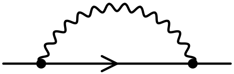
# Quark self-energy



$\Sigma(p) = \not{p}\,\Sigma_V(p^2)$

$$\Sigma_V(p^2) = -C_F \frac{g_0^2(-p^2)^{-\varepsilon}}{(4\pi)^{d/2}} \frac{d-2}{2} a_0 G_1$$

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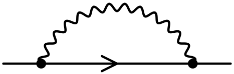

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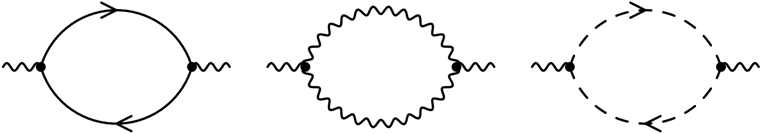
$$Z_q = 1 - C_F a \frac{\alpha_s}{4\pi\varepsilon} + \cdots \qquad \gamma_q = 2 C_F a \frac{\alpha_s}{4\pi} + \cdots$$

# Gluon self-energy

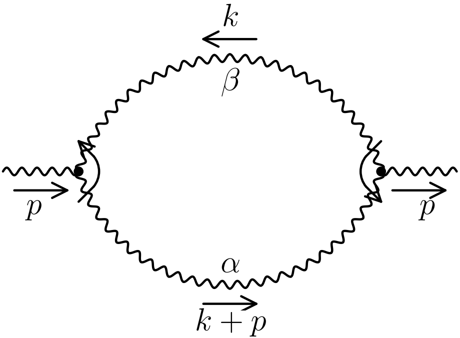


$$i\delta^{ab}\Pi_{\mu\nu}(p) \qquad \Pi_{\mu\nu}(p) = (p^2 g_{\mu\nu} - p_\mu p_\nu)\Pi(p^2)$$

Quark contribution — like in QED

$$\Pi_q(p^2) = -T_F n_f \frac{g_0^2(-p^2)^{-\varepsilon}}{(4\pi)^{d/2}} 2 \frac{d-2}{d-1} G_1$$

# Gluon contribution




Symmetry factor $\frac{1}{2}$. Colour factor $C_A$. Feynman gauge $a_0 = 1$

$$\Pi_{1\,\mu}^{\ \mu} = -i\frac{1}{2}C_A g_0^2 \int \frac{d^d k}{(2\pi)^d} \frac{N}{k^2(k+p)^2}$$

$$N = V_{\mu\alpha\beta}(p, -k-p, k)V^{\mu\beta\alpha}(-p, -k, k+p)$$

# Gluon contribution

$$V_{\mu\alpha\beta}(p, -k-p, k) = (2k+p)_\mu g_{\alpha\beta} - (k-p)_\alpha g_{\beta\mu} - (k+2p)_\beta g_{\mu\alpha}$$

$V^{\mu\beta\alpha}(-p, -k, k+p)$ — the same

$$N = d\left[(2k+p)^2 + (k-p)^2 + (k+2p)^2\right]$$
$$- 2(2k+p)\cdot(k-p) - 2(2k+p)\cdot(k+2p) + 2(k+2p)\cdot(k-p)$$

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$$p^2 = -1 \qquad k^2 = -D_2 \Rightarrow 0 \qquad p\cdot k = \frac{1}{2}(1 - D_1 + D_2) \Rightarrow \frac{1}{2}$$

$$N \Rightarrow -3(d-1)$$

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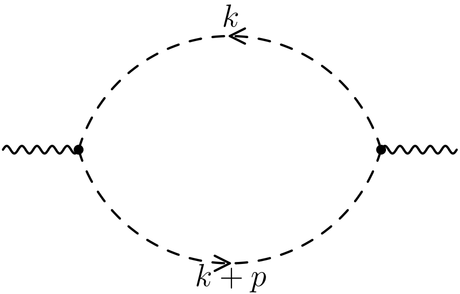
$$\Pi_{1\mu}^{\mu} = -\frac{3}{2} C_A \frac{g_0^2(-p^2)^{1-\varepsilon}}{(4\pi)^{d/2}} G_1 (d-1)$$

# Ghost contribution




Fermion loop gives $-1$. Colour factor $C_A$

$$\Pi_{2\,\mu}^{\mu} = iC_A g_0^2 \int \frac{d^d k}{(2\pi)^d} \frac{k \cdot (k+p)}{k^2 (k+p)^2} = -\frac{1}{2} C_A \frac{g_0^2 (-p^2)^{1-\varepsilon}}{(4\pi)^{d/2}} G_1$$

# Result

$$\Pi_g(p^2) = -\frac{\Pi_1{}^{\mu}_{\mu} + \Pi_2{}^{\mu}_{\mu}}{(d-1)(-p^2)} = C_A \frac{g_0^2(-p^2)^{-\varepsilon}}{(4\pi)^{d/2}} G_1 \frac{3d-2}{2(d-1)}$$

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In an arbitrary covariant gauge

$$\Pi_g(p^2) = C_A \frac{g_0^2(-p^2)^{-\varepsilon}}{(4\pi)^{d/2}} \frac{G_1}{2(d-1)} \times \left[3d - 2 + (d-1)(2d-7)\xi - \frac{1}{4}(d-1)(d-4)\xi^2\right]$$

# Gluon field renormalization

$$p^2 D_\perp(p^2) = 1 + \frac{\alpha_s(\mu)}{4\pi\varepsilon} e^{-L\varepsilon} \left[ -\frac{1}{2}\left(a - \frac{13}{3}\right) C_A - \frac{4}{3} T_F n_f \right.$$
$$\left. + \left( \frac{9a^2 + 18a + 97}{36} C_A - \frac{20}{9} T_F n_f \right) \varepsilon \right]$$

$$L = \log \frac{-p^2}{\mu^2}$$

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$$Z_A = 1 - \frac{\alpha_s}{4\pi\varepsilon} \left[ \frac{1}{2}\left(a - \frac{13}{3}\right) C_A + \frac{4}{3} T_F n_f \right]$$
$$\gamma_A = \left[ \left(a - \frac{13}{3}\right) C_A + \frac{8}{3} T_F n_f \right] \frac{\alpha_s}{4\pi} + \cdots$$

# Quark–gluon vertex

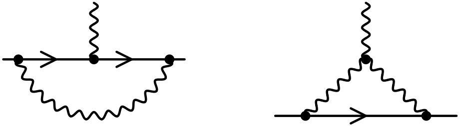
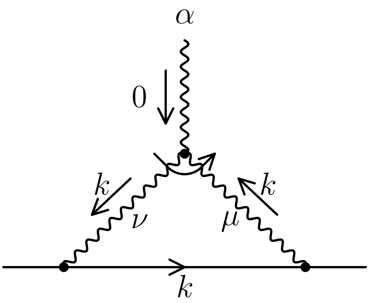


Diagram 1: like in QED, but with a colour factor

$$\Lambda_1^\alpha = \left( C_F - \frac{C_A}{2} \right) \frac{\alpha_s}{4\pi\varepsilon} \gamma^\alpha$$

# Diagram 2




Feynman gauge $a_0 = 1$
Colour factor $C_A/2$

$$\Lambda_2^\alpha = i\frac{C_A}{2} g_0^2 \int \frac{d^d k}{(2\pi)^d} \frac{\gamma_\mu \not{k} \gamma_\nu}{(k^2)^3} V^{\alpha\nu\mu}(0, -k, k)$$

$$V^{\alpha\nu\mu}(0, -k, k) = 2k^\alpha g^{\mu\nu} - k^\mu g^{\nu\alpha} - k^\nu g^{\mu\alpha}$$

$$\Lambda_2^\alpha = i\frac{C_A}{2} g_0^2 \int \frac{d^d k}{(2\pi)^d} \frac{2\gamma_\mu \not{k} \gamma^\mu k^\alpha - 2k^2 \gamma^\alpha}{(k^2)^3}$$

# Result

Averaging $\not{k} k^\alpha \to (k^2/d)\gamma^\alpha$

$$\Lambda_2^\alpha = \frac{3}{2} C_A \frac{\alpha_s}{4\pi\varepsilon} \gamma^\alpha$$

# Result

Averaging $\not{k} k^\alpha \to (k^2/d)\gamma^\alpha$

$$\Lambda_2^\alpha = \frac{3}{2} C_A \frac{\alpha_s}{4\pi\varepsilon} \gamma^\alpha$$

Arbitrary covariant gauge

$$\Lambda_2^\alpha = \frac{3}{4}(1+a) C_A \frac{\alpha_s}{4\pi\varepsilon} \gamma^\alpha$$

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$$\Lambda^\alpha = \left( C_F a + C_A \frac{a+3}{4} \right) \frac{\alpha_s}{4\pi\varepsilon} \gamma^\alpha$$

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$$Z_\Gamma = 1 + \left( C_F a + C_A \frac{a+3}{4} \right) \frac{\alpha_s}{4\pi\varepsilon}$$

# Coupling constant renormalization

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$C_F$ cancelled as in QED

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$$Z_\alpha = 1 - \left( \frac{11}{3} C_A - \frac{4}{3} T_F n_f \right) \frac{\alpha_s}{4\pi\varepsilon}$$

$$\beta_0 = \frac{11}{3} C_A - \frac{4}{3} T_F n_f$$

# Coupling constant renormalization

$$a_s = \frac{\alpha_s}{4\pi} \qquad \beta(a_s) = \beta_0 a_s + \beta_1 a_s^2 + \beta_2 a_s^3 + \cdots$$

RG equation at $\varepsilon = 0$

$$\frac{d \log a_s(\mu)}{d \log \mu} = -2\beta(a_s(\mu))$$

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$$\left( \frac{1}{2\beta(a_s)} - \frac{1}{2\beta_0 a_s} + \frac{\beta_1}{2\beta_0^2} + \frac{1}{2\beta_0 a_s} - \frac{\beta_1}{2\beta_0^2} \right) \frac{da_s}{a_s} = -d \, \log \mu$$

# Coupling constant renormalization

$$\frac{1}{2\beta_0 a_s(\mu)} - \frac{\beta_1}{2\beta_0^2} \log\left[\beta_0 a_s(\mu)\right] + \int_0^{a_s(\mu)} \left( \frac{1}{2\beta(a_s)} - \frac{1}{2\beta_0 a_s} + \frac{\beta_1}{2\beta_0^2} \right) \frac{da_s}{a_s} = -\log \frac{\mu}{\Lambda_{\overline{\mathrm{MS}}}}$$

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$$\Lambda_{\overline{\mathrm{MS}}} = \mu \exp\left( -\frac{1}{2\beta_0 a_s(\mu)} \right) \left[\beta_0 a_s(\mu)\right]^{-\beta_1/(2\beta_0^2)} K(a_s(\mu))$$

$$K(a_s) = \exp \int_0^{a_s} \left( \frac{1}{2\beta(a_s)} - \frac{1}{2\beta_0 a_s} + \frac{\beta_1}{2\beta_0^2} \right) \frac{da_s}{a_s} = 1 - \frac{\beta_0\beta_2 - \beta_1^2}{2\beta_0^3} a_s + \cdots$$