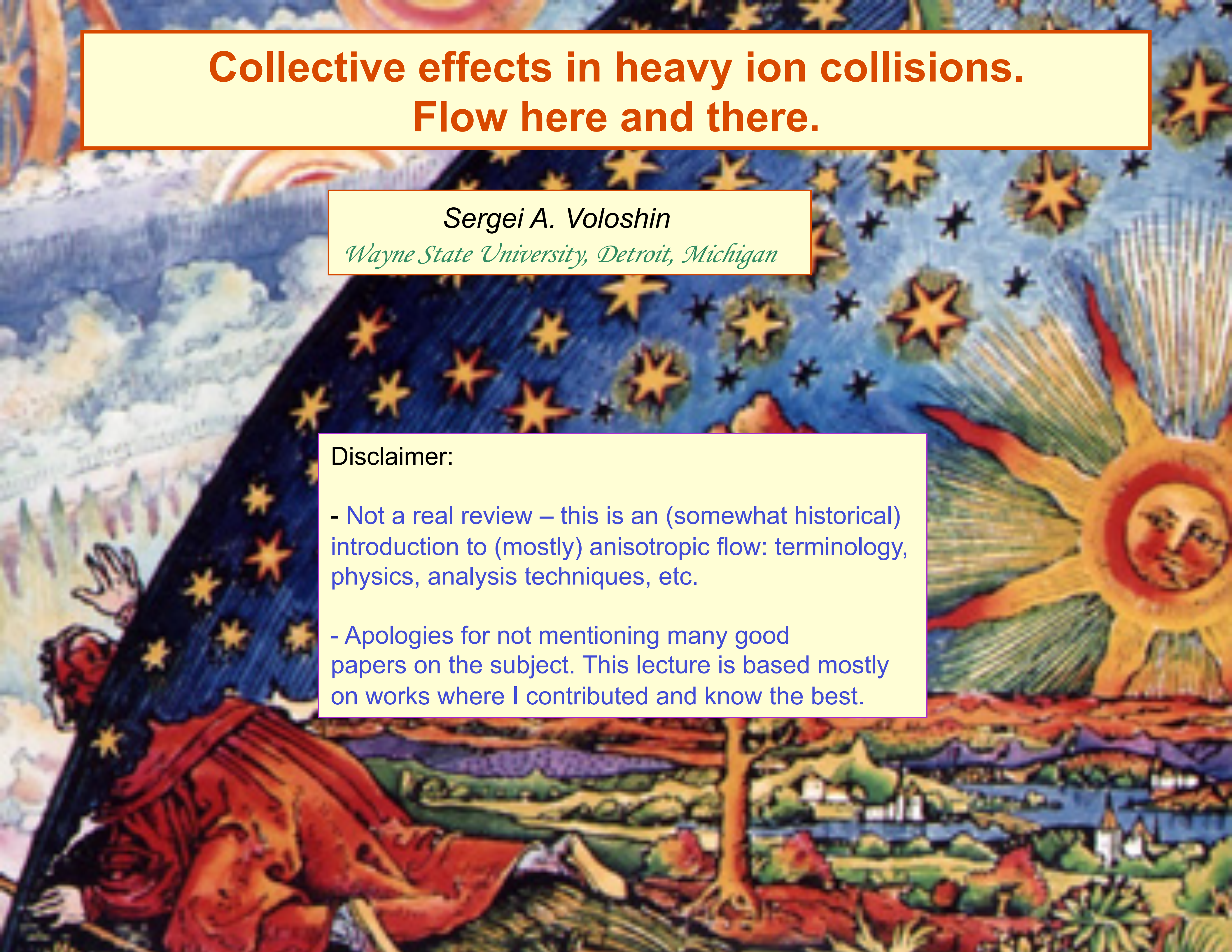


Collective effects in heavy ion collisions. Flow here and there.

Sergei A. Voloshin

Wayne State University, Detroit, Michigan

Disclaimer:

- Not a real review – this is an (somewhat historical) introduction to (mostly) anisotropic flow: terminology, physics, analysis techniques, etc.

- Apologies for not mentioning many good papers on the subject. This lecture is based mostly on works where I contributed and know the best.

Outline

Part I.

1. Introduction. Definitions.
   - Flow and "non-flow", Flow vectors.
     Non-flow estimates
   - General properties. "Early times"

2. v_2/ε plot
   - Low Density and "Hydro" Limits

3. Coalescence and flow.
   - Directed flow of light nuclei
   - Constituent quark number scaling.

4. Interplay with radial flow
   - Directed flow "wiggle"
   - Blast wave. "Mass splitting"
   - "Ridge" formation

5. Eccentricity fluctuations
   - Gaussian model

6. Eccentricity in CGC model.
   Viscous effects

Not here (yet).

1. Elliptic flow at high pt
2. Elliptic flow or rare probes
3. Higher harmonics
4. …

In the news

Universe May Have Begun as Liquid, Not Gas

Associated Press
Tuesday, April 19, 2005; Page A05

The Washington Post

New results from a particle collider suggest that the universe behaved like a liquid in its earliest moments, not the fiery gas that was thought to have per

Early Universe was a liquid

Quark-gluon blob surprises particle physicists.

nature

by Mark Peplow
news@nature.com

The Universe consisted of a perfect liquid in its first m
results from an atom-smashing experiment.

New State of Matter Is 'Nearly Perfect' Liquid

Physicists working at Brookhaven National Laboratory announced today that they have created what appears to be a new state of matter out of the building blocks of atomic nuclei, quarks and gluons. The researchers unveiled their findings--which could provide new insight into the composition of the universe just moments after the big bang--today in Florida at a meeting of the American Physical Society.

SCIENTIFIC AMERICAN

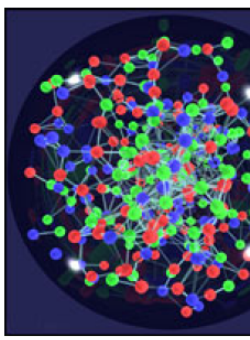
Image: BNL

There are four collaborations, dubbed BRAHMS, PHENIX, PHOBOS and STAR, working at Brookhaven's Relativistic Heavy Ion Collider (RHIC). All of them study what happens when two interacting beams of gold ions smash into one another at great velocities, resulting in thousands of subatomic collisions every second. When the researchers analyzed the
found that the particles produ

Early Universe was 'liquid-like'

Physicists say they have created a new state of hot, dense matter by crashing together the nuclei of gold atoms.

BBC NEWS



The impression is of matter th
more strongly interacting than
pred

The high-energy collisions prised open the nuclei to reveal their most basic particles, known as quarks and gluons.

The researchers, at the US Brookhaven National Laboratory, say these particles were seen to behave as an almost p

社会 asahi.comトップ > 社会 > その他・話題

宇宙の始まりはしずく？　「クオークは液体」と発表

2005年04月18日23時34分

宇宙誕生の大爆発「ビッグバン」直後に相当する超高温・高密度の状態を再現する実験をしてきた日米などの国際チームは１８日、物質を形づくる究極の基本粒子クオークは超高温でバラバラになるが、気体のように自由に跳び回るのでなく、しずくのような液体状

Early Universe Went With the Flow

Posted April 18, 2005 5:57PM

Between 2000 and 2003 the lab's Relativistic Heavy Ion Collider repeatedly smashed the nuclei of gold atoms together with such force that their energy briefly generated trillion-degree temperatures. Physicists think of the collider as a time machine, because those extreme temperature conditions last prevailed in the universe less than 100 millionths of a second after the big bang.

Science — Iran Daily — April 20, 2005

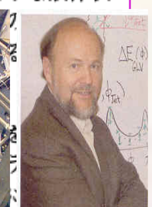
...niverse Liq...

gold atoms together with such force that their energy briefly generated trillion-degree temperatures. Physicists think of the collider as a time machine, because those extreme temperature conditions last prevailed in the universe less than 100 millionths of a second after the big bang.

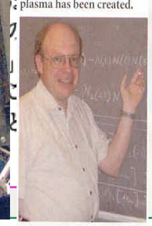
Everything was so hot then that quarks and glu-ons, which are now almost inextricably bound into the proton and neutrons inside atomic nuclei, were thought to have flown around like BBs in a blender.

But by reproducing the conditions of the early universe, RHIC has shown that uncon-strained quarks and glu-ons don't fly away in all

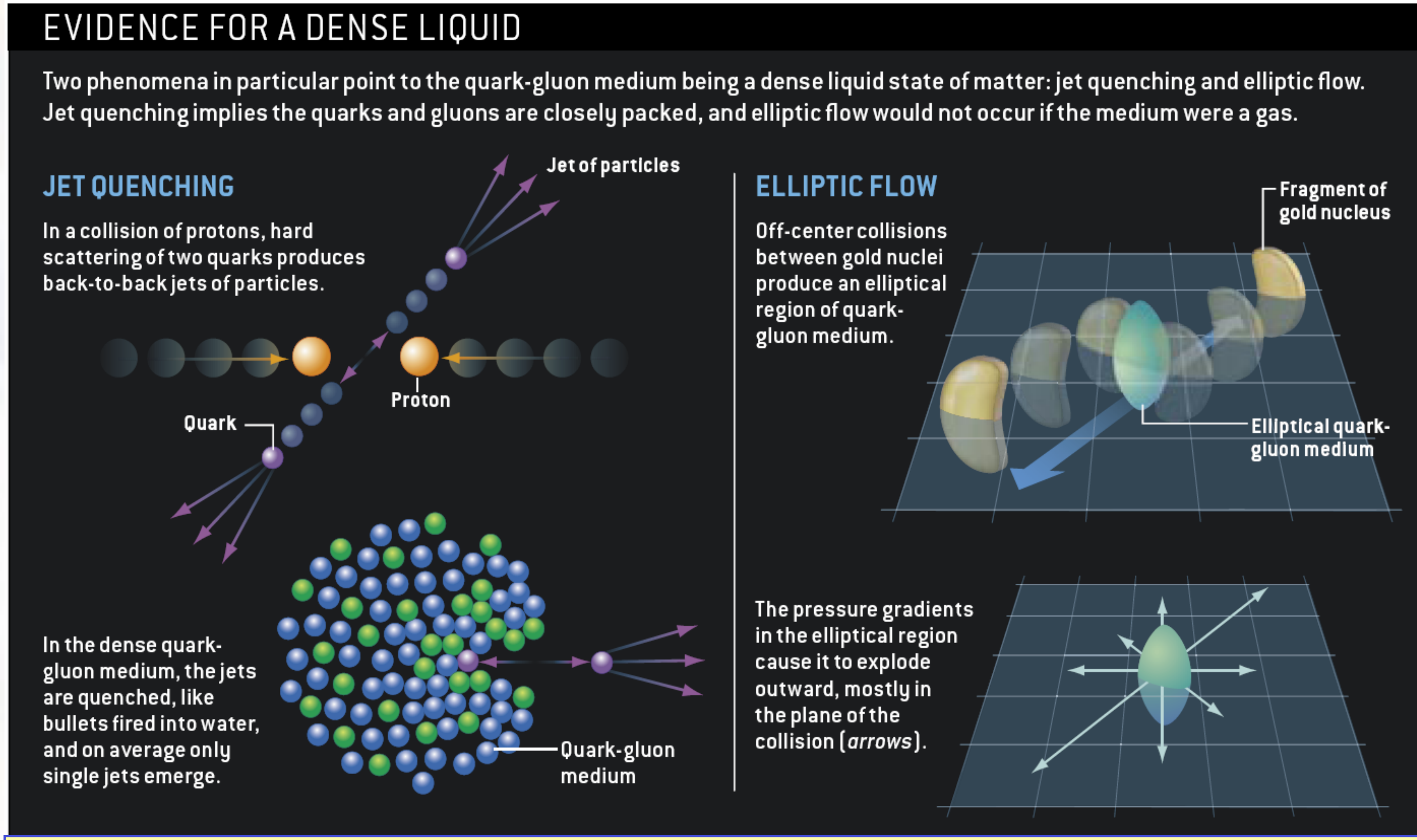
offers opportunities to exciting questions," said smashed the nuclei of ... tion-like property that ... that material swallowed RHIC.



Theorists Miklos Gyulassy (above) and Ulrich Heinz believe that a quark–gluon plasma has been created.



Major RHIC discoveries



"The physical picture emerging from the four (RHIC) experiments is consistent and surprising. The quarks and gluons indeed break out of confinement and behave collectively, if only fleetingly. But this hot mélange acts like a liquid, not the ideal gas theorists had anticipated."

M. Riordan, W. Zajc, Sci. Am., May 2006, 34-41.

Three major RHIC discoveries (my view):

1. Large elliptic flow
2. Jet quenching
3. Constituent quark scaling

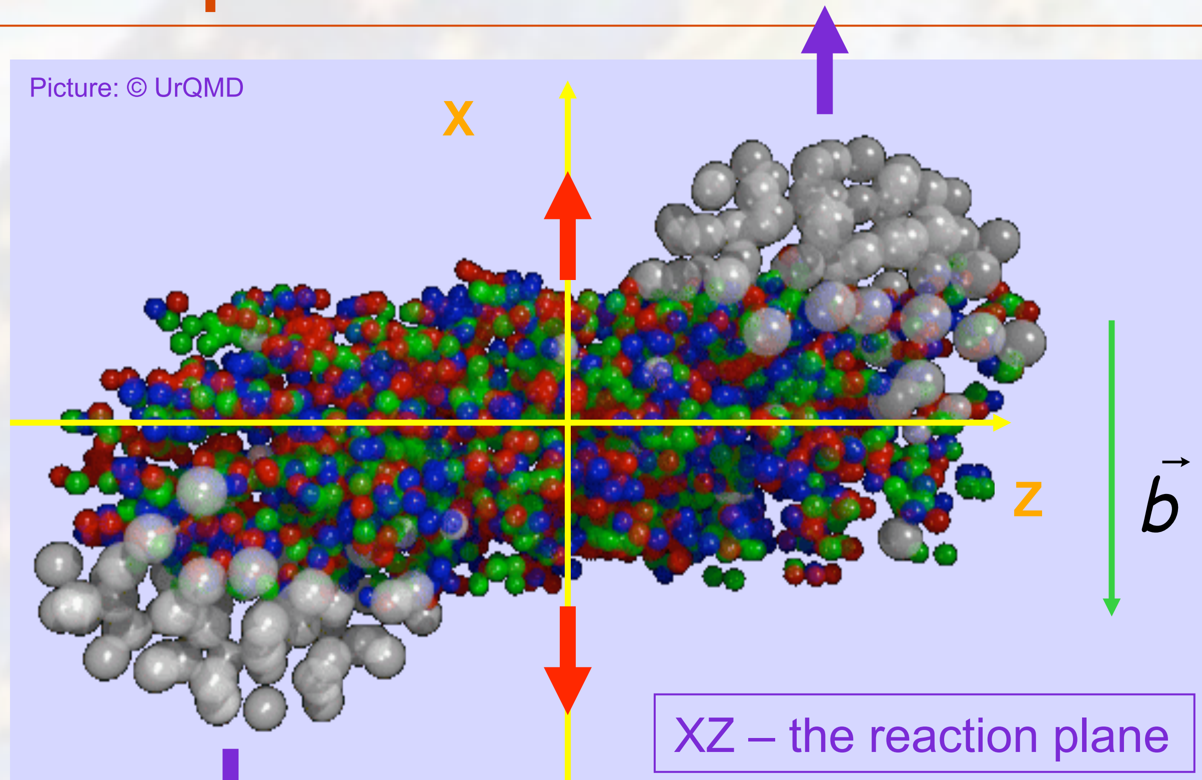
Introduction

Anisotropic flow

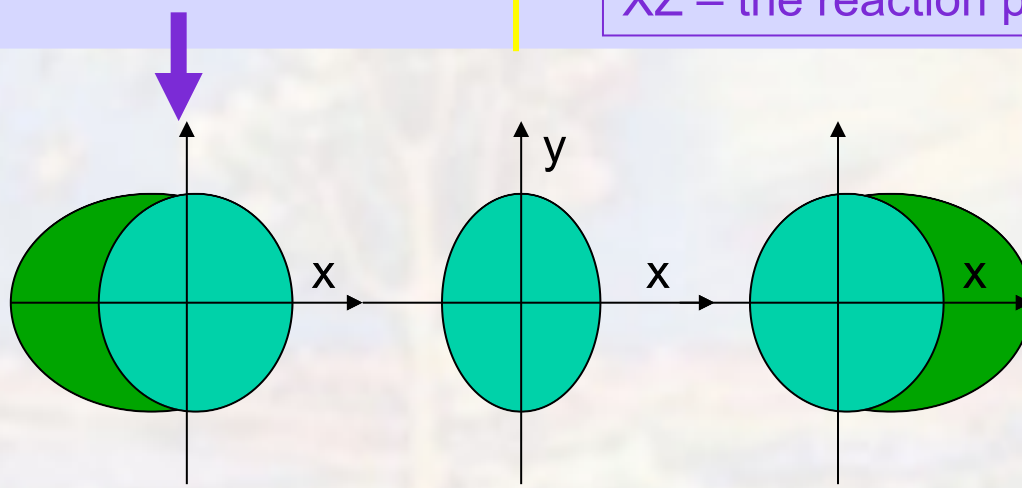
Anisotropic flow = correlations with respect to the reaction plane

More general: system response to the initial spatial anisotropy (below)

Term “flow” does not mean necessarily “hydro” flow – used only to emphasize the collective behavior ←→ multiparticle azimuthal correlation.
Newer trend: “event anisotropy”



No symmetry between “x” and ”-x”, except midrapidity
Symmetry between “y” and and ”-y”
(Otherwise – parity violation)



Asymmetry in the configuration space $\Rightarrow$ anisotropy in the momentum space : $\frac{dN}{d\phi} \neq const$

What is that good about anisotropic flow?

Short answer: to learn more about system properties, evolution dynamics, hadronization –
Anisotropic flow – as a measure of interactions in the system.

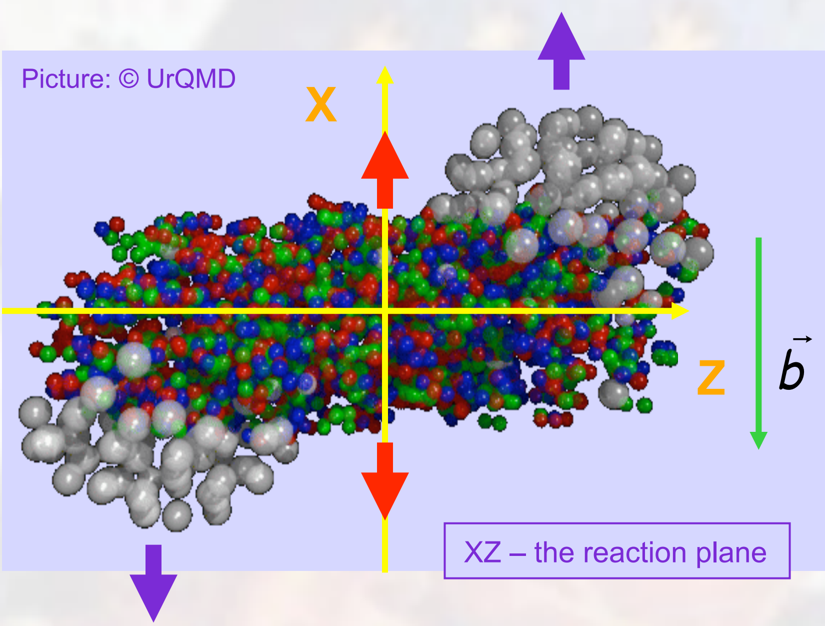
Example: Does hydrodynamic model works? 1992 paper of J.-Y. Ollitrault

PRD 46 (1992) 229

Collide Au+Au, is it enough to create QGP?

- Is the system dense enough?
- Does the system equilibrate?

To answer these questions we need to study the system early in the collision
- hard (rare) probes: J/Psi, jets, dileptons,…
- anisotropic flow !



Q: How what to to if theory is not "ready"?

A: Study "qualitative" features.

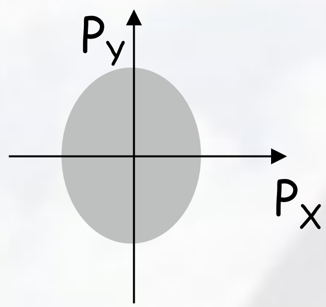
e.g. Can we describe flow assuming some properties of the collective motion?

Directed flow: does it look like particle emission from
a) Moving source?
b) Screened source (shadowing)?

Elliptic flow:
- Surface emission?
- Anisotropically expanding source?
- Rescattering?

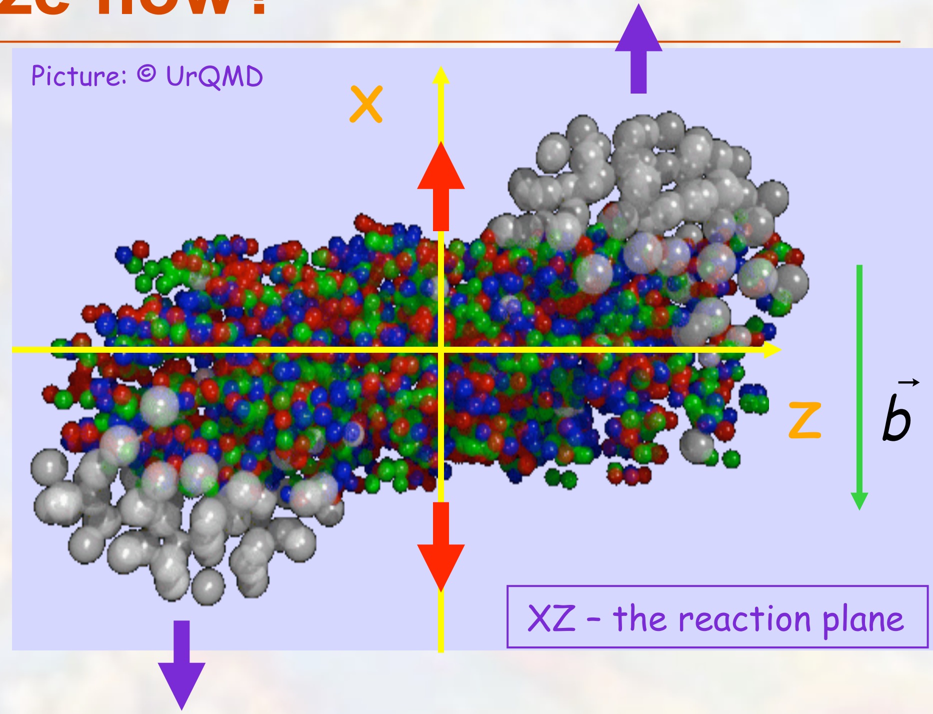
How to characterize flow?

Early days: $\langle p_x \rangle$, 3d sphericity, 2d shericity



$$S_{ij} = \langle p_i p_j \rangle$$

$i, j = 1, 2, 3$ or $1, 2$



Fourier decomposition of single particle (semi) inclusive spectra:

$$\frac{d^3N}{dp_t\,dy\,d\varphi} = \frac{d^2N}{dp_t\,dy}\frac{1}{2\pi}(1 + 2v_1\cos(\varphi) + 2v_2\cos(2\varphi) + \ldots)$$

Directed flow (v_1) — Elliptic flow (v_2)

S.V., Y. Zhang
hep-ph/9407282
Z.Phys. C70 (1996) 665

Advantages:
- Describes different kind of anisotropies in a common way
- Possibility to fully correct the results ➔ compare directly with theory and other experiments

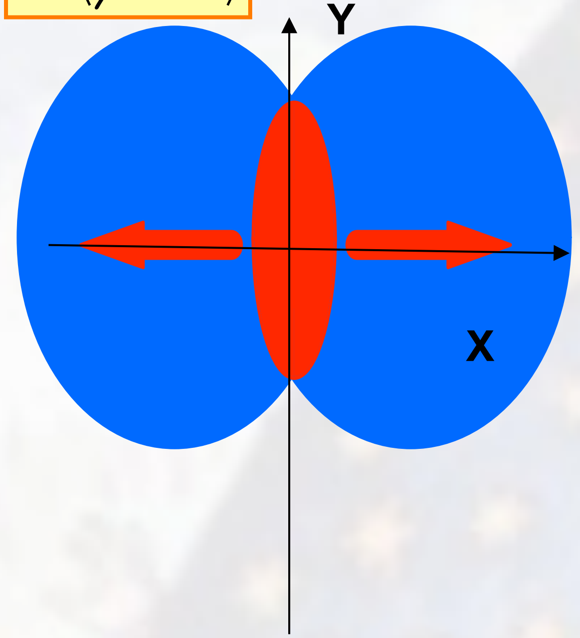
The situation is more complicated for 2-particle spectra. Not discussed here.

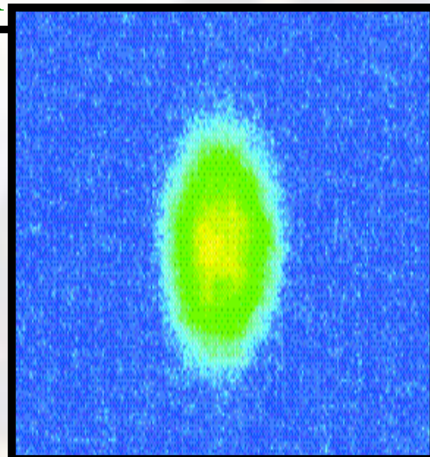
Second harmonic: Elliptic Flow

XZ-plane - the reaction plane

Transverse Plane

$$\varepsilon = \frac{\langle y^2 - x^2 \rangle}{\langle y^2 + x^2 \rangle}$$



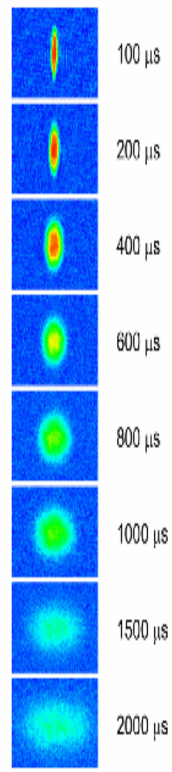


Sensitive to the physics of constituent interactions (needed to convert space to momentum anisotropy) at early times due to
a) decrease in spatial anisotropy
b) decrease in spatial particle density

Elliptic flow with trapped Li^6 atoms:

K.M.O'Hara et al, Science 298,2179, 2002

T.Bourdel et al, PRL 91 020402 , July 11 2003



Magnetic field $B \sim 800G$ shifts (via the Feschbach resonance $|f = 1/2, m_f = 1/2 \rangle \Longleftrightarrow |f = 1/2, m_f = -1/2 \rangle$) and makes the 38-th vibrational Li_2 state to exactly zero energy => infinite scattering length a, very large size and lifetime ~ 1 sec.

Normally gas is transparent, $l \ll L$, and expands without collisions isotropically

But in the strong coupling regime $l \ll L$ it explodes hydrodynamically !, see the figure

Cross section can be changed by many orders of magnitude, but the EoS changes by $\sim 20\%$ only ! (like in QGP and CFT... why?)

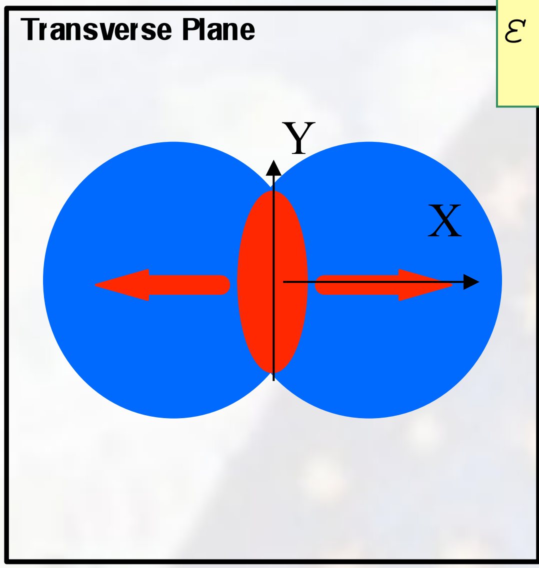
$$v_2 = \langle \frac{p_x^2 - p_y^2}{p_x^2 + p_y^2} \rangle = \langle \cos(2\phi) \rangle$$

$v_2 > 0$, E877, PRL 73 (1994) 2532

When the elliptic flow is formed

XZ-plane - the reaction plane

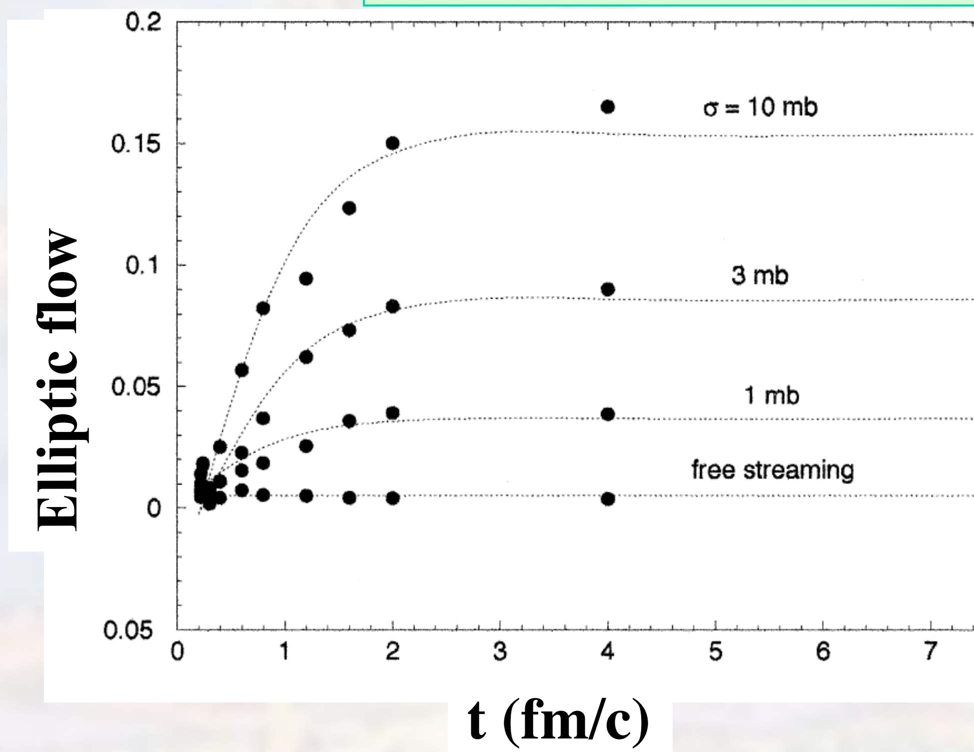
$$\varepsilon = \frac{\langle y^2 - x^2 \rangle}{\langle y^2 + x^2 \rangle}$$



$$v_2 = \langle \frac{p_x^2 - p_y^2}{p_x^2 + p_y^2} \rangle = \langle \cos(2\varphi) \rangle$$

Sensitive to the physics of constituent interactions (needed to converts space to momentum anisotropy) at early times (free-streaming kills the initial space anisotropy)

Zhang, Gyulassy, Ko, PL B455 (1999) 45



The characteristic time scale of 2-4 fm is similar in any model: parton cascade, hydro, etc.

Flow induced correlations. Non-flow.

$$\frac{d^2 n}{d\varphi_1 d\varphi_2} \propto \int \frac{d\Psi_{RP}}{2\pi}\{1+\sum_n 2v_n \cos[n(\varphi_1-\Psi_{RP})]\}\{1+\sum_n 2v_n \cos[n(\varphi_2-\Psi_{RP})]\} =$$

$$= 1+\sum_n 2v_n^2 \cos[n(\varphi_1-\varphi_2)]\} \quad \Rightarrow \left\langle \cos^2[n(\varphi_1-\varphi_2)]\right\rangle = v_n^2 + \delta$$

q-distribution, effect of non-flow correlations

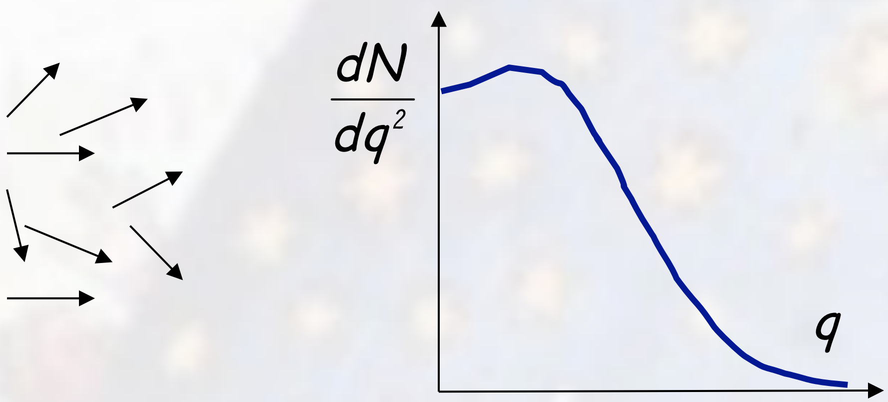
Distribution in the magnitude of the flow vector

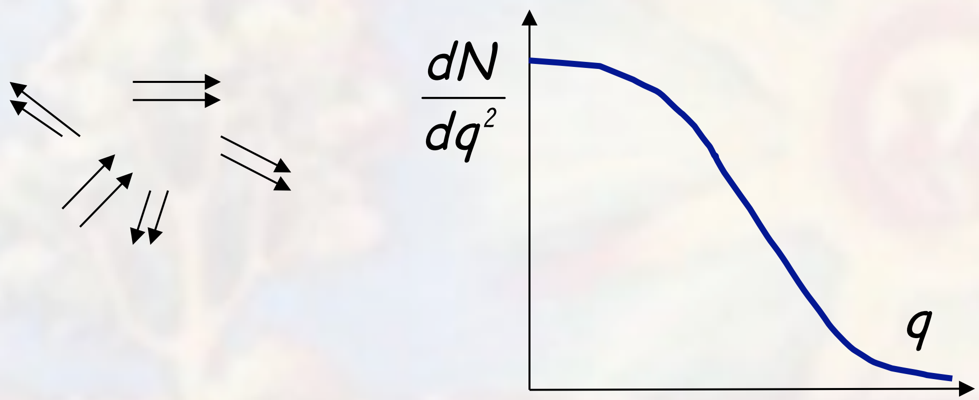
$$u = e^{i\phi}; \quad Q = \sum u; \quad Q_n = \sum u^n = \sum e^{in\phi} = |Q_n| e^{in\Psi_n} = X_n + iY_n = \sum \cos n\varphi + i \sum \sin n\varphi$$

q-distributions $q = |Q| / \sqrt{M}$

$$Q_2 = \sum e^{i2\phi} = |Q_2| e^{i2\Psi_2}$$

$$\langle u_1 u_2^* \rangle = v_2^2 + \delta;$$





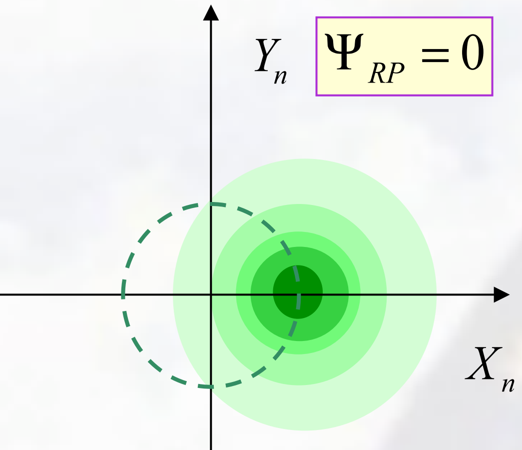
Correlations due to flow

Non-flow contribution

Fit to q-distribution yields flow results mostly free of non-flow effects!

First measurement of flow at AGS

$$u = e^{i\phi}; \quad Q = \sum u; \quad Q_n = \sum u^n = \sum e^{in\phi} = |Q_n| e^{in\Psi_n} = X_n + iY_n = \sum \cos n\varphi + i \sum \sin n\varphi$$



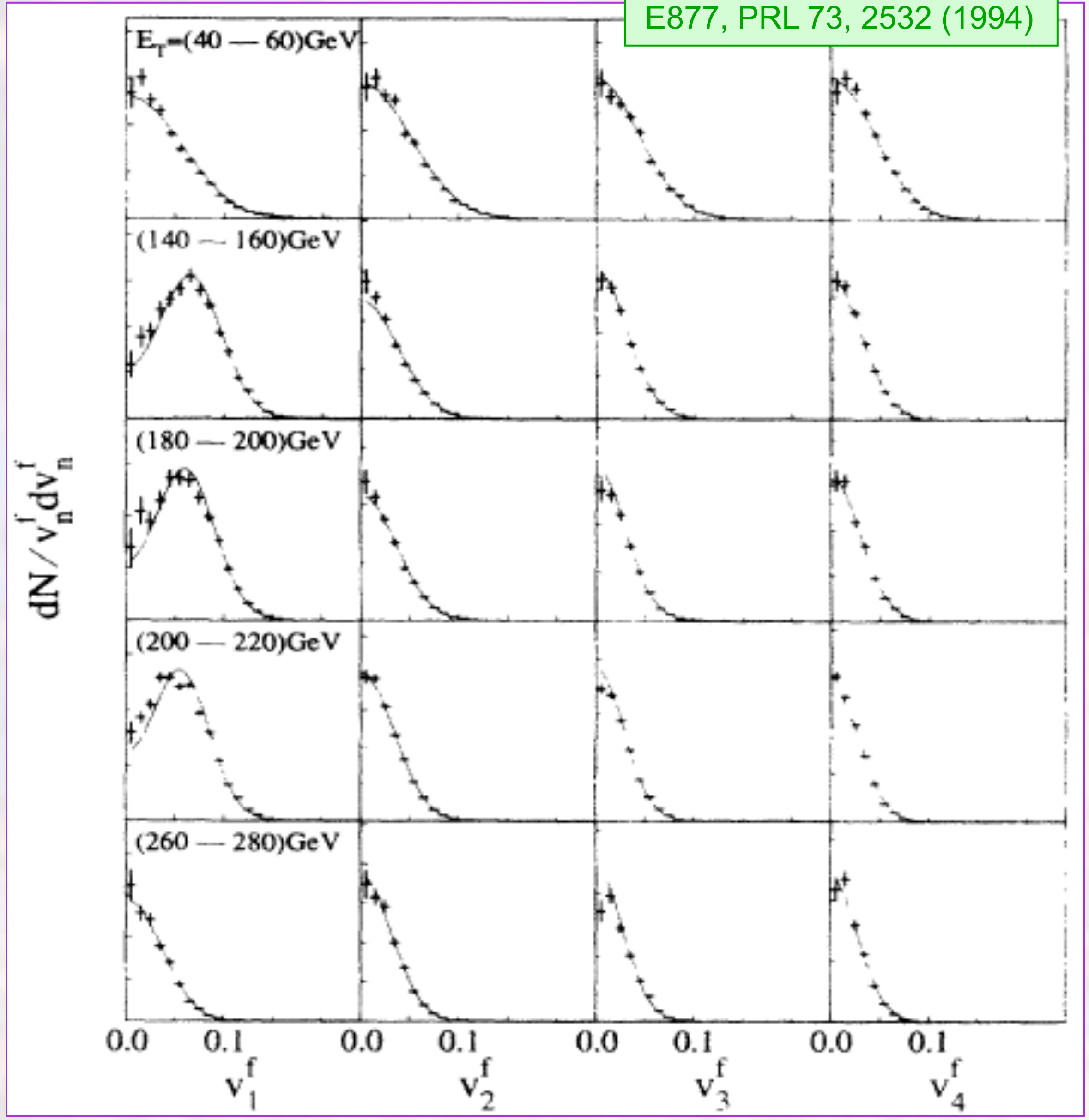
$$\Psi_{RP} = 0$$

$$\langle X \rangle = M v_1; \ \langle Y \rangle = 0$$

$$r_n = Q_n / N$$

$$\frac{dP}{r_n \, dr_n} = \frac{1}{\sigma^2} \exp\left(-\frac{\bar{v}_n^2 + r_n^2}{2\sigma^2} \right) I_0\left(\frac{r_n \bar{v}_n}{\sigma^2} \right)$$

E877, PRL 73, 2532 (1994)



Differential flow : First observation of v_2 > 0

E877, PRC 55 (1997) 1420

$$Q_n = \sum e^{in\phi};\ Q_n = |Q_n| e^{in\Psi_n} = X_n + iY_n$$

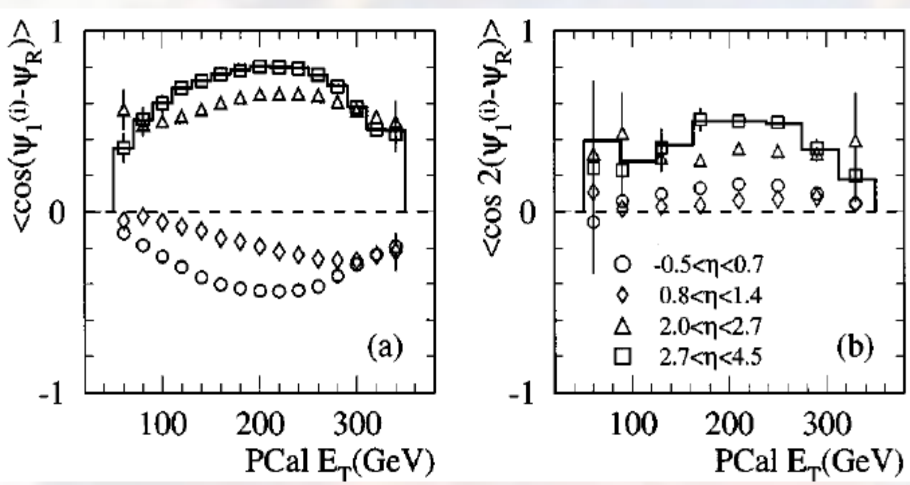
Ψ_n – n-th harmonic Event Plane

$$\langle \cos(\Psi_1^{(i)} - \Psi_1^{(j)}) \rangle = \langle \cos(\Psi_1^{(i)} - \Psi_R) \rangle \langle \cos(\Psi_1^{(j)} - \Psi_R) \rangle$$

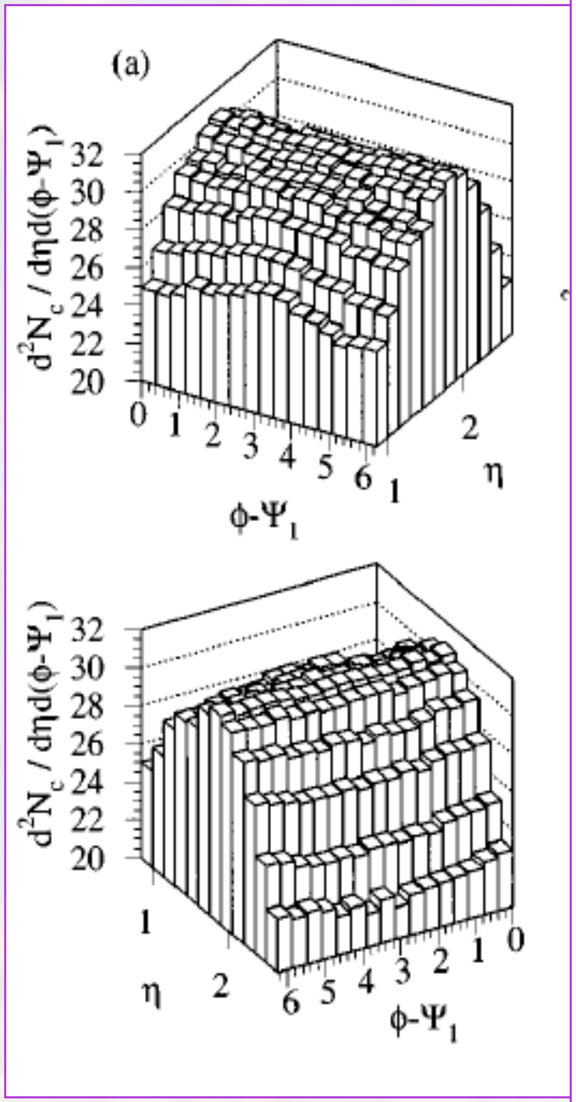
$$v_n^{obs} = \langle \cos[n(\Psi_m - \Psi_r)] \rangle$$

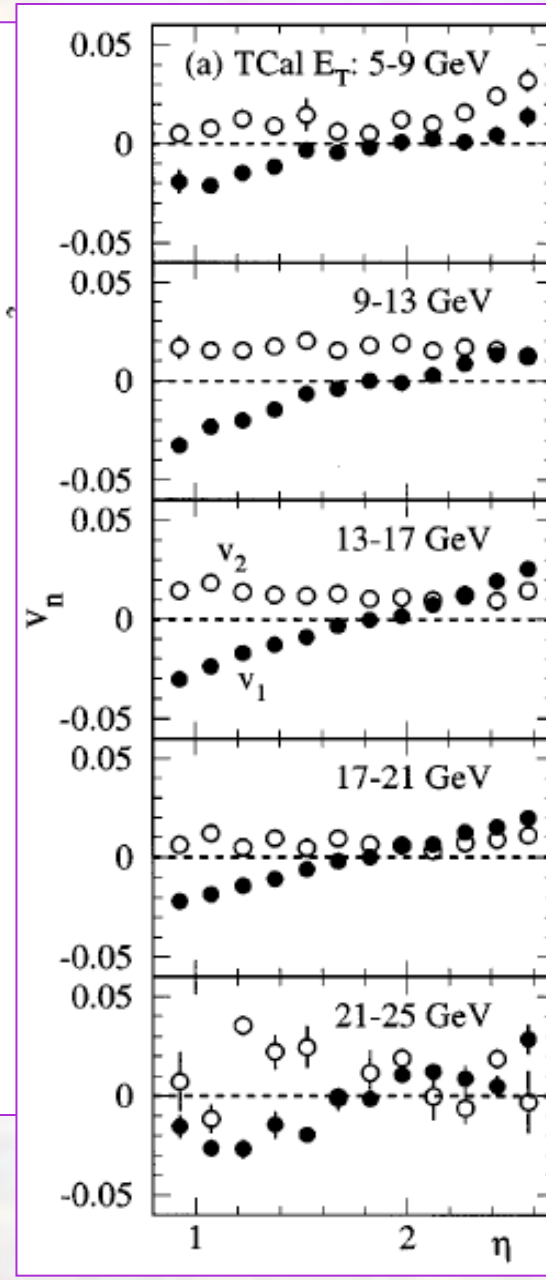
$$v_n = v_n^{obs} / \langle \cos[km(\Psi_m - \Psi_r)] \rangle$$

$$\langle \cos[n(\Psi_m^a - \Psi_r)] \rangle = \sqrt{\langle \cos[n(\Psi_m^a - \Psi_m^b)] \rangle}$$



Distribution of hits in the silicon pad detector wrt RP determined by calorimeters.





WAYNE STATE UNIVERSITY

"Non-flow" and flow fluctuations

$$\langle u_{n,a} u^*_{n,b} \rangle = \langle \cos[n(\phi_a - \phi_b)] \rangle \equiv v_{n,a} v_{n,b} + \delta_n$$

Flow "non-flow"

$$\delta_n \ll v_n^2 \quad ?$$

"Non-flow" – azimuthal correlations of any other origin except the correlation with respect to the reaction plane.
It combines the possible contributions from resonance decay,
inter and intra jet correlations, etc.
If general, two effects do not factorize; then the above equations would serve as a definition of "non-flow", with "v"'s defined as on previous slides.

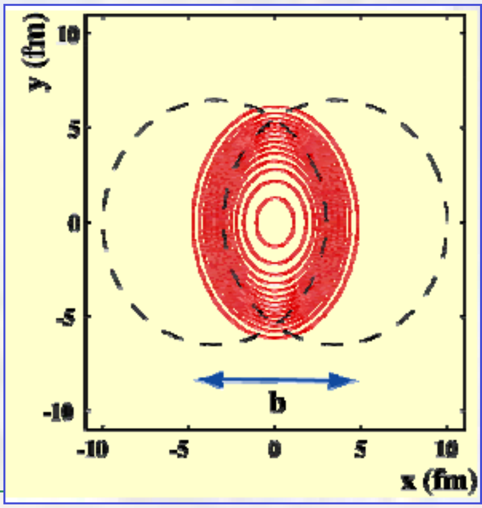
$$\langle v_{n,a} v_{n,b} \rangle \neq \langle v_{n,a} \rangle \langle v_{n,b} \rangle$$

Effect of flow fluctuations

An example: $v_2 \propto \varepsilon \;\rightarrow\; \sigma_v \propto \sigma_\varepsilon$

$$\sigma_\varepsilon^2 = \langle \varepsilon^2(b) \rangle - \langle \varepsilon(b) \rangle^2$$

Also, fluctuations in particle density (number of particles, area), etc.

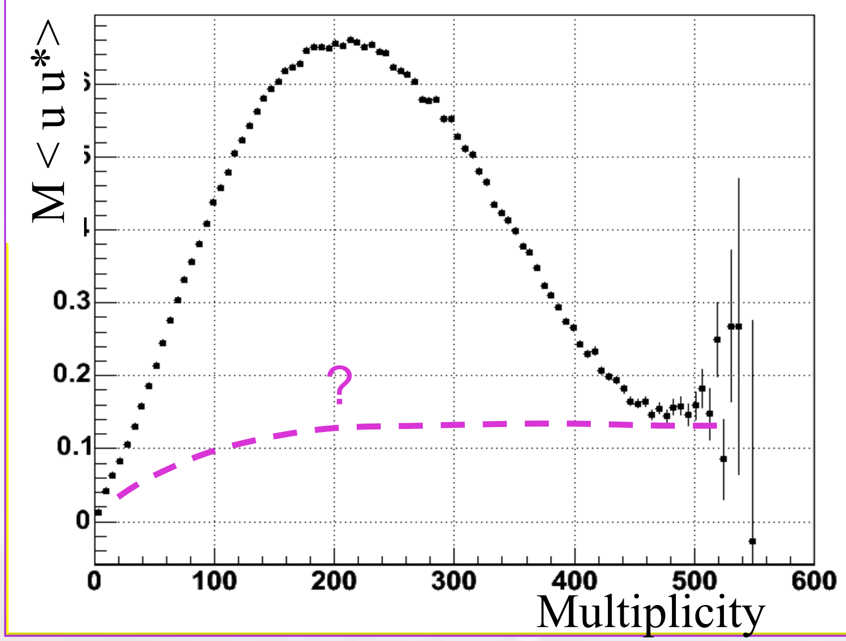


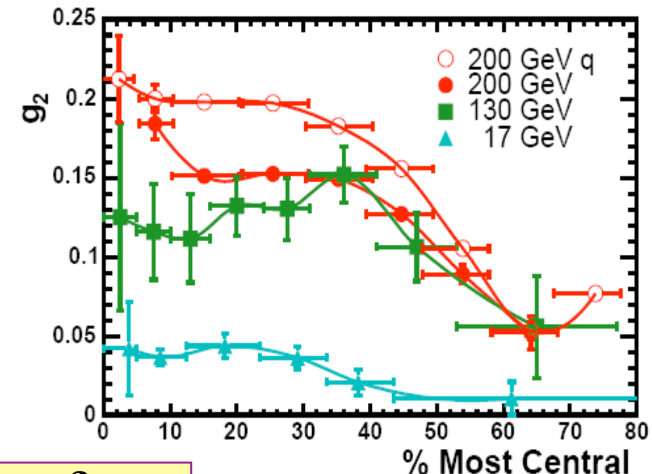
$$v_2 \equiv \langle \cos(2(\varphi_i - \Psi_{RP})) \rangle$$

$$\varepsilon = \frac{\langle y^2 - x^2 \rangle}{\langle y^2 + x^2 \rangle}$$

Non-flow estimates. Centrality dependence.

$$\langle uQ^* \rangle = M \langle uu^* \rangle = M(v^2 + \delta) = Mv^2 + \tilde{\delta}$$





$$g_2 = \delta \cdot N_{part}$$

STAR nucl-ex/0409033

FIG. 31: (color online). The nonflow parameter, g_2, as a function of centrality. The solid points are from the cumulant method. The open circles are from the q-distribution method.

Suppressing non-flow with multiparticle correlations

$$\langle u_1 u_2^* \rangle = v_2^2 + \delta; \quad u \equiv e^{i2\varphi}$$

$$\langle u_1 u_2 u_3^* u_4^* \rangle = v_2^4 + 2 \cdot 2\delta v_2^2 + 2\delta^2$$

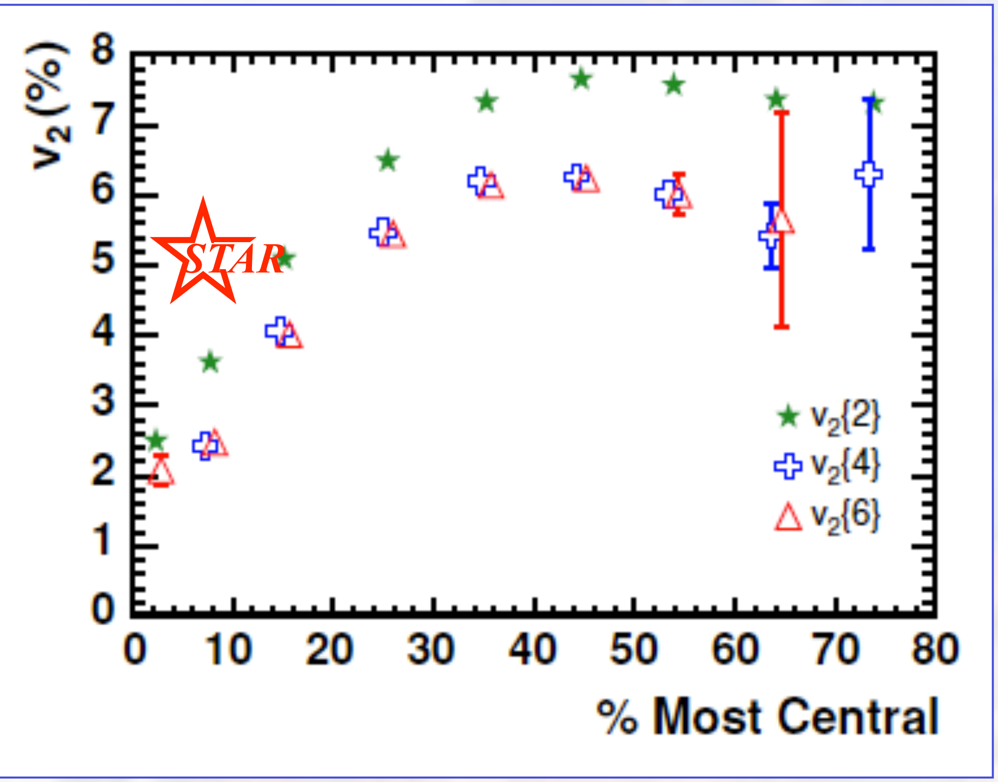
$$\langle u_1 u_2 u_3^* u_4^* \rangle - 2\langle u_1 u_2^* \rangle = -v_2^4$$

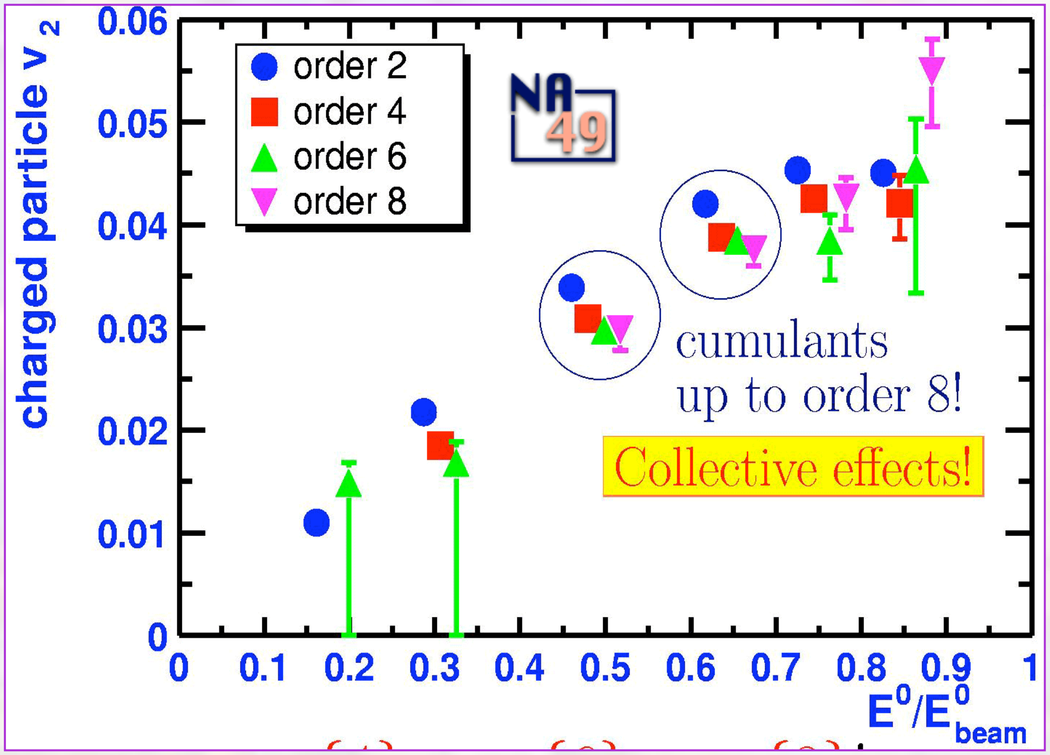
flow*nflow + nflow*flow

(1,3)(2,4)+(1,4)(2,3)

$$\langle u_i u_j u_k^* u_l^* \rangle = \left\langle \frac{1}{N(N-1)(N-2)(N-3)} \left[(Q^2 Q^{*2} - N) - 2N(N-1) - 4(N-2)(QQ^* - N) - 2(Q^2 Q_2^* - N) + (Q_2 Q_2^* - N) \right] \right\rangle$$

Application: Generating functions (Borghini, Ding, Ollitrault)
+ **many other ways**





Effect of flow fluctuations discussed later

Methods – just mention a few

No discussion of the detector effects

$$\frac{d^3N}{dp_t\,dy\,d\Delta\varphi} = \frac{d^2N}{dp_t\,dy}\frac{1}{2\pi}\left(1 + 2v_1\cos(\Delta\varphi) + 2v_2\cos(2\Delta\varphi) + \ldots\right)$$

Directed flow

Elliptic flow

$$\Delta\varphi = \varphi - \Psi_{RP}$$

2-particle correlations, $v_n\{2\}$

$$v_n\{2\}^2 = \langle\cos[n(\phi_1 - \phi_2)]\rangle = \langle u_{n,1}u^*_{n,2}\rangle$$

$$u_n \equiv e^{in\phi}$$ unit flow vector

"Standard" method, $v_n\{EP\}$

$$Q_{n,x} = \sum_i w_i\cos(n\phi_i) = \mathrm{Q}_n\cos(n\Psi_n),$$
$$Q_{n,y} = \sum_i w_i\sin(n\phi_i) = \mathrm{Q}_n\sin(n\Psi_n),$$

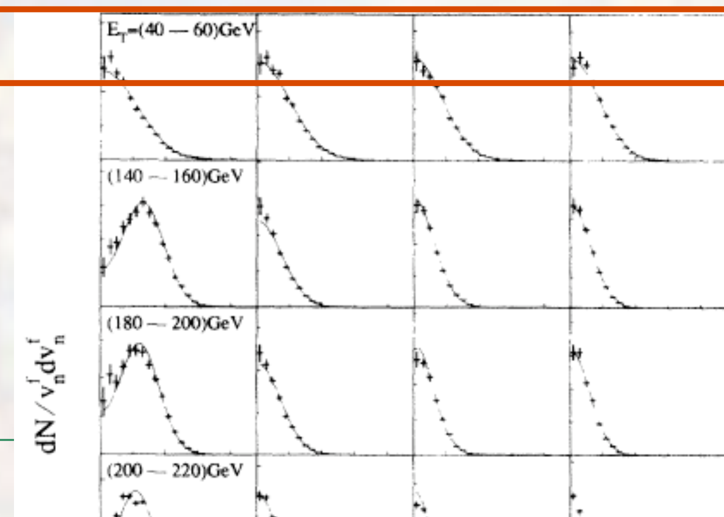
$$v_n^{obs}(p_T, y) = \langle\cos[n(\phi_i - \Psi_n)]\rangle$$
$$\mathscr{R}_n = \langle\cos[n(\Psi_n - \Psi_{RP})]\rangle$$
$$\mathscr{R}_{n,sub} = \sqrt{\langle\cos[n(\Psi_n^A - \Psi_n^B)]\rangle}$$

$$v_n = \frac{v_n^{obs}}{\mathscr{R}_n}.$$

Fitting q-distribution, $v_n\{q\text{-dist}\}$

$$\frac{dN}{dq_n} = \frac{q_n}{\sigma_n^2}e^{-\frac{v_n^2M + q_n^2}{2\sigma_n^2}} I_0\left(\frac{q_n v_n\sqrt{M}}{\sigma_n^2}\right)$$

$$\mathrm{q}_n = \mathrm{Q}_n/\sqrt{M}.$$



q-distributions

$$\langle \cos(2\Delta\varphi) \rangle = v_2{}^2 + \delta_2$$

δ – does not depend efficiency; does depend on centrality (approx. as 1/# of clusters) .

$$u = e^{i\phi}; \quad Q = \sum u; \quad Q_n = \sum u^n = \sum e^{in\phi} = |Q_n| e^{in\Psi_n} = X_n + iY_n = \sum \cos n\varphi + i \sum \sin n\varphi$$

$$q \equiv Q / \sqrt{M}$$

$$\sigma_X^2 = \langle q_x^2 \rangle - \langle q_x \rangle^2 = \frac{1}{2}\left(1 + v_{2n} - 2v_n{}^2 + (M-1)\delta\right)$$

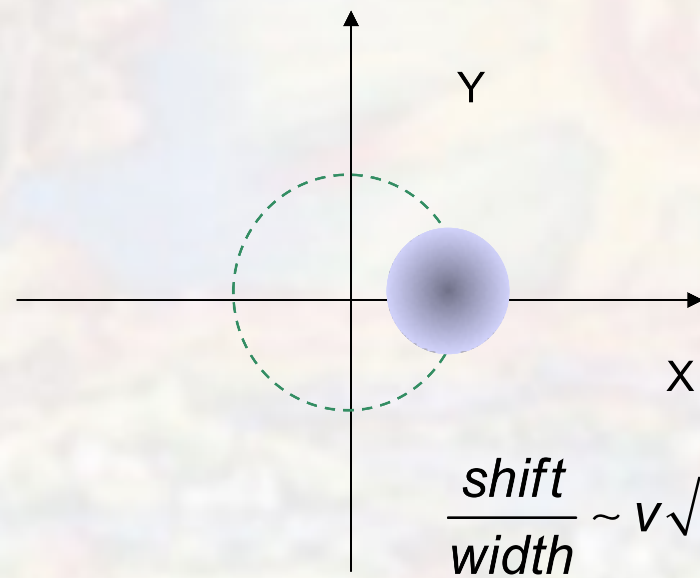
$$\sigma_Y^2 = \langle q_y^2 \rangle - \langle q_y \rangle^2 = \frac{1}{2}\left(1 - v_{2n} + (M-1)\delta\right)$$

$$\langle q_y^2 \rangle = \frac{1}{M}\left\langle \left(\sum_{i=1}^{M} \sin(2\varphi_i) \right)^2 \right\rangle = \frac{1}{M}\left\langle \sum_{j=1}^{M} \sin(2\varphi_j) \sum_{i=1}^{M} \sin(2\varphi_i) \right\rangle$$

$$= \frac{1}{M}\left\langle \sum_{i=1}^{M} \sin^2(2\varphi_i) \right\rangle + \frac{1}{M}\left\langle \sum_{i \neq j} \sin(2\varphi_i)\sin(2\varphi_j) \right\rangle$$

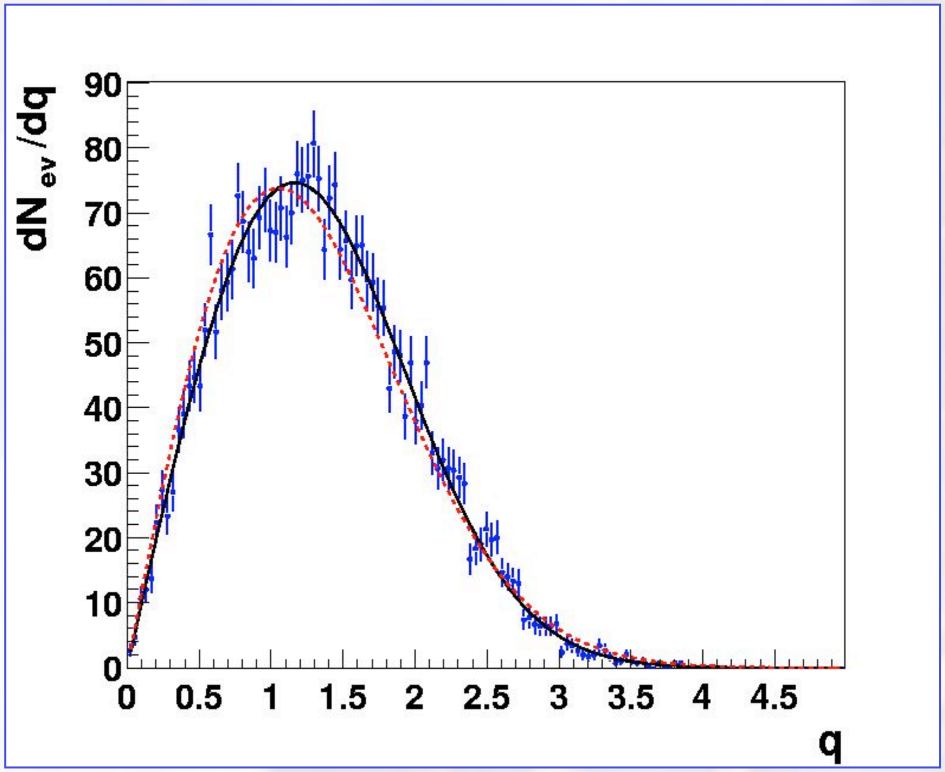
$$\frac{dP}{q_n dq_n} = \frac{1}{\sigma_n^2} \exp\left(-\frac{v_n^2 M + q_n^2}{2\sigma_n^2} \right) I_0\left(\frac{q_n v_n \sqrt{M}}{\sigma_n^2} \right), \qquad (8)$$

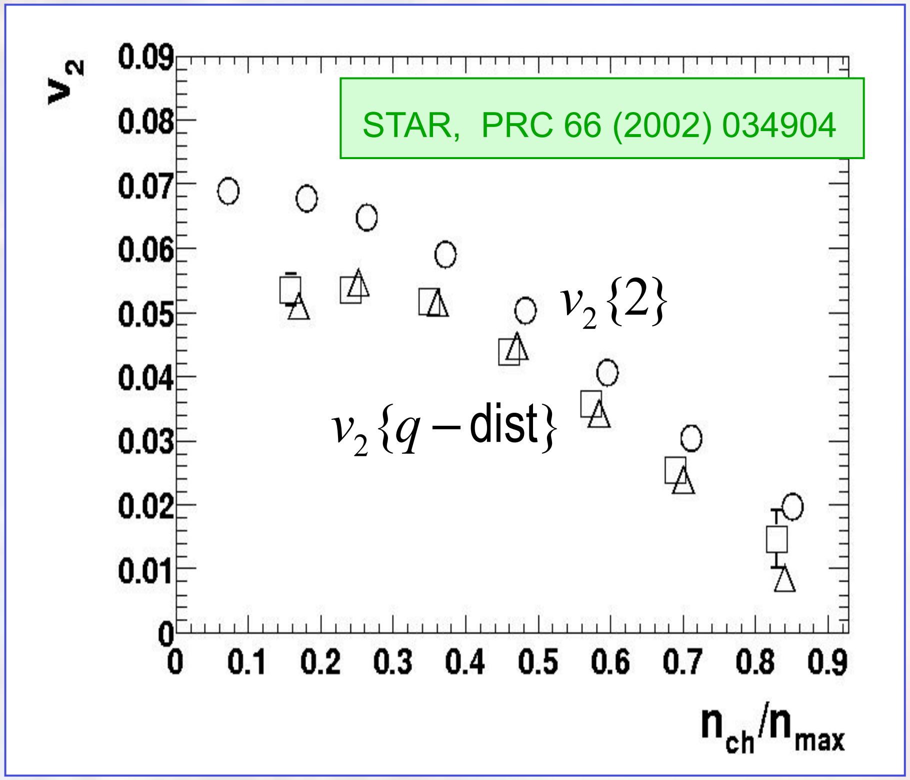
where I_0 is the modified Bessel function. We have introduced the variable $q_n = Q_n / \sqrt{M}$, which greatly reduces the effect on the shape of the distribution from averaging over events with different multiplicities. In a more general case



$$\frac{\textit{shift}}{\textit{width}} \sim v\sqrt{M}$$

v_2 from q-distributions





$$\frac{dP}{q_n dq_n} = \frac{1}{\sigma_n^2} \exp\left(-\frac{v_n^2 M + q_n^2}{2\sigma_n^2} \right) I_0\left(\frac{q_n v_n \sqrt{M}}{\sigma_n^2} \right), \qquad (8)$$

where I_0 is the modified Bessel function. We have introduced the variable $q_n = Q_n / \sqrt{M}$, which greatly reduces the effect on the shape of the distribution from averaging over events with different multiplicities. In a more general case

$$Q_n \cos(n\Psi_n) = \sum [w_i \cos(n\phi_i)]$$
$$Q_n \sin(n\Psi_n) = \sum [w_i \sin(n\phi_i)],$$

Fourier transform of the distribution in flow vector components

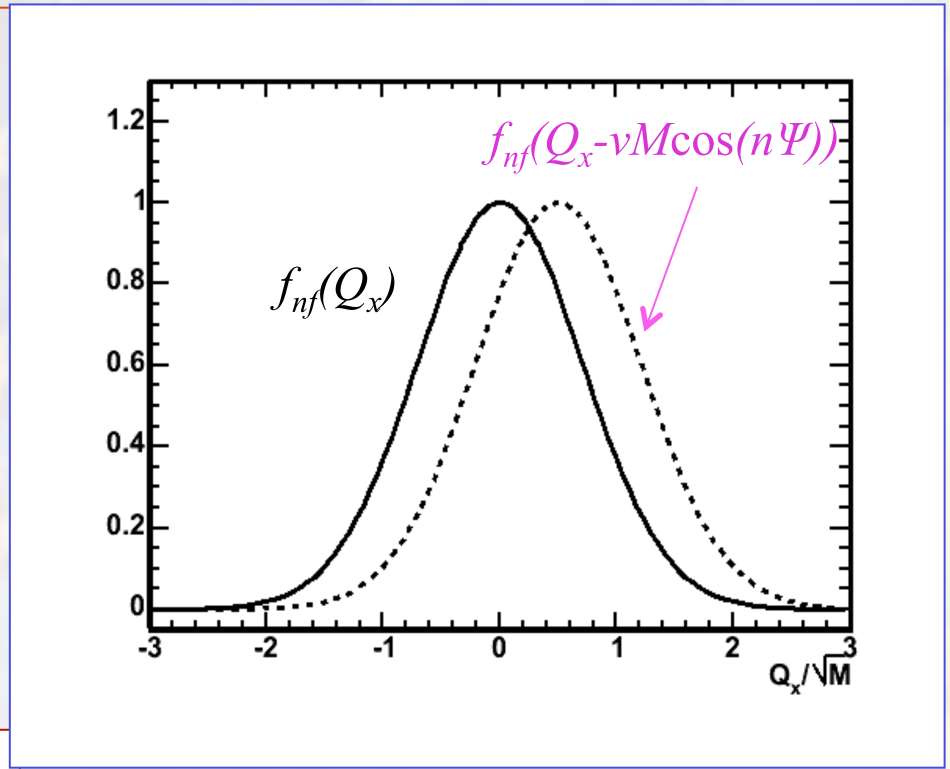
x component of flow vector

$$Q_x = \sum_i \cos(n\phi_i)$$

$$f(Q_x) \equiv \frac{dn}{dQ_x} = \int \frac{d\Psi}{2\pi} f_{nf}(Q_x - vM\cos(n\Psi))$$

Fourier transform of this function:

$$\begin{aligned}\tilde{f}(k) &= \int dQ_x e^{ikQ_x} \int \frac{d\Psi}{2\pi} f_{nf}(Q_x - vM\cos(n\Psi)) \\ &= \int \frac{d\Psi}{2\pi} \int dQ_x e^{ikQ_x} f_{nf}(Q_x - vM\cos(n\Psi)) \\ &= \int \frac{d\Psi}{2\pi} e^{ikvM\cos(n\Psi)} \int dt e^{ikt} f_{nf}(t) \\ &= J_0(kvM)\tilde{f}_{nf}(k)\end{aligned}$$



$$\text{shift} = v\cos(\Psi_{RP})\sqrt{M}$$

Due to symmetry (no acceptance effects!) only real part is non-zero

General strategy:
Let x_{01} be the first root of equation $J_0(x_{0i})=0$.
x_{01}~=2.045.

Then: $v = k_{01}/M$, where
k_{01} is the first zero of f(k)

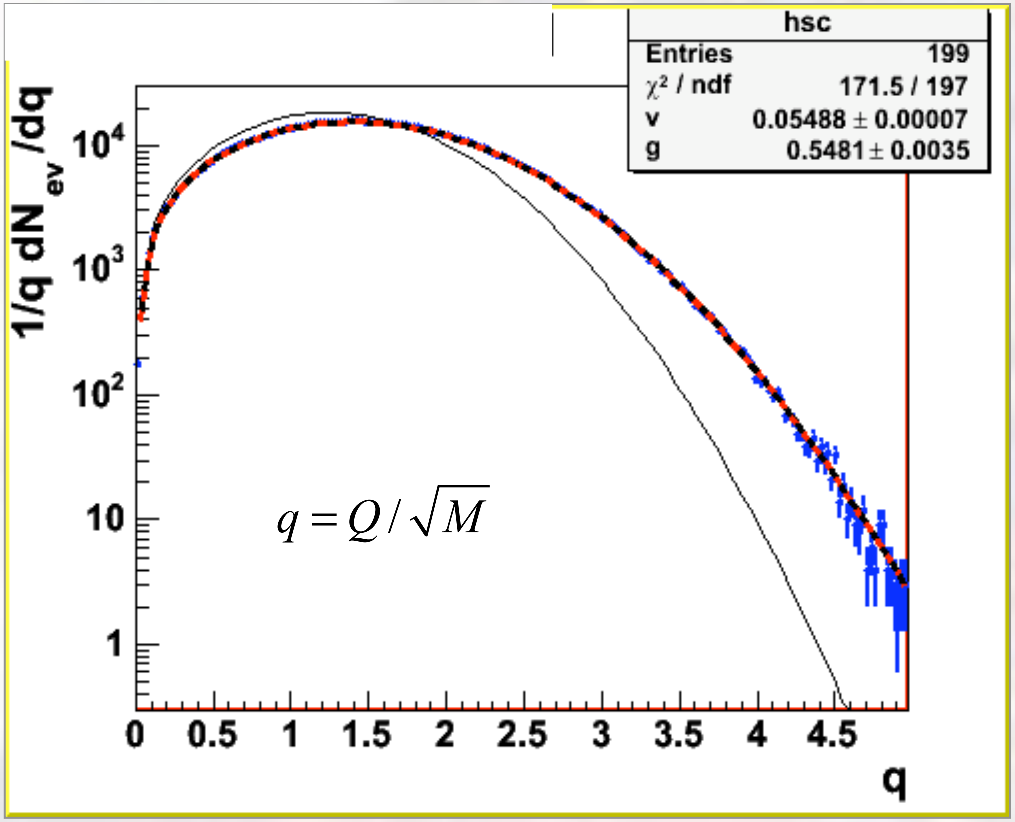
Using Bessel transform

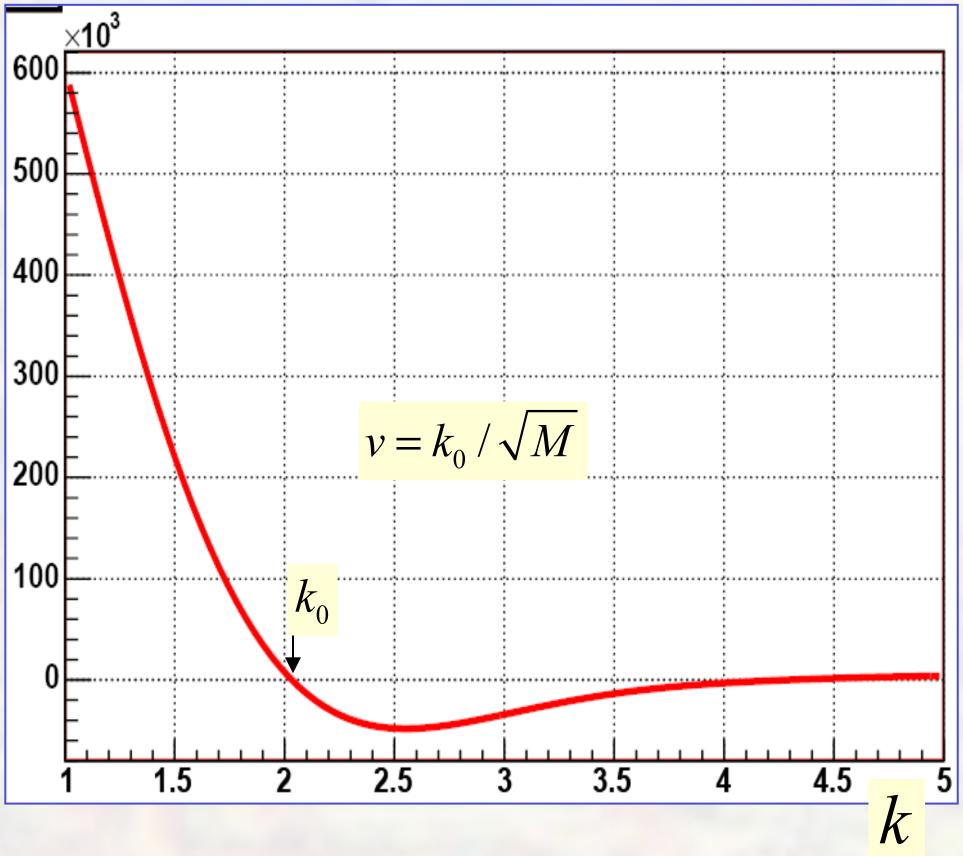
2-dimensional distribution. Then

$$\tilde{f}(k) = \int dQ_x e^{ik_x Q_x} dQ_y e^{ik_y Q_y} \frac{d^2 n}{dQ_x dQ_y}$$
$$= \int dQ\, J_0(kQ) \frac{dn}{dQ}$$
$$\sim J_0(kvM)$$

LYZ ==
== Fourier transform of distribution in Q_x, and/or Q_y
== Bessel transform of the distribution in Q
== Fitting of Q-distribution (used first by E877) !?

net results of the so-called "sum" generating function
Lee-Yang Zeroes method





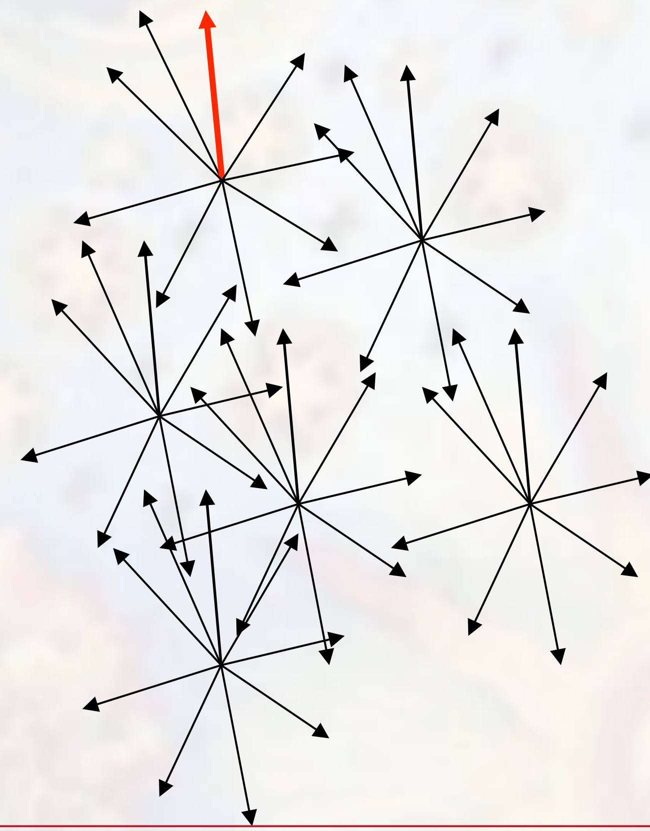
Non-flow: pp vs. AA

Consider correlations of particles from some “bin” (POI) with all particles from a “pool”

$$Q = \sum_{i \in "pool"} u_i; \quad u_i = e^{i2\varphi_i}$$

$$\langle u_b Q^* \rangle = (v_b v_p + \delta_{bp}^{AA}) M^{AA}$$

$$\delta_{bp}^{AA} \approx \frac{\delta_{bp}^{pp}}{N_{coll}} \approx \frac{\delta_{bp}^{pp} M^{pp}}{M^{AA}}$$



$$\langle u_b Q^* \rangle^{AA} \approx v_b v_p M^{AA} + \langle u_b Q^* \rangle^{pp}$$

Non-flow looks exactly the same in pp and AA ➔ Results - directly “correctible”.

Notations:

v_b – Flow in a particular p_t/eta “bin”

v_p – Average flow in the pt/eta region used to define RP (or “pool” of particles)

δ_{bp}^{pp} – Azimuthal correlations in *pp* ($\langle u_a u_b^* \rangle; \quad u \equiv e^{i2\phi}$)

δ_{pr}^{AA} – Non-flow part in 2-part azimuthal correlation in AA

N_{coll} – Number of “independent NN collisions”, a la $N_{part}/2$.

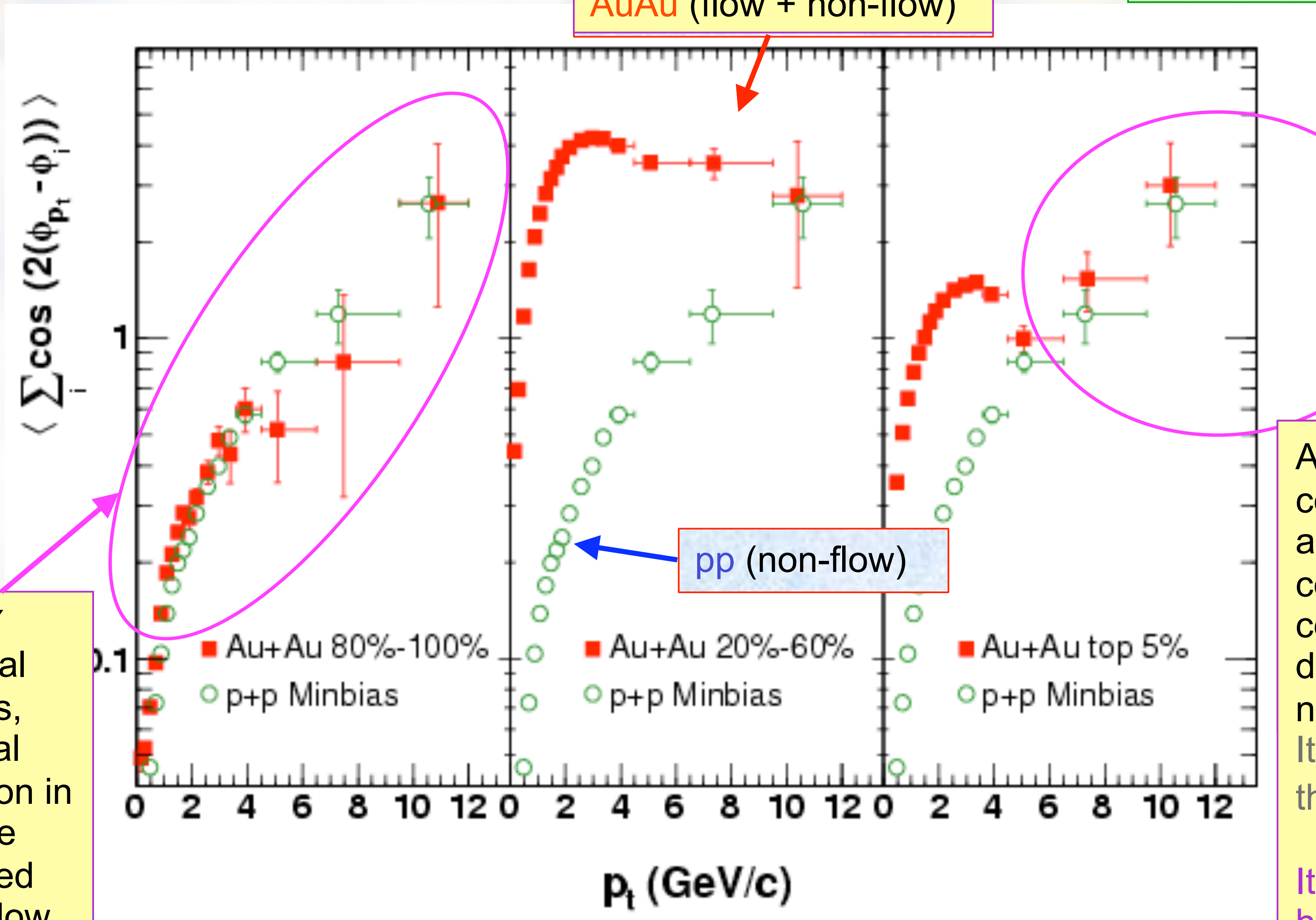
Check if non-flow estimates/measurements reported or *Au+Au* are consistent with measurements in *pp*. (Expect the difference of the order of factor of <~2. Extra particles in jets ➔ non-flow contribution increases B-to-B jet suppression – non-flow goes down)

Use *pp* data to estimate non-flow effects in *Au+Au* when other methods do not work (high p_t, K and Lambda flow, etc.)

Comparison: pp & AuAu

STAR: PRL 93(2004)252301

AuAu (flow + non-flow)



pp (non-flow)

In VERY peripheral collisions, azimuthal correlation in AuAu are dominated by non-flow.

At high p_t in central collisions, azimuthal correlation in AuAu could be dominated by nonflow.

It does not mean that v_2 is zero!

It is remarkable how well they agree

Analysis can be more differential: charge combination dependence, identified particles…

v_2/ε plot. Low density and "hydro" limits.

Low density limit

(called "collisionless" in the original paper of Heiselberg and Levy)
Below - my own derivation of Heiselberg's results

Heiselberg & Levy, PRC 59 (1999) 2716

$$\frac{dN}{d\varphi} = \left(\frac{dN}{d\varphi}\right)_0 + \Delta\left(\frac{dN}{d\varphi}\right) \qquad n(\mathbf{v}) = n_0(\mathbf{v}) + \Delta n(\mathbf{v})$$

Change in the particle flux is proportional to the probability for the particle to interact.

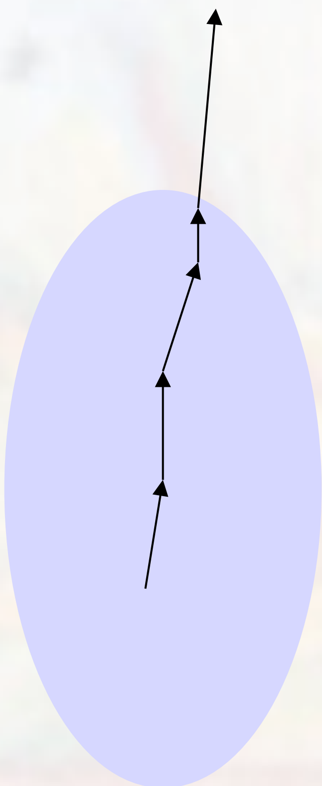
$$\Delta n(\mathbf{v}) \propto \int d\mathbf{r}_0 \rho(\mathbf{r}_0) \int dt\, \rho(\mathbf{r}_0 + \mathbf{v}t, t)$$

Integrations over: a) particle emission point
b) Over the trajectory of the particle (time) with weight proportional to the density of other particles --"scattering centers"

$$\rho(\mathbf{r}_0 + \mathbf{v}t, t) \propto \int d\mathbf{v}' \rho(\mathbf{r}_0 - \mathbf{v}'t)$$

Particle density at time *t* assuming free streaming

$$\Delta n(\mathbf{v}) \propto \int d\mathbf{r}_0 \rho_0(\mathbf{r}_0) \int dt \int d\mathbf{v}' \rho_0(\mathbf{r}_0 - (\mathbf{v} - \mathbf{v}')t)$$



$$\rho_0 \propto \exp\left(-\frac{x^2}{2\langle x^2\rangle} - \frac{y^2}{2\langle y^2\rangle}\right)$$

$$v_2^i = \frac{\varepsilon}{16\pi R_x R_y} \sum_j \left\langle v_{ij} \sigma^{ij}_{transport} \right\rangle \frac{dN_j}{dy} \frac{v_{i\perp}^2}{v_{i\perp}^2 + \langle v_{j\perp}^2 \rangle}$$

$$v_2 \propto \varepsilon \frac{1}{S} \frac{dN}{dy} \qquad S = \pi \sqrt{\langle x^2\rangle \langle y^2\rangle}$$

Note:
(x-vt) changes very little over the entire history

J.-Y. Ollitrault, PRD 46 (1992) 229

First hydro calculations

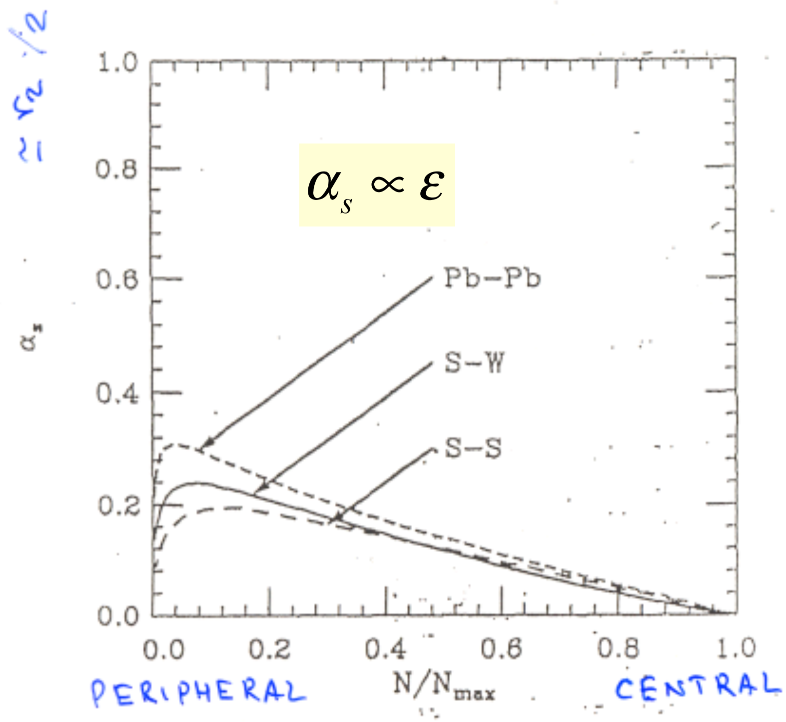


FIG. 3. Spatial anisotropy for various colliding systems. α_s, defined by Eq. (4.18), is plotted against the number of participating nucleons, scaled to its maximum value (reached for a central collision) N_{max}. Short dashes: lead-lead collision ($N_{max} \simeq 395$). Long dashes: sulfur-sulfur collision ($N_{max} \simeq 51$). Solid line: sulfur-tungsten collision ($N_{max} \simeq 121$).

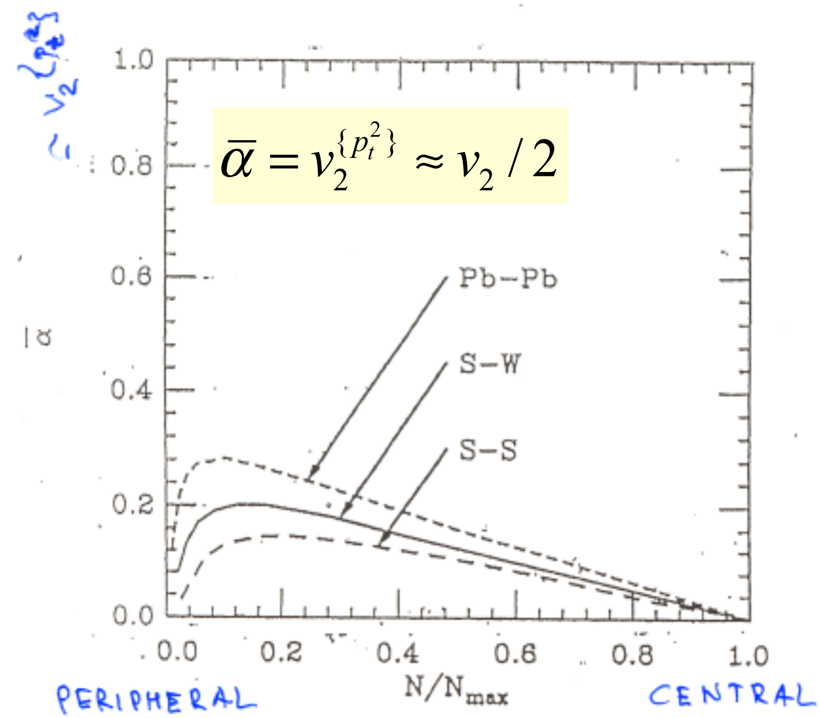
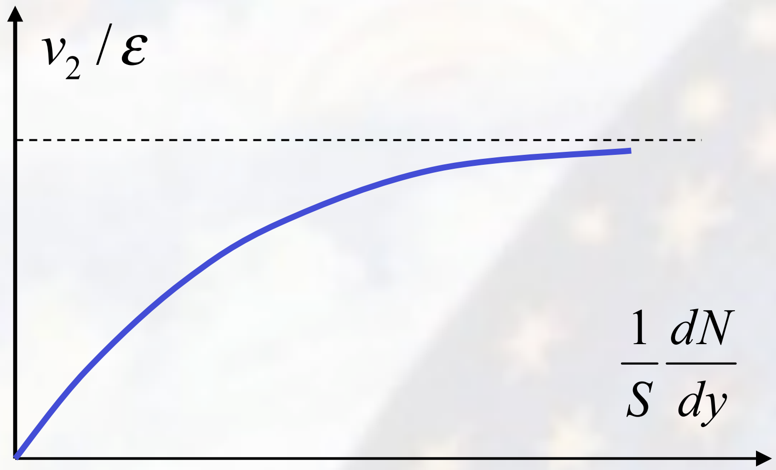


FIG. 6. Comparison between various colliding systems. $\bar{\alpha}$ is plotted against the number of participating nucleons scaled to its maximum value N_{max}, as in Fig. 4. The decoupling temperature is $T_d \doteq 150$ MeV and the initial time $t_0 =$fm/c for the three curves.

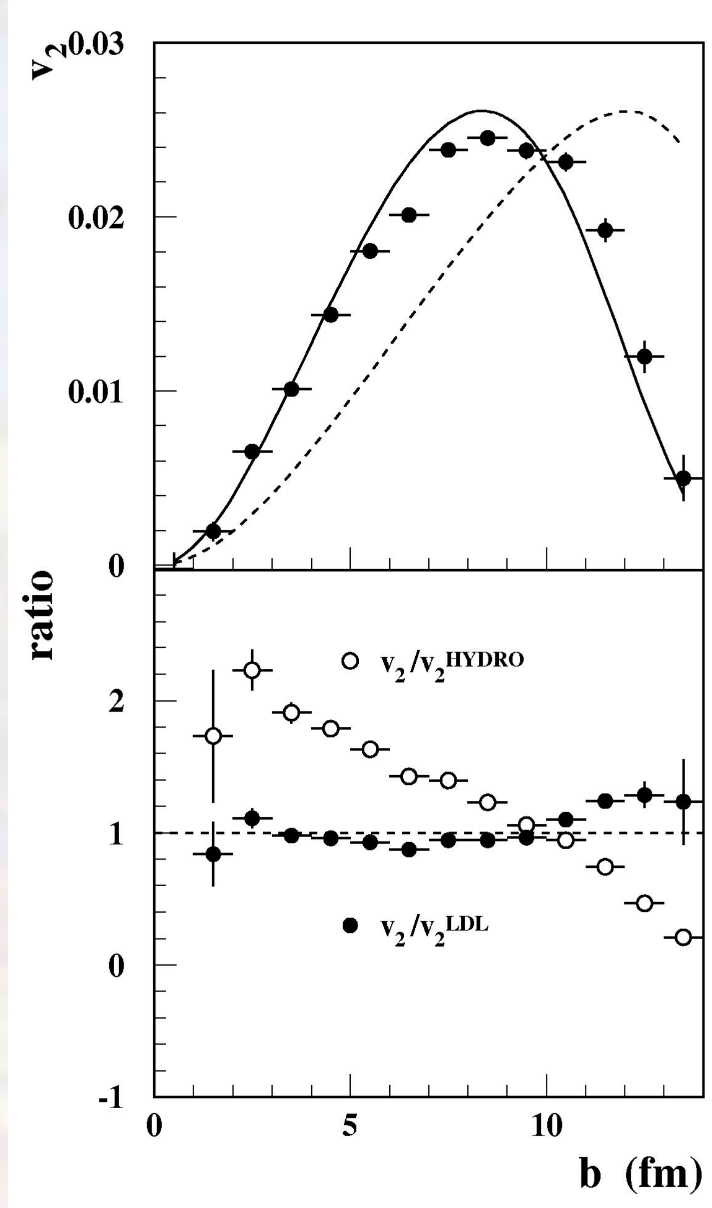
In hydro, where mean free path is by assumption much less than the size of the system, there is no other parameters than the system size (time scales may enter, see below). Then elliptic flow must follow closely the initial eccentricity.

Centrality dependence

S.V. & A. Poskanzer, PLB 474 (2000) 27



Points – RQMD, dashed curve - ε(b)



LDL parameters : mean free path, ε

Hydro : only ε

"HYDRO limit"

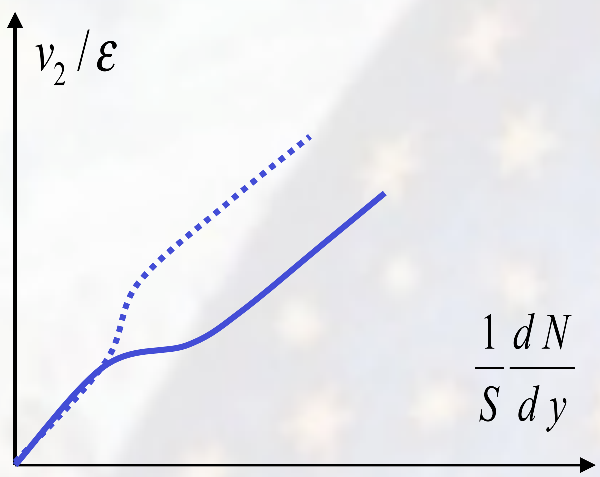
Ollitrault: $\frac{v_2}{\varepsilon} \approx \frac{v_2^{\{p_t^2\}}}{2\varepsilon} \approx 0.27 \div 0.35$

Heinz et al.: $(v_2/\varepsilon)_{HYDRO} \approx 0.21 \div 0.23$

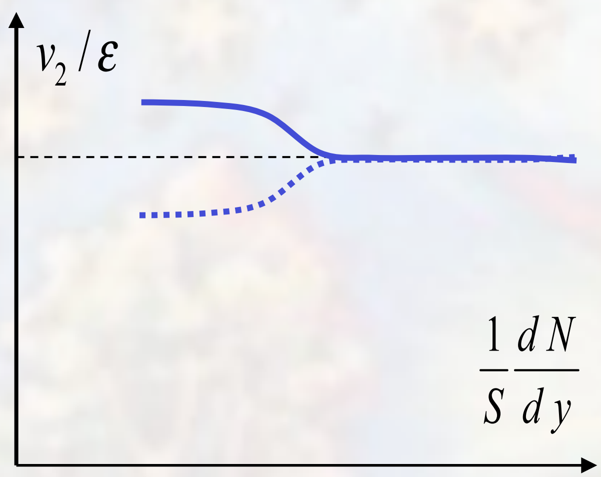
v_2/ε and phase transition

After original ideas of : Sorge, PRL 82 (1999) 2048, Heiselberg & Levy, PRC 59 (1999) 2716

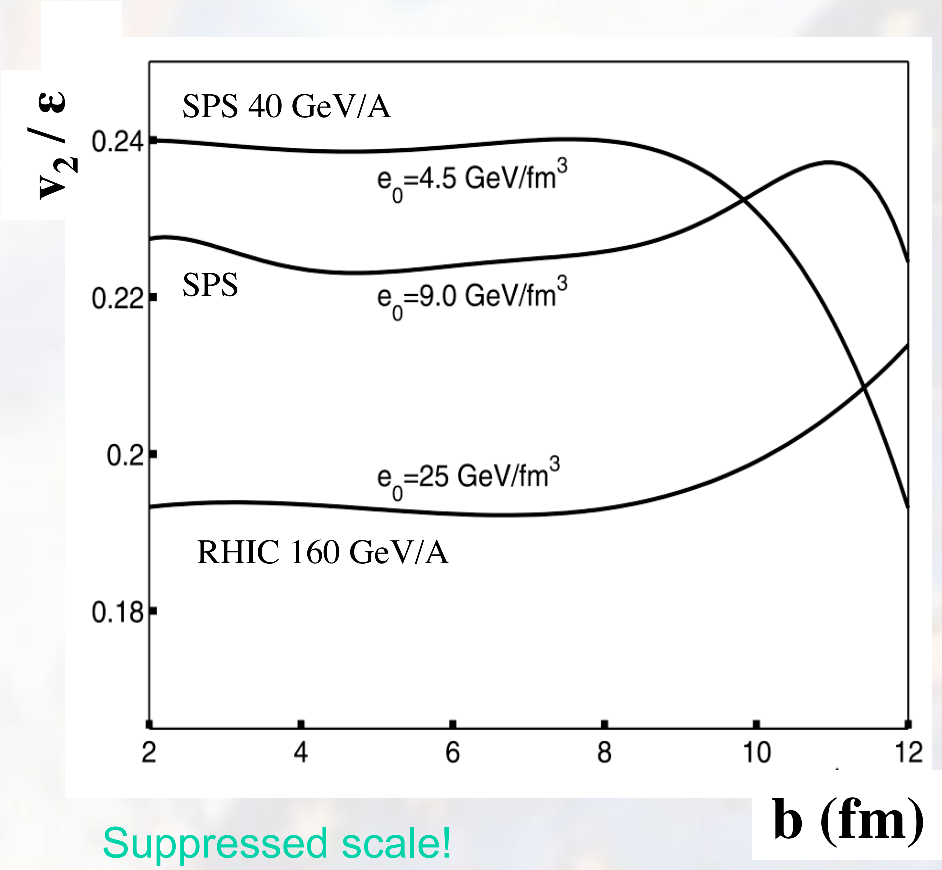
LDL



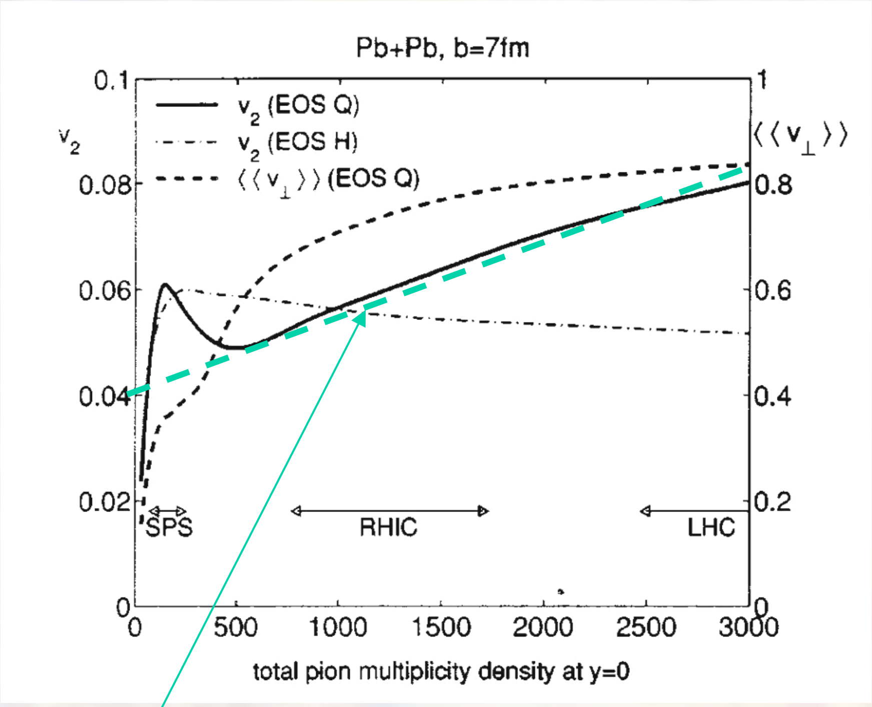
HYDRO



More on "hydro limits"



Suppressed scale!



$$v_2 = 0.04 + 0.04 * \frac{dN}{dy} / 3000$$

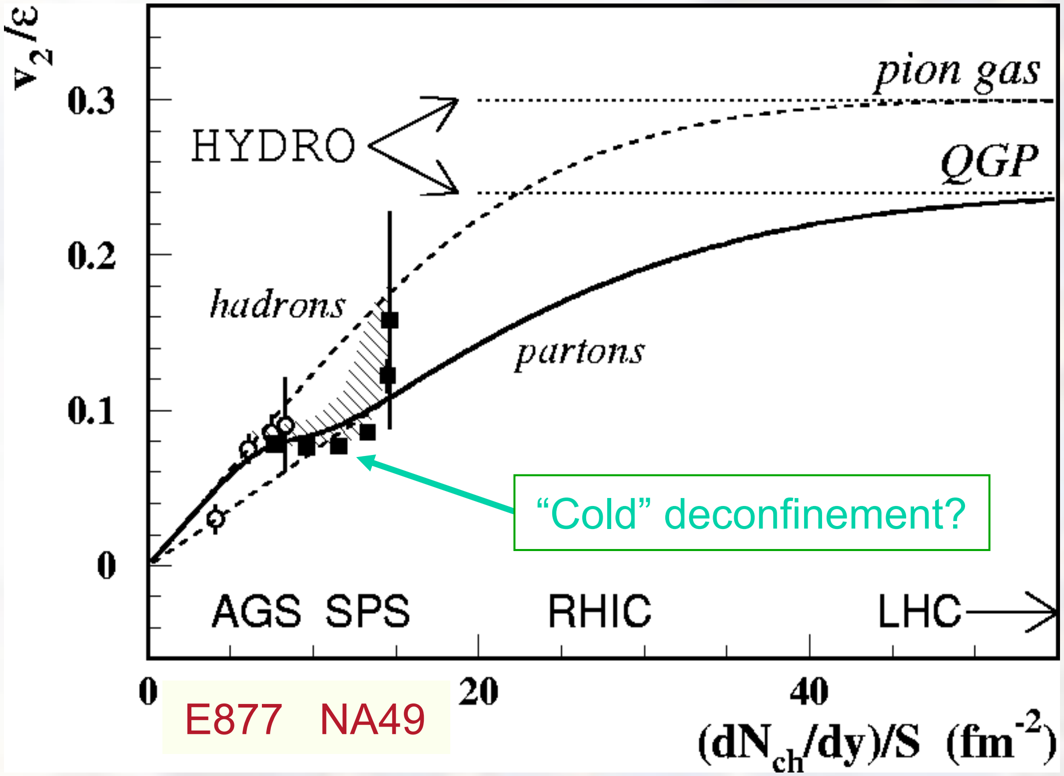
4. P.F. Kolb, J. Sollfrank, and U. Heinz, Phys. Rev. C 62 (2000) 054909.

Minimum in v2/ε due to softening of the EoS at phase transition

WAYNE STATE UNIVERSITY

v_2/ε vs particle density

S.V. & A. Poskanzer, PLB 474 (2000) 27



Uncertainties:
Hydro limits: slightly depend on initial conditions
Data: no systematic errors, shaded area –uncertainty in centrality determinations.
Curves: "hand made"

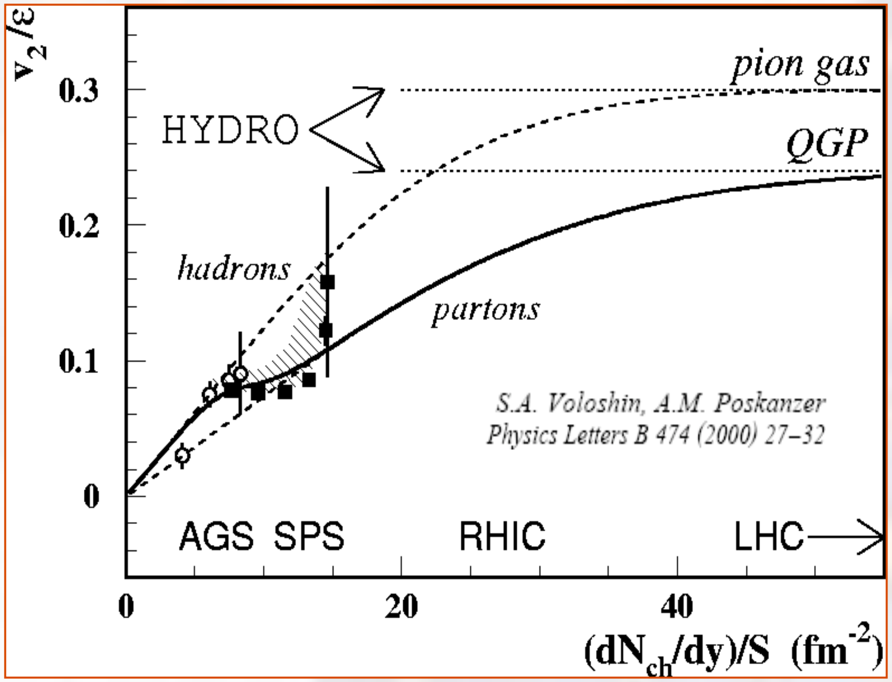
What to expect for different nuclei, e.g. Cu+Cu?
First guess would be:

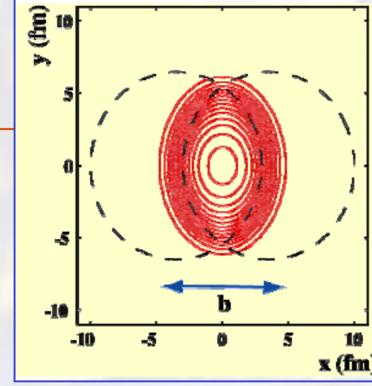
$$v_2 \propto \varepsilon \frac{1}{S}\frac{dN}{dy}\frac{1}{R_A}$$

… but there is no such factor in the LDL:

$$v_2^i = \frac{\varepsilon}{16\pi R_x R_y}\sum_j \left\langle v_{ij}\sigma^{ij}_{transport}\right\rangle \frac{dN_j}{dy}\frac{v_{i\perp}^2}{v_{i\perp}^2 + \left\langle v_{j\perp}^2\right\rangle}$$

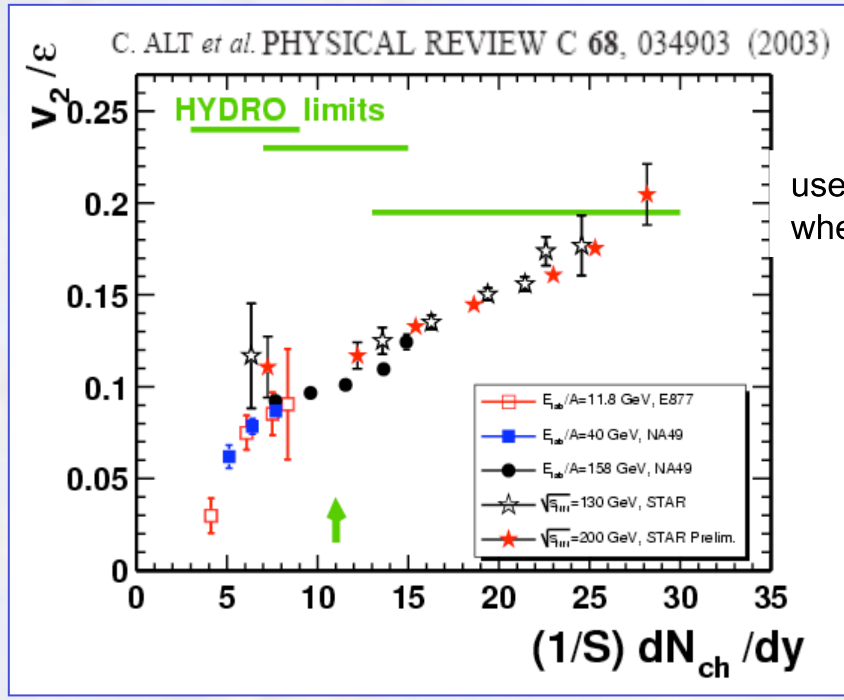
v_2/ε plot evolution



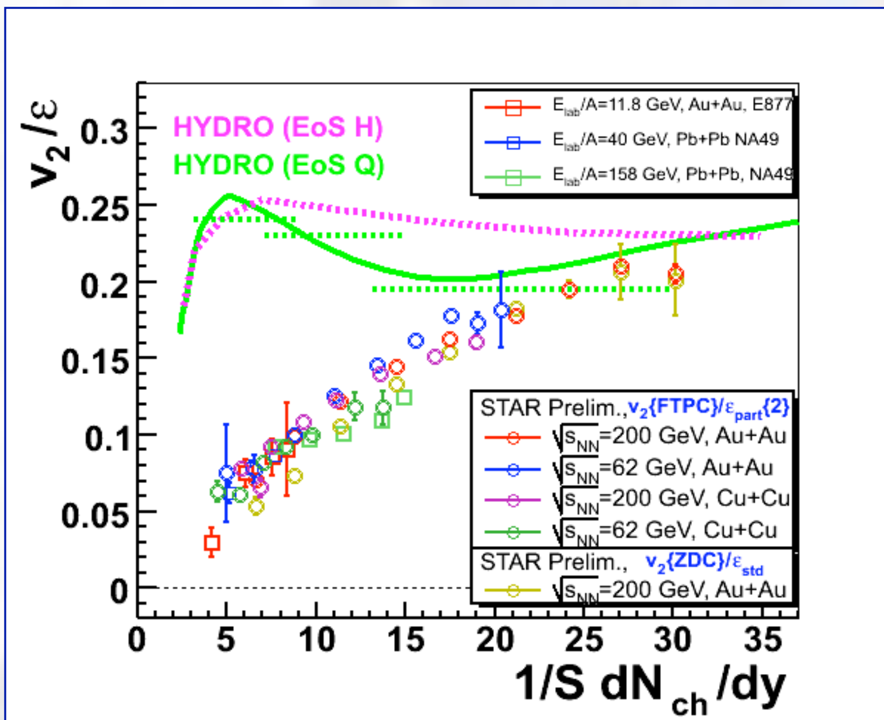


$$v_2 \equiv \left\langle \cos\left(2\left(\varphi_i - \Psi_{RP}\right)\right)\right\rangle$$

$$\varepsilon = \frac{\left\langle y^2 - x^2 \right\rangle}{\left\langle y^2 + x^2 \right\rangle}$$



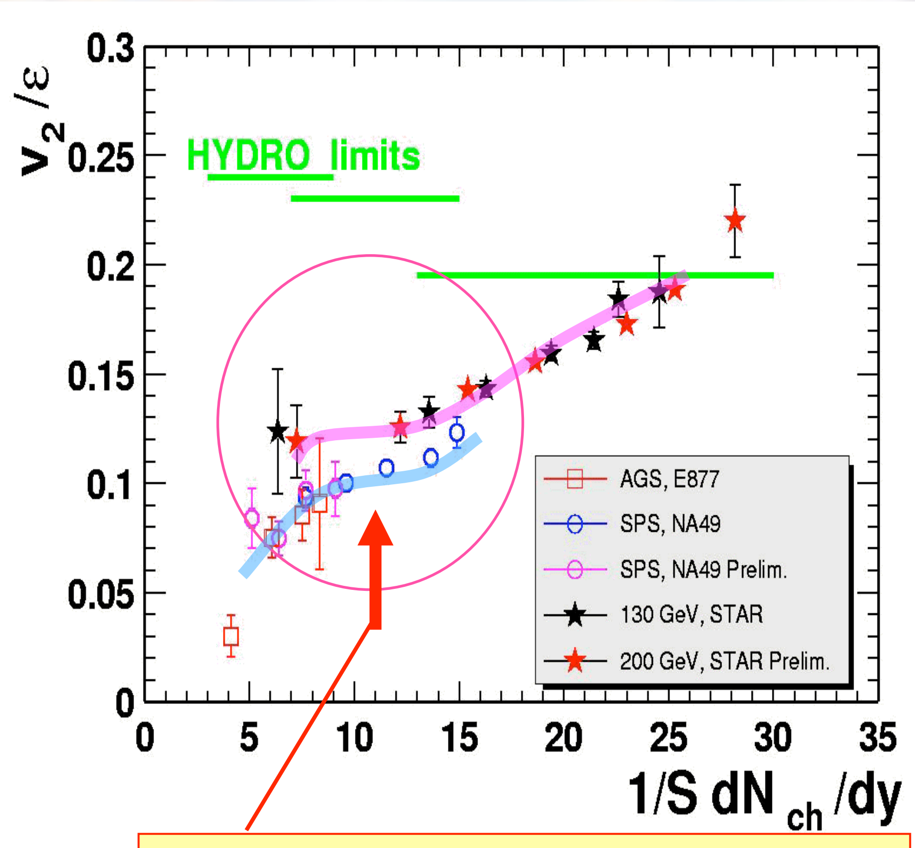
used v2{4} whenever possible



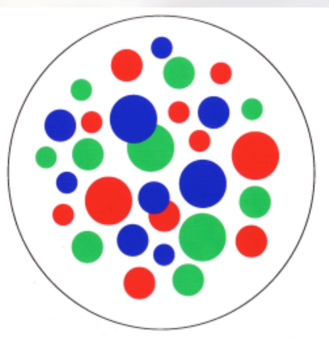
Findings:
- Viscosity leads to decrease of flow → $(v_2/\varepsilon)_{theor}$ down ~30%
- Initial eccentricity is likely higher than thought → $(v_2/\varepsilon)_{theor}$ up ~<50%
- "proper" selection of parameters in hydro → $(v_2/\varepsilon)_{theor}$ up ~20%
- "Correcting for flow fluctuations → $(v_2/\varepsilon)_{exp}$ down <~ 20%
- Initial flow field → $(v_2/\varepsilon)_{theor}$ up ~?

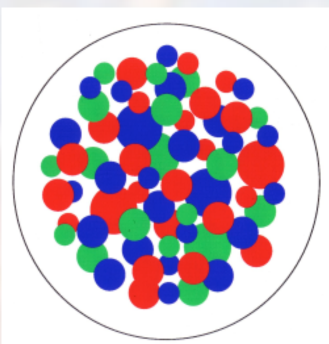
Why does it work that well? (no ε^2 terms?)

"Cold" deconfinement, color percolation?



Percolation point by H. Satz, QM2002





CERN SPS energies b ~ 4 fm
RHIC: b ~ 7 fm

There is a need for the "next generation" of this plot: better estimates of epsilon, adding more data (in particular 62 GeV)

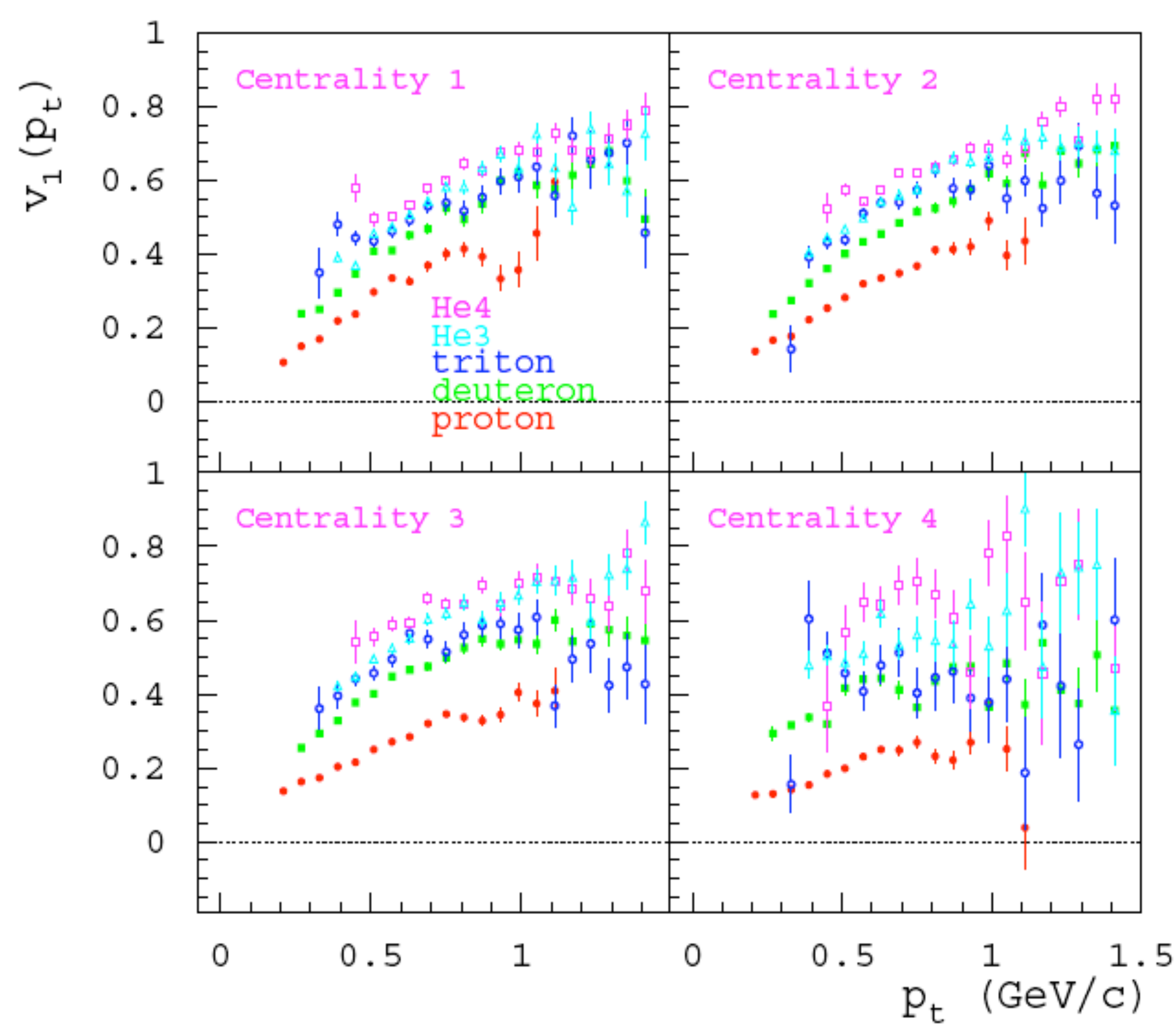
It is a real pity that NA49 measurements have so large systematic uncertainty. Need detector with better azimuthal acceptance (could be just a simple extra detector used to determine the RP) .

BES at RHIC !?

Coalescence and anisotropic flow.
Constituent quark number scaling.

Coalescence I. E877 light nuclei flow.

S.V and E877, Nucl Phys A638 (1998) 455c
E877 PRC 59(1999) 884



$$\frac{d^3N_d}{d^3p}(p) \propto B\left(\frac{d^3N_p}{d^3p}(p/2)\right)^2 \to$$

$$v_{1,d}(p_t) \approx 2v_{1,p}(p_t/2)/\left(1+2v_{1,p}^2\right)$$

What is needed for the equation above to work?

a) "Rare" process
b) B=const only if the configuration space density does not depend on the orientation wrt RP

Note! The coalescence picture itself can have much larger region of applicability than the equation above. We/I just do not know how to describe coalescence in the case of "not rare processes".

E877 conclusion:
Configuration space density increases in the direction of flow.

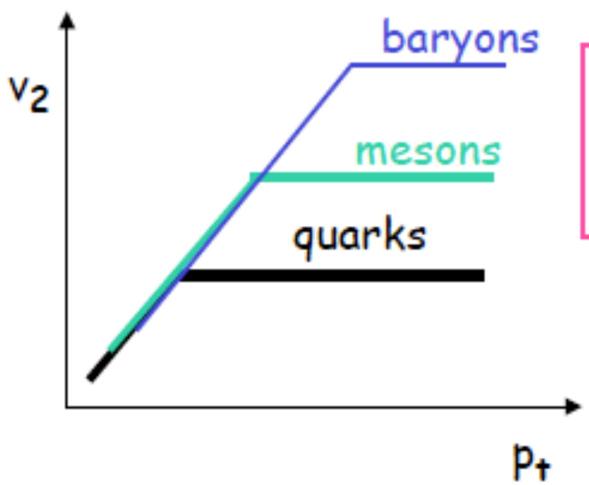
Note v_1 values > 0.5 ➔ there must be other non-zero harmonics!

Constituent quark model + coalescence

coalescence — fragmentation

Low p_t quarks — High p_t quarks

Coalescence in the intermediate region (rare products):

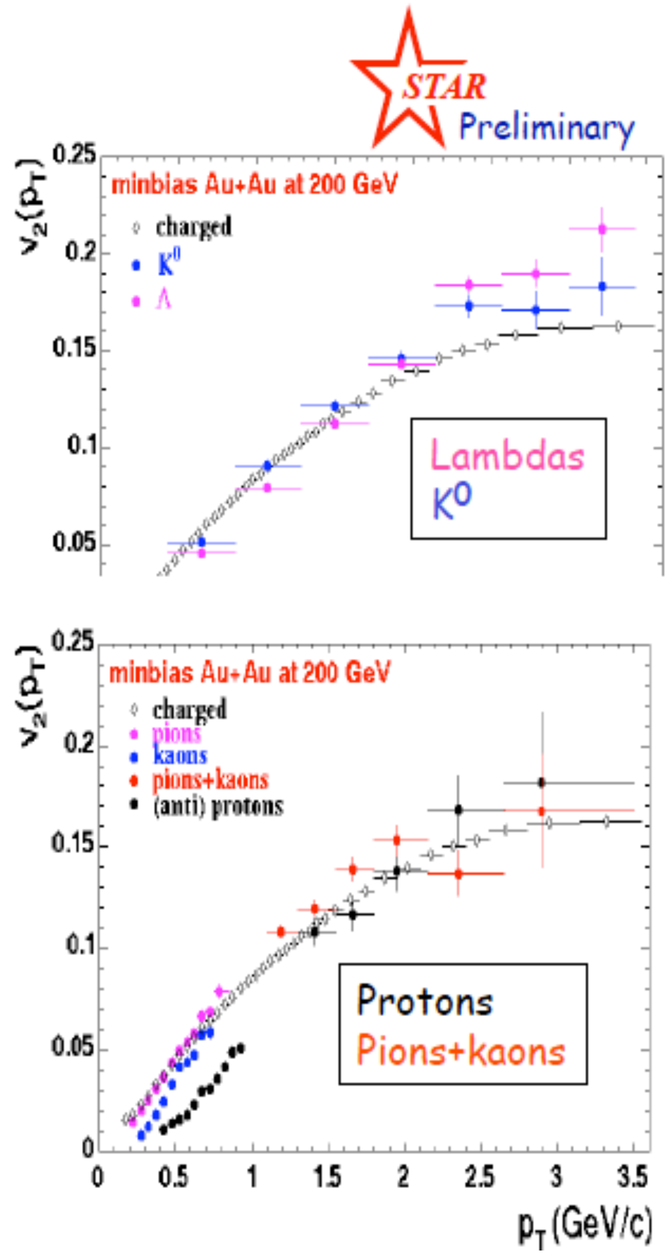


$$\frac{d^3 n_M}{d^3 p_M} \propto \left[\frac{d^3 n_q}{d^3 p_q}\left(p_q \approx p_M / 2\right)\right]^2$$

Side-notes:

a) more particles produced via coalescence vs parton fragmentation ➔ larger mean p_t...

b) ➔ higher baryon/meson ratio



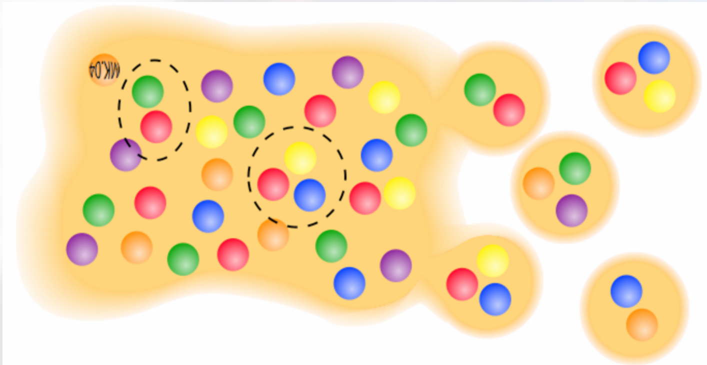
Constituent quark coalescence

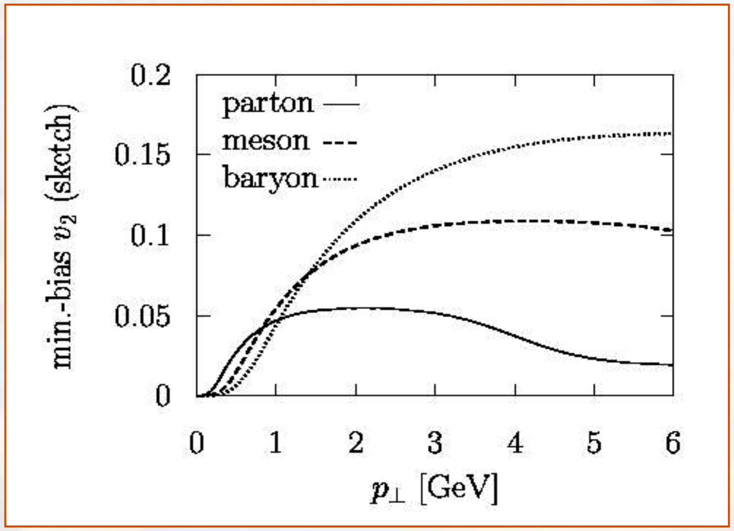
coalescence
fragmentation
Low p_t quarks
High p_t quarks

S.V., QM2002
D. Molnar, S.V., PRL 2003

In the intermediate region coalescence can be described by:

$$\frac{d^3 n_M}{d^3 p_M} \propto \left[\frac{d^3 n_q}{d^3 p_q} \left(p_q \approx p_M / 2 \right) \right]^2 \quad \rightarrow \quad \begin{aligned} v_{2,M}(p_t) &\approx 2 v_{2,q}(p_t/2) \\ v_{2,B}(p_t) &\approx 3 v_{2,q}(p_t/3) \end{aligned}$$



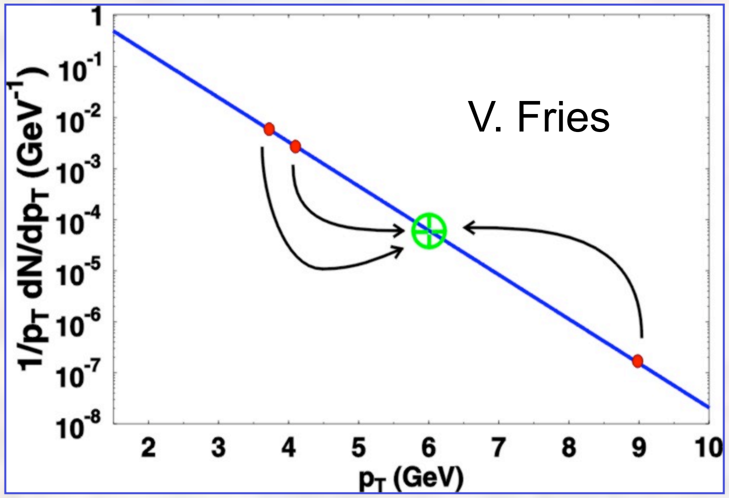


In the low pt region density is large, most quarks coalesce: N hadron ~ N quark

$$e^{-Bp_t^2} \prec (e^{-Bp_t^2/4})^2$$

In the high pt region fragmentation eventually wins:

$$p_t^{-n} \succ ((p_t/2)^{-n})^2$$

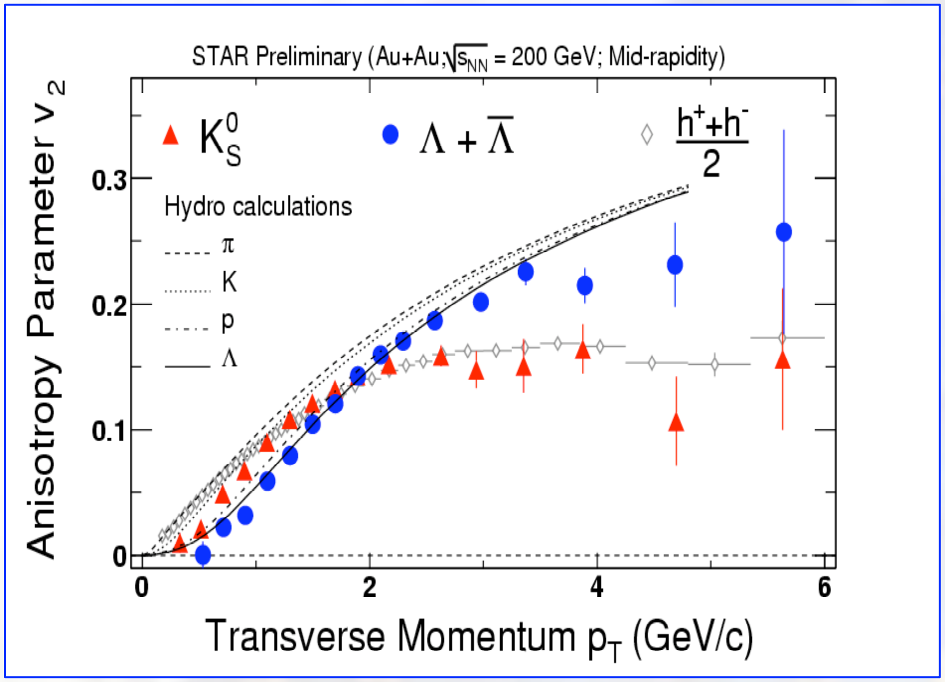


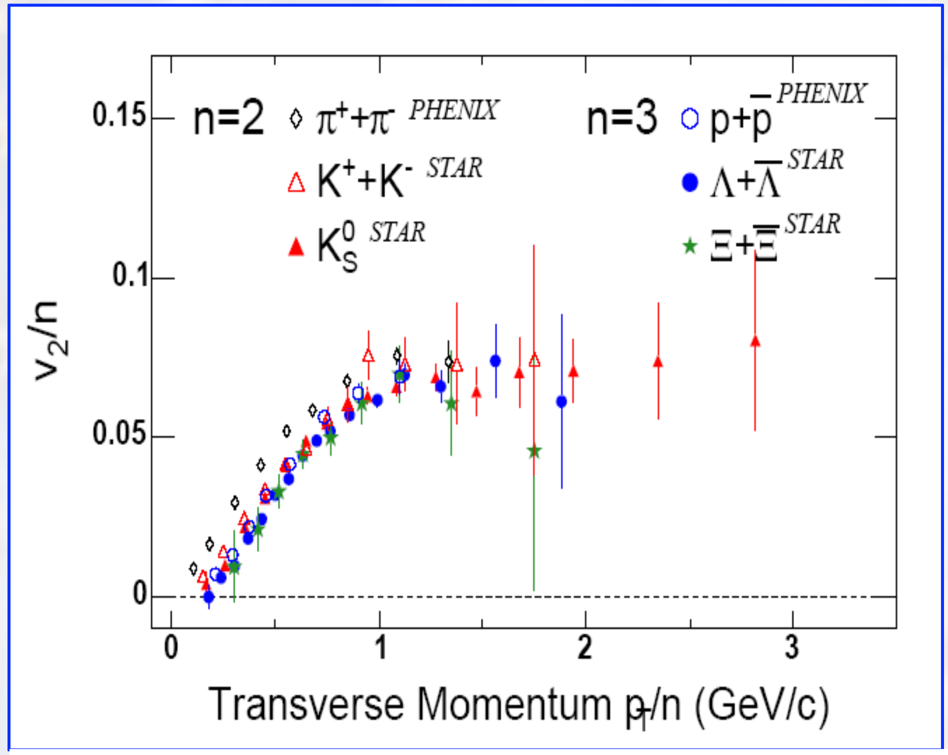
"Side-notes":
a) more particles produced via coalescence vs parton fragmentation ➔ larger mean p_t…
b) ➔ higher baryon/meson ratio
c) ➔ lower multiplicity per "participant"

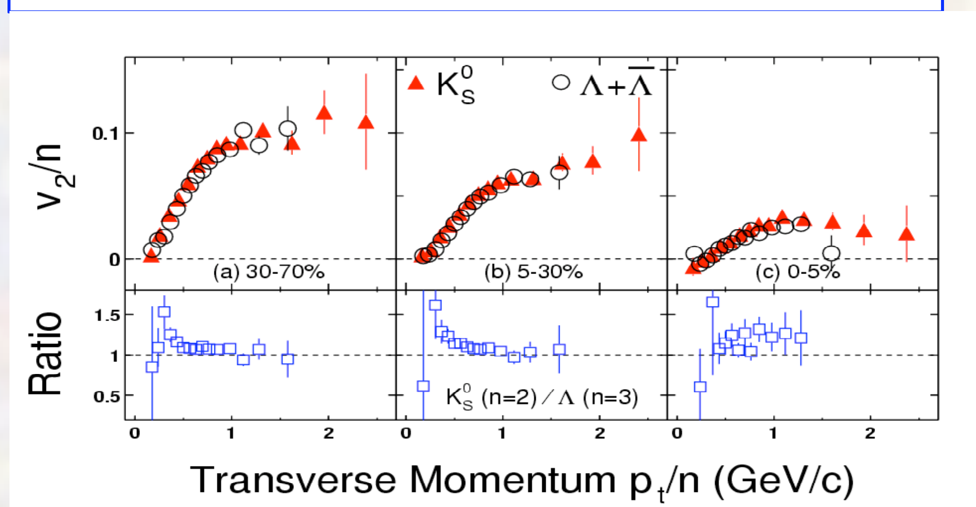
-> D. Molnar, QM2004
-> Bass, Fries, Mueller. Nonaka; Levai, Ko; …
-> Eremin, S.V., PRC2005

Constituent quark scaling

STAR PRL 92(2004)052302



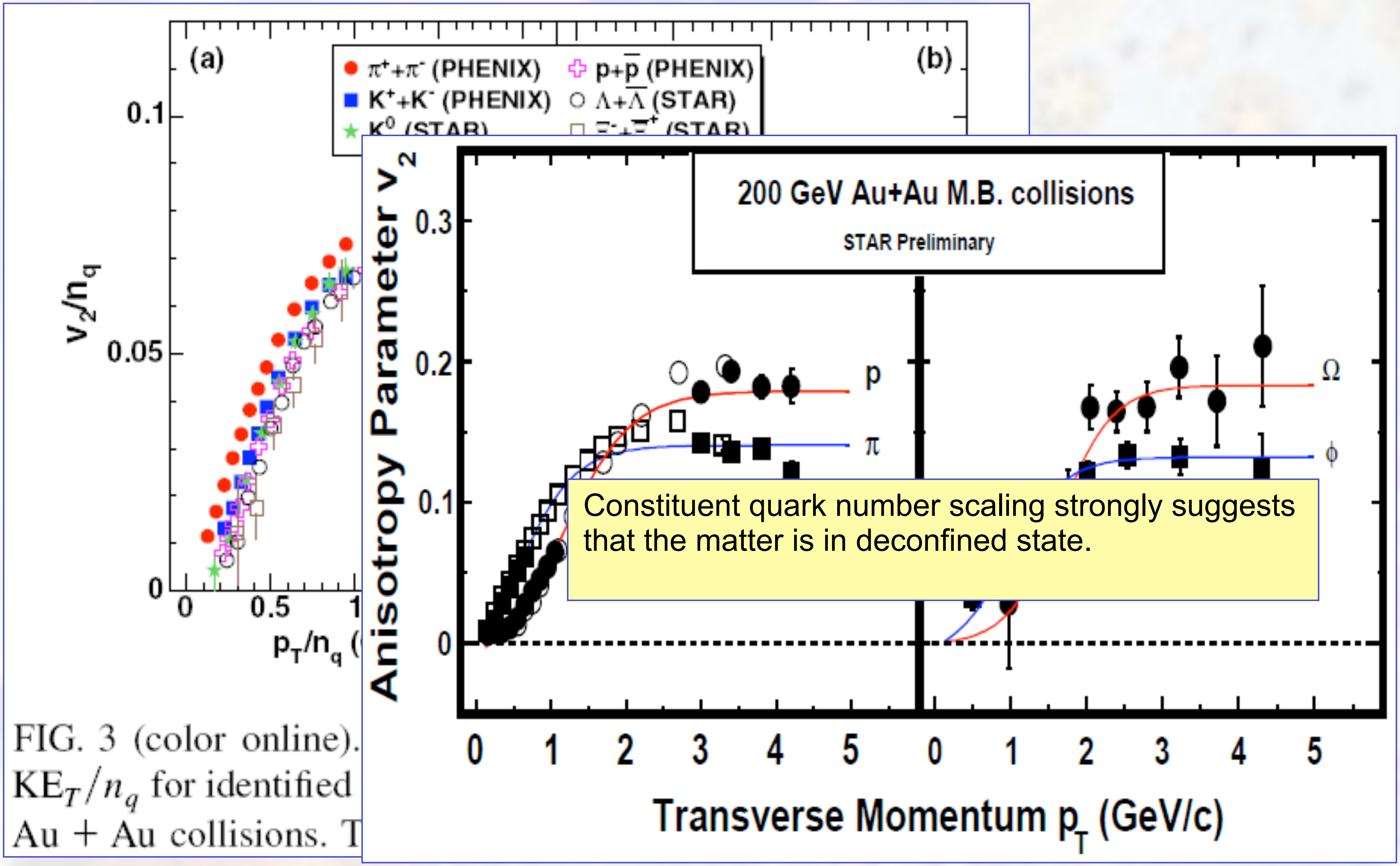




- Constituent quark scaling holds very well. Deviations are where expected.
- Elliptic flow saturates at pt ~ 1 GeV, just at constituent quark scale. An accident?

Gas of free(?) constituent quarks – deconfinement !

Number of constituent quark scaling



Constituent quark number scaling strongly suggests that the matter is in deconfined state.

interplay with radial flow

Directed flow "wiggle" in cascade models

Snellings, Poskanzer, S.V., nucl-ex/9904003

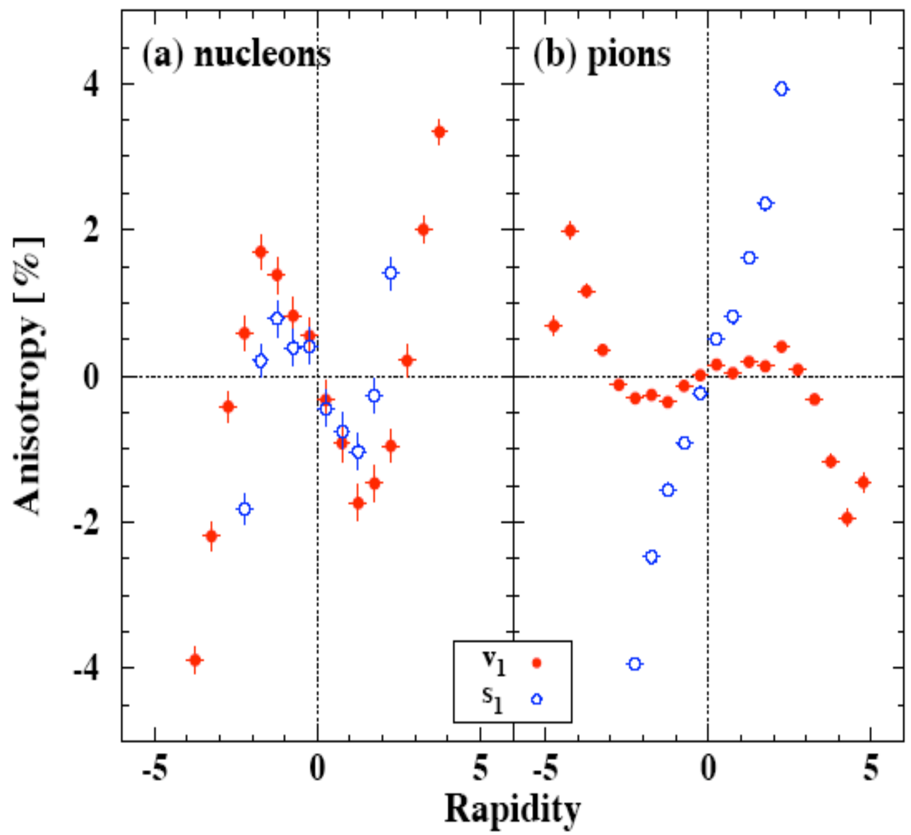
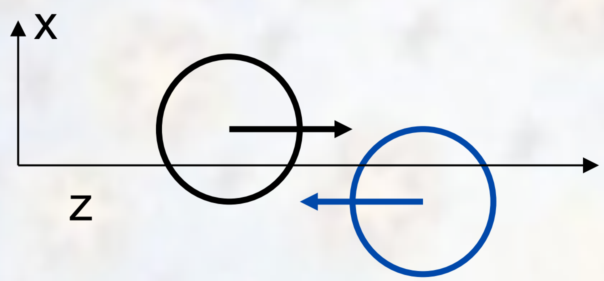
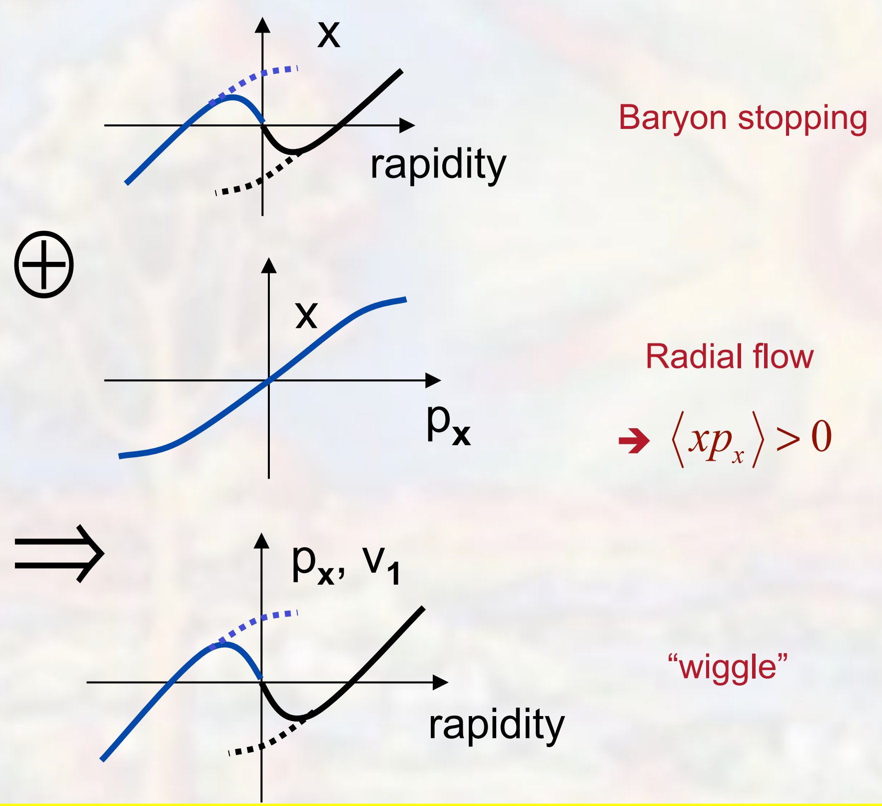


FIG. 2. RQMD calculations of v_1 (filled circles) and s_1 (open circles) for nucleons (left panel) and pions (right panel).

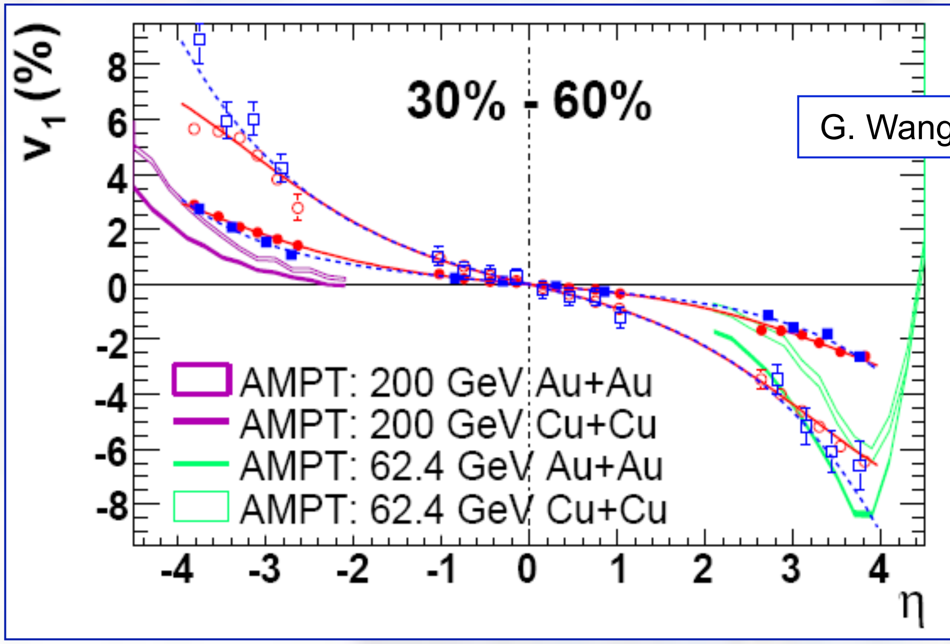
The wiggle is pronounced only at high energies

Snellings, Sorge, S.V., F. Wang, Nu Xu, PRL 84 (2000) 2803

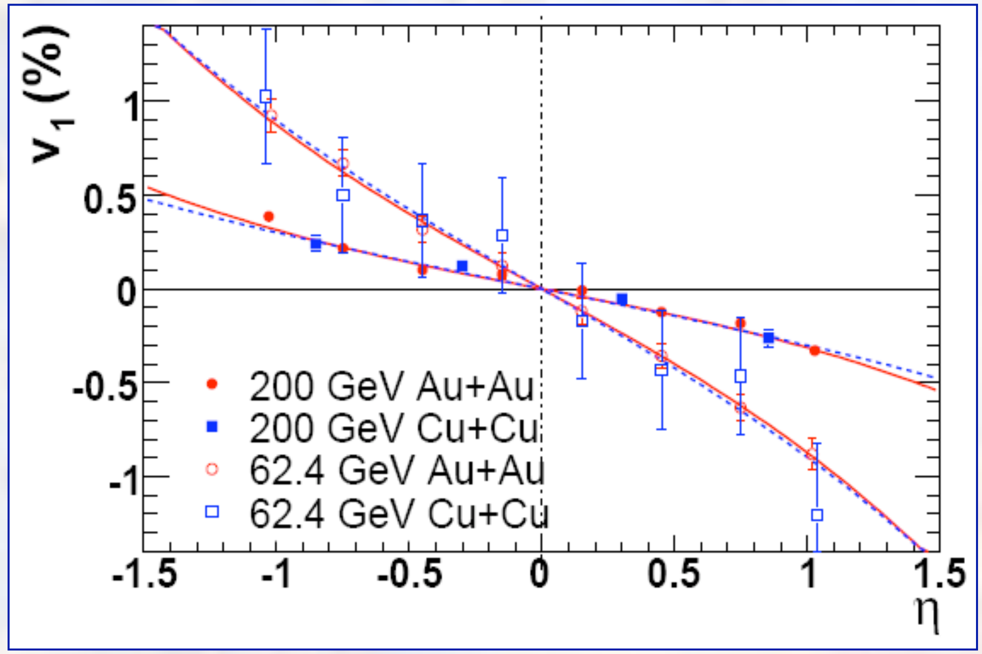




Directed flow



G. Wang (STAR), QM2006



Not quantitatively explained by any model (even close)!
Error-bars at the level of $< 10^{-3}$!

AuAu and CuCu are very similar at the same centrality!

(Magenta curves are polynomial fits to guide the eye)

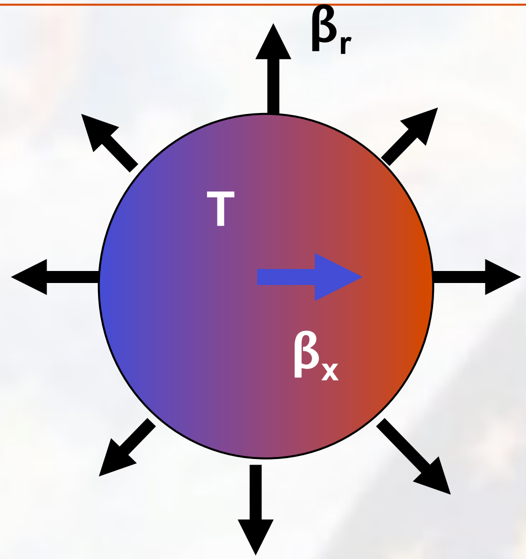
High accuracy of these measurements achieved by
- using STAR ZDC-SMD ("spectator neutrons")
- 3 –particle correlations (mixed harmonics)

3 particle correlations measure the difference in correlations projected into the reaction plane and out-of-plane directions making use of strong elliptic flow to define the plane..

$$\langle \cos(\phi_\alpha + \phi_\beta - 2\Psi_{RP}) \rangle$$
$$= \langle \cos(\phi_\alpha - \Psi_{RP})\cos(\phi_b - \Psi_{RP}) \rangle - \langle \sin(\phi_\alpha - \Psi_{RP})\sin(\phi_b - \Psi_{RP}) \rangle$$
$$\approx v_{1,a} v_{1,b}$$

Interplay of radial and anisotropic (directed) flow

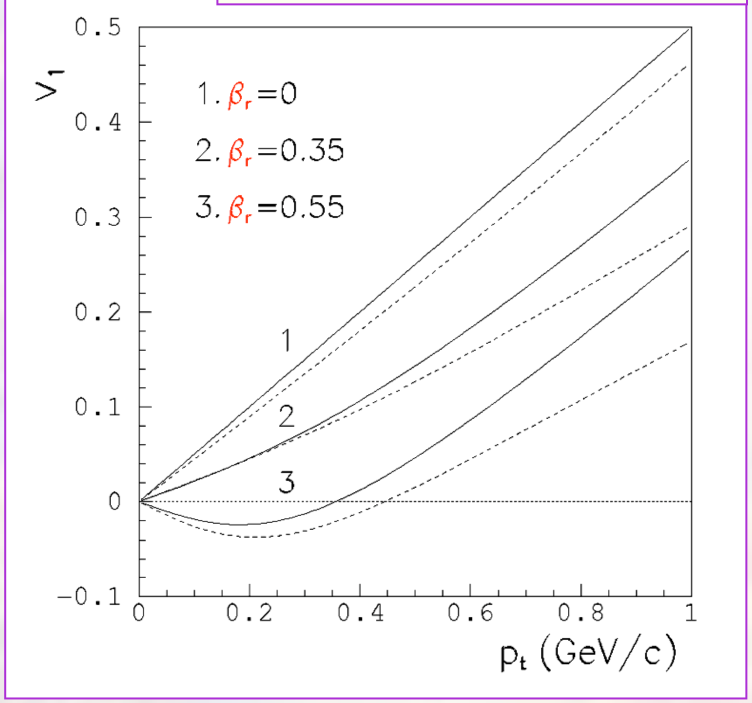
S.V., PRC 55 (1997) 1630



High pt particles are produced mostly in the red region; low pt particles – in the blue region

$$v_1(p_t) \approx \frac{p_t \beta_x}{2T}\left(1 - \frac{m\beta_r}{p_t}\frac{I_1(\beta_r p_t / T)}{I_0(\beta_r p_t / T)}\right)$$

Solid lines: non-relativistic case



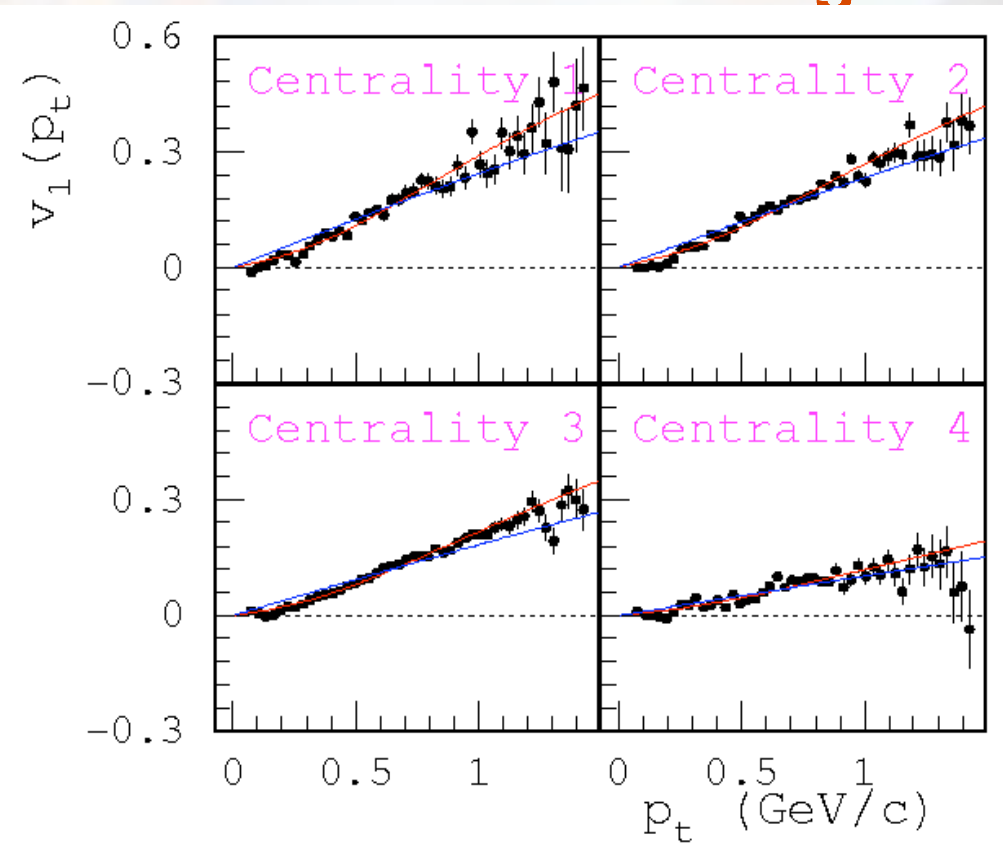
$\beta_x = 0.1 \quad T = 120$ MeV $(\beta_{thermal} \approx 0.5)$

Interplay of three velocities:
1) Thermal velocity
2) Radial expansion mean velocity
3) Anisotropic (modulation in radial expansion)

Note! Similar formalism can be achieved with totally different interpretation (not requiring thermalization), where the role of
Temperature plays mean pt change due to scattering
Radial flow velocity – mean radial component of particle velocity
Anisotropic flow velocity – the modulation in the above

The effect is larger for larger mass

Fitting the real data

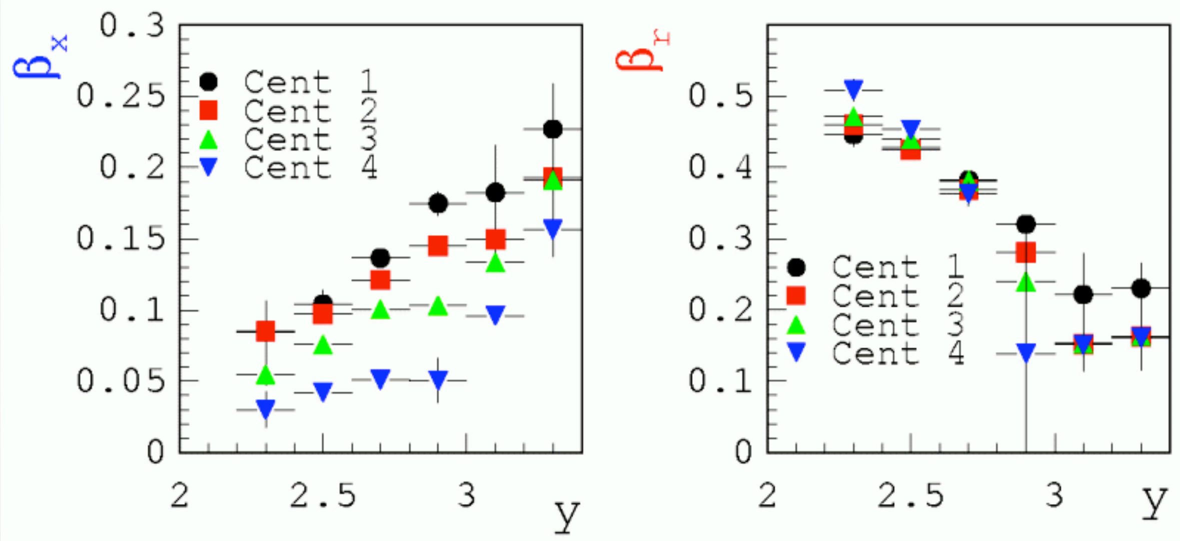


S.V and E877, Nucl Phys. A638, 455c (1998)

Directed flow of protons in Au+Au collisions at E_{lab}=11.4 GeV, 2.6 < y < 2.8

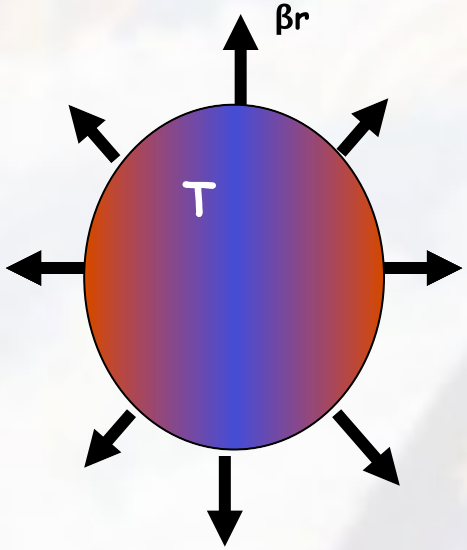
$$v_1(p_t) \approx \frac{p_t \beta_x}{2T}\left(1 - \frac{m\beta_r}{p_t}\frac{I_1(\beta_r p_t / T)}{I_0(\beta_r p_t / T)}\right)$$

$T = 110$ MeV, $y - y^* = 0.5$



$v_2(p_t)$ dependence. Blast wave model

$v_2(p_t)$ - Houvinen, Kolb, Heinz, Ruuskanen, S.V., PLB 503 (2001) 58

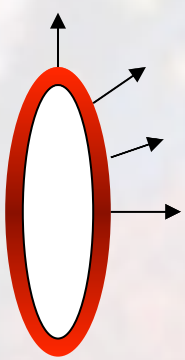


$$v_2(p_t) = \frac{\int_0^{2\pi} d\phi_s \cos(2\phi_s) I_2(\alpha_t) K_1(\beta_t)}{\int_0^{2\pi} d\phi_s I_0(\alpha_t) K_1(\beta_t)}$$

$$\alpha_t = (p_t / T)\sinh(\rho);\ \beta_t = (m_t / T)\cosh(\rho);$$

$$\rho = \rho_0 + \rho_a \cos(2\phi_s)$$

$v_2(p_t)$ - STAR Collaboration, PRL 87 (2001) 182301



$\oplus$ Elementary source density -

$$\propto 1 + 2 s_2 \cos(2\varphi_s)$$

Note:

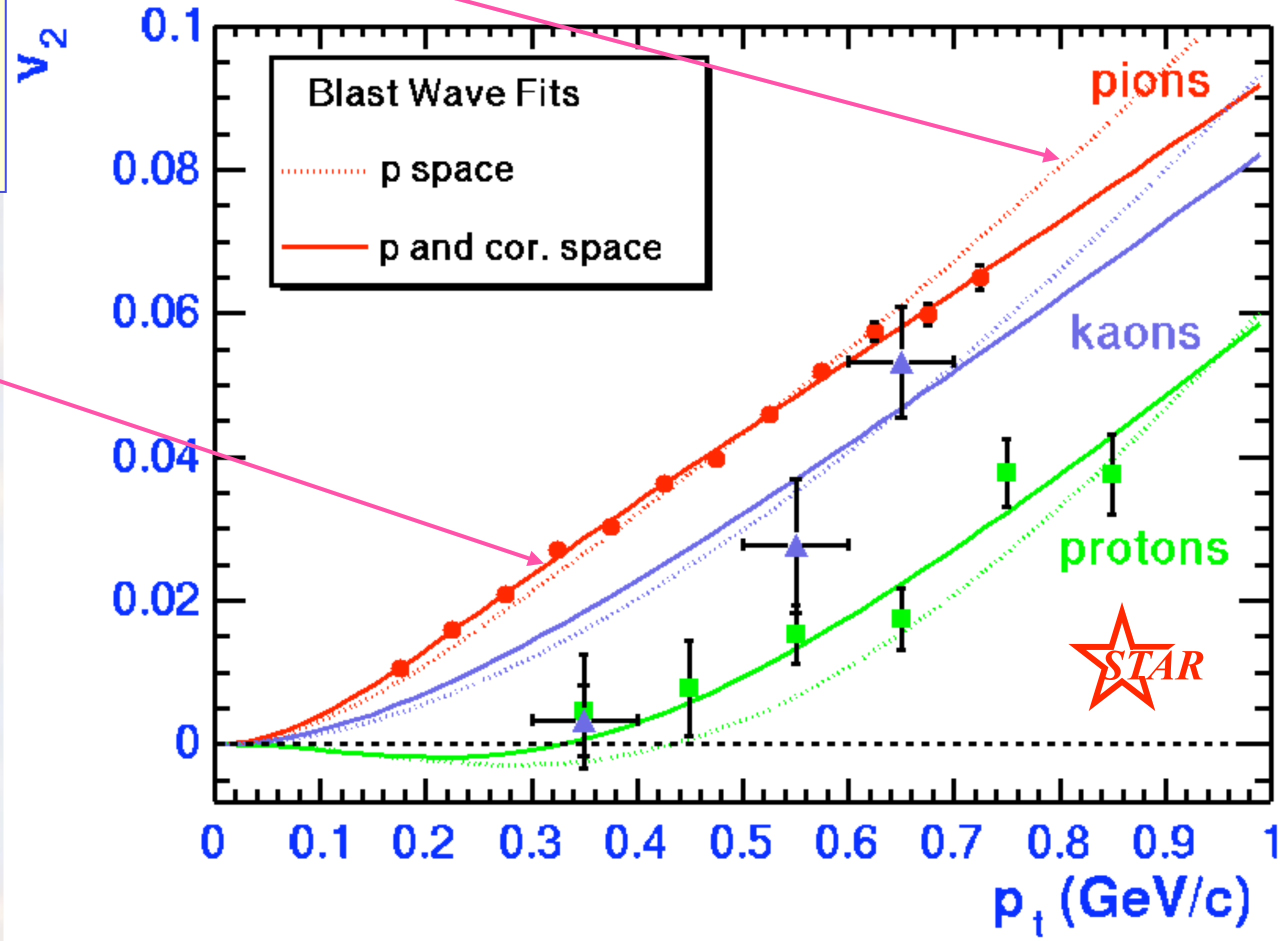
a) The possibility different interpretation of the parameters (other than "hydro-like")

b) The possibility of different "realization" of the parameter s_2. There is no strict correspondence between this parameter and the shape of the source at freeze-out.

Fit to data

Parameters:
T – temperature
ρ_0 - radial expansion rapidity
ρ_2 - amplitude of azimuthal variation in expansion rapidity

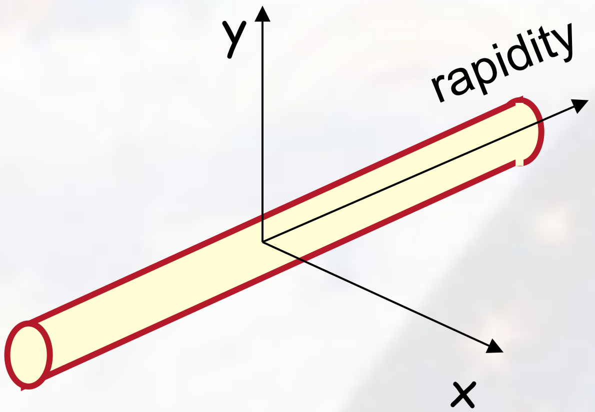
⊕ Elementary source density - $\propto 1 + 2 s_2 \cos(2\varphi_s)$



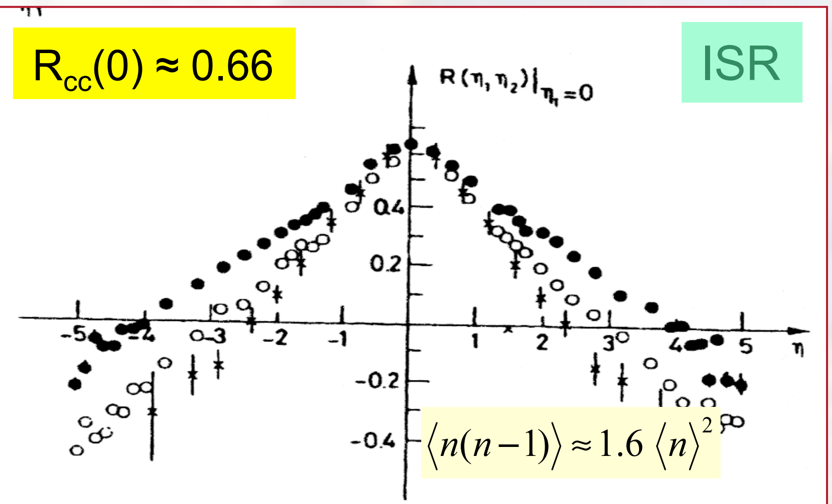
T (MeV)	dashed 135 ± 20	solid 100 ±24
ρ_0	0.52 ± 0.02	0.54 ± 0.03
ρ_a	0.09 ± 0.02	0.04 ± 0.01
s_2	0.0	0.04 ± 0.01

- model fits data well
- shape (s_2 parameter) agrees with the interferometry measurements (see below) under assumption that flow velocity field is normal to the surface

"Elementary" NN-collision. Inclusive correlation functions.



Correlations are due to local charge(s) conservation, resonances, fluctuations in number of produced strings, e.g. number of qq-collisions in const. quark approach



At midrapidity, the probability to find a particle is about 60% larger if one particle has been already detected.

Inclusive

$$\int dy\, \rho_1(y) = \langle n \rangle$$

$$\int dy_1 \int dy_2 \rho_2(y_1, y_2) = \langle n(n-1) \rangle$$

Distribution of "correlated" pairs:

$$C(y_1, y_2) = \rho_2(y_1, y_2) - \rho_1(y_1)\rho_1(y_2)$$

Distribution of "associated" particles (2) per "trigger" particle (1), "Balance function"

$$B(y_1, y_2) = \frac{C(y_1, y_2)}{\rho_1(y_1)}$$

"Probability" to find a "correlated" pair

$$R(y_1, y_2) = \frac{C(y_1, y_2)}{\rho_1(y_1)\rho_1(y_2)}$$

Semi- inclusive (topological) $\rho_k \to \rho_k^{(n)}$

might use "probability density" correlation functions: $\rho_k^{(n)} \to p_k^{(n)}$; e.g. $p_2^{(n)} = \rho_2^{(n)} / n(n-1)$

Production via N_c clusters [e.g. independent NN collisions]

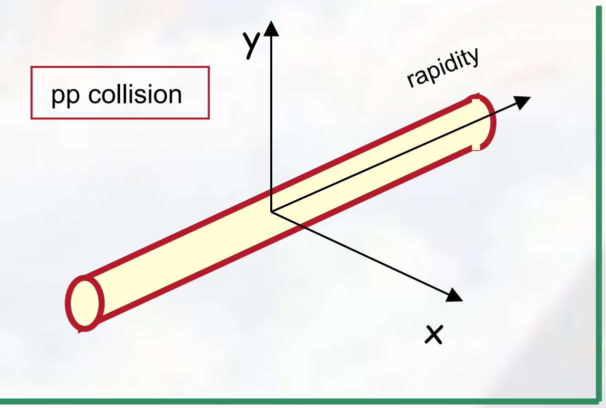
$$\rho_1^{\{Nc\}}(y) = N_c \rho_1^{\{1\}}(y);$$

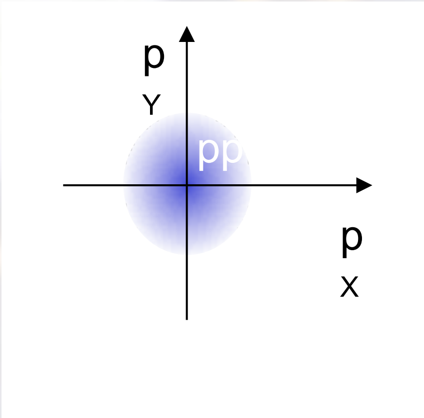
$$\rho_2^{\{Nc\}}(y_1, y_2) = N_c \rho_2^{\{1\}}(y_1, y_2) + N_c(N_c - 1)\rho_1^{\{1\}}(y_1)\rho_1^{\{1\}}(y_2)$$

$$R^{\{Nc\}} = \frac{N_c \rho_2^{\{1\}}(y_1, y_2) + N_c(N_c - 1)\rho_1^{\{1\}}(y_1)\rho_1^{\{1\}}(y_2) - N_c^2 \rho_1^{\{1\}}(y_1)\rho_1^{\{1\}}(y_2)}{N_c^2 \rho_1^{\{1\}}(y_1)\rho_1^{\{1\}}(y_2)} = \frac{R^{\{1\}}}{N_c}$$

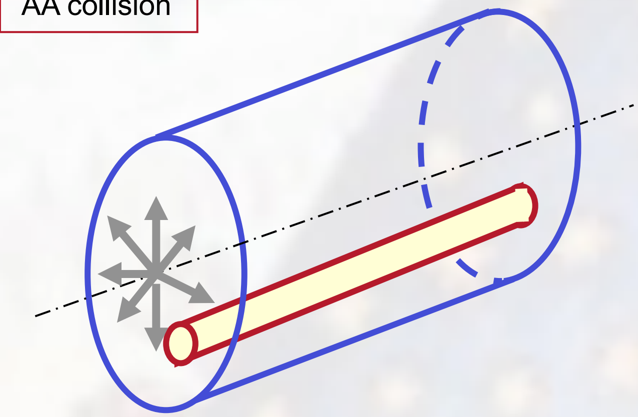
Radial expansion ➔ 2-part azimuthal correlations

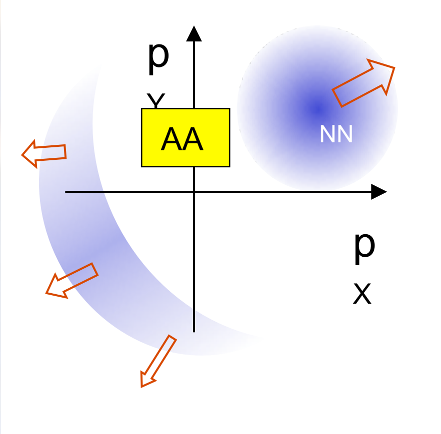
[arXiv:nucl-th/0312065]





All particles produced in the same NN-collision (qq-string) experience the transverse radial "push" that is
(a) in the same direction (leads to correlations in phi)
(b) the same in magnitude (→ correlations in p_t)





Particle correlations existed in pp – become modified.

→ Long range rapidity correlations become narrow in phi –"ridge" develops
→ Stronger 2-particle pt correlation in narrow phi bins.
→ Narrowing of the charge balance function ($\Delta p_z \approx m_t \sinh(\Delta y)$-- increase in m_t → decrease in rapidity separation)
→ Charge correlations become narrow in phi. Azimuthal Balance function
→ stronger in-plane than out-of-plane, etc.

Radial expansion ➔ 2-part azimuthal correlations

S.A. Voloshin / Physics Letters B 632 (2006) 490–494 493

[arXiv:nucl-th/0312065]

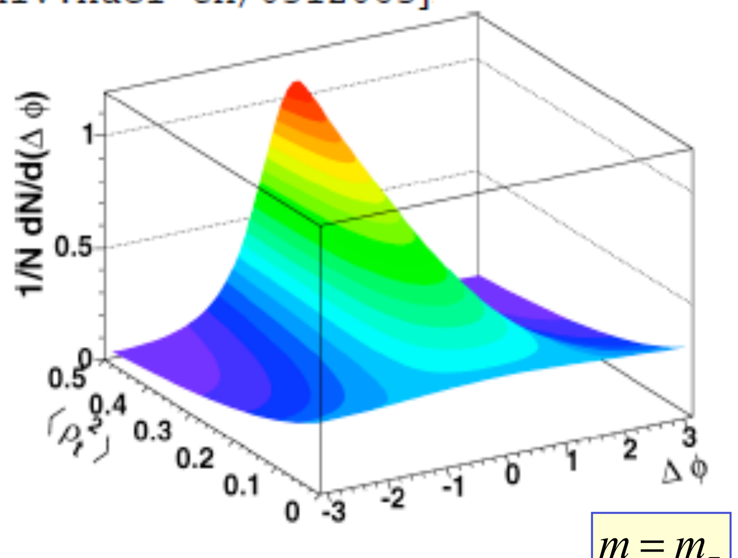


Fig. 3. (Color online.) Two pion $\Delta\phi$ distribution as function of $\langle \rho_t^2 \rangle$ in the blast wave model. Linear velocity profile and $T = 110$ MeV have been assumed.

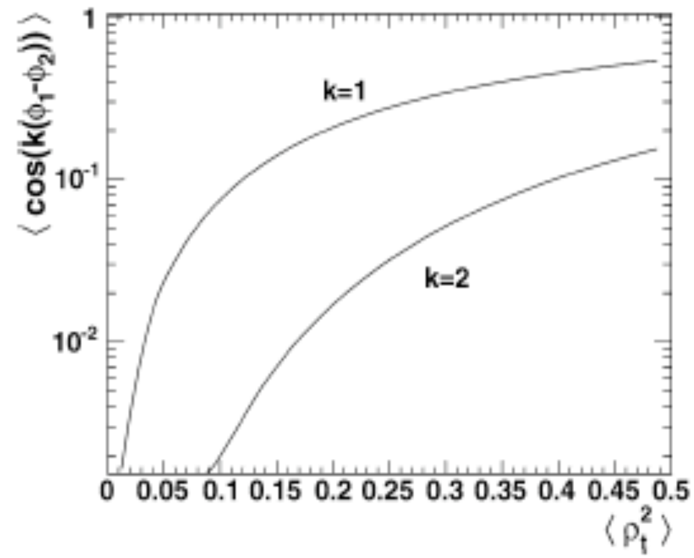
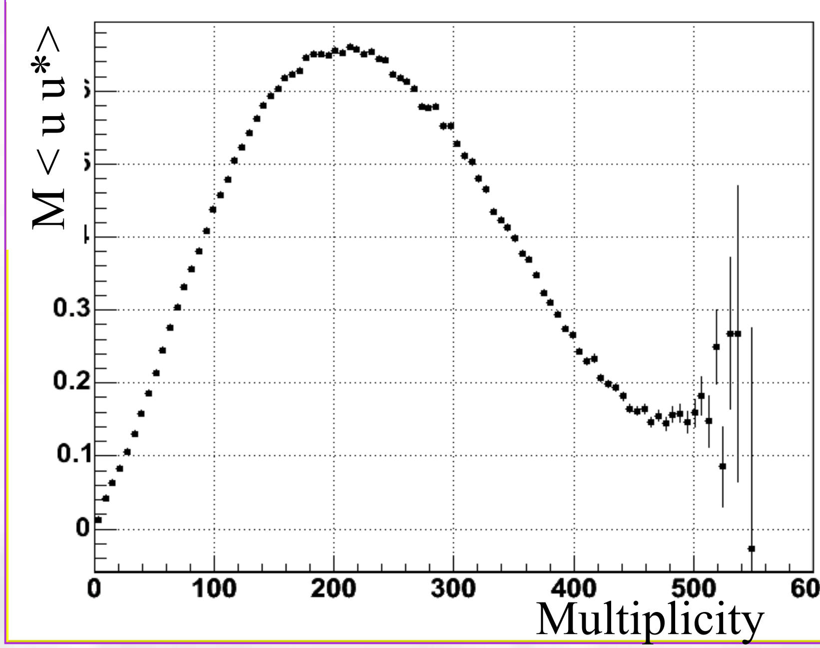


Fig. 4. The average values of $\cos(\Delta\phi)$ and $\cos(2\Delta\phi)$ for the distribution shown in Fig. 3.



Figures are shown for particles from the same NN collision. Dilution factor to be applied!

!!! - the large values of transverse flow, $\rho_t^2 > 0.25$, would contradict "non-flow"

estimates in elliptic flow measurements

If the momentum conservation effect is approximated by the first harmonic, the amplitude can be estimated from the momentum of the tag particle + "associates" from the same NN collision

PHOBOS' correlation functions

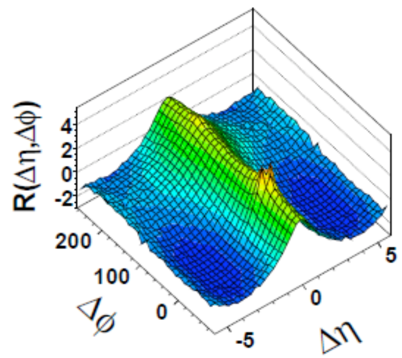


Figure 1: Inclusive two-particle correlation function in $(\Delta\eta,\ \Delta\phi)$ for p+p collisions at $\sqrt{s} = 200$ GeV [3].

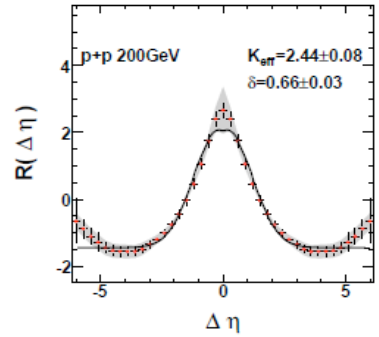


Figure 2: 1D two-particle pseudorapidity correlation function in $\Delta\eta$ for p+p collisions at $\sqrt{s} = 200$ GeV together with a fit from a cluster model [3].

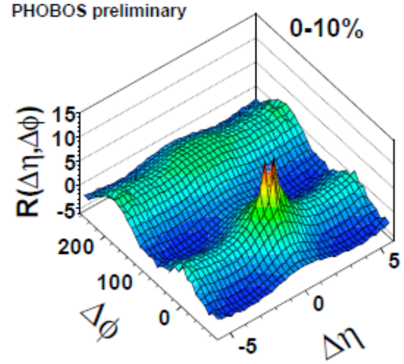


Figure 3: Two-particle correlation function in $(\Delta\eta,\ \Delta\phi)$ for the most central 10% of the Au+Au collisions at $\sqrt{s_{NN}} = 200$ GeV.

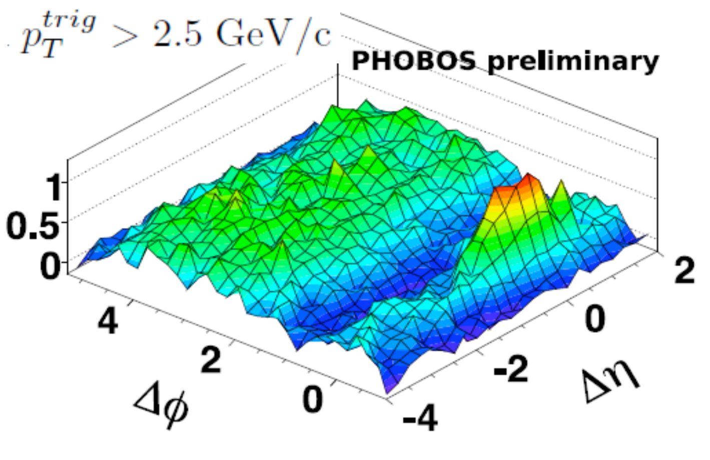
arXiv:0804.2471v2 [nucl-ex] 25 Apr 2008

$$R(\Delta\eta,\Delta\phi) = \left\langle (n-1)\left(\frac{F_n(\Delta\eta,\Delta\phi)}{B_n(\Delta\eta,\Delta\phi)} - 1\right)\right\rangle$$

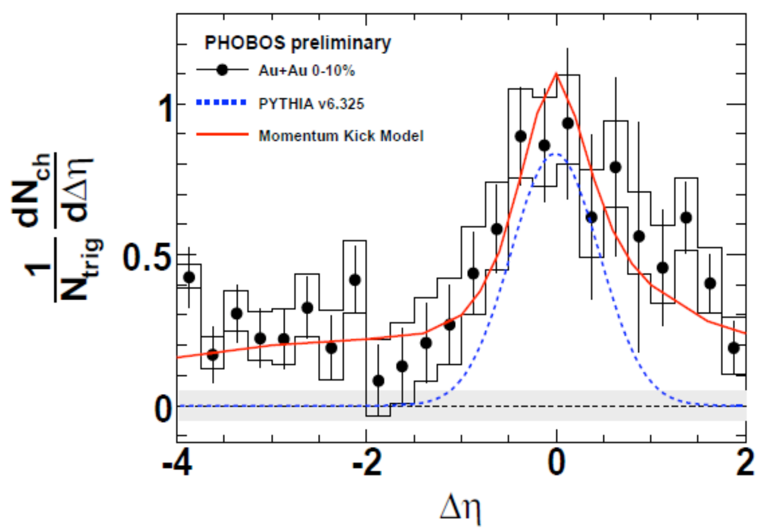
arXiv:0804.3038v3 [nucl-ex] 19 May 2008

$$\frac{1}{N_{trig}}\frac{d^2N_{ch}}{d\Delta\phi d\Delta\eta} = B(\Delta\eta)\cdot\left[\frac{s(\Delta\phi,\Delta\eta)}{b(\Delta\phi,\Delta\eta)} - a(\Delta\eta)[1+2V(\Delta\eta)cos(2\Delta\phi)]\right]$$

$p_T^{trig} > 2.5$ GeV/c



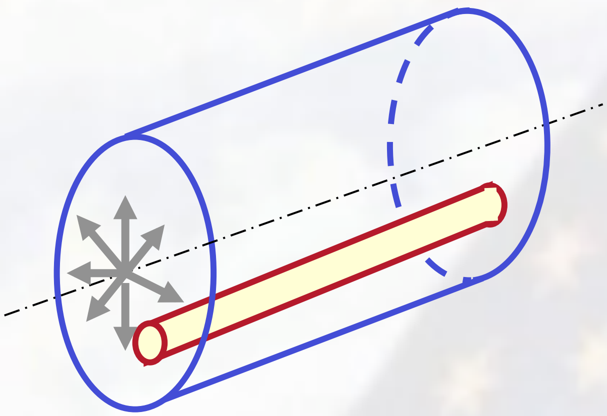
(b) Au+Au 0-30%

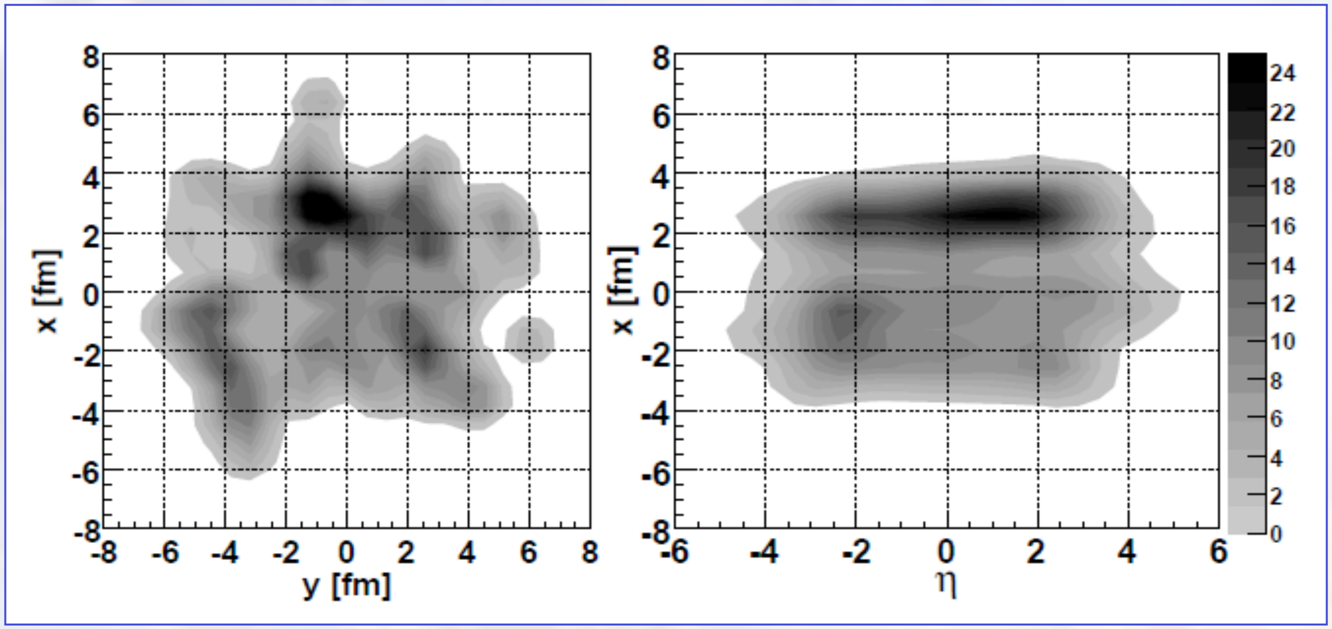


$a(\Delta\eta)$ ← (ZYAM) !?

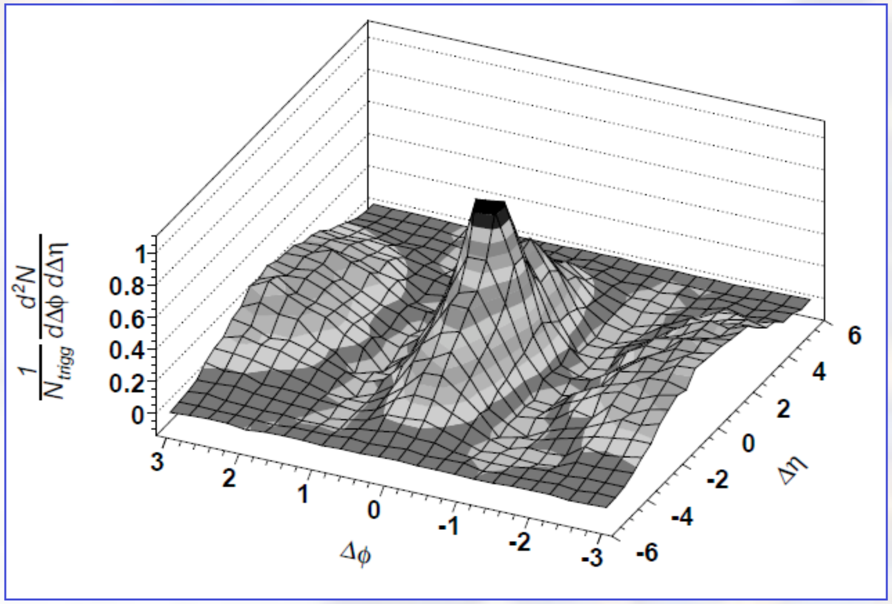
Fluctuating initial conditions

J. Takahashi *et al.*,arXiv:0902.4870v1

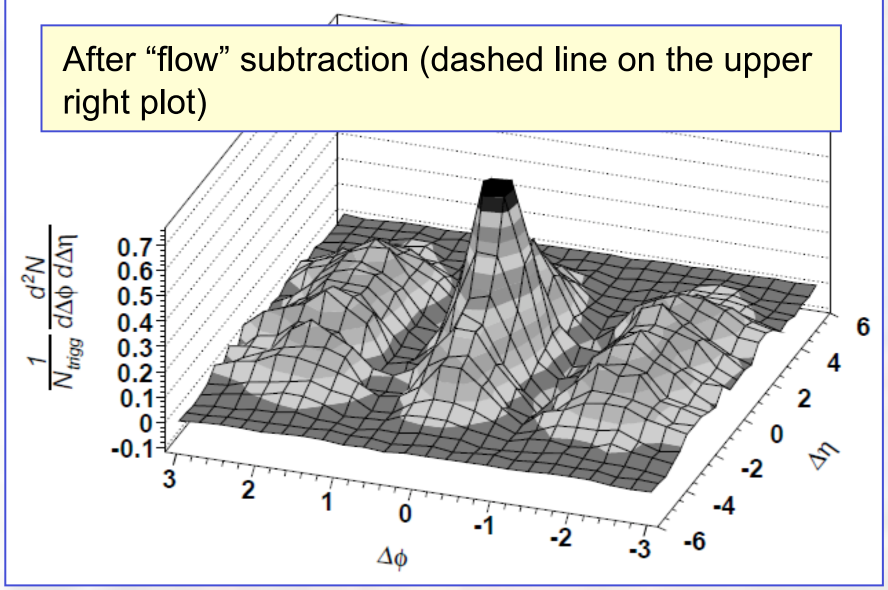


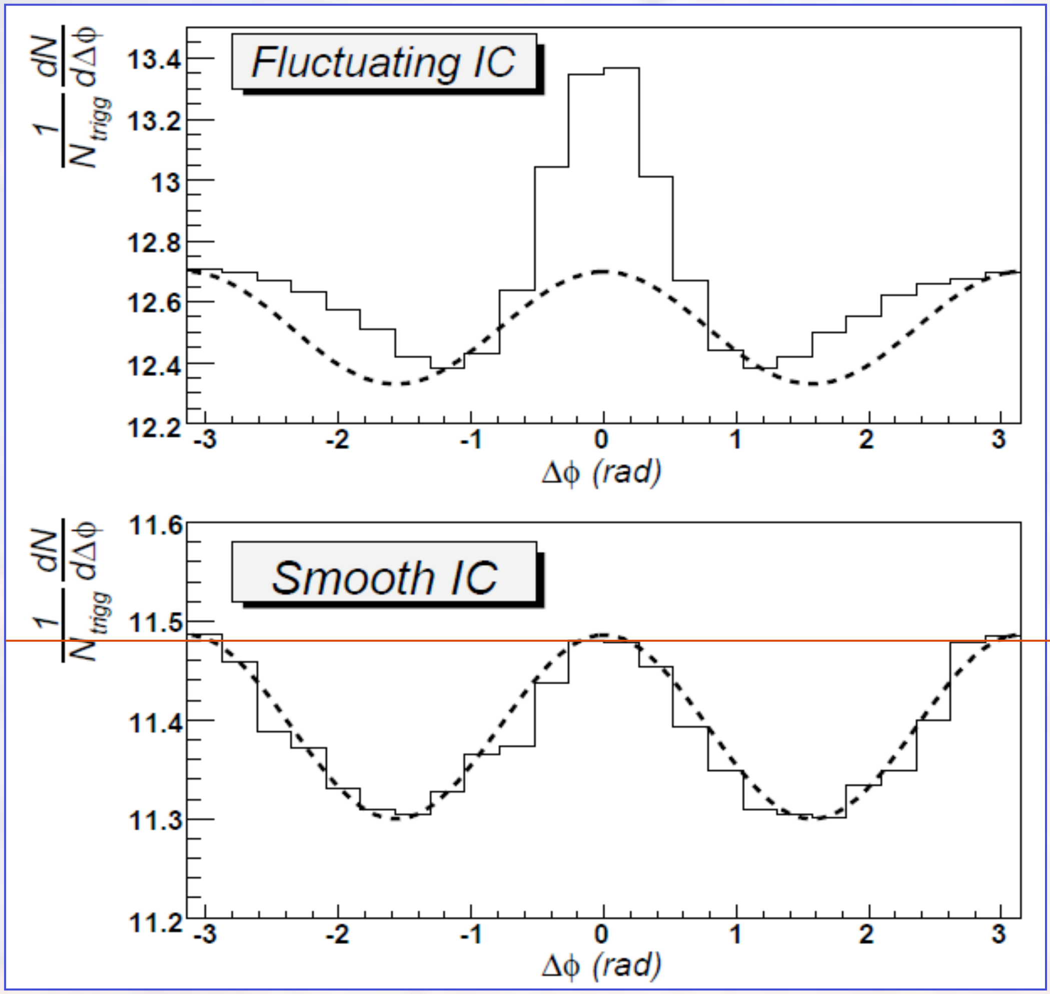


Correlation function



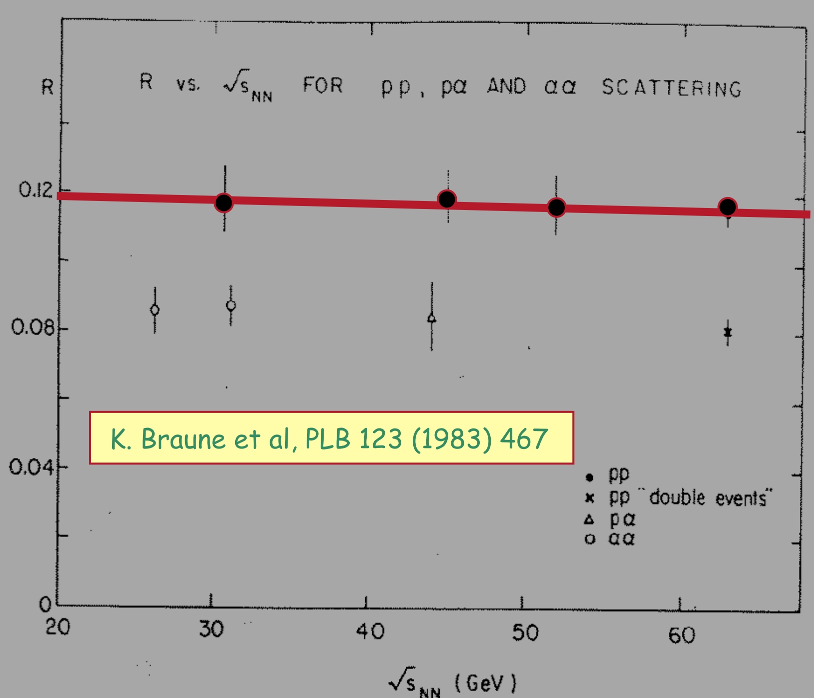
After "flow" subtraction (dashed line on the upper right plot)





Correlations (particle interactions) modify background ("flow").
Ignoring such modification produces perfect "Mach" cone

2-particle p_t correlations: $\langle \delta p_{t,1} \delta p_{t,2} \rangle; \quad \delta p_{t,i} = p_{t,i} - \langle p_t \rangle$



K. Braune et al, PLB 123 (1983) 467

Fig. 2. Normalized dis[...]f average trans-verse momentum dist[...] reactions as a function of cms energ[...]ollision $\sqrt{s}$NN. Contributions to R from statistical fluctuations of $\bar{p}_T$ are removed by the method described in the text.

$$R \approx \frac{\sqrt{\langle \delta p_{t,1} \delta p_{t,2} \rangle}}{\langle p_t \rangle}$$

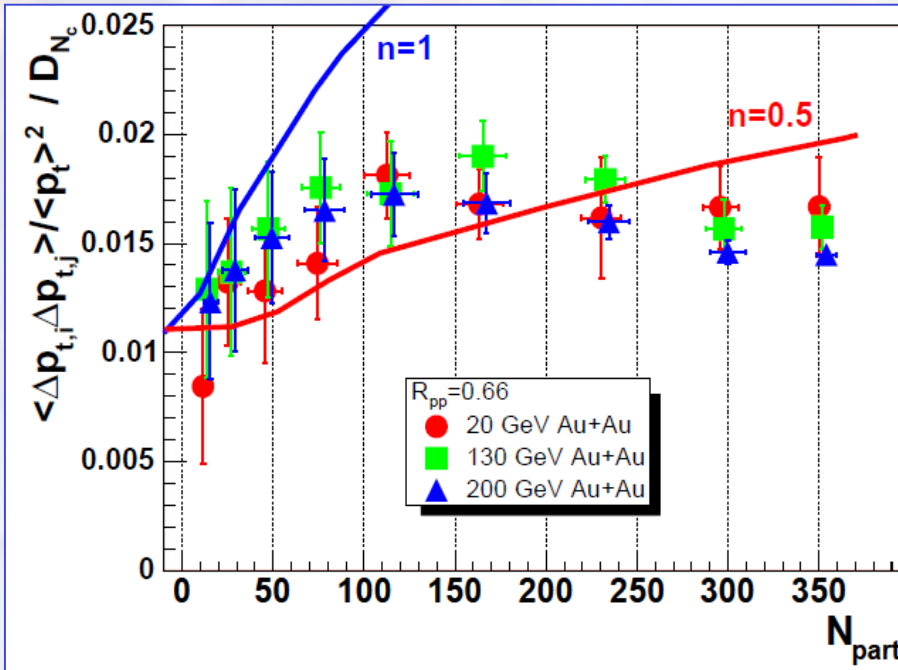


Figure 2. Comparison of Blast Wave calculations for two different velocity profiles with preliminary STAR data [7]. Relation $\langle n(n-1) \rangle_{NN} = 1.66 \langle n \rangle_{NN}^2$ has been used.

➔ $\langle \delta p_{t,1} \delta p_{t,2} \rangle / \langle p_t \rangle^2 \approx 0.014$

Production via N_c clusters (N_c~N_{part}/2)

$$\langle \delta p_{t,1} \delta p_{t,2} \rangle_{AA} = D_{N_{coll}} \langle \delta p_{t,1} \delta p_{t,2} \rangle_{NN}$$

$$D_{N_{coll}} = \frac{\langle n(n-1) \rangle_{NN}}{(N_{coll}-1)\langle n \rangle_{NN}^2 + \langle n(n-1) \rangle_{NN}}$$

WAYNE STATE UNIVERSITY

eccentricity fluctuations

Reaction, "participant", and event (flow vector) planes

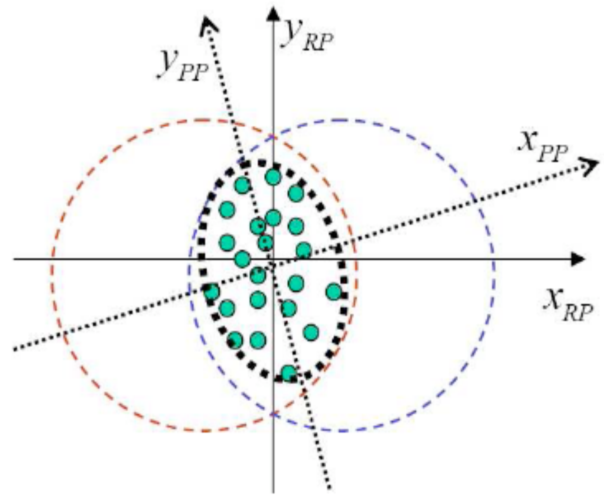


FIG. 1: The definition of the RP and PP coordinate systems.

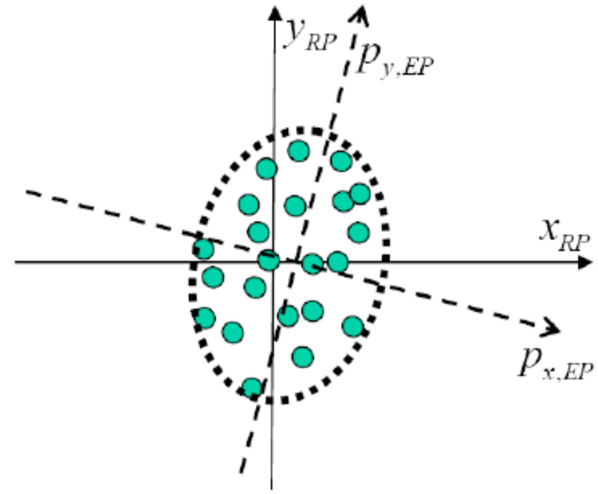
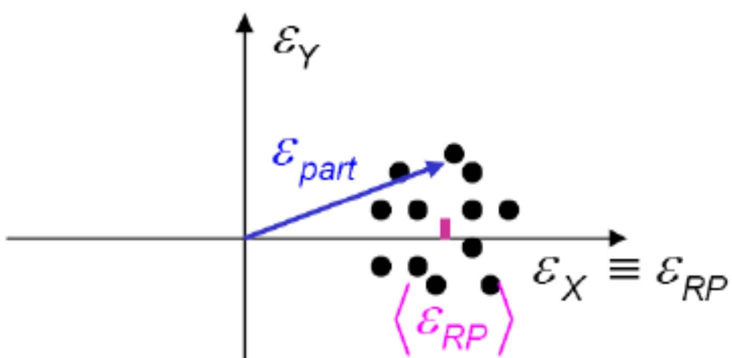


FIG. 2: The definitions of the EP coordinate system

$$v = \kappa \varepsilon$$

$$\varepsilon = \{\varepsilon_x, \varepsilon_y\} = \left\{ \left\langle \frac{\sigma_y^2 - \sigma_x^2}{\sigma_x^2 + \sigma_y^2} \right\rangle_{part}, \left\langle \frac{2\sigma_{xy}}{\sigma_x^2 + \sigma_y^2} \right\rangle_{part} \right\}$$



$$\sqrt{\varepsilon_x^2 + \varepsilon_y^2} \equiv \varepsilon_{PP}$$

FIG. 3: Definition of ε_{part}.

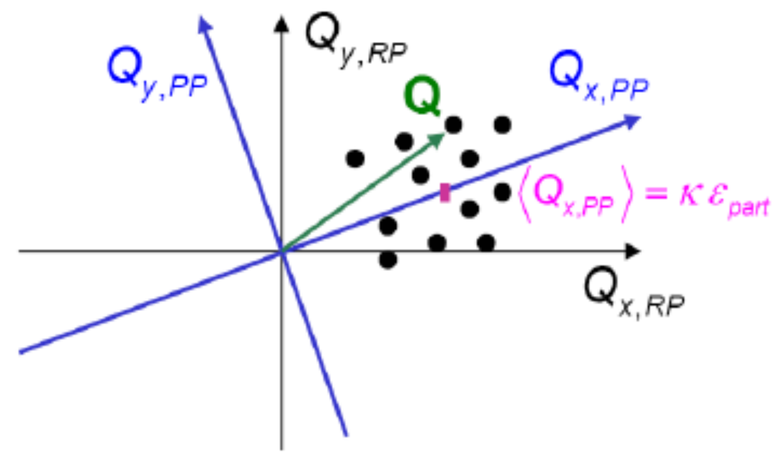
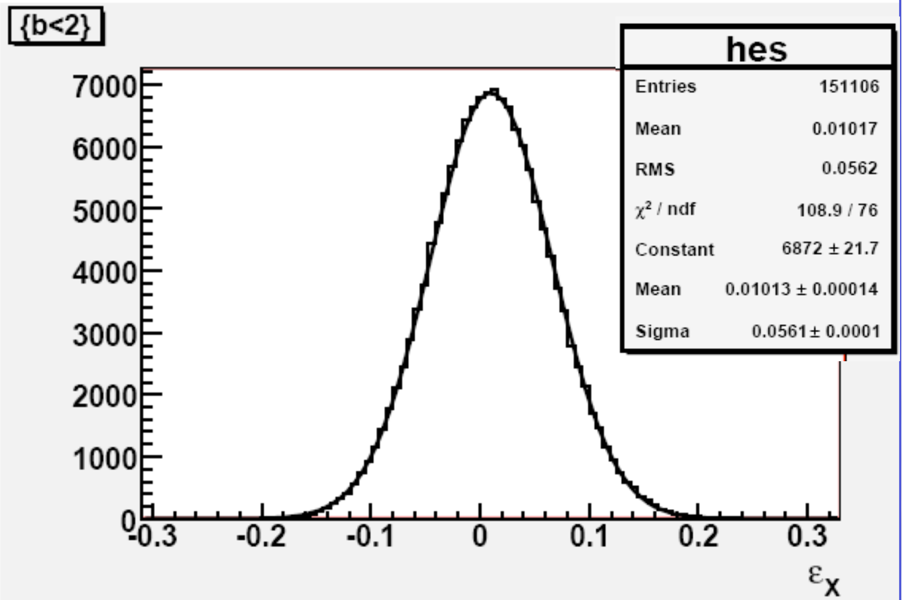
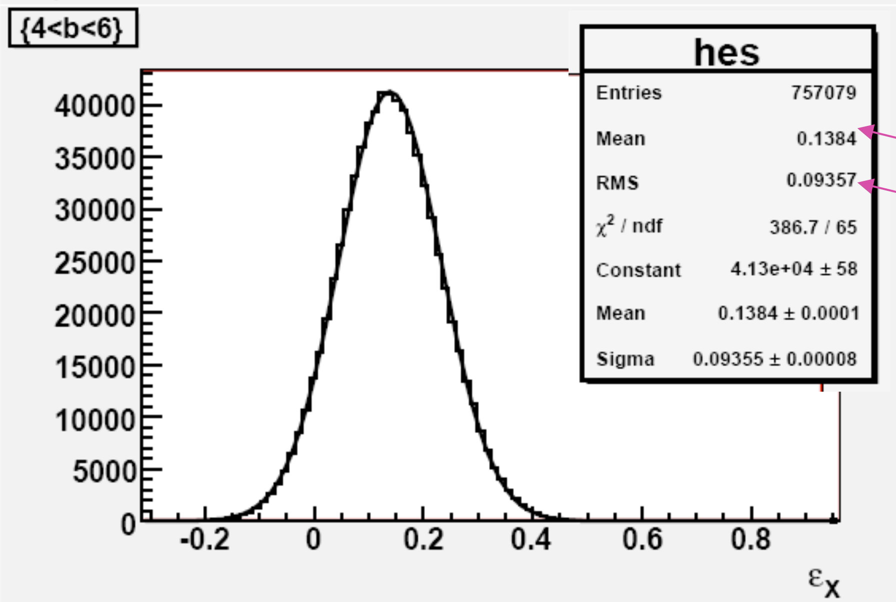


FIG. 4: Flow vector distribution in events with fixed ε

Distributions in ε_x , ε_y





$$\varepsilon = \{\varepsilon_x, \varepsilon_y\} = \left\{ \left\langle \frac{\sigma_y^2 - \sigma_x^2}{\sigma_x^2 + \sigma_y^2} \right\rangle_{part}, \left\langle \frac{2\sigma_{xy}}{\sigma_x^2 + \sigma_y^2} \right\rangle_{part} \right\}$$

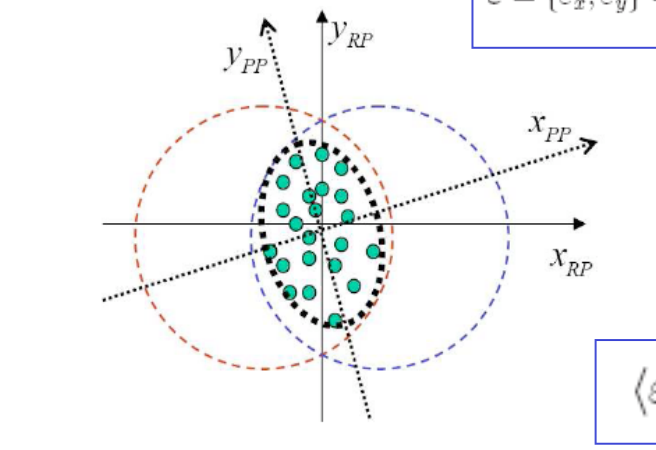
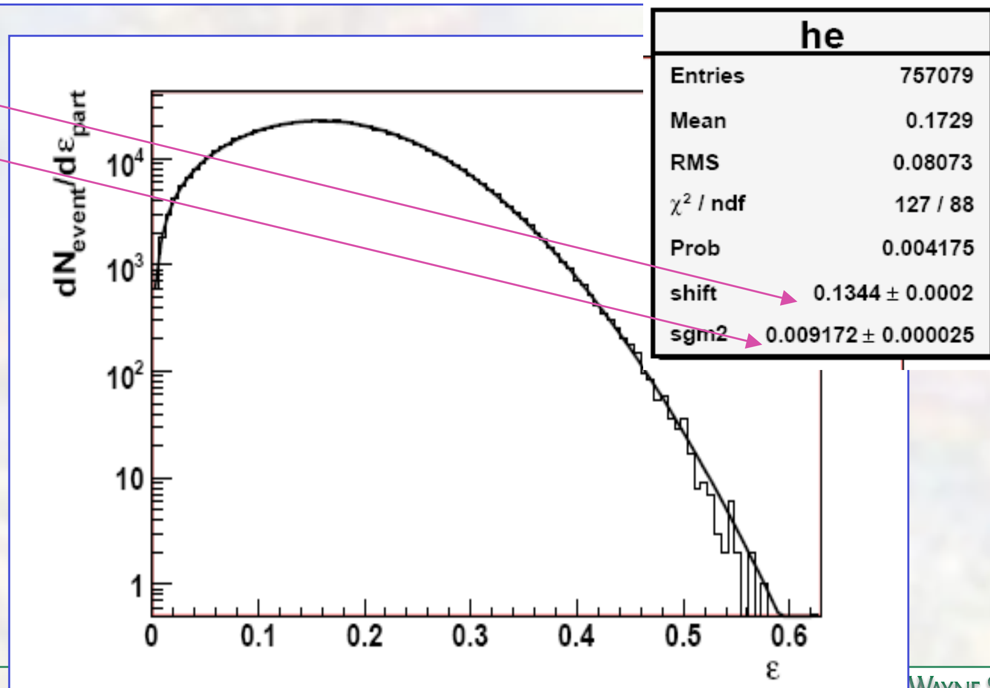


FIG. 1: The definition of the *RP* and *PP* coordinate systems.

$$\langle \varepsilon_x \rangle \equiv \bar{\varepsilon} \approx \varepsilon_{opt}.$$

$$\frac{dn}{d\varepsilon_{part}} = \frac{\varepsilon_{part}}{\sigma_\varepsilon^2} I_0\left(\frac{\varepsilon_{part}\langle\varepsilon_{RP}\rangle}{\sigma_\varepsilon^2}\right) \exp\left(-\frac{\varepsilon_{part}^2 + \langle\varepsilon_{RP}\rangle^2}{2\sigma_\varepsilon^2}\right) \equiv \mathrm{BG}(\varepsilon_{part}; \bar{\varepsilon}, \sigma_\varepsilon),$$



WAYNE STATE UNIVERSITY

Gaussian model of eccentricity fluctuations

Sergei A. Voloshin [a,*], Arthur M. Poskanzer [b], Aihong Tang [c], Gang Wang [c] Physics Letters B 659 (2008) 537–541

Model assumes Gaussian form for the distributions in ε_x and ε_y, (which is a very good approximation of MC Glauber calculations).

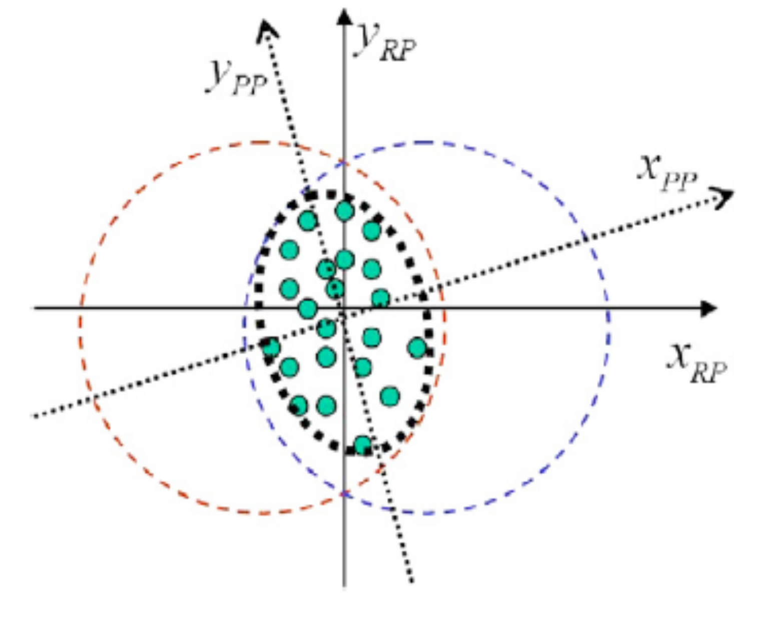


Fig. 1. The definitions of the *RP* and *PP* coordinate systems.

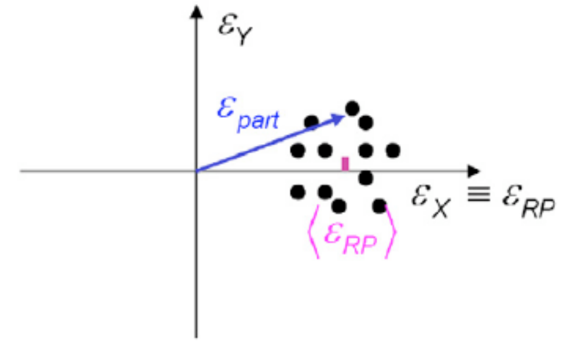


Fig. 3. Definition of ε_{part}.

$$v_2\{2\}^2 \equiv \langle \cos(2(\varphi_1-\varphi_2)) \rangle = \langle v_2^2 \rangle + \delta = \langle v_2 \rangle^2 + \sigma_v^2 + \delta$$

$$v_2\{4\}^4 \equiv 2\langle \cos(2(\varphi_1-\varphi_2)) \rangle^2 - \langle \cos(2(\varphi_1+\varphi_2-\varphi_3-\varphi_4)) \rangle \approx 2\langle v_2^2 \rangle^2 - \langle v_2^4 \rangle$$

$$v_2 \propto \varepsilon; \quad v_2\{2\} = \langle v_2 \rangle \sqrt{\langle \varepsilon^2 \rangle} / \langle \varepsilon \rangle = \langle v_2 \rangle \varepsilon_2\{2\} / \langle \varepsilon \rangle$$

$$\boldsymbol{\varepsilon} = \{\varepsilon_x, \varepsilon_y\} = \left\{ \left\langle \frac{\sigma_y^2 - \sigma_x^2}{\sigma_x^2 + \sigma_y^2} \right\rangle_{\rm part}, \left\langle \frac{2\sigma_{xy}}{\sigma_x^2 + \sigma_y^2} \right\rangle_{\rm part} \right\}$$

$$v_2\{2\}^2 = \kappa^2 (\langle \varepsilon_{RP} \rangle^2 + 2\sigma_\varepsilon^2) + \delta = \langle v_{RP} \rangle^2 + 2\sigma_{vx}^2 + \delta$$

$$v_2\{4\}^4 = 2\langle v_2^2 \rangle^2 - \langle v_2^4 \rangle = \bar{v}_2^4 = \langle v_{RP} \rangle^4$$

$$v_2\{6\}^6 = (\langle v_2^6 \rangle - 9\langle v_2^4 \rangle \langle v_2^2 \rangle + 12\langle v_2^2 \rangle^3)/4 = \langle v_{RP} \rangle^6$$

In this model it is not possible to separate flow fluctuations and non-flow effects (this can be traced to the fact that the Gaussian distribution has all cumulants higher than rank 2 equal to zero)

➔ $v_2\{4\}$ (and higher cumulants, v2{LYZ}, v2{q-dist}) measures “true” elliptic flow (wrt reaction plane) – exactly what is needed for comparison with theory!

Higher cumulant results, data and UrQMD calculations

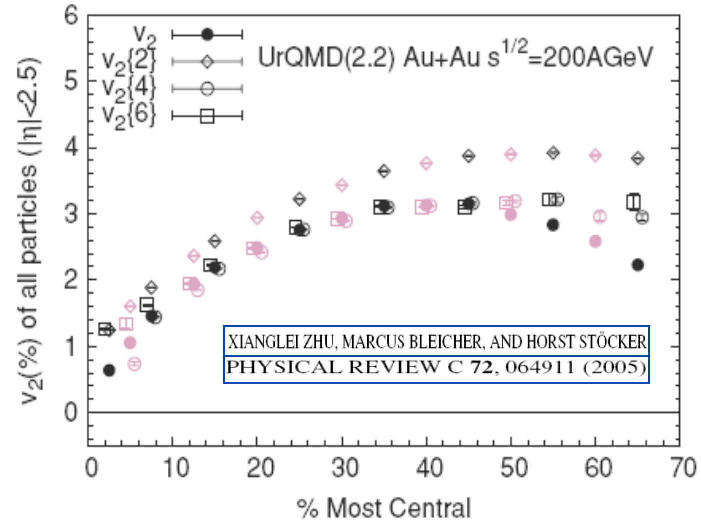
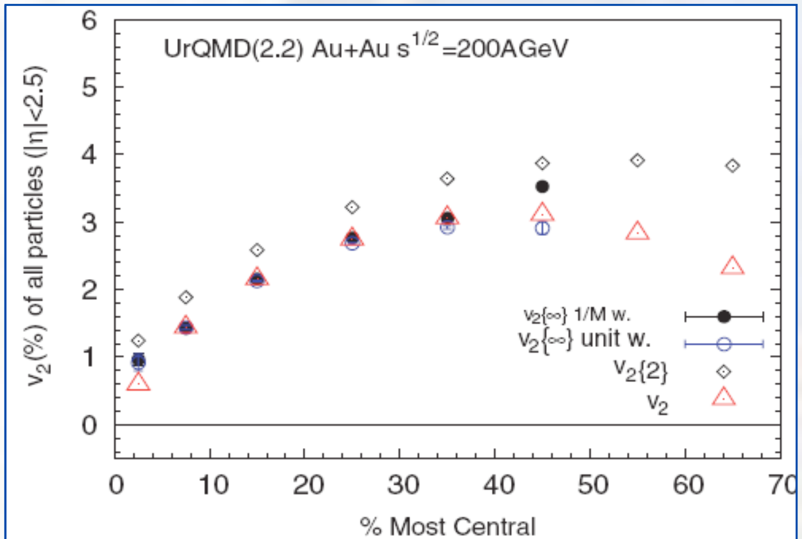
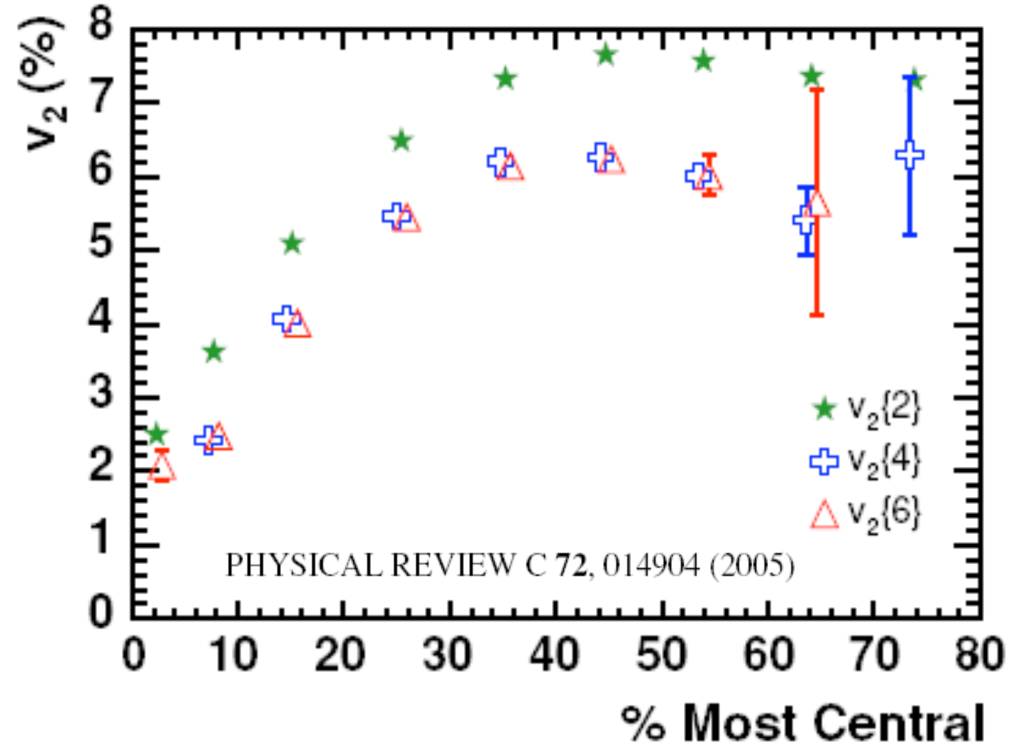


FIG. 4. (Color online) The integral v_2 results ($v_2\{2\}$, $v_2\{4\}$, and $v_2\{6\}$) from the cumulant method are compared to the exact v_2 in different centrality bins. The pink (gray) points are the corresponding results from the enlarged centrality bins which merge two of the original bins.



Results from the Lee–Yang zeros method ($v_2\{\infty\}$)

Xianglei Zhu[1,2,3], Marcus Bleicher[2] and Horst Stöcker
J. Phys. G: Nucl. Part. Phys. 32 (2006) 2181



v2{2}=v2{4}=v2{6}=v2{LYZ}=v2{ZDC-SMD)

How it works under simple assumption

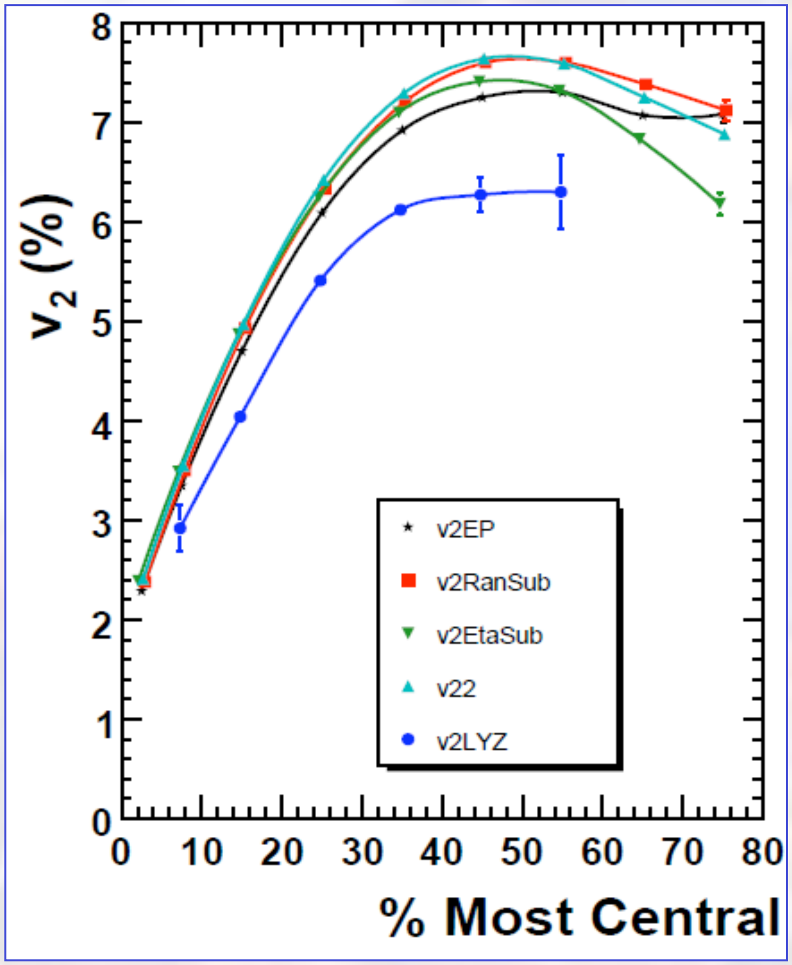
$$\sigma_{v2} = \frac{\sigma_\varepsilon}{\langle\varepsilon\rangle}\langle v_2\rangle$$

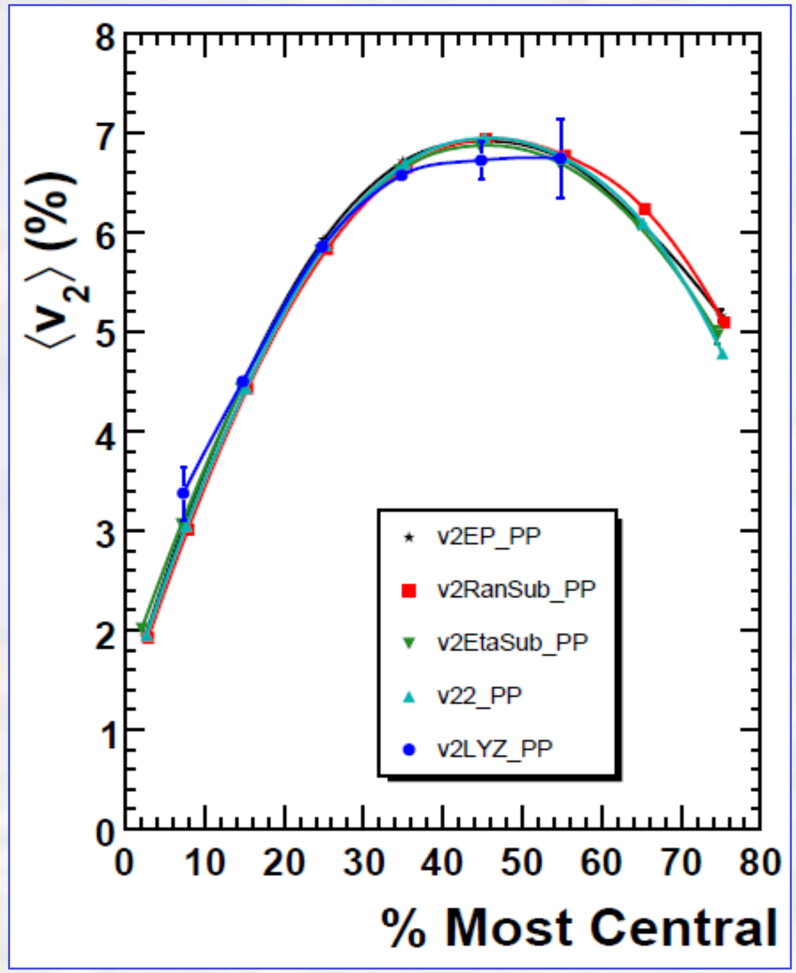
MC Glauber ε participant

$$\delta_2 = 2\ \delta_{pp}/N_{\rm part} \qquad \delta_{pp} = 0.0145$$

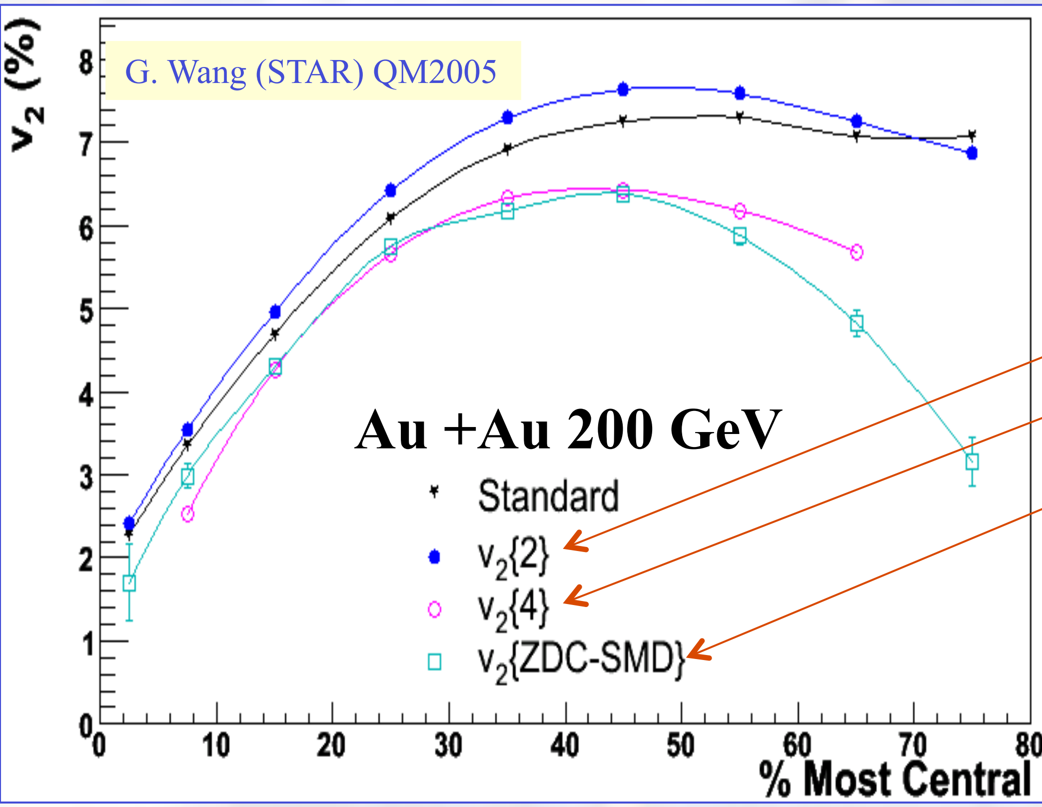
$$\delta_{\rm etaSub} = 0.5\ \delta_2$$

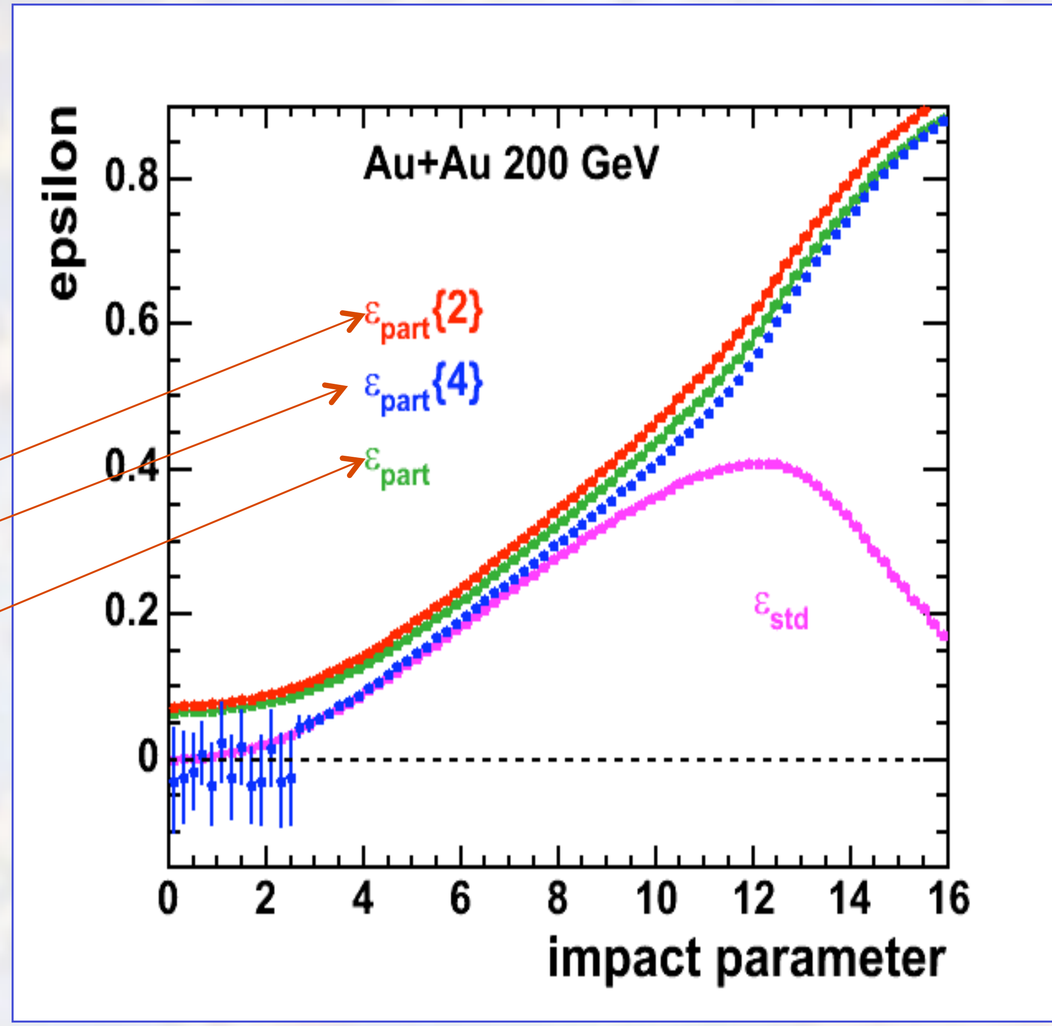
less nonflow





v_2{ZDC-SMD} and eccentricity





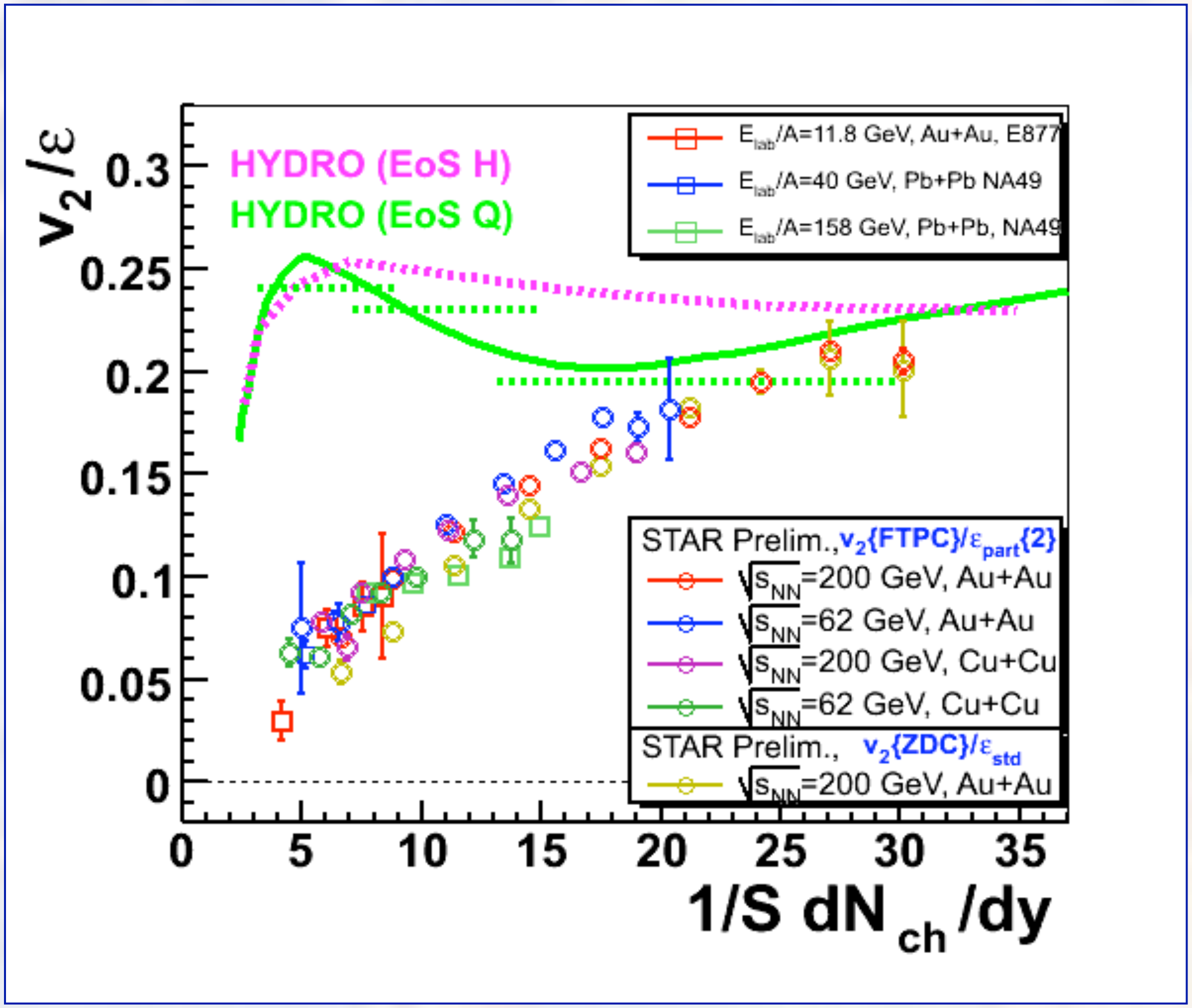
Note that $\varepsilon_{part}\{4\}$ approaches ε_{std} .

For each method it's own epsilon

If $v_2 \propto \varepsilon$, then $v_2\{2\} \propto \varepsilon\{2\} \equiv \sqrt{\langle \varepsilon^2 \rangle}$

$v_2\{4\} \propto \varepsilon\{4\}$, etc.

$$\varepsilon^2\{2\} \equiv \langle \varepsilon^2 \rangle = \langle \varepsilon \rangle^2 + \sigma_\varepsilon^2$$
$$\varepsilon^4\{4\} \equiv 2\langle \varepsilon^2 \rangle^2 - \langle \varepsilon^4 \rangle$$



Main idea: use proper $\varepsilon\{n\}$ to rescale corresponding $v_2\{n\}$:

$$v\{n\} = \langle v \rangle\, \varepsilon\{n\} / \langle \varepsilon \rangle$$

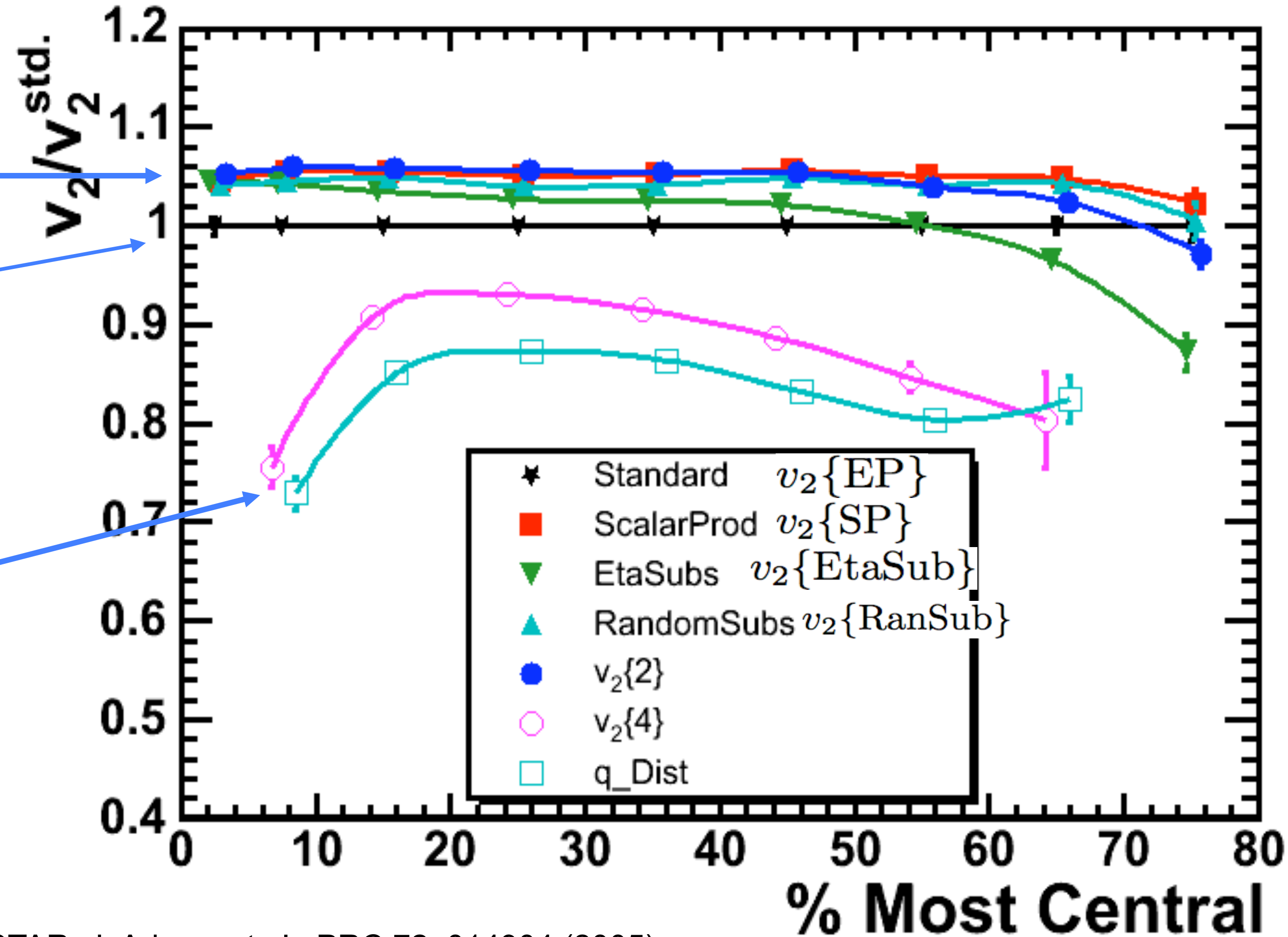
What is so special about $v_2\{EP\}$?

2-part. methods

std. method

5% low!

multi-part. methods



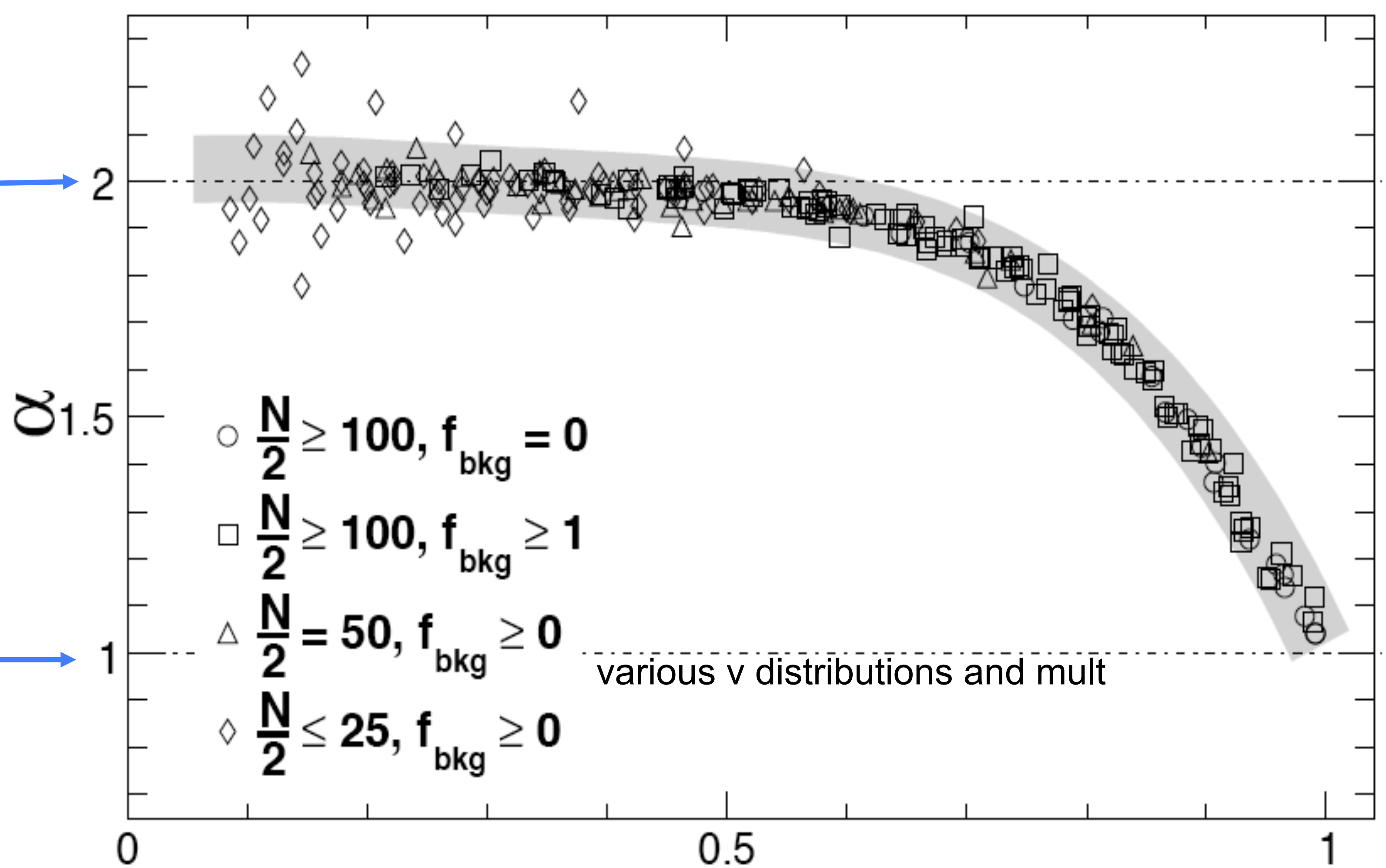
STAR, J. Adams et al., PRC **72**, 014904 (2005)

PHOBOS Simulations

$$v_2\{\ \} = \langle v^\alpha \rangle^{1/\alpha}$$

root-mean-square

mean



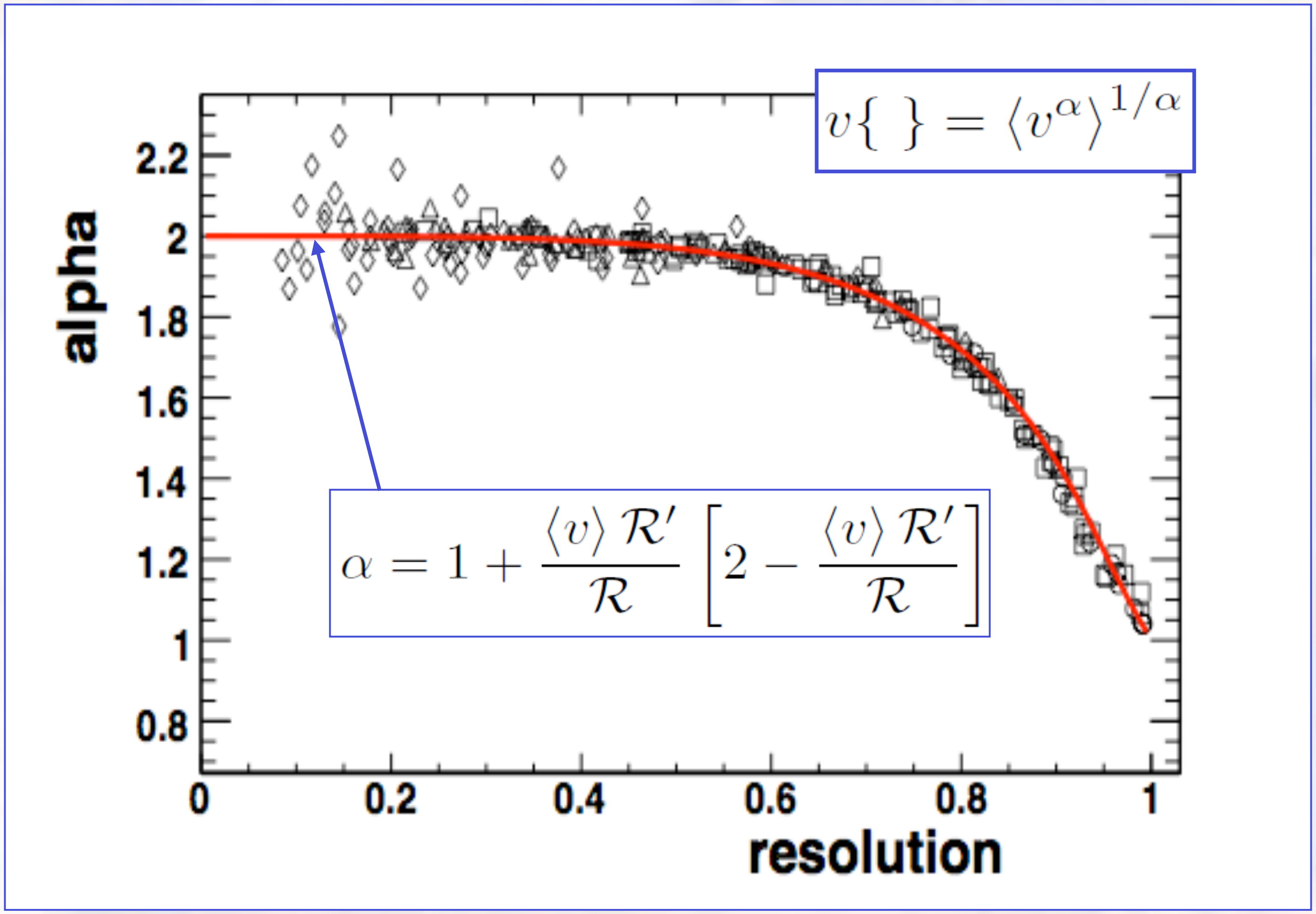
PHOBOS+Heinz, PRC **77**, 014906 (2008)

event plane resolution

more complicated case: v{EP}

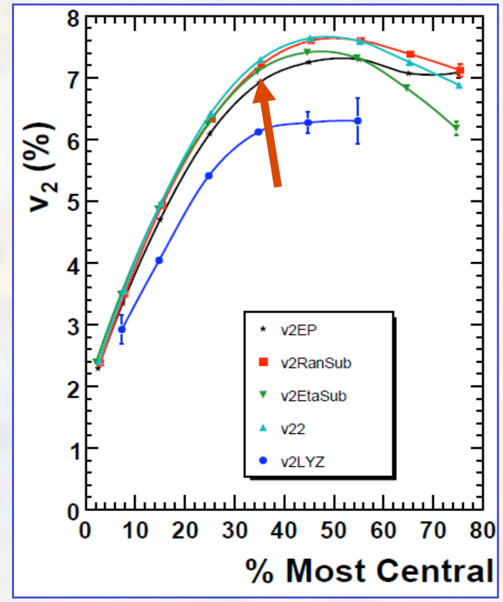
PHYSICAL REVIEW C 80, 014904 (2009)

Jean-Yves Ollitrault,[1] Arthur M. Poskanzer,[2] and Sergei A. Voloshin[3]



$$v\{\ \} = \langle v^{\alpha} \rangle^{1/\alpha}$$

$$\alpha = 1 + \frac{\langle v \rangle \mathcal{R}'}{\mathcal{R}} \left[2 - \frac{\langle v \rangle \mathcal{R}'}{\mathcal{R}} \right]$$



$$v\{\mathrm{EP}\} \equiv \frac{\langle \cos(\phi - \Psi_R) \rangle}{R}$$

$$R = \sqrt{\langle \cos(\Psi_A - \Psi_B) \rangle}$$

$$q \cos \Psi_R = \frac{Q}{\sqrt{N}} \cos \Psi_R = \frac{1}{\sqrt{N}} \sum_{j=1}^{N} \cos \phi_j$$

$$q \sin \Psi_R = \frac{Q}{\sqrt{N}} \sin \Psi_R = \frac{1}{\sqrt{N}} \sum_{j=1}^{N} \sin \phi_j$$

$$\mathcal{R}(\chi) = \frac{\sqrt{\pi}}{2} e^{-\chi^2/2} \chi \left[I_0\left(\frac{\chi^2}{2}\right) + I_1\left(\frac{\chi^2}{2}\right) \right]$$

$$\chi_s = v\sqrt{N_s}$$

… but it is still not possible to separate the effect of fluctuations from non-flow.

Analytic Correction

to flow fluctuations

$\chi = v_2 \sqrt{M}$ $\quad I_{0,1} = I_{0,1}(\chi^2/2)$ $\quad i_{0,1} = I_{0,1}(\chi_s^2/2)$

$$v_2\{2\}^2 = \langle v \rangle^2 + \sigma_v^2 \qquad v_2\{\text{subEP}\}^2 = \langle v \rangle^2 + \left(1 - \frac{4\, i_1^2}{(i_0 + i_1)^2}\right) \sigma_v^2$$

$$v_2\{\text{EP}\}^2 = \langle v \rangle^2 + \left(1 - \frac{2(I_0 - I_1)}{I_0 + I_1}\left(\chi^2 - \chi_s^2 + \frac{2 i_1^2}{i_0^2 - i_1^2}\right)\right) \sigma_v^2$$

and non-flow

$$v_2\{2\}^2 = \langle v \rangle^2 + \delta$$

$$v_2\{4\} = \langle v \rangle$$

$$v_2\{\text{EP}\}^2 = \langle v \rangle^2 + \left(1 - \frac{(I_0 - I_1)}{(I_0 + I_1)}\left(\chi^2 - \chi_s^2 + \frac{2 i_1^2}{(i_0^2 - i_1^2)}\right)\right) \delta$$

$$v_2\{\text{subEP}\}^2 = \langle v \rangle^2 + \left(1 - \frac{2 i_1^2}{(i_0 + i_1)^2}\right) \delta$$

$$\langle v \rangle = v$$

Differences of Measured v_2 Values

$$v_2\{2\}^2 - v_2\{4\}^2 = \delta + 2\sigma_v^2$$

$$v_2\{2\}^2 - v_2\{\mathrm{EP}\}^2 = \frac{(I_0 - I_1)}{(I_0 + I_1)}\left(\chi^2 - \chi_s^2 + \frac{2i_1^2}{(i_0^2 - i_1^2)}\right)(\delta + 2\sigma_v^2)$$

$$v_2\{2\}^2 - v_2\{\mathrm{subEP}\}^2 = \frac{2i_1^2}{(i_0 + i_1)^2}(\delta + 2\sigma_v^2)$$

$$v_2\{\mathrm{subEP}\}^2 - v_2\{\mathrm{EP}\}^2 = \frac{(I_0 - I_1)}{(I_0 + I_1)}\left(\chi^2 - \chi_s^2 + \frac{2i_1^2}{(i_0^2 - i_1^2)} - \frac{2I_1^2}{(I_0^2 - I_1^2)}\right)(\delta + 2\sigma_v^2)$$

All differences proportional to

$$\sigma_{\mathrm{tot}}^2 = \delta_2 + 2\sigma_{v2}^2$$

Without additional assumptions
can not separate non-flow and fluctuations

eccentricity in CGC model. viscous effects

Initial eccentricity

ADIL, DRESCHER, DUMITRU, HAYASHIGAKI, AND NARA
PHYSICAL REVIEW C **74**, 044905 (2006)

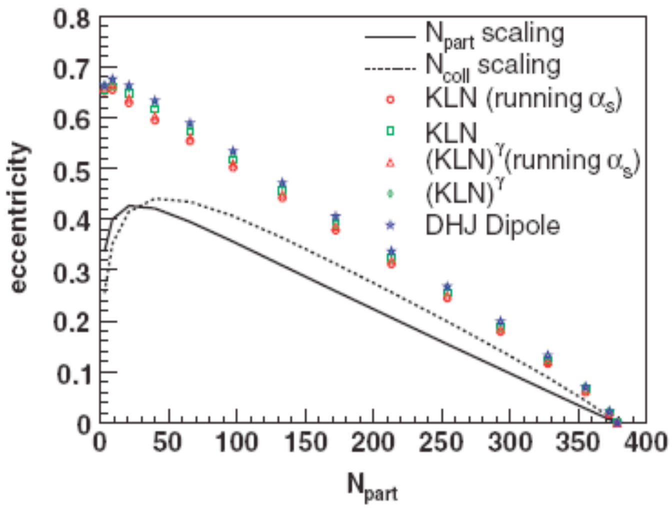


FIG. 2. (Color online) Initial spatial eccentricity ε at midrapidity as a function of the number of participants for 200 A GeV Au + Au collisions from various models. For comparison, we also show initial conditions where the initial parton density at midrapidity scales with the transverse density of wounded nucleons (solid line) and of binary collisions (dotted line) [19].

Tetsufumi Hirano [a,*], Ulrich Heinz [b], Dmitri Kharzeev [c], Roy Lacey [d], Yasushi Nara [e]
Physics Letters B 636 (2006) 299–304

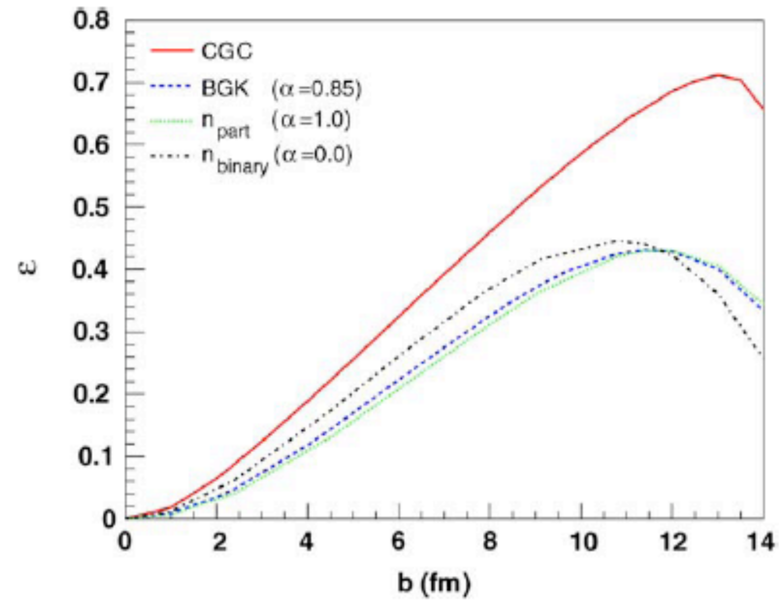


Fig. 1. (Color online.) Initial spatial eccentricity $\varepsilon = \frac{\langle y^2 - x^2 \rangle}{\langle y^2 + x^2 \rangle}$ at midrapidity as a function of impact parameter b, for 200 A GeV Au + Au collisions with CGC (solid red) and BGK (dashed blue) initial conditions. For comparison we also show initial conditions where the initial parton density at midrapidity scales with the transverse density of wounded nucleons (dotted green) and of binary collisions (dash-dotted black) [20].

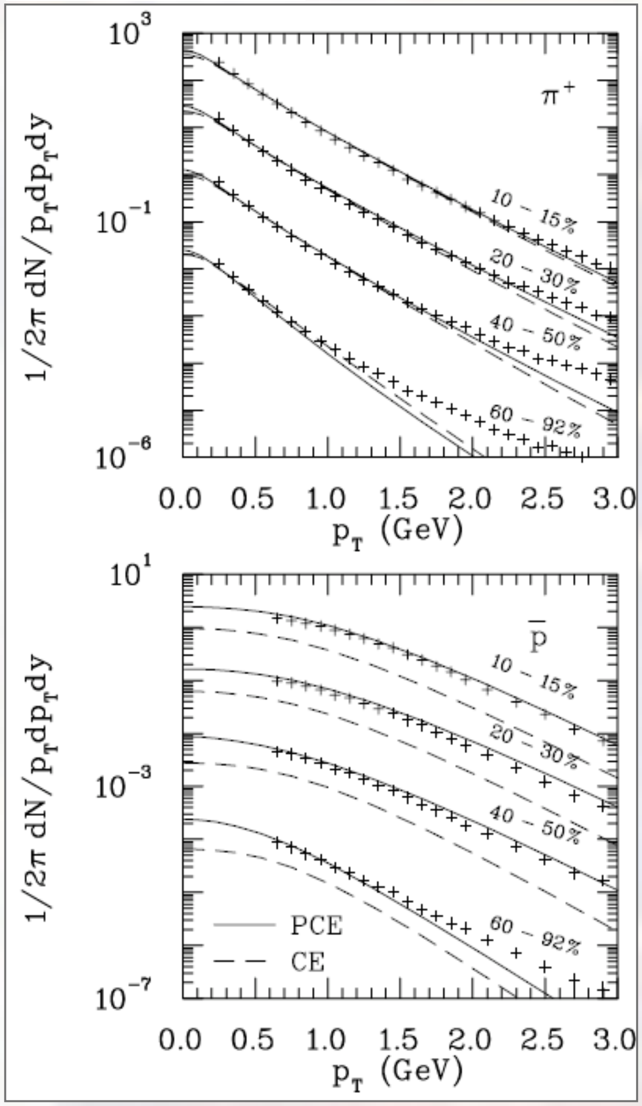
Gluon saturation picture (CGC) leads to a noticeably larger (~ 50%)
initial eccentricity, and consequently larger elliptic flow.

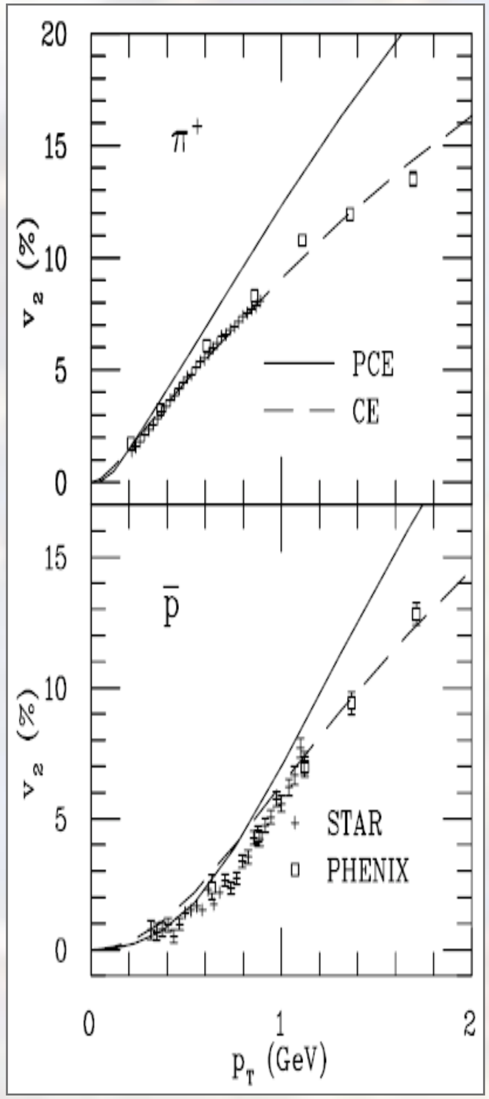
Ideal Hydro → Viscous hydro

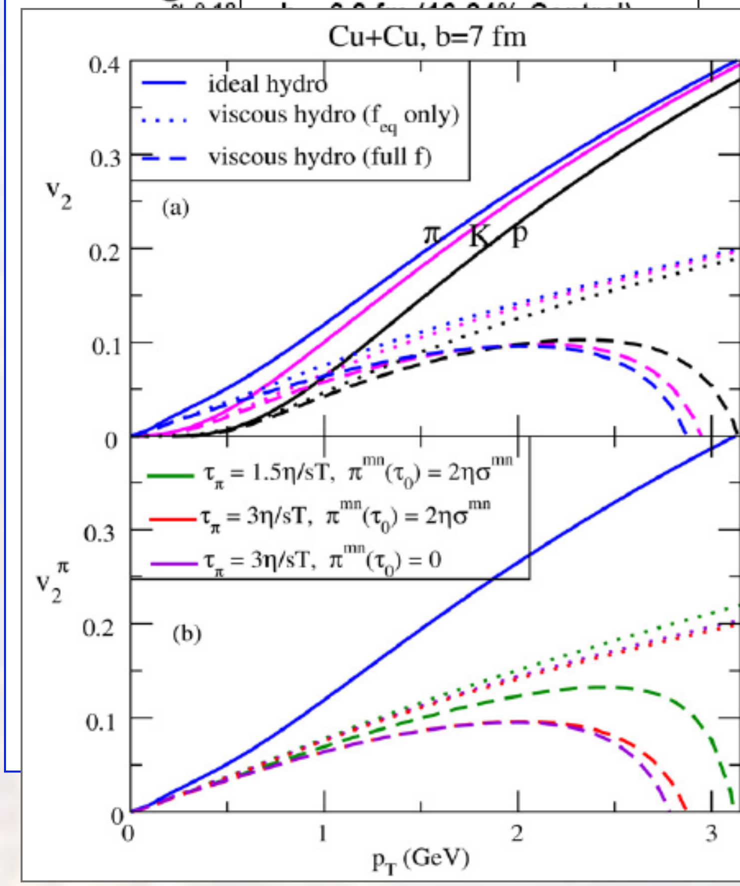
Huichao Song[a], Ulrich Heinz
Physics Letters B 658 (2008) 279–283

It is probably fair to say that the process of verification and validation of these numerical codes is still ongoing: While different initial conditions and evolution parameters used by the different groups of authors render a direct comparison of their results difficult, it seems unlikely that accounting for these differences will bring the various published numerical results in line with each other.

Pasi Huovinen
arXiv:0710.4379v1





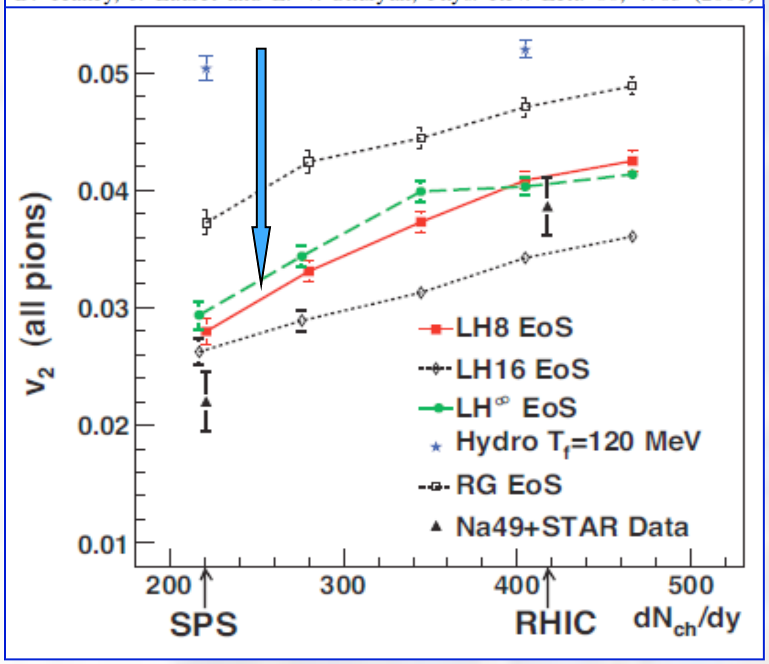


arXiv:0712.3715v1

Ideal hydro, if tuned to spectra, over predicts elliptic flow!
Including viscosity might improve agreement with data.

Viscous effects. Eccentricity in CGC model.

D. Teaney, J. Lauret and E. V. Shuryak, Phys. Rev. Lett. **86**, 4783 (2001)



LH8 denotes the results obtained for an EoS with latent heat 0.8 GeV/fm^3.

DEREK TEANEY PHYSICAL REVIEW C **68**, 034913 (2003)

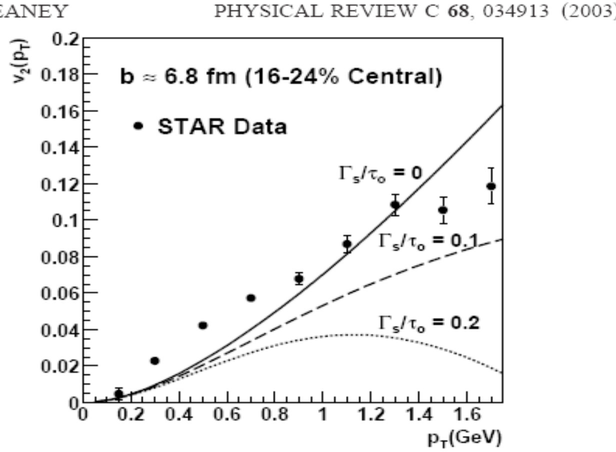


FIG. 3. Elliptic flow v_2 as a function of p_T for different values of Γ_s/τ_o. The data points are four-particle cumulant data from the STAR Collaboration [3]. Only statistical errors are shown. The difference between the ideal and viscous curves is linearly proportional to Γ_s/τ_o.

Tetsufumi Hirano[a,*], Ulrich Heinz[b], Dmitri Kharzeev[c], Roy Lacey[d], Yasushi Nara[e]
Physics Letters B 636 (2006) 299–304

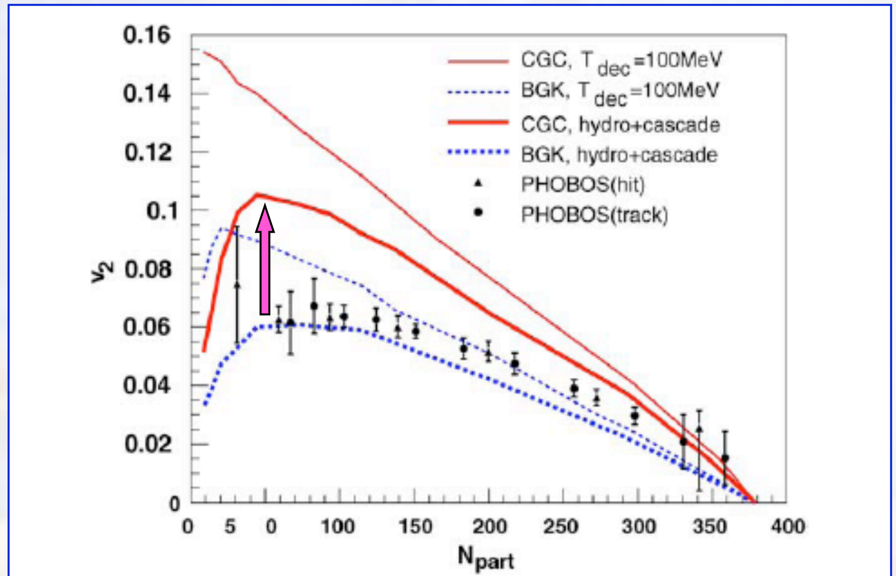


Fig. 2. (Color online.) p_T-integrated elliptic flow for charged hadrons at midrapidity ($|\eta| < 1$) from 200*A* GeV Au + Au collisions, as a function of the number N_{part} of participating nucleons. The thin lines show the prediction from ideal fluid dynamics with a freeze-out temperature T_{dec} = 100 MeV, for CGC (solid red) and BGK (dashed blue) initial conditions. The thick lines (solid red for CGC and dashed blue for BGK initial conditions) show the corresponding results from the hydro+cascade hybrid model. The data are from the PHOBOS Collaboration [19].

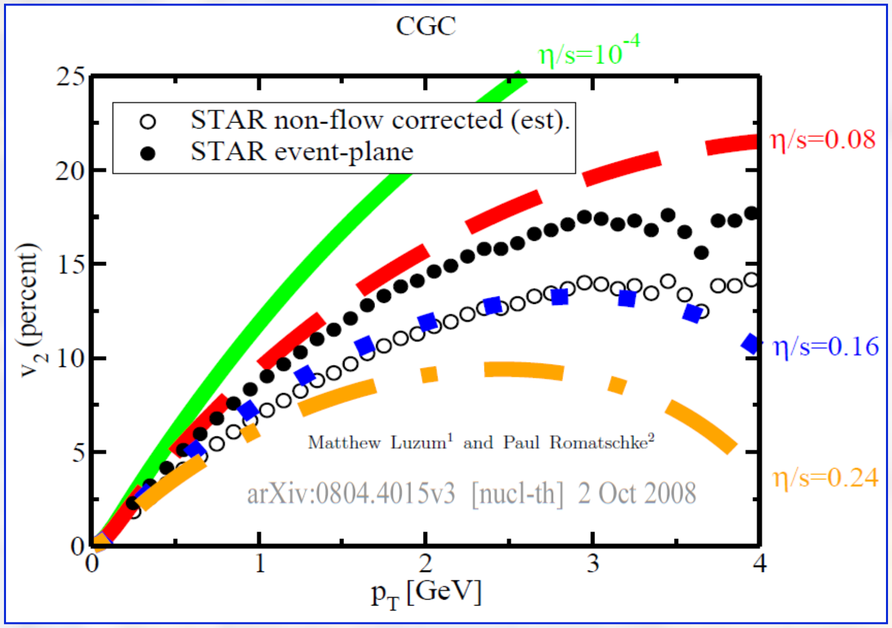
"late viscosity" was simulated by hydro+cascade MC.

The details depend in particular on transverse coordinate dependence of the saturation scale, if entropy or energy density is used as a weight,…
Lappi, Venugopalan **Phys.Rev.C74:054905,2006**

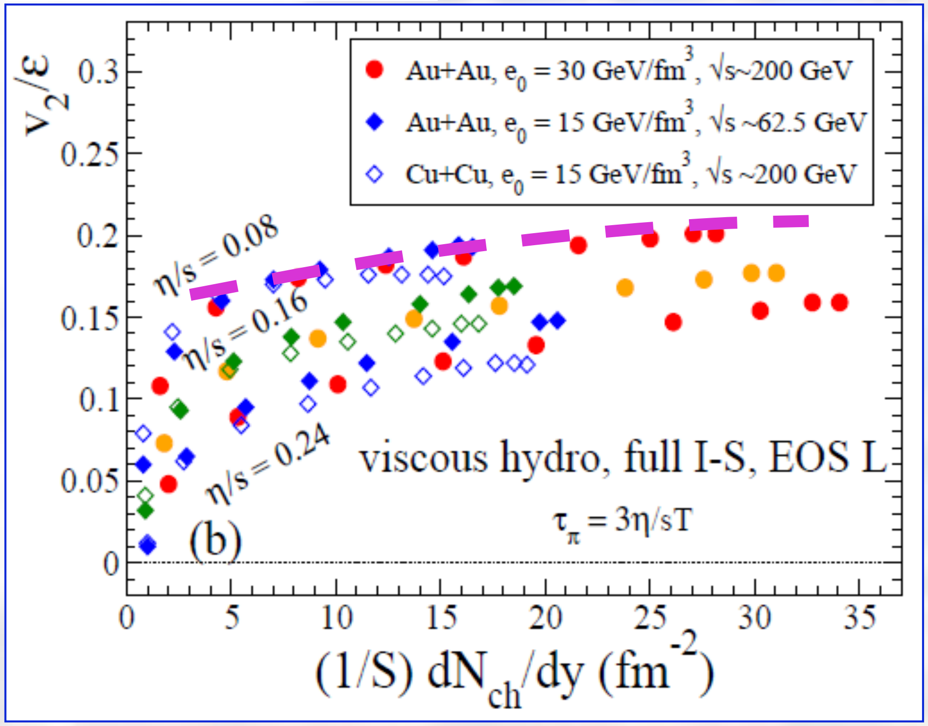
Viscous hydro calculations vs data

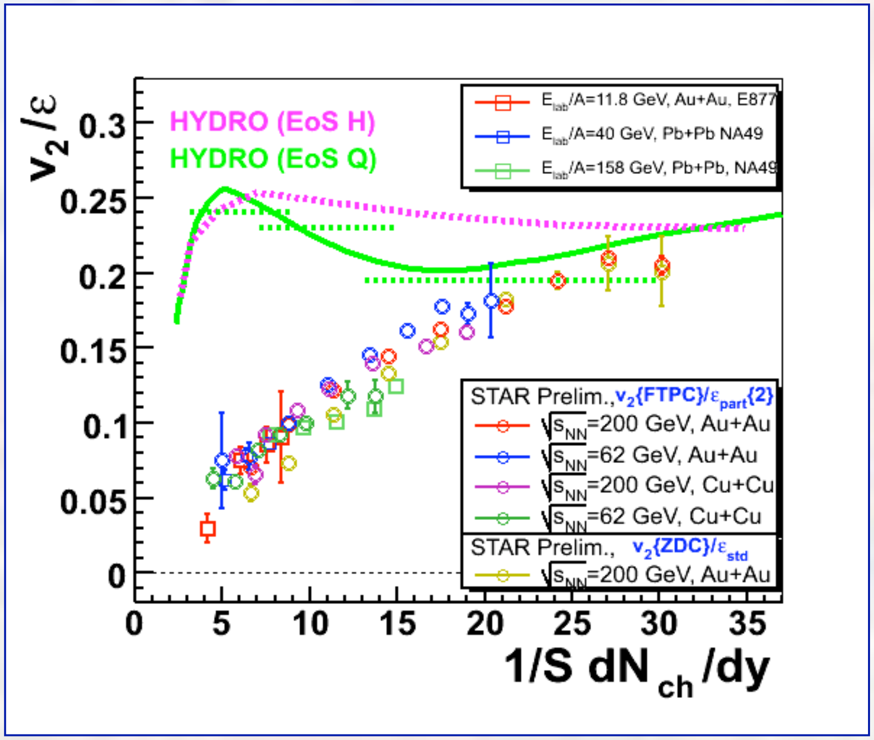
➔ η/s is about factor of 2 larger compared to the conjectured low limit of 1/(4π),

It is still the most "perfect" liquid!



Huichao Song[1,*] and Ulrich Heinz[1,2]
arXiv:0805.1756v2 [nucl-th] 3 Jul 2008





viscosity II

Paul Romatschke[1] and Ulrike Romatschke[2] PRL **99**, 172301 (2007)

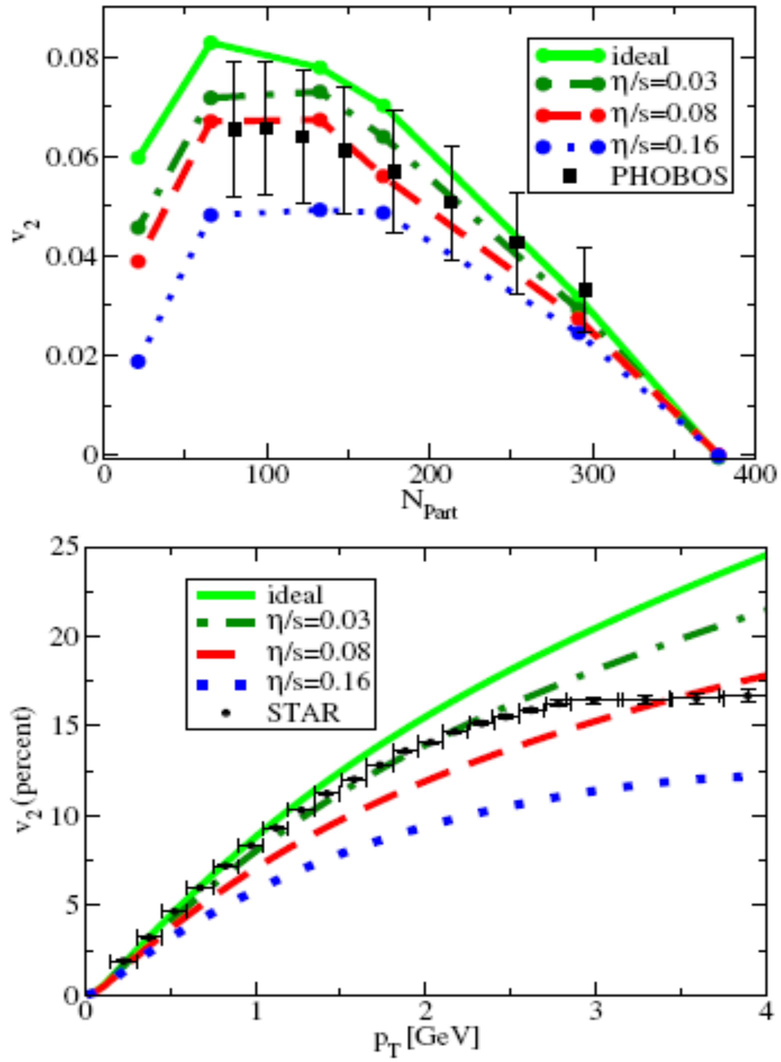


FIG. 3 (color online). PHOBOS [41] data on p_T integrated v_2 and STAR [30] data on minimum bias v_2, for charged particles in Au + Au collisions at $\sqrt{s} = 200$ GeV, compared to our hydrodynamic model for various viscosity ratios η/s. Error bars for PHOBOS data show 90% confidence level systematic errors, while for STAR only statistical errors are shown.

For standard (Glauber-type) initial conditions, while data on the integrated elliptic flow coefficient v_2 are consistent with a ratio of viscosity over entropy density up to $\eta/s \simeq 0.16$, data on minimum bias v_2 seem to favor a much smaller viscosity over entropy ratio, below the bound from the anti−de Sitter conformal field theory conjecture. Some caveats on this result are discussed.

Questions?