The Standard Model

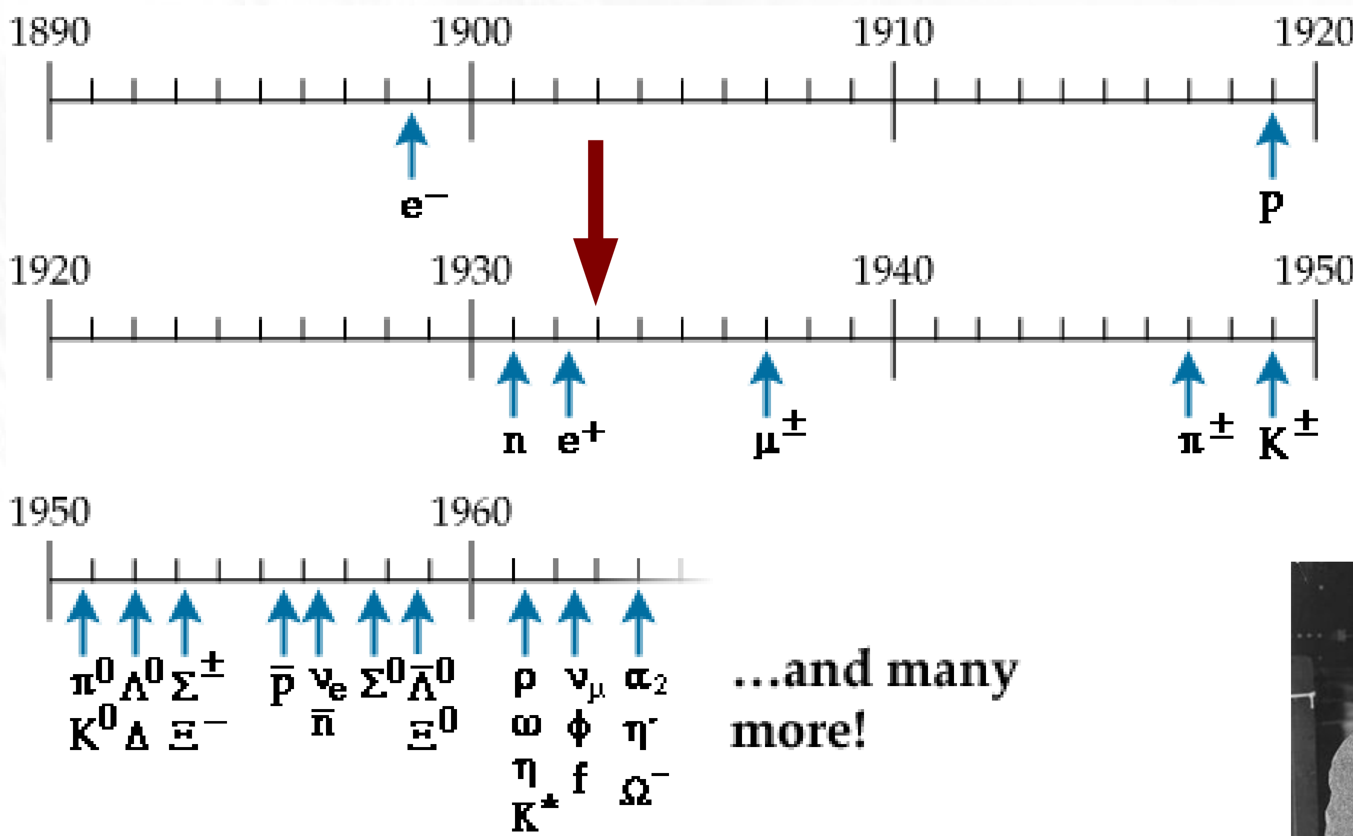
Lecture 3

(Flavor in the SM)

A.V. Bednyakov

X Winter School on Theoretical Physics, February, 2012

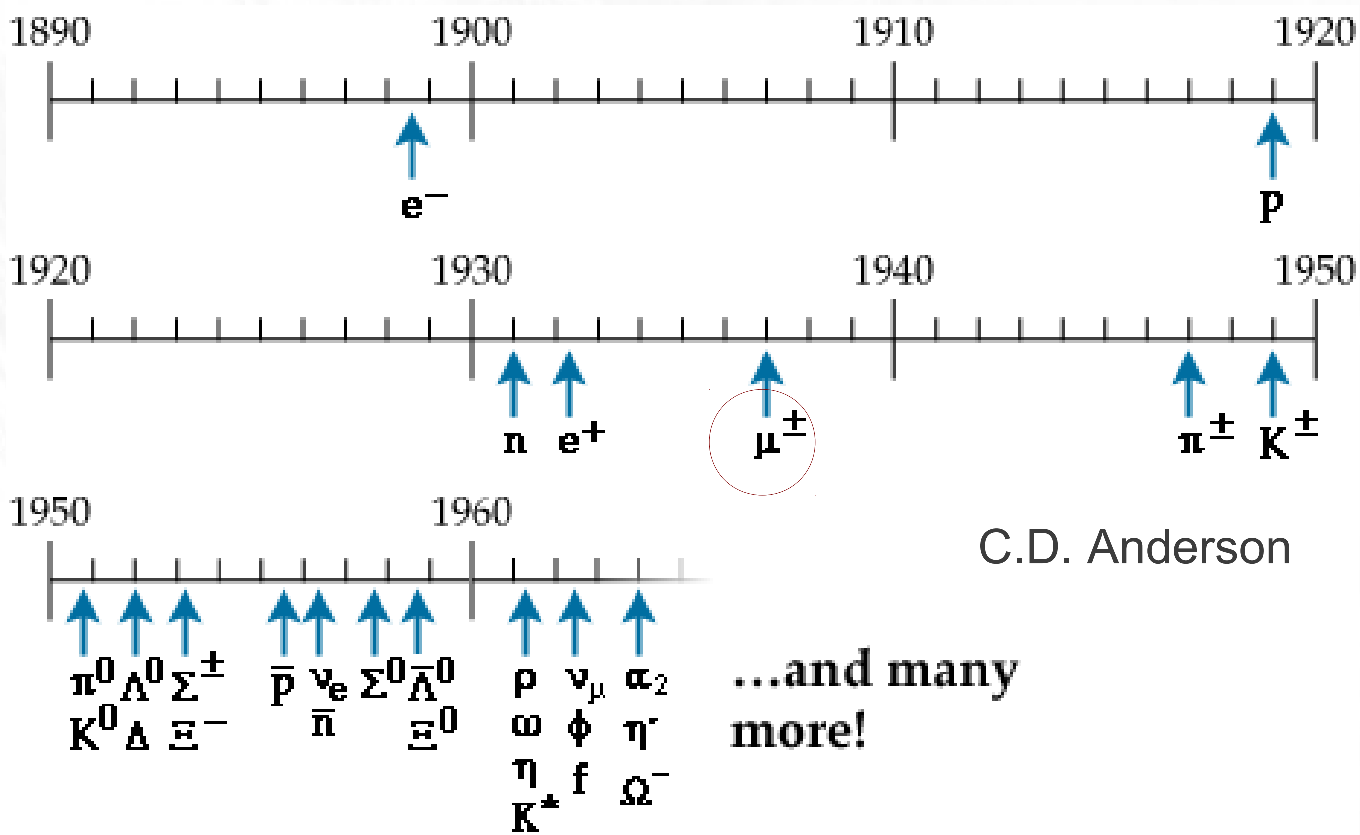
Brief history of (Flavor) Physics



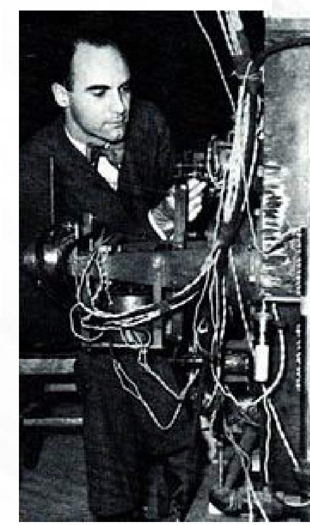
Fermi theory of beta decay

E. Fermi

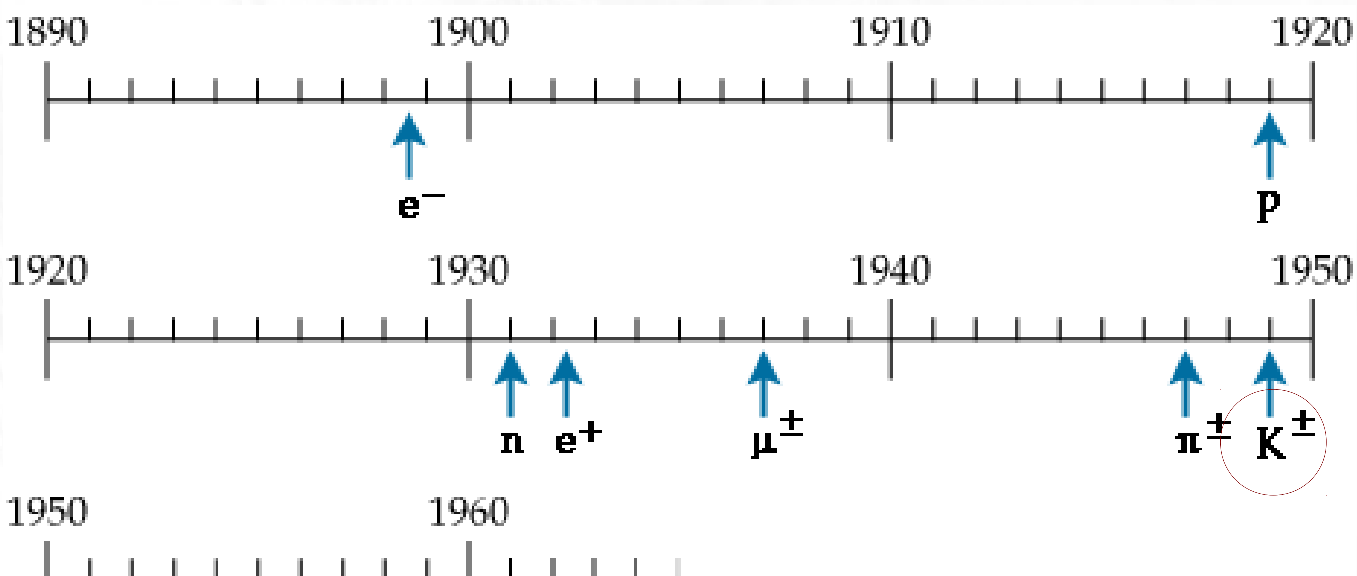
Brief history of (Flavor) Physics



C.D. Anderson



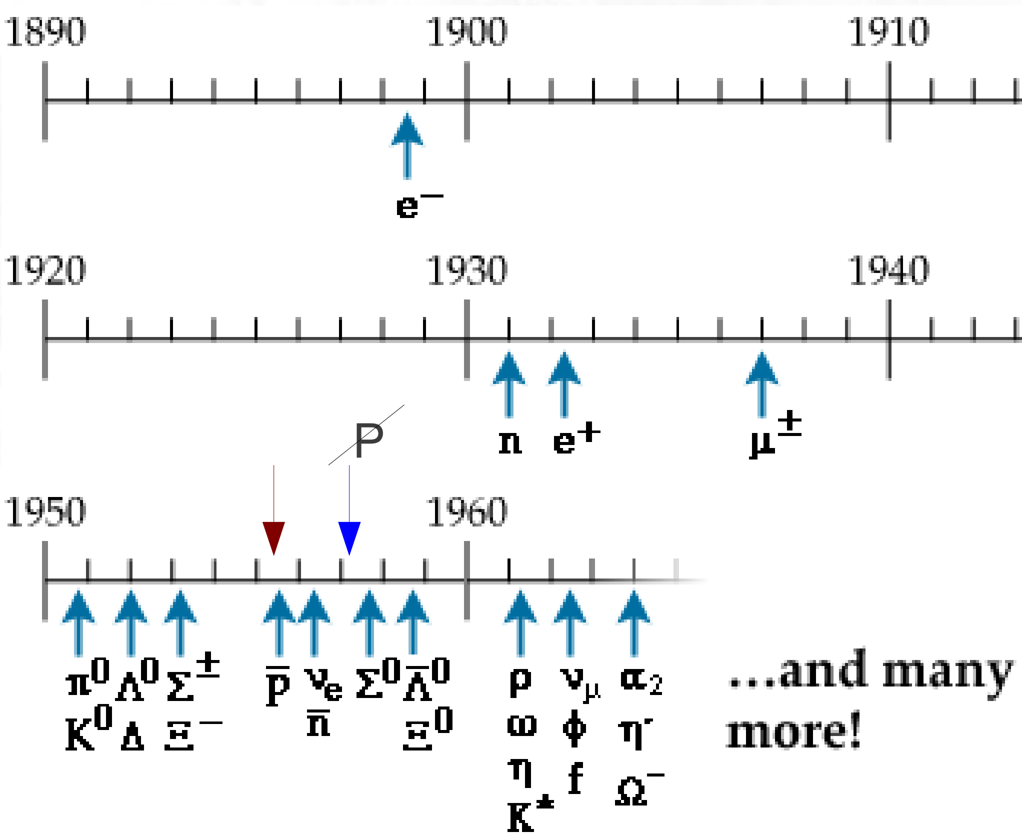
Brief history of (Flavor) Physics



G.D. Rochester

C.C. Butler

Brief history of (Flavor) Physics

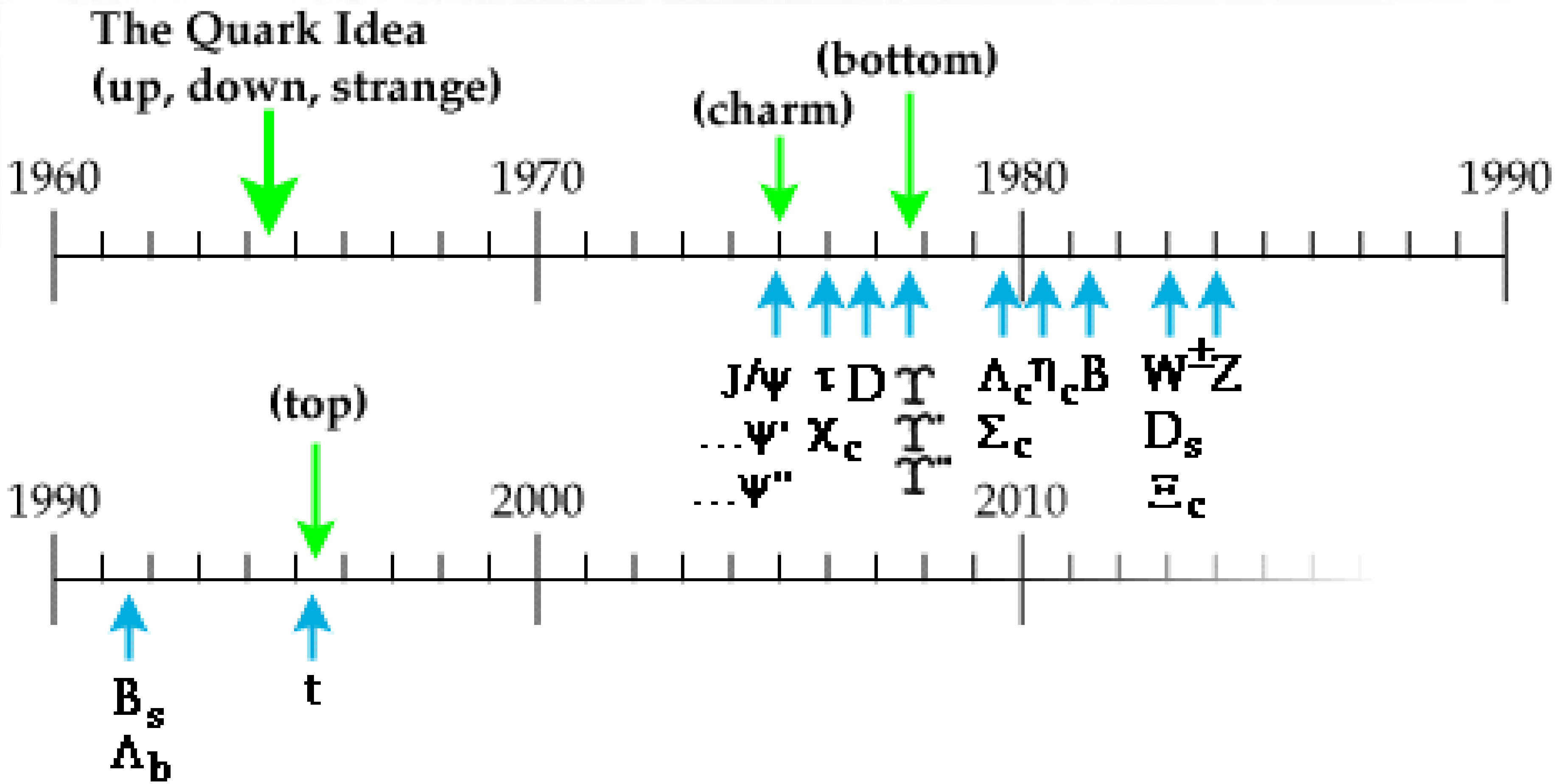


Strangeness
Gell-Mann, Nishijima
1954-55

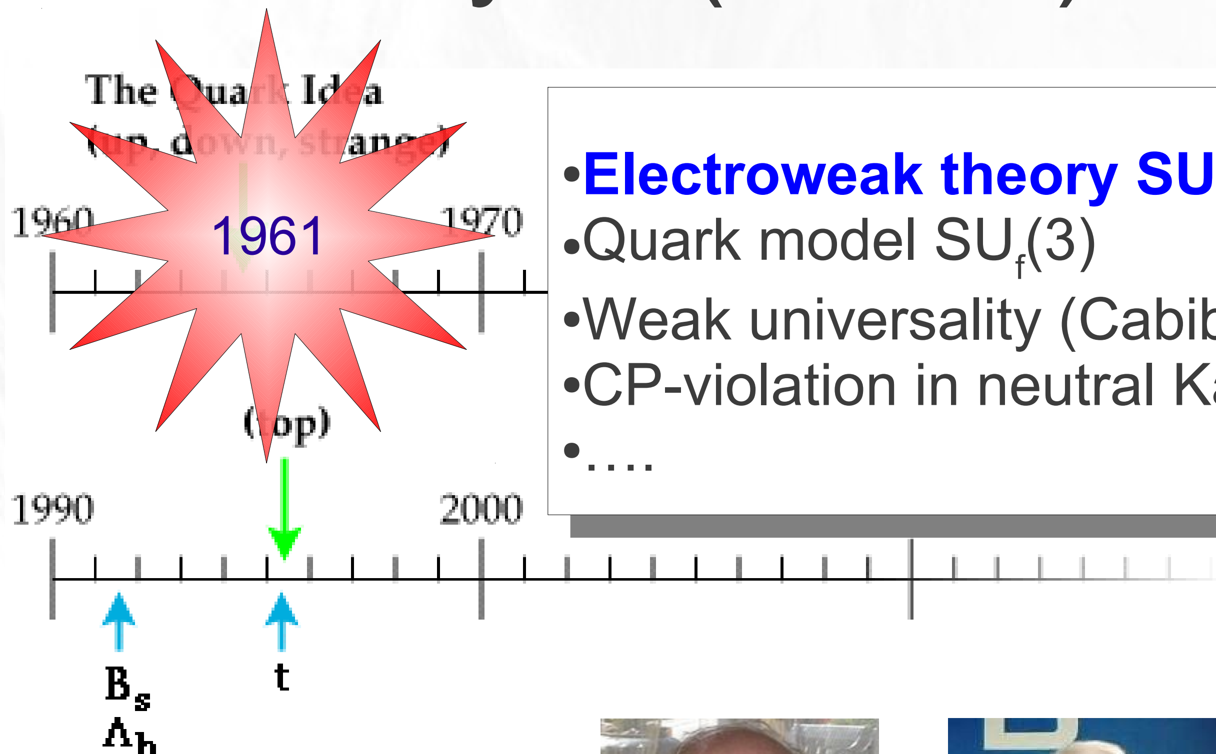
G.D. Rochester

C.C. Butler

Brief history of (Flavor) Physics



Brief history of (Flavor) Physics



- **Electroweak theory SU(2)xU(1)**
- Quark model $SU_f(3)$
- Weak universality (Cabibbo angle)
- CP-violation in neutral Kaons
- ….

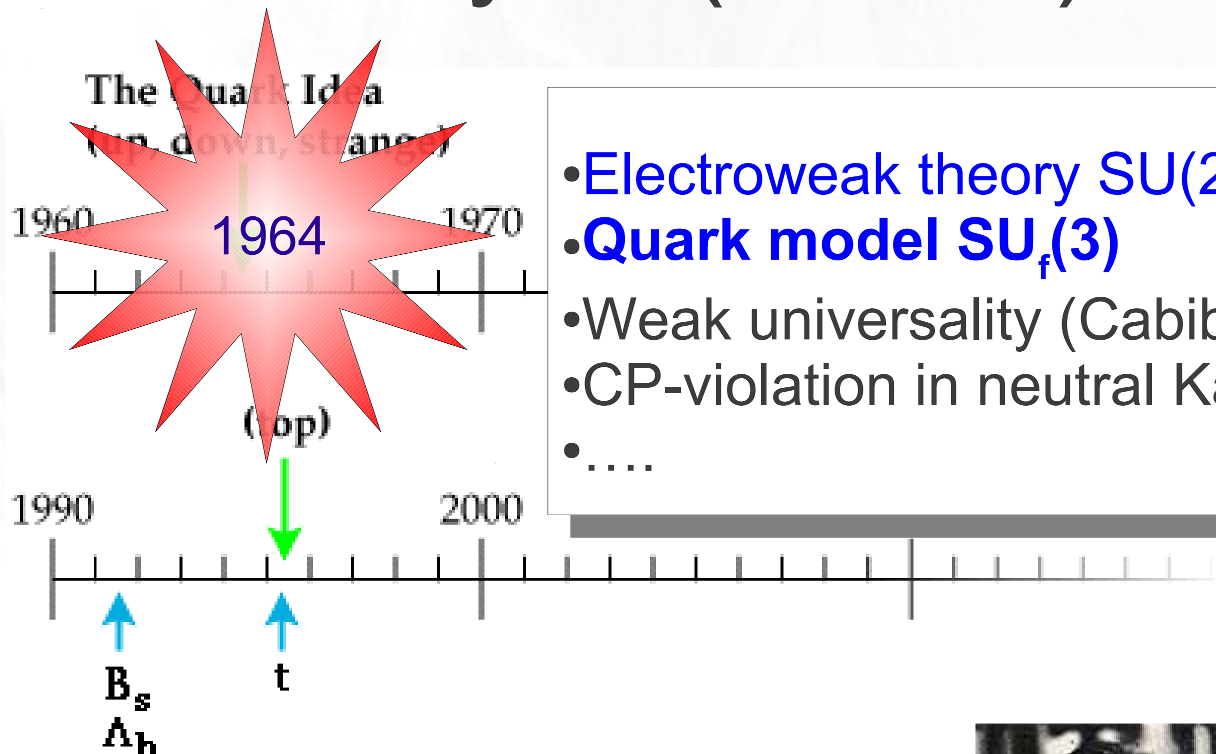
Massive
W,Z bosons
(Higgs mechanism)

S. Weinberg

S. Glashow

A. Salam

Brief history of (Flavor) Physics



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- ….

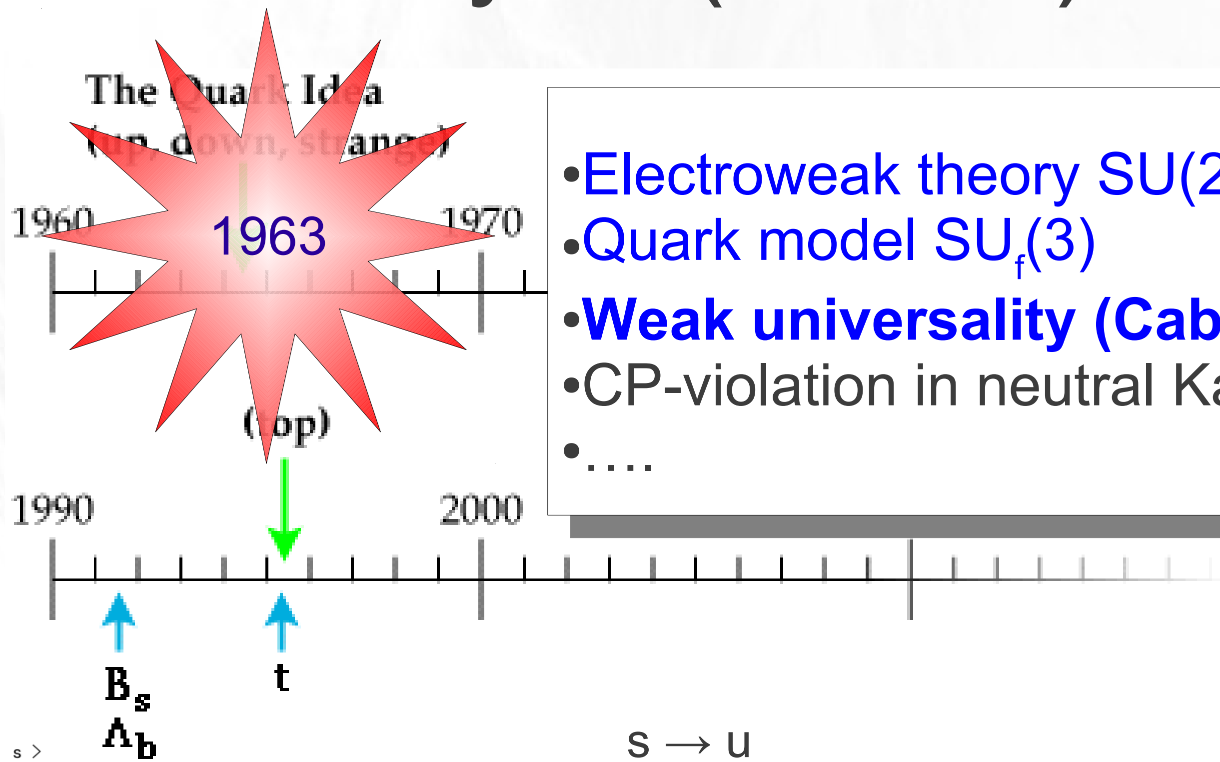
SU(3) flavor symmetry
Classification of hadrons

QUARKS!

G. Zweig

M. Gellman

Brief history of (Flavor) Physics



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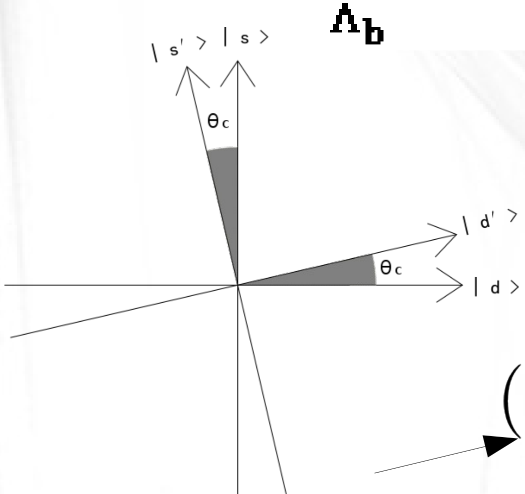
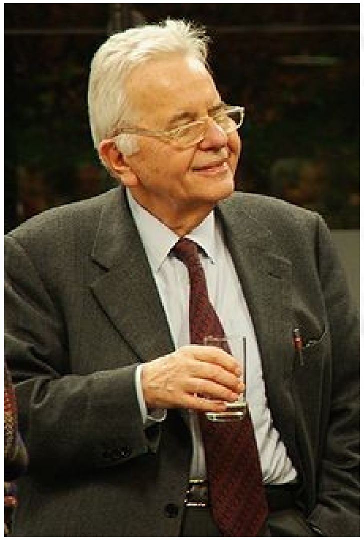


s → u
is suppressed
by a factor of ~20
in comparison with
d → u



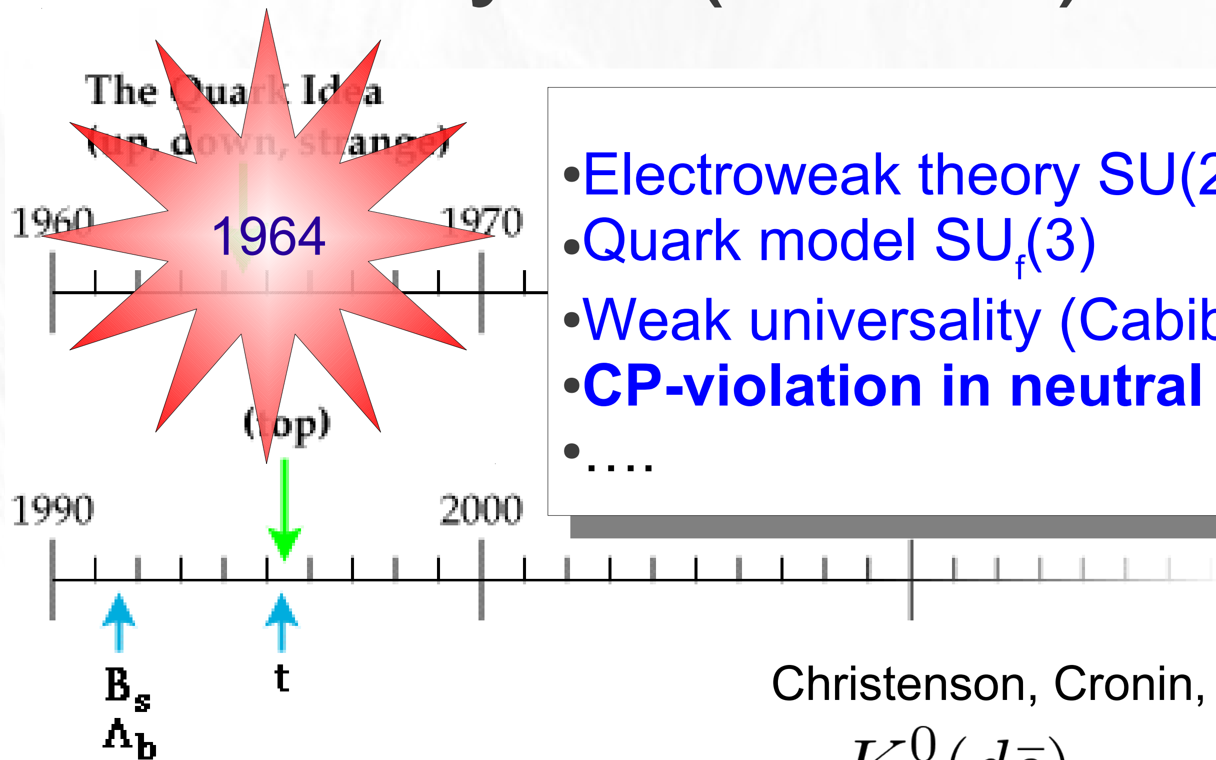
N. Cabbibo

$$(u, d') = (u, d\cos\theta_c + s\sin\theta_c)$$

SU(2) doublet that couples to W

Ex: $\sin\theta_c \simeq ?$

Brief history of (Flavor) Physics



- Electroweak theory SU(2)xU(1)
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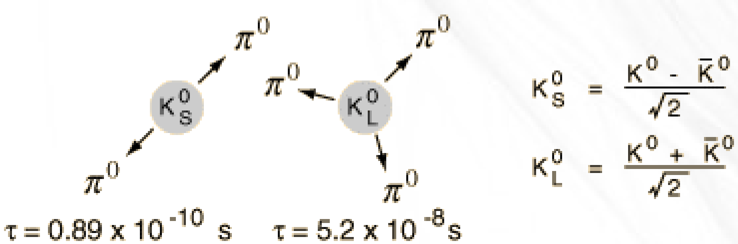
- **CP-violation in neutral Kaons**
- ….

Christenson, Cronin, Fitch, Turlay

$$K^0(d\bar{s}), \qquad \bar{K}^0(\bar{d}s)$$

$$\mathbf{CP}\, K^0 \equiv -\bar{K}^0$$

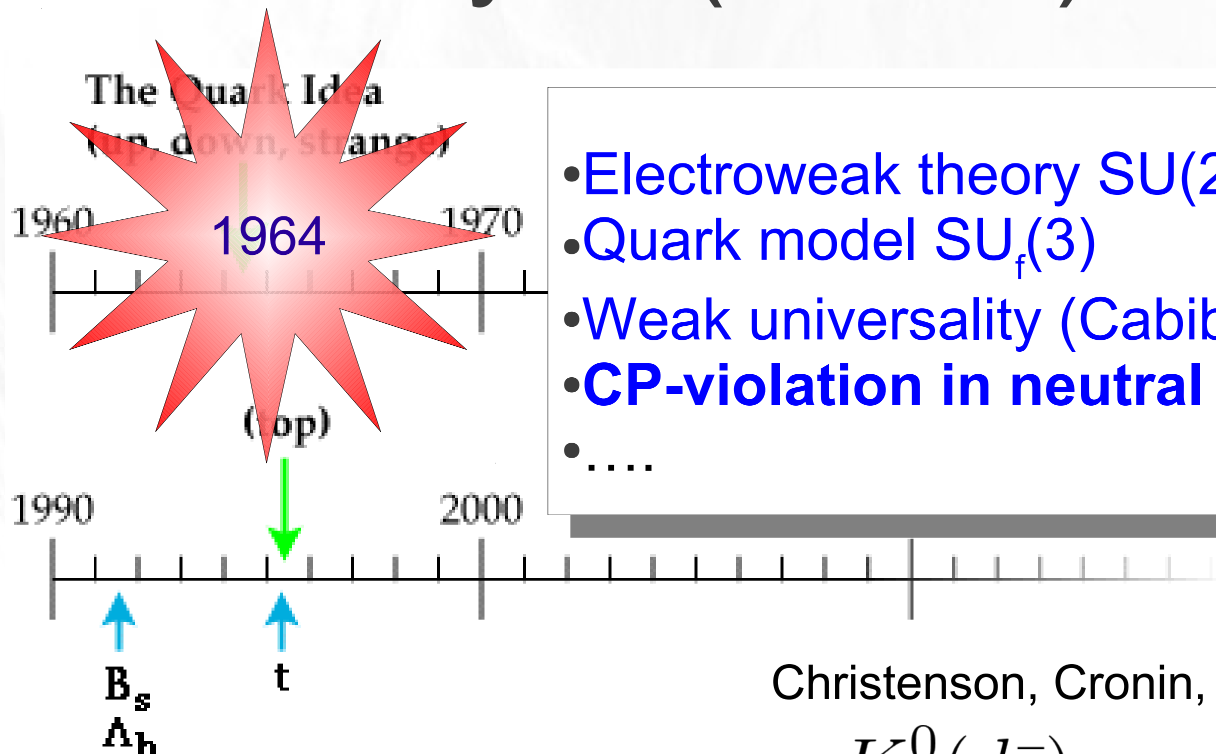


$$\mathbf{CP}|\pi\pi\rangle = |\pi\pi\rangle$$

$$\mathbf{CP}|\pi\pi\pi\rangle = -|\pi\pi\pi\rangle$$

$$\mathbf{CP}: K_L \not\to \pi\pi$$

Brief history of (Flavor) Physics



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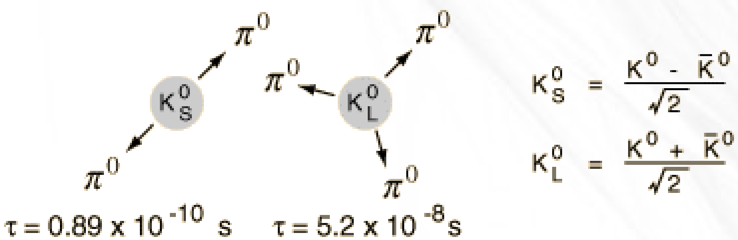
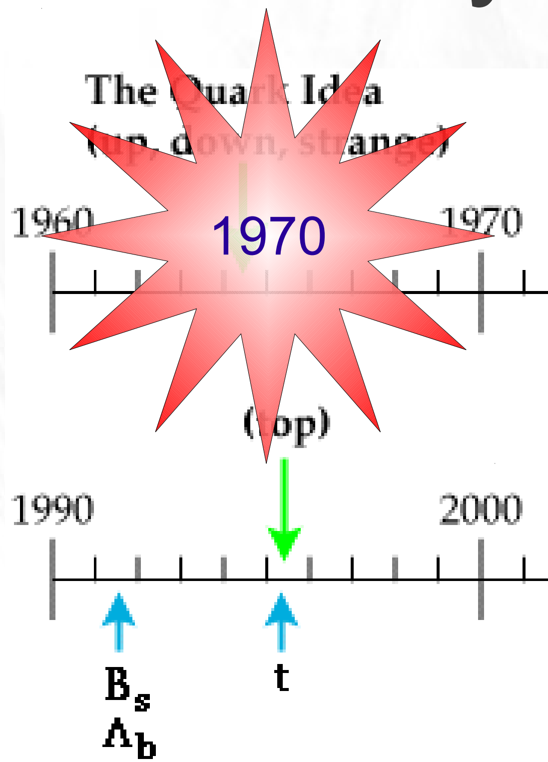


! $\not{\mathbf{CP}} : K_L \to \pi\pi$

Brief history of (Flavor) Physics



- ....
- GIM mechanism
- Renormalizability of EW theory

Suppression of Flavor-Changing Neutral Current (FCNC) interactions

charm prediction

S. Glashow

J. Iliopoulos

L. Maniani

FCNC suppression (GIM)

$$\Delta\mathcal{L} = Z_\mu J_Z^\mu = g_Z \bar{q}_i \gamma_\mu \left(T_3 - Q_q \sin^2\theta_W\right) q_i Z_\mu$$

Rotation (d,s) → (d',s') (weak eigenstates → mass eigenstates)
Does NOT produce tree-level FCNC if d and s
have the same quantum numbers with respect to SU(2)xU(1)

No charm → $s_{L,}$ should be a SU(2) singlet.

Tree-level FCNC interactions!

$$K^+(u\bar{s}) \to \mu^+\nu$$

Allowed!

$$K^0(d\bar{s}) \to \mu^+\mu^-$$

Exp: suppressed!
Br ~ 10^{-9}

Another aspect: two charged FC current → FCNC!

FCNC suppression (GIM)

Suppression of FCNC via loops

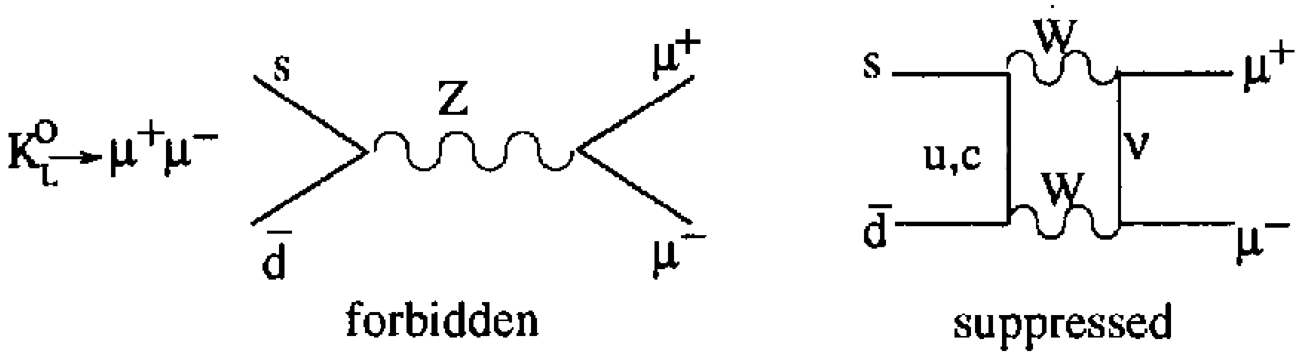
$$\begin{pmatrix} d' \\ s' \end{pmatrix} = \begin{pmatrix} \cos\theta_c & \sin\theta_c \\ -\sin\theta_c & \cos\theta_c \end{pmatrix} \begin{pmatrix} d \\ s \end{pmatrix}$$

$$J^{\mu}_{c.c} = \bar{u}\gamma^{\mu}(1-\gamma_5)d' + \bar{c}\gamma^{\mu}(1-\gamma_5)s'$$

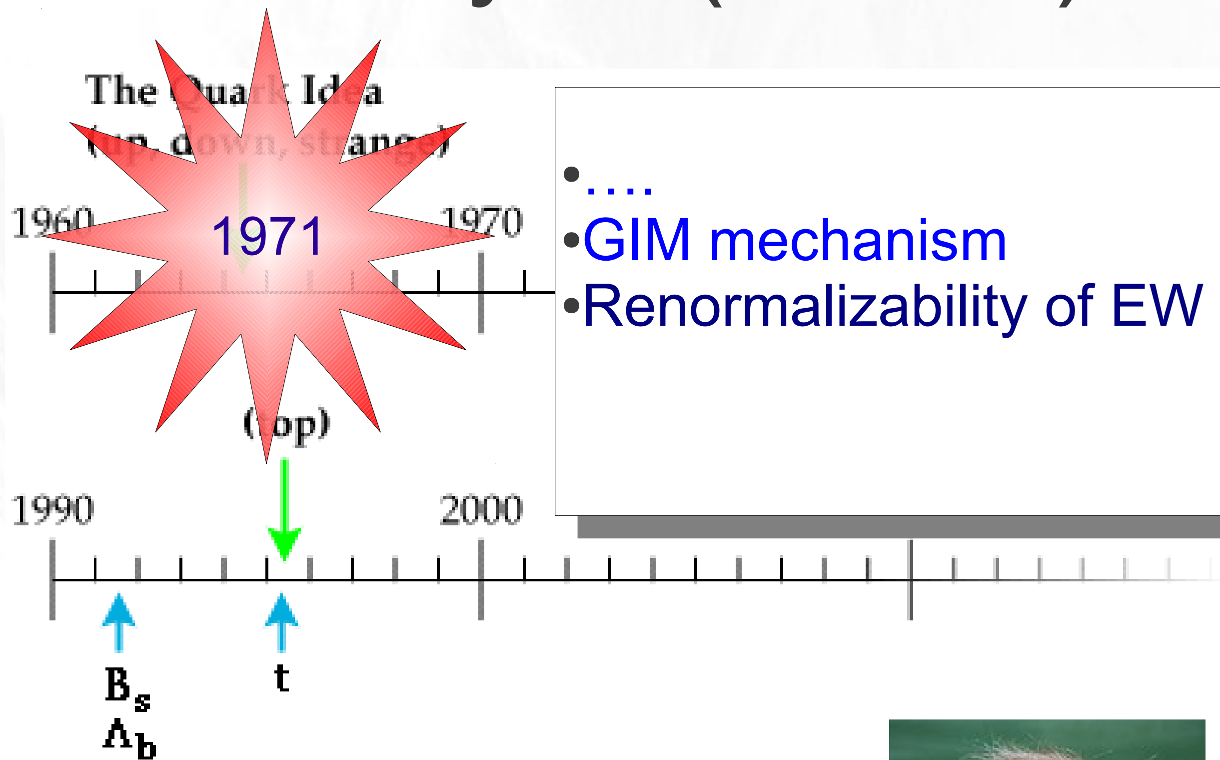
Weak basis (couplings to W,Z)

Mass basis (propagators)

$$\frac{\sin\theta_c\cos\theta_c}{q^2 - m_u^2} - \frac{\cos\theta_c\sin\theta_c}{q^2 - m_c^2}$$



Brief history of (Flavor) Physics

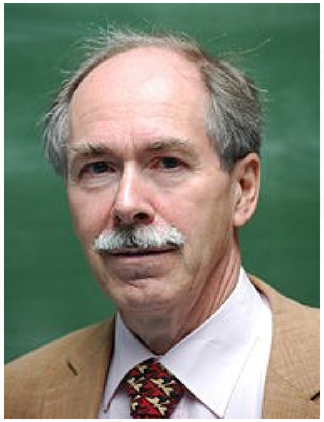


- ….
- GIM mechanism
- Renormalizability of EW theory

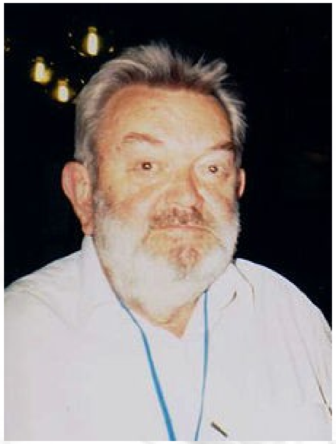
Invention of a powerful tool to perform Perturbation theory in gauge theories

$$\int d^4k \rightarrow \mu^{2\epsilon} \int d^{4-2\epsilon}k$$

Dimensional regularization

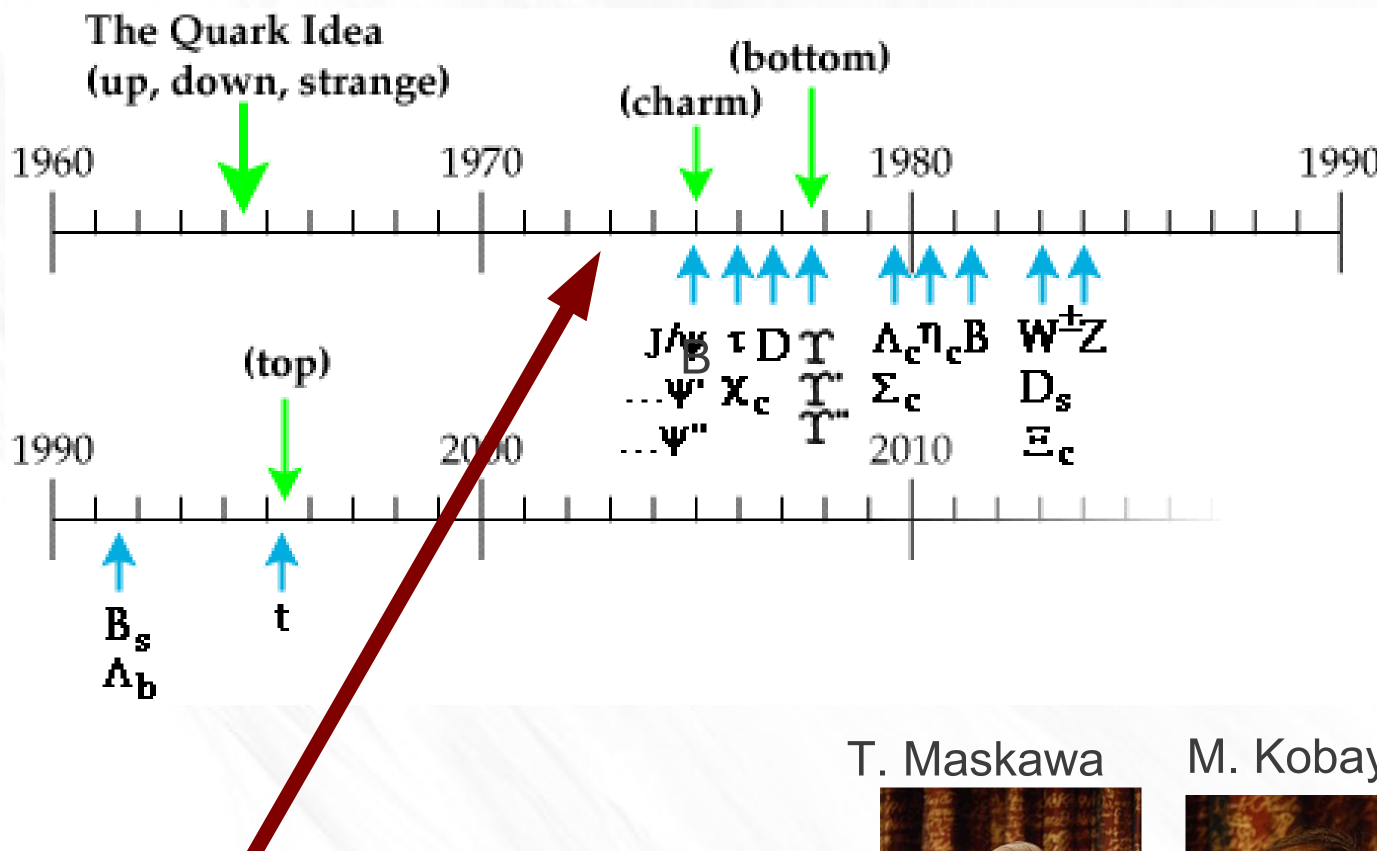


G. 't Hooft



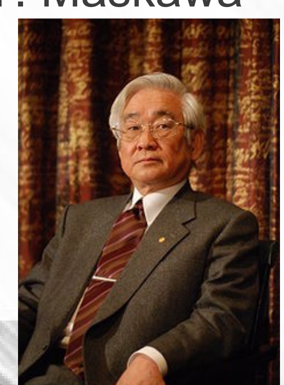
M. Veltman

Brief history of (Flavor) Physics



CP violation requires at least
three generations

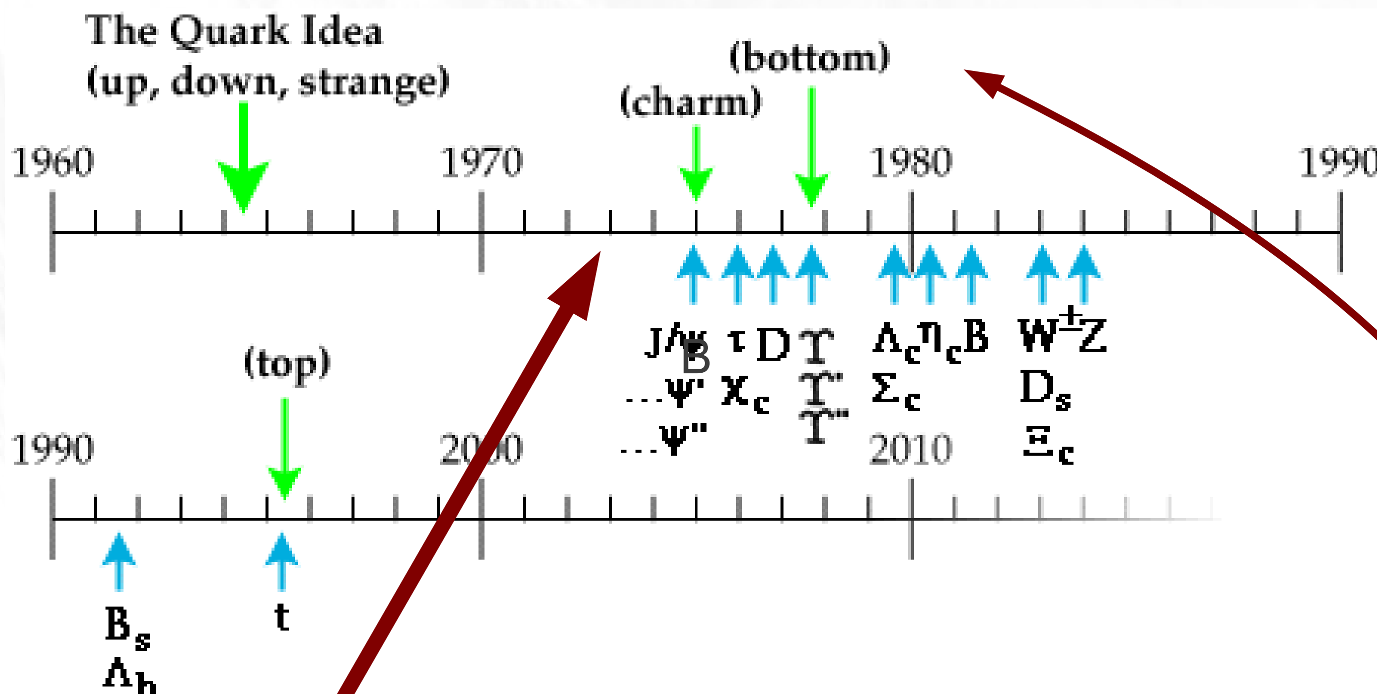
T. Maskawa



M. Kobayashi

Prediction of 3rd generation!

Brief history of (Flavor) Physics



CKM matrix!

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = V_{CKM} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

CP violation requires at least three generations

Prediction of 3rd generation!

Why CP violation and the number of generations are connected?

”CP violation is given by a phase”

$$\mathcal{H} = a\mathcal{O} + \mathrm{h.c.} = a\mathcal{O} + a^*\mathcal{O}^\dagger$$

coupling — operator

$$\mathbf{CP}\mathcal{O}\mathbf{CP}^{-1} = \mathcal{O}^\dagger$$

CP trasformation interchanges two operators.

If a is real then CP is a symmetry of Hamiltonian

In the SM Yukawa interactions with Higgs bosons is the only place for complex couplings

- For each generation we have one left-handed SU(2) doublet, and two right-handed singlets

SU(3) triplet — hypercharge $Q-T_3$

$$Q_L^I = \begin{pmatrix} U_L^I \\ D_L^I \end{pmatrix} = (3,2)_{+1/6}, \quad u_R^I = (3,1)_{+2/3}, \quad d_R^I = (3,1)_{-1/3}.$$

SU(2) doublet $\phi = (1,2)_{1/2}, \tilde{\phi} = (1,2)_{-1/2}$

- Quarks interact with Higgs field via Yukawa coupling

$$\mathcal{L}_Y = -Y_{ij}^d \overline{Q_{Li}^I} \phi d_{Rj}^I - Y_{ij}^u \overline{Q_{Li}^I} \tilde{\phi} u_{Rj}^I + \text{H.c.}$$

Generic complex matrix of yukawa coupling constants

- Quarks acquire mass through because of spontaneous symmetry breaking

$$\mathcal{L}_M = -\sqrt{\frac{1}{2}} v Y_{ij}^d \overline{d_{Li}^I} d_{Rj}^I - \sqrt{\frac{1}{2}} v Y_{ij}^u \overline{u_{Li}^I} u_{Rj}^I + \text{H.c.}$$

$$\mathbf{M}_d = Y^d v/\sqrt{2}, \quad \mathbf{M}_u = Y^u v/\sqrt{2}.$$

Mass matrices for up and down quarks. Elements are complex!

CP violation in the SM is related to complex Yukawa couplings!

How many parameters are physical?

- "Unphysical" parameters are those that can be set to zero by a basis rotation
- General theorem

$$N(\text{Phys}) = N(\text{tot}) - N(\text{broken})$$

$N(\text{Phys})$, number of physical parameters

$N(\text{tot})$, total number of parameters

$N(\text{broken})$, number of broken generators

- Without the new terms the global symmetry is large, and the new terms break part of it. It is the breaking that can be "used" to find a batter basis

Parameter counting
An example:

A hydrogen atom with weak magnetic field

The magnetic field add one new physical parameter, B

$$V(r) = \frac{-e^2}{r} \qquad V(r) = \frac{-e^2}{r} + B_x\hat{x} + B_y\hat{y} + B_z\hat{z}$$

But there are 3 total new parameters

The magnetic field breaks explicitly: $SO(3) \to SO(2)$

2 broken generators, can be "used" to define the z axis

$$N(\text{Phys}) = N(\text{tot}) - N(\text{broken}) \quad \Rightarrow \quad 1 = 3 - 2$$

Parameter counting in the SM quark sector

- Without Yukawa terms we have accidental global symmetry

$$U(N)_Q \times U(N)_D \times U(N)_U$$

of quark kinetic term

Rotation of up and down left-handed quarks should be correlated

- Yukawa couplings break this symmetry down to $U(1)_B$ (all quark fields should be rotated simultaneously)

Parameter counting in the SM quark sector

- A unitary matrix has

$$N \times N = \frac{N(N-1)}{2} + \frac{N(N+1)}{2}$$

parameters

phases

angles

Parameter counting in the SM quark sector

- A unitary matrix has

$$N \times N = \frac{N(N-1)}{2} + \frac{N(N+1)}{2}$$

parameters

$$U(N)_Q \times U(N)_D \times U(N)_U \to U(1)_B$$

$$N_{\text{broken}} = 3 \times \frac{N(N-1)}{2} + \left[3 \times \frac{N(N+1)}{2} - 1\right]$$

Parameter counting in the SM quark sector

- We have two Yukawa matrices $Y^d, \; Y^u$

$$N_{\text{total}} = 2N \times N + 2N \times N$$

- So we have

$$N_{\text{physical}} = \frac{N(N+3)}{2} + \frac{(N-2)(N-1)}{2}$$

For N=1,2 all the phases can be absorbed into definition of the quark fields $\Rightarrow$ $N \geq 3$

Parameter counting in the SM quark sector

- For N = 3 we have 9 real parameters (6 masses + 3 angles) and 1 CPV phase.
- In the mass basis angles and CPV phase appear in the CKM matrix

Weak (interaction) eigenstates

$$-\Delta\mathcal{L}_2 = \bar{d}_{Li} M^d_{ij} d_{Rj} + \bar{u}_{Li} M^u_{ij} u_{Rj} + \mathrm{h.c}$$

Parameter counting in the SM quark sector

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Weak Eigenstates (interact)

$$-\Delta\mathcal{L}_2 = \bar{d}'_{Li} M^d_{ij} d'_{Rj} + \bar{u}'_{Li} M^u_{ij} u'_{Rj} + \mathrm{h.c}$$

$$q_{Li} = (V_{qL})_{ij} q'_{Lj}, \quad q_{Ri} = (V_{qR})_{ij} q'_{Rj}$$

Mass eigenstates (propagate)

$$V_{qL} M^q V^\dagger_{qR} = M^q_{\mathrm{diag}}, \qquad q = u, d$$

The global symmetry in RH sector allows us to get rid of V_{uR}, V_{dR}

In the SM gauge interactions do not mix RH u-type and d-type quarks...

Parameter counting in the SM quark sector

- For N = 3 we have 9 real parameters (6 masses + 3 angles) and 1 CPV phase.
- In the mass basis angles and CPV phase appear in the CKM matrix

Weak Eigenstates (interact)

$$\Delta\mathcal{L}_W = \frac{g_2}{\sqrt{2}}\bar{u}'_{Li}\gamma^\mu d'_{Li}W^+ + \mathrm{h.c}$$

$$u_{Li} = (V_{uL})_{ij}u'_{Lj}, \quad d_{Li} = (V_{dL})_{ij}d'_{Lj}$$

Mass eigenstates (propagate)

Due to the gauge symmetry we are NOT allowed to rotate up and down LH weak eigenstates independently!

Parameter counting in the SM quark sector

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Mass eigenstates

$$\Delta\mathcal{L}_W = \frac{g_2}{\sqrt{2}}\bar{u}_{Li}\gamma^\mu(V_L^u V^{d\dagger})_{ij} d_j W^+ + \mathrm{h.c}$$

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Mass eigenstates (propagate)

Due to the gauge symmetry we are NOT allowed to rotate up and down LH weak eigenstates independently!

CKM matrix ↔ Flavor Physics (in SM)

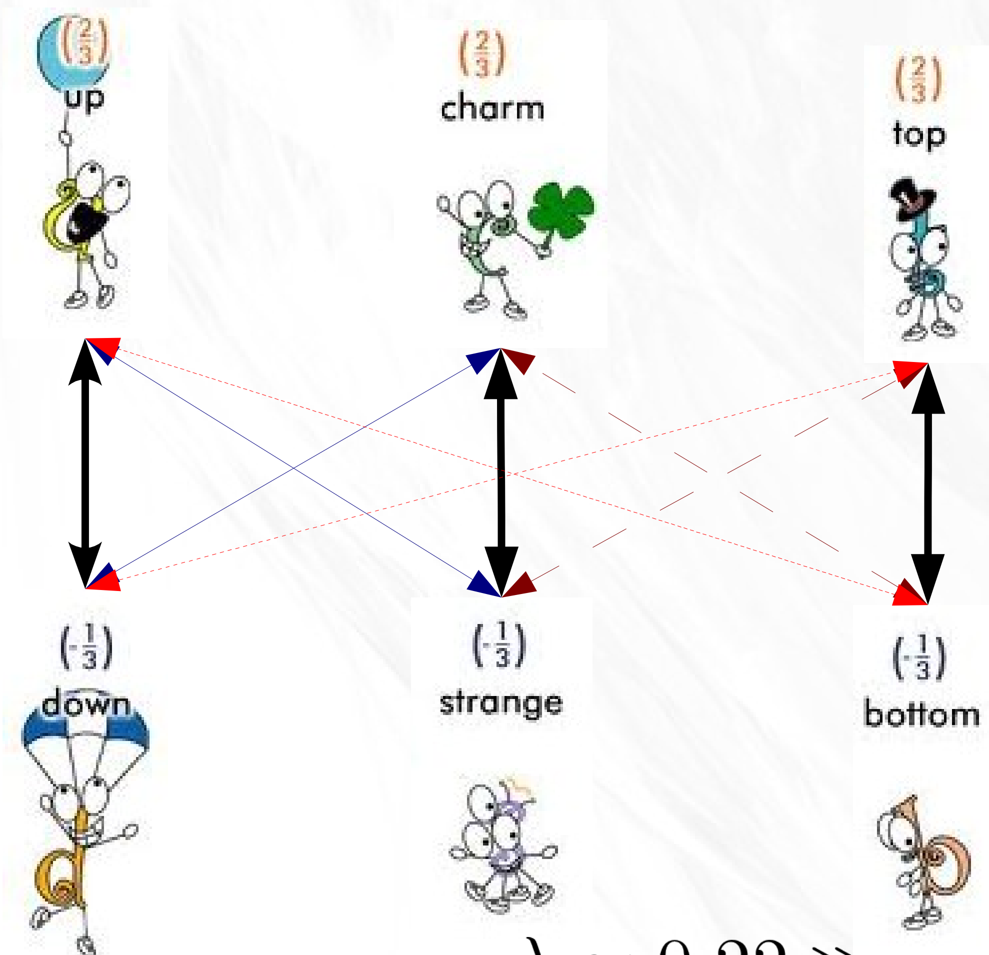
$$V_{\rm CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \qquad V_{\rm CKM}^\dagger V_{\rm CKM} = 1$$

«Standard parametrization»

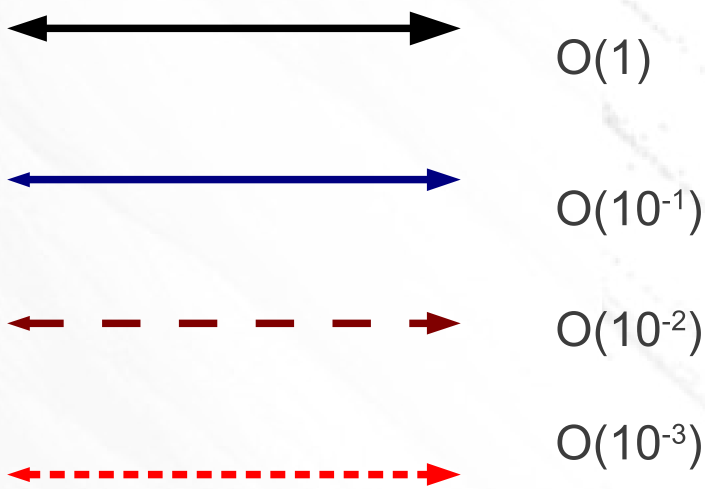
$$V_{\rm CKM} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

Only one physical phase in CKM. What happen with the remaining five phases?

CKM matrix ↔ Flavor Physics (in SM)



Hierachy of charged current process



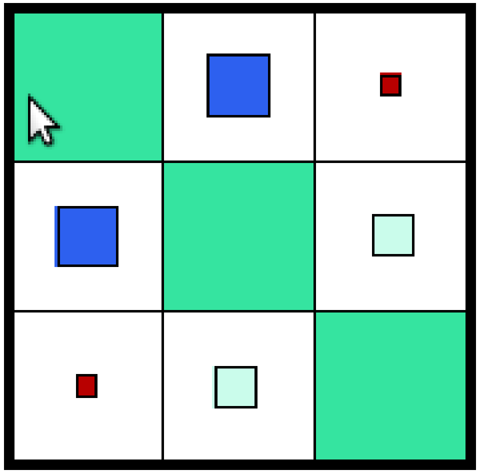
Why? Flavor problem!

$$s_{12} = \lambda \simeq 0.22 \gg s_{23} \gg s_{13}$$

CKM matrix ↔ Flavor Physics (in SM)

Wolfenstein parametrization reflects hierarchy in CKM mixing

$$V_{\rm CKM} = \begin{pmatrix} 1-\frac{1}{2}\lambda^2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\frac{1}{2}\lambda^2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4)$$



$$V^\dagger_{\rm CKM} V_{\rm CKM} = 1$$

CKM matrix $\leftrightarrow$ Flavor Physics (in SM)

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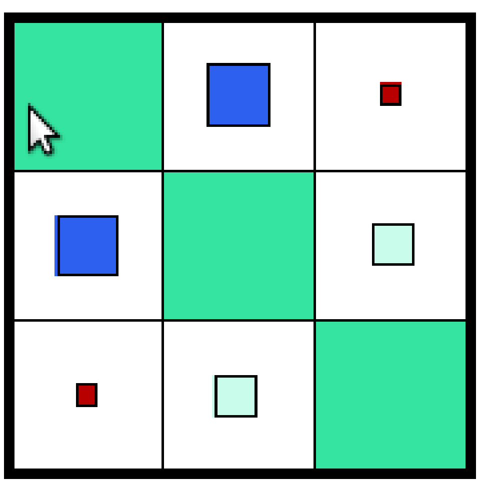

$$s_{13}e^{-i\delta} = A\lambda^3(\rho - i\eta)$$

Cabbibo mixing!

$$\lambda \simeq \sin\theta_c$$

CKM matrix $\leftrightarrow$ Flavor Physics (in SM)

Wolfenstein parametrization reflects hierarchy in CKM mixing

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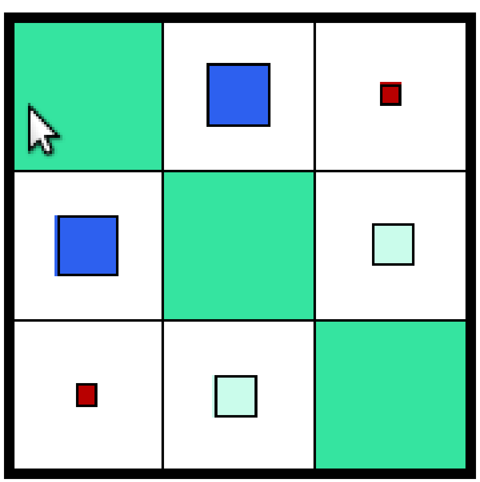

$$s_{13}e^{-i\delta} = A\lambda^3(\rho - i\eta)$$

Cabbibo mixing!

$$\lambda \simeq \sin\theta_c$$

$$\lambda = 0.22,\ \ A = 0.8,\ \ \rho = 0.13,\ \ \eta = 0.34$$

Hierarchy in Unitarity relations

$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

$$\underset{\mathcal{O}(\lambda)}{V_{ud}\, V_{us}^*} \quad + \quad \underset{\mathcal{O}(\lambda)}{V_{cd}\, V_{cs}^*} \quad + \quad \underset{\mathcal{O}(\lambda^5)}{V_{td}\, V_{ts}^*} \quad = 0$$

Hierarchy in Unitarity relations

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$$\underset{\mathcal{O}(\lambda^4)}{V_{us}\, V_{ub}^*} + \underset{\mathcal{O}(\lambda^2)}{V_{cs}\, V_{cb}^*} + \underset{\mathcal{O}(\lambda^2)}{V_{ts}\, V_{tb}^*} = 0$$

Hierarchy in Unitarity relations

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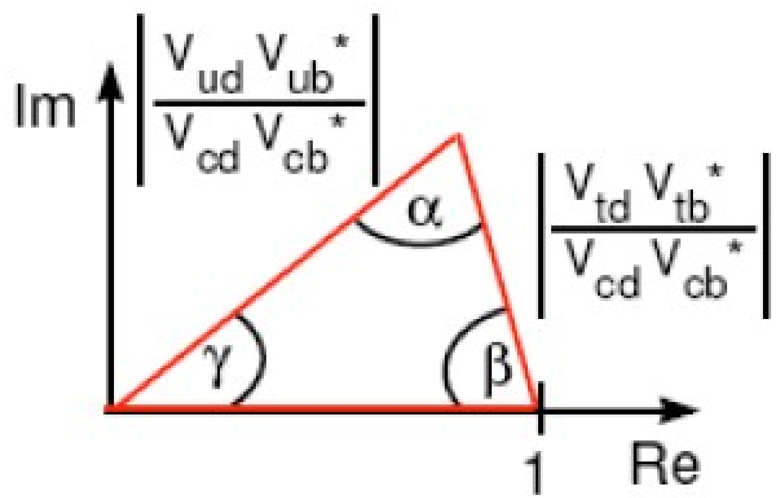
THE unitarity triangle

The unitarity triangle

$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

$$\underset{A\lambda^3(\rho - i\eta)}{V_{ud}\,V_{ub}^*} + \underset{-A\lambda^3}{V_{cd}\,V_{cb}^*} + \underset{A\lambda^3(1-\rho-i\eta)}{V_{td}\,V_{tb}^*} = 0$$



$$\alpha \equiv \arg\left(-\frac{V_{td}V_{tb}^*}{V_{ud}V_{ub}^*}\right)$$

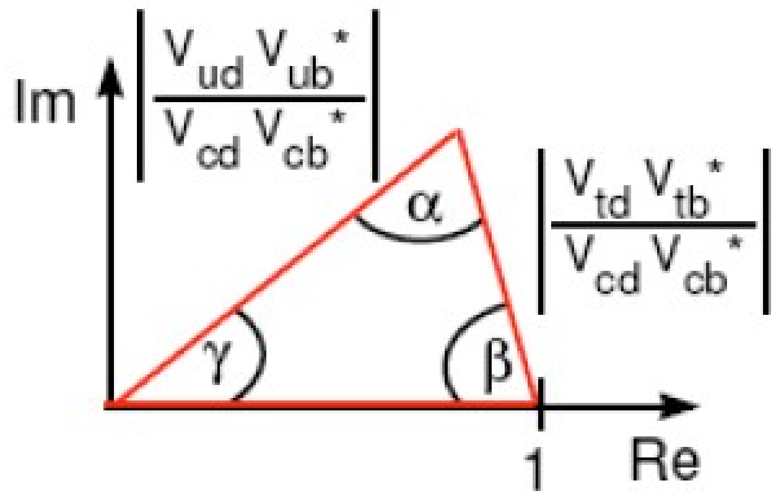
$$\beta \equiv \arg\left(-\frac{V_{cd}V_{cb}^*}{V_{td}V_{tb}^*}\right)$$

$$\gamma \equiv \arg\left(-\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}\right)$$

The unitarity triangle

$$\begin{pmatrix} |V_{ud}| & |V_{us}| & |V_{ub}|e^{-i\gamma} \\ |V_{cd}|e^{i\pi} & |V_{cs}| & |V_{cb}| \\ |V_{td}|e^{-i\beta} & |V_{ts}|e^{i\pi} & |V_{tb}| \end{pmatrix}$$

$$\begin{matrix} V_{ud}\,V_{ub}^* & + & V_{cd}\,V_{cb}^* & + & V_{td}\,V_{tb}^* & =0 \\ A\lambda^3(\rho - i\eta) & & -A\lambda^3 & & A\lambda^3(1-\rho-i\eta) & \end{matrix}$$



$$\alpha \equiv \arg\left(-\frac{V_{td}V_{tb}^*}{V_{ud}V_{ub}^*}\right)$$

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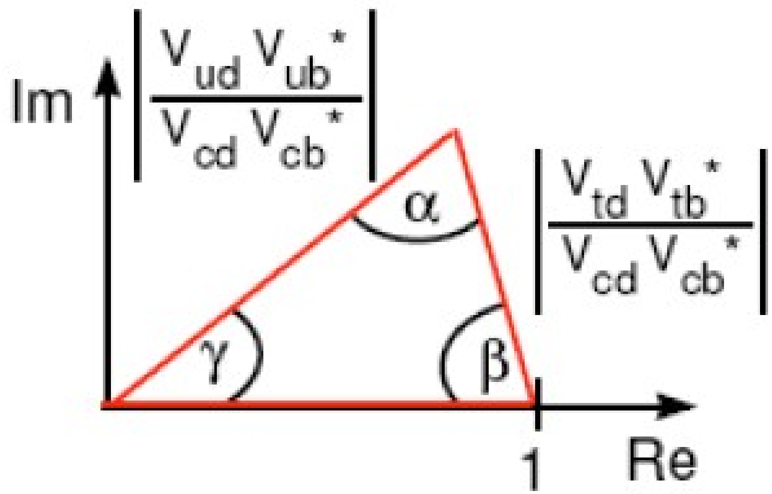
The unitarity triangle



The area of all unitarity triangles
Is connected to the CPV phase

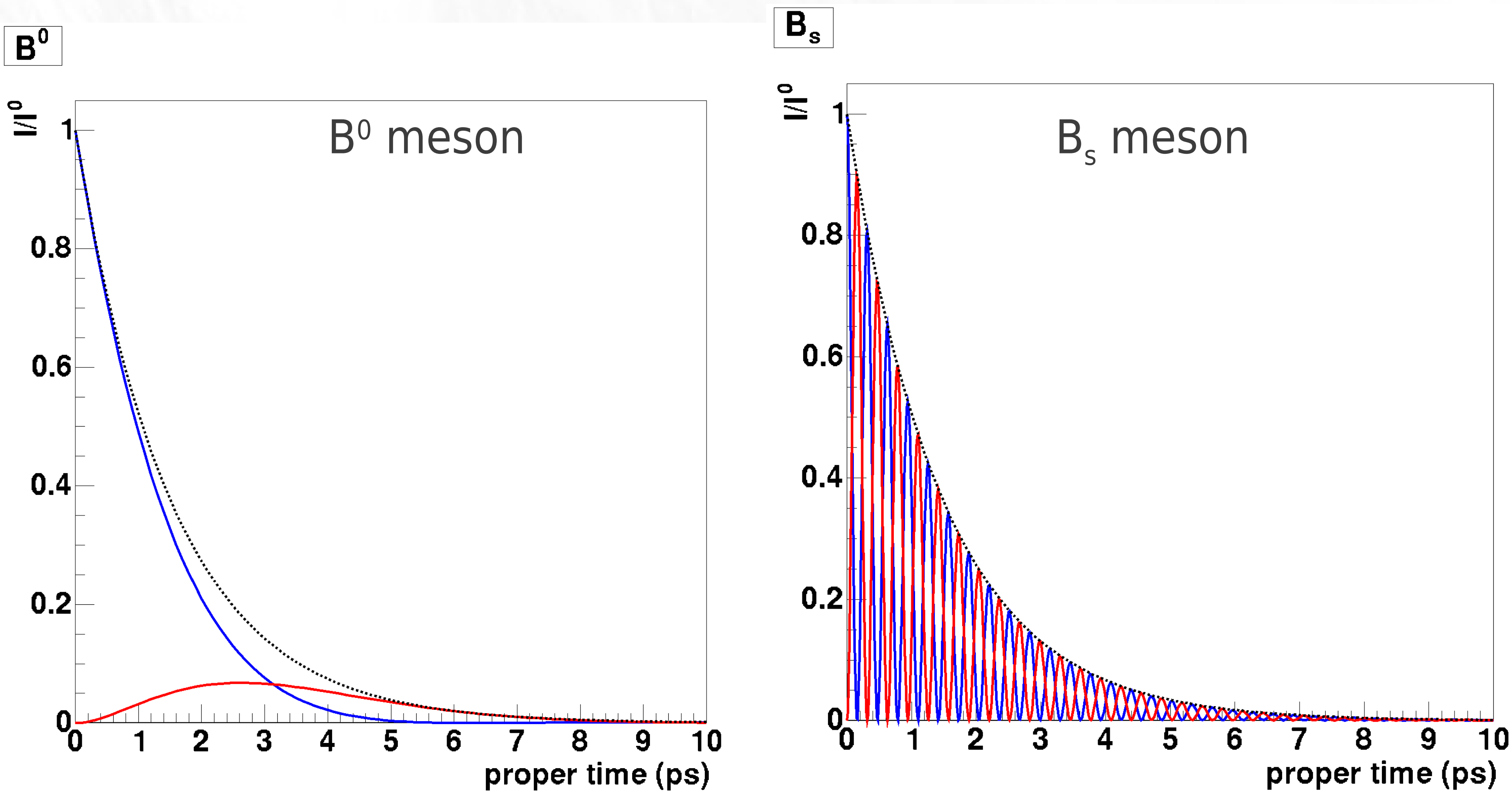
$$S = \frac{1}{2}J, \qquad \mathrm{Im}[V_{ij}V_{kl}V_{il}^*V_{jl}^*] = J\sum_{m,n}\epsilon_{ikm}\epsilon_{njl}$$

Jarlskog invariant

$$J \simeq \lambda^6 A^2 \eta = \mathcal{O}(10^{-5})$$

Main phenomena influenced by CKM

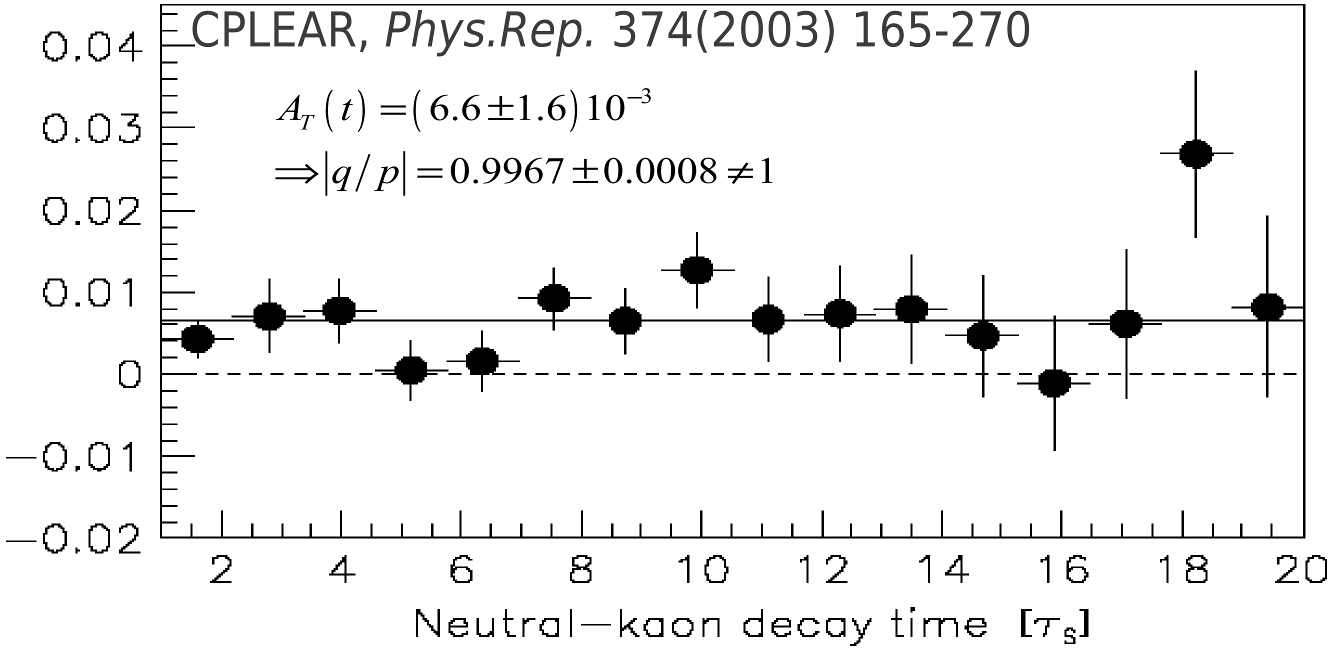
- Neutral meson mixing (oscillations)



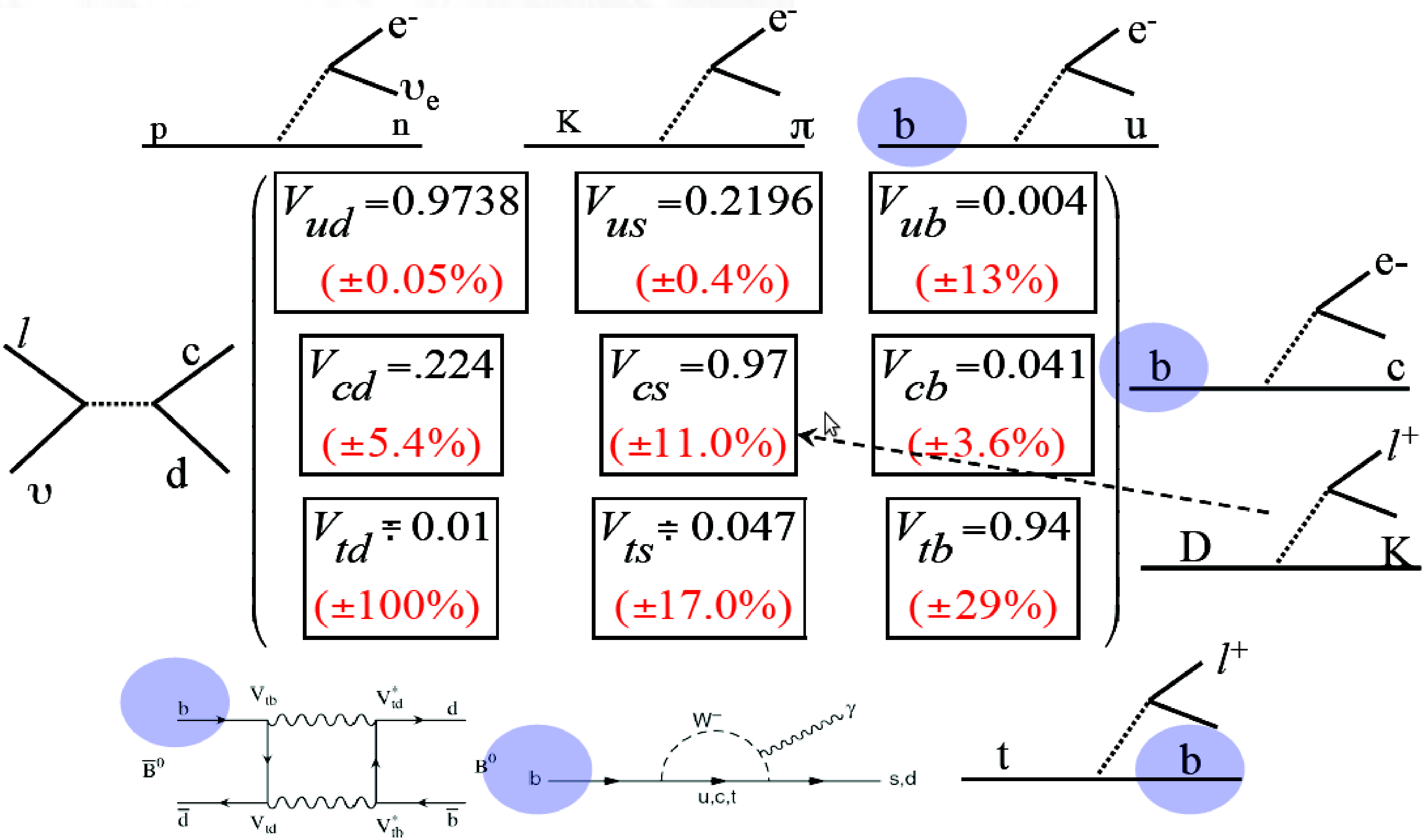
Main phenomena influenced by CKM

- CP violation

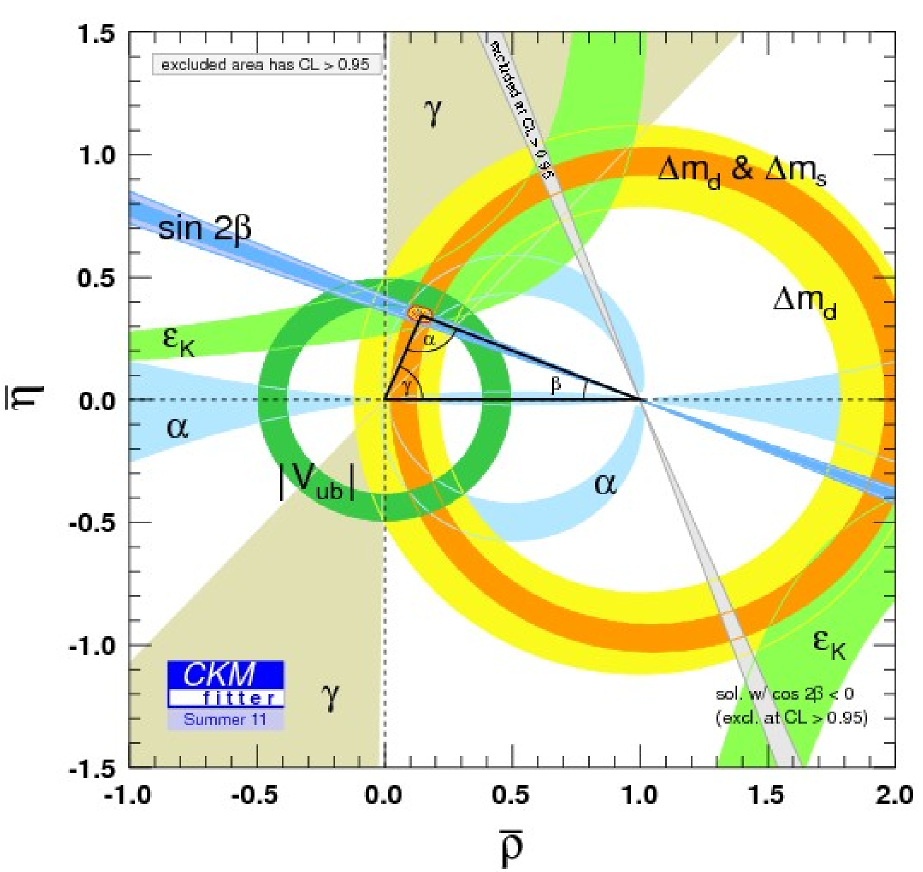
$$A_{+-} \equiv \frac{R\left(K_L^0 \to e^+\pi^-\nu_e\right) - R\left(K_L^0 \to e^-\pi^+\overline{\nu}_e\right)}{R\left(K_L^0 \to e^+\pi^-\nu_e\right) + R\left(K_L^0 \to e^-\pi^+\overline{\nu}_e\right)} = \frac{1-|q/p|^4}{1+|q/p|^4} = 4\Re\varepsilon$$



Determination of the unitarity triangle



Determination of the unitarity triangle



All the measurements seem to be consistent with the SM predictions

Flavor physics serves as a severe constraint on New Physics scenarios, espcially in FCNC effects, since in the SM they are very suppressed and generated at loop level.

NB: the extraction of CKM parameters is complicated by the fact that we are dealing
With mesons istead of quarks ← Non-perturbative effects have to be estimated!

Importance of B-physics

- Large mass m_b
  - Variety of final states to decay to
    - Determination of several CKM elements
  - allows us to use expansion in $1/m_b$ to estimate non-perturbative effects systematically
- CPV phase in V_{ub} → CPV effects
- Rare decays of B-mesons due to loops: important NP constraints

$$B \to X_s\gamma, \qquad B_s \to \mu^+\mu^-$$

Lecture 3 summary

- CKM description of Flavor Physics in quark sector turns to be very successful

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- Still it is just a description!

- There should be something behind.....Flavor problem...