

The Big Bang Cosmology:  
Lecture #5  
Unsolved problems

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# Outline

## 1 The list of problems

## 2 Dark Matter

- General properties and candidates
- WIMPs
- Sterile neutrinos
- Gravitino
- Axion

# Big Bang within GR and SM: problems

- Dark Matter
- Baryogenesis
- Horizon, Entropy, Flatness, . . . problems
- Heavy relics
- Initial fluctuations
- Dark Energy
- Coincidence problem:  $\Omega_B \sim \Omega_{DM} \sim \Omega_\Lambda$

# Dark Matter Properties

$$p = 0$$

If particles:

- 1 stable on cosmological time-scale
- 2 nonrelativistic before RD/MD-transition (either **Cold** or **Warm**)
- 3 (almost) collisionless
- 4 (almost) electrically neutral
- 5 unsuppressed fluctuations @ scales  $l_0 \gtrsim 100$  kpc

If were in **thermal equilibrium**:

$$M_X \gtrsim 1 \text{ keV}$$

If not:

for bosons

$$\lambda = 2\pi / (M_X v_X), \text{ in a galaxy } v_X \sim 0.5 \cdot 10^{-3} \longrightarrow M_X \gtrsim 3 \cdot 10^{-22} \text{ eV}$$

for fermions

Pauli blocking:

$$M_X \gtrsim 750 \text{ eV}$$

$$f(\mathbf{p}, \mathbf{x}) = \frac{\rho_X(\mathbf{x})}{M_X} \cdot \frac{1}{\left(\sqrt{2\pi} M_X v_X\right)^3} \cdot e^{-\frac{p^2}{2M_X^2 v_X^2}} \Bigg|_{\mathbf{p}=0} \leq \frac{g_X}{(2\pi)^3}$$

# Dark Matter Candidates

- WIMPs (neutralino, ...)
- sterile neutrinos
- axion
- gravitino
- Heavy relics
- (Topological) defects
- Massive Astrophysical Compact Heavy Objects
- Primordial black hole remnants

# Weakly Interacting Massive Particles

Assumptions:

- 1 no  $X - \bar{X}$  asymmetry
- 2 @  $T < M_X$  in thermal equilibrium with plasma

$$n_X = n_{\bar{X}}$$

$$n_X = n_{\bar{X}} = g_X \left( \frac{M_X T}{2\pi} \right)^{3/2} e^{-M_X/T}$$

$X\bar{X} \rightarrow$  light particles

freeze-out temperature  $T_f$

$$\frac{1}{n_X} \frac{1}{\langle \sigma_{\text{ann}} v \rangle} = H^{-1}(T_f) \rightarrow T_f = \frac{M_X}{\ln \left( \frac{g_X M_X M_{\text{pl}}^* \sigma_0}{(2\pi)^{3/2}} \right)}.$$

Bethe formulae:

s-wave:  $\sigma_{\text{ann}} = \frac{\sigma_0}{v}$

# Weakly Interacting Massive Particles

density after freeze-out:

$$n_X(T_f) = \frac{T_f^2}{M_{\text{Pl}}^* \sigma_0}$$

today's density:

$$n_X(T_0) = \left(\frac{a(T_f)}{a(T_0)}\right)^3 n_X(T_f) = \left(\frac{s_0}{s(T_f)}\right) n_X(T_f)$$

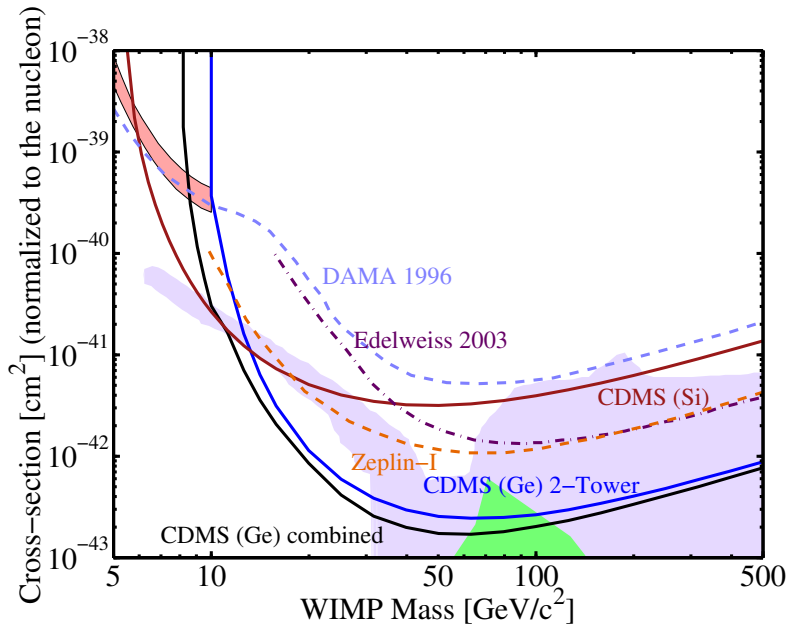
$X + \bar{X}$  contribution to critical density:

$$\begin{aligned} \Omega_X &= 2 \frac{M_X n_X(T_0)}{\rho_c} = 7.6 \frac{s_0 \ln\left(\frac{g_X M_{\text{Pl}}^* M_X \sigma_0}{(2\pi)^{3/2}}\right)}{\rho_c \sigma_0 M_{\text{Pl}} \sqrt{g_*(T_f)}} \\ &= 0.1 \cdot \left(\frac{(10 \text{ TeV})^{-2}}{\sigma_0}\right) \frac{0.3}{\sqrt{g_*(T_f)}} \ln\left(\frac{g_X M_{\text{Pl}}^* M_X \sigma_0}{(2\pi)^{3/2}}\right) \cdot \frac{1}{2h^2} \end{aligned}$$

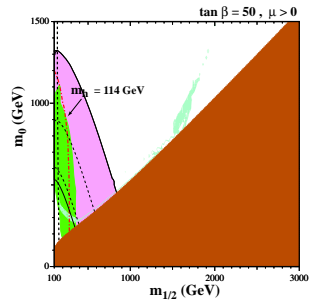
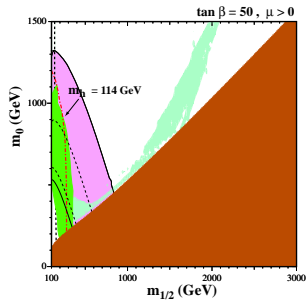
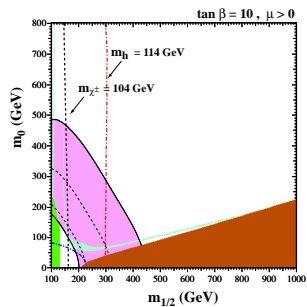
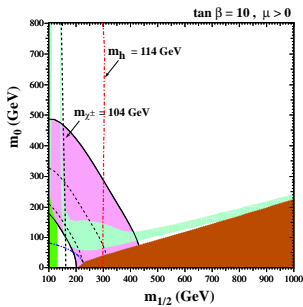
naturally "light"

naturally dark matter

$$\sigma_0 \lesssim \frac{4\pi}{M_X^2} \longrightarrow M_X \lesssim 100 \text{ TeV}$$







# DM keV neutrino constraints

Sterile neutrino of keV scale mass provides the Warm Dark Matter

Bounds on mass

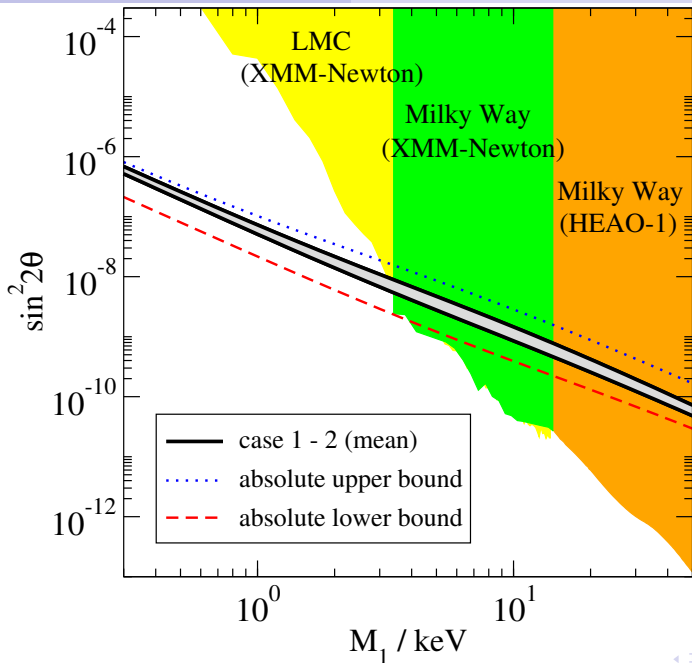
- Tremaine-Gunn bound  $M_1 \gtrsim 0.3 \text{ keV}$
- Lyman- $\alpha$  bound  $M_1 \gtrsim 11.6 \text{ keV}$  or  $8 \text{ keV}$

Bound on mass and mixing angle

- X-ray observation:  $N_1 \rightarrow \nu + \gamma$

Production mechanism

- Dodelson-Widrow (thermal) scenario
- Primordial abundance: physics at higher energies
  - ▶ Lepton asymmetries
  - ▶ Production from inflaton decay
  - ▶ etc.



# DM keV neutrino constraints

Sterile neutrino of keV scale mass provides the Warm Dark Matter

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Bound on mass and mixing angle

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# Gravitino production

$$\mathcal{L} = \frac{1}{F} \partial^\mu \psi \cdot J_\mu^{SUSY}, \quad \tilde{G}_\mu \rightarrow \tilde{G}_\mu + i\sqrt{4\pi} \frac{M_{Pl}}{F} \partial_\mu \psi$$

$$m_{3/2} = \sqrt{\frac{8\pi}{3}} \frac{F}{M_{Pl}}$$

$$1 \text{ TeV} \lesssim \sqrt{F} \lesssim M_{Pl}, \quad 2 \cdot 10^{-4} \text{ eV} \lesssim m_{3/2} \lesssim M_{Pl}.$$

## LSP in low scale SUSY breaking models

$$2 \cdot 10^{-4} \text{ eV} \lesssim m_{3/2} \lesssim 100 \text{ GeV} \longrightarrow \sqrt{F} \lesssim 10^{10} \text{ GeV}$$

Thermal equilibrium is forbidden:

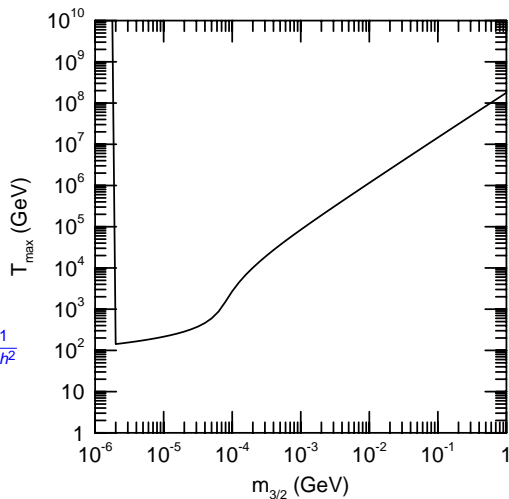
$$\Omega_{3/2} = \frac{m_{3/2} \cdot n_{3/2}}{\rho_c} = 0.2 \frac{m_{3/2}}{200 \text{ eV}} \left( \frac{g_{3/2}}{2} \right) \cdot \left( \frac{210}{g_*(T_f)} \right) \cdot \frac{1}{2h^2}$$

$$\tilde{X}_i \rightarrow \tilde{G} + X_i, \quad X_i + X_j \rightarrow X_k + \tilde{G}$$

# Gravitino non-thermal production

$$\frac{dn_{3/2}}{dt} + 3Hn_{3/2} = \sum_i \Gamma_{\tilde{X}_i} \cdot \gamma_i^{-1} \cdot n_{\tilde{X}_i} + \langle \sigma_{tot} \rangle \cdot n_\gamma^2,$$

$$\Omega_{3/2} \sim \left( \frac{200}{m_{3/2}} \right) \cdot \left( \frac{T_{max}}{10} \right) \cdot \left( \frac{M_S}{1} \right)^2 \cdot \left( \frac{15}{\sqrt{g_*(T_{max})}} \right) \cdot \frac{1}{2h^2}$$



# Axion as Cold Dark Matter

## nonperturbative CP-violation in QCD

$$L_\theta = \frac{\alpha_s}{8\pi} \left( \theta_0 + \text{Arg} \left( \text{Det} \hat{M}_q \right) \right) G_{\mu\nu}^a \tilde{G}^{\mu\nu a} \equiv \frac{\alpha_s}{8\pi} \cdot \theta \cdot G_{\mu\nu}^a \tilde{G}^{\mu\nu a} .$$

$$\theta \rightarrow \bar{\theta}(x) = \theta + C_g \frac{a(x)}{f_{PQ}} .$$

$$m_a(T) \simeq 0.1 \cdot m_a(0) \cdot \left( \frac{\Lambda_{\text{QCD}}}{T} \right)^{3.7}, \quad T > \Lambda_{\text{QCD}}$$

$$\mathcal{L} = \frac{f_{PQ}^2}{2} \cdot \left( \frac{d\bar{\theta}}{dt} \right)^2 - \frac{m_a^2(T)}{2} f_{PQ}^2 \bar{\theta}^2 ,$$

$$\Omega_a \simeq 0.2 \cdot \bar{\theta}_i^2 \cdot \left( \frac{4 \cdot 10^{-6} \text{ eV}}{m_a} \right)^{1.2} \cdot \frac{1}{2h^2}$$

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