BOUND STATES AND SUPERCONDUCTIVITY IN DENSE MATTER (I)

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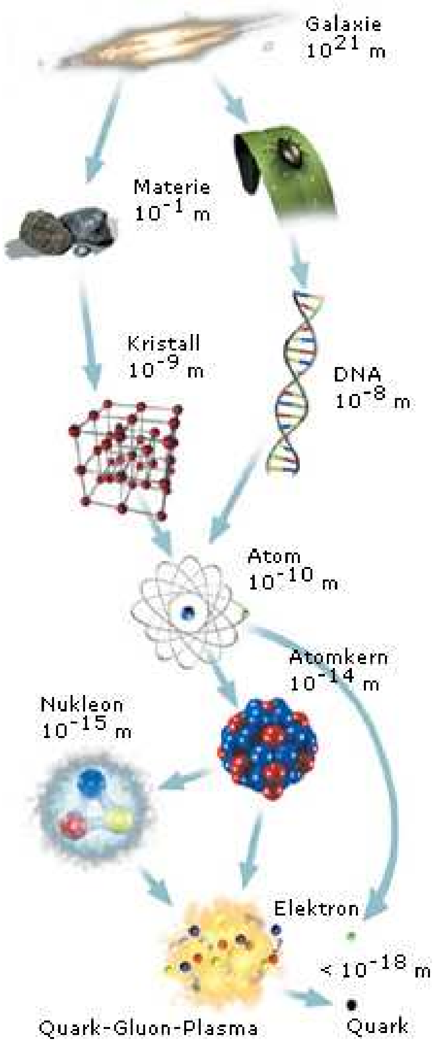
- Introduction: Many-particle Systems and Quantum Field Theory
- Partition function for QCD: Lattice Simulations vs. Resonance Gas
- Bound states and Mott effect, Color superconductivity
  - Heavy Quarkonia - Schrödinger Equation
  - Chiral quark model - Color superconductivity
  - Pions, Kaons, D-mesons - Chiral Quark Model
- Application 1: J/ψ suppression in Heavy-Ion Collisions
- Application 2: Quark Matter in Compact Stars
- Summary / Outlook to Lecture II: BCS/BEC crossover in Quark Matter

Many Particle Systems & Quantum Field Theory



Elements	Bound states	System
humans, animals	couples, groups, parties	society
molecules, crystals	(bio)polymers	animals, plants
atoms	molecules, clusters, crystals	solids, liquids, ...
ions, electrons	atoms	plasmas
nucleons, mesons	nuclei	nuclear matter
quarks, anti-quarks	nucleons, mesons	quark matter

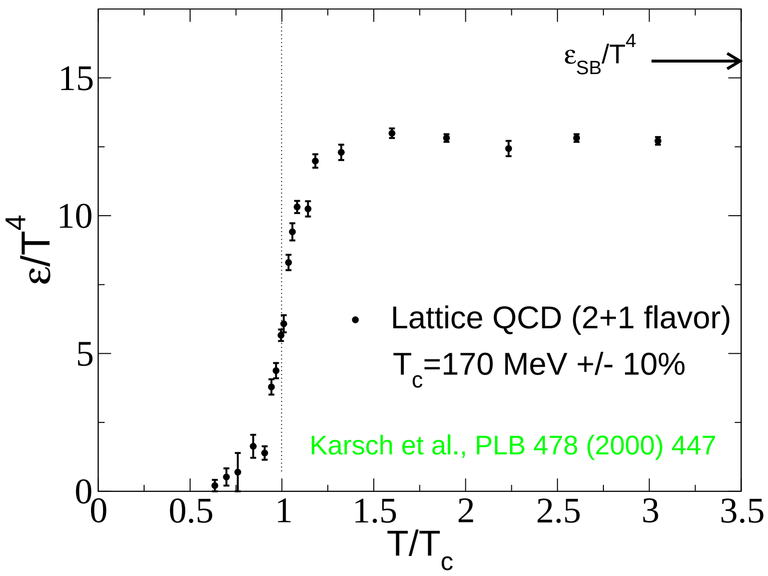
Partition function: $Z = \mathrm{Tr}\left\{ \mathrm{e}^{-\beta(H-\mu_i Q_i)} \right\}$

PARTITION FUNCTION FOR QUANTUM CHROMODYNAMICS (QCD)

- Partition function as a Path Integral (imaginary time $\tau = i\,t$, $0 \le \tau \le \beta = 1/T$)

$$Z[T,V,\mu] = \int \mathcal{D}\bar{\psi}\mathcal{D}\psi\mathcal{D}A \exp\left\{-\int_0^\beta d\tau \int_V d^3x\ \mathcal{L}_{QCD}(\psi,\bar{\psi},A)\right\}$$

- QCD Lagrangian, non-Abelian gluon field strength:
- Numerical evaluation: Lattice gauge theory simulations (Bielefeld group)

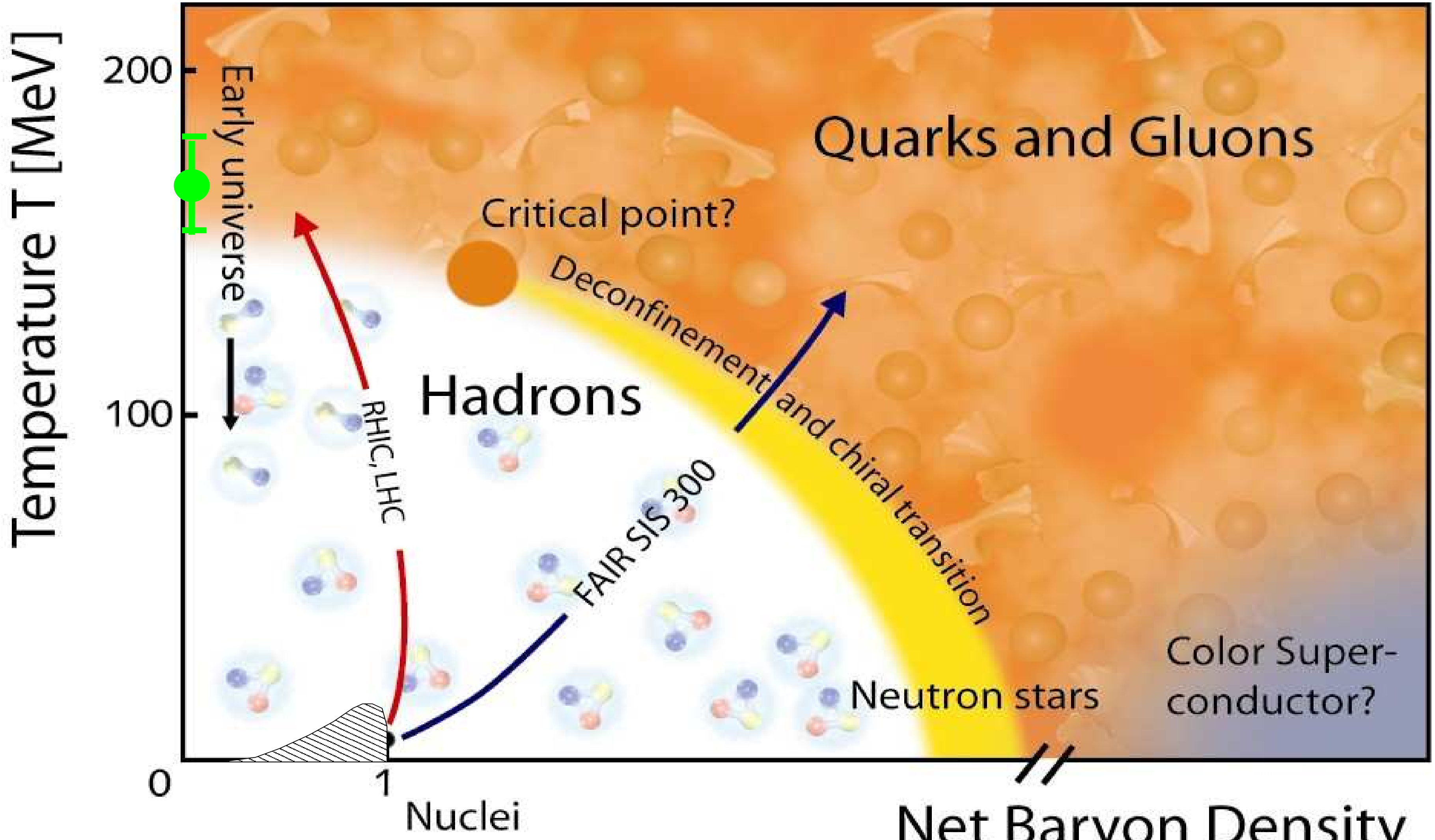


- Equation of state: $\varepsilon(T) = -\partial \ln Z[T,V,\mu]/\partial\beta$
- Phase transition at $T_c = 170$ MeV
- Problem: Interpretation ?

$\varepsilon/T^4 = \frac{\pi^2}{30}N_\pi \sim 1$ (ideal pion gas)

$\varepsilon/T^4 = \frac{\pi^2}{30}(N_G + \frac{7}{8}N_Q) \sim 15.6$ (quarks and gluons)

PHASEDIAGRAM OF QCD: LATTICE SIMULATIONS



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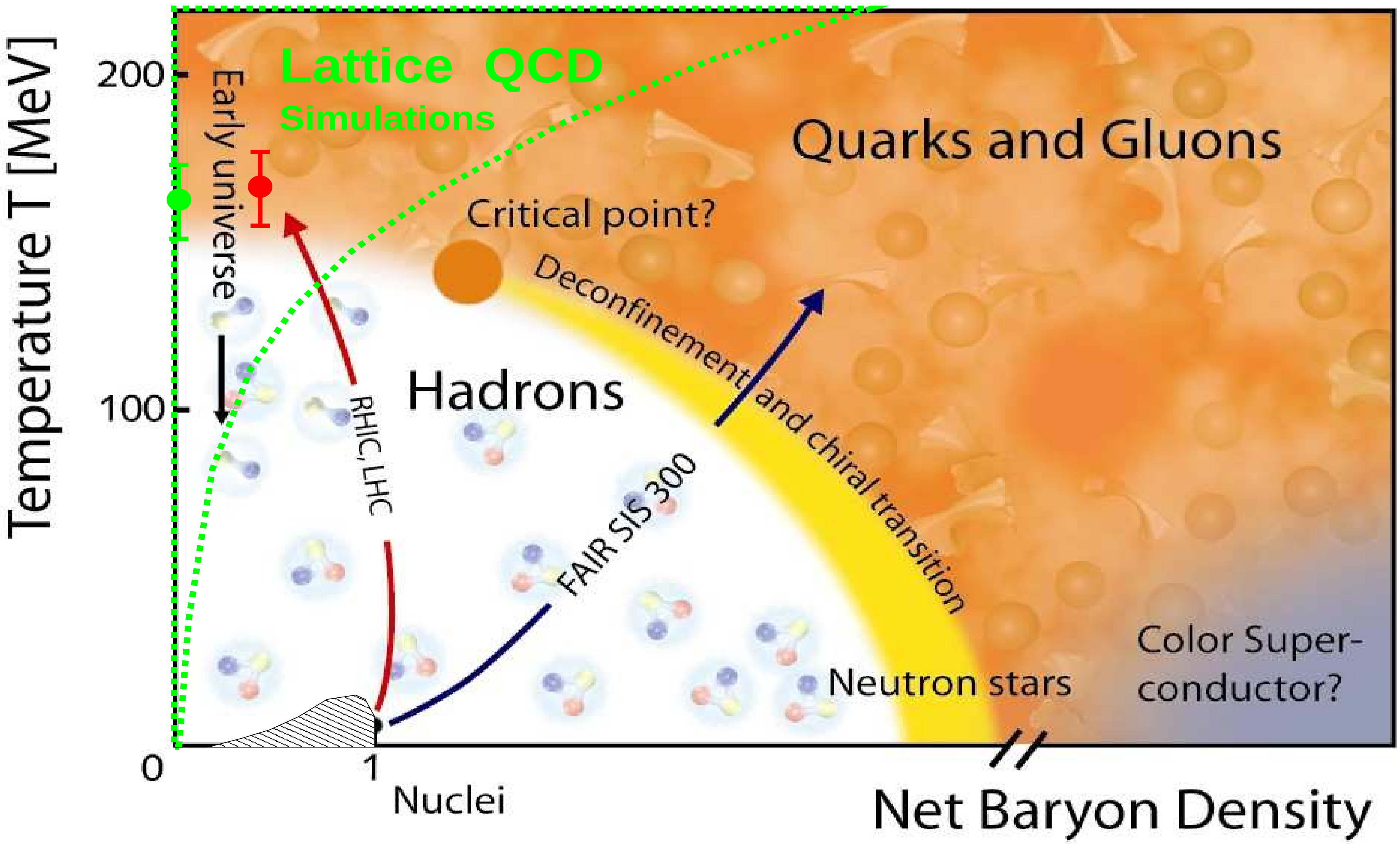
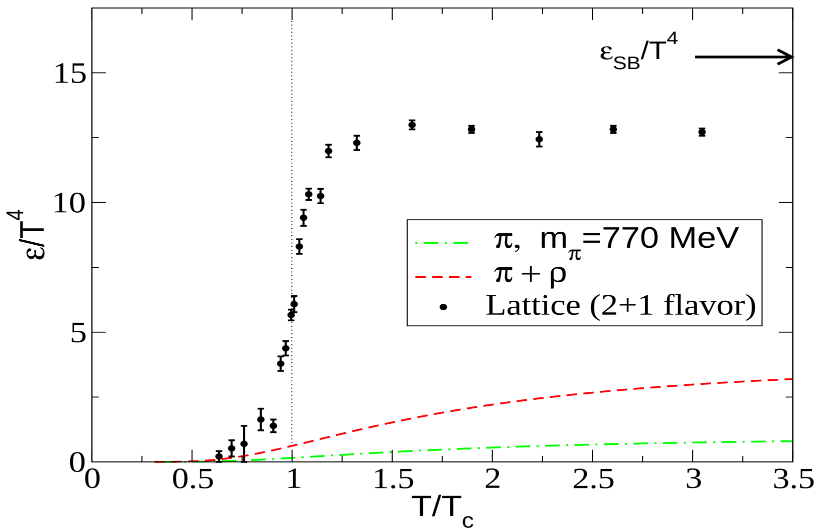


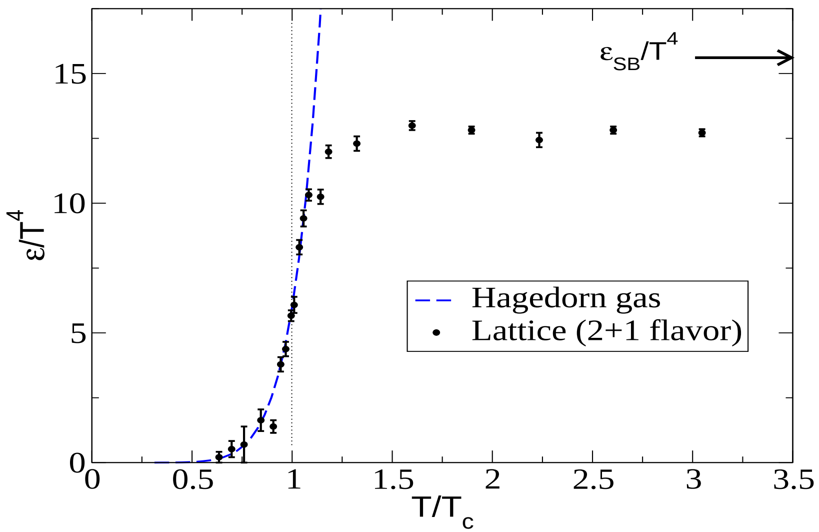
LATTICE QCD EOS VS. RESONANCE GAS



Ideal hadron gas mixture ...

$$\varepsilon(T) = \sum_{i=\pi,\rho,\ldots} g_i \int \frac{d^3p}{(2\pi)^3} \frac{\sqrt{p^2+m_i^2}}{\exp(\sqrt{p^2+m_i^2}/T)+\delta_i}$$

missing degrees of freedom below and above T_c



Resonance gas ...

Karsch, Redlich, Tawfik, Eur.Phys.J. C29, 549 (2003)

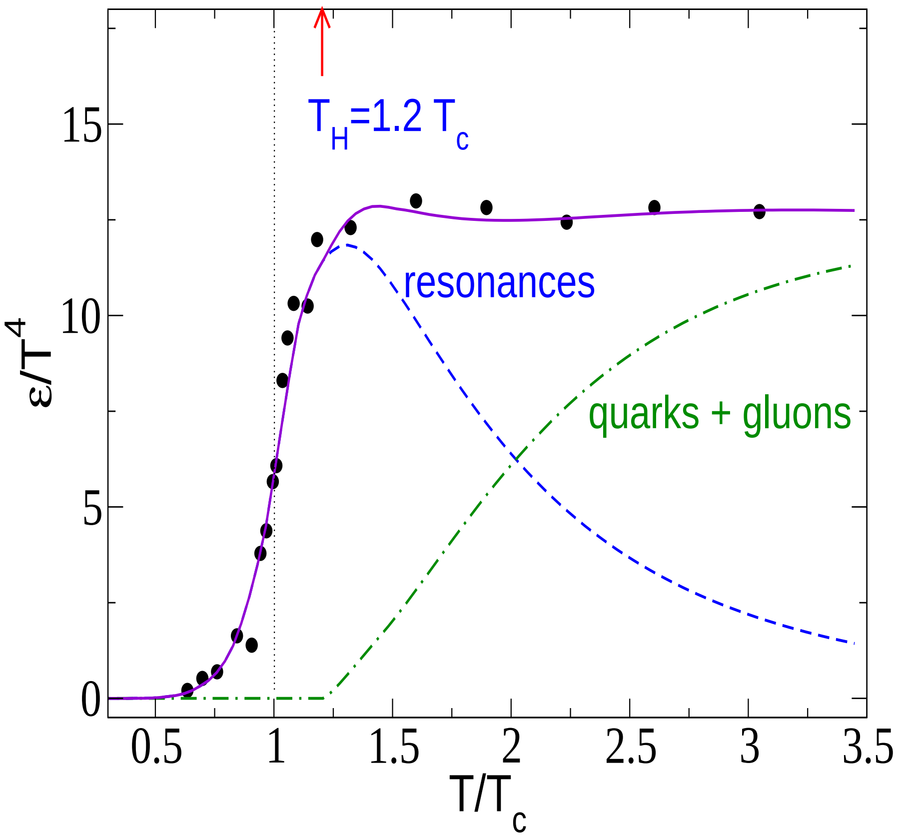
$$\begin{aligned}\varepsilon(T) &= \sum_{i=\pi,\rho,\ldots} \varepsilon_i(T) \\ &+ \sum_{r=M,B} g_r \int dm\, \rho(m) \int \frac{d^3p}{(2\pi)^3} \frac{\sqrt{p^2+m^2}}{\exp(\sqrt{p^2+m^2}/T)+\delta_r}\end{aligned}$$

$\rho(m) \sim m^\beta \exp(m/T_H)$ ... Hagedorn Massenspektrum

too many degrees of freedom above T_c

Lattice QCD EoS and Mott-Hagedorn Gas

$$\varepsilon_{\rm R}(T,\{\mu_j\}) = \sum_{i=\pi,K,\dots} \varepsilon_i(T,\{\mu_i\}) + \sum_{r=M,B} g_r \int_{m_r} dm \int ds\, \rho(m) A(s,m;T) \int \frac{d^3p}{(2\pi)^3} \frac{\sqrt{p^2+s}}{\exp\left(\frac{\sqrt{p^2+s}-\mu_r}{T}\right)+\delta_r}$$



Hagedorn mass spectrum: $\rho(m)$

Spectral function for heavy resonances:

$$A(s,m;T) = N_s \frac{m\Gamma(T)}{(s-m^2)^2 + m^2\Gamma^2(T)}$$

Ansatz with Mott effect at $T = T_H = 180$ MeV:

$$\Gamma(T) = B\Theta(T-T_H)\left(\frac{m}{T_H}\right)^{2.5}\left(\frac{T}{T_H}\right)^{6}\exp\left(\frac{m}{T_H}\right)$$

No width below T_H: Hagedorn resonance gas

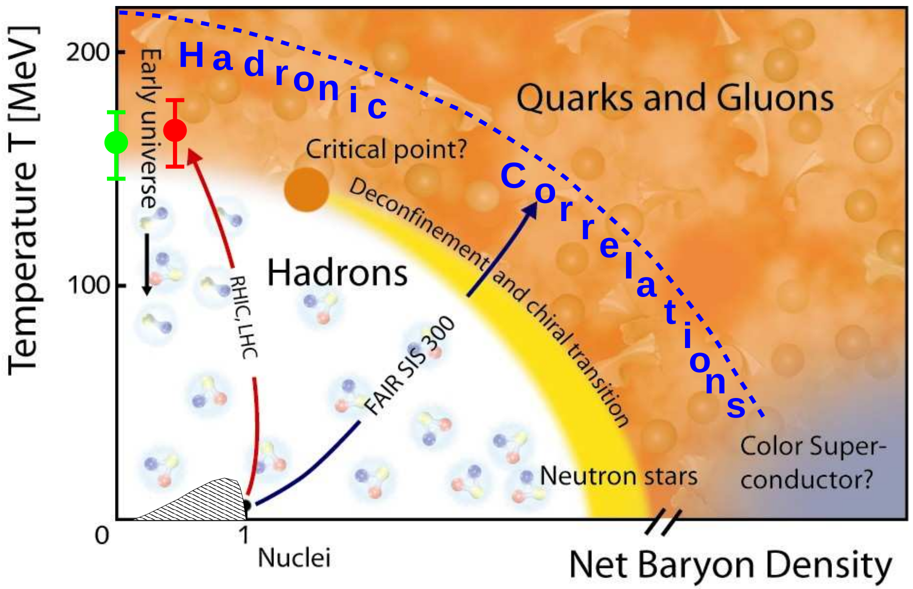
Apparent phase transition at $T_c \sim 150$ MeV

Blaschke & Bugaev, Fizika B13, 491 (2004)

Prog. Part. Nucl. Phys. 53, 197 (2004)

Blaschke & Yudichev, in preparation

Hadronic Correlations above T_c: Lattice QCD



Hadron correlators $G_H \Longrightarrow$ spectral densities $\rho_H(\omega, T)$

$$G_H(\tau, T) = \int_0^\infty d\omega \rho_H(\omega, T) \frac{\cosh(\omega(\tau - T/2))}{\sinh(\omega/2T)}$$

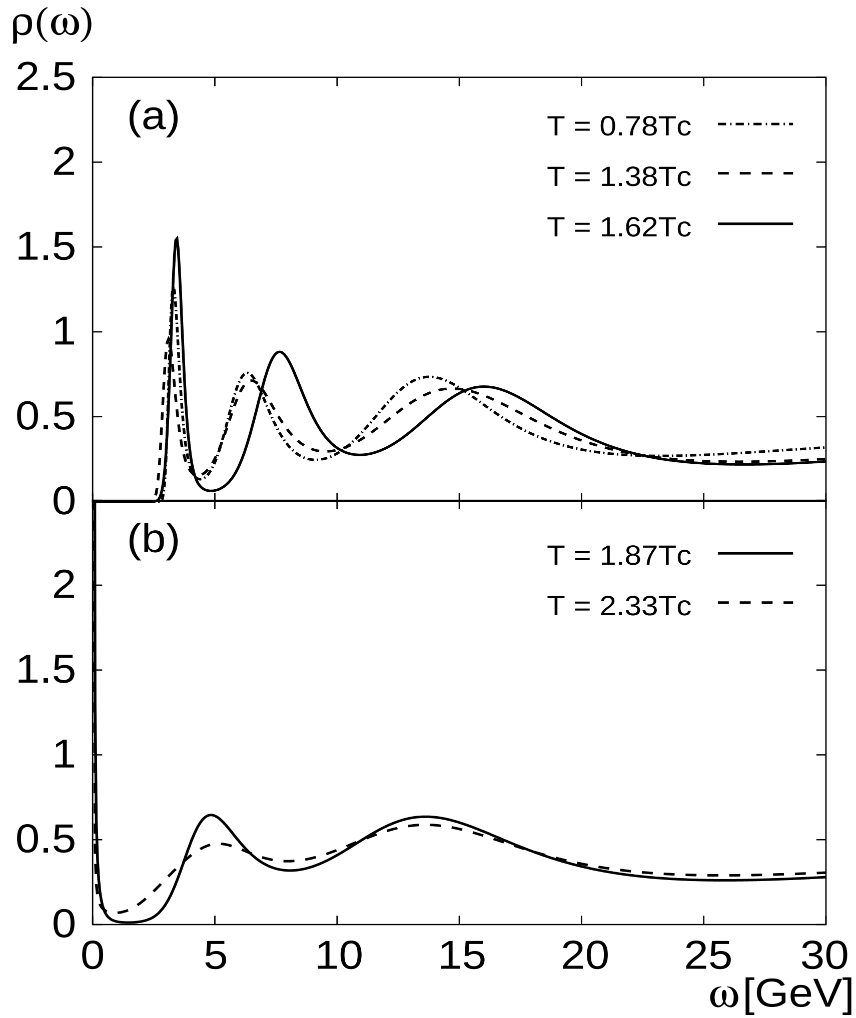
Maximum entropy method

Karsch et al. PLB 530 (2002) 147

Result:

Correlations persist above T_c !

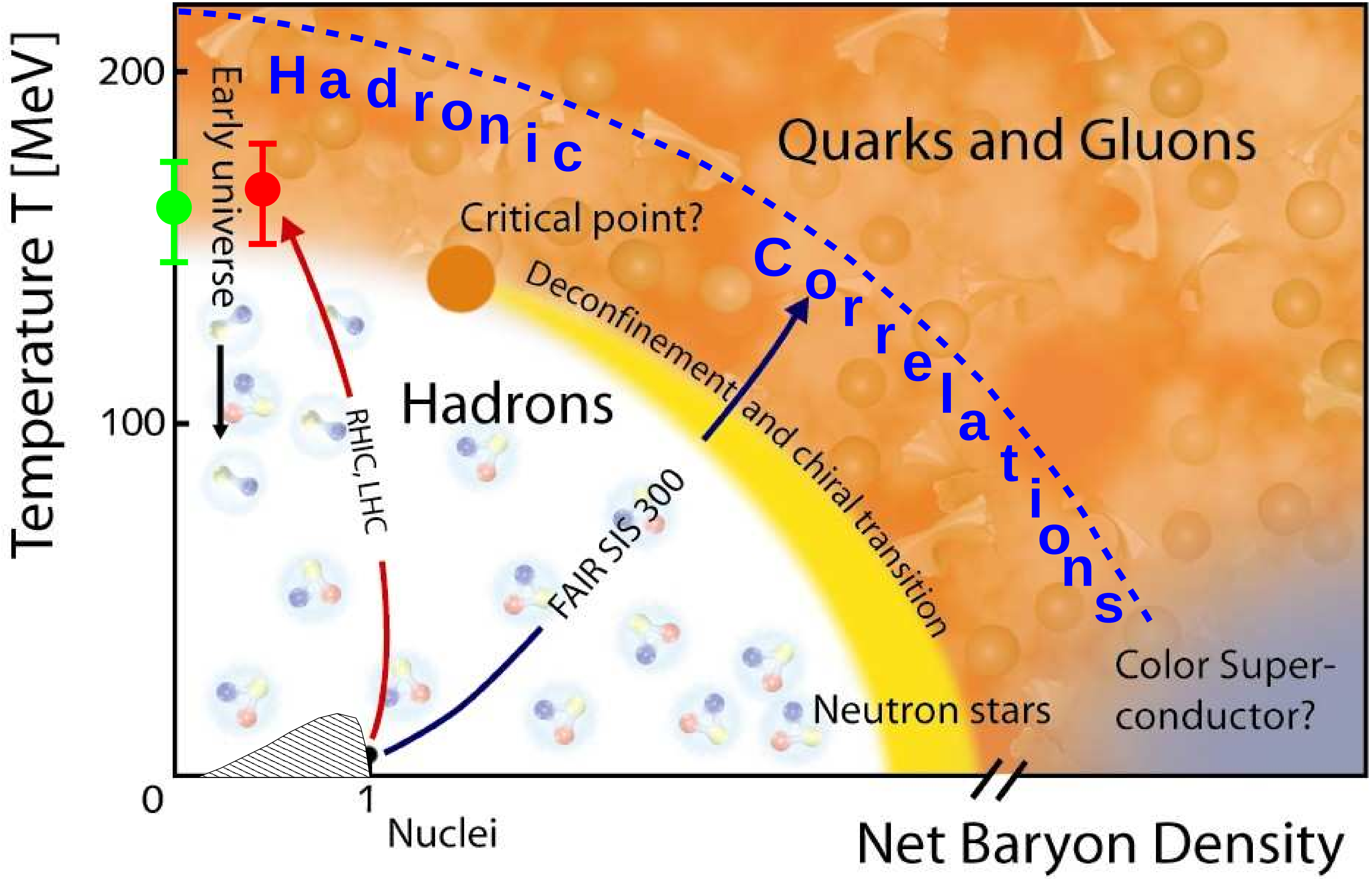
Karsch et al. NPA 715 (2003)



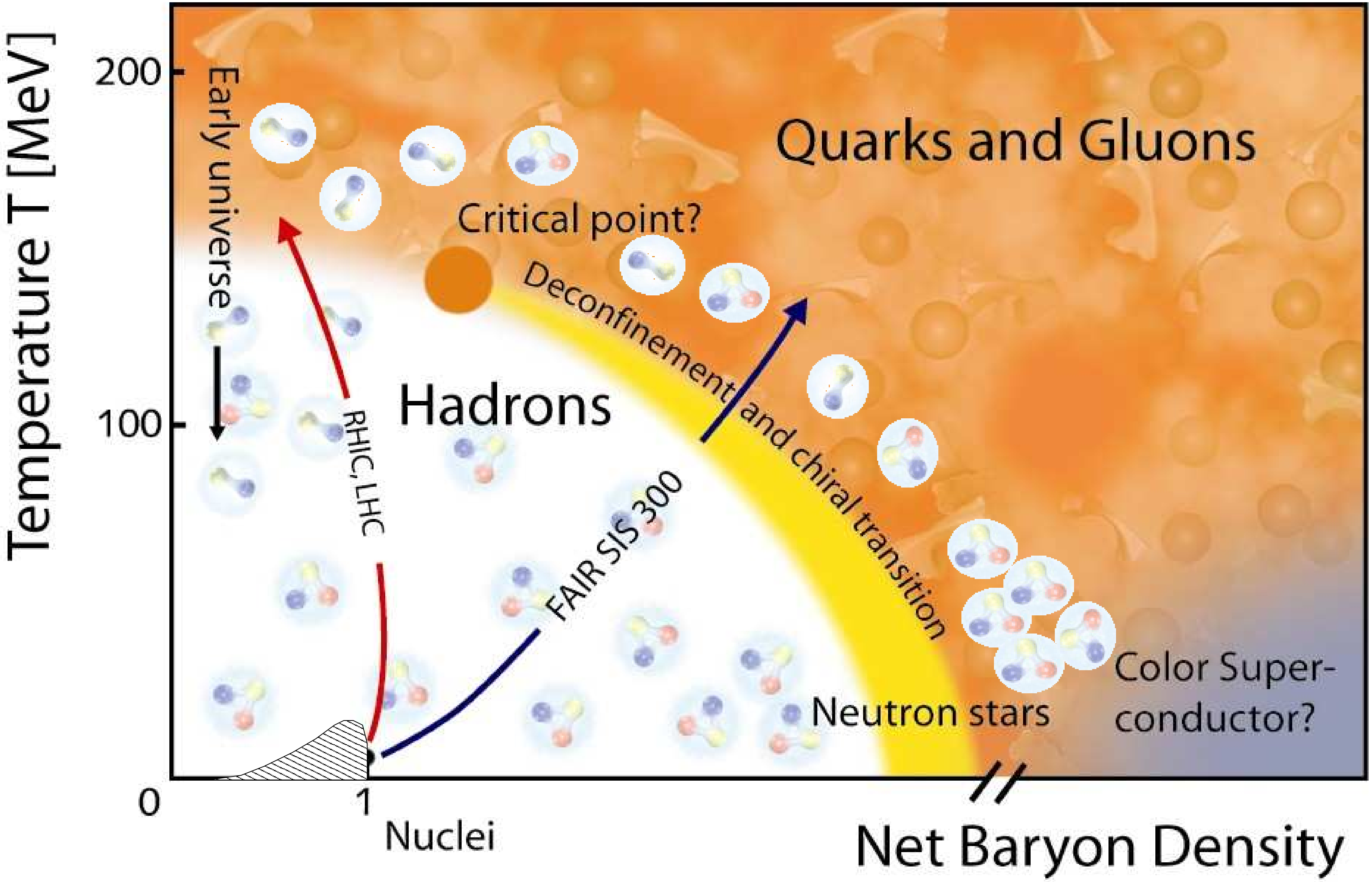
J/ψ and η_c survive up to $T \sim 1.6 T_c$

Asakawa, Hatsuda; PRL 92 (2004) 012001

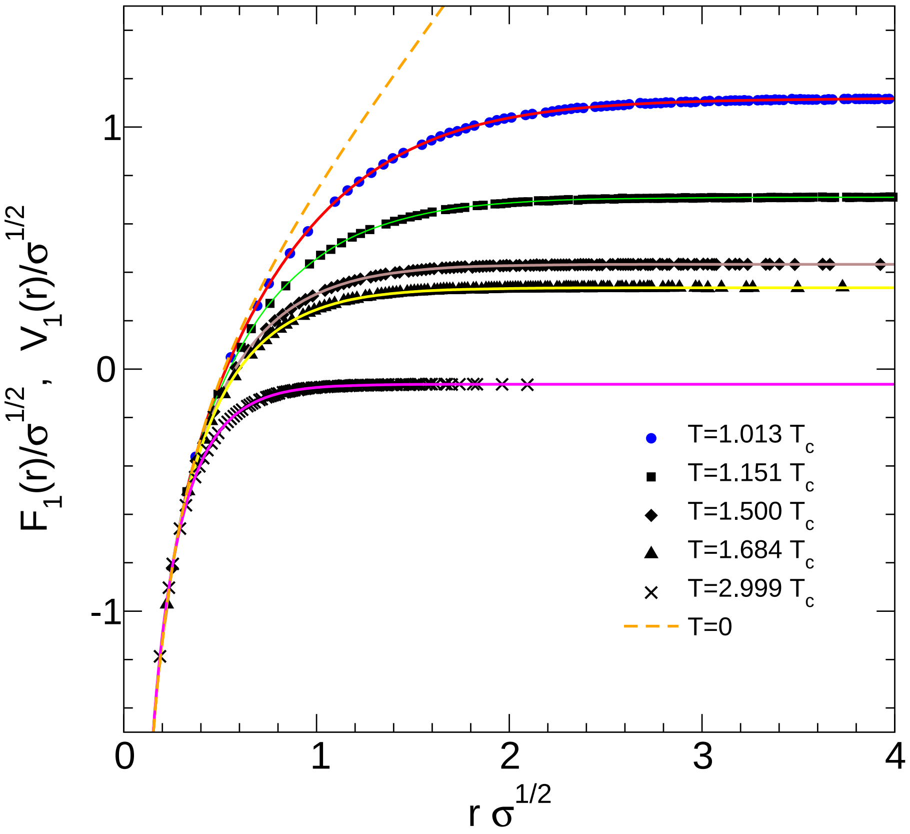
Hadronic Correlations in the Phasediagram of QCD



Hadronic Correlations in the Phasediagram of QCD



Heavy Quark Potential from Lattice QCD



Blaschke, Kaczmarek, Laermann, Yudichev, EPJC 43, 81 (2005); [hep-ph/0505053]

Color-singlet free energy F_1 in quenched QCD

$$\langle \mathrm{Tr}[L(0)L^\dagger(r)]\rangle = \exp[-F_1(r)/T]$$

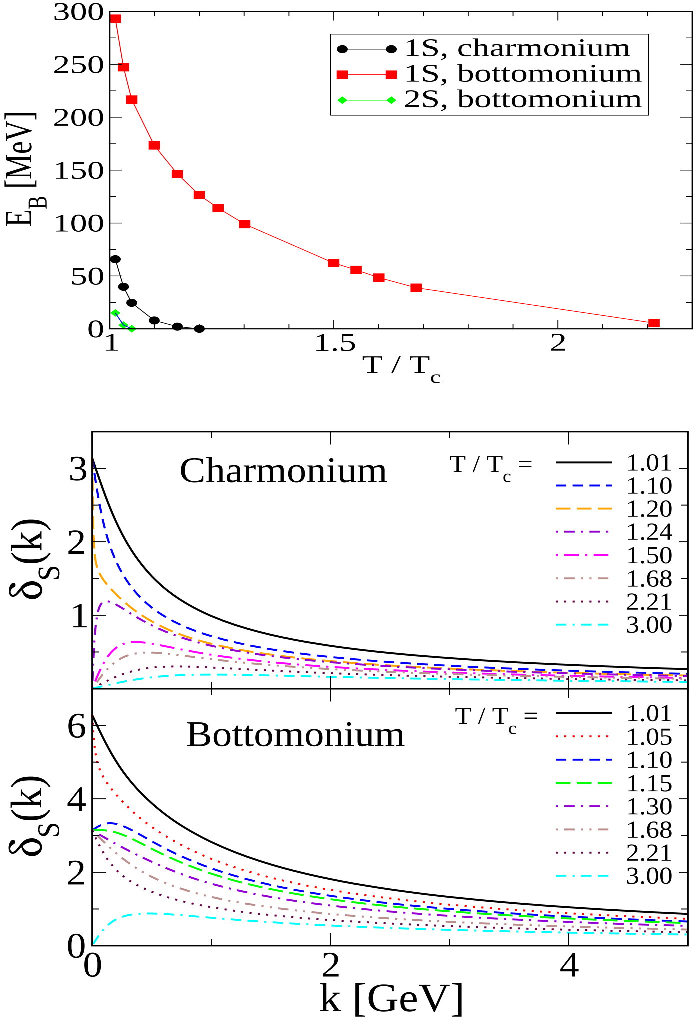
Long- and short- range parts

$$F_1(r,T) = F_{1,\mathrm{long}}(r,T) + V_{1,\mathrm{short}}(r)e^{-(\mu(T)r)^2}$$

$$\begin{aligned} F_{1,\mathrm{long}}(r,T) &= \text{'screened' confinement pot.} \\ V_{1,\mathrm{short}}(r) &= -\frac{4}{3}\frac{\alpha(r)}{r},\ \alpha(r) = \text{running coupl.} \end{aligned} \quad (1)$$

Quarkonium ($Q\bar{Q}$)	1S	$1P_1$	2S
Charmonium ($c\bar{c}$)	J/ψ(3097)	χ_{c1}(3510)	ψ' (3686)
Bottomonium ($b\bar{b}$)	Υ (9460)	χ_{b1} (9892)	Υ' (10023)

SCHROEDINGER EQN: BOUND & SCATTERING STATES



Quarkonia bound states at finite T:

$$[-\nabla^2/m_Q + V_{\text{eff}}(r,T)]\psi(r,T) = E_B(T)\psi(r,T)$$

Binding energy vanishes $E_B(T_{\text{Mott}}) = 0$: Mott effect

Scattering states:

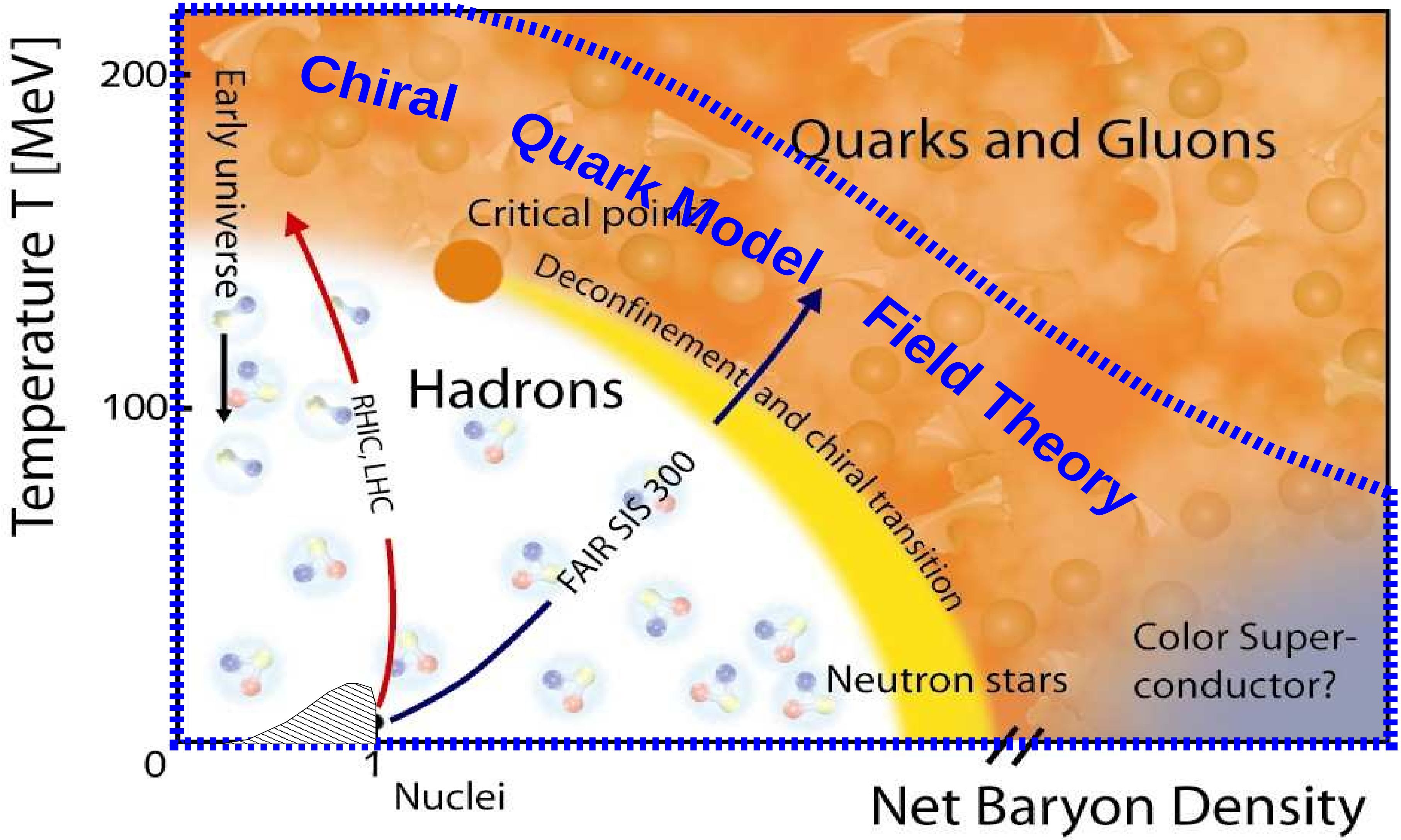
$$\frac{d\delta_S(k,r,T)}{dr} = -\frac{m_Q V_{\text{eff}}}{k}\sin(kr + \delta_S(k,r,T))$$

Levinson theorem:

Phase shift at threshold jumps by π when bound state $\rightarrow$ resonance at $T = T_{\text{Mott}}$

Blaschke, Kaczmarek, Laermann, Yudichev
EPJC 43, 81 (2005); [hep-ph/0505053]

Phasediagram of QCD: Chiral Model Field Theories



Chiral Model Field Theory for Quark Matter

- Partition function as a Path Integral (imaginary time $\tau = i\,t$)

$$Z[T,V,\mu] = \int \mathcal{D}\bar{\psi}\mathcal{D}\psi \exp\left\{-\int^{\beta} d\tau \int_V d^3x [\bar{\psi}(i\gamma^\mu\partial_\mu - m - \gamma^0\mu)\psi - \mathcal{L}_{\rm int}]\right\}$$

- Current-current interaction (4-Fermion coupling)

$$\mathcal{L}_{\rm int} = \sum_{M=\pi,\sigma,\dots} G_M(\bar{\psi}\Gamma_M\psi)^2 + \sum_D G_D(\bar{\psi}^C\Gamma_D\psi)^2$$

- Bosonization (Hubbard-Stratonovich Transformation)

$$Z[T,V,\mu] = \int \mathcal{D}M_M\mathcal{D}\Delta_D^\dagger\mathcal{D}\Delta_D \exp\left\{-\sum_M \frac{M_M^2}{4G_M} - \sum_D \frac{|\Delta_D|^2}{4G_D} + \frac{1}{2}\mathrm{Tr}\,\ln S^{-1}[\{M_M\},\{\Delta_D\}]\right\}$$

- Collective (stochastic) fields: Mesons (M_M) and Diquarks (Δ_D)
- Systematic evaluation: Mean fields + Fluctuations
  - Mean-field approximation: order parameters for phase transitions (gap equations)
  - Lowest order fluctuations: hadronic correlations (bound & scattering states)
  - Higher order fluctuations: hadron-hadron interactions

NJL MODEL FOR NEUTRAL 3-FLAVOR QUARK MATTER

$$\Omega(T,\mu) = \frac{\phi_u^2+\phi_d^2+\phi_s^2}{8G_S} + \frac{|\Delta_{ud}|^2+|\Delta_{us}|^2+|\Delta_{ds}|^2}{4G_D} - T\sum_n \int \frac{d^3p}{(2\pi)^3}\frac{1}{2}\mathrm{Tr}\ln\left(\frac{1}{T}S^{-1}(i\omega_n,\vec{p})\right) + \Omega_e - \Omega_0.$$

Inverse propagator of Nambu-Gorkov spinors

$$S^{-1}(i\omega_n,\vec{p}) = \begin{bmatrix} \not{p} - M + \mu\gamma^0 & \widehat{\Delta} \\ \widehat{\Delta}^\dagger & \not{p} - M - \mu\gamma^0 \end{bmatrix},$$

with diquark gaps ($\Delta_{ur}=\Delta_{ds}$, ...)

$$\Delta_{k\gamma} = 2G_D\langle \bar{q}_{i\alpha} i\gamma_5 \epsilon_{\alpha\beta\gamma}\epsilon_{ijk} q^C_{j\beta}\rangle.$$

as elements of the gap matrix

$$\widehat{\Delta} = i\gamma_5\epsilon_{\alpha\beta\gamma}\epsilon_{ijk}\Delta_{k\gamma}.$$

Fermion determinant (Tr ln D = ln det D)

$$\mathrm{ln det}\left(\frac{1}{T}S^{-1}(i\omega_n,\vec{p})\right) = 2\sum_{a=1}^{18}\ln\left(\frac{\omega_n^2+\lambda_a(\vec{p})^2}{T^2}\right).$$

Result for thermodynamic potential

$$\Omega(T,\mu) = \frac{\phi_u^2+\phi_d^2+\phi_s^2}{8G_S} + \frac{|\Delta_{ud}|^2+|\Delta_{us}|^2+|\Delta_{ds}|^2}{4G_D} - \int\frac{d^3p}{(2\pi)^3}\sum_{a=1}^{18}\left(\lambda_a + 2T\ln\left(1+e^{-\lambda_a/T}\right)\right) + \Omega_e - \Omega_0.$$

Neutrality conditions: $n_Q = n_8 = n_3 = 0$,

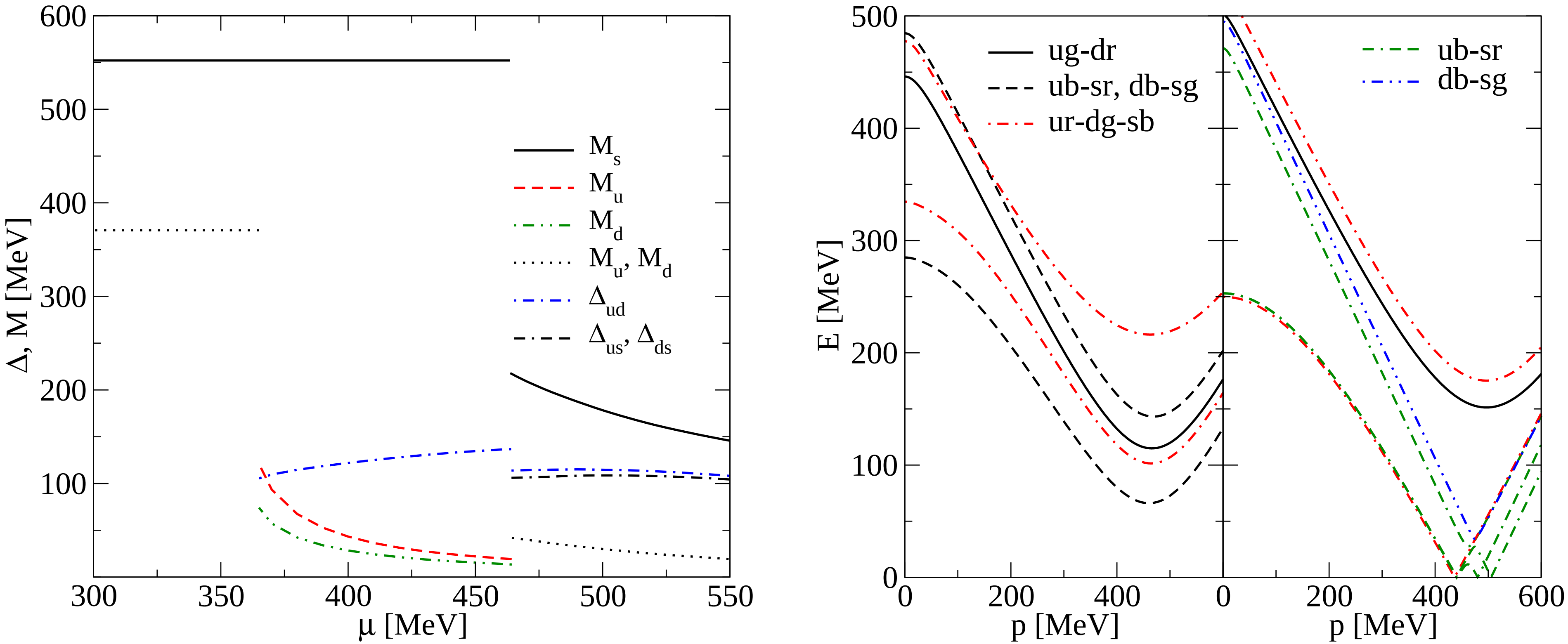
$$n_i = -\frac{\partial\Omega}{\partial\mu_i} = 0,$$

Equation of state: $P = -\Omega$, etc.

Order Parameters: Masses and Diquark Gaps

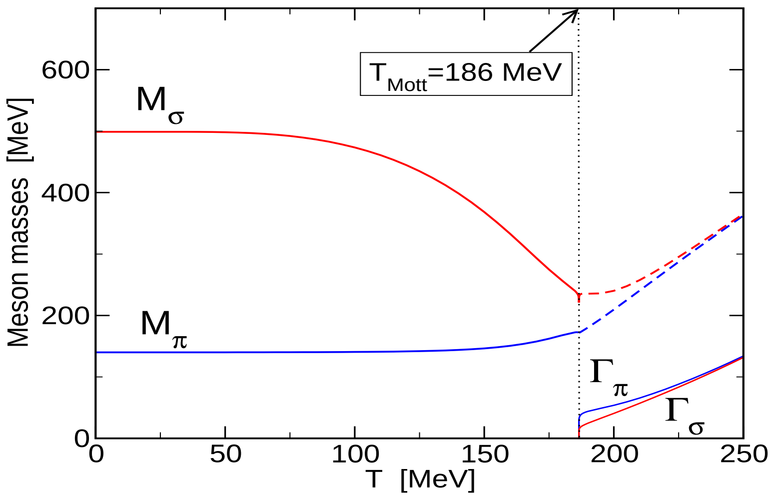
Masses (M) and Diquark gaps (Δ) as a function of the chemical potential at $T = 0$

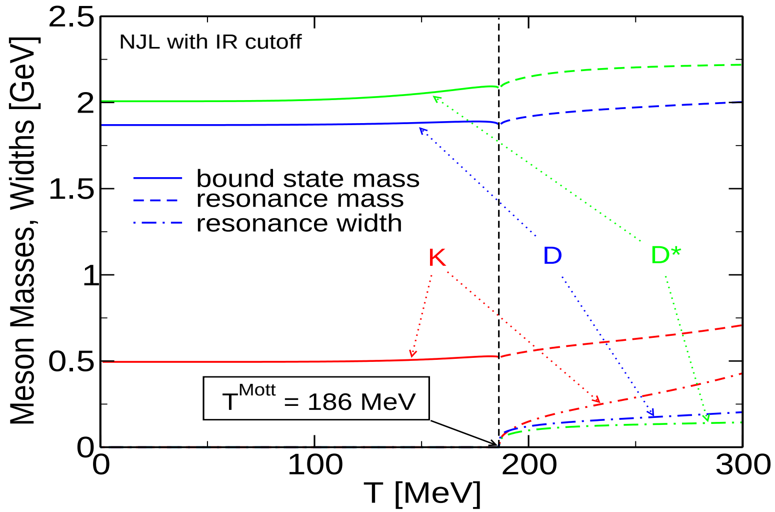
Left: Gap in excitation spectrum ($T = 0$)
Right: 'Gapless' excitations ($T = 60$ MeV)



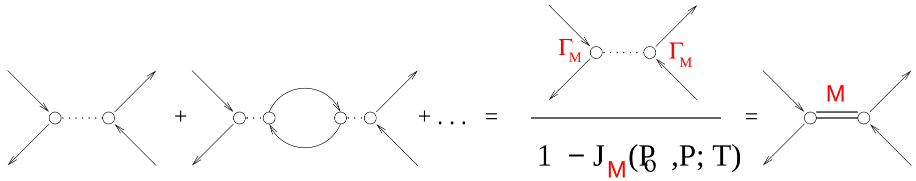
$\Longrightarrow$ Buballa (DM 1, 6, 6+)

Mott effect: NJL model primer





RPA-type resummation of quark-antiquark scattering in the mesonic channel M,



defines Meson propagator

$$D_M(P_0, P; T) \sim [1 - J_M(P_0, P; T)]^{-1},$$

by the complex polarization function J_M
$\longrightarrow$ Breit-Wigner type spectral function

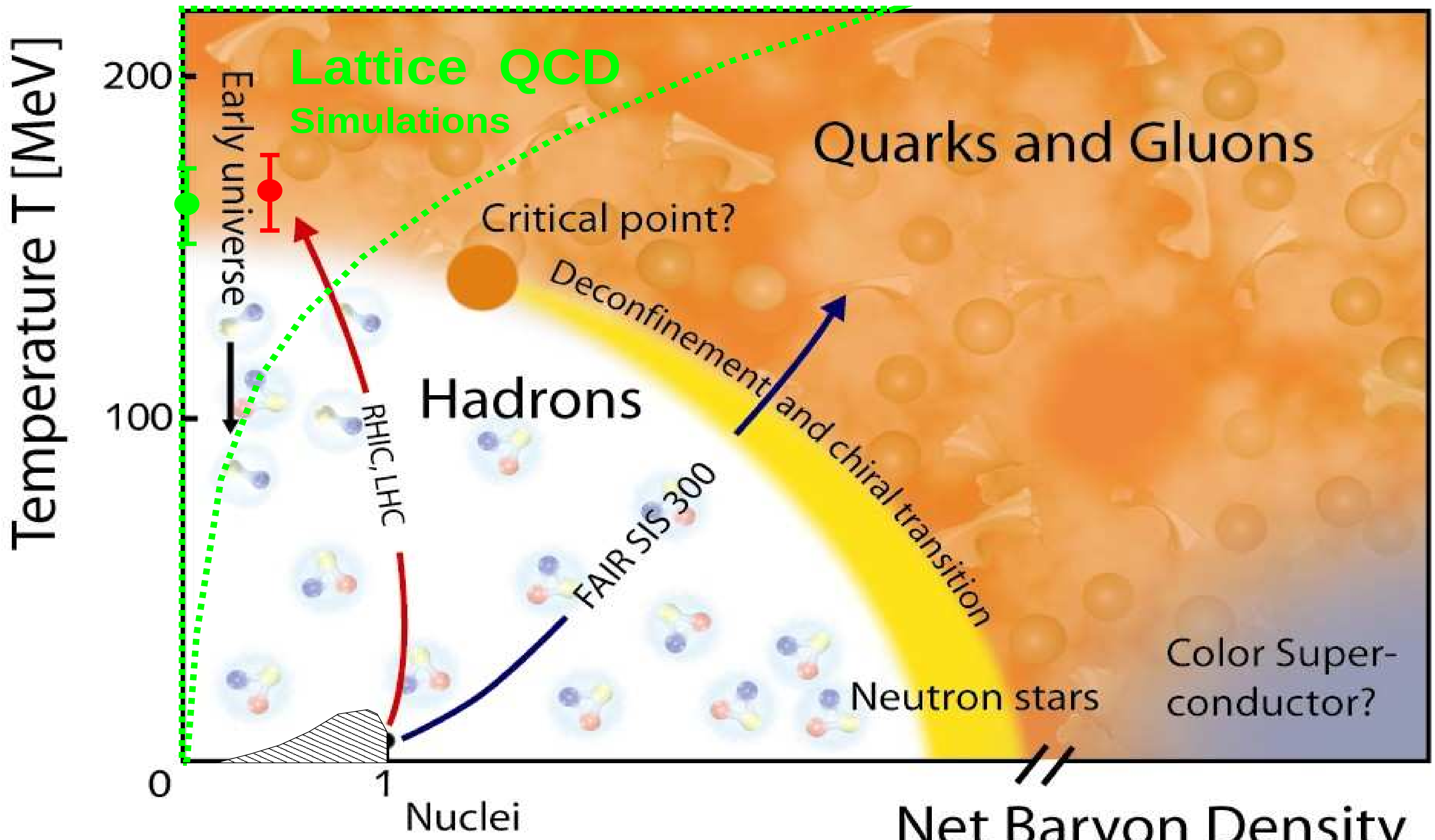
$$\begin{aligned} \mathcal{A}_M(P_0, P; T) &= \frac{1}{\pi}\mathrm{Im}\ D_M(P_0, P; T) \\ &\sim \frac{1}{\pi}\frac{\Gamma_M(T)M_M(T)}{(s - M_M^2(T))^2 + \Gamma_M^2(T)M_M^2(T)} \end{aligned}$$

For $T < T_{\rm Mott}$: $\Gamma \to 0$, i.e. bound state
$\mathcal{A}_M(P_0, P; T) = \delta(s - M_M^2(T))$

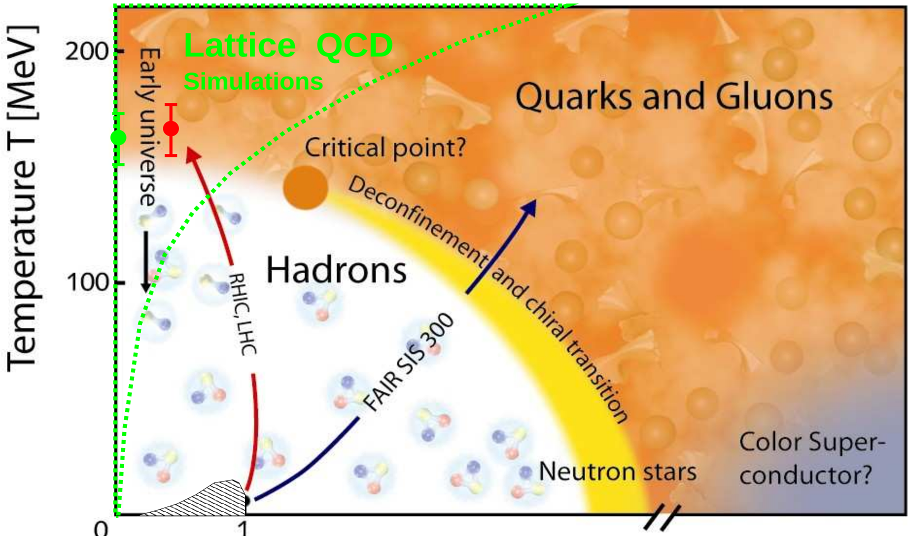
Light meson sector:
Blaschke, Burau, Volkov, Yudichev: EPJA **11** (2001) 319

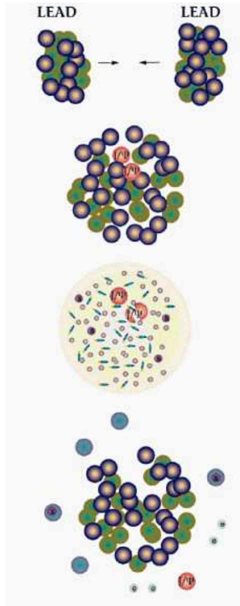
Charm meson sector:
Blaschke, Burau, Kalinovsky, Yudichev,
Prog. Theor. Phys. Suppl. **149** (2003) 182

PHASEDIAGRAM OF QCD: HEAVY-ION COLLISIONS

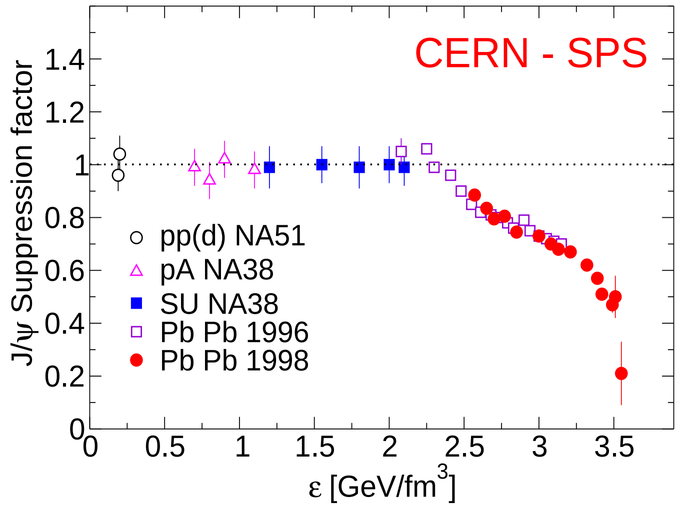


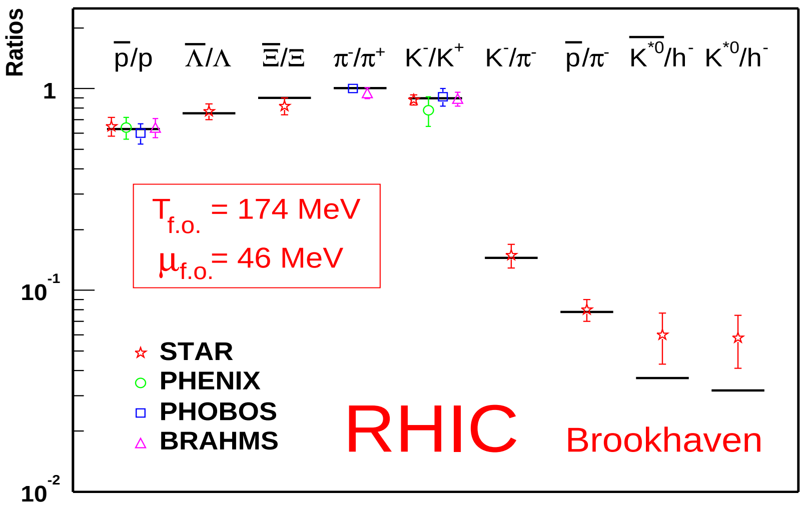
Phasediagram of QCD: Lattice vs. Heavy-Ion Collisions





QGP Signal: Anomalous J/ψ suppression





Statistical model describes composition of hadron yields in Heavy-Ion Collisions with few freeze-out parameters.

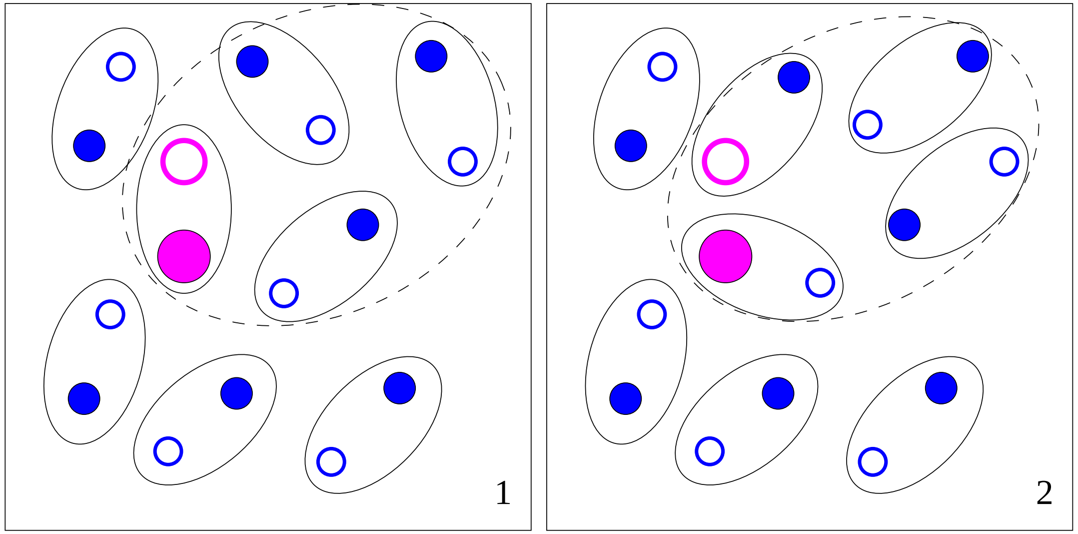
$$\begin{aligned}\ln Z[T,V,\{\mu\}] &= \pm V \sum_i \frac{g_i}{2\pi^2}\int_0^\infty dp\, p^2 \ln[1 \pm \lambda_i \exp(-\beta\varepsilon_i(p))] \\ \lambda_i(T,\{\mu\}) &= \exp[\beta(\mu_B B_i + \mu_S S_i + \mu_Q Q_i)]\end{aligned}$$

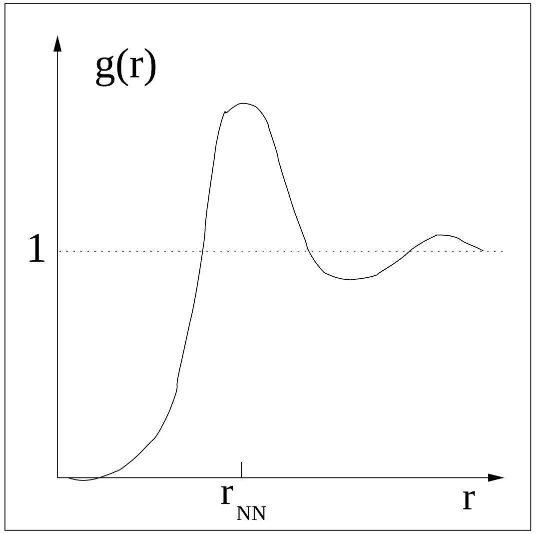
Braun-Munzinger, Redlich, Stachel, in *QGP III* (2003)

$\Longrightarrow$ Turko (DM 9,11), $\Longrightarrow$ Gorenstein (HIC 6, 8)

A SNAPSHOP OF THE sQGP

The Picture: String-flip (Rearrangement) $\Longleftrightarrow$ Pair correlation



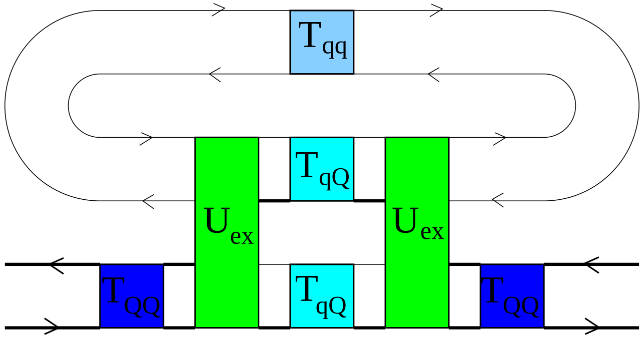


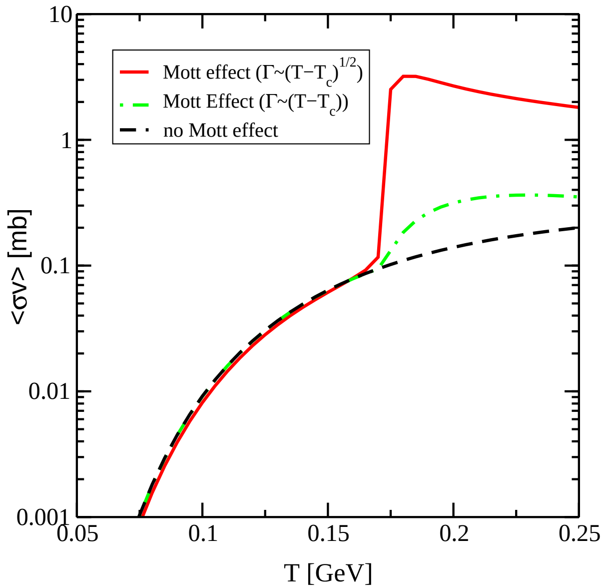
Horowitz et al. PRD (1985), D.B. et al. PLB (1985), Röpke, Blaschke, Schulz, PRD (1986)

Thoma, Quark Matter '05; [hep-ph/0509154]

- Strong correlations present: hadronic spectral functions above T_c (lattice QCD)
- Finite width due to rearrangement collisions (higher order correlations)
- Liquid-like pair correlation function (nearest neighbor peak)

QUANTUM KINETIC APPROACH TO J/ψ BREAKUP





Inverse lifetime for Charmonium states

$$\tau^{-1}(p) = \Gamma(p) = \Sigma^{>}(p) \mp \Sigma^{<}(p)$$

$$\Sigma^{\gtrless}(p,\omega) = \int_{p'}\int_{p_1}\int_{p_2}(2\pi)^4\delta_{p,p';p_1,p_2}\,|\mathcal{M}|^2\,G_\pi^{\lessgtr}(p')\,G_{D_1}^{\gtrless}(p_1)\,G_{D_2}^{\gtrless}(p_2)$$

$G_h^{>}(p) = [1 \pm f_h(p)]A_h(p)$ and $G_h^{<}(p) = f_h(p)A_h(p)$

$$\tau^{-1}(p) = \int\frac{d^3\mathbf{p}'}{(2\pi)^3}\int ds' \quad f_\pi(\mathbf{p}',s')\,A_\pi(s')v_{\rm rel}\,\sigma^*(s)$$

In-medium breakup cross section

$$\sigma^*(s) = \int ds_1\,ds_2\,A_{D_1}(s_1)\,A_{D_2}(s_2)\,\sigma(s;s_1,s_2)$$

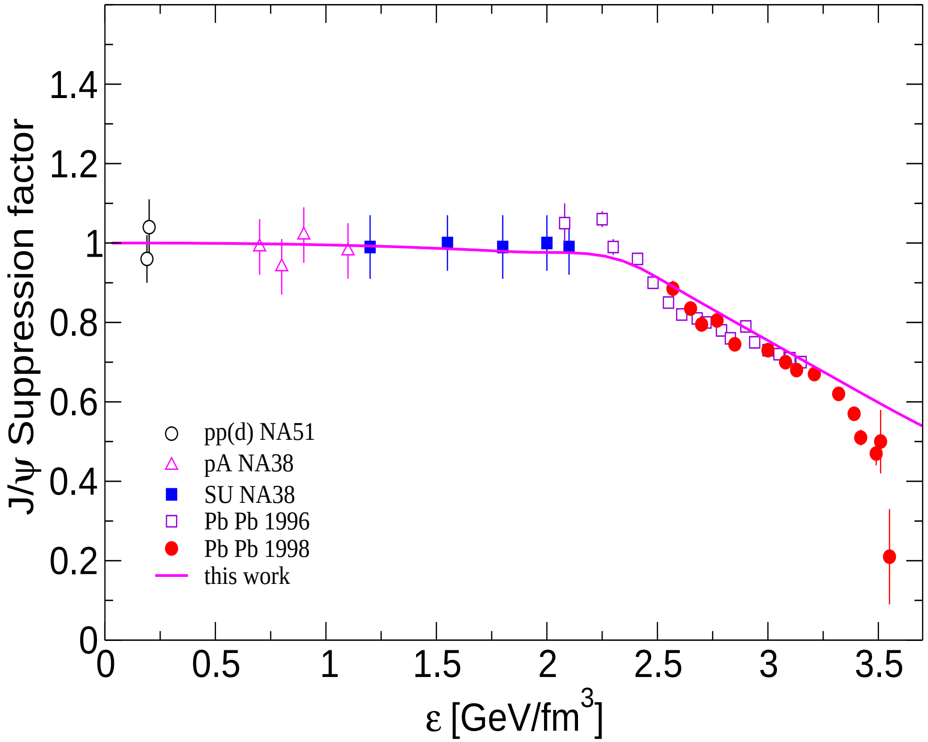
Medium effects in spectral functions A_h and $\sigma(s;s_1,s_2)$

$$A_h(s) = \frac{1}{\pi}\frac{\Gamma_h(T)\,M_h(T)}{(s-M_h^2(T))^2+\Gamma_h^2(T)M_h^2(T)} \longrightarrow \delta(s-M_h^2)$$

resonance $\Leftarrow$ $\boxed{\text{Mott-effect}}$ $\Leftarrow$ bound state

Blaschke et al., Heavy Ion Phys. 18 (2003) 49

"Anomalous" J/ψ suppression in Mott-Hagedorn gas



Survival probability for J/ψ

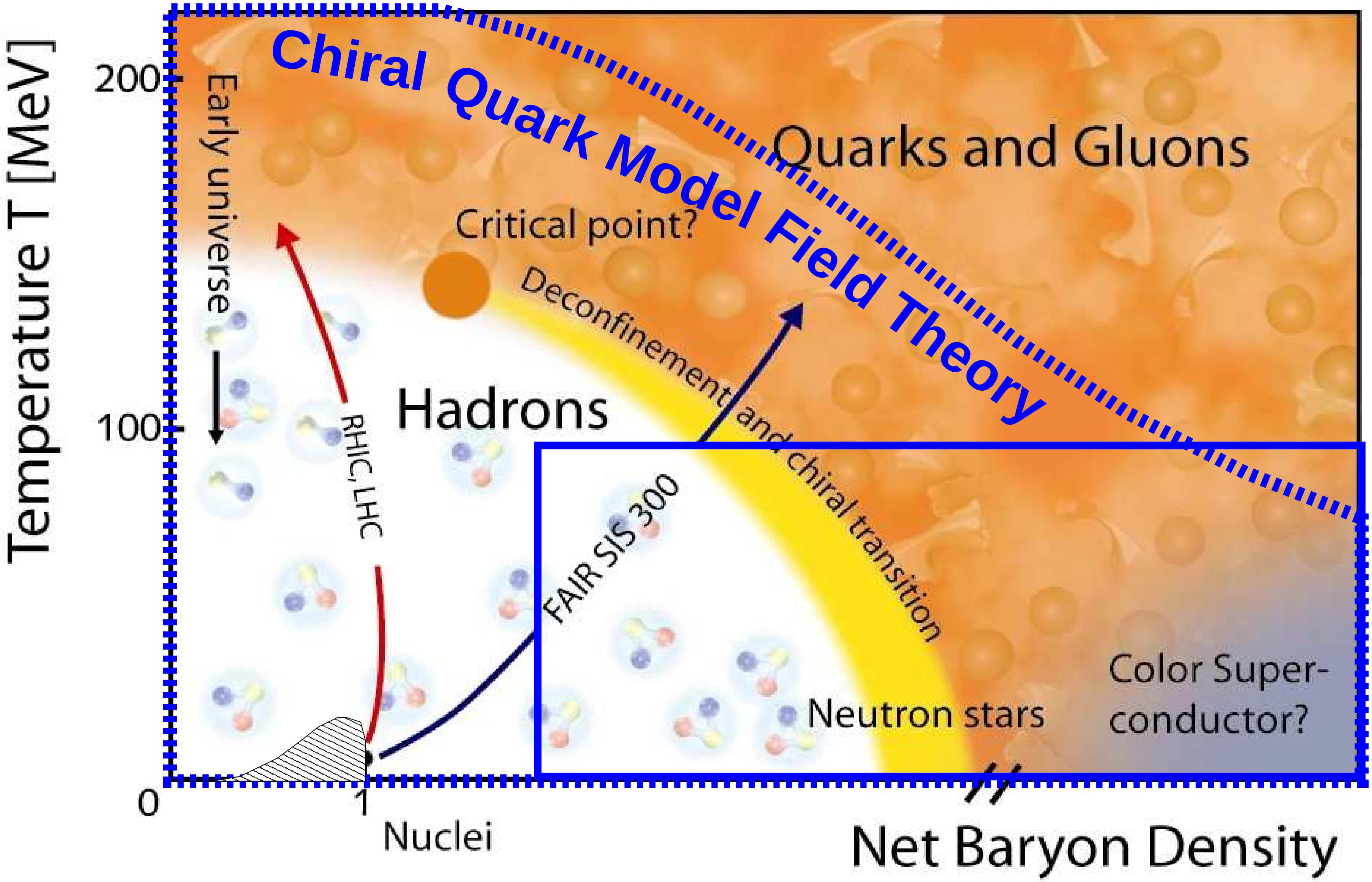
$$S(E_T)/S_N(E_T) \;=\; \exp\left[-\int_{t_0}^{t_f} dt\; \tau^{-1}(n(t))\right]$$

Threshold: Mott effect for hadrons

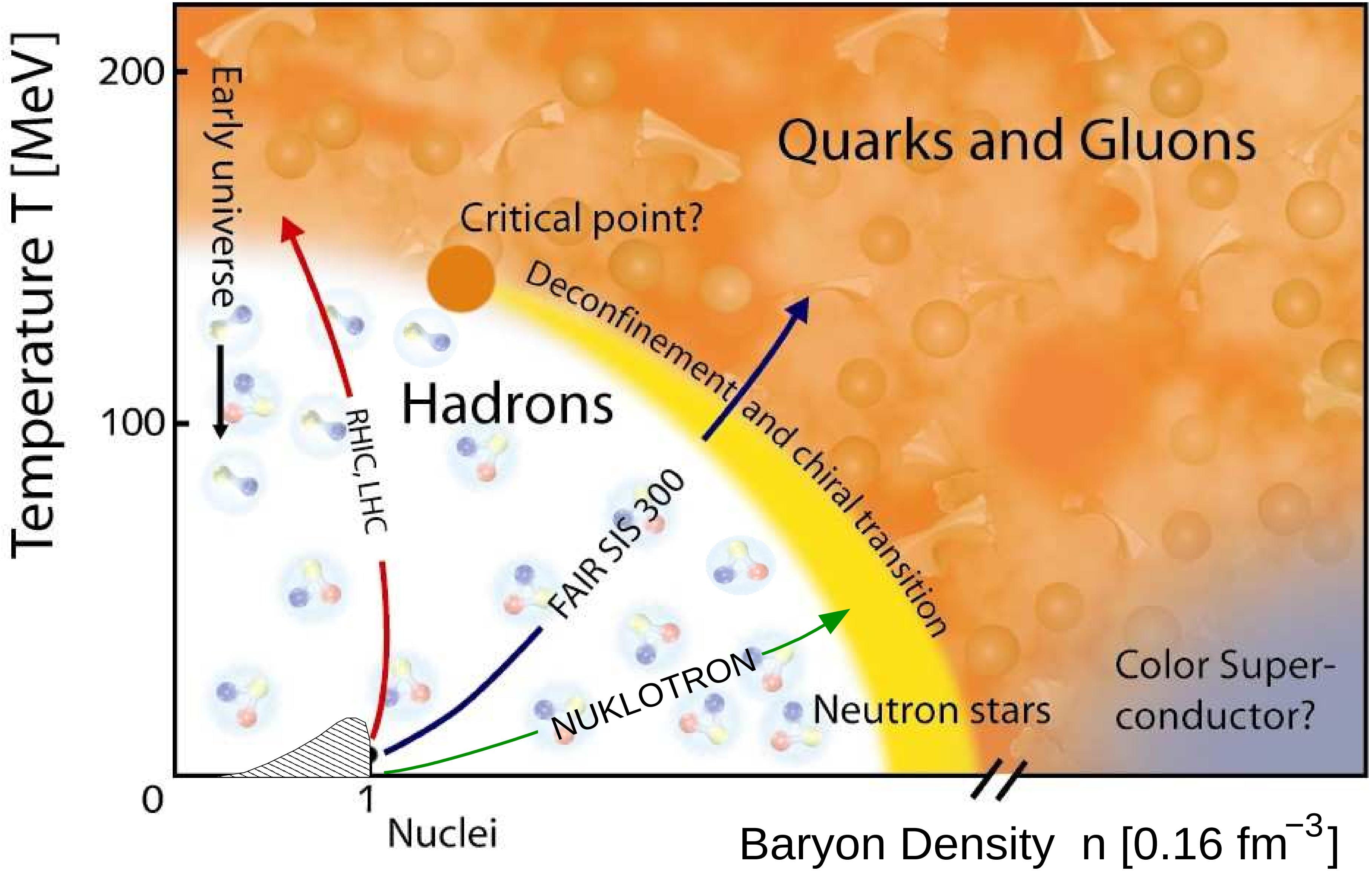
Blaschke and Bugaev, Prog. Part. Nucl. Phys. 53 (2004) 197

In progress: full kinetics with gain processes (D-fusion), HIC simulation

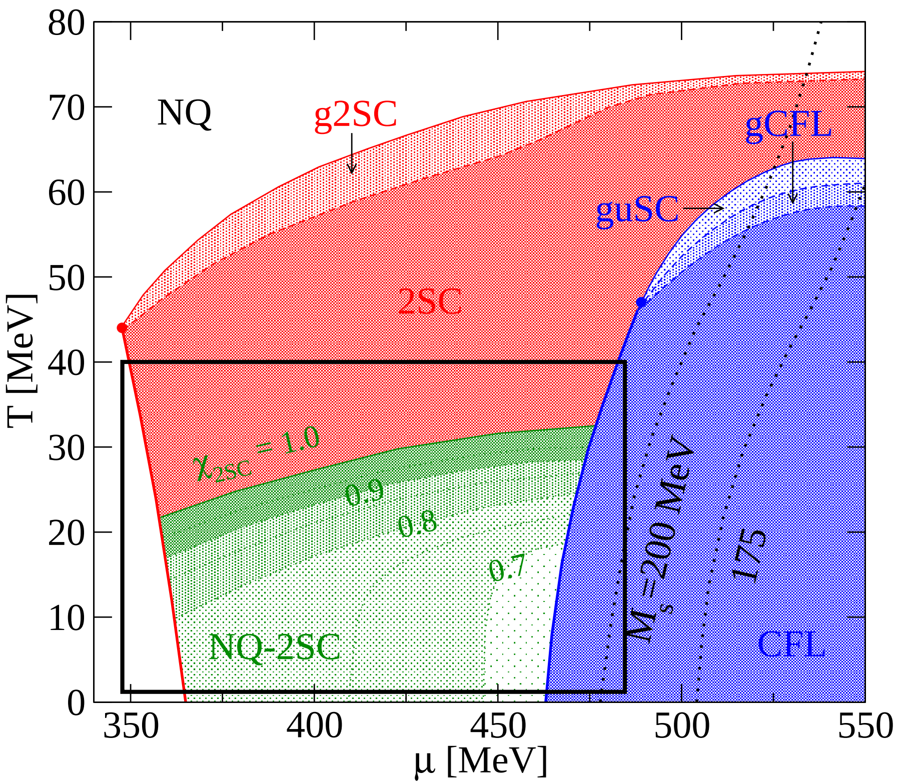
PHASEDIAGRAM OF DEGENERATE QUARK MATTER



PHASEDIAGRAM OF DEGENERATE QUARK MATTER



Quark matter in compact stars



Rüster et al: PRD 72 (2005) 034004
Blaschke et al: PRD 72 (2005) 065020

The phases are characterized by 3 gaps:

- NQ: $\Delta_{ud} = \Delta_{us} = \Delta_{ds} = 0$;
- NQ-2SC: $\Delta_{ud} \neq 0$, $\Delta_{us} = \Delta_{ds} = 0$, 0$\leq \chi_{2SC} \leq$1;
- 2SC: $\Delta_{ud} \neq 0$, $\Delta_{us} = \Delta_{ds} = 0$;
- uSC: $\Delta_{ud} \neq 0$, $\Delta_{us} \neq 0$, $\Delta_{ds} = 0$;
- CFL: $\Delta_{ud} \neq 0$, $\Delta_{ds} \neq 0$, $\Delta_{us} \neq 0$;

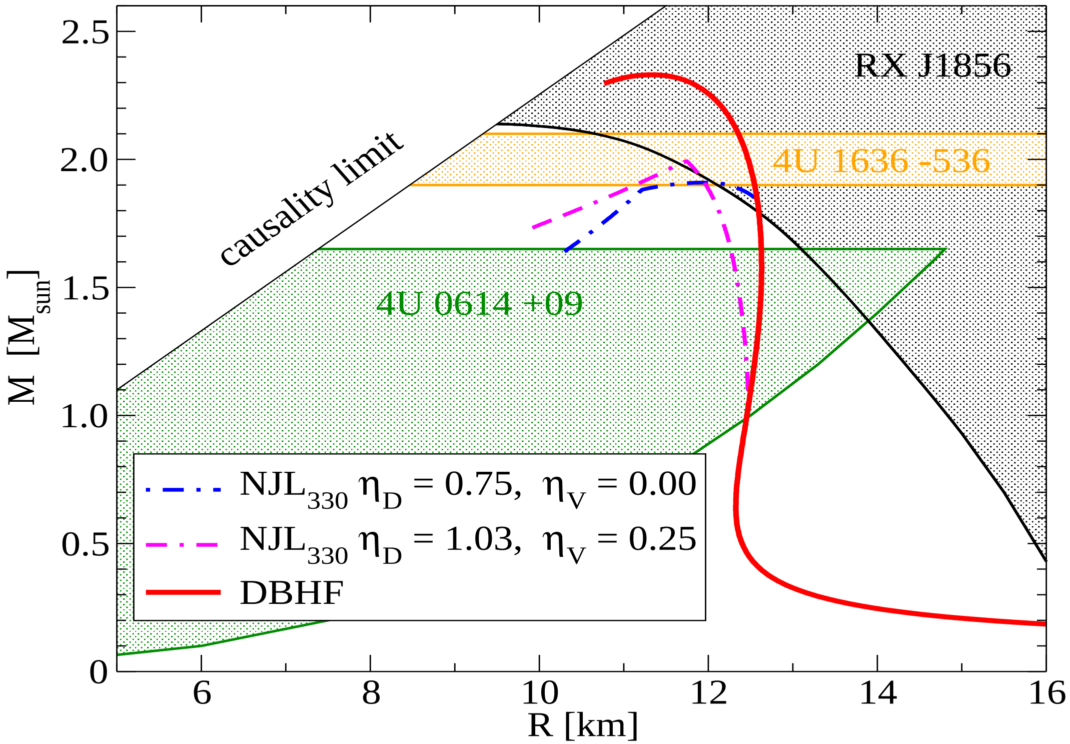
Result:

- Gapless phases only at high T,
- CFL only at high chemical potential,
- At T $\leq$25-30 MeV: mixed NQ-2SC phase,
- Critical point (T_c,μ_c)=(48 MeV, 353 MeV),
- Strong coupling, $\eta = 1$, changes?.

$\Longrightarrow$ Buballa (DM 1, 6, 6+)

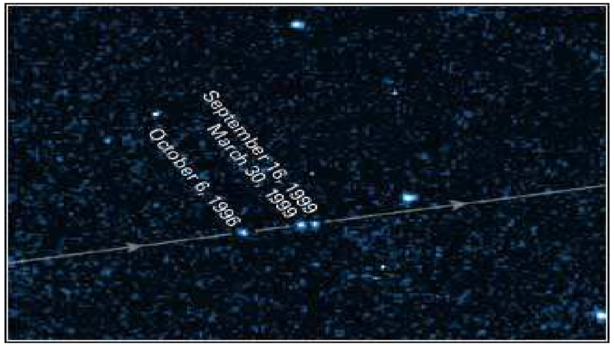
QUARK MATTER IN COMPACT STARS: MASS-RADIUS CONSTRAINT

Solve TOV Eqn. → Hybrid stars fulfill constraint!

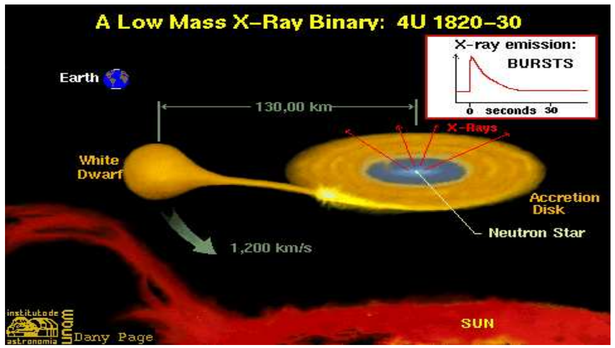


Klähn et al: Constraints on the high-density EoS ...
PRC 74 (2006); [nucl-th/0602038], [astro-ph/0606524]
⟹ Grigorian (Ast 5)

- Isolated Neutron star RX J1856: M-R constraint from thermal emission

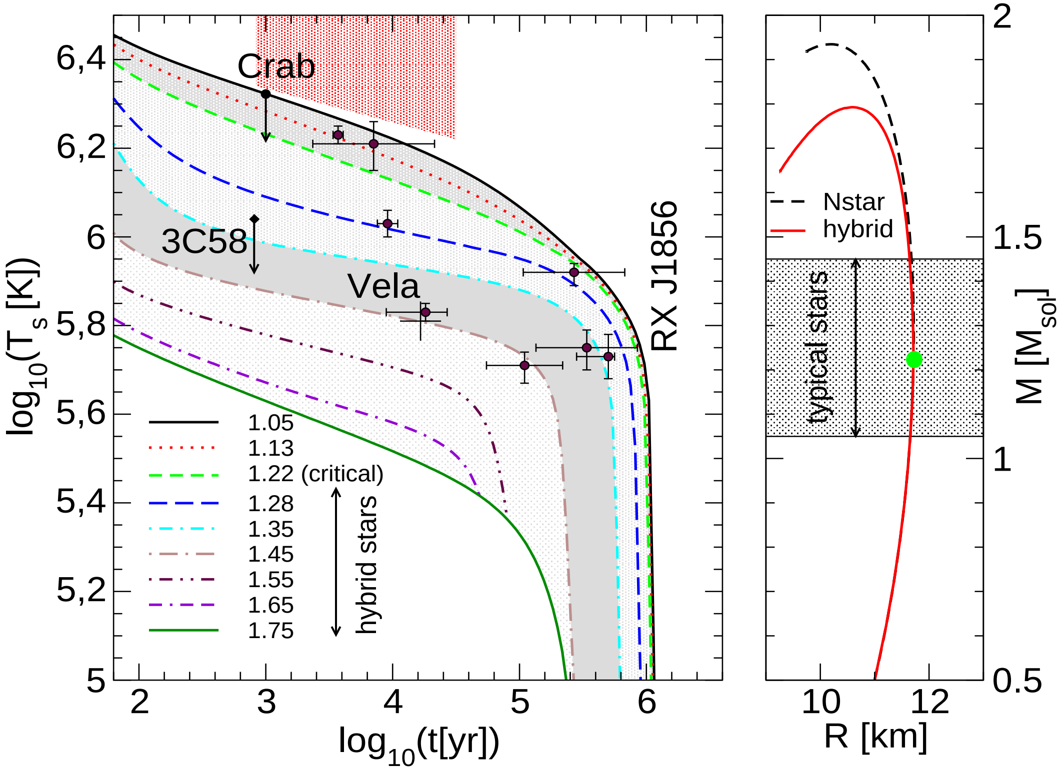


- Low-mass X-ray binary 4U 0614: Mass constraint from ISCO obs.



Quark matter in compact stars: Cooling constraint

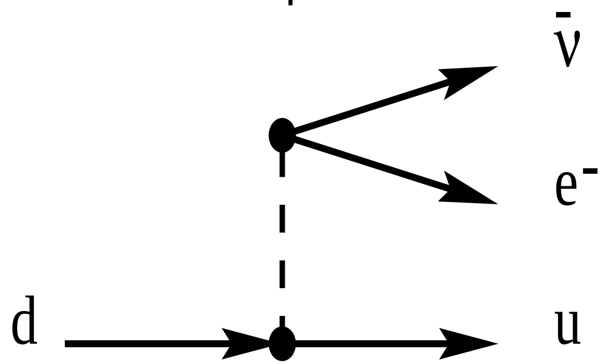
Quark matter in compact stars: color superconducting



- Neutrinos carry energy off the star, Cooling evolution (schematic) by

$$\frac{dT(t)}{dt} = -\frac{\epsilon_\gamma + \sum_{j=Urca,\dots} \epsilon_\nu^j}{\sum_{i=q,e,\gamma,\dots} c_V^i}$$

- Most efficient process: Urca



- Exponential suppression by pairing gaps! $\Delta \sim 10 \dots 100$ keV

Popov et al: Neutron star cooling constraints ...
PRC 74, 025803 (2006); [nucl-th/0512098]

$\Longrightarrow$ Popov (Ast 6, 8)
$\Longrightarrow$ Grigorian (Ast 9)

SUMMARY

- Mott-Hagedorn model as alternative interpretation of Lattice data
- Microscopic formulation of the hadronic Mott effect within a chiral quark model
- Mesonic (hadronic) correlations important for $T > T_c$
- Step-like enhancement of threshold processes due to Mott effect
- Reaction kinetics for strong correlations in plasmas applicable @ SPS and RHIC
- Prospects for LHC: Plasma diagnostics with bottomonium

LECTURE II: BCS/BEC CROSSOVER IN QUARK MATTER

- Nozieres–Schmitt-Rink theory for relativistic fermion system
- Thermodynamics of the BEC/BCS crossover
- Pair fluctuation transport: Gross-Pitaevskii equation and shear viscosity
- Density dependence of T_c and role of quantum fluctuations