p^2 - ip},$$

with the s-wave scattering length a_s and the effective range r_e to be extracted from $\Gamma_{0,0}(2E_p,\mathbf{0})$.

THOULESS CRITERION AND RENORMALIZATION OF COUPLING (II)

The effective range is UV finite, while the inverse scattering length is quadratically divergent

$$-\frac{m}{4\pi a_s} = \frac{1}{G} - 2\int\frac{d\boldsymbol{q}}{(2\pi)^3}\left(\frac{1}{E_q - m} + \frac{1}{E_q + m}\right) = \frac{1}{G} - \chi_{0,0}(2m, \mathbf{0}) \equiv -\frac{1}{G_R},$$

Thouless criterion with renormalization of coupling and pair correlation reads

$$-\frac{1}{G_R} - \chi^{\rm Ren}_{\mu,T_c}(0,\mathbf{0}) = 0, \quad \chi^{\rm Ren}_{\mu,T}(\omega,\boldsymbol{P}) \equiv \chi_{\mu,T}(\omega,\boldsymbol{P}) - \chi_{0,0}(2m,\mathbf{0}).$$

Bound state or resonance ? In vacuum ($\mu = T = 0$) from condition: $-1/G_R - \chi^{\rm Ren}_{0,0}(\omega,\mathbf{0}) = 0$. Since $\chi^{\rm Ren}_0(2m,0) = 0$ follows: if $-1/G_R > 0$ then a resonance pole is located at $|\omega| > 2m$; otherwise a *stable bound state* is located at $|\omega| < 2m$, because $\mathrm{Im}\,\chi(|\omega| < 2m,\mathbf{0}) = 0$.
Then the critical coupling $G_R = G_{c1}$ for the zero binding is given by the condition

$$-\frac{1}{G_{c1}} - \chi^{\rm Ren}_0(2m,\mathbf{0}) = -\frac{1}{G_{c1}} = 0,$$

so that $1/a_s = 0$, so-called unitary limit.
For the relativistic case, a second critical coupling exists at $G_R = G_{c2}$, when the bound state becomes massless: $-1/G_{c2} = \chi^{\rm Ren}_0(0,\mathbf{0}) < 0$.

PARTICLE NUMBER CONSERVATION (I)

In-medium spectral function and the in-medium phase shift from

$$\rho^{\mathrm{Ren}}_{\mu,T}(\omega,\boldsymbol{P}) = \mathrm{Im}\,\Gamma^{\mathrm{Ren}}_{\mu,T}(\omega,\boldsymbol{P}),\, \delta^{\mathrm{Ren}}_{\mu,T}(\omega,\boldsymbol{P}) = \mathrm{Arg}\left[\Gamma^{\mathrm{Ren}}_{\mu,T}(\omega,\boldsymbol{P})\right].$$

Total quark number density from the thermodynamic potential $n_{\mathrm{tot}}(\mu,T) = -\frac{\partial\Omega(\mu,T)}{\partial\mu}$

$$n_{\mathrm{tot}}(\mu,T) = 2N_cN_f\int\frac{d\boldsymbol{q}}{(2\pi)^3}[f_F(E_q-\mu)+f_F(E_q+\mu)] + d_B\int_{-\infty}^{\infty}\frac{d\omega}{\pi}\frac{d\boldsymbol{P}}{(2\pi)^3}\tilde{f}_B(\omega)\frac{\partial\delta^{\mathrm{Ren}}_{\mu,T}(\omega,\boldsymbol{P})}{\partial\mu},$$

The NSR contribution can be transformed to

$$n_{\mathrm{NSR}}(\mu,T) = d_B\int\frac{d\boldsymbol{P}}{(2\pi)^3}\left[2-\frac{\partial E^{\mu,T}_{B\boldsymbol{P}}}{\partial\mu}\right]\tilde{f}_B(E^{\mu,T}_{B\boldsymbol{P}}-2\mu) - d_B\int\frac{d\boldsymbol{P}}{(2\pi)^3}\left[2+\frac{\partial E^{\mu,T}_{\bar{B}\boldsymbol{P}}}{\partial\mu}\right]\tilde{f}_B(E^{\mu,T}_{\bar{B}\boldsymbol{P}}+2\mu)$$

and contains unstable pair correlations besides the bound (anti-)boson ones.
The critical chemical potential μ_c is defined by

$$N_cN_f\frac{k_F^3}{3\pi^2} \equiv n_{\mathrm{tot}}(\mu_c,T) = n_{\mathrm{MF}}(\mu_c,T) + n_{\mathrm{NSR}}(\mu_c,T),$$

Particle number conservation (II)

Estimate of critical parameters for Bose condensation from bound state dominance.

(1) Nonrelativistic BEC: boson mass ($2\mu_c$) much larger than kinetic energy

$$2\mu_c \gg N_{\rm tot}^{2/3}/4\mu_c \implies E_{B\boldsymbol{P}}^{\mu_c,T} \sim 2\mu_c + \frac{P^2}{4\mu_c}.$$

Boson pole dominates and one gets

$$n_{\rm tot} = N_c N_f \frac{k_F^3}{3\pi^2} = 2d_B \left(\frac{\mu_c T}{\pi}\right)^{3/2} \zeta(3/2) \implies T < T_{\rm BEC}^{\rm NR} = \frac{k_F^2}{\pi^{1/3}\mu_c} \left(\frac{N_f}{N_c - 1} \frac{1}{3\zeta(3/2)}\right)^{2/3}.$$

(2) Relativistic BEC: $2\mu_c \ll n_{\rm tot}^{1/3}$; boson and anti-boson with dispersion $\sqrt{4\mu_c^2 + P^2}$ contribute.

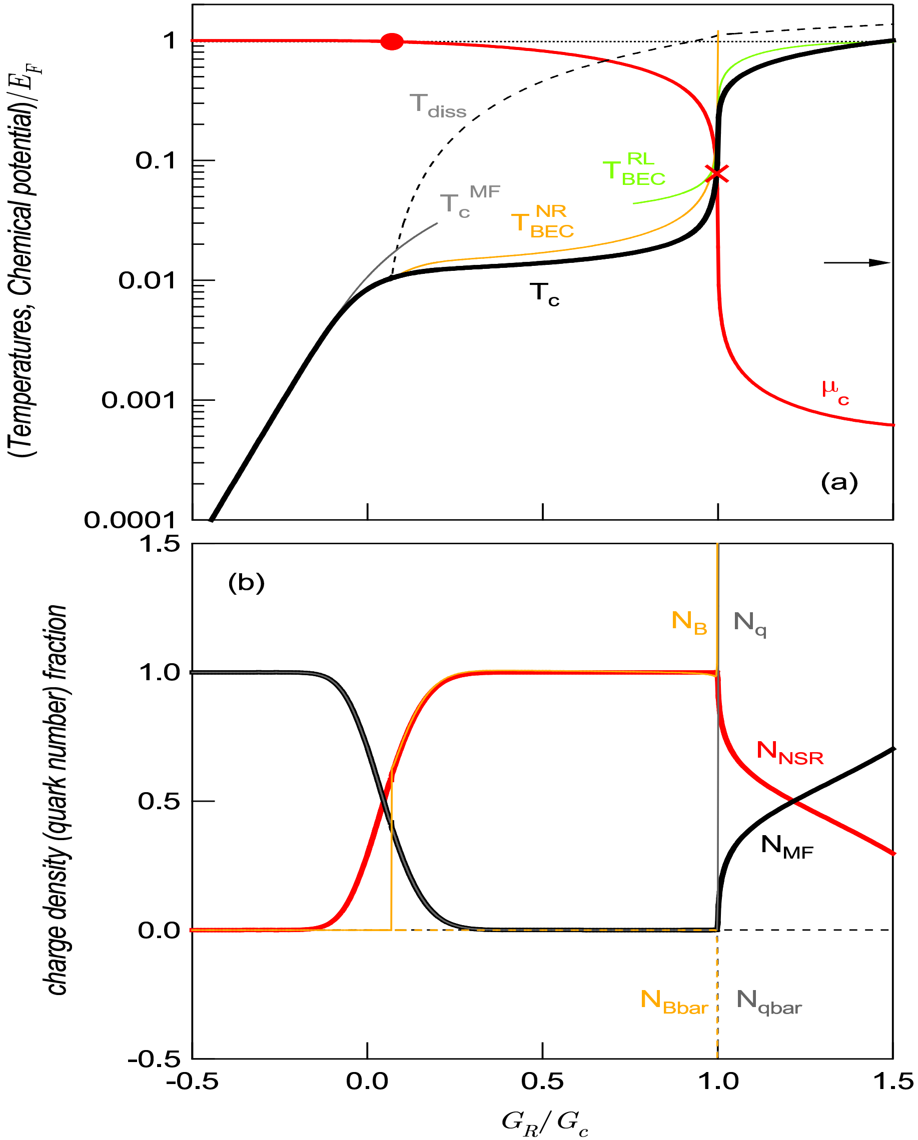
$$n_{\rm tot} = N_c N_f \frac{k_F^3}{3\pi^2} = 2d_B \frac{2\mu_c T^2}{3} \implies T_{\rm BEC}^{\rm RL} = \frac{k_F}{\pi}\sqrt{\frac{k_F}{\mu_c}\frac{N_f}{2(N_c-1)}}.$$

The critical temperature is of order $(n_{\rm tot}/2\mu_c)^{1/2}$, i.e. much larger than the boson mass $2\mu_c$

(3) Crossover regime: Thouless criterion and number conservation

$$0 = {\rm Re}\left[\Gamma_{\mu_c,T_c}^{\rm Ren}(0,\mathbf{0})^{-1}\right]; \quad N_c N_f \frac{k_F^3}{3\pi^2} = n_{\rm MF}(\mu_c, T_c) + \int_{-\infty}^{\infty} \frac{d\omega}{\pi} \frac{d\boldsymbol{P}}{(2\pi)^3} \tilde{f}_B(\omega) \frac{\partial \delta_{\mu_c,T_c}^{\rm Ren}(\omega,\boldsymbol{P})}{\partial \mu_c}.$$

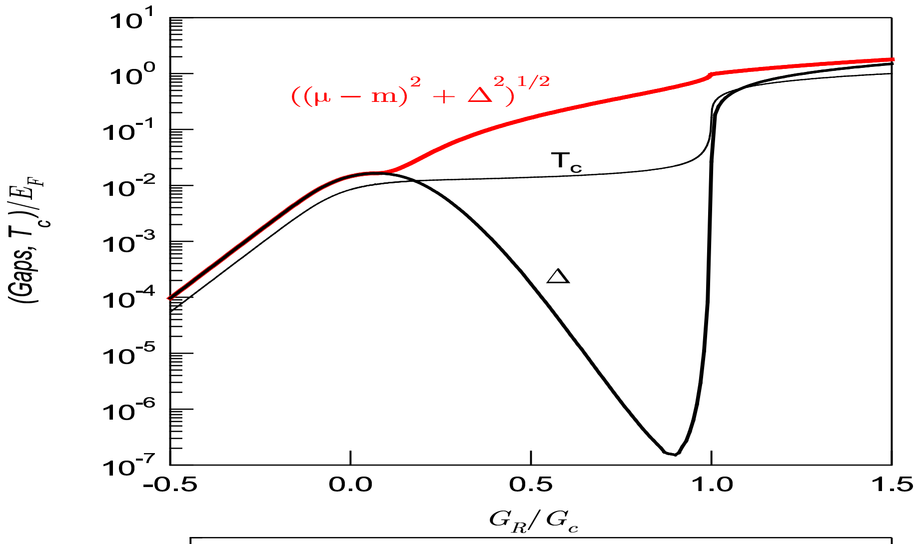
THERMODYNAMICS OF THE BCS-TO-BEC CROSSOVER



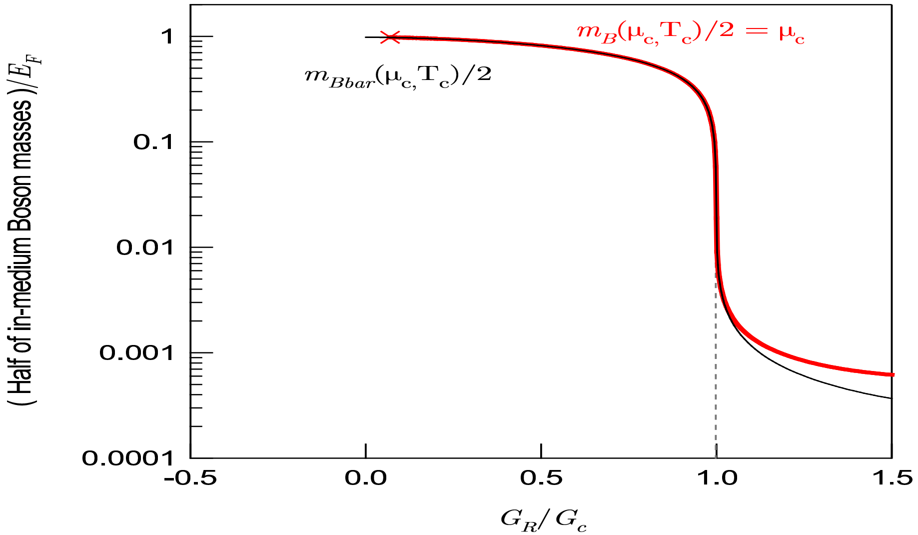
(a) The critical temperature T_c and the critical chemical potential μ_c as a function of attractive coupling G_R. The dissociation temperature of pre-formed pair in the strong coupling regime is also depicted by dashed line.

(b) The quark number fractions as a function of coupling G_R.

THERMODYNAMICS OF THE BCS-TO-BEC CROSSOVER (II)

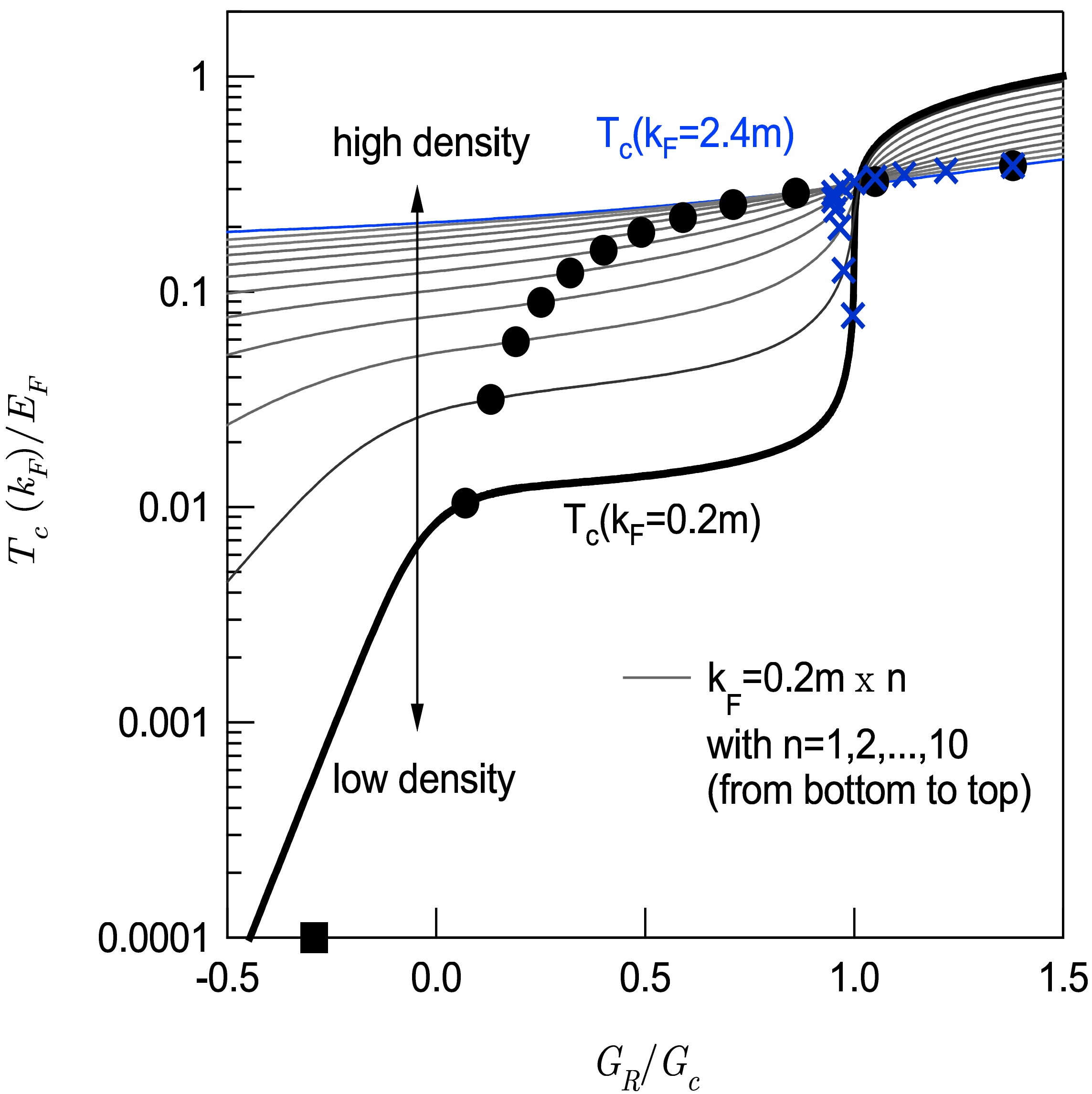


The zero temperature gap parameter Δ_0, and T_c as a function of G_R. The mean field gap parameter Δ_0 is calculated at $(\mu = \mu_c, T = 0)$. A measure of physical gap in the single quark excitation at $T = 0$, $\sqrt{\Delta_0^2 + \max.(m - \mu_c, 0)^2}$, is also shown by the bold red line.



The in-medium boson and antiboson masses $(M_B, M_{\bar{B}})$ along the critical line $(\mu_c(G_R), T_c(G_R))$. The cross indicates the point $G_R/G_c \cong 0.07$ where the bound boson forms, i.e., $M_B(\mu_c, T_c) = 2\mu_c = 2m$ holds at this point.

THERMODYNAMICS OF THE BCS-TO-BEC CROSSOVER



The k_F dependence of the critical temperature as a function of G_R/G_c. From bottom to top, the Fermi momentum increases as $k_F = 0.2m, 0.4m, \cdots, 2.4m$. The large point located on each critical line represents the BCS/BEC crossover point where $\mu_c = m$ holds, while the large cross corresponds to the BEC/RBEC boundary where the $T_c = \mu_c$. The large square put on the horizontal axis indicates the standard choice of the diquark attraction, i.e., $G/4 \sim \frac{3}{4}G_s$ with $G_s = 2.17/\Lambda^2$ set to reproduce the dynamical quark mass $M_q = 400$ MeV in vacuum.

Abuki, arxiv:hep-ph/0605081

Ginzburg-Landau and Gross-Pitaevskii equation

Time-dependent Ginzburg-Landau (TDGL) equation in the BCS regime:

$$\left[\frac{\operatorname{Im} d}{c}\partial_t + \frac{a_T}{c} - \frac{\nabla_{\boldsymbol{x}}^2}{4m} + \frac{b_0}{c^2}\sum_{\xi}|\Psi_\xi(t,\boldsymbol{x})|^2\right]\Psi_\eta(t,\boldsymbol{x}) = 0$$

Gross-Pitaevskii (GP) equation

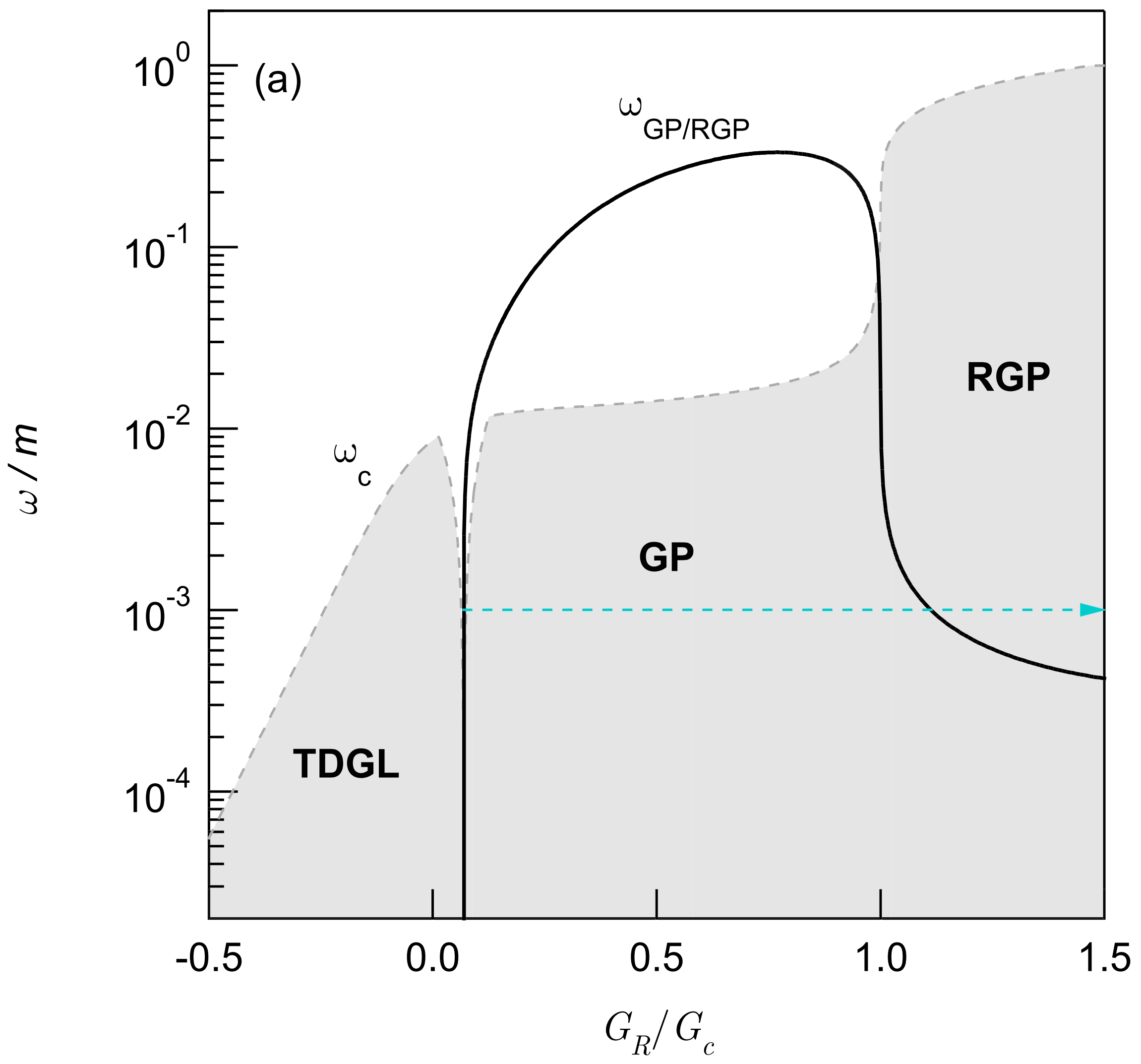
$$\left[-i\partial_t + \frac{a_T}{d} - \frac{c\nabla_{\boldsymbol{x}}^2}{4md} + \frac{b_0}{d^2}\sum_{\xi}|\Psi_\xi(t,\boldsymbol{x})|^2\right]\Psi_\eta(t,\boldsymbol{x}) = 0.$$

Relativistic Gross-Pitaevskii (RGP) equation (Klein-Gordon equation with Φ^4 interaction)

$$\left[\partial_t^2 - v_s^2\nabla_{\boldsymbol{x}}^2 + M^2 + \lambda\sum_{\xi}|\Psi_\xi(t,\boldsymbol{x})|^2\right]\Psi_\eta(t,\boldsymbol{x}) = 0.$$

Abuki, arxiv:hep-ph/0605081

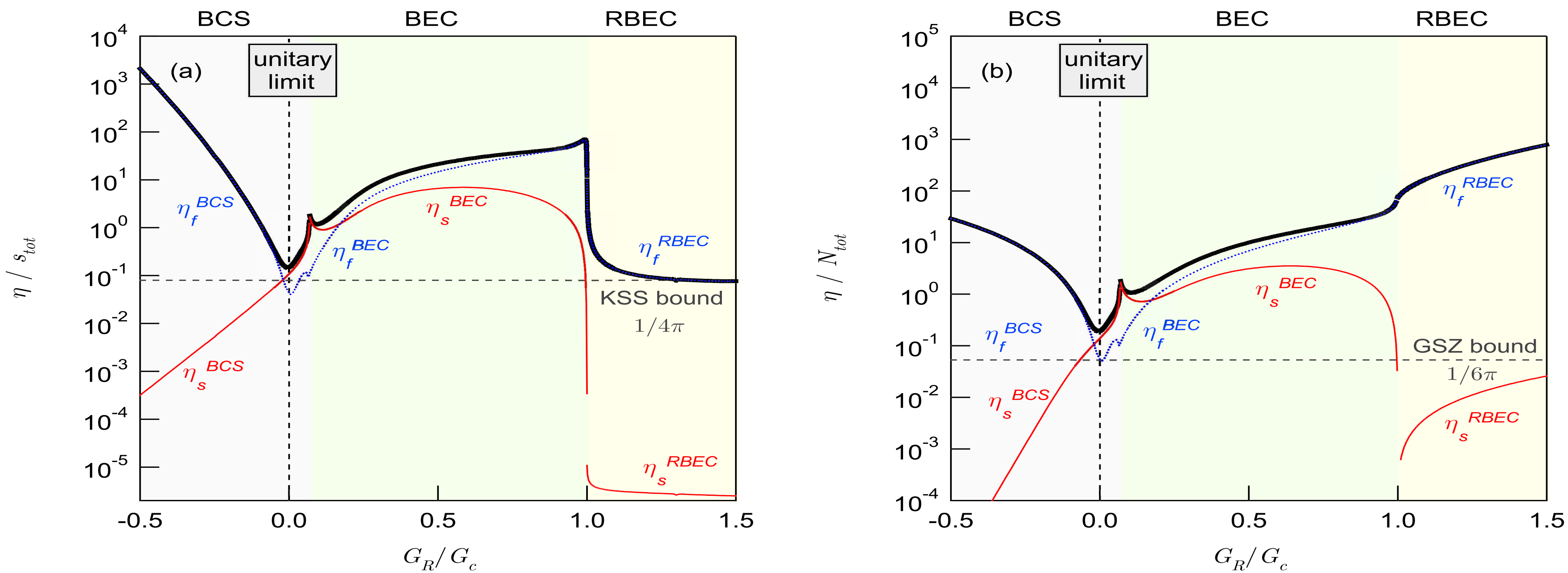
GINZBURG-LANDAU AND GROSS-PITAEVSKII EQUATION



The three physically different ω-regions where the fluctuation is described by TDGL, GP, and RGP equations. The low energy expansion of the pair susceptibility is allowed for the shaded area where $\omega < \omega_c \equiv \min.(T_c, |\mu_c - m|)$.

Abuki, arxiv:hep-ph/0605081

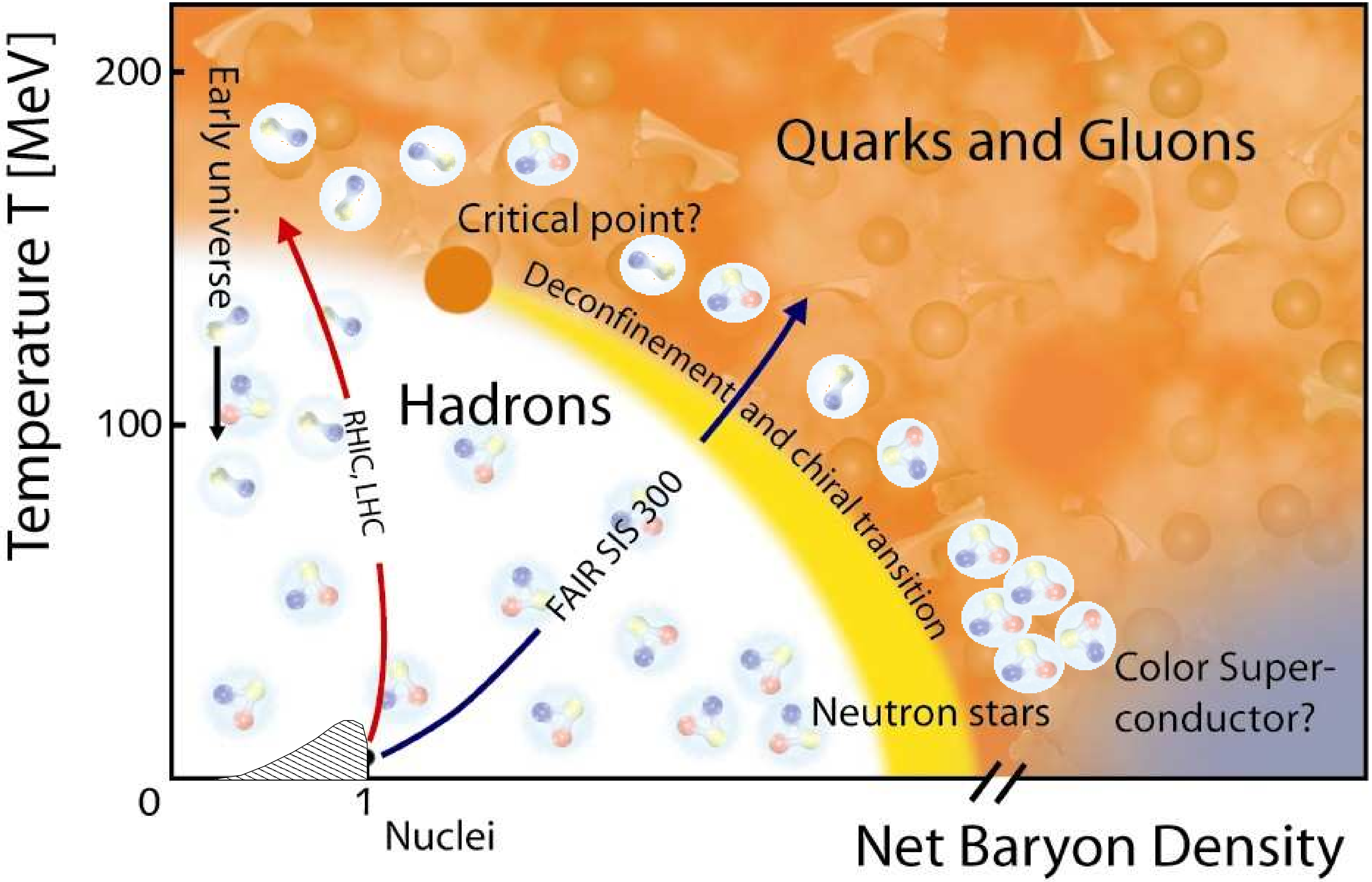
TRANSPORT PROPERTIES: PERFECT LIQUID!



(a) The shear viscosity to entropy ratios from the damping (or scattering) process of soft modes, $\eta_s^{\rm BCS}$, $\eta_s^{\rm BEC}$, and $\eta_s^{\rm RBEC}$ as a function of four-fermion coupling G_R (thin solid lines). The dashed lines indicated by $\eta_f^{\rm BCS}$ and $\eta_f^{\rm RBEC}$ correspond to the shear viscosities from fermion binary scattering in BCS and RBEC regimes. The “KSS bound”, the minimum bound $\left(\frac{1}{4\pi}\right)$ proposed by Kovtun, Son and Starinets

(b) The shear viscosity to baryon charge ratios as a function of G_R. The horizontal dashed line $\left(\frac{1}{6\pi}\right)$ indicated by “GSZ bound” is the minimum bound proposed by Gelman, Shuryak and Zahed

HADRONIC CORRELATIONS IN THE PHASEDIAGRAM OF QCD



SUMMARY

- Nozieres–Schmitt-Rink theory for relativistic fermion system
- Thermodynamics of the BEC/BCS crossover
- Pair fluctuation transport: Gross-Pitaevskii equation and shear viscosity
- Density dependence of T_c and role of quantum fluctuations

OUTLOOK

- Dynamic pair susceptibilities from a chiral quark model with mesonic channels and gluonic dof (e.g. Polyakov loop)
- Investigation of the transport properties in the phase diagram
- Step-like enhancement of threshold processes due to Mott effect
- Reaction kinetics for strong correlations @ SPS, RHIC and FAIR: observable effects?