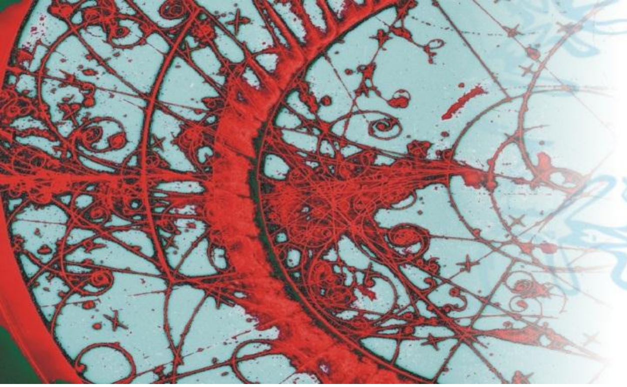

Dubna International Advanced School on Theoretical Physics / DIAS-TH

X Winter School on Theoretical Physics

PHYSICS AT THE LARGE HADRON COLLIDER

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Some notes about the PT series in the non-PT region

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Motivation

This talk is motivated by the recent four-loop phenomenological analysis *[Khandramai, Pasechnik, Shirkov, Teryaev, Solovtsova (PLB, 2012)]* **of**

● the high precision Jefferson Lab (Newport News, USA) data on the Bjorken sum rule amplitude at **low *Q*** in the wide range ***0.22 GeV <Q< 1.8 GeV*** *[Prok et al. (2009)] + old data SLAC (E80, E130, E142, E143, E154, E155), CERN (EMC, SMC), DESY (HERMES, COMPASS)*

by using recently available

● the four-loop expression for the perturbative QCD contribution to the Bjorken sum rule *[Chetyrkin et al. (2010)]*.

The aim of this talk is answer on question:
how can the standard PT allows penetrate in the low-energy region?

BSR at finite momentum transfer

$$\Gamma_1^{p-n}(Q^2) = \int_0^1 \left[g_1^p(x,Q^2) - g_1^n(x,Q^2) \right] dx = \frac{g_A}{6} C_{Bj}(Q^2) + \sum_{i=2}^{\infty} \frac{\mu_{2i}^{p-n}}{Q^{2i-2}}$$

$$C_{Bj}(Q^2) = 1 - \Delta^{Bj}(Q^2) \qquad g_A = 1.267 \pm 0.004$$ [PDG (2010)]

$$\Delta^{Bj}(Q^2) = 0.31831\alpha_S + 0.36307\alpha_S^2 + 0.65197\alpha_S^3 + \mathbf{1.8042}\alpha_S^4$$ [Chetyrkin, et al. (2010)]

[Bethke (2009), PDG (2010)]

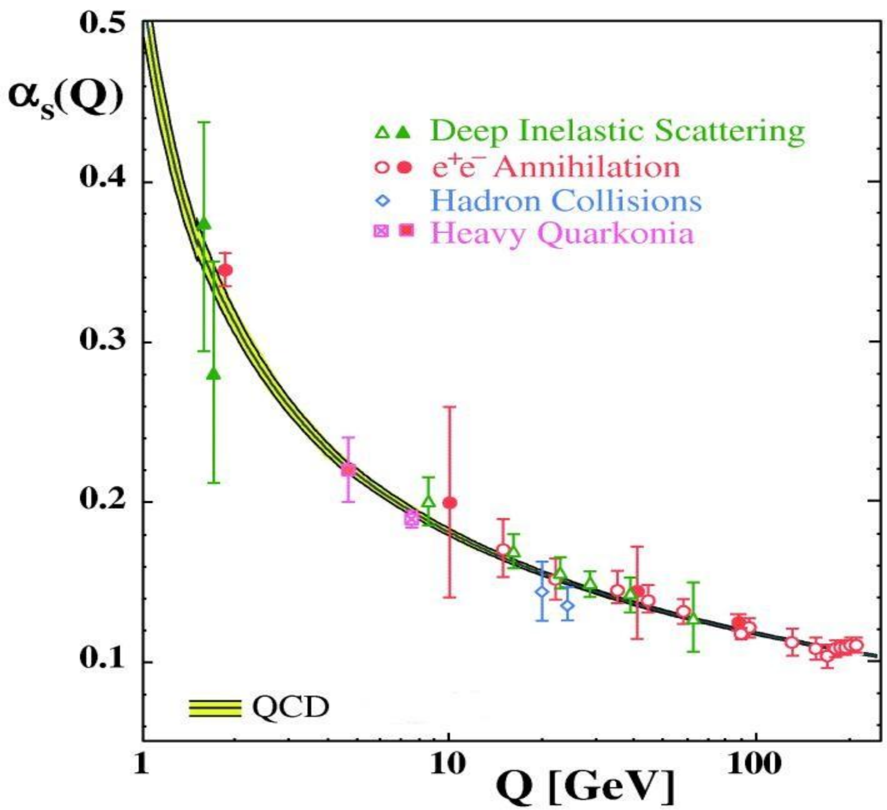
$$\alpha_S(M_Z) = 0.1184 \pm 0.0007$$

for n_f=3

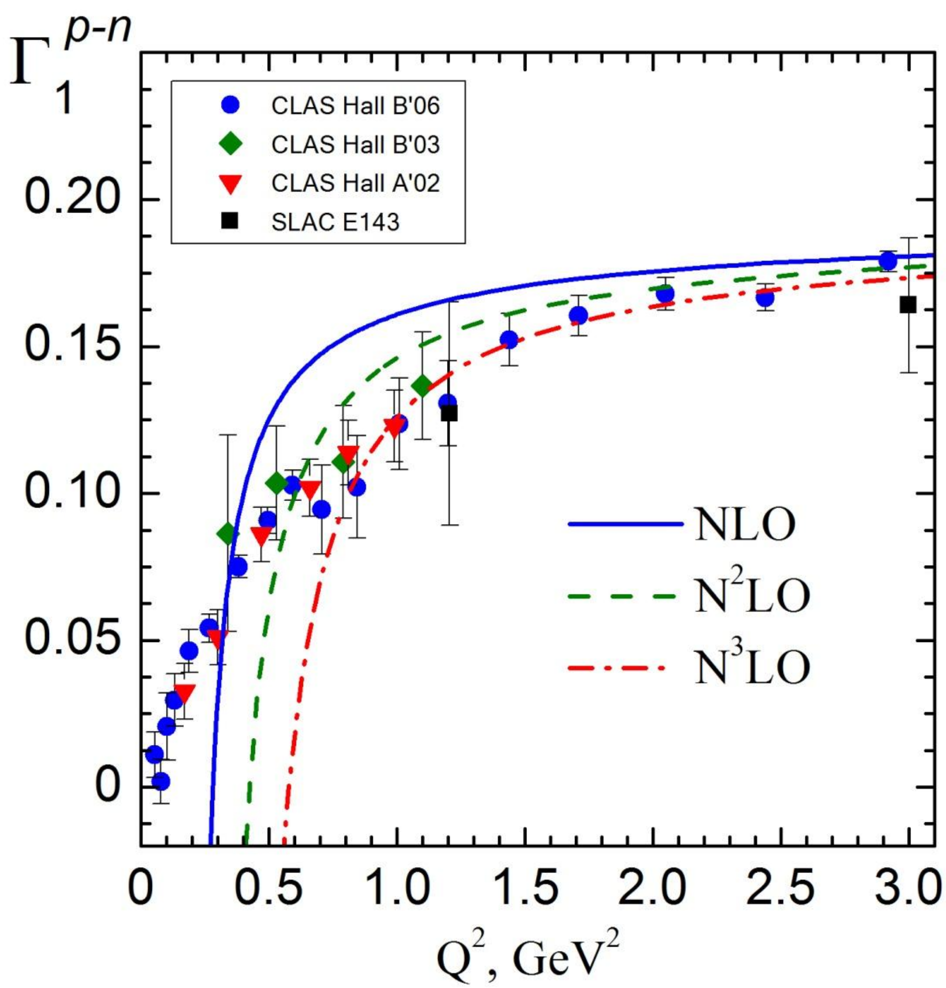
$$\Lambda^{\rm NLO} = (394 \pm 13)\,\mathrm{MeV}$$

$$\Lambda^{\rm N^2LO} = (348 \pm 11)\,\mathrm{MeV}$$

$$\Lambda^{\rm N^3LO} = (336 \pm 10)\,\mathrm{MeV}$$



The perturbative part of BSR



$0.47\ \ GeV^2 < Q^2 < 0.7\ \ GeV^2$

$$NLO: \quad \chi^2_{d.o.f} = 27.4$$

$$N^2LO: \chi^2_{d.o.f} = 14.9$$

$$N^3LO: \chi^2_{d.o.f} = 735.8$$

$Q^2 > 0.7\ \ GeV^2$

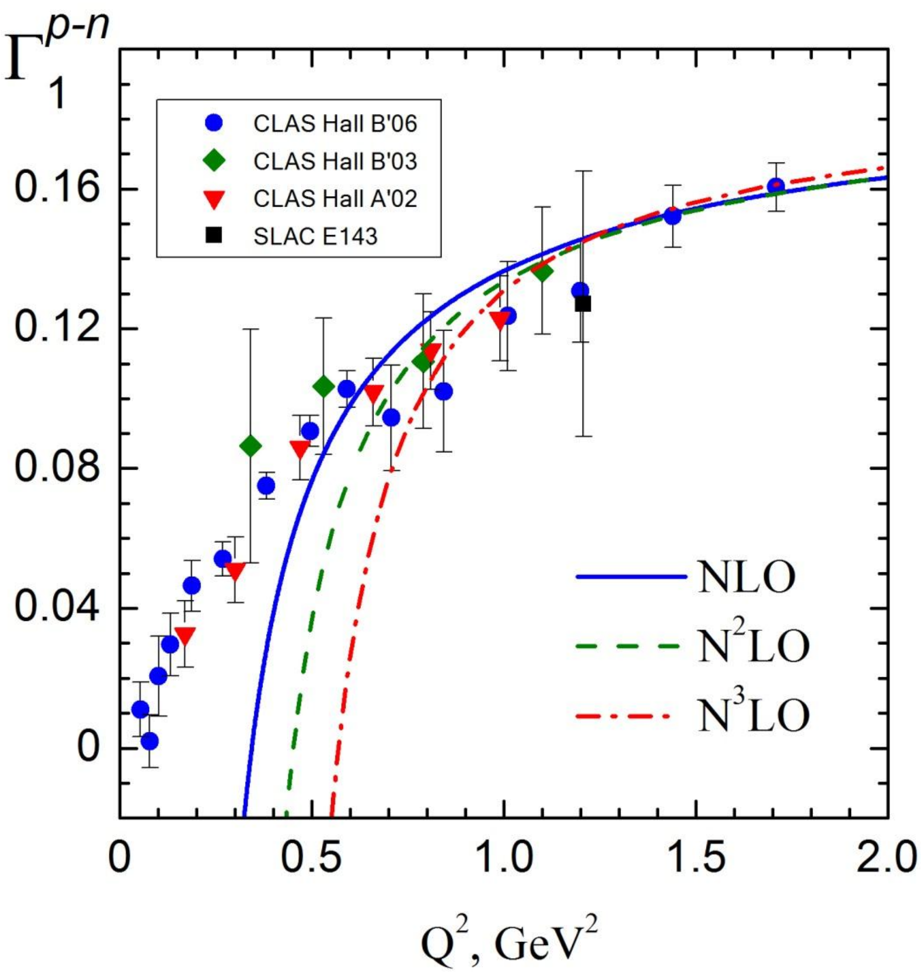
$$NLO: \quad \chi^2_{d.o.f} = 2.41$$

$$N^2LO: \chi^2_{d.o.f} = 0.88$$

$$N^3LO: \chi^2_{d.o.f} = 0.43$$

Results of μ_4-extraction with left border Q_{min}^2 [in GeV^2]

	Q^2_{min}	μ_4/M^2, GeV^2	χ^2/D.o.f
NLO PT	0.5	-0.028±0.005	0.80
N^2LO PT	0.66	-0.014±0.007	0.59
N^3LO PT	0.707	0.006±0.009	0.51

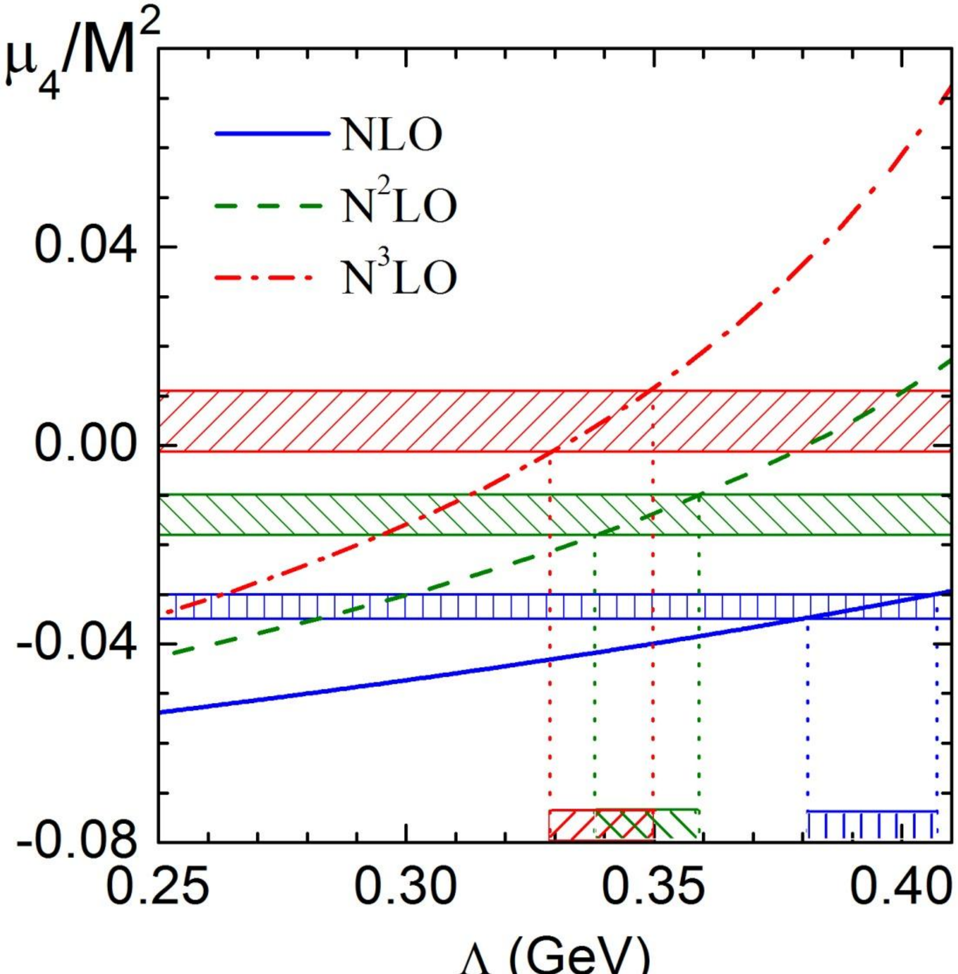


$$\Gamma_1^{p-n}\left(Q^2\right) = \frac{g_A}{6} C_{Bj}\left(Q^2\right) + \frac{\mu_4^{p-n}}{Q^2}$$

- *The lower border shifts up* to higher Q^2 scales with increasing of the PT expansion order.
- *The coefficients of higher-twists μ_4 strongly changes* from order to order
- The absolute value of *μ_4 decreases with the order of PT* and just at four-loop order becomes compatible to zero.

QCD scale parameter dependency

Results of μ_4-extraction with different Λ

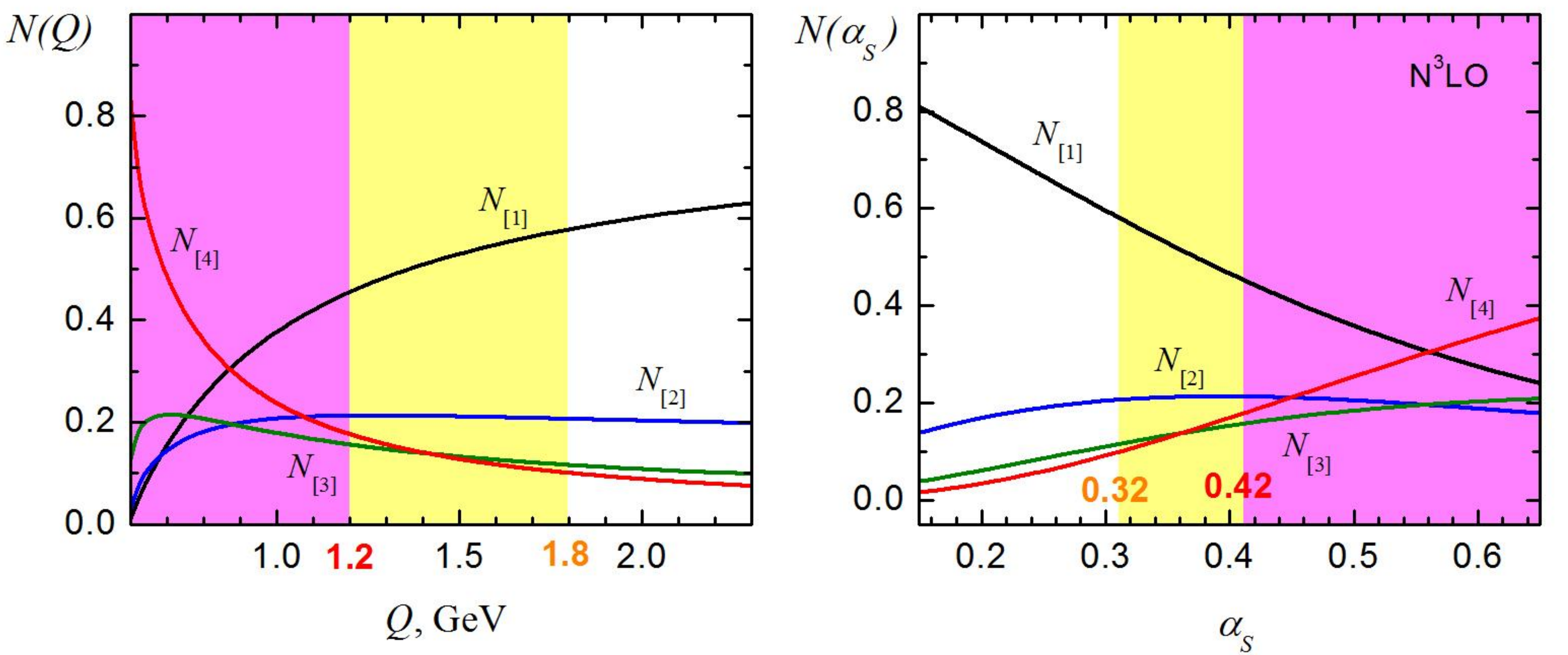


✔ The PT does not lead to a stable results: the extracted coefficient μ_4 changes quite strongly between different orders of the PT expansion. The sensitivity to Λ-variation arises at higher PT orders.

The relative contributions

$$N_{[i]} = \frac{s_i \alpha_S^i}{\Delta_{[4]}^{Bj}} \qquad \Delta_{[n]}^{Bj}\left(\alpha_S\right) = s_1\alpha_S + s_2\alpha_S^2 + \ldots + s_n\alpha_S^n$$

$$s_1 = 0.31831; \quad s_2 = 0.36307; \quad s_3 = 0.65197; \quad s_4 = 1.8042$$



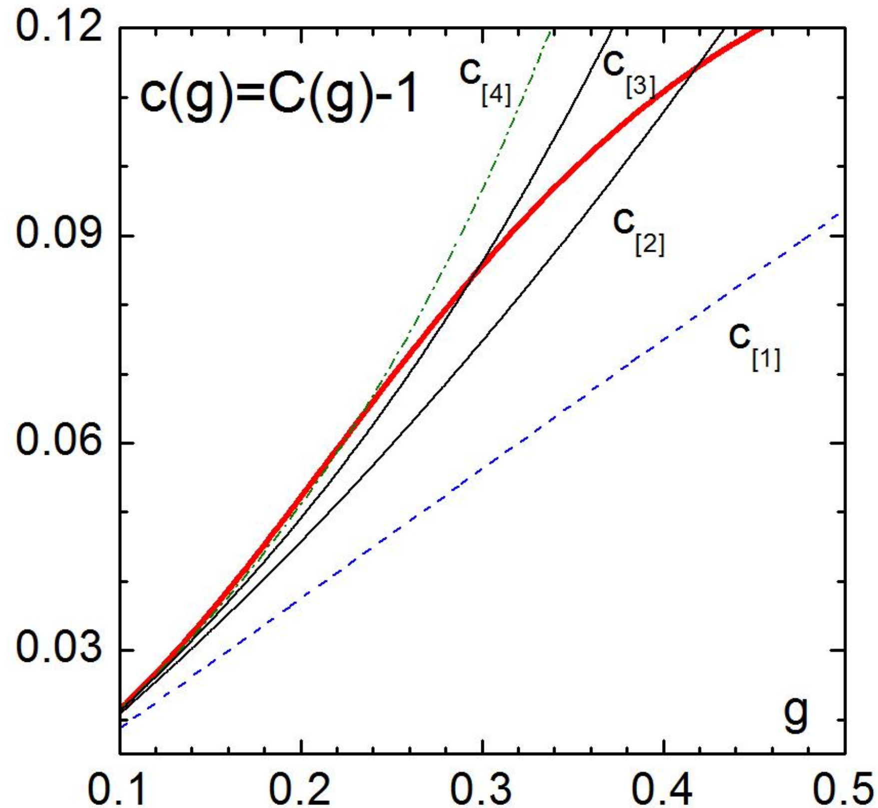
Explicit Illustration

from [Kazakov, Shirkov (1980)]

$$C(g)=\frac{1}{\sqrt{\pi}}\int_{-\infty}^{\infty}dx\exp\left[-x^2\left(1-\frac{\sqrt{g}}{4}x\right)^2\right]$$

$$C(g)=\frac{\sqrt{\pi}e^{-\frac{1}{2g}}}{2\sqrt{g}}\left[I_{1/4}\left(\frac{1}{2g}\right)+I_{-1/4}\left(\frac{1}{2g}\right)\right]\Bigg|_{g\to 0}$$

$$\to\sum_k g^k C_k, C_k=\frac{\Gamma(2k+1/2)}{4^k\Gamma(k+1)}\Bigg|_{k>>1}\to\frac{\Gamma(k)}{\sqrt{2\pi}}$$



$$c_{[5]}(g)=C(g)-1=0.1875\left(\boldsymbol{g}+1!\cdot 1.094\boldsymbol{g^2}+2!\cdot(1.062)^2\boldsymbol{g^3}+3!\cdot(1.047)^3\boldsymbol{g^4}\right)+5.204\boldsymbol{g^5}$$

$$\Delta_{[4]}^{Bj}(\alpha_S)=0.3183\left(\boldsymbol{\alpha_S}+1!\cdot 1.141\boldsymbol{\alpha_S^2}+2!\cdot(1.03)^2\boldsymbol{\alpha_S^3}+3!\cdot(0.982)^3\boldsymbol{\alpha_S^4}\right)$$

Summary

- Natural boundary between the PT and non-PT regions can be considered as a Q ~ 1 GeV.
- For lower $Q \leq 0.8$ GeV the four-loop PT contribution does not help to describe the data on BSR amplitude.
- The possible solution of this problem is to apply the Analytic Perturbative Theory. *For an overview on the APT concept and results see [Shirkov, Solovtsov (2007)]*

The Asymptotic Series «summation» is an Art