Dense QCD Phases in Heavy Ion Collisions

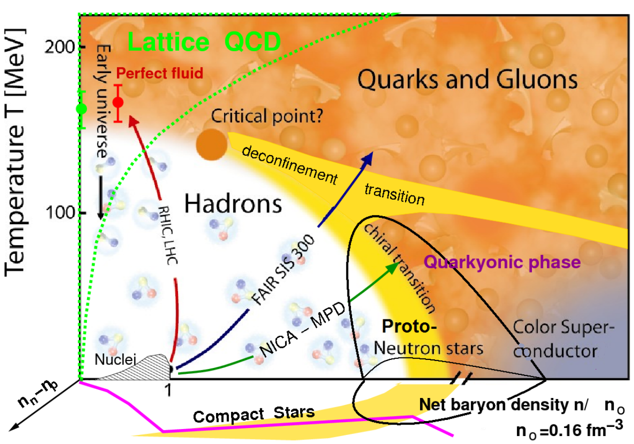
JINR, Dubna, August 21 – September 4, 2010

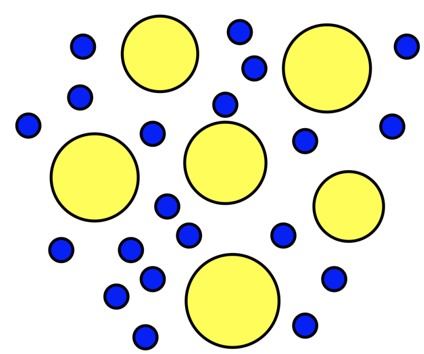
Phase Transitions & Instabilities

Jørgen Randrup (LBNL)

Lecture #1: Phase coexistence

Lecture #2: Phase separation





Phase Transitions & Instabilities

Static

Phase coexistence

Illustrative examples

Finite range effects

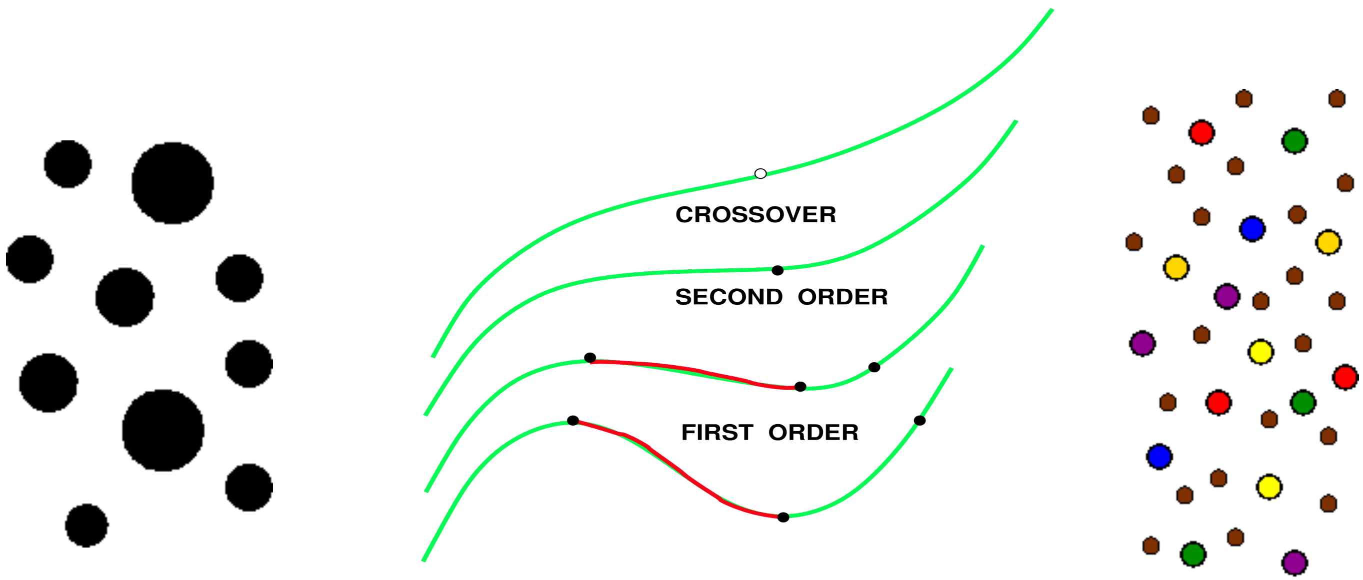
Dynamic

Phase crossing

Mean field instabilities

Instabilities in fluid dynamics

Instabilities in chiral dynamics


CROSSOVER
SECOND ORDER
FIRST ORDER

Transport model: Dissipative fluid dynamics

Energy-momentum tensor:

$$T^{00} \approx \varepsilon \quad \& \quad T^{0i} \approx (\varepsilon + p)v^i + q^i \quad \&$$

A Muronga, PRC 76, 014909 (2007)

$$T^{ij} \approx \delta_{ij}p - \eta[\partial_i v^j + \partial_j v^i - \tfrac{2}{3}\delta_{ij}\partial^k v^k] - \zeta\delta_{ij}\partial^k v^k$$

$$|\rho_k| \ll \rho_0 \;\Rightarrow\; |v| \ll 1$$

$$\boldsymbol{\nabla}\cdot\boldsymbol{T} \;\approx\; \boldsymbol{\nabla}p - \eta\Delta\boldsymbol{v} - [\tfrac{1}{3}\eta+\zeta]\boldsymbol{\nabla}(\boldsymbol{\nabla}\cdot\boldsymbol{v}) \;\asymp\; \partial_x p - [\tfrac{4}{3}\eta+\zeta]\partial_x^2 v$$

Eckart frame

Equations of motion:

$$C:\quad \partial_t\rho \;\doteq\; -\rho_0\boldsymbol{\nabla}\cdot\boldsymbol{v} \;\Rightarrow\; \omega\rho_k \doteq \rho_0 k v_k \qquad \text{charge}$$

$$\boldsymbol{M}:\quad h_0\partial_t\boldsymbol{v} \;\doteq\; -\boldsymbol{\nabla}[p-\zeta\boldsymbol{\nabla}\cdot\boldsymbol{v}] - \boldsymbol{\nabla}\cdot\boldsymbol{\pi} - \partial_t\boldsymbol{q} \qquad \text{momentum}$$

$$E:\quad \partial_t\varepsilon \;\doteq\; -h_0\boldsymbol{\nabla}\cdot\boldsymbol{v} - \boldsymbol{\nabla}\cdot\boldsymbol{q} \qquad \text{energy}$$

Sound equation:

$$\partial_t E - \boldsymbol{\nabla}\cdot\boldsymbol{M}:\quad h_0\partial_t^2\varepsilon \;\doteq\; \Delta[p-\zeta\boldsymbol{\nabla}\cdot\boldsymbol{v}] + \boldsymbol{\nabla}\cdot(\boldsymbol{\nabla}\cdot\boldsymbol{\pi})$$

$$\omega^2\varepsilon_k \;\doteq\; k^2 p_k - i[\tfrac{4}{3}\eta+\zeta]\frac{\omega}{\rho_0}k^2\rho_k \qquad \xi \equiv \tfrac{4}{3}\eta+\zeta$$

Heat flow:

$$\boldsymbol{q} \approx -\kappa[\boldsymbol{\nabla}T + T_0\partial_t\boldsymbol{v}]:\quad q_k = -i\kappa[kT_k - \frac{T_0}{\rho_0}\frac{\omega^2}{k}\rho_k] \qquad T_k \approx \frac{1}{1+i\kappa k^2/\omega c_v}\frac{T_0}{\rho_0}\left(\frac{\partial p}{\partial\varepsilon}\right)_\rho \rho_k$$

Equation of state:

$$p_T(\rho) \;\Rightarrow\; p_k = \left(\frac{\partial p}{\partial\varepsilon}\right)_\rho c_v T_k + \frac{h_0}{\rho_0}v_T^2\rho_k$$

Dispersion equation:

$$\omega^2 \;\doteq\; v_T^2 k^2 + C\frac{\rho_0^2}{h_0}k^4 - i\xi\frac{\omega}{h_0}k^2 + \frac{v_s^2 - v_T^2}{1+i\kappa k^2/\omega c_v}k^2$$

Heiselberg, Pethick, Ravenhall, Ann. Phys. 233, 37 (1993)

Ideal fluid dynamics

Energy-momentum tensor: $T^{\mu\nu} = (\varepsilon + p)u^\mu u^\nu - pg^{\mu\nu}$ $\quad u^\mu = (\gamma, \gamma\mathbf{v})$

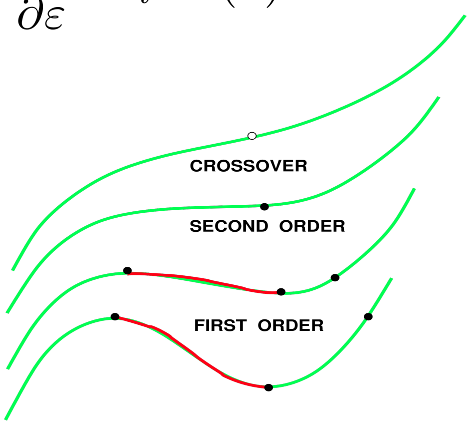
$$0 = \partial_\mu T^{\mu\nu} \begin{cases} \nu = 0: & 0 = \partial_\mu T^{\mu 0} = \partial_t(\varepsilon + pv^2)\gamma^2 + \partial_i(\varepsilon + p)\gamma^2 v^i \\ \nu = i: & 0 = \partial_\mu T^{\mu i} = \partial_t(\varepsilon + p)\gamma^2 v^i + \partial_j(\varepsilon + p)\gamma^2 v^j v^i + \partial^i p \end{cases} \quad \begin{matrix} \text{E} \\ \text{M} \end{matrix}$$

Non-relativistic flow ($v \ll 1$):
$$\begin{cases} \nu = 0: & \partial_t \varepsilon = -\partial_i(\varepsilon + p)v^i \\ \nu = i: & \partial_t(\varepsilon + p)v^i = -\partial^i p \end{cases} \quad \begin{matrix} \text{E} \\ \text{M} \end{matrix}$$

$$\partial_t E - \partial_i M: \quad \boxed{\partial_t^2 \varepsilon(x) = \partial_i \partial^i p(x)}$$ *Sound equation*

Equation of state: $p_0(\varepsilon)$ $\quad p(x) = p_0(\varepsilon(x)) \Rightarrow \partial_i \partial^i p(x) = \frac{\partial p_0(\varepsilon)}{\partial \varepsilon} \partial_i \partial^i \varepsilon(x)$

$$\boxed{\partial_t^2 \varepsilon = v_s^2 \nabla^2 \varepsilon} \qquad v_s^2 \equiv \partial_\varepsilon p_0$$ *(sound speed)²*



Evolution of small disturbances

=> $v \ll 1$

Small disturbance in a uniform stationary fluid

$$\varepsilon(x,t) = \varepsilon_0 + \delta\varepsilon(x,t)\;,\;\delta\varepsilon \ll \varepsilon_0$$

First order in $\delta\varepsilon$:

$$\begin{cases} \partial_t \delta\varepsilon(x,t) \approx (\varepsilon_0 + p_0)\partial_x v_x(x,t) \\ (\varepsilon_0 + p_0)\partial_t v_x(x,t) \approx \partial_x p(x,t) \approx \dfrac{\partial p_0}{\partial \varepsilon_0}\,\partial_x \delta\varepsilon(x,t) \end{cases}$$

Sound equation:

$$\boxed{\partial_t^2 \delta\varepsilon(x,t) = \frac{\partial p_0}{\partial \varepsilon_0}\,\partial_x^2 \delta\varepsilon(x,t)}$$

$$v_s^2 = \frac{\partial p}{\partial \epsilon}$$

Harmonic disturbance: $\delta\varepsilon_k(x,t) \sim \mathrm{e}^{ikx - i\omega_k t}$

Dispersion relation:

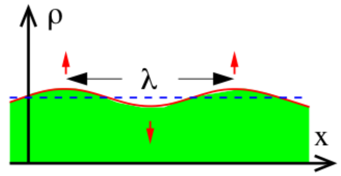
$$\boxed{\omega_k^2 = v_s^2\,k^2}$$

$$\begin{cases} v_s^2 > 0: \quad \omega_k = \pm v_s k \\ v_s^2 < 0: \quad \omega_k = \pm i\gamma_k = \pm i|v_s|k \end{cases}$$

Diverges for large k!

Ideal fluid dynamics with one conserved charge

η, ζ, κ = 0



$$\left.\begin{array}{l}\varepsilon(x) = \varepsilon_0 + \delta\varepsilon(x) \\ \rho(x) = \rho_0 + \delta\rho(x)\end{array}\right\} \; |v(x)| \ll 1$$

$T^{\mu\nu}(x)$: $\quad T^{00} \approx \varepsilon \qquad T^{i0} = T^{0i} \approx (\varepsilon + p)v^i \qquad T^{ij} = T^{ji} \approx \delta_{ij}p$

E $\quad 0 \doteq \partial_\mu T^{\mu 0} = \partial_t T^{00} + \partial_i T^{i0} = \partial_t \varepsilon + (\varepsilon + p)\partial_i v^i \quad \Rightarrow \quad \omega\varepsilon_k \doteq (\varepsilon_0 + p_0)k v_k$

M $\quad 0 \doteq \partial_\mu T^{\mu i} = \partial_t T^{0i} + \partial_j T^{ji} = (\varepsilon + p)\partial_t v^i + \partial_j T^{ji}$

C $\quad \partial_t \rho \doteq -\rho \partial_i v^i \quad \Rightarrow \quad \omega \rho_k \doteq \rho_0 k v_k$ *Continuity equation*

E & C => $\quad (\varepsilon_0 + p_0)\rho_k = \rho_0 \varepsilon_k$ *ρ tracks ε when κ=0*

$\partial_t E - \partial_i M \quad \Rightarrow \quad \partial_t^2 \varepsilon = \partial_i \partial_j T^{ji} = \partial_i \partial^i p$ => $\omega^2 \varepsilon_k = k^2 p_k$ *Sound equation*

$$p(\varepsilon, \rho) \Rightarrow \quad p_k = \frac{\partial p}{\partial \varepsilon}\varepsilon_k + \frac{\partial p}{\partial \rho}\rho_k = [\frac{\partial p}{\partial \varepsilon} + \frac{\rho_0}{\varepsilon_0 + p_0}\frac{\partial p}{\partial \rho}]\varepsilon_k = v_s^2 \varepsilon_k \qquad v_s^2 \equiv \frac{\rho}{\varepsilon + p}\left(\frac{\partial p}{\partial \rho}\right)_s$$

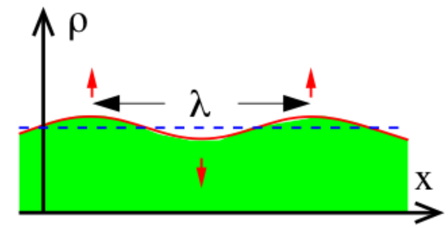
=> $\quad \omega_k^2 = v_s^2 k^2$ => $\boxed{\gamma_k = |v_s| k}$ *Dispersion relation*

Diverges for large k!

JRandrup: Dubna School, 2010

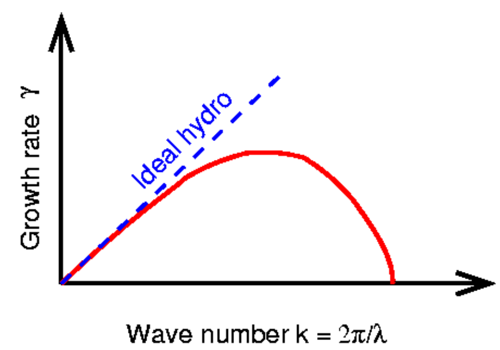
Inclusion of gradient correction

$$p(\boldsymbol{r}) \approx p_0(\varepsilon(\boldsymbol{r}), \rho(\boldsymbol{r})) - C\rho_0\nabla^2\rho(\boldsymbol{r})$$



$$\rho(r,t) = \rho_0 + \delta\rho(x,t) \doteq \ \rho_0 + \rho_k \mathrm{e}^{ikx - i\omega t}$$

=> $$p_k \ \rightarrow \ p_k + C\rho_0 k^2 \rho_k$$

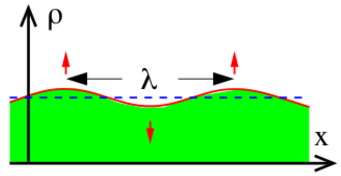


=> $$\omega_k^2 = v_s^2 k^2 + C\frac{\rho_0^2}{\varepsilon_0 + p_0} k^4$$

=> $$\gamma_k^2 = |v_s^2| k^2 - C\frac{\rho_0^2}{\varepsilon_0 + p_0} k^4$$

Viscous fluid dynamics with one conserved charge

$\kappa = 0$



$$\left.\begin{aligned}\varepsilon(x) &= \varepsilon_0 + \delta\varepsilon(x)\\ \rho(x) &= \rho_0 + \delta\rho(x)\end{aligned}\right\} \quad |v(x)| \ll 1$$

$T^{\mu\nu}(x)$:

$$\begin{cases} T^{00} \approx \varepsilon \qquad T^{i0} = T^{0i} \approx (\varepsilon + p)v^i \\ T^{ij} = T^{ji} \approx \delta_{ij}p - \eta[\partial_i v^j + \partial_j v^i - \frac{2}{3}\delta_{ij}\boldsymbol{\nabla}\cdot\boldsymbol{v}] - \zeta\delta_{ij}\boldsymbol{\nabla}\cdot\boldsymbol{v} \end{cases}$$

=> $\boldsymbol{\nabla}\cdot\boldsymbol{T} \approx \boldsymbol{\nabla}p - \eta\boldsymbol{\Delta}v - [\frac{1}{3}\eta + \zeta]\boldsymbol{\nabla}(\boldsymbol{\nabla}\cdot\boldsymbol{v})$

E $\quad 0 \doteq \partial_\mu T^{\mu 0} = \partial_t T^{00} + \partial_i T^{i0} = \partial_t\varepsilon + (\varepsilon + p)\partial_i v^i \quad \Rightarrow \quad \omega\varepsilon_k \doteq (\varepsilon_0 + p_0)kv_k$

M $\quad 0 \doteq \partial_\mu T^{\mu i} = \partial_t T^{0i} + \partial_j T^{ji} = (\varepsilon + p)\partial_t v^i + \partial_j T^{ji}$

C $\quad \partial_t\rho \doteq -\rho\partial_i v^i \quad \Rightarrow \quad \omega\rho_k \doteq \rho_0 k v_k$ — *Continuity equation*

E & C => $\quad (\varepsilon_0 + p_0)\rho_k = \rho_0\varepsilon_k$ — *ρ tracks ε when κ=0*

$\partial_t E - \partial_i M \quad \Rightarrow \quad \partial_t^2\varepsilon = \partial_i\partial_j T^{ji} = \partial_i\partial^i[p - (\frac{4}{3}\eta + \zeta)\partial_j v^j]$ — *Sound equation*

=> $\quad \omega^2\varepsilon_k = k^2 p_k - i\xi k^3 v_k = v_s^2 k^2\varepsilon_k - i\xi\frac{\omega}{\varepsilon_0 + p_0}k^2\varepsilon_k \qquad \xi \equiv \frac{4}{3}\eta + \zeta$

=> $$\gamma_k^2 = |v_s^2|k^2 - C\frac{\rho_0^2}{\varepsilon_0 + \rho_0}k^4 - \xi\frac{k^2}{\varepsilon_0 + p_0}\gamma_k$$

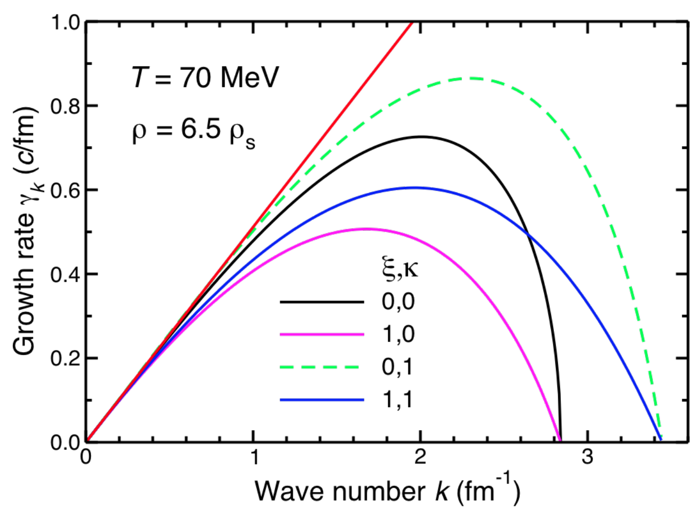
Dispersion relation

Viscous fluid dynamics with one conserved charge

$$\gamma_k^2 = |v_s^2|k^2 - C\frac{\rho_0^2}{\varepsilon_0+\rho_0}k^4 - \xi\frac{k^2}{\varepsilon_0+p_0}\gamma_k$$

$$\xi \equiv \tfrac{4}{3}\eta + \zeta$$

$$\gamma_k \approx \left[|v_s^2|k^2 - C\frac{\rho_0^2}{\varepsilon_0+\rho_0}k^4\right]^{\frac{1}{2}} - \tfrac{1}{2}\xi\frac{k^2}{\varepsilon_0+p_0}$$



Scaling expansions .... on the board!

Phase Transitions & Instabilities

Static

Dynamic

Phase coexistence

Illustrative examples

Finite range effects

Phase crossing

Mean field instabilities

Instabilities in fluid dynamics

Instabilities in chiral dynamics

Linear sigma model: Semi-classical treatment

Chiral O(4) field: $\boldsymbol{\phi}(x) = (\sigma(x), \boldsymbol{\pi}(x))$

$$\phi(x)^2 = \boldsymbol{\phi}(x)\cdot\boldsymbol{\phi}(x) = \phi_\mu(x)\phi^\mu(x) = \sigma(x)^2 + \boldsymbol{\pi}(x)\cdot\boldsymbol{\pi}(x)$$

Lagrangian: $\mathcal{L}(x) = \frac{1}{2}\partial_\mu\boldsymbol{\phi}(x)\cdot\partial^\mu\boldsymbol{\phi}(x) - \frac{\lambda}{4}[\phi(x)^2 - v^2]^2 + H\sigma(x)$

Hamiltonian: $\mathcal{H}(x) = \frac{1}{2}\psi(x)^2 + \frac{1}{2}\phi(x)^2 + \frac{\lambda}{4}[\phi(x)^2 - v^2]^2 - H\sigma(x)$ $\qquad \boldsymbol{\psi} = \partial_t\boldsymbol{\phi}$

Equation of motion: $\partial_\mu\partial^\mu\boldsymbol{\phi}(x) + \lambda(\phi^2 - v^2)\boldsymbol{\phi} = H\hat{\boldsymbol{\sigma}}$

Separation into order parameter and quasi-particles

Periodic box

$$\boldsymbol{\phi}(\boldsymbol{r},t) = \underline{\boldsymbol{\phi}}(t) + \delta\boldsymbol{\phi}(\boldsymbol{r},t)$$

$$\boldsymbol{\phi}(x) = (\sigma(x), \boldsymbol{\pi}(x))$$

$$\rightarrow \underline{\boldsymbol{\phi}}(t) = (\phi_0(t), \mathbf{0})$$

Order parameter:

$$\underline{\boldsymbol{\phi}}(t) = \langle \boldsymbol{\phi}(\boldsymbol{r},t) \rangle = \int \frac{d^3\boldsymbol{r}}{\Omega} \boldsymbol{\phi}(\boldsymbol{r},t)$$

Quasi particles:

$$\delta\boldsymbol{\phi} = (\delta\phi_{\parallel}, \delta\boldsymbol{\phi}_{\perp})$$

Field fluctuations:

$$\prec \delta\phi^2 \succ = \prec \delta\phi_{\parallel}^2 \succ + 3 \prec \delta\phi_{\perp}^2 \succ$$

Equations of motion:

Order parameter:

$$\partial_t^2 \underline{\boldsymbol{\phi}} + \lambda[\phi_0^2 + \prec \delta\phi^2 \succ + 2 \prec \delta\phi_{\parallel}^2 \succ - v^2]\boldsymbol{\phi} = H\hat{\boldsymbol{\sigma}}$$

Quasi particles:

$$\begin{cases} [\partial_\mu \partial^\mu + \mu_{\parallel}^2]\delta\phi_{\parallel} &= 0 \\ [\partial_\mu \partial^\mu + \mu_{\perp}^2]\delta\boldsymbol{\phi} &= \mathbf{0} \end{cases}$$

Quasi-particle modes

Quasi particles:

$$\begin{cases} [\partial_\mu \partial^\mu + \mu_\parallel^2]\delta\phi_\parallel &= 0 \\ [\partial_\mu \partial^\mu + \mu_\perp^2]\delta\boldsymbol{\phi} &= \mathbf{0} \end{cases}$$

$$\underline{\boldsymbol{\phi}} = (\phi_0, \mathbf{0})$$

$$\delta\boldsymbol{\phi} = (\delta\phi_\parallel, \delta\boldsymbol{\phi}_\perp)$$

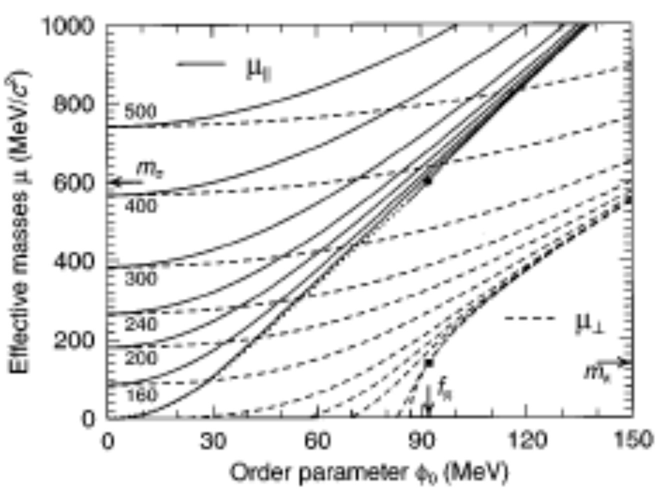
Effective masses are determined by the gap equations:

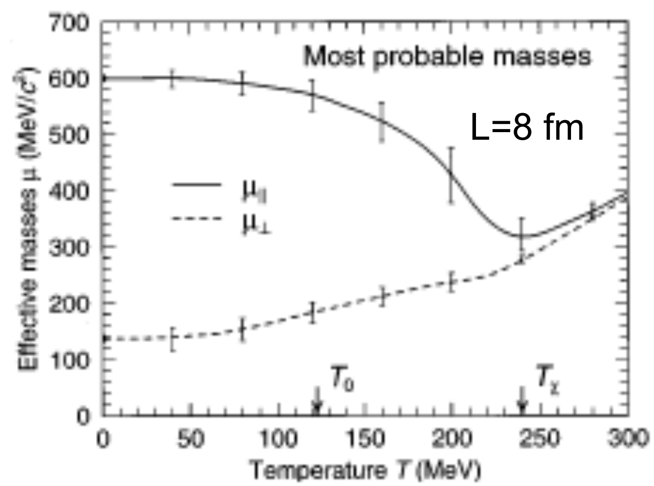
$$\mu_\parallel^2 = \lambda[3\phi_0^2 + \prec \delta\phi^2 \succ + 2 \prec \delta\phi_\parallel^2 \succ - v^2]$$

$$\mu_\perp^2 = \lambda[\phi_0^2 + \prec \delta\phi^2 \succ + 2 \prec \delta\phi_\perp^2 \succ - v^2]$$

Thermal fluctuations:

$$\prec \delta\phi_i^2 \succ = \frac{1}{\Omega} \sum_{\boldsymbol{k}}{}' \frac{1}{\epsilon_k^{(i)}} \frac{1}{\mathrm{e}^{\epsilon_k^{(i)}/T} - 1}$$





Effective potential for the order parameter

Separation of the total energy:

$$\mathcal{E}(t) = \int d^3\boldsymbol{r}\,\mathcal{H}(\boldsymbol{r},t) = \Omega\langle\tfrac{1}{2}\psi^2 + \tfrac{1}{2}(\nabla\boldsymbol{\phi})^2 + V\rangle = \Omega(E_0 + E_{\rm qp} + \delta V)$$

Bare energy:

$$E_0 = \tfrac{1}{2}\psi_0^2 + \tfrac{\lambda}{4}(\phi_0^2 - v^2)^2 - H\phi_0\cos\chi_0 = K_0 + V_0 \qquad \boldsymbol{\psi} = \partial_t\boldsymbol{\phi}$$

Quasi particles:

$$E_{\rm qp} = \sum_{j=0}^{3}\tfrac{1}{2}\langle(\partial_t\delta\phi_j)^2 + (\nabla\phi_j)^2 + \mu_j\delta\phi_j^2\rangle = \sum_{j=0}^{3}\tfrac{1}{2}\sum_{\boldsymbol{k}}{}'(|\psi_{\boldsymbol{k}}^{(j)}|^2 + (k^2+\mu_j^2)|\phi_{\boldsymbol{k}}^{(j)}|^2)$$

Correction:

$$\delta V = \tfrac{\lambda}{4}\langle\delta\phi^4\rangle - \tfrac{\lambda}{2}\langle\delta\phi^4\rangle_G \approx -\tfrac{\lambda}{4}\langle\delta\phi^4\rangle_G$$

$$\boxed{V_T(\phi_0,\chi_0) = V_0 + \langle E_{\rm qp}\rangle + \langle\delta V\rangle}$$

$$\begin{aligned}\delta V &= \tfrac{\lambda}{4}\langle\delta\phi^4\rangle - \tfrac{\lambda}{2}\langle\delta\phi^4\rangle_G \approx -\tfrac{\lambda}{4}\langle\delta\phi^4\rangle_G\\ &= \langle\delta\phi^2\rangle^2 + 2\,\mathrm{Tr}(\langle\boldsymbol{\delta\phi\delta\phi}\rangle\circ\langle\boldsymbol{\delta\phi\delta\phi}\rangle)\\ &\approx -\tfrac{3}{4}\lambda\,[\prec\delta\phi_\parallel^2\succ^2 + 2\prec\delta\phi_\parallel^2\succ\prec\delta\phi_\perp^2\succ + 5\prec\delta\phi_\perp^2\succ^2] \equiv \delta V_T\end{aligned}$$

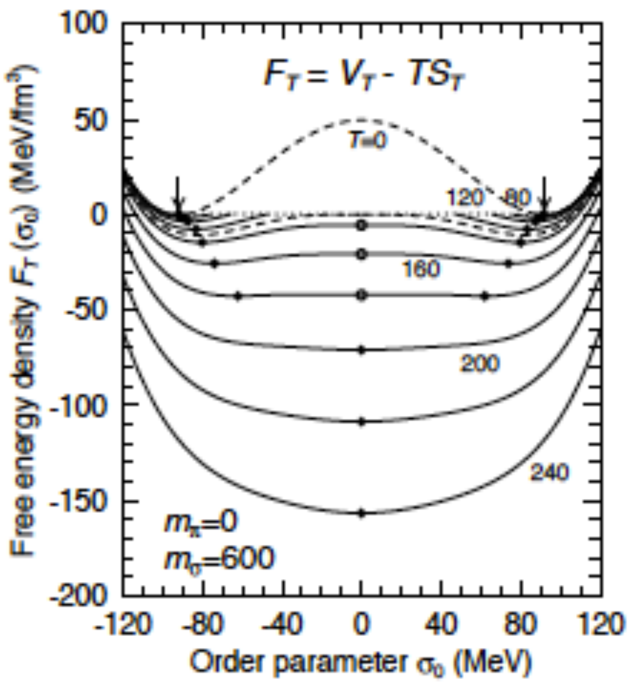
$$E_{\rm qp} = \tfrac{1}{2}\langle\boldsymbol{\delta\psi}\circ\boldsymbol{\delta\psi} + \nabla\boldsymbol{\delta\phi}\circ\nabla\boldsymbol{\delta\phi} + \boldsymbol{\delta\phi}\circ\boldsymbol{M}\circ\boldsymbol{\delta\phi}\rangle$$

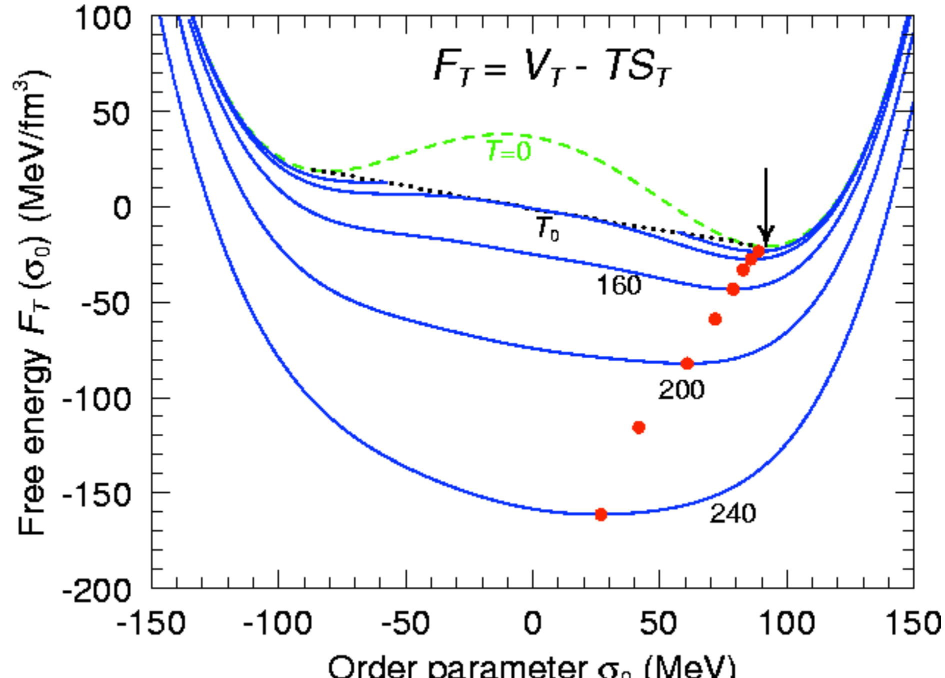
Free energy of the order parameter

$$\mathcal{E}(t) = \int d^3\boldsymbol{r}\,\mathcal{H}(\boldsymbol{r},t) = \Omega\langle \tfrac{1}{2}\psi^2 + \tfrac{1}{2}(\nabla\boldsymbol{\phi})^2 + V\rangle = \Omega(E_0 + E_{\rm qp} + \delta V) \qquad \boldsymbol{\psi} = \partial_t\boldsymbol{\phi}$$

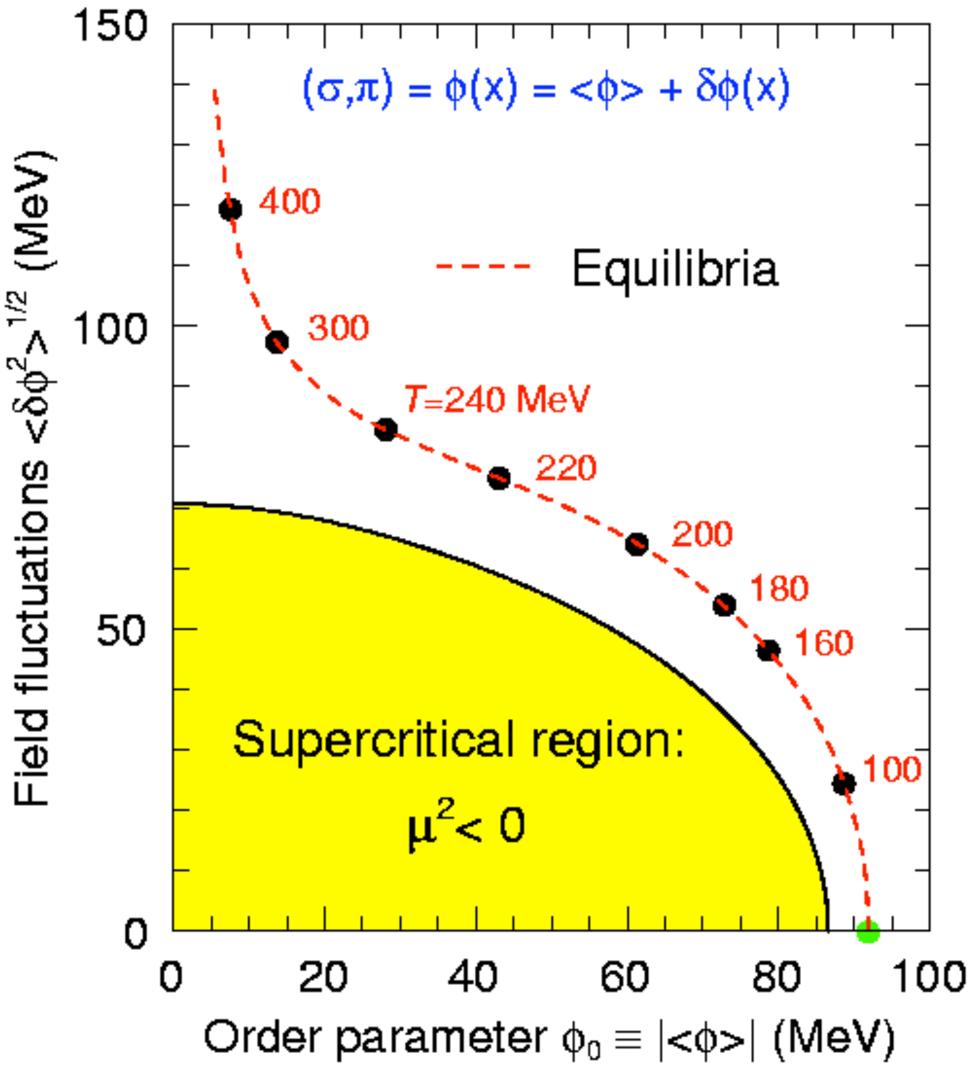
$$F_T = V_T(\phi_0, \chi_0) - TS_T(\phi_0)$$

$$\begin{aligned} F_T \;=\; & \tfrac{1}{4}\lambda(\phi_0^2 - v^2)^2 \;-\; H\phi_0\cos\chi_0 \\ + \; & \frac{T}{\Omega}{\sum_{\mathbf{k}}}'[\ln(1-\mathrm{e}^{-\epsilon_k^{\parallel}/T}) + 3\ln(1-\mathrm{e}^{-\epsilon_k^{\perp}/T})] \\ - \; & \tfrac{3}{4}\lambda\,[\prec\delta\phi_\parallel^2\succ^2 \;+ 2\prec\delta\phi_\parallel^2\succ\prec\delta\phi_\perp^2\succ \;+ 5\prec\delta\phi_\perp^2\succ^2] \end{aligned}$$





Chiral phase diagram

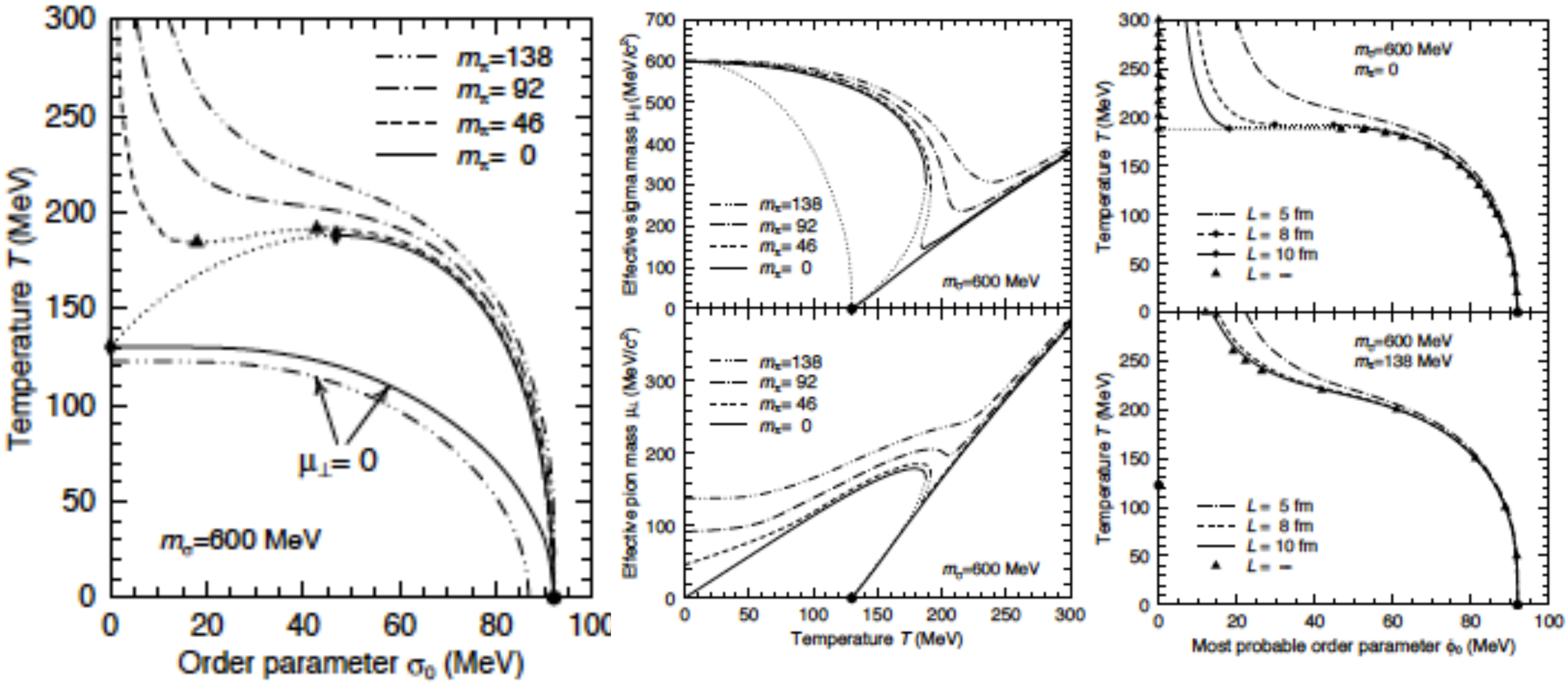


JRandrup: Dubna School, 2010

Phase structure for different pion masses

First-order transition emerges below about half the physical mass

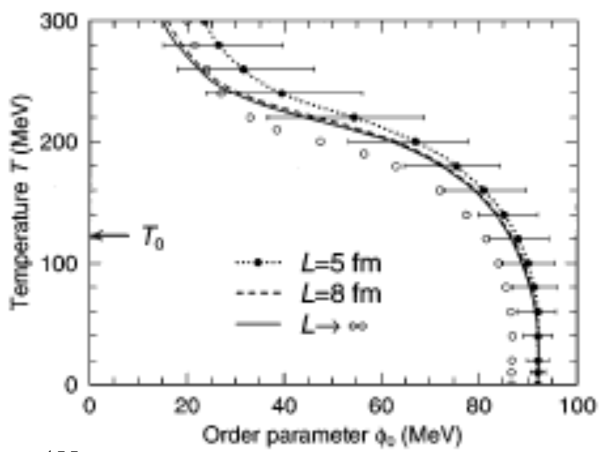
The phase structure washes out when the volume is reduced

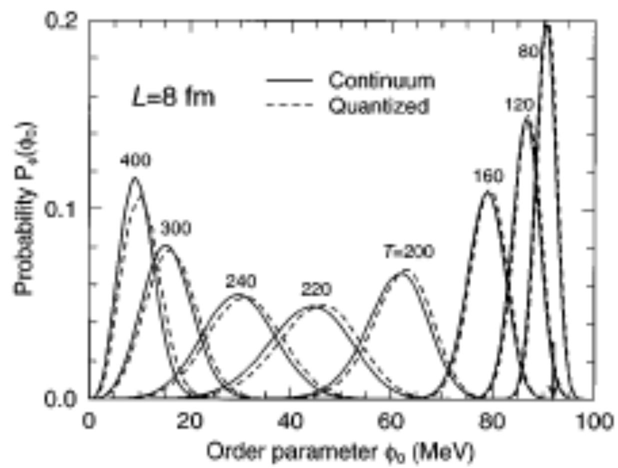


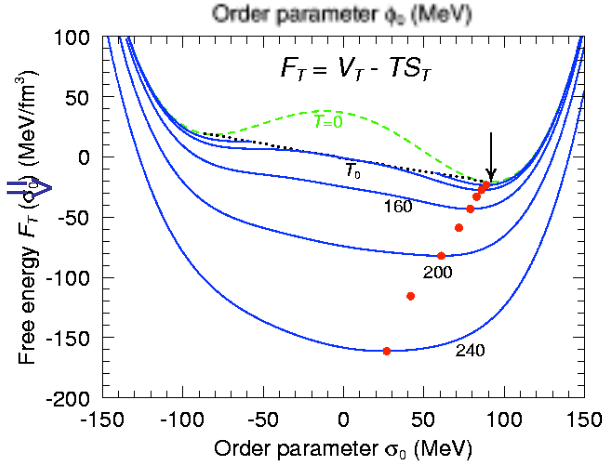
Thermal distribution of the order parameter

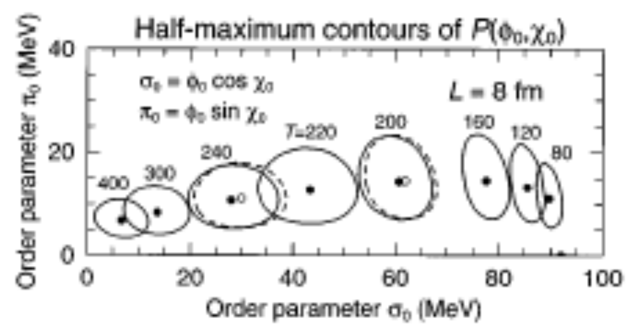
$$\mathcal{Z}_T = \int \mathcal{D}[\boldsymbol{\phi}(\boldsymbol{r}),\boldsymbol{\psi}(\boldsymbol{r})]\ \mathrm{e}^{-\frac{\Omega}{T}E[\boldsymbol{\phi}(\boldsymbol{r}),\boldsymbol{\psi}(\boldsymbol{r})]} \quad \Rightarrow \quad \mathcal{Z}_T = \int d^4\underline{\boldsymbol{\psi}}\ \mathrm{e}^{-\frac{\Omega}{T}K_0} \int d^4\underline{\boldsymbol{\phi}}\ \mathrm{e}^{-\frac{\Omega}{T}(V_0+\delta V_T)} \int \mathcal{D}[\delta\boldsymbol{\phi}(\boldsymbol{r}),\delta\boldsymbol{\psi}(\boldsymbol{r})]\ \mathrm{e}^{-\frac{\Omega}{T}E_{qp}}$$

$$\mathcal{Z}_T = \int d^4\underline{\phi}\ \mathrm{e}^{-\Omega F_T(\phi_0,\chi_0)/T}$$







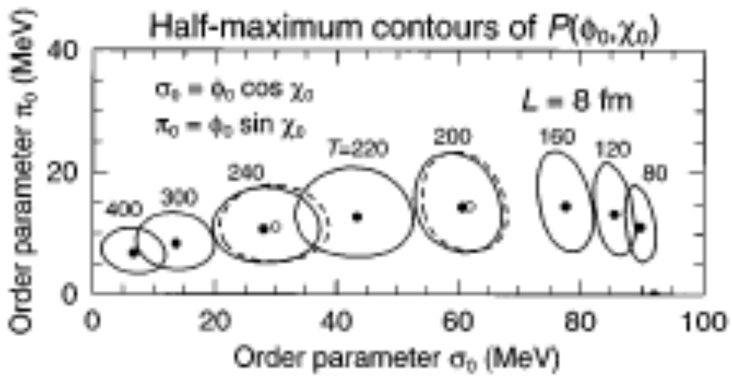


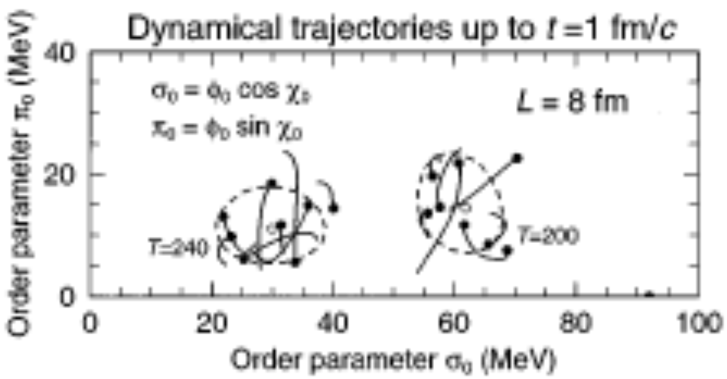
JRandrup: Dubna School, 2010

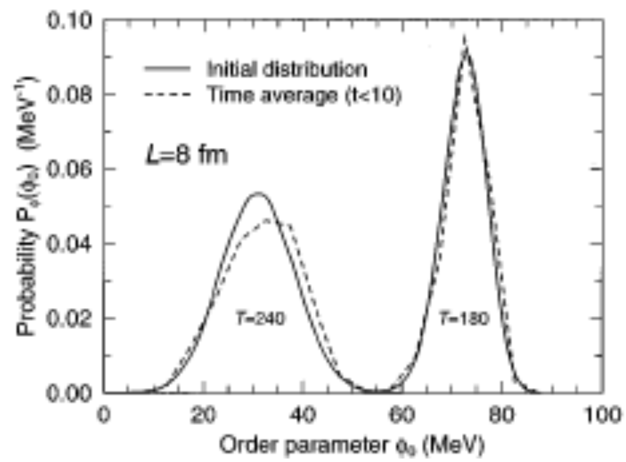
Is the equilibrium consistent with the dynamics?

Initialize a sample of fields from the calculated thermal distribution

Then follow their dynamical evolution to check for consistency

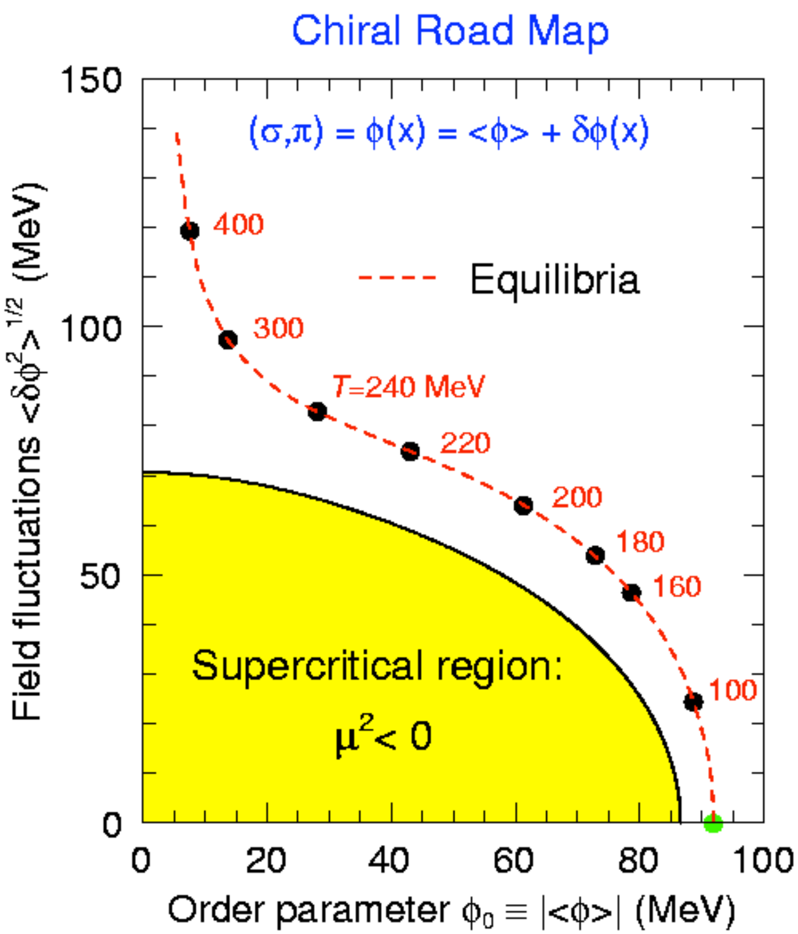






The calculated equilibrium distribution is indeed reproduced by the dynamics

Chiral dynamics

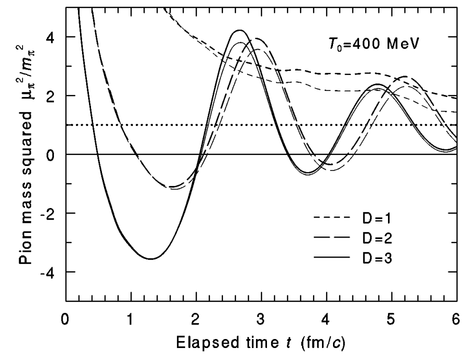


Prepare sample of initial fields

Follow dynamical evolutions

Pseudo Bjorken expansions in D dimensions

Augment the equation of motion
with a dissipative term that
emulates the Bjorken expansion:



$$\partial_\mu\partial^\mu \boldsymbol{\phi}(x) + \lambda(\phi^2 - v^2)\boldsymbol{\phi} = H\hat{\boldsymbol{\sigma}} - \frac{D}{t}\partial_t\boldsymbol{\phi}$$