# Neutrino emissivities in color-superconducting quark matter

Jens Berdermann (University Rostock)

DIAS-TH: Dubna International Advanced School of Theoretical Physics and Helmholtz International Summer School

*"Dense Matter in HIC and Astrophysics"*

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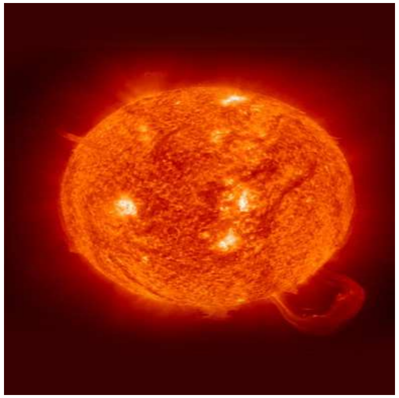
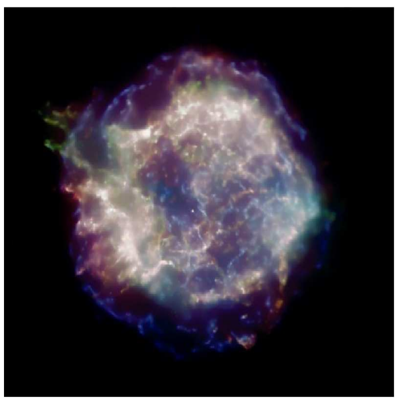
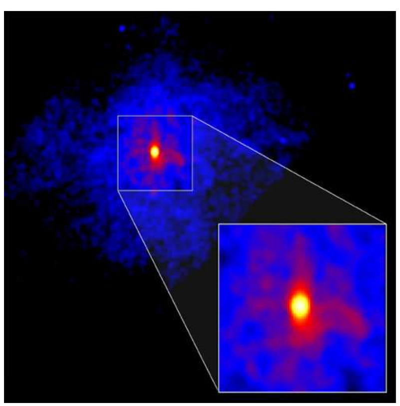
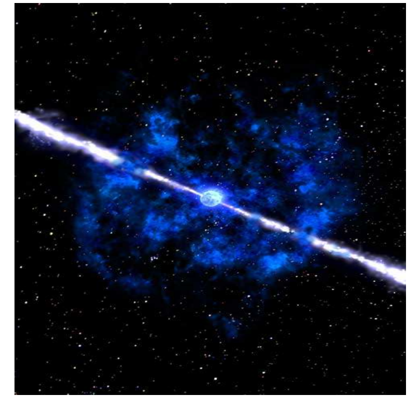
# Content

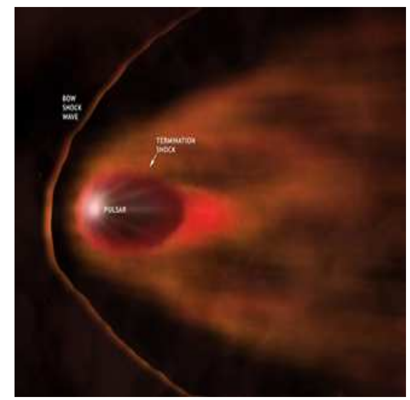


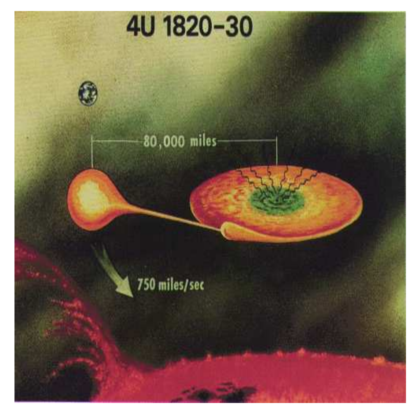




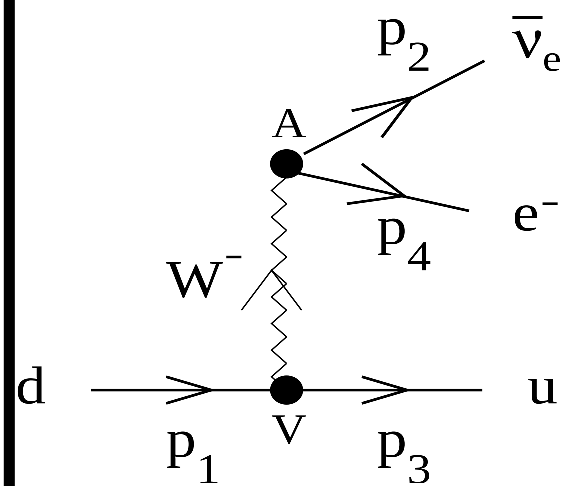
- Introduction
- Urca process Iwamoto's formalism
- Limitations
- Nambu-Gorkov formalism
- First results
- Summary/Outlook



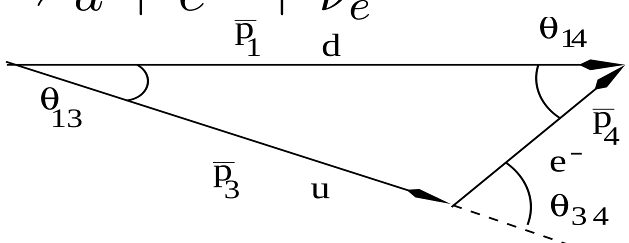




# URCA-Process (Iwamoto Ann.o.Phys 1[1982])




$$d \longrightarrow u + e^- + \bar{\nu}_e$$




$$
\begin{aligned}
\varepsilon &= 6 \prod_{i=1}^{4} \int \frac{d^3\, p_i}{(2\pi)^3\, 2E_i}\, E_2\, W_{fi}\, n(\mathbf{p_1})[1 - n(\mathbf{p_3})][1 - n(\mathbf{p_4})] \\
W_{fi} &= (2\pi)^4 \delta^{(4)}(p_1 - p_2 - p_3 - p_4) \mid M \mid^2 \\
\mid M \mid^2 &\equiv \frac{1}{2} \sum_{\sigma_d, \sigma_u, \sigma_{e^-}} \mid M_{fi} \mid^2 = 64\, G^2 \cos^2\theta_c\, (p_1 \cdot p_2)\, (p_3 \cdot p_4) \\
&= 64\, G^2 \cos^2\theta_c\, E_1 E_2 E_3 E_4 (1 - \cos\theta_{34})
\end{aligned}
$$

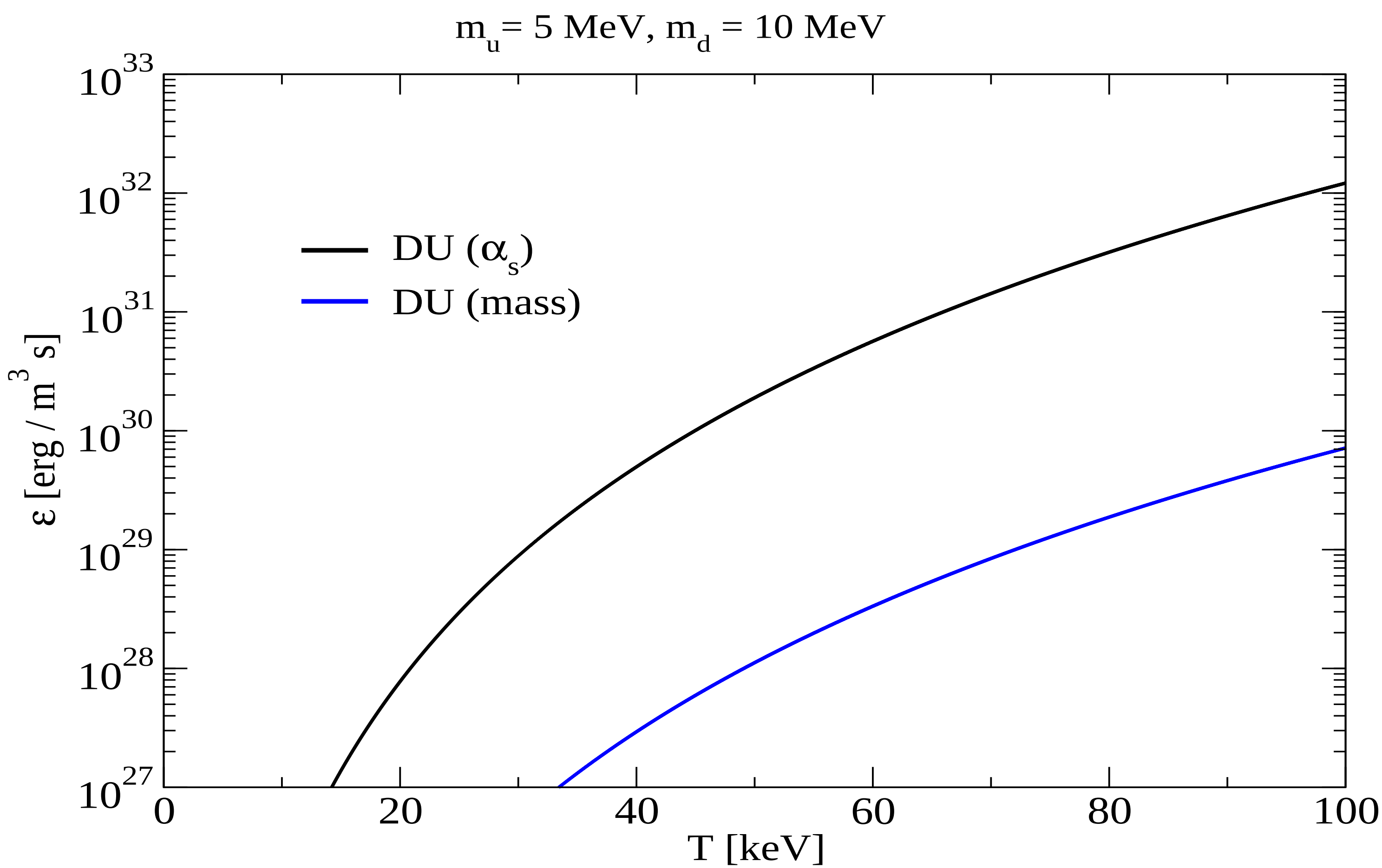
$$
\begin{aligned}
\mu_i &= p_F^i [1 + (2/3\pi)\alpha_s], \qquad i = u, d \\
\mu_i &\simeq p_F^i [1 + 0.5(m_i/p_F^i)^2], \qquad i = u, d, e
\end{aligned}
$$

$$
\begin{aligned}
p_F^d - p_F^u - p_F^e &\simeq -\frac{1}{2} p_F^e \theta_{14}^2, \qquad \theta_{14} \simeq \theta_{34} \\
p_F^d - p_F^u - p_F^e &\simeq -p_F^e (2/3\pi)\alpha_s, \qquad \alpha_s = g^2/4 \\
p_F^d - p_F^u - p_F^e &\simeq -\frac{1}{2} p_F^e \left[ \left(\frac{p_F^d}{p_F^e}\right) \left(\frac{m_d}{p_F^d}\right)^2 - \left(\frac{p_F^u}{p_F^e}\right) \left(\frac{m_u}{p_F^u}\right)^2 - \left(\frac{m_e}{p_F^e}\right)^2 \right]
\end{aligned}
$$

# Emissivities (perturbative)

$$\varepsilon^{\alpha_s} \simeq (457/630)\, G^2 \cos^2\theta_c\, \alpha_s\, p_F^d\, p_F^u\, p_F^e\, T^6$$

$$\varepsilon^{m} \simeq (457\pi/1680)\, G^2 \cos^2\theta_c\, m_d^2\, f\, p_F^u\, T^6, f \equiv 1 - (m_u/m_d)^2(p_F^d/p_F^u) - (m_e/m_d)^2(p_F^d/p_F^e)$$



# QCD phase diagram

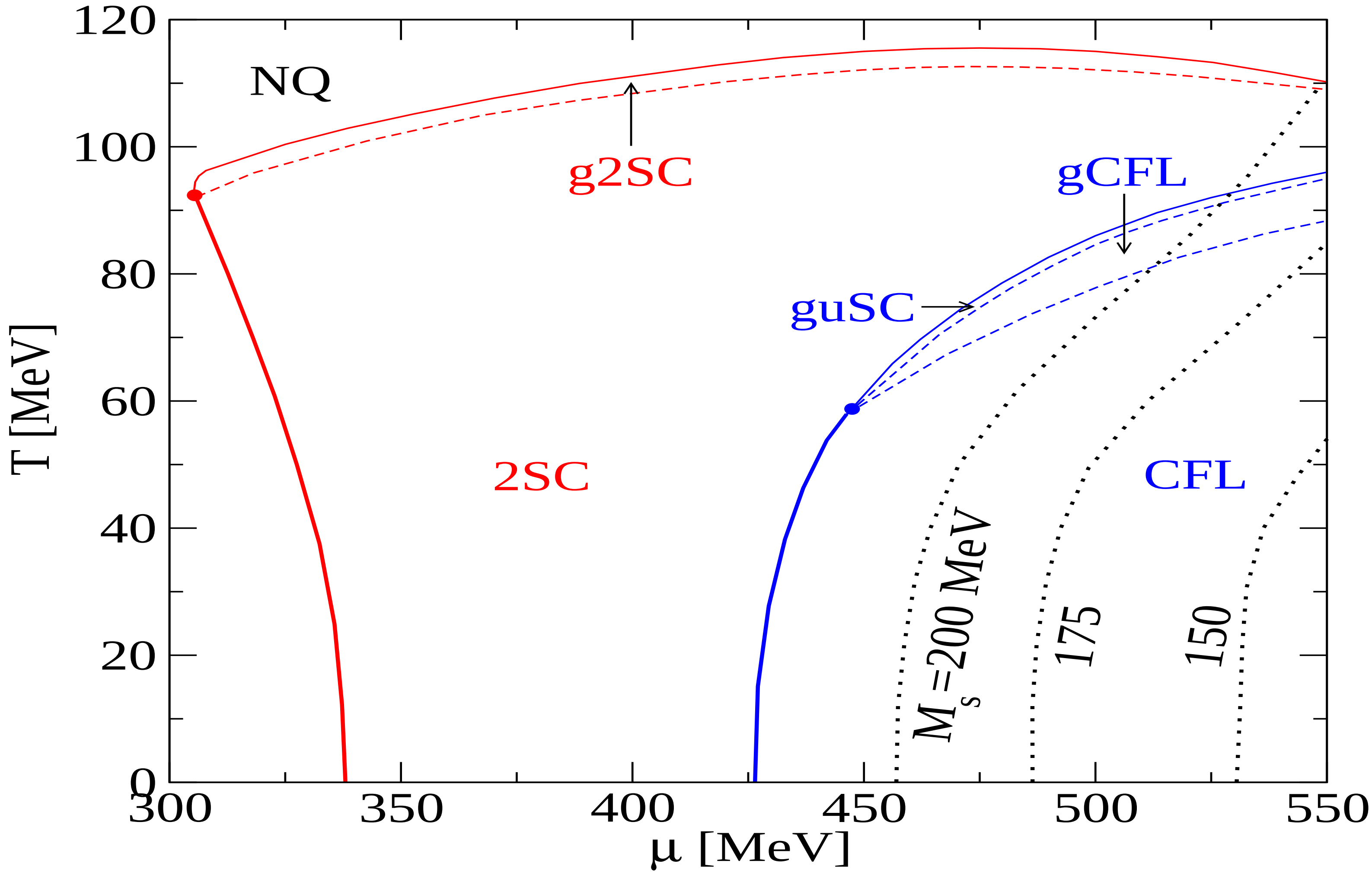



Figure F.Sandin [Blaschke et. al. Phys.Rev D **72**,065020 (2005)]

# Quark mass and diquark gap

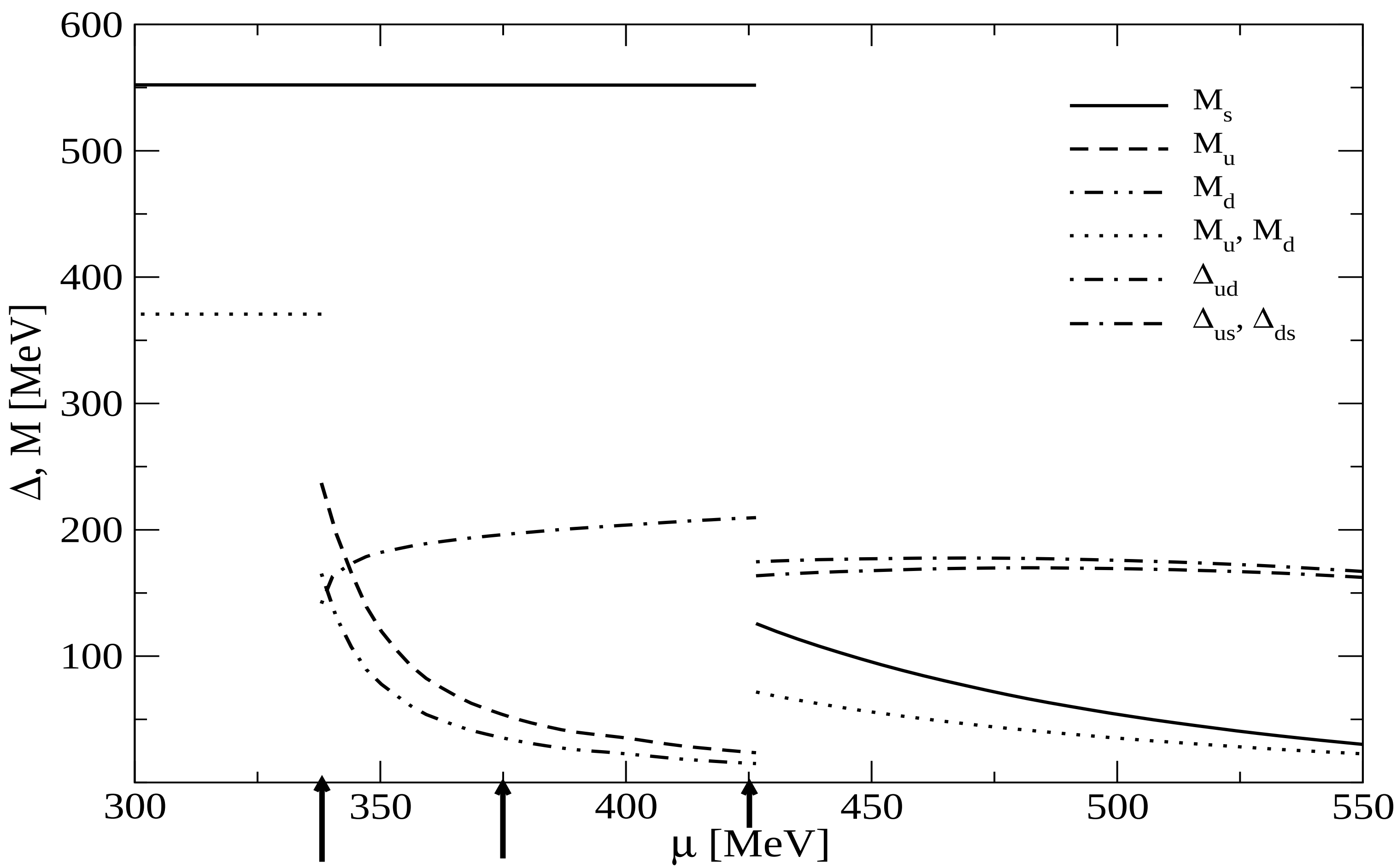
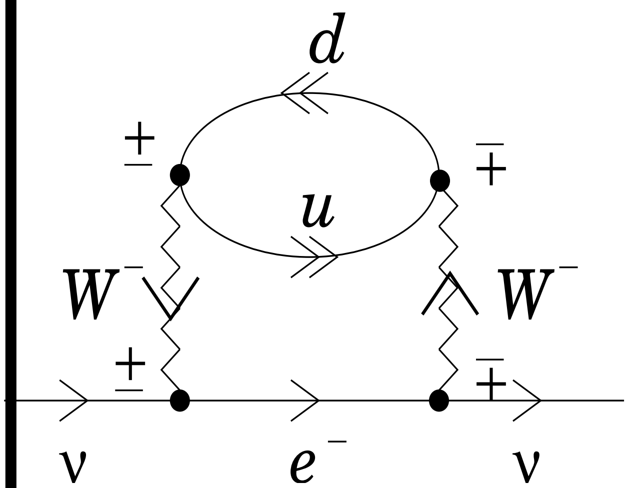



Figure F.Sandin [Blaschke et. al. Phys.Rev D **72**,065020 (2005)]

# Kinetic equation

$$i\partial_x^\mu \mathrm{Tr}[\gamma_\mu G_\nu^<(X,q_2)] = -\mathrm{Tr}[G_\nu^>(X,q_2)\Sigma_\nu^<(X,q_2) - \Sigma_\nu^>(X,q_2)G_\nu^<(X,q_2)], \quad X=(t,\mathbf{x})$$




$$\Sigma_\nu^<(t,q_2) = \frac{G_F^2}{2}\int \frac{d^4\ q_1}{(2\pi^4)}\gamma^\mu(1-\gamma_5)(\gamma^\alpha q_{1,\alpha}+\mu_e\gamma_0)\gamma^\nu(1-\gamma_5)$$
$$\times\ \Pi_{\mu\nu}^>(q_1-q_2)\frac{\pi}{q_1}f_e(t,\mathbf{q_1})\delta(q_1^0+\mu_e-|\mathbf{q}_1|),$$

$$\Sigma_\nu^>(t,q_2) = \frac{G_F^2}{2}\int \frac{d^4\ q_1}{(2\pi^4)}\gamma^\mu(1-\gamma_5)(\gamma^\alpha q_{1,\alpha}+\mu_e\gamma_0)\gamma^\nu(1-\gamma_5)$$
$$\times\ \Pi_{\mu\nu}^<(q_1-q_2)\frac{\pi}{q_1}[1-f_e(t,\mathbf{q_1})]\delta(q_1^0+\mu_e-|\mathbf{q}_1$$

$$iG_\nu^<(t,q_2) = -(\gamma^\beta q_{2,\beta}+\mu_\nu\gamma_0)\frac{\pi}{q_2}\{f_\nu(t,\mathbf{q_2})\delta(p_2^0+\mu_\nu-|\mathbf{q_2}|)$$
$$-[1-f_{\bar\nu}(t,-\mathbf{q_2})]\delta(q_2^0+\mu_\nu+|\mathbf{q_2}|)\}$$

$$iG_\nu^>(t,q_2) = (\gamma^\beta q_{2,\beta}+\mu_\nu\gamma_0)\frac{\pi}{q_2}\{[1-f_\nu(t,\mathbf{q_2})]\delta(q_2^0+\mu_\nu-|\mathbf{q_2}|)$$
$$-f_{\bar\nu}(t,-\mathbf{q_2})\delta(q_2^0+\mu_\nu+|\mathbf{q_2}|)\}$$

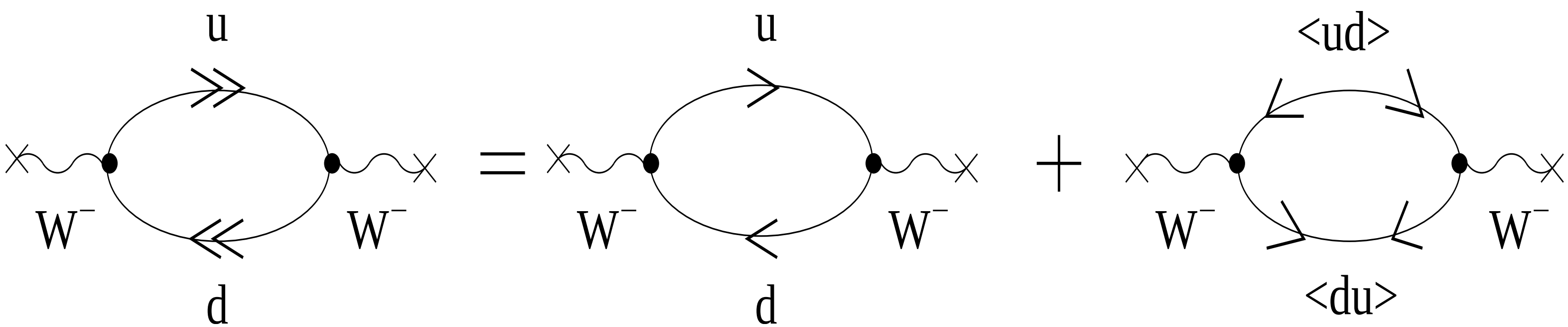
# Kinetic equation

$$
\begin{aligned}
\frac{\partial}{\partial t} f_\nu(t,\mathbf{q_2}) &= -i\frac{G_F^2}{16}\int \frac{d^3\mathbf{q_1}}{(2\pi)^3|\mathbf{q_1}||\mathbf{q_2}|}\mathcal{L}^{\mu\nu}(q_1,q_2)\{[1-f_\nu(t,\mathbf{q_2})]f_e(t,\mathbf{q_1})\Pi^{>}_{\mu\nu}(q) \\
&\quad - f_\nu(t,\mathbf{q_2})[1-f_e(t,\mathbf{q_1})]\Pi^{<}_{\mu\nu}(q)\} \\
&= \frac{G_F^2}{8}\int \frac{d^3\mathbf{q_1}}{(2\pi)^3|\mathbf{q_1}||\mathbf{q_2}|}\mathcal{L}^{\mu\nu}(q_1,q_2)\, n_F(|\mathbf{q_1}|-\mu_e)n_B(|\mathbf{q_2}|+\mu_e-|\mathbf{q_1}|)\mathrm{Im}\Pi^R_{\mu\nu}(
\end{aligned}
$$

$$
\begin{aligned}
\Pi^{>}(q) &= -2i[1+n_B(q_0)]\mathrm{Im}\Pi_R(q) \\
\Pi^{<}(q) &= -2i\, n_B(q_0)\mathrm{Im}\Pi_R(q) \\
n_B(\omega) &\equiv 1/(e^{\omega/T}-1)\ \ \text{(Bose)} \\
n_F(\omega) &\equiv 1/(e^{\omega/T}+1)\ \ \text{(Fermi)}
\end{aligned}
$$

$$
\mathcal{L}^{\mu\nu}(q_1,q_2) = \mathrm{Tr}[\gamma^\mu(1-\gamma_5)\,\not{q}_1\gamma^\nu(1-\gamma_5)\,\not{q}_2] = 8[q_1^\mu q_2^\nu - g^{\mu\nu}(q_1\cdot q_2) + q_1^\nu q_2^\mu - i\epsilon^{\mu\alpha\nu\beta}q_{1\alpha}q_{2\beta}]
$$

# Hadronic loop




$$\Pi_{\mu\nu}(q) = -\frac{i}{2}\int \frac{d^4p}{(2\pi)^4}\mathrm{Tr}_{\mathrm{Z,D}}\,[\Gamma^Z_\mu\; S_p\;\Gamma^Z_\nu\; S_{p+q}], \qquad u,d \to p, p+q \to \hat{p}, \hat{k}$$

$$\Gamma^Z_i = \begin{pmatrix} \Gamma^-_i & 0 \\ 0 & \Gamma^+_i \end{pmatrix} \qquad \Gamma^\pm_i = \gamma_i(1 \pm g_A\gamma_5) \quad i = \mu, \nu \;;\; S_j = \begin{pmatrix} G^+_j & F^-_j \\ F^+_j & G^-_j \end{pmatrix} \qquad j = p, p+q.$$

$$S^{-1}S = \mathbf{1} \quad \Rightarrow \quad \begin{pmatrix} [S^+_0]^{-1} & \Delta^- \\ \Delta^+ & [S^-_0]^{-1} \end{pmatrix}\begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

# Nambu-Gorkov propagators

$$\begin{aligned} \mathrm{I}: \quad [S_0^+]^{-1}\, A + \Delta^-\, C &= 1 \\ \mathrm{II}: \quad [S_0^+]^{-1}\, B + \Delta^-\, D &= 0 \\ \mathrm{III}: \quad \Delta^+\, A + [S_0^-]^{-1}\, C &= 0 \\ \mathrm{IV}: \quad \Delta^+\, B + [S_0^-]^{-1}\, D &= 1 \end{aligned} \quad \Longrightarrow \quad \begin{aligned} A &= [(S_0^+)^{-1} - \Sigma^+]^{-1} = G^+ \\ B &= -S_0^+ \Delta^- G^- = F^- \\ C &= -S_0^- \Delta^+ G^+ = F^+ \\ D &= [(S_0^-)^{-1} - \Sigma^-]^{-1} = G^- \end{aligned}$$

$$\begin{aligned} S_0^\pm(p_0, \mathbf{p}) &= \frac{\gamma_0 \tilde{\Lambda}_p^-}{p_0 - E_p^\mp} + \frac{\gamma_0 \tilde{\Lambda}_p^+}{p_0 + E_p^\pm} \\ \left[S_0^\pm(p_0, \mathbf{p})\right]^{-1} &= \gamma_0(p_0 \pm \mu) - \gamma \mathbf{p} - m = \gamma_0(p_0 - E_p^\mp)\Lambda_p^+ + \gamma_0(p_0 + E_p^\pm)\Lambda_p^- \end{aligned}$$

- $E_p^\mp = E_p \mp \mu$ (particle/hole), $E_p^\pm = E_p \pm \mu$ (antiparticle/antihole), $E_p = \sqrt{\mathbf{p}^2 + m^2}$
- $\Delta^- = -i\Delta\varepsilon^{ik}\epsilon^{\alpha\beta b}\gamma_5$,$\Delta^+ = \gamma_0(\Delta^-)^\dagger\gamma_0$ (diquark condensat)
- $\Sigma^\pm = \Delta^\mp S_0^\mp \Delta^\pm$

# Nambu-Gorkov propagators

$$G^{\pm} = [(S_0^{\pm})^{-1} - \Delta^{\mp} S_0^{\mp} \Delta^{\pm}]^{-1} = \frac{p_0 + E_p^{\mp}}{p_0^2 - (\xi_p^{\mp})^2} \gamma_0 \tilde{\Lambda}_p^{-} + \frac{p_0 - E_p^{\pm}}{p_0^2 - (\xi_p^{\pm})^2} \gamma_0 \tilde{\Lambda}_p^{+}$$

$$F^{\pm} = -S_0^{\mp} \Delta^{\pm} G^{\pm} = \frac{\Delta^{\pm}}{p_0^2 - (\xi_p^{\pm})^2} \tilde{\Lambda}_p^{+} + \frac{\Delta^{\pm}}{p_0^2 - (\xi_p^{\mp})^2} \tilde{\Lambda}_p^{-}$$

- Pole $p_0 = \pm \xi_p^{-}$ und $p_0 = \mp \xi_p^{+}$ mit $(\xi_p^{\pm})^2 = (E_p^{\pm})^2 + \Delta^2$ (Quasiparticle/Quasihole- and Quasiantiparticle/Quasiantihole excitation energies)
- Energy projectors

$$\left.\begin{aligned} \Lambda_p^{\pm} &= \tfrac{1}{2}(1 \pm \gamma_0\, \mathcal{S}_p^{+}) \\ \tilde{\Lambda}_p^{\pm} &= \tfrac{1}{2}(1 \pm \gamma_0\, \mathcal{S}_p^{-}) \end{aligned}\right\} \quad \mathcal{S}_p^{\pm} = \vec{\gamma}\, \hat{p} \pm \hat{m}, \quad \hat{p} = \frac{\mathbf{p}}{E_p} \quad \hat{m} = \frac{m}{E_p}$$

# Polarisation tensor

$$\Pi_{\mu\nu}(q) = -i\frac{T}{2}\sum_n \int \frac{d^3p}{(2\pi)^3}\mathrm{Tr}_{\mathrm{Z,D}}\,[\Gamma^Z_\mu\; S_p\; \Gamma^Z_\nu\; S_{p+q}]$$

$$\mathrm{Tr}_{\mathrm{D}}[\Gamma^-_\mu G^+_p \Gamma^-_\nu G^+_{p+q} + \Gamma^+_\mu G^-_p \Gamma^+_\nu G^-_{p+q} + \Gamma^-_\mu F^-_p \Gamma^+_\nu F^+_{p+q} + \Gamma^+_\mu F^+_p \Gamma^-_\nu F^-_{p+q}] \quad =$$

$$\frac{(p_0+E^-_p)(p_0+q_0+E^-_k)}{[p_0^2-(\xi^-_p)^2][(p_0+q_0)^2-(\xi^-_k)^2]}\{\mathcal{T}^+_{\mu\nu}(\hat p,\hat k) + g_A^2\widetilde{\mathcal{T}}^+_{\mu\nu}(\hat p,\hat k) - g_A[\widetilde{\mathcal{W}}^+_{\mu\nu}(\hat p,\hat k) + \mathcal{W}^+_{\mu\nu}(\hat p,\hat k)]\} \quad +$$

$$\frac{(p_0-E^-_p)(p_0+q_0-E^-_k)}{[p_0^2-(\xi^-_p)^2][(p_0+q_0)^2-(\xi^-_k)^2]}\{\mathcal{T}^-_{\mu\nu}(\hat p,\hat k) + g_A^2\widetilde{\mathcal{T}}^-_{\mu\nu}(\hat p,\hat k) + g_A[\widetilde{\mathcal{W}}^-_{\mu\nu}(\hat p,\hat k) + \mathcal{W}^-_{\mu\nu}(\hat p,\hat k)]\} \quad -$$

$$\frac{\Delta^2}{[p_0^2-(\xi^-_p)^2][(p_0+q_0)^2-(\xi^-_k)^2]}\{[\mathcal{T}^-_{\mu\nu}(\hat p,\hat k) + \mathcal{T}^+_{\mu\nu}(\hat p,\hat k)] + g_A^2[\widetilde{\mathcal{T}}^-_{\mu\nu}(\hat p,\hat k) + \widetilde{\mathcal{T}}^+_{\mu\nu}(\hat p,\hat k)] \quad -$$

$$g_A[\widetilde{\mathcal{W}}^+_{\mu\nu}(\hat p,\hat k) + \mathcal{W}^+_{\mu\nu}(\hat p,\hat k) - \widetilde{\mathcal{W}}^-_{\mu\nu}(\hat p,\hat k) - \mathcal{W}^-_{\mu\nu}(\hat p,\hat k)]\}$$

- $p_0 = i(2n+1)\pi T$, $q_0 = i2m\pi T$ fermionic and bosonic Matsubara frequencies
- $\widetilde{\mathcal{T}}^\pm_{\mu\nu}(\hat p,\hat k) = \mathrm{Tr}[\gamma_0\gamma_\mu\widetilde{\Lambda}^\pm_p\gamma_0\gamma_\nu\Lambda^\pm_k]$, $\widetilde{\mathcal{W}}^\pm_{\mu\nu}(\hat p,\hat k) = \mathrm{Tr}[\gamma_0\gamma_\mu\widetilde{\Lambda}^\pm_p\gamma_0\gamma_\nu\Lambda^\pm_k\gamma_5]$,
  $\mathcal{T}^\pm_{\mu\nu}(\hat p,\hat k) = \mathrm{Tr}[\gamma_0\gamma_\mu\Lambda^\pm_p\gamma_0\gamma_\nu\Lambda^\pm_k]$, $\mathcal{W}^\pm_{\mu\nu}(\hat p,\hat k) = \mathrm{Tr}[\gamma_0\gamma_\mu\Lambda^\pm_p\gamma_0\gamma_\nu\Lambda^\pm_k\gamma_5]$
  (hadronic tensors)

# Polarisation tensor

$$\Pi_{\mu\nu}(q_0,\mathbf{q}) = -\frac{i}{2}\int\frac{d^3\mathbf{p}}{(2\pi)^3}A^+(E_p,E_k)\{\mathcal{T}^+_{\mu\nu}(\hat{p},\hat{k})+\widetilde{\mathcal{T}}^+_{\mu\nu}(\hat{p},\hat{k})-[\widetilde{\mathcal{W}}^+_{\mu\nu}(\hat{p},\hat{k})+\mathcal{W}^+_{\mu\nu}(\hat{p},\hat{k})]$$
$$+A^-(E_p,E_k)\{\mathcal{T}^-_{\mu\nu}(\hat{p},\hat{k})+\widetilde{\mathcal{T}}^-_{\mu\nu}(\hat{p},\hat{k})+[\widetilde{\mathcal{W}}^-_{\mu\nu}(\hat{p},\hat{k})+\mathcal{W}^-_{\mu\nu}(\hat{p},\hat{k})]\}$$
$$-\Delta^2 B(E_p,E_k)\{\mathcal{T}^-_{\mu\nu}(\hat{p},\hat{k})+\mathcal{T}^+_{\mu\nu}(\hat{p},\hat{k})+\widetilde{\mathcal{T}}^-_{\mu\nu}(\hat{p},\hat{k})+\widetilde{\mathcal{T}}^+_{\mu\nu}(\hat{p},\hat{k})$$
$$-[\widetilde{\mathcal{W}}^+_{\mu\nu}(\hat{p},\hat{k})+\mathcal{W}^+_{\mu\nu}(\hat{p},\hat{k})-\widetilde{\mathcal{W}}^-_{\mu\nu}(\hat{p},\hat{k})-\mathcal{W}^-_{\mu\nu}(\hat{p},\hat{k})]\}$$

$$A^\pm(E_p,E_k) = -\frac{1}{2\xi_p^- 2\xi_k^-}\sum_{s_1 s_2=\pm}\frac{(\xi_p^-+s_1E_p^-)(\xi_k^-+s_2E_k^-)}{q_0\pm s_1\xi_p^-\mp s_2\xi_k^-}\frac{n_F(\pm s_1\xi_p^-)n_F(\mp s_2\xi_k^-)}{n_B(\pm s_1\xi_p^-\mp s_2\xi_k^-)}$$

$$B(E_p,E_k) = -\frac{1}{2\xi_p^- 2\xi_k^-}\sum_{s_1 s_2=\pm}\frac{1}{q_0+s_1\xi_p^--s_2\xi_k^-}\frac{n_F(s_1\xi_p^-)n_F(-s_2\xi_k^-)}{n_B(s_1\xi_p^--s_2\xi_k^-)}$$

# Neutrino emissivities

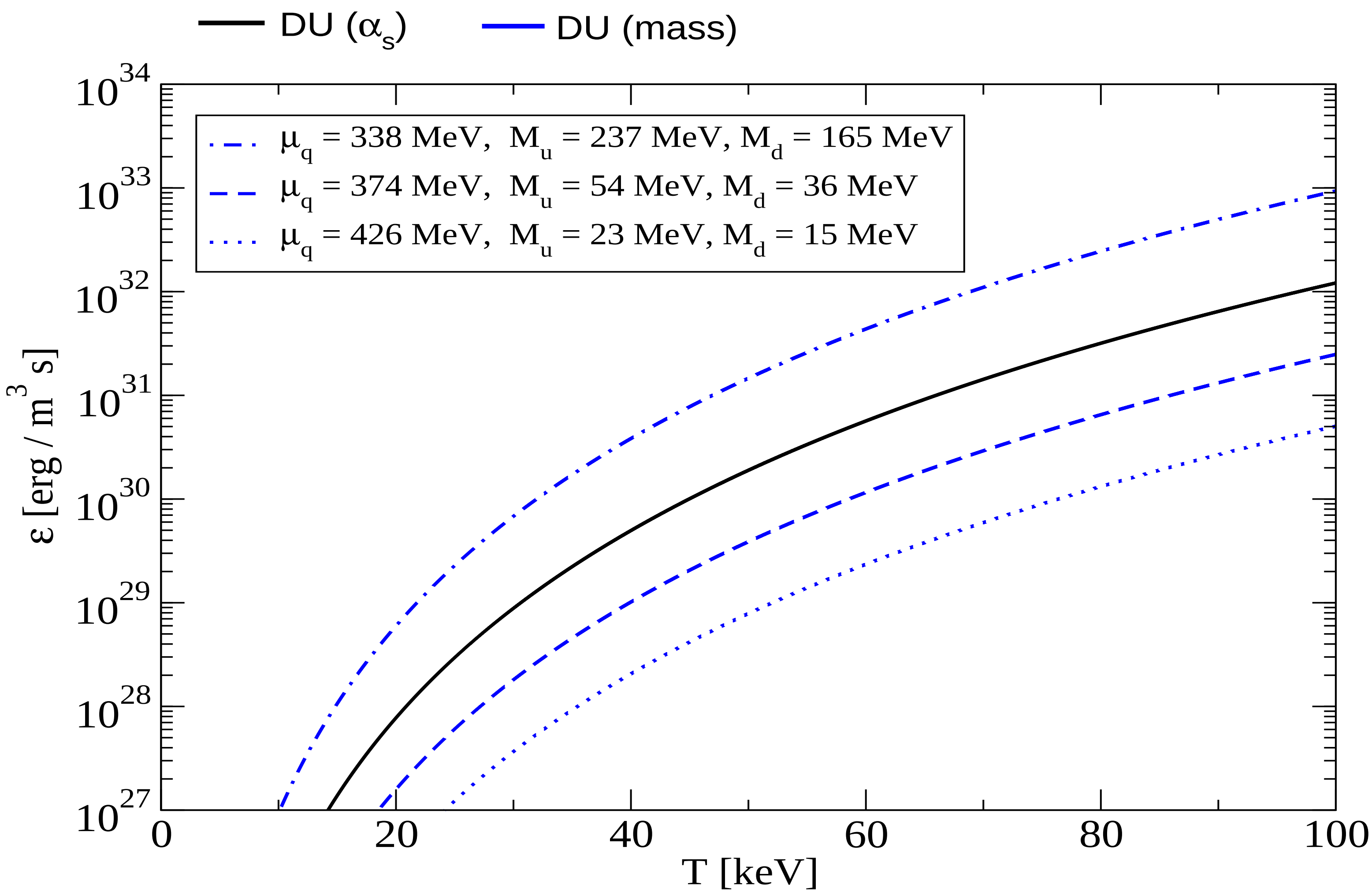
$$\varepsilon_\nu \equiv -\frac{\partial}{\partial t}\int \frac{\mathrm{d}^3\mathbf{q_2}}{(2\pi)^3}|\mathbf{q_2}|[f_\nu(t,\mathbf{q_2}) + f_{\bar\nu}(t,\mathbf{q_2})] = -2\frac{\partial}{\partial t}\int \frac{\mathrm{d}^3\mathbf{q_2}}{(2\pi)^3}\, p_{F,\nu}\, f_\nu(t,\mathbf{q_2})$$

$$\frac{\partial}{\partial t} f_\nu(t,\mathbf{q_2}) = \frac{G_F^2}{8}\int \frac{d^3\mathbf{q_1}}{(2\pi)^3\, p_{F,e}\, p_{F,\nu}}\mathcal{L}^{\mu\nu}(q_1,q_2)\, n_F(p_{F,e}-\mu_e) n_B(p_{F,\nu}+\mu_e-p_{F,e})\mathrm{Im}\Pi^R_{\mu\nu}(q)$$

$$\begin{aligned}\varepsilon_\nu = \frac{\pi}{8}G_F^2\cos^2\theta_c \int \frac{d^3\,\mathbf{q_2}}{(2\pi)^3}\, p_{F,\nu} \sum_{s_1 s_2=\pm}\int \frac{d^3\mathbf{q_1}}{(2\pi)^3\, p_{F,e}\, p_{F,\nu}}\int \frac{d^3\mathbf{p}}{(2\pi)^3}\, n_F(p_{F,e}-\mu_e)\\ \times n_B(p_{F,\nu}+\mu_e-p_{F,e})\Big[2\mathcal{B}_p^{s_1}\mathcal{B}_k^{s_2}\mathcal{L}^{\mu\nu}(q_1,q_2)\,\mathcal{H}^{(n)}_{\mu\nu}(\hat p,\hat k) - \frac{\Delta^2}{2\xi_p^- 2\xi_k^-}\mathcal{L}^{\mu\nu}(q_1,q_2)\,\mathcal{H}^{(a)}_{\mu\nu}(\hat p,\hat k)\\ \times \delta(q_0 + s_1\xi_p^- - s_2\xi_k^-)\frac{n_F(s_1\xi_p^-)n_F(-s_2\xi_k^-)}{n_B(s_1\xi_p^- - s_2\xi_k^-)}.\end{aligned}$$

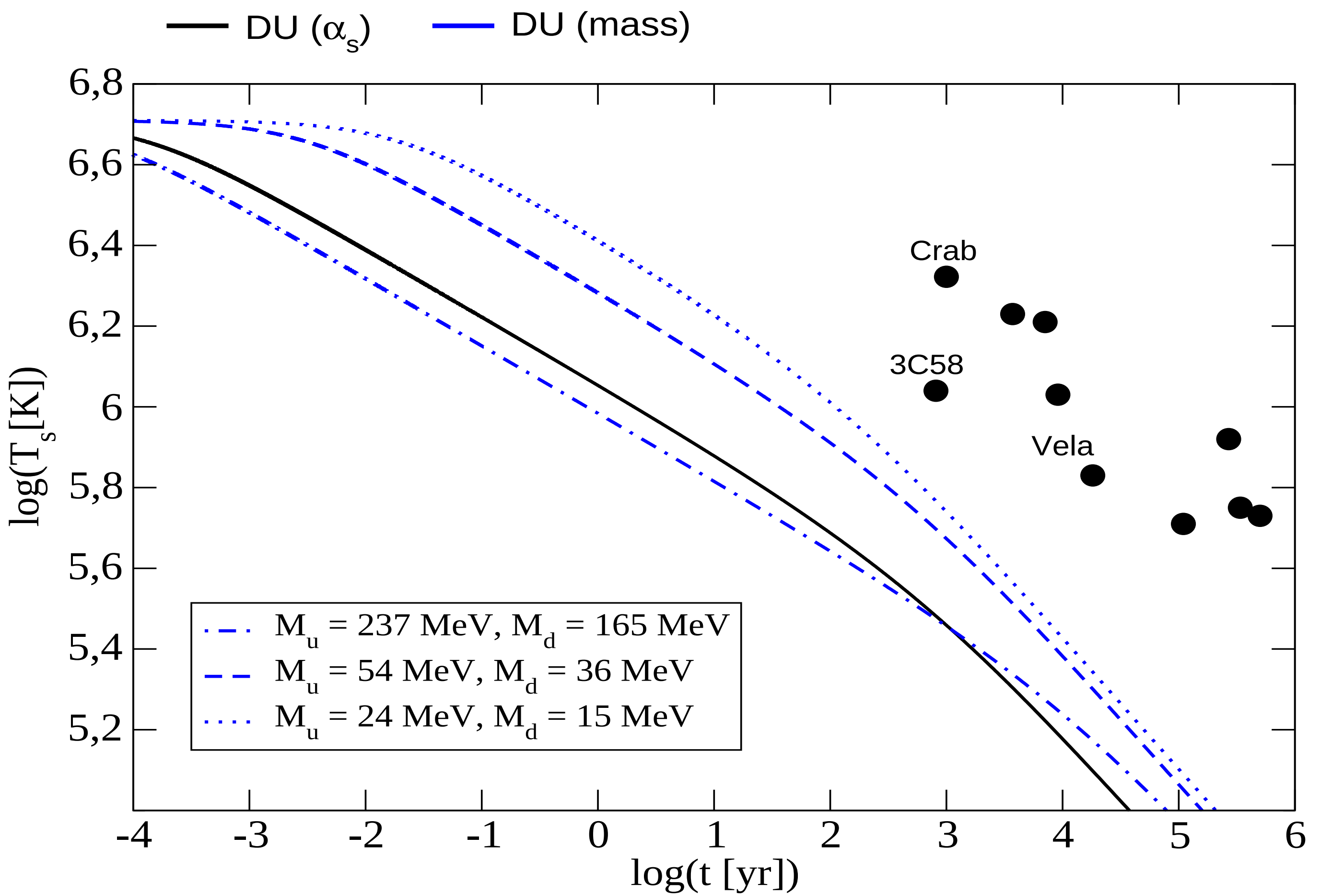
$$\mathcal{L}^{\mu\nu}(q_1,q_2)\mathcal{H}^{(\mathrm{n})}_{\mu\nu}(\hat p,\hat k) = 64 q_1^0 q_2^0(1-\hat q_1\cdot\hat p)(1-\hat q_2\cdot\hat k)$$

# Emissivities (Quark mass effect)

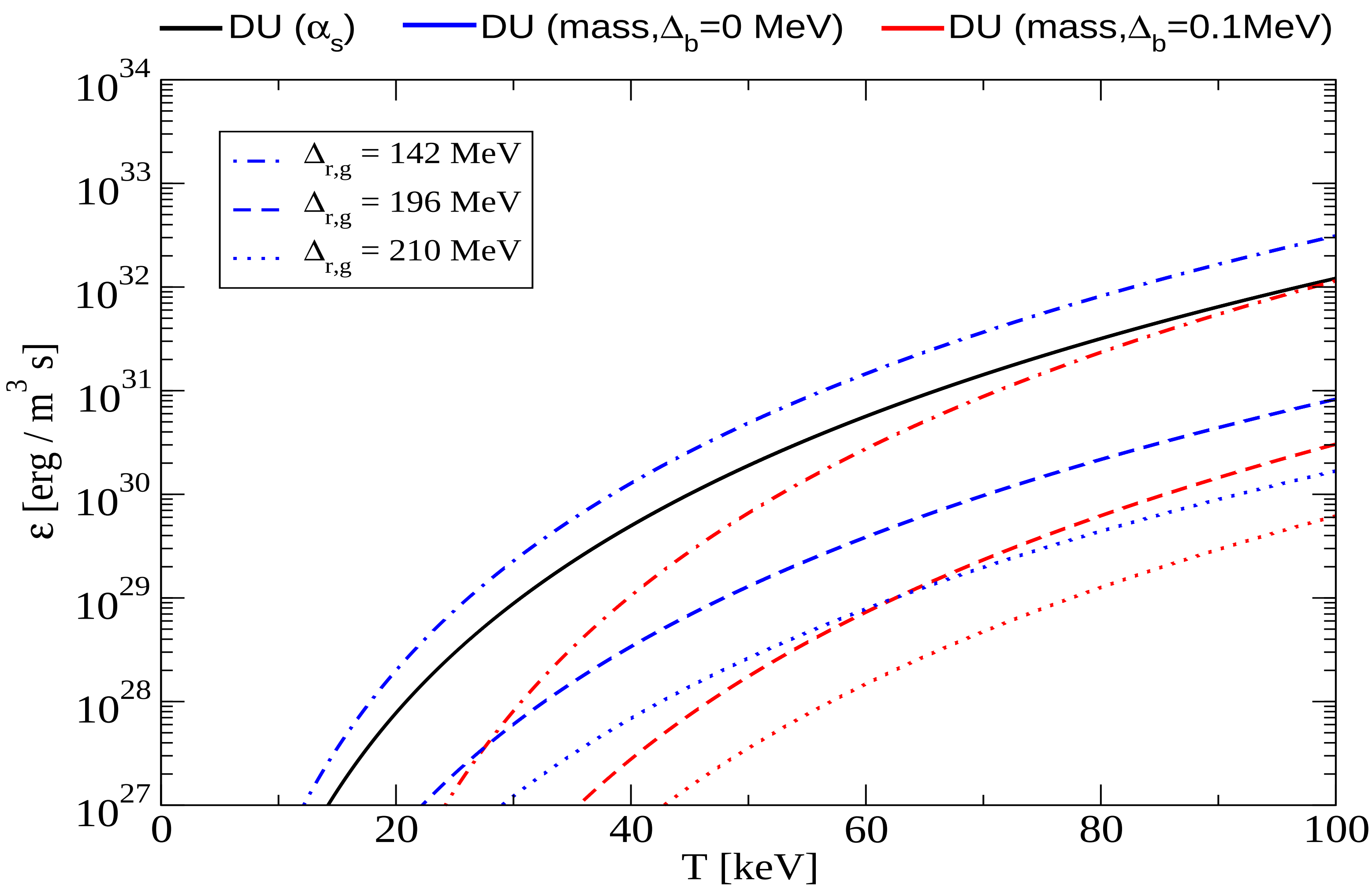


# Cooling

$$t = -\int_{T_i}^{T_f} \sum_{i,j} \frac{C_v^i(T)}{L_j(T)}\, \mathrm{d}T; \qquad i = \mathrm{quark}, e^-, \gamma, \mathrm{gluon}, \nu; \qquad j = \nu, \gamma$$



# Diquark and X-Gap effect



# Conclusion

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- (non-)perturbativ regime
- Mass effekt due to chiral phase transition
- Influence to the cooling of quark stars
- Color-superconductivity (diquark-,X-gap) estimations

# Outlook

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- High temperatures (neutrino trapping, mean free path)
- Cooling equation with transport
- Phenomenology (Pulsar Kicks,Gamma-Ray-Bursts,Supernova)

# leptonic tensors

$$
\begin{aligned}
\mathcal{L}^{00}(q_1, q_2) &= 8(q_1^0 q_2^0 + \mathbf{q_1} \cdot \mathbf{q_2}) \\
\mathcal{L}^{0i}(q_1, q_2) &= 8[q_1^0 q_2^i + q_1^i q_2^0 - i\epsilon^{ijk} q_{1j} q_{2k}] \\
\mathcal{L}^{i0}(q_1, q_2) &= 8[q_1^0 q_2^i + q_1^i q_2^0 + i\epsilon^{ijk} q_{1j} q_{2k}] \\
\mathcal{L}^{ij}(q_1, q_2) &= 8[\delta^{ij}(q_1^0 q_2^0 - \mathbf{q_1} \cdot \mathbf{q_2}) + q_1^i q_2^j + q_1^j q_2^i - i\epsilon^{ijkl} q_{1k} q_{2l}]
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{T}^{\pm}_{00}(\hat{p}, \hat{k}) &= 1 + \hat{p} \cdot \hat{k} + \hat{m}_u \hat{m}_d & \mathcal{W}^{\pm}_{00}(\hat{p}, \hat{k}) &= 0 \\
\mathcal{T}^{\pm}_{0i}(\hat{p}, \hat{k}) &= \pm(\hat{p}_i + \hat{k}_i) & \mathcal{W}^{\pm}_{0i}(\hat{p}, \hat{k}) &= -i\epsilon_{ijk} \hat{p}^j \hat{k}^k \\
\mathcal{T}^{\pm}_{i0}(\hat{p}, \hat{k}) &= \pm(\hat{p}_i + \hat{k}_i) & \mathcal{W}^{\pm}_{i0}(\hat{p}, \hat{k}) &= +i\epsilon_{ijk} \hat{p}^j \hat{k}^k \\
\mathcal{T}^{\pm}_{ij}(\hat{p}, \hat{k}) &= \delta_{ij}(1 - \hat{p} \cdot \hat{k} - \hat{m}_u \hat{m}_d) + \hat{p}_i \hat{k}_j + \hat{k}_i \hat{p}_j & \mathcal{W}^{\pm}_{ij}(\hat{p}, \hat{k}) &= \mp i\epsilon_{ijk}(\hat{p}^k - \hat{k}^k) + i\epsilon_{ijkl} \hat{p}^k \hat{k} \\
\widetilde{\mathcal{T}}^{\pm}_{00}(\hat{p}, \hat{k}) &= 1 + \hat{p} \cdot \hat{k} - \hat{m}_u \hat{m}_d & \widetilde{\mathcal{W}}^{\pm}_{00}(\hat{p}, \hat{k}) &= 0 \\
\widetilde{\mathcal{T}}^{\pm}_{0i}(\hat{p}, \hat{k}) &= \pm(\hat{p}_i + \hat{k}_i) & \widetilde{\mathcal{W}}^{\pm}_{0i}(\hat{p}, \hat{k}) &= -i\epsilon_{ijk} \hat{p}^j \hat{k}^k \\
\widetilde{\mathcal{T}}^{\pm}_{i0}(\hat{p}, \hat{k}) &= \pm(\hat{p}_i + \hat{k}_i) & \widetilde{\mathcal{W}}^{\pm}_{i0}(\hat{p}, \hat{k}) &= +i\epsilon_{ijk} \hat{p}^j \hat{k}^k \\
\widetilde{\mathcal{T}}^{\pm}_{ij}(\hat{p}, \hat{k}) &= \delta_{ij}(1 - \hat{p} \cdot \hat{k} + \hat{m}_u \hat{m}_d) + \hat{p}_i \hat{k}_j + \hat{k}_i \hat{p}_j & \widetilde{\mathcal{W}}^{\pm}_{ij}(\hat{p}, \hat{k}) &= \mp i\epsilon_{ijk}(\hat{p}^k - \hat{k}^k) + i\epsilon_{ijkl} \hat{p}^k \hat{k}
\end{aligned}
$$

# Projection operator

- Main- and transformation properties

$$\begin{aligned} \Lambda_p^{\pm}\Lambda_p^{\pm} &= \Lambda_p^{\pm} \\ \Lambda_p^{\pm}\Lambda_p^{\mp} &= 0 \\ \Lambda_p^{+} + \Lambda_p^{-} &= 1 \end{aligned} \qquad \text{and} \qquad \begin{aligned} \gamma_0\Lambda_p^{\pm}\gamma_0 &= \widetilde{\Lambda}_p^{\mp} \\ \gamma_5\Lambda_p^{\pm}\gamma_5 &= \widetilde{\Lambda}_p^{\pm} \end{aligned}$$

# Polarisation tensor

$$\Pi_{\mu\nu}(q_0,\mathbf{q}) = -\frac{i}{2}\int\frac{d^3\mathbf{p}}{(2\pi)^3}A^+(E_p,E_k)\{\mathcal{T}^+_{\mu\nu}(\hat{p},\hat{k}) + \widetilde{\mathcal{T}}^+_{\mu\nu}(\hat{p},\hat{k}) - [\widetilde{\mathcal{W}}^+_{\mu\nu}(\hat{p},\hat{k}) + \mathcal{W}^+_{\mu\nu}(\hat{p},\hat{k})]\}$$
$$+ A^-(E_p,E_k)\{\mathcal{T}^-_{\mu\nu}(\hat{p},\hat{k}) + \widetilde{\mathcal{T}}^-_{\mu\nu}(\hat{p},\hat{k}) + [\widetilde{\mathcal{W}}^-_{\mu\nu}(\hat{p},\hat{k}) + \mathcal{W}^-_{\mu\nu}(\hat{p},\hat{k})]\}$$
$$- \Delta^2 B(E_p,E_k)\{\mathcal{T}^-_{\mu\nu}(\hat{p},\hat{k}) + \mathcal{T}^+_{\mu\nu}(\hat{p},\hat{k}) + \widetilde{\mathcal{T}}^-_{\mu\nu}(\hat{p},\hat{k}) + \widetilde{\mathcal{T}}^+_{\mu\nu}(\hat{p},\hat{k})$$
$$- [\widetilde{\mathcal{W}}^+_{\mu\nu}(\hat{p},\hat{k}) + \mathcal{W}^+_{\mu\nu}(\hat{p},\hat{k}) - \widetilde{\mathcal{W}}^-_{\mu\nu}(\hat{p},\hat{k}) - \mathcal{W}^-_{\mu\nu}(\hat{p},\hat{k})]\}$$

$$A^\pm(E_p,E_k) = -\frac{1}{2\xi_p^- 2\xi_k^-}\sum_{s_1 s_2=\pm}\frac{(\xi_p^- + s_1E_p^-)(\xi_k^- + s_2E_k^-)}{q_0 \pm s_1\xi_p^- \mp s_2\xi_k^-}\,\frac{n_F(\pm s_1\xi_p^-)n_F(\mp s_2\xi_k^-)}{n_B(\pm s_1\xi_p^- \mp s_2\xi_k^-)}$$

$$B(E_p,E_k) = -\frac{1}{2\xi_p^- 2\xi_k^-}\sum_{s_1 s_2=\pm}\frac{1}{q_0 + s_1\xi_p^- - s_2\xi_k^-}\,\frac{n_F(s_1\xi_p^-)n_F(-s_2\xi_k^-)}{n_B(s_1\xi_p^- - s_2\xi_k^-)}$$

$$\mathrm{Im}\Pi_{\mu\nu}(q_0,\mathbf{q}) = \frac{\pi}{2}\cos^2\theta_c\int\frac{d^3\mathbf{p}}{(2\pi)^3}\left(2\,A^*(E_p,E_k)\mathcal{H}^{(\mathrm{n})}_{\mu\nu} - \Delta^2\,B^*(E_p,E_k)\mathcal{H}^{(\mathrm{a})}_{\mu\nu}\right),$$

$$A^*(E_p,E_k) = -\sum_{s_1 s_2=\pm}\left(\frac{\xi_p^- + s_1E_p^-}{2\xi_p^-}\right)\left(\frac{\xi_k^- + s_2E_k^-}{2\xi_k^-}\right)\delta(q_0 + s_1\xi_p^- - s_2\xi_k^-)\,\frac{n_F(s_1\xi_p^-)n_F(-}{n_B(s_1\xi_p^- - s_2}$$

$$B^*(E_p,E_k) = -\frac{1}{2\xi_p^- 2\xi_k^-}\sum_{s_1 s_2=\pm}\delta(q_0 + s_1\xi_p^- - s_2\xi_k^-)\,\frac{n_F(s_1\xi_p^-)n_F(-s_2\xi_k^-)}{n_B(s_1\xi_p^- - s_2\xi_k^-)}$$