Quarkyonic Matter

Toru Kojo (RIKEN/BNL)

Based on the collaboration with

Y. Hidaka, L. McLerran, R. D. Pisarski , A.M. Tsvelik.

Preface

This lecture:

- is very conceptual, qualitative, and suggestive.
- is Not quantitative (**I apologize** before criticized).

I will consider 1/Nc = 1/3 expansion because:

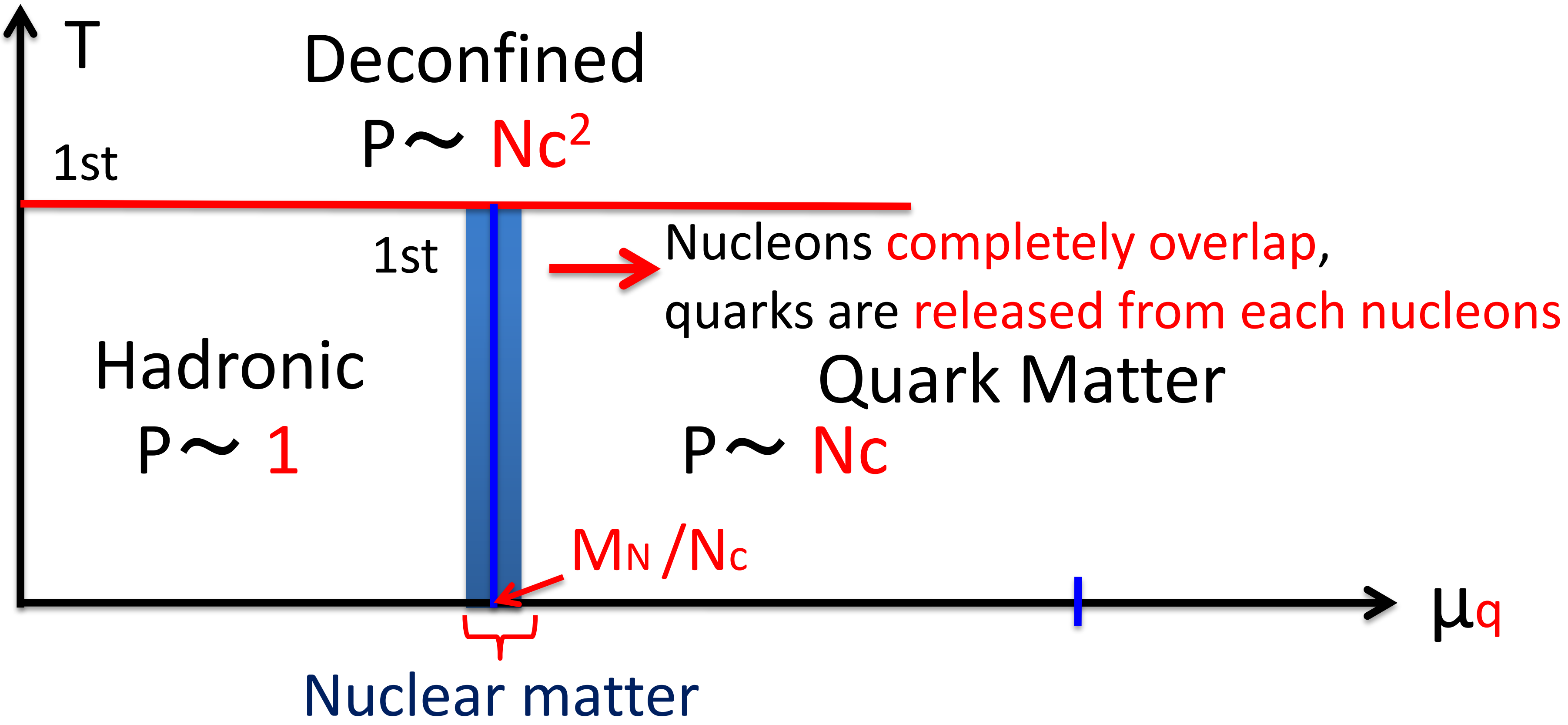
- it is a useful classification method.
- it allows us step by step arguments.

  (deeply related to hierarchy of colored fluctuations)
- it **formulates problems** & **questions** in solid term.

  (No solid formulation of problems, no clear answers)
- it gives reasonable portraits for QCD vac,
  meson sector, and maybe even for baryon sector.

Large Nc Phase Diagram : McLerran & Pisarski (2007)

(2-flavor)



Deconfined

P∼ Nc^2

Nucleons completely overlap,
quarks are released from each nucleons

Hadronic

P∼ 1

Quark Matter

P∼ Nc

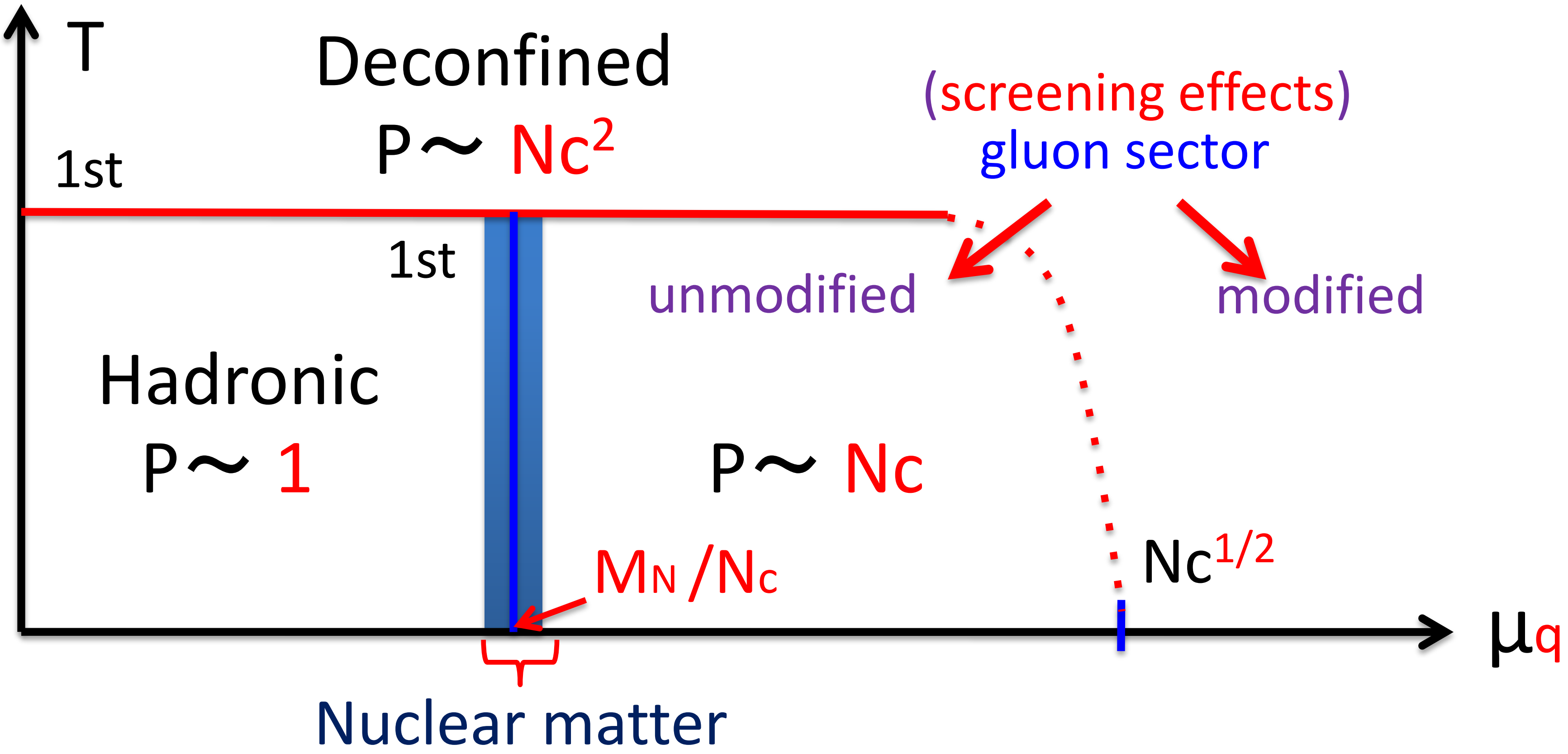
M_N / N_c

Nuclear matter

Note: Chiral restoration line is NOT plotted yet.

Large Nc Phase Diagram : McLerran & Pisarski (2007)

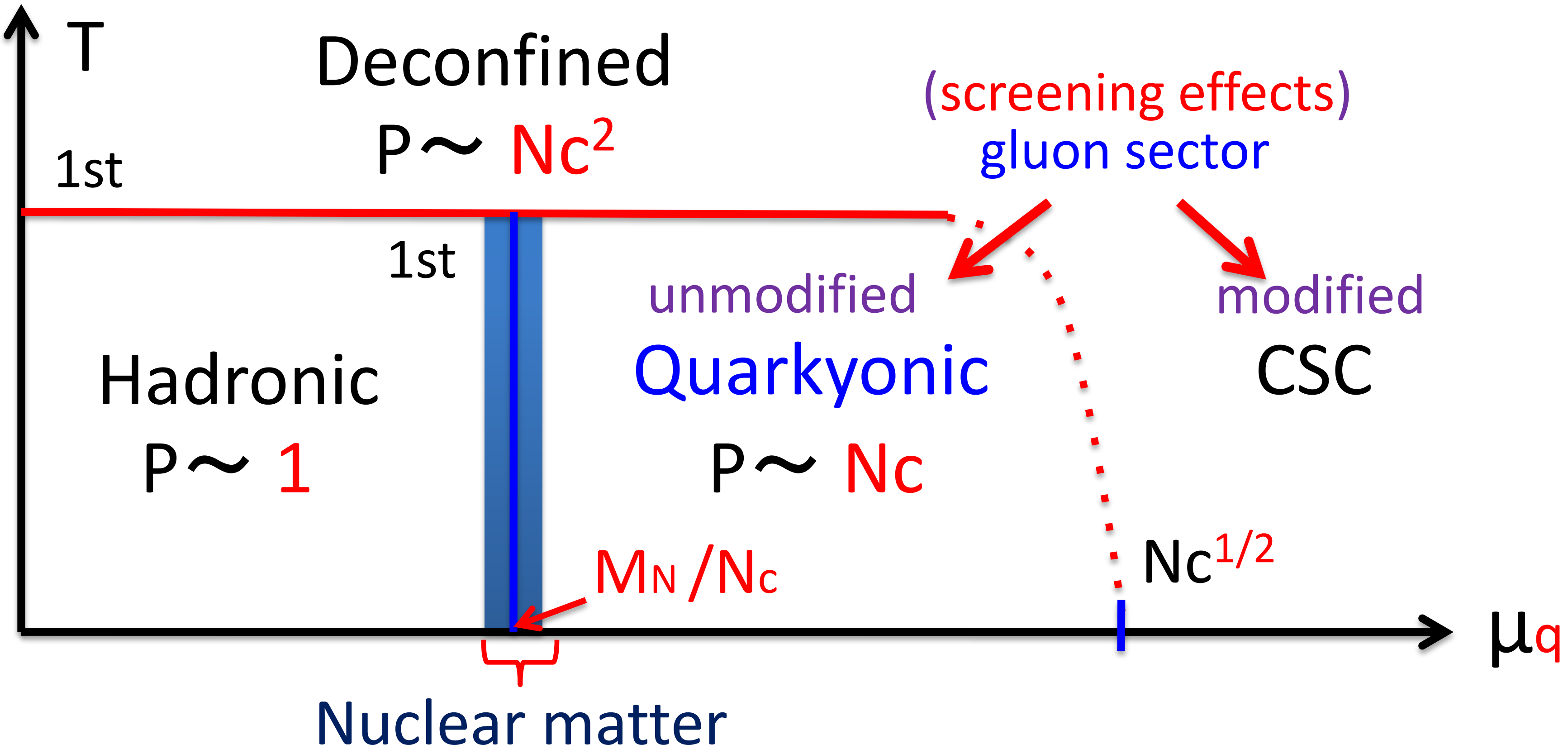
(2-flavor)



Note: Chiral restoration line is NOT plotted yet.

Large Nc Phase Diagram : McLerran & Pisarski (2007)

(2-flavor)

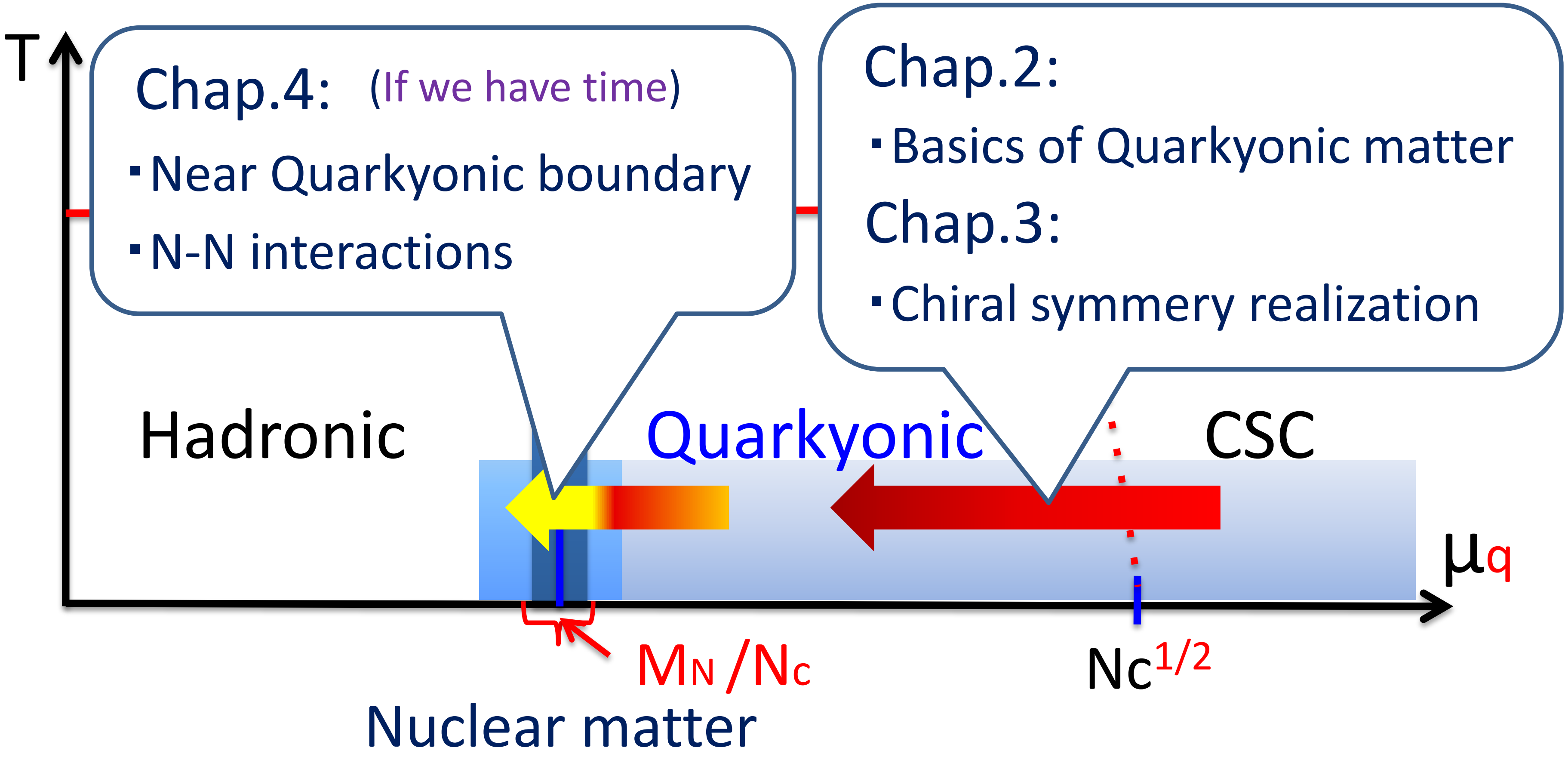


Note: Chiral restoration line is NOT plotted yet.

Plan of this lecture

- Main Topics: T～0 region of 2-flavor massless QCD

Chap.1: 1/Nc expansion : Quick review



Chap.4: (If we have time)

- Near Quarkyonic boundary
- N-N interactions

Chap.2:

- Basics of Quarkyonic matter

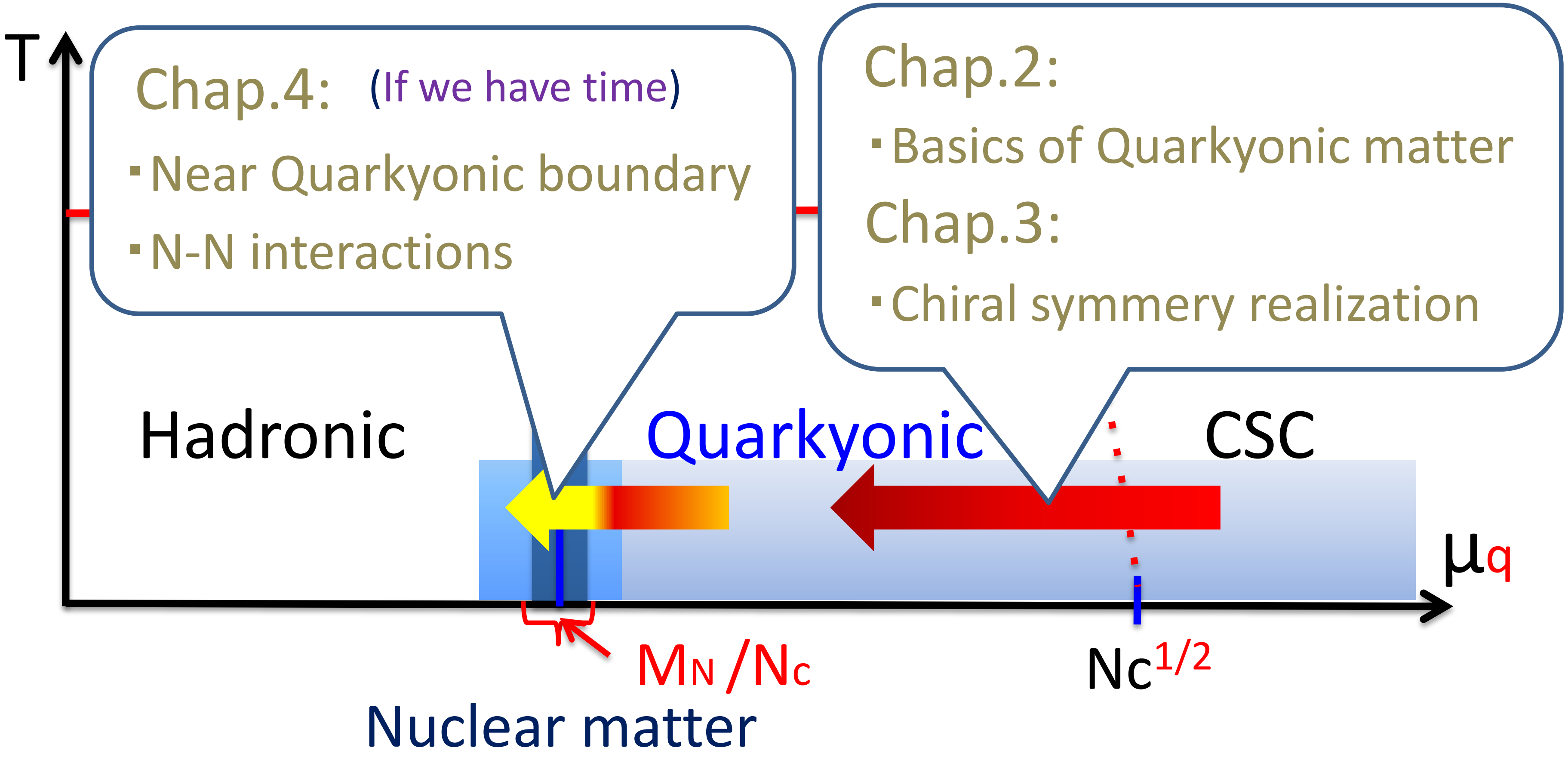
Chap.3:

- Chiral symmery realization

Plan of this lecture

- Main Topics: T～0 region of 2-flavor massless QCD

Chap.1: 1/Nc expansion : Quick review



Chap.4: (If we have time)

- Near Quarkyonic boundary
- N-N interactions

Chap.2:

- Basics of Quarkyonic matter

Chap.3:

- Chiral symmery realization

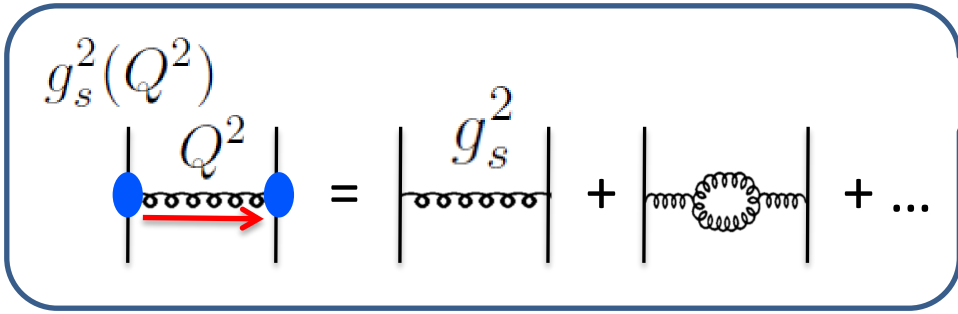
Massless QCD: 1-parameter theory

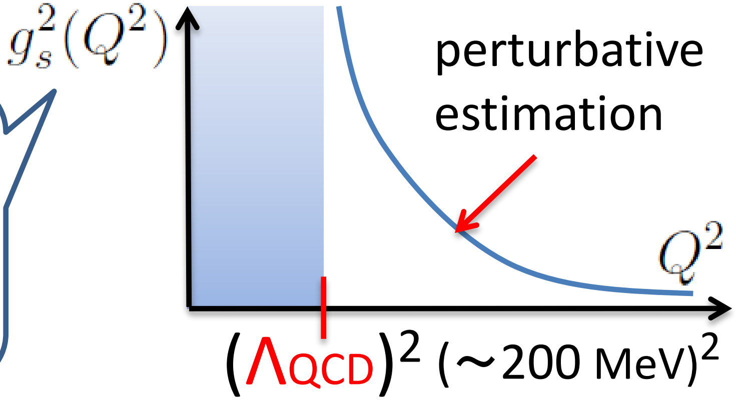
$$\mathcal{L}_{\mathrm{QCD}} = -\frac{1}{2}\mathrm{tr}[G_{\mu\nu}G^{\mu\nu}] + \sum_{f=u,d} \bar{q}_f \left[i\gamma^\mu (\partial_\mu + i\underline{g_s} A_\mu^a t^a) \right] q_f$$

(Classical level: No scale, gs can be scaled out)

- Quantum level: Scale is introduced via renormalization

e.g.) Quantum correction





- Λ_{QCD} : Scale for non-perturbative physics

Quantities with dimension → function of Λ_{QCD}

(Chiral condensate, string tension, etc.)

1/Nc (= 1/3) : Hidden expansion parameter

- Large Nc limit: $N_c g_s^2 = \text{fixed} \longrightarrow g_s \sim N_c^{-1/2}$ (weak?)

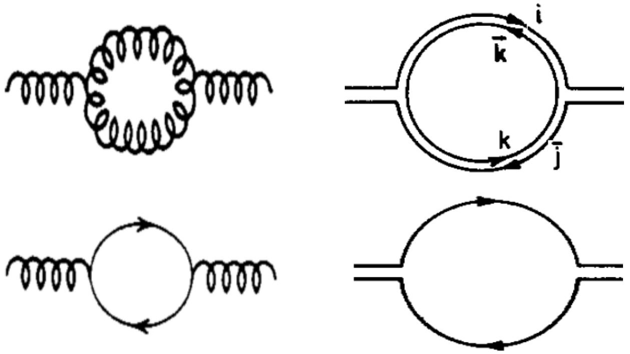
('tHooft 74, Witten 79) (condition to keep similarity with Nc=3 theory)

- Summation of color indices are relevant:

$$A^a_\mu (t^a)^j_k = (A_\mu)^{j}_{\underline{k}}$$

$(j, k = 1 \sim N_c)$

q^j



$\sim g_s^2 \underline{N_c} \sim 1$

$\sim g_s^2 \sim 1/N_c$

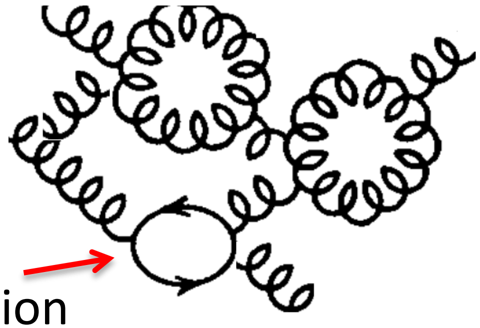
- Quantum fluctuations: Strength ∼ Num. of d.o.f.

Gluons → $O(N_c^2)$

Quarks → $O(N_c)$

gluon dominance in vacuum

(cf: quenched-lattice results)



quark loop: small fraction

Implications for Meson sector

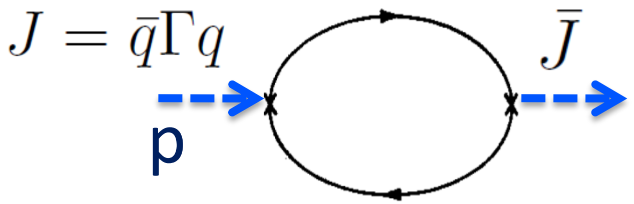
- Mesonic parameters can be estimated step by step:

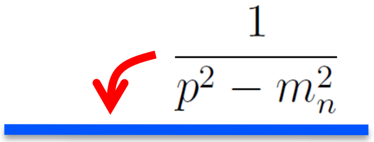
(Confinement & Meson mass: No big difference from Nc=3)

quark-side

Duality

hadronic-side

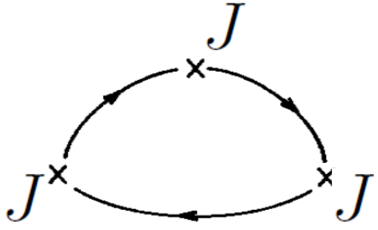




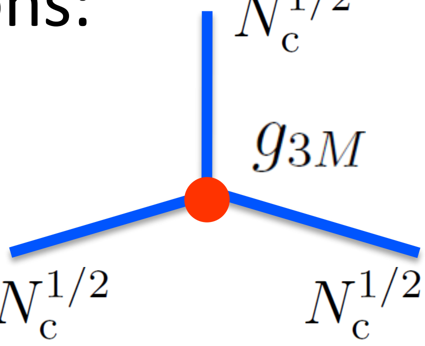
$$\sim N_c \sim \sum_n \lambda_n \frac{1}{p^2-m_n^2} \lambda_n$$

For consistency: $\boxed{\lambda_n = \langle 0|J|n\rangle \sim N_c^{1/2}}$

Interactions among mesons:



$$\sim N_c \sim$$



For consistency: $\boxed{g_{3M} \sim N_c^{-1/2}}$

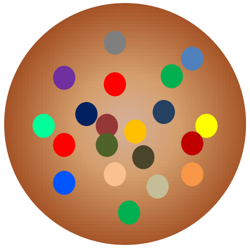
- Interactions are Weak & decay is suppressed by 1/Nc.

$\Gamma_n/M_n \sim 1/N_c$ → Mesons as Quasi-particles

Implications for Baryon sector

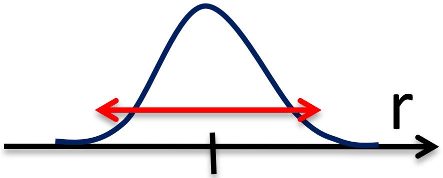
- Baryon = Bounded Nc quarks:

naïve picture



quantum w.f.

(single quark w.f.)



$1/\Lambda_{QCD}$

- All quarks have different colors

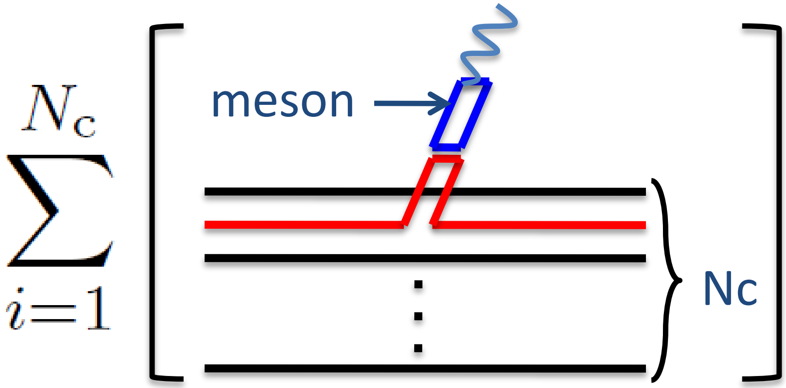
→ Can occupy same spatial orbit

- Size: $1/\Lambda_{QCD}$

kin. energy ~ potential (string tension)

- Mass: Nc × Λ_{QCD} (single quark energy)

- Baryon-Meson vertices:



Standard:

quark-meson coupling

$N_c^{-1/2}$ × N_c → $N_c^{1/2}$ (max)

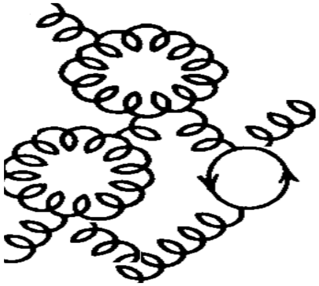
With cancellations:

$N_c^{-1/2}$ × 1 → $N_c^{-1/2}$

Interactions strongly depends on baryon w.f. !

Summary of Chapter.1

- 1, Λ_{QCD} : Typical scale for Non-perturbative dynamics
- 2, 1/Nc expansion → Hierarchy of color fluctuations



d.o.f.

- gluons: $O(N_c^2)$ → Confinement, etc.
- quarks: $O(N_c)$ → Impurity for gluon dynamics

- 3, Mesons: mass $\sim\Lambda_{QCD}$, size $\sim 1/\Lambda_{QCD}$, stable

- Interactions among mesons: suppressed by 1/Nc

- 4, Baryons: mass $\sim N_c\,\Lambda_{QCD}$, size $\sim 1/\Lambda_{QCD}$

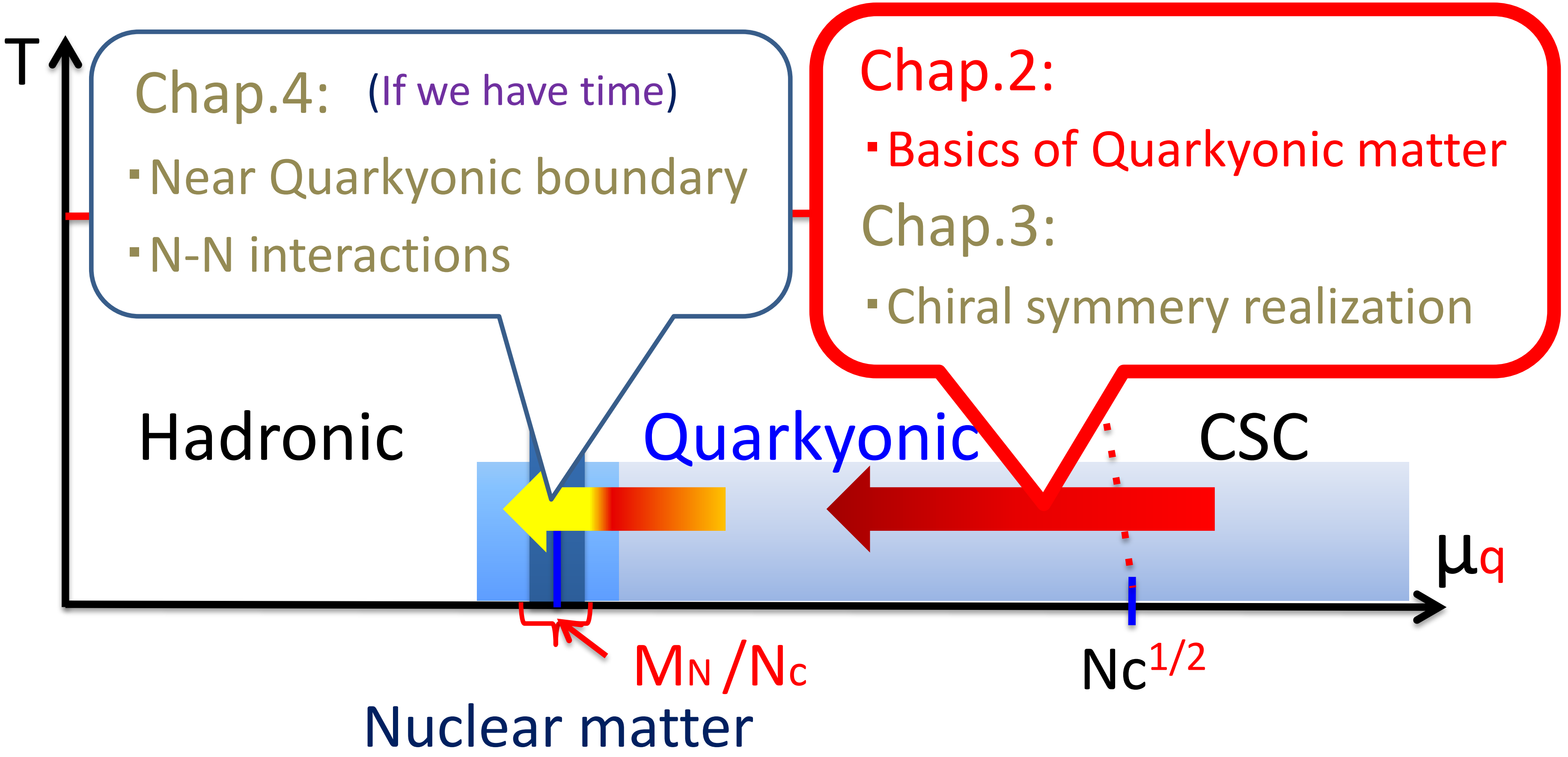
Baryon-Meson coupling: from $N_c^{-1/2}$ (min) to $N_c^{1/2}$ (max)

→ Large state dependence of Meson-mediated B-B int.

Plan of this lecture

- Main Topics: T～0 region of 2-flavor massless QCD

Chap.1: 1/Nc expansion : Quick review

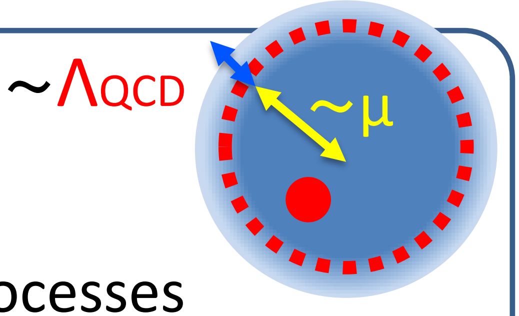


Chap.4: (If we have time)
- Near Quarkyonic boundary
- N-N interactions

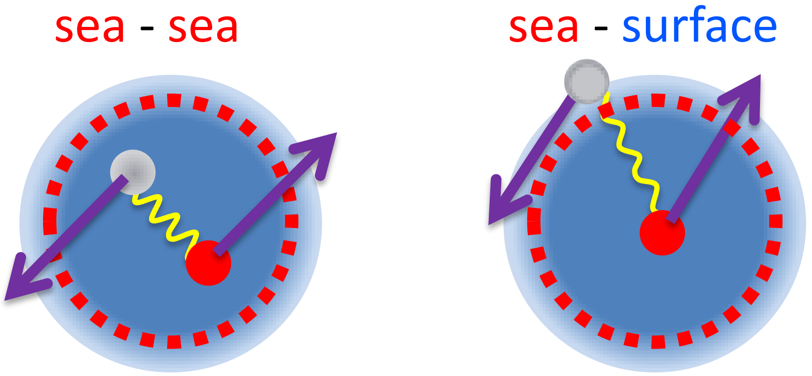
Chap.2:
- Basics of Quarkyonic matter

Chap.3:
- Chiral symmery realization

Fermi sea in asymptotic free theories. 1



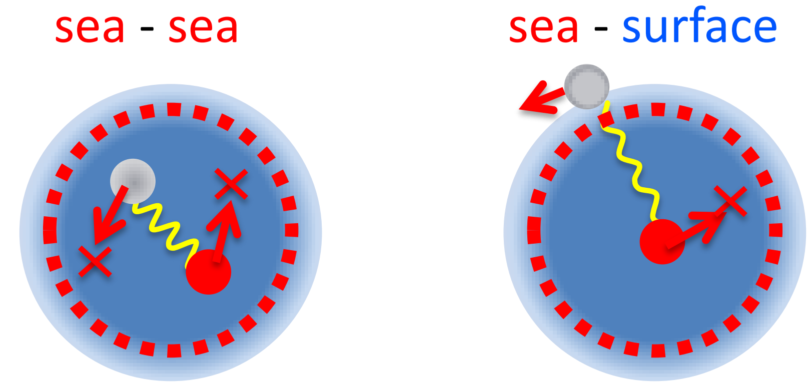
- 1, Deep inside of the Fermi sea
  - a) Hard momentum transfer processes:



Hard processes

- Typical in high density
- Perturbative

- b) Soft momentum transfer processes:



Soft processes

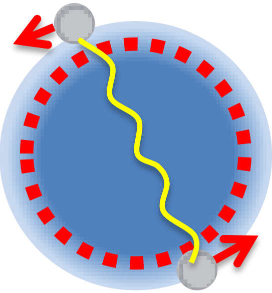
- Mostly Pauli-blocked for quarks deep inside of sea

- Hard & Soft int. do not strongly affect deep inside of sea

Fermi sea in asymptotic free theories. 2

▪2, Near the Fermi surface

surface - surface



- Surface → small fraction of total sea

  → relatively rare processes

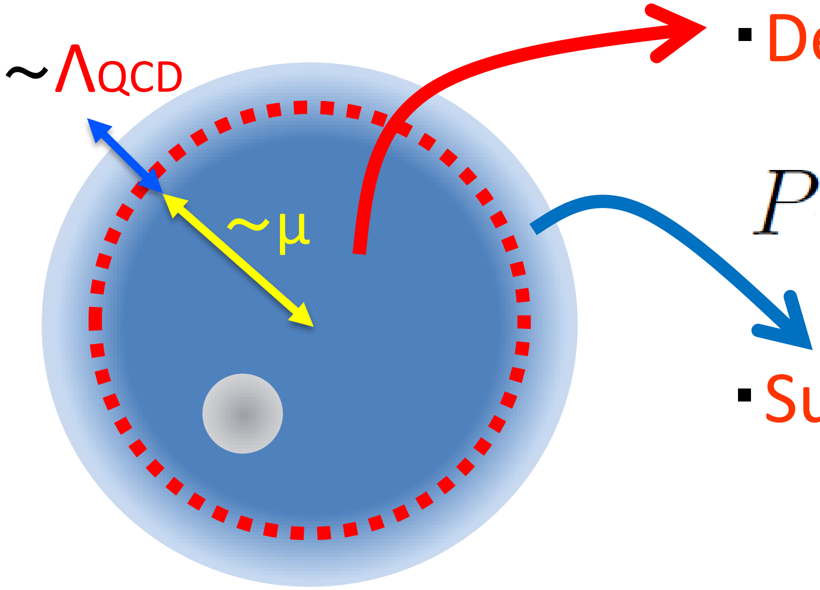
- Non-perturbative

Implications for Quark properties

- Deep inside of the Fermi sea:
  - Little chance to have soft interactions → pert. picture OK
  - Number of d.o.f. is very large.
- Near the Fermi surface:
  - Soft interactions exist, Non-pert. treatments are necessary.
  - Number of d.o.f. is small.

Fermi sea in asymptotic free theories. 3

- Free energy, Pressure:



- Deep inside: large fraction
  Perturbative

$$P = c[1 + c_1 \underline{\alpha_s(\mu)} + \ldots]\underline{\mu^4}$$

- Surface: small fraction
  Non- perturbative

$$+ O(\underline{\mu^2 \Lambda_{\mathrm{QCD}}^2})$$

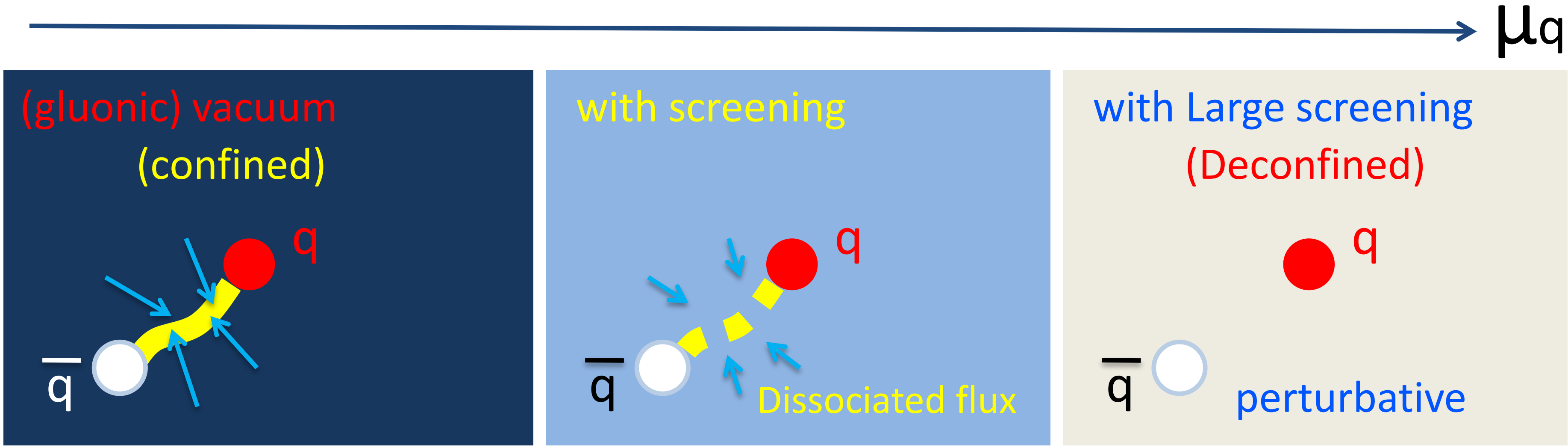
- Surface contributions are deeply connected with properties of excitation modes

→ Phase structures, transport properties, etc.

Excitation modes & gluonic vacuum

- Strength of Quantum Fluctuations:

gluons >> quarks $O(N_c^2)$ $O(N_c)$	gluons ~ quarks $O(N_c^2)$ $O(N_c) \times f(\mu)$	gluons << quarks $O(N_c^2)$ $O(N_c) \times f(\mu)$



- Note: What screens non-pert. soft gluons is:

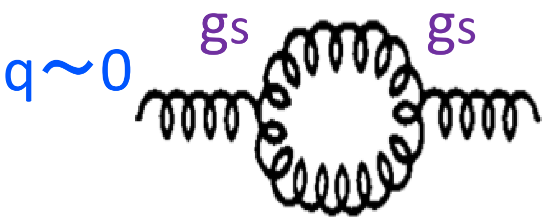
→ Soft color non-singlet $q\bar{q}$ fluctuations

(Color singlet fluctuations do not strongly couple to gluons)

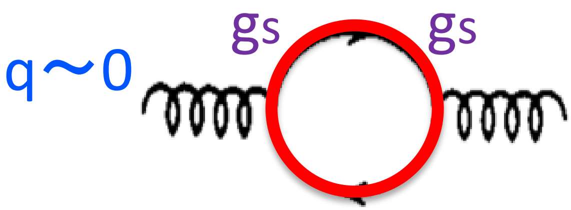
Screening effects : Vac. V.S. finite density

Allowed phase space differs → Different screening strengths

e.g.) 1-loop vacuum contributions V.S. 1-loop (electric) screening



$g_s^2 \times N_c \sim 1$



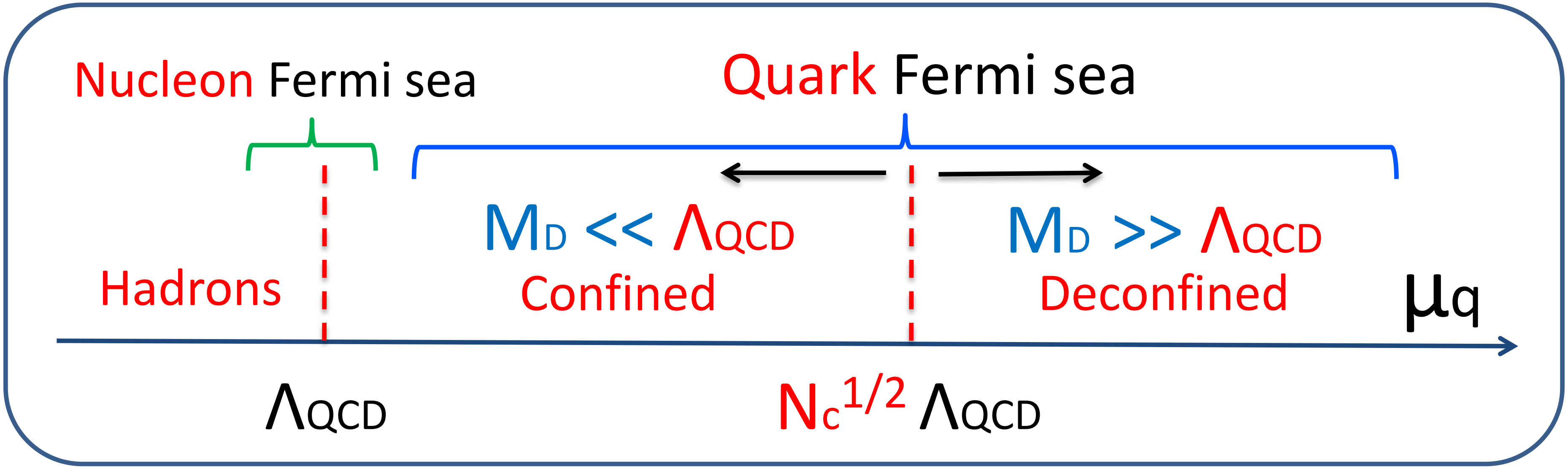
$g_s^2 \times f(\mu) \sim 1/N_c \times f(\mu)$

What is the parametric behavior of $f(\mu)$?

- Gapless quarks → $f(\mu) \sim \mu^2 \to M_D(\mu) \sim N_c^{-1/2}\,\mu$ (pert. estimate)
- Gapped quarks → $f(\mu) \sim$? (screening effects would be reduced?)

Three characteristic regions

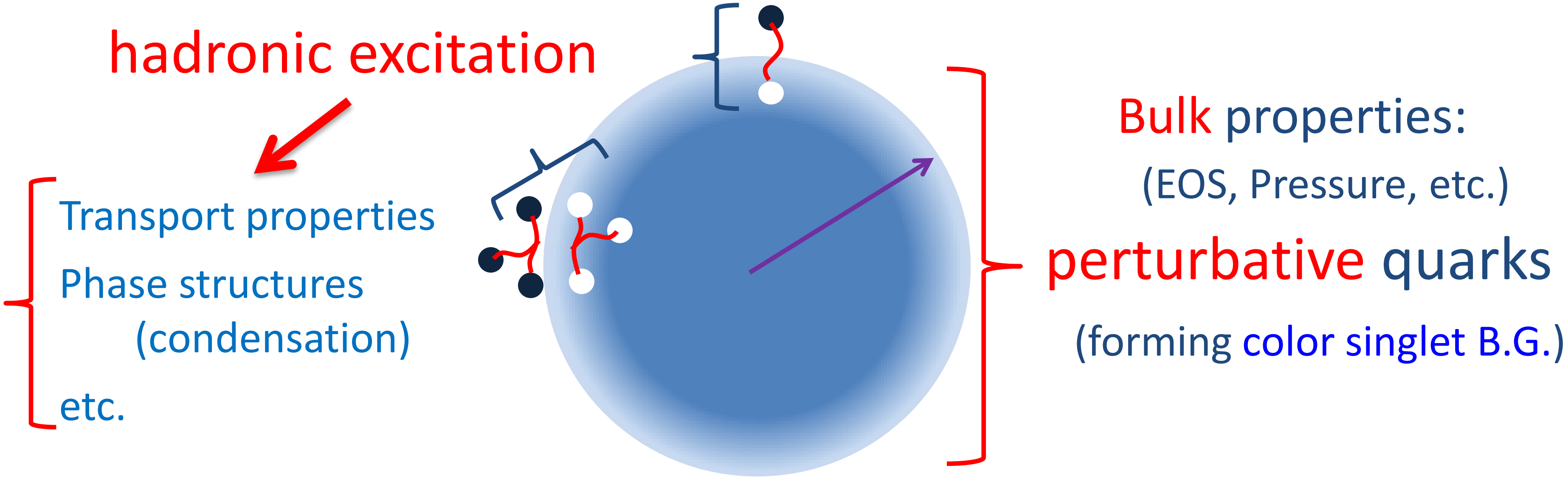
- 3-characteristic scales
  - Λ_{QCD} : intrinsic scale of QCD
  - μ : scale introduced externally
  - M_D : induced scale by external parameter



Remarks:

- In high density region, baryon-based picture breaks down because of hard core.
- Non-pert. gluons with momenta below M_D are killed.
- The scale M_D is larger than that of Nuclear matter, ~ Λ_{QCD}.

Summary of Chapter.2



hadronic excitation

- Transport properties
- Phase structures (condensation)
- etc.

Bulk properties:

(EOS, Pressure, etc.)

perturbative quarks

(forming color singlet B.G.)

Quark Fermi sea + baryonic Fermi surface → Quarkyonic

(hadronic)

- Large Nc: screening by quarks : $M_D \sim N_c^{-1/2} \to 0$

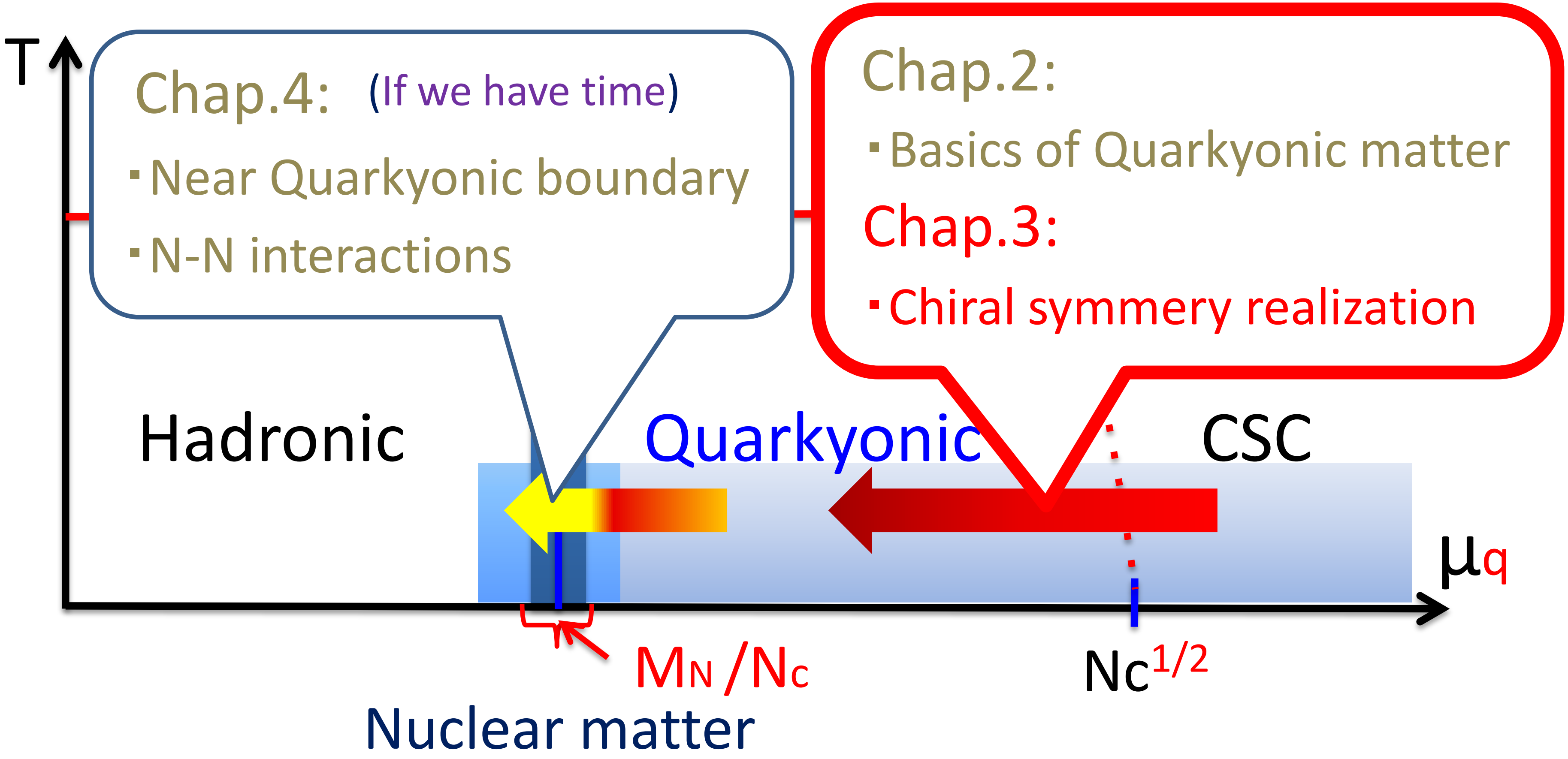
  → gluon sector unchanged.

Quarkyonic regime holds for $\mu_q \sim Nc^{1/2}\,\Lambda_{QCD}$.

Plan of this lecture

- Main Topics: T～0 region of 2-flavor massless QCD

Chap.1: 1/Nc expansion : Quick review



Chap.4: (If we have time)

- Near Quarkyonic boundary
- N-N interactions

Chap.2:

- Basics of Quarkyonic matter

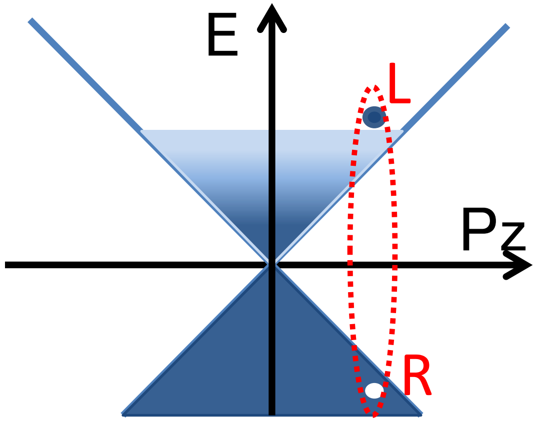
Chap.3:

- Chiral symmery realization

How is Chiral Symmetry realized ?

- Candidates which spontaneously break Chiral Symmetry

Dirac Type

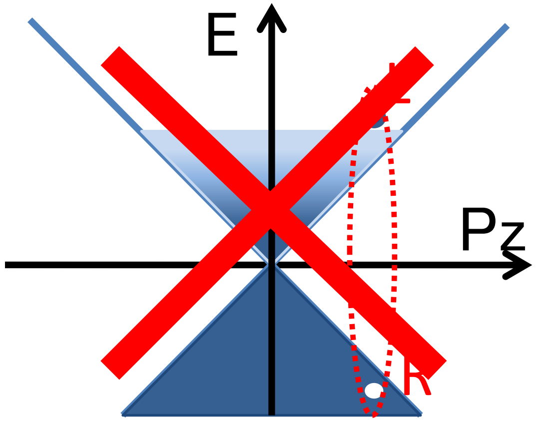


$P_{Tot}=0$ (uniform)

How is Chiral Symmetry realized ?

- Candidates which spontaneously break Chiral Symmetry

Dirac Type



$P_{Tot}=0$ (uniform)

It costs large energy,
so does not occur spontaneously.

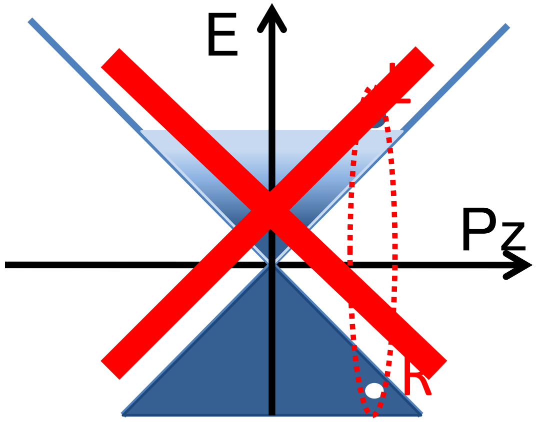
&

For most part of Fermi sea,
we need not consider sym. breaking effects.

How is Chiral Symmetry realized ?

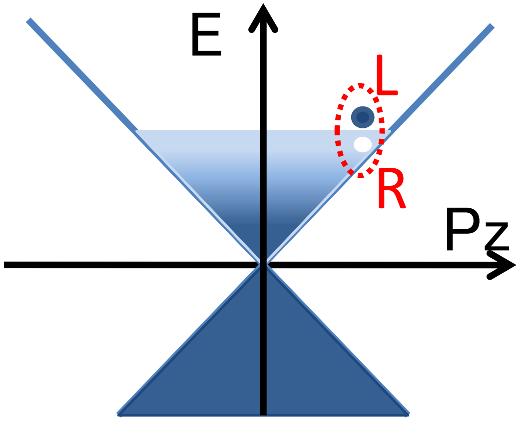
- Candidates which spontaneously break Chiral Symmetry

Dirac Type



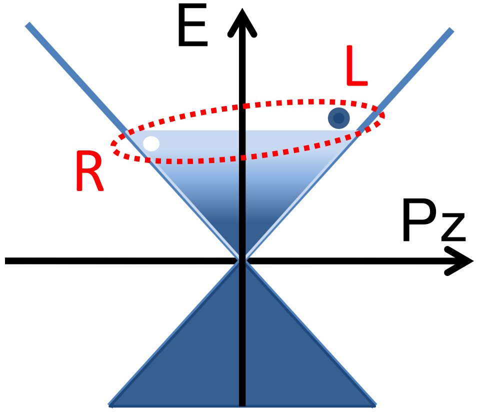
$P_{Tot}=0$ (uniform)

Exciton Type



$P_{Tot}=0$ (uniform)

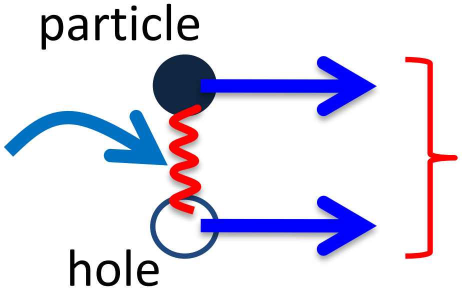
Density wave



$P_{Tot}=2\mu$ (nonuniform)

- Several model analyses → Density wave is preferred

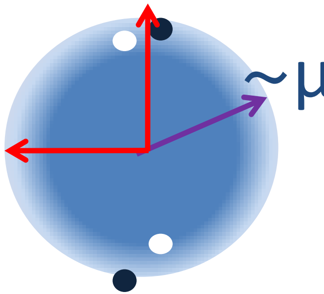
(Pert. analyses, (P)NJL, Model with confining propagators)



exchange momenta $\sim \Lambda_{QCD}$

$P_{Tot} \sim 2\mu$

Dim. reduction & Flavor Doubling



- At leading order of $1/N_c$ & Λ_{QCD}/μ

Dimensional reduction of Non-pert. self-consistent eqs :

4D “QCD” in Coulomb gauge ⟷ 2D QCD in $A_1=0$ gauge

(confining model)

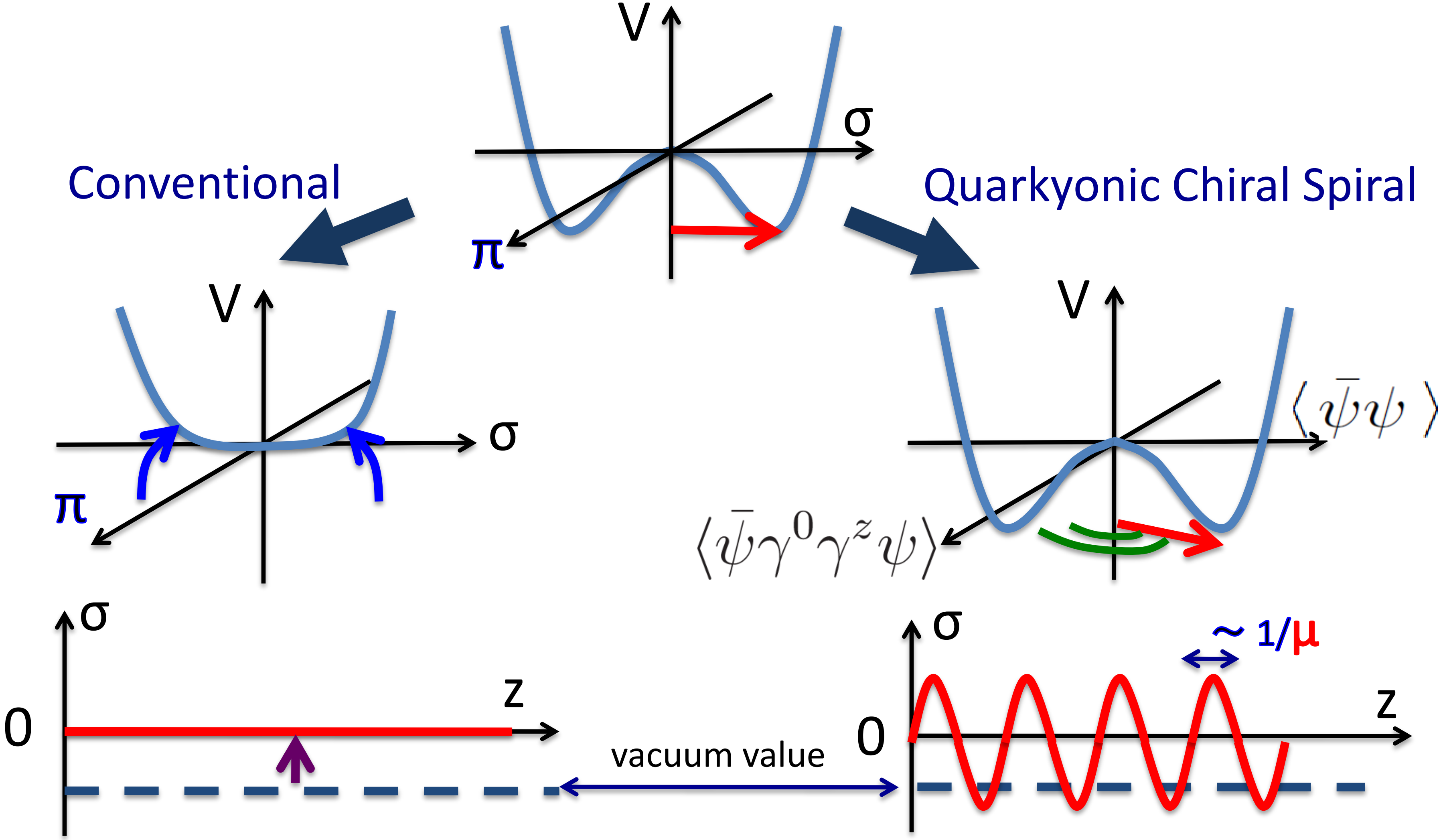
- Consequence of “flat” Fermi surface: $\Delta P_T / P_L \rightarrow 0$

Absence of γ_1, γ_2 → Absence of spin mixing

suppression of spin mixing		no angular d.o.f in (1+1) D
spin SU(2) x SU(Nf)	⟷	SU(2Nf)
(3+1)-D side		(1+1)-D side

cf) Shuster & Son, NPB573, 434 (2000)

Chiral sym. breaking/restoration



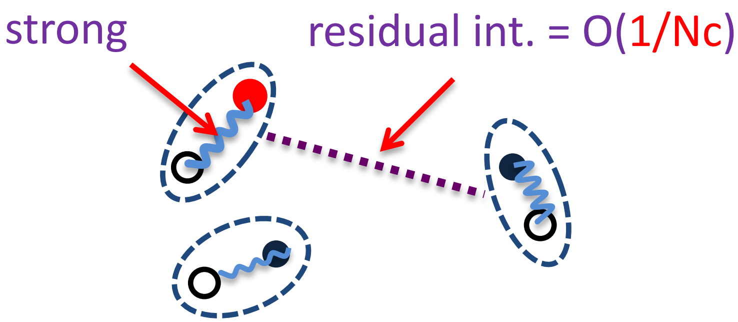
zero everywhere.

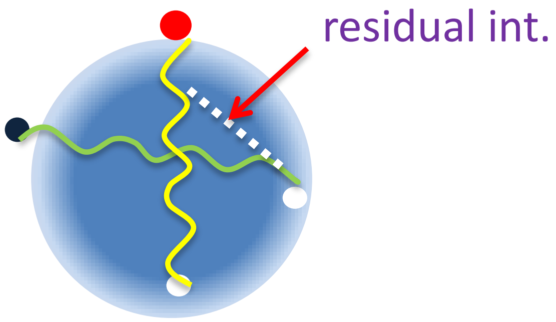
nonzero locally, but globally zero!

(cf: chiral sym. restoration in Skyrme model)

Multiple patch: Chiral Crystals

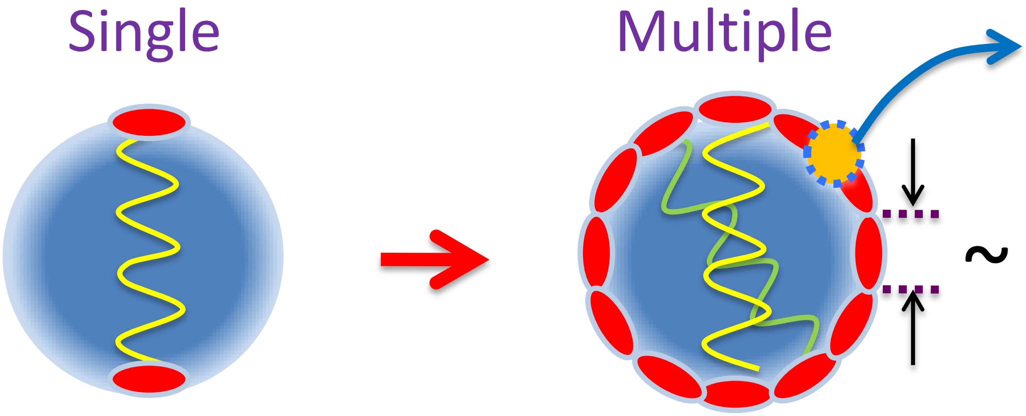
- Special properties of confining models:





Gap → weakly density dep. ~ Λ_{QCD}

- Multiple QCSs ~ Incoherent sum of single QCSs
  (+ residual interactions b.t.w. patches)



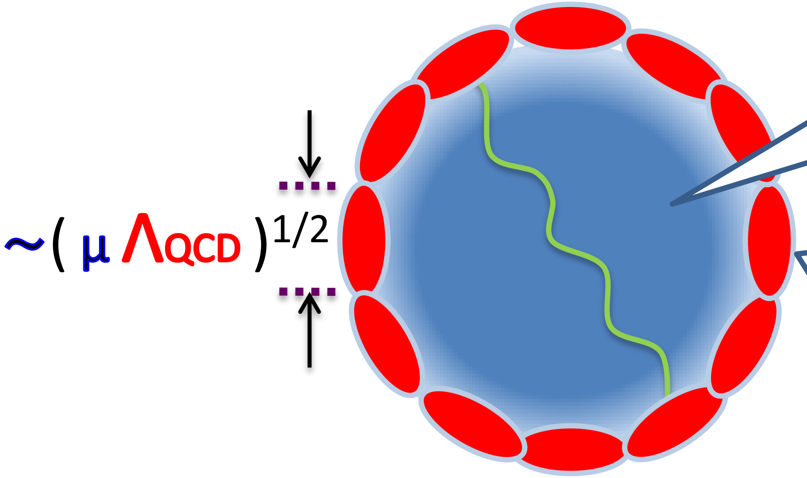
Near edge: gaps start to close

$(\Delta P_T)^2/2\mu \sim \Lambda_{QCD}$

$\sim (\mu \Lambda_{QCD})^{1/2}$

transverse size of single patch

Summary of Chapter.3



- Deep inside:

  Very likely Chiral sym. restored

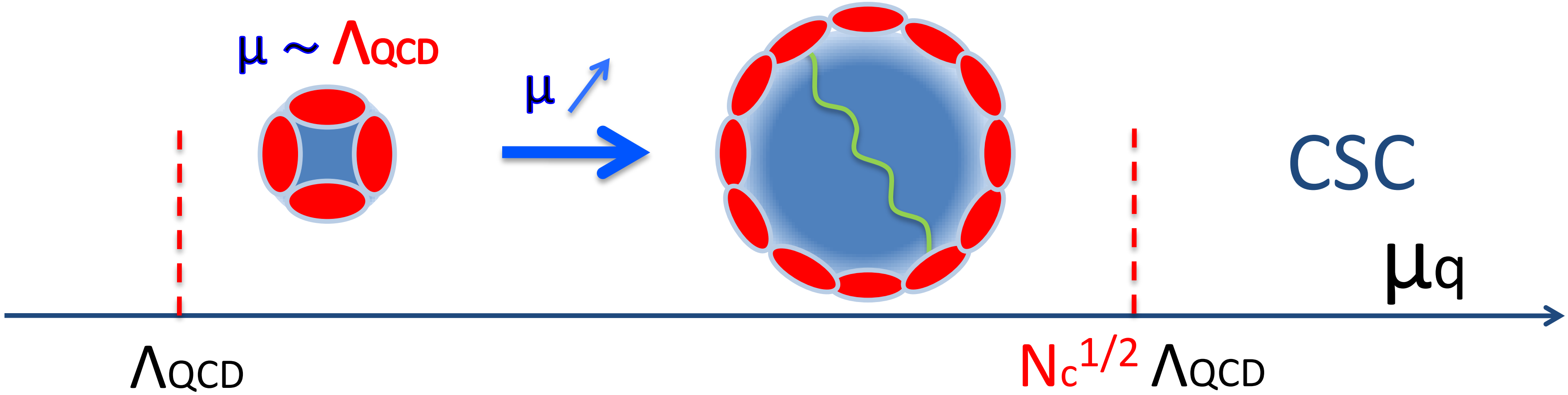
- Near Fermi surface:

  Quarkyonic Chiral Spirals

  → Local violation of P & Chiral sym.

- Number of Patches $\sim$ $\mu^2 / \mu\,\Lambda_{QCD}$ $\sim$ μ/Λ_{QCD}

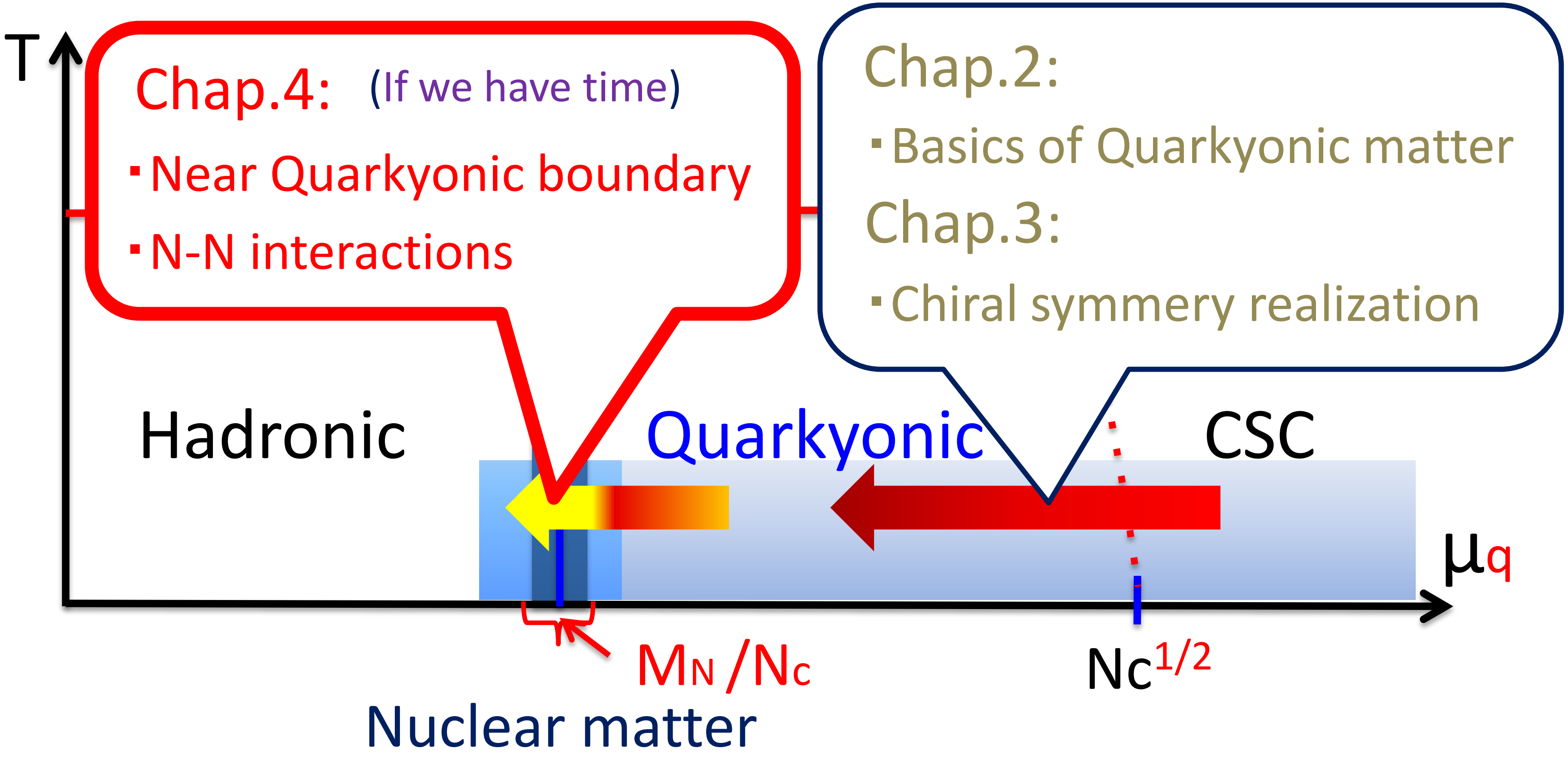
  total surface area ↗ ↖ 1-patch area



Plan of this lecture

- Main Topics: T〜0 region of 2-flavor massless QCD

Chap.1: 1/Nc expansion : Quick review

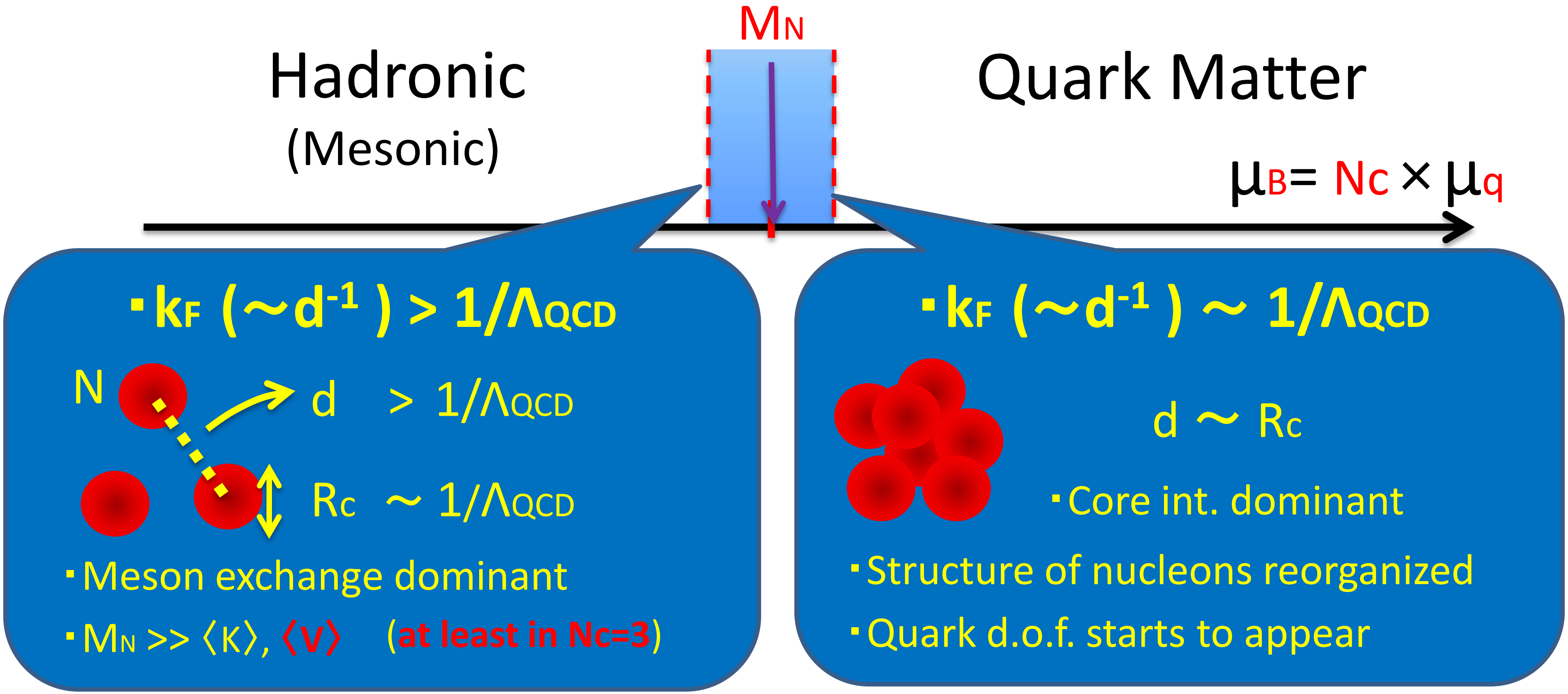


Chap.4: (If we have time)
- Near Quarkyonic boundary
- N-N interactions

Chap.2:
- Basics of Quarkyonic matter

Chap.3:
- Chiral symmery realization

Near Nuclear boundary: Rapid changes



M_N

Hadronic

(Mesonic)

Quark Matter

$\mu_B = N_c \times \mu_q$

- k_F ($\sim d^{-1}$) > $1/\Lambda_{QCD}$

N

d > $1/\Lambda_{QCD}$

R_c ~ $1/\Lambda_{QCD}$

- Meson exchange dominant
- M_N >> ⟨K⟩, ⟨V⟩ (at least in Nc=3)

- k_F ($\sim d^{-1}$) ~ $1/\Lambda_{QCD}$

d ~ R_c

- Core int. dominant
- Structure of nucleons reorganized
- Quark d.o.f. starts to appear

- $\Delta\mu_B \sim \Delta(k_F^2/N_c)$: k_F changes rapidly by small change in μ_B

→ Change from dilute to dense regime occurs within small μ_B window

(Tacit assumptions: ⟨V⟩ ~ 1/Nc, M_N = const.)

Nc dependence of N-N interactions

- Recall: Meson mediated B-B int. strongly depends on baryon w.f.

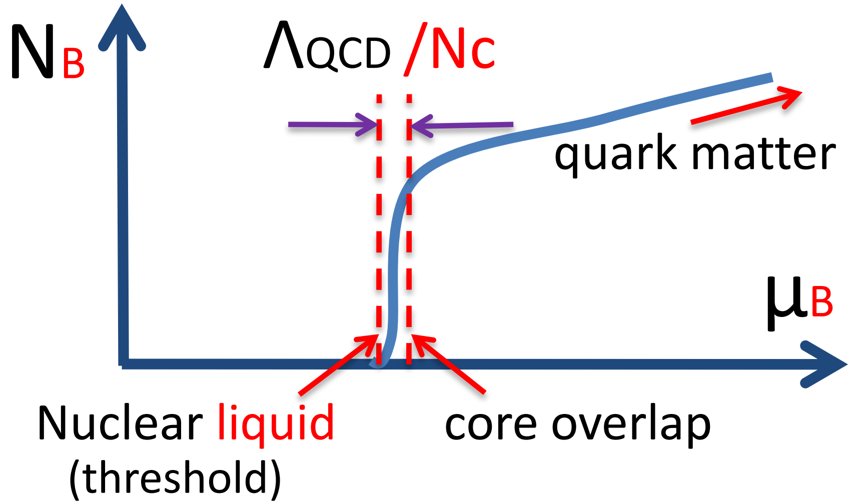
- Its impact on the phase boundary:

O(1/Nc)
(with cancellations)

O(Nc)
(Standard)

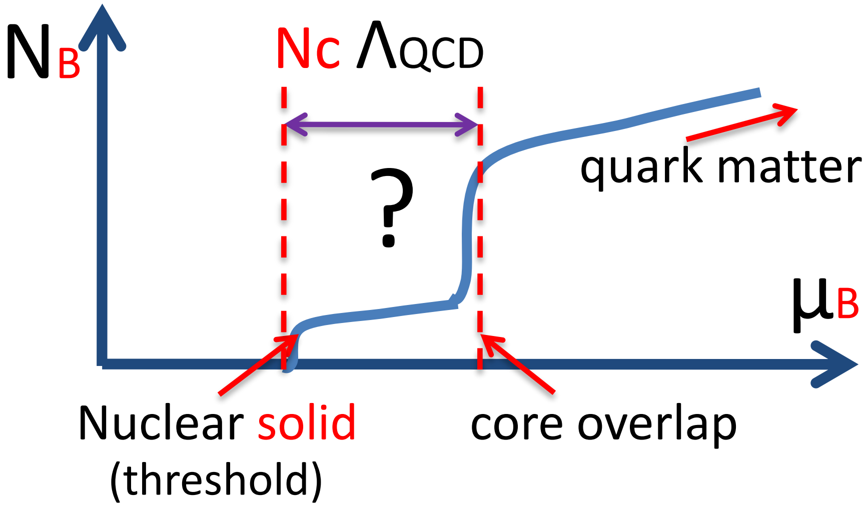
- B.E. ～ 1/Nc : ⟨V⟩ ～ ⟨K⟩

(Liquid like in low density)

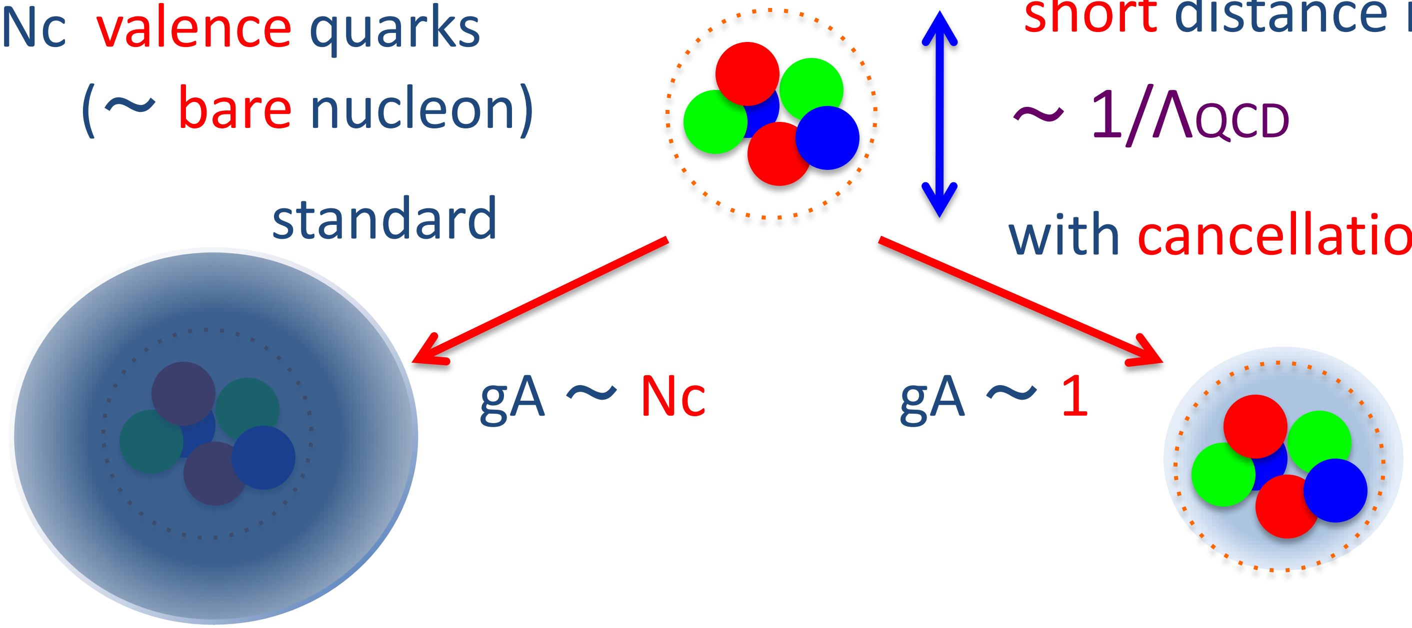


- B.E. ～ Nc : ⟨V⟩ >> ⟨K⟩

(Solid like in low density)



Axial charge gA : coupling to pions



Nc valence quarks
(～ bare nucleon)

short distance region
～ 1/ΛQCD

standard

with cancellations

gA ～ Nc

gA ～ 1

coherent

O(Nc)
(coherent overlap)

<K> << <V>
(solid - like)

pion clouds

NN potential
(long distance)

low density regime

quantum

O(1/Nc)
(one pion exchange)

<K>～<V>～1/Nc
(liquid - like)

Summary of Chapter.4

- 1, Subtle issues exist in Quark-Nuclear matter boundary, especially related to N-N interactions in long distance.

  (Mass shift would make situations more complicated.)

- 2, Axial charge gA is key quantity to measure long range force, and picture of large Nc nucleons.

  (example of small gA w.f. → Hidaka-Kojo-McLerran-Pisarski10)

- 3, IF baryon w.f. has reasonable order of charges, large Nc phen. does not strongly differ from Nc=3 case.

O(1) charges of nucleons give O(1/Nc) int. (e.g., ω, ρ, ... exchange)

Quasi-particle picture of baryons (due to weak int.)

Summary

1/Nc expansion

- is useful **classification** method.
- allows us to **formulate** very **simple**, nevertheless, sufficiently **nontrivial** **problems** & **questions**.

Quarkyonic Matter offers

- chances to reconsider basic concepts in Dense QCD.

  (conventional arguments except treatments of Fermi surface region, remain unchanged).

- interesting & educational, theoretical lab.

Appendix

A simple model of linear confinement

- Confining propagator for quark-antiquark (quark-hole):

$$D_{\mu\nu} = C_F \times g_{\mu 0} g_{\nu 0} \times \frac{\sigma}{(\vec{p}^2)^2}$$ (linear rising type)

strong IR enhancement

cf) leading part of Coulomb gauge propagator (ref: Gribov, Zwanziger)

- Absence of q$\bar{\text{q}}$ continuum in mesonic channel

→ linear confinement

- We will apply nonperturbative treatments:

Schwinger-Dyson & Bethe-Salpeter equations.

- We dimensionally reduce these from (3+1)D to (1+1)D.

(Pert. regime; Deryagin-Grigoriev-Rubakov ’92, Shuster-Son 99, etc)

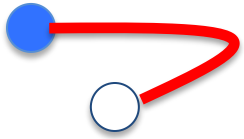
On the IR prescription

$$\frac{\sigma}{(\vec{k}^2)^2} \xrightarrow{\text{IR cut}} \frac{\sigma}{(\vec{k}^2+\Lambda_{\mathrm{IR}}^2)^2} \xrightarrow{\text{F.T.}} \underline{-\frac{\sigma}{\Lambda_{\mathrm{IR}}}} + \overset{\text{linear potential}}{\sigma r} + O(\Lambda_{\mathrm{IR}} r^2)$$

- Probe colored objects:

IR div.: const. from naïve IR cutoff

- Color singlet sector:



IR const. → irrelevant.
(Linear conf. without IR const.)

- As far as color-singlet sector is concerned,
  we can get the same results even if we drop off div. const.
  (principal value IR regulation; e.g., Coleman, Aspects of Symmetry)

- S-D eqs. → just sub-diagrams in B-S eqs.
- Div. of poles will be used as color selection rules at best.

e.g.) Dim. reduction of Schwinger-Dyson eq. 1

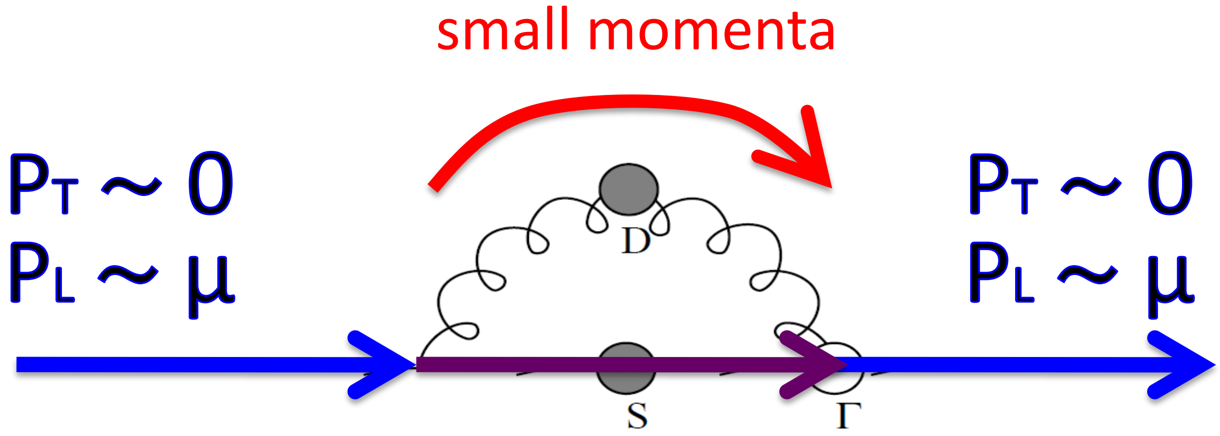
quark self-energy

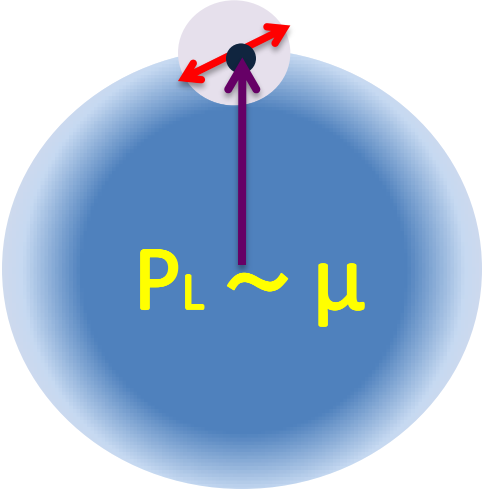
including ∑

$$\not{\Sigma}(p) + \Sigma_m(p) = \int \frac{dk_4 dk_z d^2\vec{k}_T}{(2\pi)^4} \gamma_4 S(k) \gamma_4 \frac{\sigma}{|\vec{p} - \vec{k}|^4}$$

- **Note1**: Mom. restriction from **confining** interaction.

$\Delta k \sim \Lambda_{QCD}$





e.g.) Dim. reduction of Schwinger-Dyson eq. 2

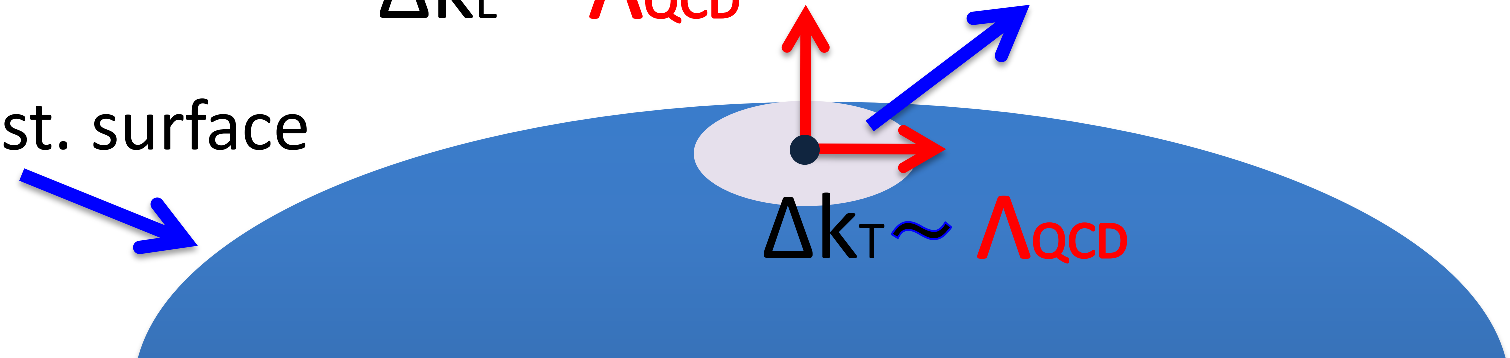
quark self-energy

$$\not{\Sigma}(p) + \Sigma_m(p) = \int \frac{dk_4 dk_z d^2\vec{k}_T}{(2\pi)^4} \gamma_4 \underline{S(k)} \gamma_4 \frac{\sigma}{|\vec{p}-\vec{k}|^4}$$

- **Note2**: Suppression of transverse part:

$$S(k) = \gamma_0 S_0 - \underbrace{\gamma_z S_z}_{\sim \mu} - \underbrace{\vec{\gamma}_T \cancel{\vec{S}_T}}_{\sim \Lambda_{QCD}} + S_m$$

- **Note3**: Quark energy is insensitive to small change of k_T :



$\Delta k_L \sim \Lambda_{QCD}$

along E = const. surface

E= const. surface

$\Delta k_T \sim \Lambda_{QCD}$

e.g.) Dim. reduction of Schwinger-Dyson eq. 3

insensitive to kT

$$\not{\Sigma}(p) + \Sigma_m(p) = \underline{\int \frac{dk_4 dk_z d^2\vec{k}_T}{(2\pi)^4} \gamma_4 S(k) \gamma_4 \frac{\sigma}{|\vec{p}-\vec{k}|^4}}$$

factorization

$$\gamma_4\Sigma_4 + \gamma_z\Sigma_z = \int \frac{dk_4 dk_z}{(2\pi)^2} \gamma_4 S(k_4, k_z, \underline{\vec{0}_T}) \gamma_4 \otimes \underline{\int \frac{d\vec{k}_T}{(2\pi)^2} \frac{\sigma}{|\vec{p}-\vec{k}|^4}}$$

smearing

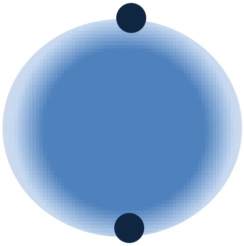
confining propagator in (1+1)D: $\boxed{\frac{\sigma}{2\pi} \frac{1}{|p_z - q_z|^2}}$

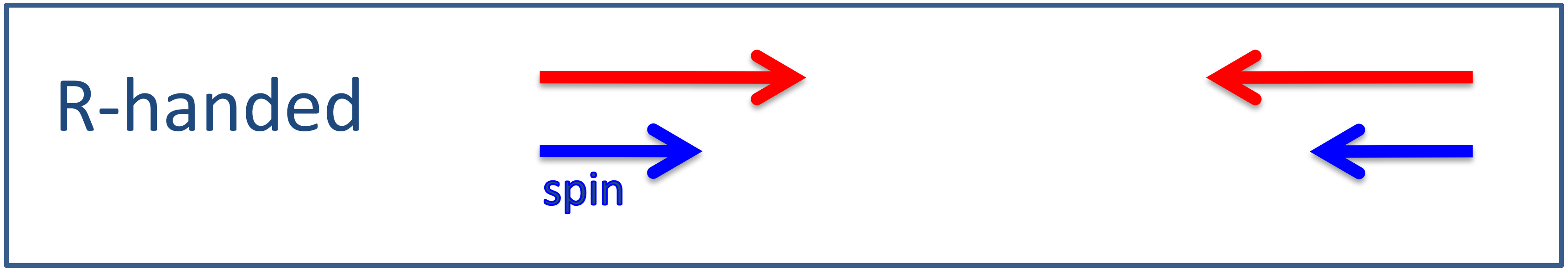
Schwinger-Dyson eq. in (1+1) D QCD in $A_1=0$ gauge

Bethe-Salpeter eq. can be also converted to (1+1)D

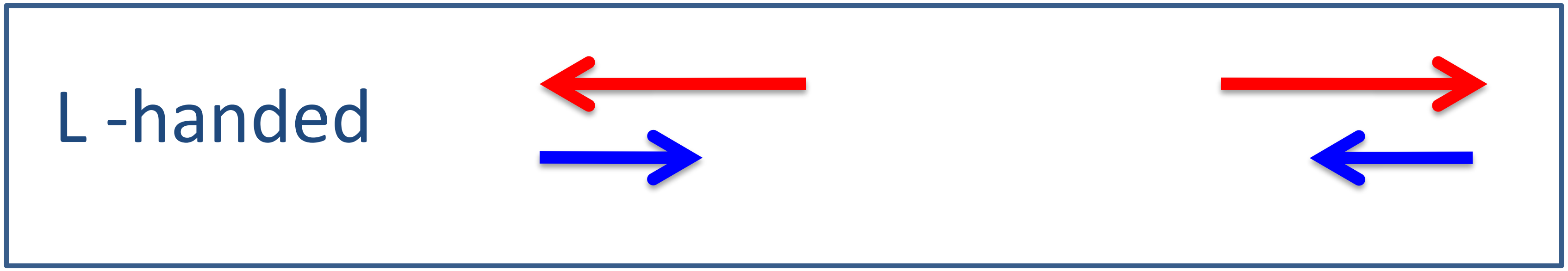
Flavor Multiplet

particle near north & south pole





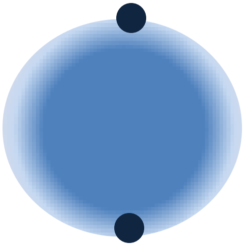
R-handed

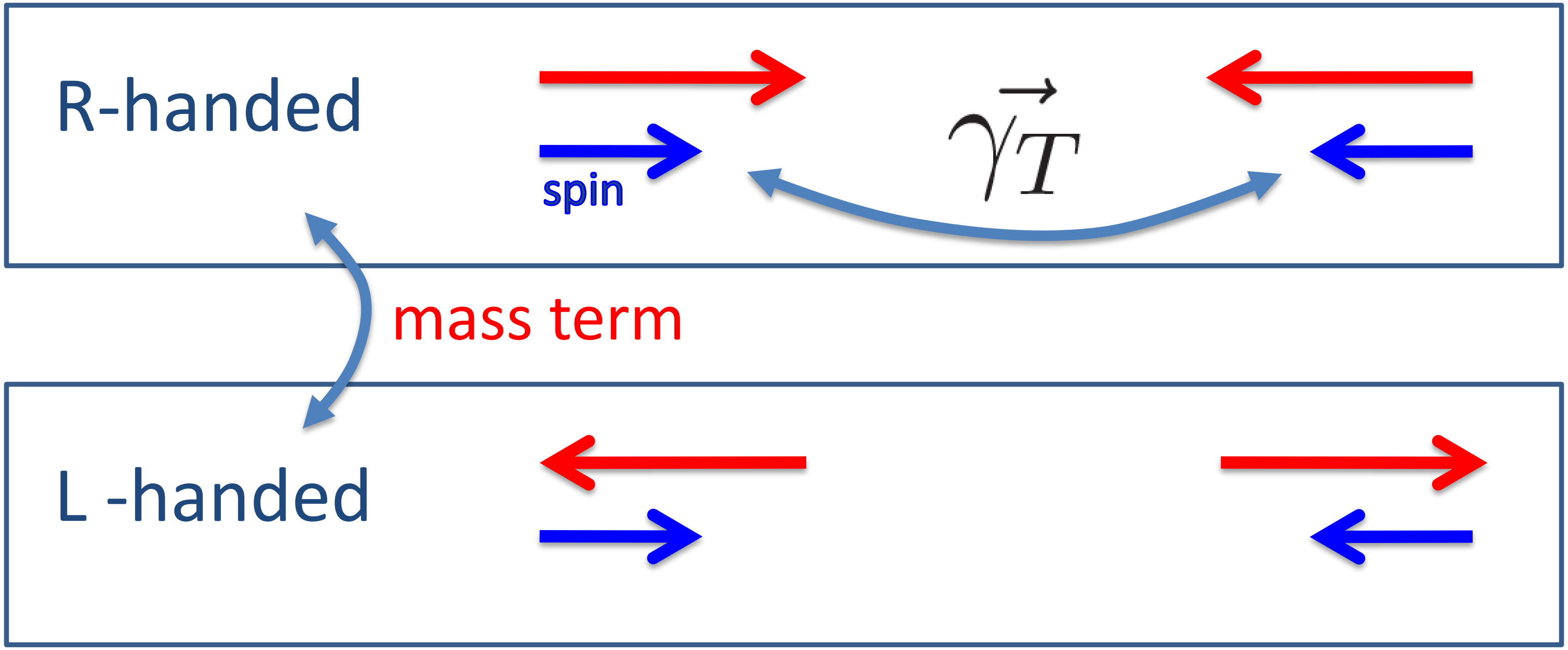


L -handed

Flavor Multiplet

particle near north & south pole





R-handed

spin

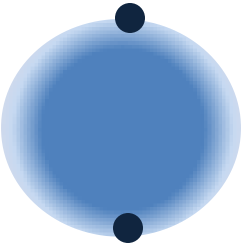
$\vec{\gamma}_T$

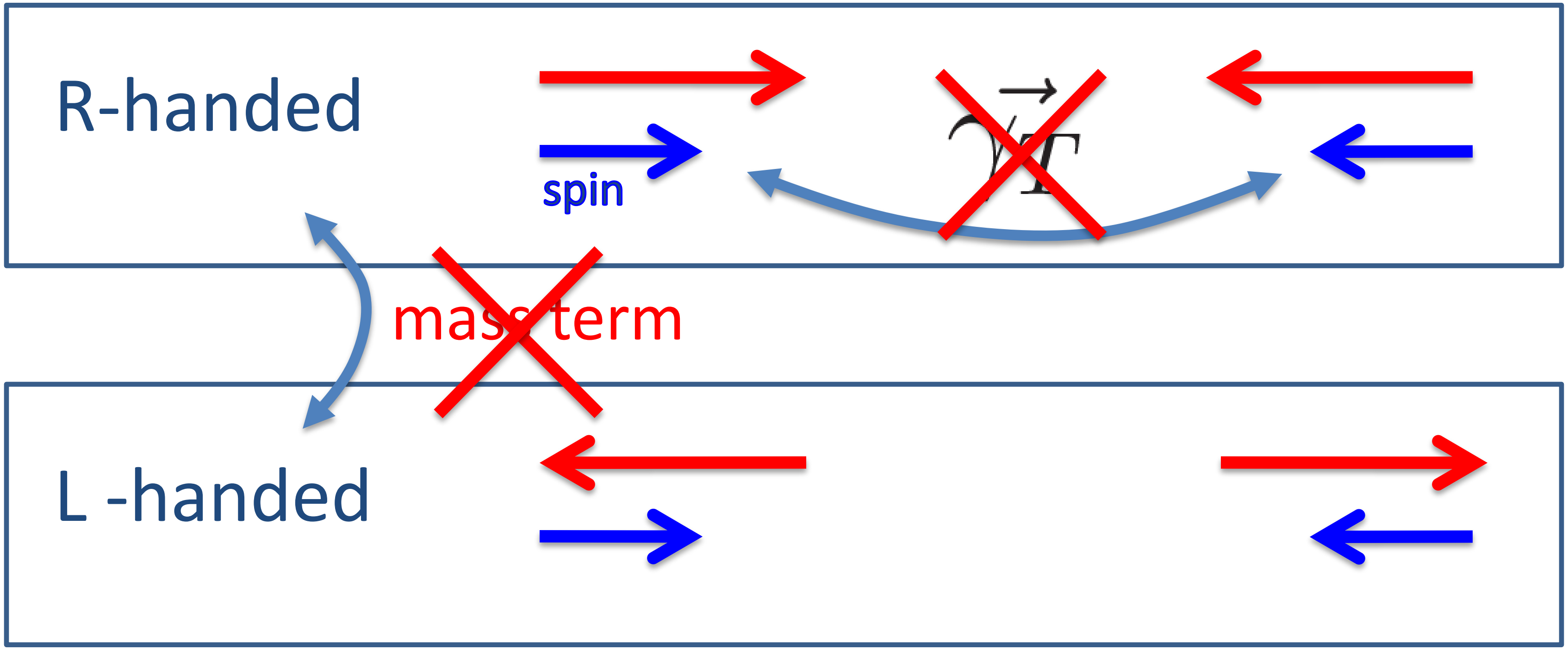
mass term

L -handed

Flavor Multiplet

particle near north & south pole





R-handed

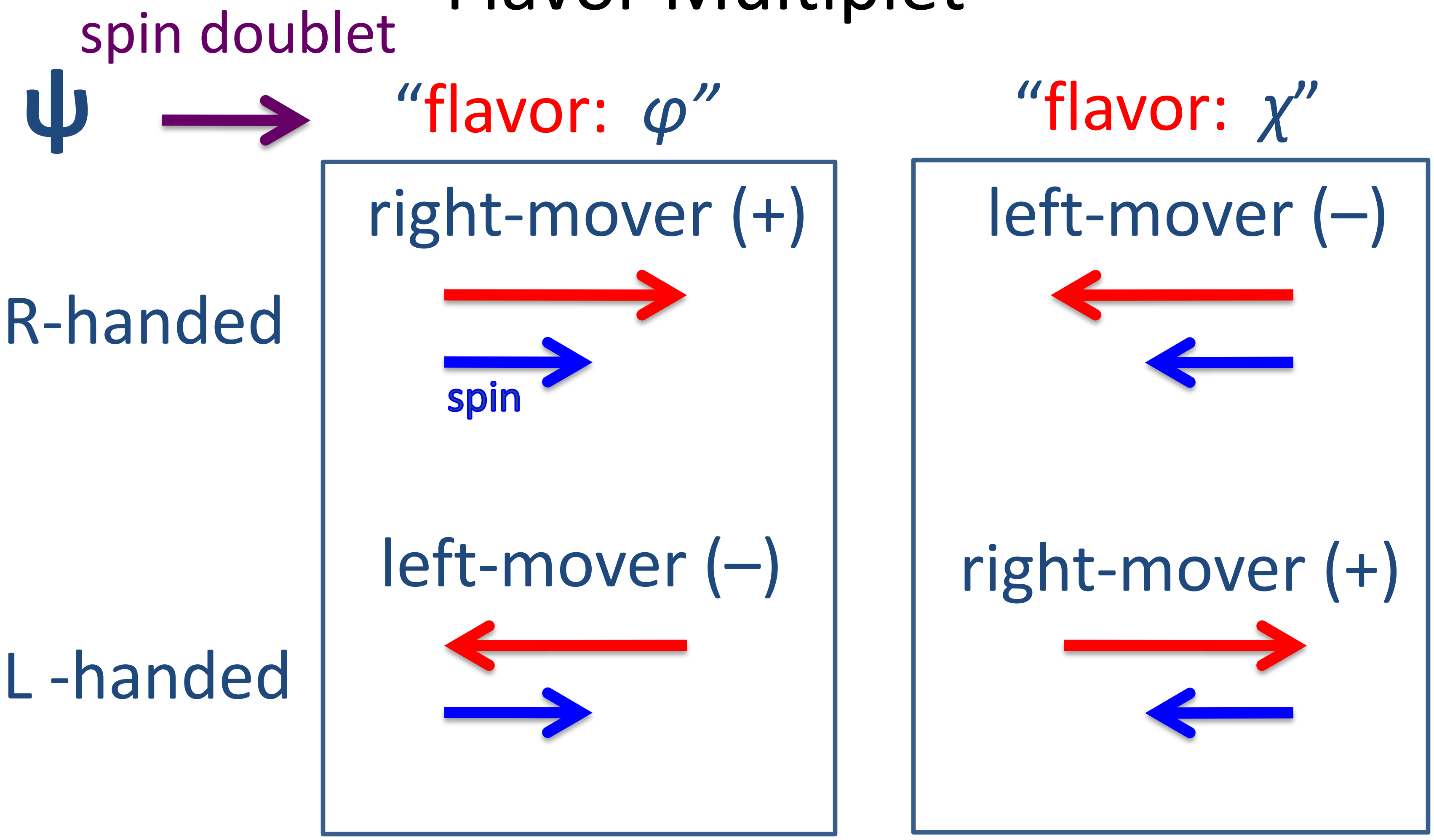
spin

$\cancel{\gamma \vec{T}}$

~~mass term~~

L -handed

Flavor Multiplet



spin doublet

ψ

“flavor: φ”

“flavor: χ”

R-handed

right-mover (+)

spin

left-mover (–)

L -handed

left-mover (–)

right-mover (+)

Moving direction: (1+1)D “chirality”

(3+1)D – CPT sym. directly convert to (1+1)D ones

Relations between composite operators

- 1-flavor (3+1)D operators without spin mixing:

$$\begin{tikzcd}
\bar{\psi}\psi \arrow[d] & \bar{\psi}\gamma^0\psi \arrow[d] & \bar{\psi}\gamma^z\psi \arrow[d] & \bar{\psi}\gamma^0\gamma^z\psi \arrow[d] \\
\bar{\Psi}\Psi & \bar{\Psi}\Gamma^0\Psi & \bar{\Psi}\Gamma^z\Psi & \bar{\Psi}\Gamma^5\Psi
\end{tikzcd}$$

Flavor singlet in (1+1)D

- All others have spin mixing:

ex)

$$\bar{\psi}\gamma^5\psi \to \bar{\Phi}\tau_3\Gamma^5\Phi, \qquad i\bar{\psi}\gamma^1\psi \to \Phi\tau_2\Gamma^5\Phi,$$

(They will show no flavored condensation)

Flavor non-singlet in (1+1)D

Dictionary: μ = 0 & μ ≠ 0 in (1+1)D

- μ ≠ 0 2D QCD can be mapped onto μ = 0 2D QCD

$$\Phi = \exp\left(-i\mu z\Gamma^5\right)\Phi'$$

: Chiral rotation

(Opposite shift of mom. for (+, -) moving states)

$$\boxed{\underset{(\mu \neq 0)}{\overline{\Phi}\left[i\Gamma^\mu\partial_\mu + \mu\Gamma^0\right]\Phi} \rightarrow \underset{(\mu = 0)}{\overline{\Phi}' i\Gamma^\mu\partial_\mu\Phi'}}$$

(due to special geometric property of 2D Fermi sea)

- Dictionary between μ = 0 & μ ≠ 0 condensates:

μ = 0 μ ≠ 0

$$\langle\overline{\Phi}'\Phi'\rangle \rightarrow \cos(2\mu z)\,\langle\overline{\Phi}\Phi\rangle - \sin(2\mu z)\,\langle\overline{\Phi}i\Gamma^5\Phi\rangle$$

$$\underset{(=0)}{\langle\overline{\Phi}'\Gamma_0\Phi'\rangle} \rightarrow \underset{(=0)}{\langle\overline{\Phi}\Gamma_0\Phi\rangle} + \underline{\frac{\mu}{2\pi}}$$

induced by anomaly

"correct baryon number"

Why Chiral Spirals in (1+1)D ?

- Key observation: Moving direction = (1+1)D Chirality

$$\langle \bar{\varphi}_- \varphi_+ \rangle = \Delta e^{\underline{2i\mu z}} \qquad \langle \bar{\varphi}_+ \varphi_- \rangle = \Delta e^{\underline{-2i\mu z}}$$

Opposite phase

$$\boxed{\langle \bar{\varphi} \Gamma_5 \varphi \rangle = \langle \bar{\varphi}_- \varphi_+ \rangle - \langle \bar{\varphi}_+ \varphi_- \rangle = \Delta i \sin 2\mu z \neq 0}$$

Density wave of $\overline{\Phi}\Phi$ inevitably accompanies $\overline{\Phi} i \Gamma^5 \Phi$

(because of phase mismatch)

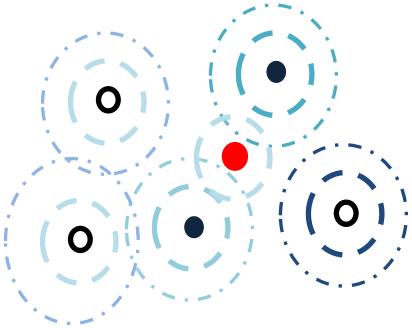
(1+1)D: Chiral Density wave → Chiral Spiral

Toward multiple patch construction. 1

One patch results may be good starting point.

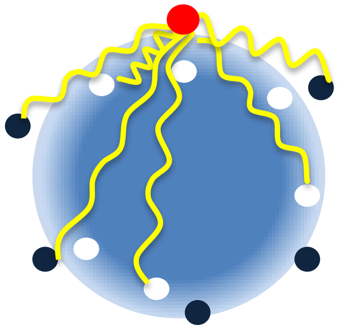
Perturbative gluons

- r - space)



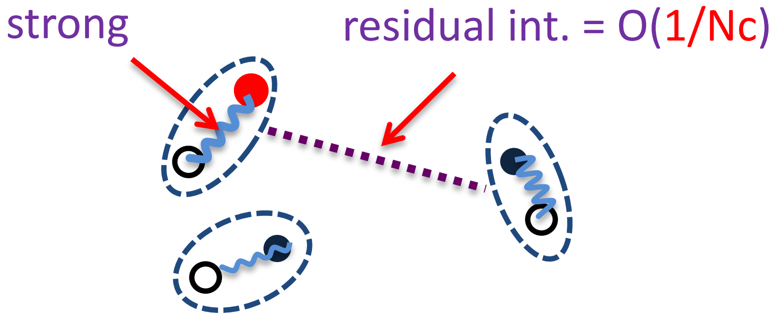
Influence by all other quarks
must be treated simultaneously.

- p - space)

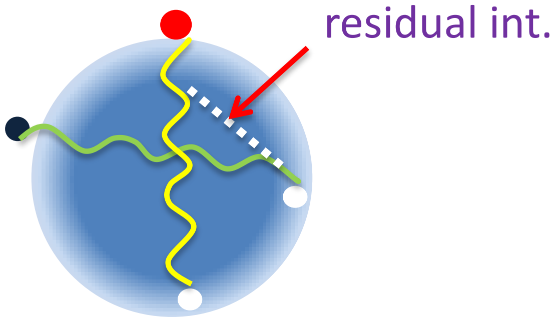


Gap → strongly density dependent.

Confining gluons



After mesonic objects are formed,
residual interactions enter.



Gap → weakly density dependent.
(confinement - origin)

Toward multiple patch construction. 2

- e.g.) Quark-Condensate int. in the presence of many QCSs

Sum over all Chiral spirals

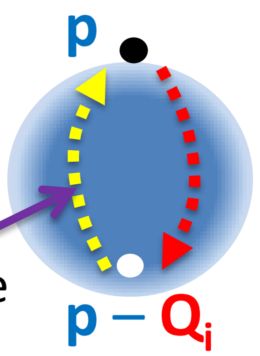
$$\sum_{i=1}^{N_p}\int\frac{d^4p}{(2\pi)^4}\bar{\psi}(p-\boldsymbol{Q}_i)M(\underline{p};\boldsymbol{Q}_i)\psi(p)$$

Space-dependent mass self-energy

- Key point: Quarks with high virtuality feel small Chiral Sym. breaking

For both of p^2 and $(p-Q_i)^2$ to be close to Minkovski region:

Angle between **p** and **Q**$_i$ ⟶ $|\theta| < \Lambda_{\rm QCD}/p_F$



e.g.) θ ~ 0 case

If angles between quark moving direction and QCS are large:

⟶ Chirality changing scatterings are suppressed.

⟶ Each QCS behaves incoherently (except matching point of patches)

Quarkyonic Chiral Spirals vs …..

1, Perturbative gluon propagator : Deryagin, Grigoriev, & Rubakov '92

- Scalar CDW (not spirals) was studied in large Nc, high density regime.
- Gaps are small, and reach ~ Λ_{QCD} when **μ ~ 100 GeV.**

2, + Screening effects : Shuster & Son 99 Park-Rho-Wirzba-Zahed 99

- Spirals (same structure as QCS) are found in large Nc.
- Screening mass develops faster than pert. gap, so no spirals in Nc=3.

3, Effective models : Nakano-Tatsumi 04, Nickel08, Carignano-Nickel-Buballa10

Ralf-Shuryak-Zahed01

- Relatively low density regime.
- CDW or CS or solitons in σ-π (not σ-Tensor) channels are studied.

4, Non-Perturbative gluon propagator : This work

- Spirals are studied in large Nc, relatively high density regime.
- gap is confinement origin ~ Λ_{QCD} (>> perp. gap) ,

  it may be possible to have QCS before screening mass fully develops.

3 possibilities to construct sensible arguments

1, To find additional hidden expansion parameters.

e.g.) 1/λ in Sakai-Sugimoto model, etc.

2, To find some cancellation mechanisms.

At the level of pion-Nucleon dynamics

e.g.) Gervais-Sakita, Dashen-Manohar

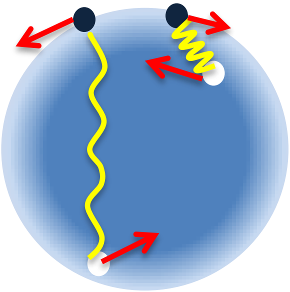
3, To change proposition of gA from O(**Nc**) to O(1).

At the level of internal structure of Nucleons

→ This work

Non-perpurbative processes

- 1) Int. b.t.w. quarks & holes in color singlet channel:



- Small mom. transfer
- Color summation

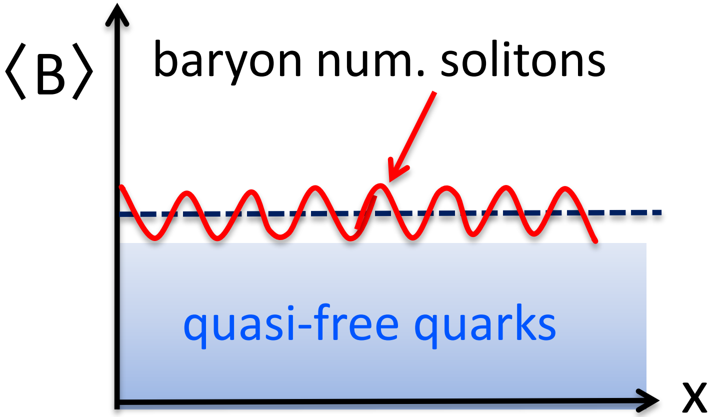
Mesonic type

Fermi surface is described by weakly interacting “Meson” gas.

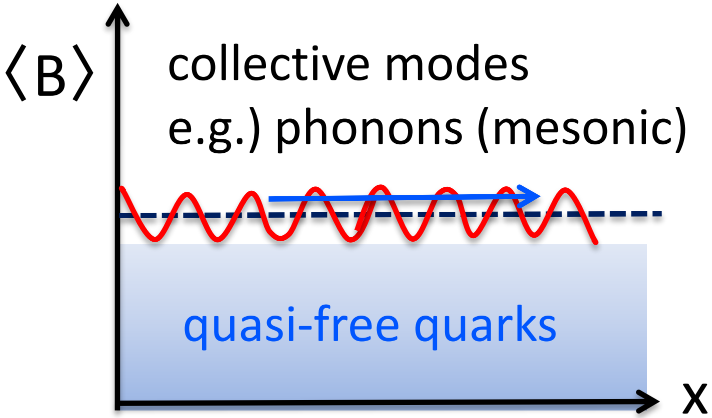
(Once correct ground state is constructed)

- 2) (Possible) Baryon Number crystals & Collective modes:

Ground state



Excitation



Nucleon Fermi sea v.s. Quark Fermi sea

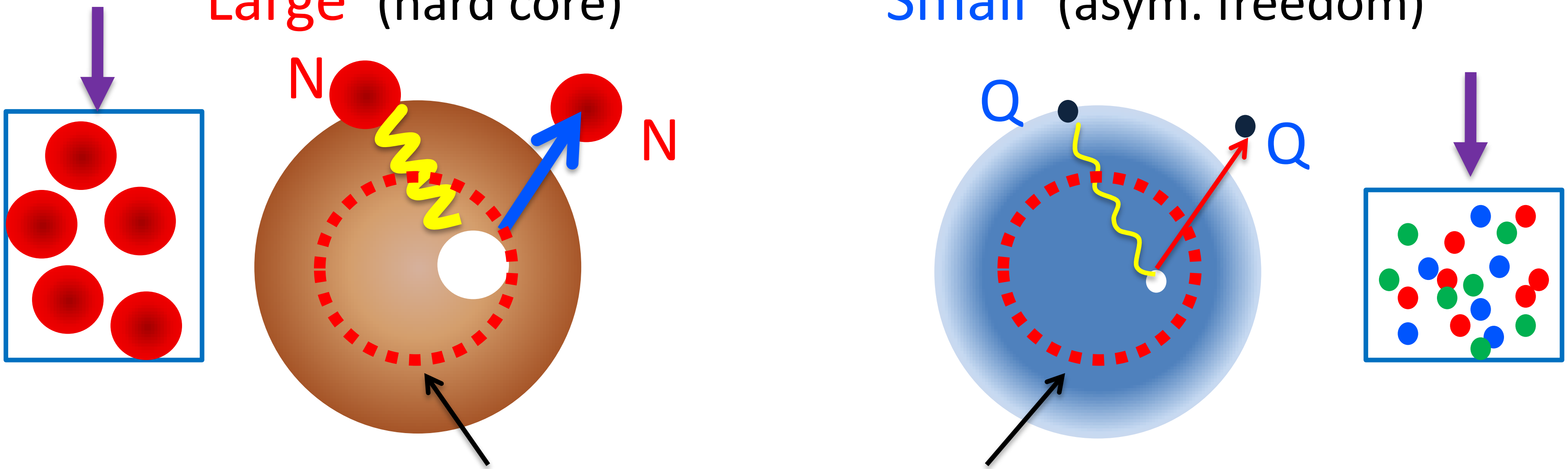
- Crucial difference: Strength of Short distance int.

Nucleon Fermi sea

Large (hard core)

Quark Fermi sea

Small (asym. freedom)

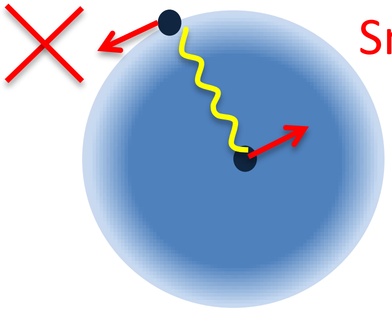


As density increases, Nucleon Surface destroys Nucleon sea:

→ Quark Fermi Sea: More natural description

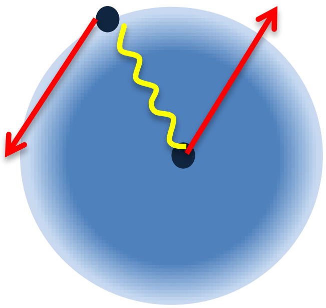
Weakly interacting processes

- 1) Interaction b.t.w. quarks near surface and deep in sea:



Small mom. transfer,

but Pauli blocked



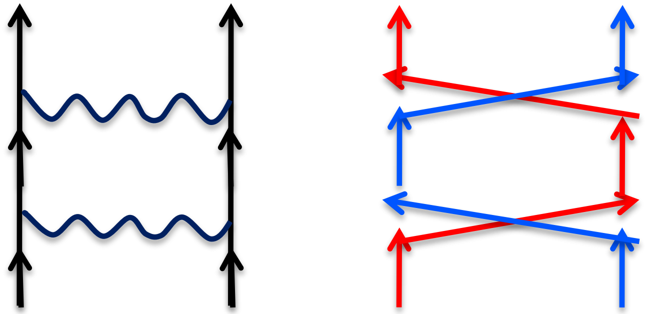
Not Pauli blocked,

but large mom. transfer

→ Guarantee Quark description of deep inside of Fermi sea

- 2) Interaction in color non-singlet channel:

e.g.) Diquark channel

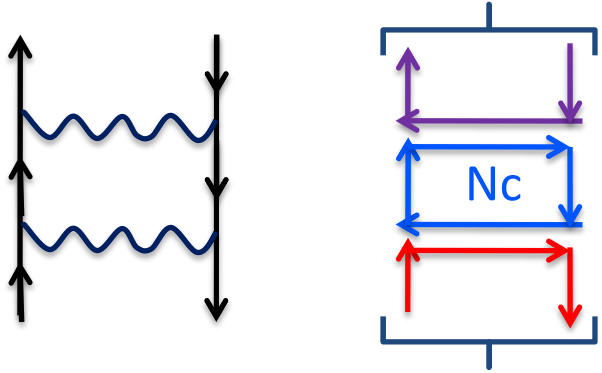


No color summation

→ suppressed in large Nc

cf) Singlet channel

color sum



color sum

More on Thermodynamic properties

- Bulk properties: Free energy, Pressure, etc.

→ Quantities to which ALL quarks contribute

Typical processes in high density → hard scattering

pert. non-pert.

$$P = c[1 + c_1\alpha_s(\mu) + \ldots]\underline{\mu^4} + O(\mu^2\Lambda_{\rm QCD}^2)$$

- Phase structures, transport properties:

→ Sensitive to Excitations near the Fermi surface

Soft scattering is NOT forbidden near the Fermi surface

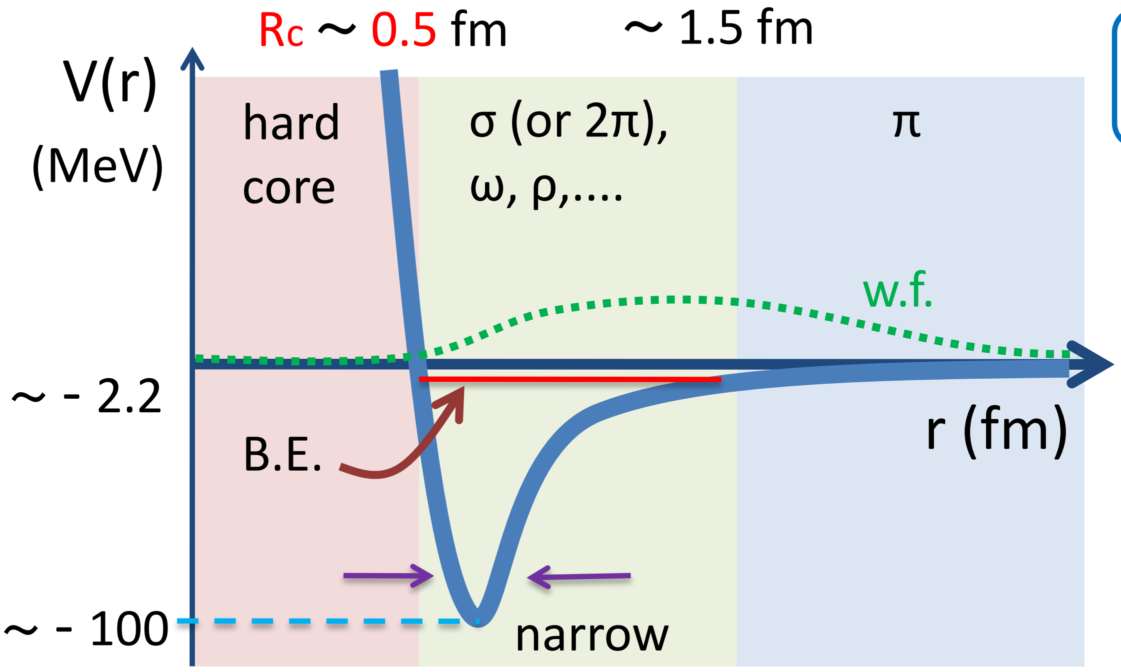
→ Confining effects

Contributions to Pressure is small compared to pert. one,

But it is this part which classifies different phase structures!

Bases for Nuclear Matter (Nc=3)

- e.g.) N-N pot. for Deuteron (p-n): ($I=0^+$, effective $^3S_1$ pot.)



- $\langle K \rangle \sim -\langle V \rangle$ ($\ll M_N$)

- zero point oscillation large
  (quantum property)
- B.E. ~ - 2.2 MeV ($\ll \Lambda_{QCD}$)
- w.f. widely spread
  (long range part: relevant)
- Typical distance: d ~ 1.6 fm

- These are bases to argue Nuclear Matter (Saturation, etc.)

- State dependence of N-N potential is very large

  (e.g., No N-N bound state except deuteron channel)

- We consider only Symmetric Nuclear matter (N=Z=A/2, or $\mu_u = \mu_d$)

Projection of moving direction & Flavor doubling

- Proj. of moving direction: $\psi_{R\pm} = \frac{1 \pm \gamma^0\gamma^z}{2}\psi_R \quad \psi_{L\pm} = \frac{1 \pm \gamma^0\gamma^z}{2}\psi_L$

$$\mathscr{L}^{\text{lightcone}}_{\text{kin}} = i[\psi^\dagger_{\underline{R+}}(\partial_0 \underline{+} \partial_z)\psi_{\underline{R+}} + \psi^\dagger_{\underline{R-}}(\partial_0 \underline{-} \partial_z)\psi_{\underline{R-}}]$$ + (Left-handed)

(At leading order: no mixing terms of moving direction & chirality)

- Spin doublet ⟶ Flavor doublet in (1+1)D

$$\underline{\varphi_\uparrow} = \begin{bmatrix} \underline{\varphi_{\uparrow}}_+ \\ \underline{\varphi_{\uparrow}}_- \end{bmatrix} = \begin{bmatrix} \psi_{R+} \\ \psi_{L-} \end{bmatrix} \quad \underline{\varphi_\downarrow} = \begin{bmatrix} \underline{\varphi_{\downarrow}}_+ \\ \underline{\varphi_{\downarrow}}_- \end{bmatrix} = \begin{bmatrix} \psi_{L+} \\ \psi_{R-} \end{bmatrix}$$

moving direction

↕

(1+1) D Chirality

- Without spin mixing ⟶ Flavor singlet op. in (1+1)D

(Only 4-candidates)

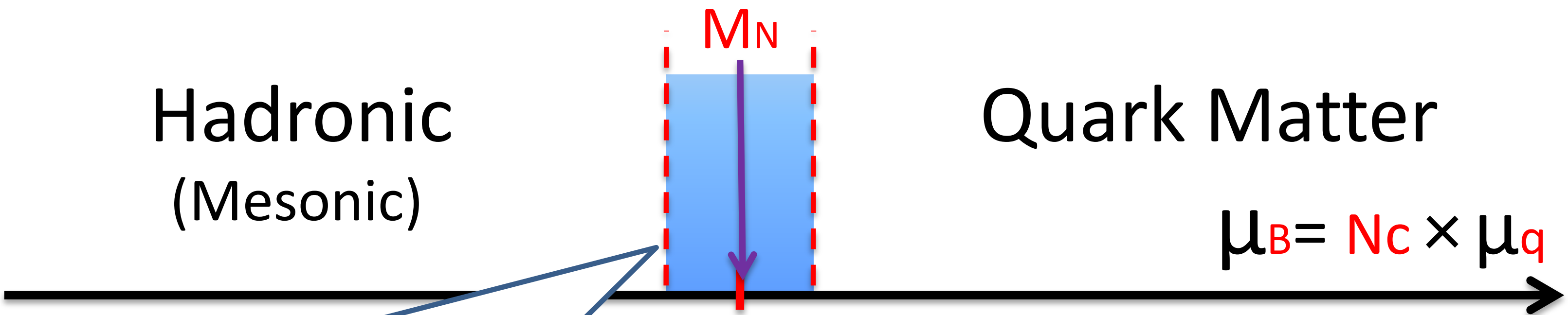
$$\bar\psi\psi \to \bar\Phi\Phi, \quad \bar\psi\gamma^0\psi \to \bar\Phi\Gamma^0\Phi, \quad \bar\psi\gamma^z\psi \to \bar\Phi\Gamma^z\Phi, \quad \underline{\bar\psi\gamma^0\gamma^z\psi \to \bar\Phi\Gamma^5\Phi}$$

- With spin mixing ⟶ Flavor non-singlet op.

e.g.) $\bar\psi\gamma^5\psi \to \bar\Phi\tau_3\Gamma^5\Phi, \qquad i\bar\psi\gamma^1\psi \to \Phi\tau_2\Gamma^5\Phi,$

Near Quarkyonic boundary: Nuclear region

- We consider symmetric (N=Z=A/2) nuclear matter (self-bounded)



- $\mu_B \sim M_N + \underbrace{k_F^2/2M_N + \langle V_N(k_F)\rangle}_{E_{bind}(k_F)\ (\,<0\,)}$

For Nc=3, at $\rho_B \sim \rho_0 = 0.17\ \mathrm{fm}^{-3}$:

$\langle K\rangle \sim -\langle V_N(k_F)\rangle \sim 50$ MeV $<< M_N$

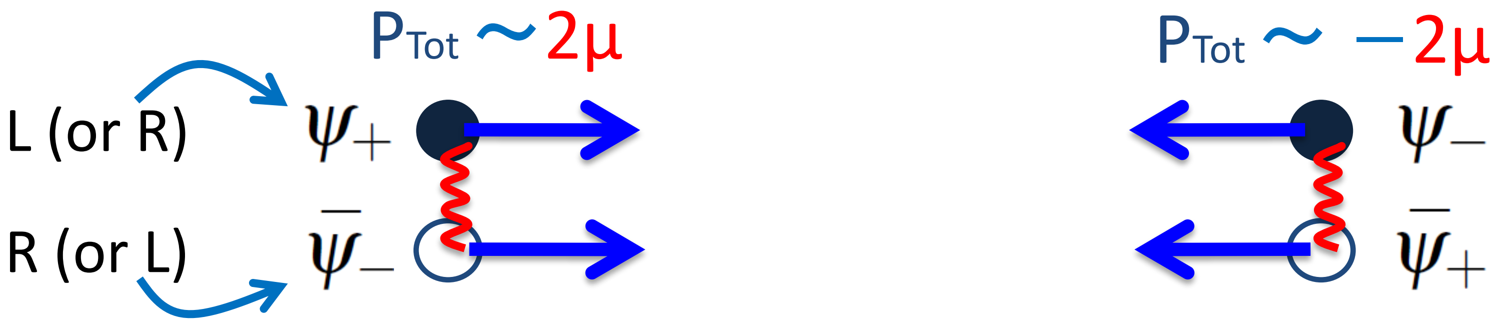
$E_{bind} \sim -16$ MeV $<< M_N$

Chiral density wave → Chiral Spirals

- Proj. of moving direction:

$$\psi_{R\pm} = \frac{1 \pm \gamma^0\gamma^z}{2}\psi_R \qquad \psi_{L\pm} = \frac{1 \pm \gamma^0\gamma^z}{2}\psi_L$$

(+ : moving to +z direction) (— : moving to —z direction)



$$\langle \bar{\psi}_- \psi_+ \rangle = \Delta e^{\underline{2i\mu z}} \qquad \langle \bar{\psi}_+ \psi_- \rangle = \Delta e^{\underline{-2i\mu z}}$$

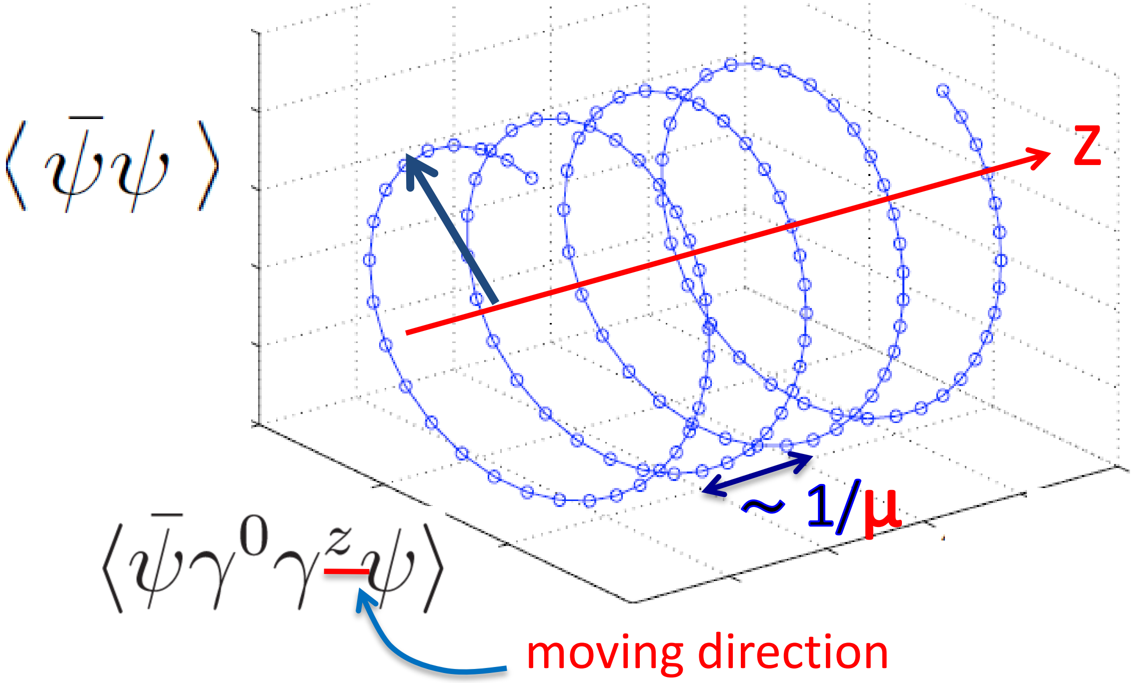
- Phase mismatch → Chiral Spirals

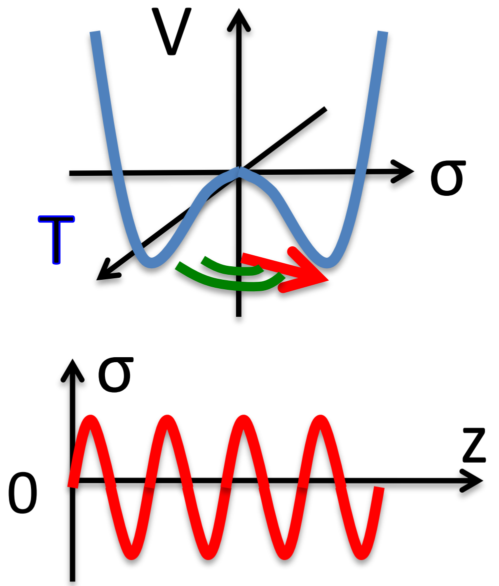
$$\langle \bar{\psi}\psi \rangle = \langle \bar{\psi}_- \psi_+ \rangle + \langle \bar{\psi}_+ \psi_- \rangle = \Delta \cos(2\mu z)$$

$$\langle \bar{\psi}\gamma^0\gamma^z\psi \rangle = \langle \bar{\psi}_- \psi_+ \rangle - \langle \bar{\psi}_+ \psi_- \rangle = i\Delta \sin(2\mu z)$$

Quarkyonic Chiral Spirals (QCSs)

- Chiral rotation evolves in the longitudinal direction:





- Chiral sym. is globally restored, but locally broken.

(cf: chiral sym. restoration in Skyrme model)

High density limit:

- $\Lambda_{QCD}/\mu \ll 1$ →
  - Baryon number is spatially constant.
  - No other condensates.