Классические и квантовые струны: основные идеи и приложения

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(ЛТФ ОИЯИ)

V Зимняя Школа по теоретической физике
«Введение в теорию фундаментальных взаимодействий»
(Дубна, 28 янв. - 6 фев. 2007г)

Fundamental Interactions

Fundamental fermions

Generations

	I	II	III	IV-?
Leptons	e^-	μ^-	τ^-	
	ν_e	ν_μ	ν_τ	
Quarks	u	c	t	
	d	s	b	

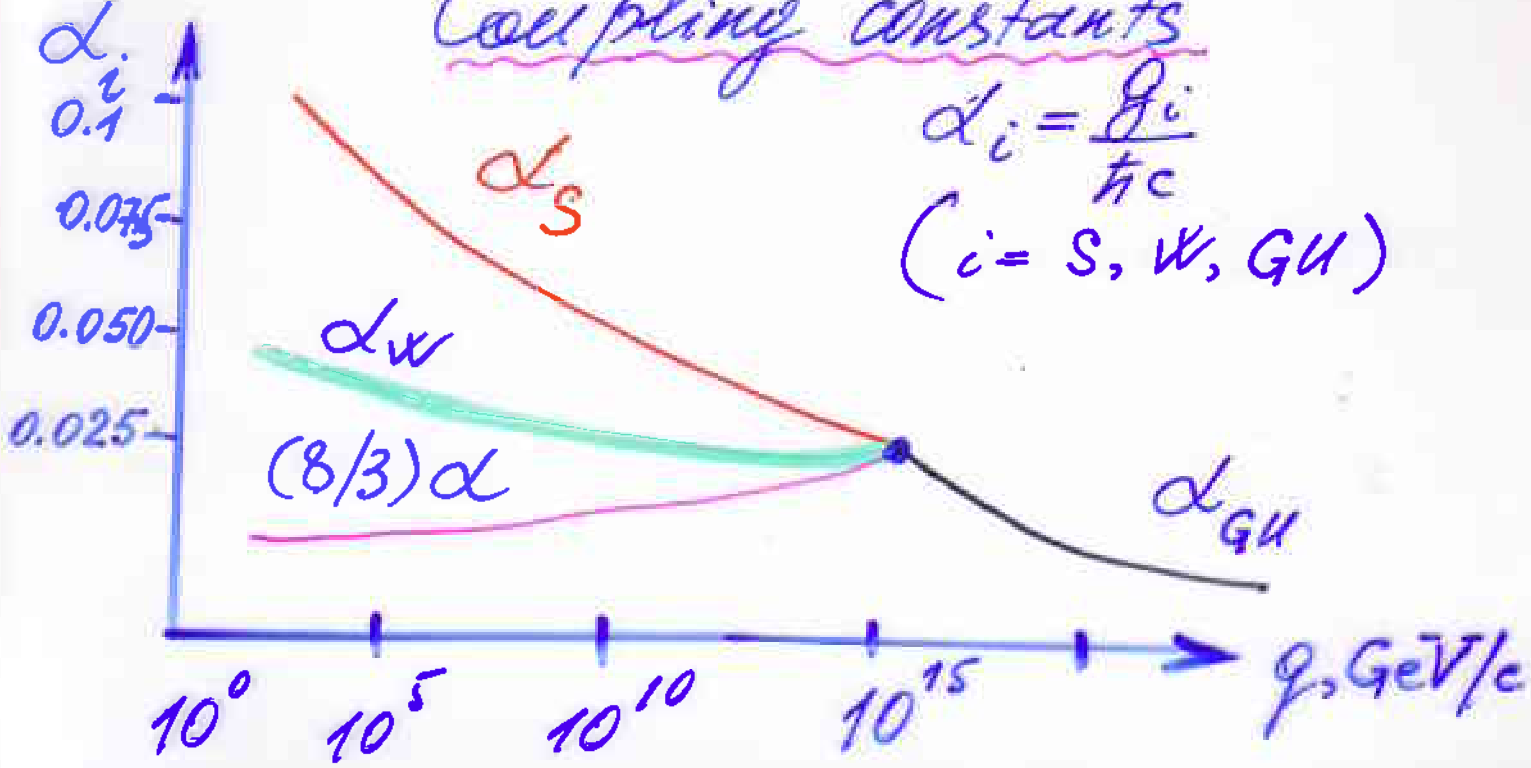
Fundamental gauge bosons

Gravity	Electro-weak interact.	Strong interact.
G_∞	$SU(2)\times U(1)$	$SU(3)$
$g_{\mu\nu}$	$A_\mu, Z^0_\mu, W^\pm_\mu$	A^a_μ
graviton	photon	$a=1,\dots,8$
$m=0$	$m=0$	gluon
$S=2$	$S=1$	$m=0$
	$m_{W^\pm}\sim 80$ GeV	$S=1$
	$m_Z \sim 90$ GeV	
	(+ Higgs bosons) $S=0$	

Coupling constants

$$\alpha_i = \frac{g_i}{\hbar c}$$

$(i = S, W, GU)$

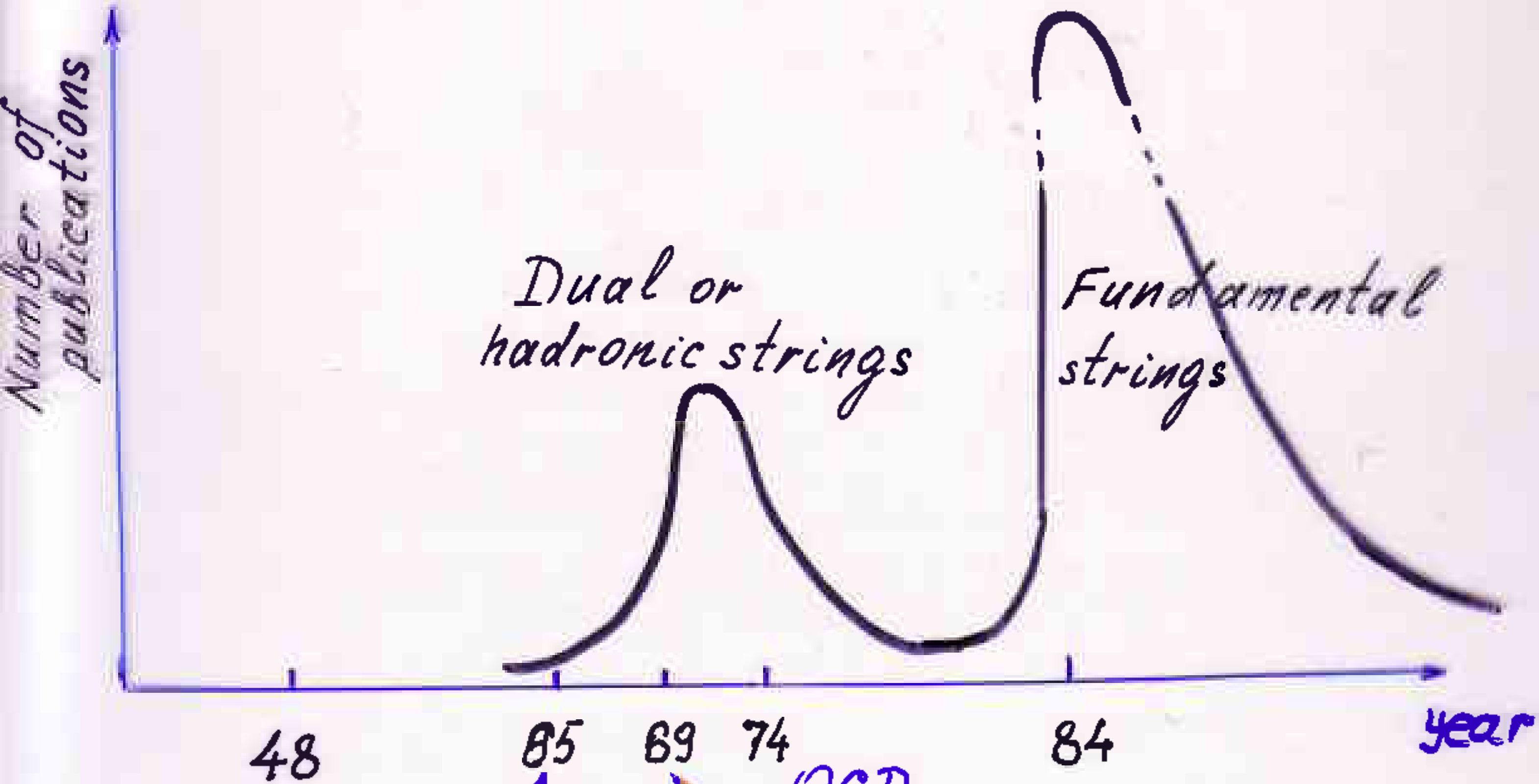


Introduction to the String Theory.

Basic Ideas and Applications.

Lecture 1.

Short history of the string models



QCD

Dirac magnetic monopoles

B. Barbashov and N. Chernikov (Dubna) (Born-Infeld models)

Y. Nambu - T. Goto (dynamical basis for dual models)

B. Green and J.H. Schwarz (unification of all fundamental interactions including gravity)

References

Hadronic strings:

1. C. Rebbi, Phys. Rep. C12 (1974) 1.
2. J. Scherk, Rev. Mod. Phys. 47 (1975) 123.
3. P. H. Frampton, Dual Resonance Models and Superstrings (World Scientific, 1986).
4. G. Veneziano, Phys. Rep. 9 (1974) 199.
5. J. H. Schwarz, Phys. Rep. 8 (1973) 269.
6. S. Mandelstam, Phys. Rep. C13 (1974) 259.
7. B. M. Barbashov, V. V. Nesterenko, Introduction to the Relativistic String Theory. (World Scientific, 1990).

Fundamental strings:

1. M. B. Green, J. H. Schwarz, and E. Witten, Superstring Theory (Cambridge University Press, Cambridge, 1987), Vols. 1 and 2.

! Resource Letter NSST-1: The Nature and Status of String Theory.
D. Marolf hep-th/0311044 v. 3
From popular articles and books to the holographic principle

http://superstringtheory.com "The Official String Theory Web Site - basic version"

Strings in Dirac's theory of magnetic poles

$$\begin{cases} \partial^\nu F_{\mu\nu} = -j_\mu^{(e)}, & j_\mu^{(e)}(z) = \sum_e e \int \frac{dx_\mu}{ds} \delta^{(4)}(z - x(s))\, ds \\ \partial^\nu \tilde{F}_{\mu\nu} = -j_\mu^{(m)} & j_\mu^{(m)}(z) = \sum_g g \int \frac{dx_\mu}{ds} \delta^{(4)}(z - x(s))\, ds \end{cases}$$

$$\tilde{F}_{\mu\nu} = \frac{1}{2} \varepsilon_{\mu\nu\rho\sigma} F^{\rho\sigma}$$

This modification of the Maxwell eqs. results in the definition of $F_{\mu\nu}(z)$:

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + \tilde{G}_{\mu\nu}$$

$$\operatorname{div} \vec{H} = \rho^{(m)} \qquad \vec{H} = \operatorname{rot} \vec{A} + \text{(string contribut.)}$$

Equation for $\tilde{G}_{\mu\nu}$:

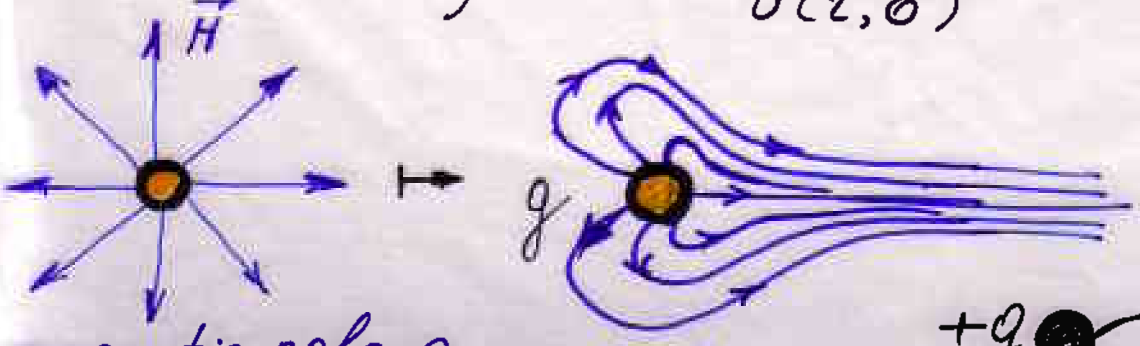
$$\partial^\nu G_{\mu\nu}(z) = j_\mu^{(m)}(z) = \sum_g g \int \frac{dx_\mu}{ds} \delta^{(4)}(z - x(s))\, ds$$

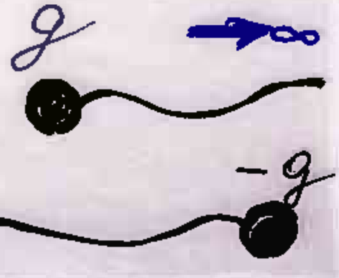
Solution to this equation is given by

$$G_{\mu\nu}(z) = \iint d\tau\, d\sigma\, \delta^{(4)}(z - \underbrace{x(\tau,\sigma)}_{\text{(string coordinates}})\, \sigma_{\mu\nu}(\tau,\sigma),$$

where

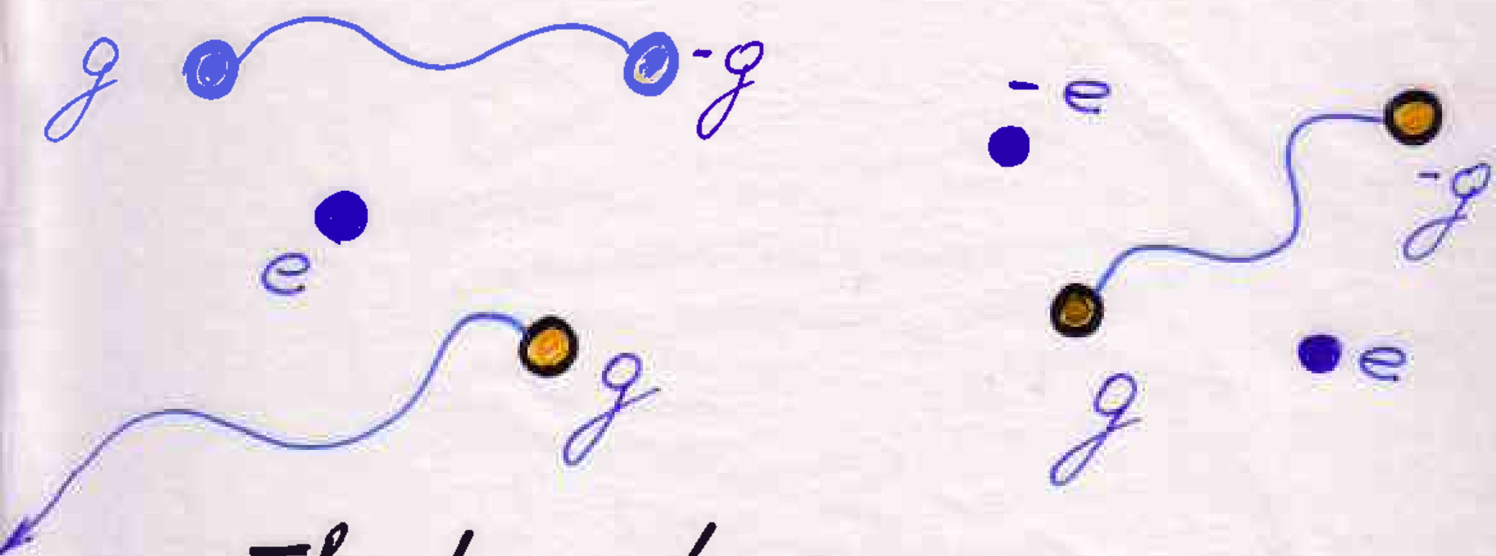
$$\sigma_{\mu\nu}(\tau,\sigma) = \frac{\partial(x_\mu, x_\nu)}{\partial(\tau,\sigma)}$$





magnetic pole g

The Dirac veto: this string does not enter the electric charges



The Lorentz eqs.

$$\begin{cases} m_e \dfrac{d^2 z_\nu}{ds^2} = e \dfrac{dz^\mu}{ds} F_{\nu\mu}(z), \\ m_g \dfrac{d^2 z_\nu}{ds^2} = g \dfrac{dz^\mu}{ds} F_{\nu\mu}(z). \end{cases}$$

The total action is

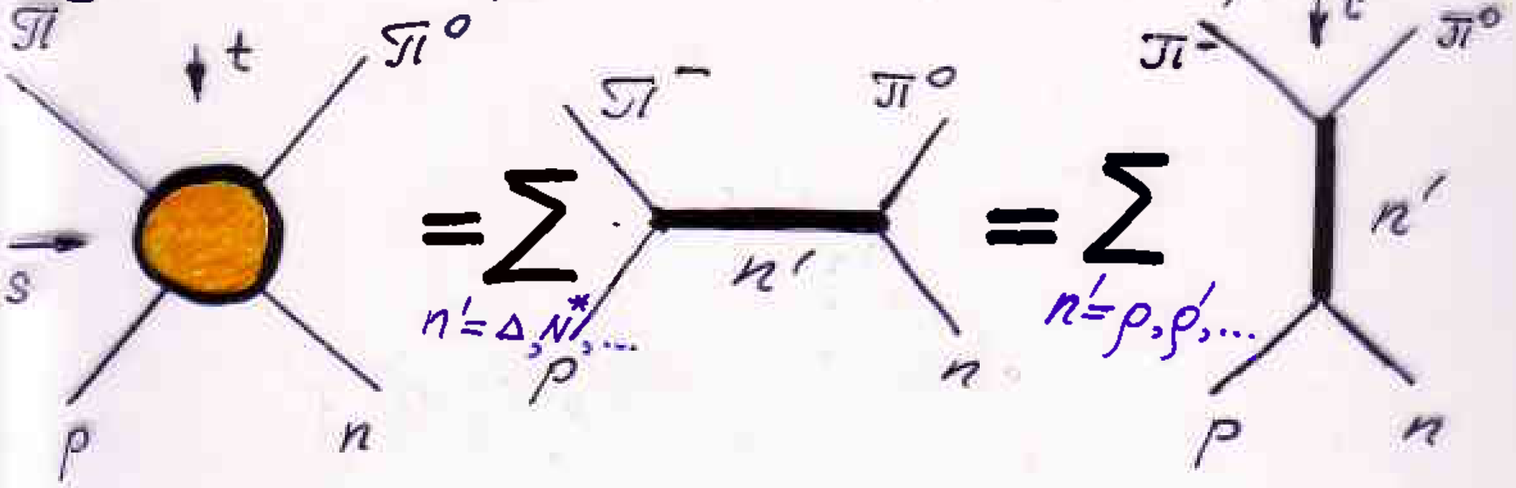
$$S = -\sum_e m_e \int ds - \sum_g m_g \int ds - \frac{1}{4}\int d^4 z\, F_{\mu\nu} F^{\mu\nu} -$$

$$- e\int A^\mu(z) \frac{dz_\mu}{ds} ds$$

No equations appear for string variables $x^\mu(\tau,\sigma)$. It reflects the unphysical nature of these variables.

Dual models and dual strings

Duality principle for hadronic amplitudes



Veneziano amplitude for meson-meson scattering obeying the duality principle

$$A(s,t,u) = F(s,t) + F(t,u) + F(u,s)$$

$$F(s,t) = g^2 \frac{\Gamma(-\alpha(s))\,\Gamma(-\alpha(t))}{\Gamma(-\alpha(s)-\alpha(t))} \equiv g^2 B(-\alpha(s),-\alpha(t))$$

Γ and B are the Euler functions,

$$\alpha(s) = \alpha(0) + \alpha' s.$$

Beta function has the following expansions

$$B(-\alpha(s),-\alpha(t)) = \sum_{n=0}^{\infty} \frac{\Gamma(n+1+\alpha(t))}{n!\,\Gamma(1+\alpha(t))} \, \frac{1}{n-\alpha(s)} =$$

$$= \sum_{n=0}^{\infty} \frac{\Gamma(n+1+\alpha(s))}{n!\,\Gamma(1+\alpha(s))} \, \frac{1}{n-\alpha(t)}.$$

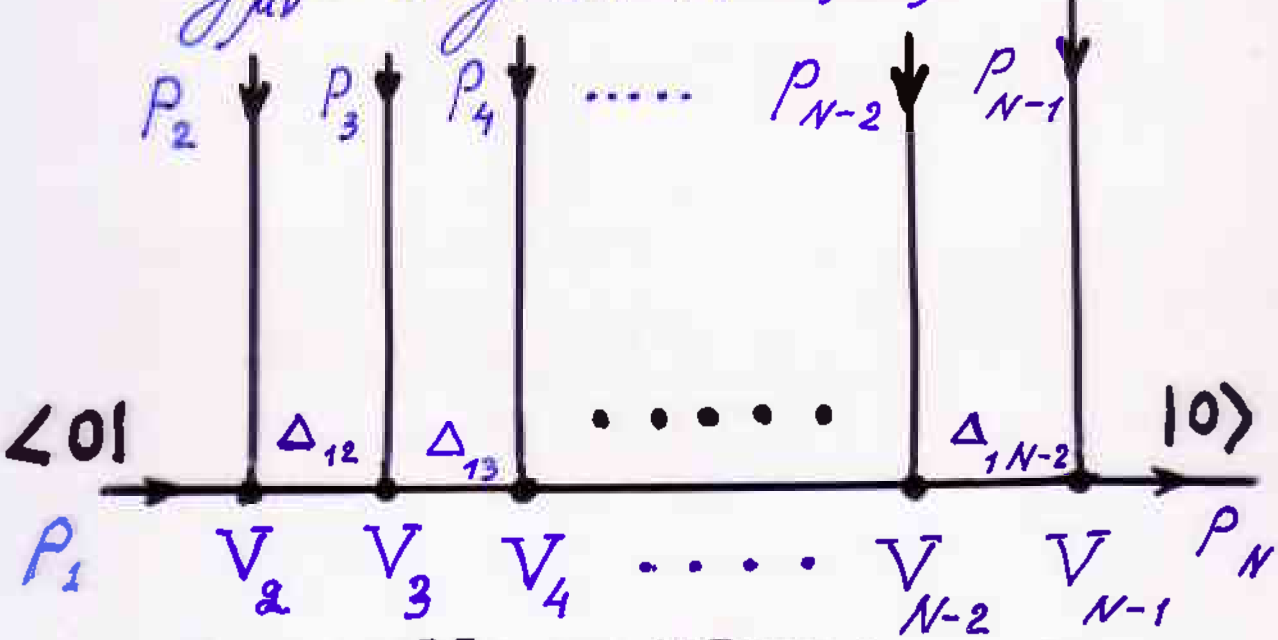
Operator formalism and diagram techniques for dual amplitudes

$$\alpha_n^\mu, \quad \mu = 0, 1, \dots, D-1; \quad n = 0, \pm 1, \pm 2, \dots$$

$$\alpha_{n\mu} = \overset{*}{\alpha}_{-n\mu}$$

$$[\alpha_{m\mu}, \alpha_{n\nu}] = -m\, g_{\mu\nu}\, \delta_{n+m,0},$$

$$g_{\mu\nu} = \mathrm{diag}\,(1, -1, \dots, -1)$$



$$B_N = \langle 0 | V(p_2) \Delta_{12} V(p_3) \Delta_{13} \dots V(p_{N-1}) | 0 \rangle$$

Vertex operators are given by

$$V(p) = \exp\left(i\sqrt{2\alpha'} \sum_{n=1}^{\infty} p^\mu \alpha^+_{n\mu}\right) \cdot \exp\left(i\sqrt{2\alpha'} \sum_{n=1}^{\infty} p^\mu \alpha_{n\mu}\right)$$

Propagator Δ_{ij} is

$$\Delta_{ij} = \left[s_{ij} + \alpha' \hat{M}^2 + \alpha(0)\right]^{-1},$$

$s_{ij} = (p_i + p_{i+1} + \dots + p_j)^2$, $\hat{M}^2 = \sum \alpha_n^\mu \alpha_{n\mu}$

$$A_N = \sum_{\substack{\text{noncyclic}\\ \text{permutations}}} B_N(p_1, p_2, \dots, p_N),$$

$$B_N = \int_0^1 \dots \int_0^1 \prod_{i=2}^{N-2} dx_i \, x_i^{-\alpha(s_{1i})-1} \prod_{2 \le i < j \le N-1} (1-x_{ij})^{-2\alpha' p_i p_j},$$

where

$$x_{ij} = x_i x_{i+1} \dots x_{j-1}, \quad \text{and} \quad \alpha(0) = 1$$

Virasoro conditions on the physical state vectors

$$L_n |\Phi\rangle = 0, \quad n = 1, 2, \dots$$

$$(L_0 - \alpha(0)) |\Phi\rangle = 0, \qquad L_0 \equiv \hat{M}^2$$

$$L_n = -\frac{1}{2} \sum_{m=-\infty}^{+\infty} : \alpha_{n-m} \alpha_m : ,$$

$$\alpha_{0\mu} = \sqrt{2\alpha'} p_\mu, \qquad \alpha_{-k} = \alpha_k^+ = \sqrt{k}\, a_k^+, \quad k = 1, 2, \dots$$

$$[a_k^\mu, a_\ell^{\nu +}] = -\delta_{k\ell} g^{\mu\nu}.$$

$$|\Phi\rangle = a_{n_1}^{+\mu_1} a_{n_2}^{+\mu_2} \dots a_{n_m}^{+\mu_m} |0\rangle$$

When $\mu_j = 0$ we have the ghost states with negative norm.

For time-like components of a_n^μ we have

$$[a_k^0, a_l^{0+}] = -\delta_{kl}$$

$|g\rangle = a_k^{0+}|0\rangle$; $\langle g|g\rangle = \langle 0|a_k^0 a_k^{0+}|0\rangle =$

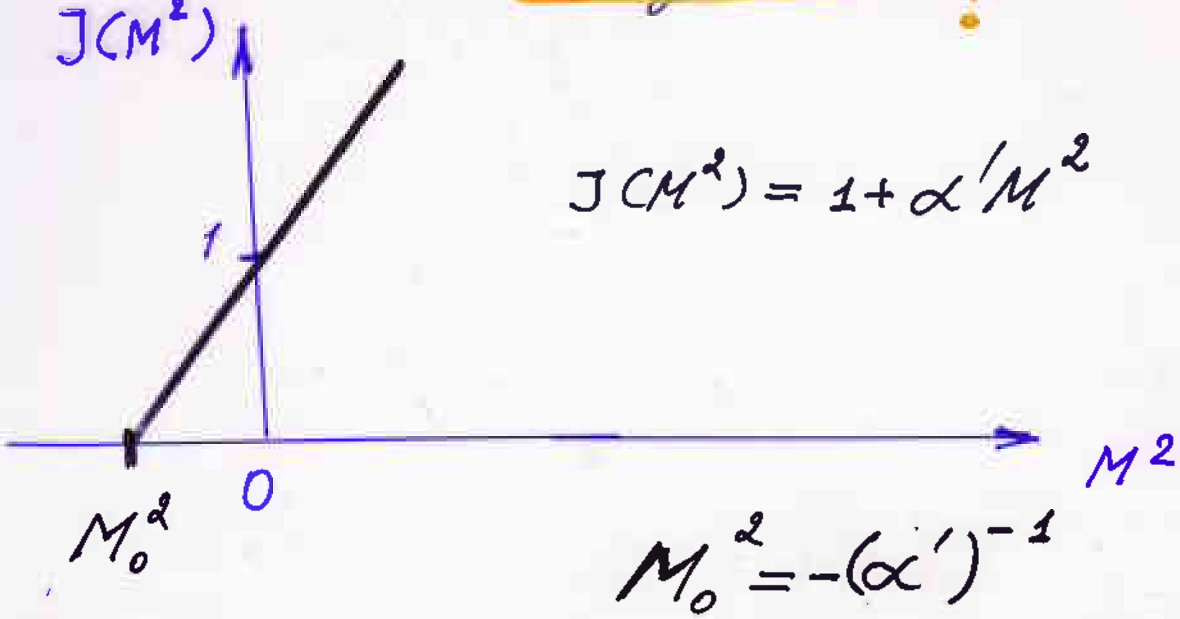
$$= -\langle 0|0\rangle = -1$$

In order to remove the ghost states one has to require

$$D = 26, \quad \alpha(0) = 1$$

$\downarrow$

ground state is tachyonic !



$$J(M^2) = 1 + \alpha' M^2$$

$$M_0^2 = -(\alpha')^{-1}$$

To obtain the set of operators

$$\alpha_n^\mu, \quad n = 0, \pm 1, \pm 2, \ldots$$
$$\mu = 0, 1, \ldots, D-1$$

one has to quantize the ONE-DIMENSIONAL RELATIVISTIC OBJECT

Linear string model with the action S_{Lin} is not suitable

$$S_{Lin} \sim \frac{1}{2} \iint d\tau d\sigma \, (\dot{x}^2 - x'^2) \Longrightarrow \ddot{x}^\mu - x''^\mu = 0$$

$$x = x^\mu(\tau,\sigma), \quad \dot{x} = \frac{\partial x}{\partial \tau}, \quad x' = \frac{\partial x}{\partial \sigma},$$

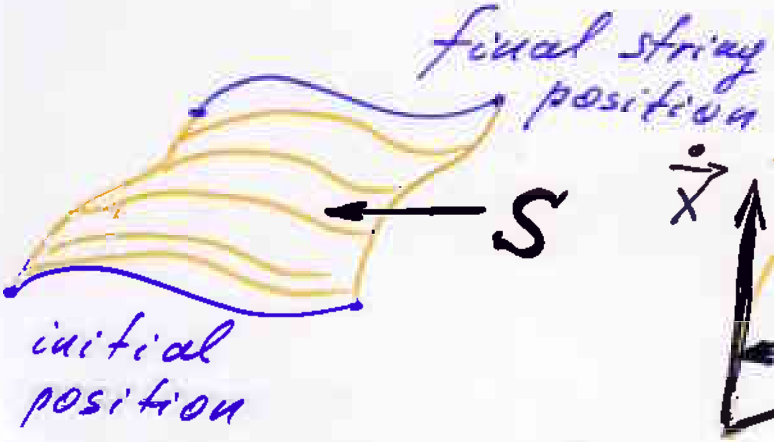
because here there are no Virasoro conditions. The required model should have a gauge symmetry to produce the Virasoro conditions in a consistent way

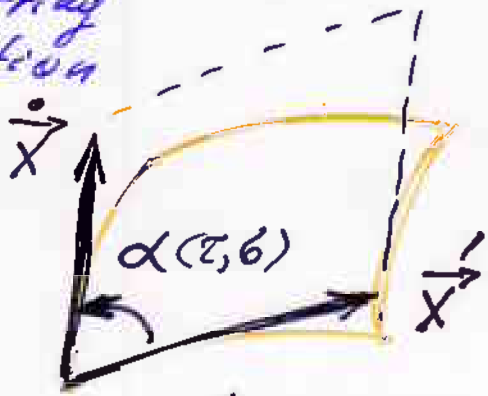
$$S_{N-G} = -\gamma \iint dS = -\gamma \int_{-\infty}^{+\infty} d\tau \int_{\sigma_0}^{\sigma_1} d\sigma \sqrt{-g}$$

$$g = \det g_{ij}, \quad g_{ij} = \partial_i x^\mu \partial_j x_\mu$$

$$-g = (\dot{x} x')^2 - \dot{x}^2 x'^2$$

$$\alpha' = (2\pi\gamma)^{-1}. \qquad \dot{x}^2 > 0, \; x'^2 < 0.$$





$$d\Sigma = |\dot{\vec{x}}| \, |\vec{x}'| \cdot \sin\alpha \, d\tau d\sigma =$$

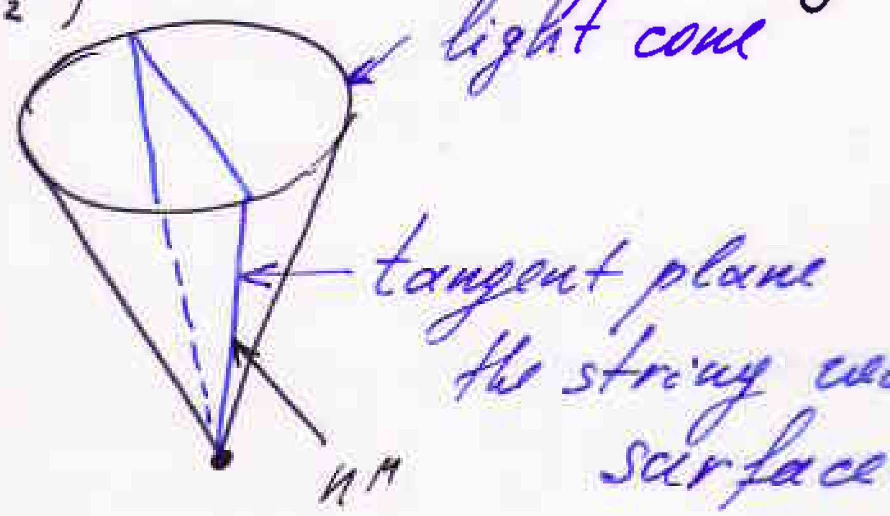
$$= |\dot{\vec{x}}| \, |\vec{x}'| \sqrt{1 - \cos^2\alpha} \, d\tau d\sigma = |\dot{\vec{x}}| \, |\vec{x}'| \sqrt{1 - \frac{(\dot{\vec{x}} \vec{x}')^2}{\dot{\vec{x}}^2 \vec{x}'^2}} \, d\tau d\sigma$$

Kinimatical conditions

$\dot{x}^2 > 0 \quad x'^2 < 0$

$-g = (\dot{x}x')^2 - \dot{x}^2 x'^2 > 0$ means the velocity of each point of the string is less than the velocity of light

$\dot{x}_\perp^2 = \left(\dot{x} - \frac{(\dot{x}x')x'}{x'^2}\right)^2 = (\dot{x}^2 x'^2 - (\dot{x}x')^2)/x'^2 > 0$ $\quad g < 0$ because $x'^2 < 0$



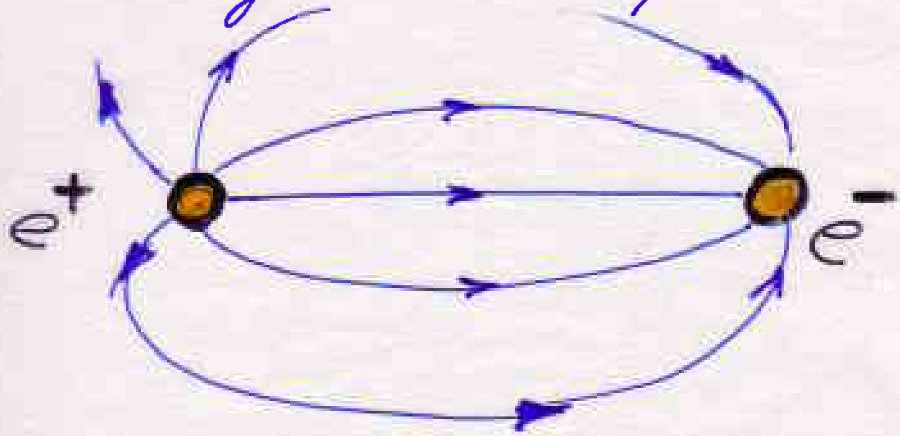
$n^\mu = a\dot{x}^\mu + b x'^\mu \qquad n^2 = 0$

$n^2 = a^2\left(\dot{x} + \frac{b}{a}x'\right)^2 = a^2\left(\dot{x}^2 + 2\dot{x}x'\frac{b}{a} + x'^2\frac{b^2}{a^2}\right) = 0$

If $-g > 0$ then this quadratic eq. has two different solutions for b/a, i.e. there are two different vectors n^μ.

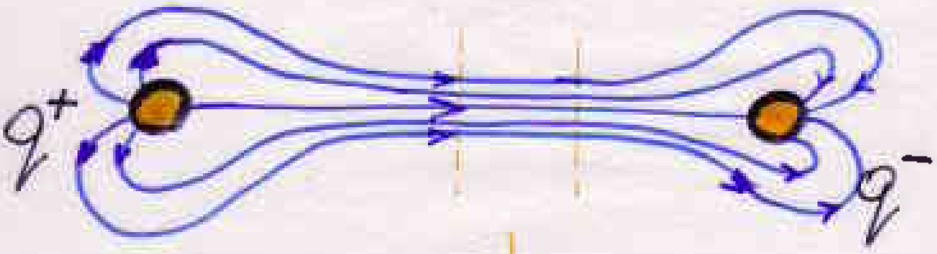
Flux-tube model and QCD

Qurk confinement can be understood more easily in the flux-tube model.



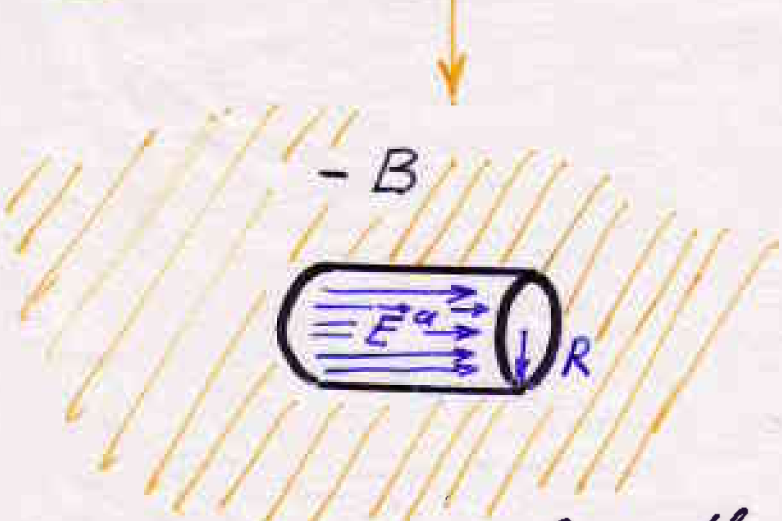
QED

$$E \underset{R\to\infty}{\sim} \frac{1}{R}$$



QCD

$$E \underset{R\to\infty}{\sim} R$$



ε is energy per unit length of the flux tube

$$\varepsilon = \frac{1}{2} |\vec{E}^a|^2 \pi R^2 + B \pi R^2$$

stable configuration $\partial\varepsilon / \partial R = 0$

$|\vec{E}^a| = \frac{\Phi}{\pi R^2}$, where Φ is the total flux of gluon field generated by quark-antiquark pair

$$R_0 = \left(\bar{\Phi}^2/(2\pi^2 B)\right)^{1/2}.$$

This configuration of the gluon tube is stable because

$$\left.\frac{\partial^2 \mathcal{E}}{\partial R^2}\right|_{R=R_0} = 8\pi B > 0.$$

Lagrangian formalism for the Nambu-Goto string

$$S = -\gamma \iint \sqrt{-g}\, du^0 du^1, \qquad u^0 = \tau,\ u^1 = \sigma$$

This action is invariant under transformations

$$\tau = f_1(\bar\tau, \bar\sigma), \quad \sigma = f_2(\bar\tau, \bar\sigma)$$

(reparametrization invariance).
According to the second Noether theorem this entailes two identities

$$L_\mu \dot{x}^\mu = L_\mu x'^\mu = 0,$$

where

$$L_\mu = L_\mu(\partial x, \partial^2 x) = \frac{\delta S}{\delta x^\mu}.$$

The rule of differentiation of the determinant

$$dg = dg_{ik}\, g^{ik} g = -g_{ik}\, dg^{ik}$$

$$\downarrow$$

$$\frac{\partial g}{\partial x_{\mu,i}} = \frac{\partial g_{lm}}{\partial x_{\mu,i}} g^{lm} \cdot g, \qquad x^{\mu}_{,i} \equiv \frac{\partial x^{\mu}}{\partial u^{i}}$$

$$\frac{\delta\sqrt{|g|}}{\delta x_{\mu}} = \sqrt{|g|}\,\Delta x^{\mu},$$ where Δ is the Laplace-Beltrami operator for induced metric g_{ij}.

$$\Delta = \nabla_i \nabla^i = \frac{1}{\sqrt{|g|}} \frac{\partial}{\partial u^i}\left(\sqrt{|g|}\, g^{ij} \frac{\partial}{\partial u^j}\right)$$

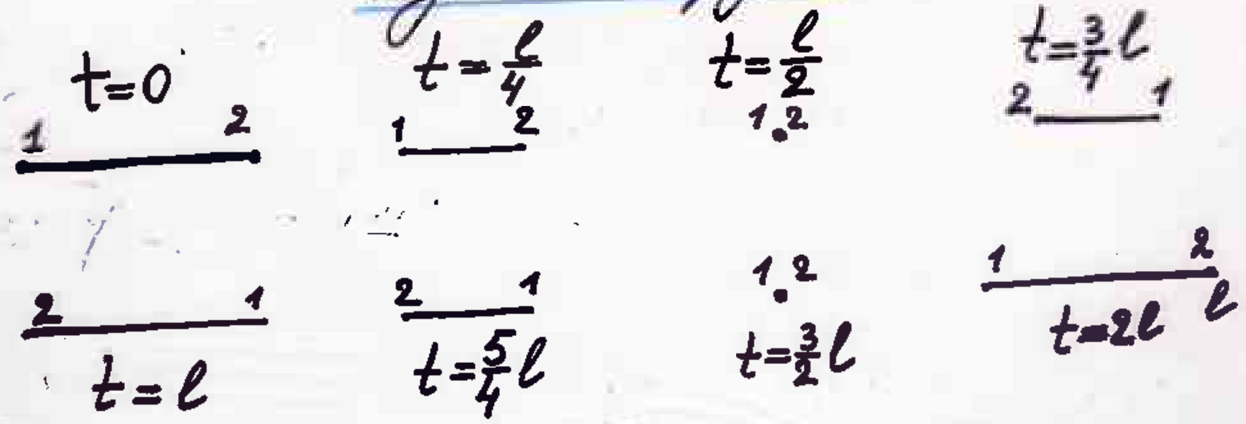
One can impose <u>two</u> gauge conditions on $x^{\mu}(\tau,\sigma)$. Orthonormal gauge conditions

$$(\dot{x} \pm x')^2 = 0$$

$$g_{00} = -g_{11}, \quad g_{10} = g_{01} = 0.$$

$$\Delta x^{\mu} \longrightarrow \quad \ddot{x}^{\mu} - x''^{\mu} = 0$$

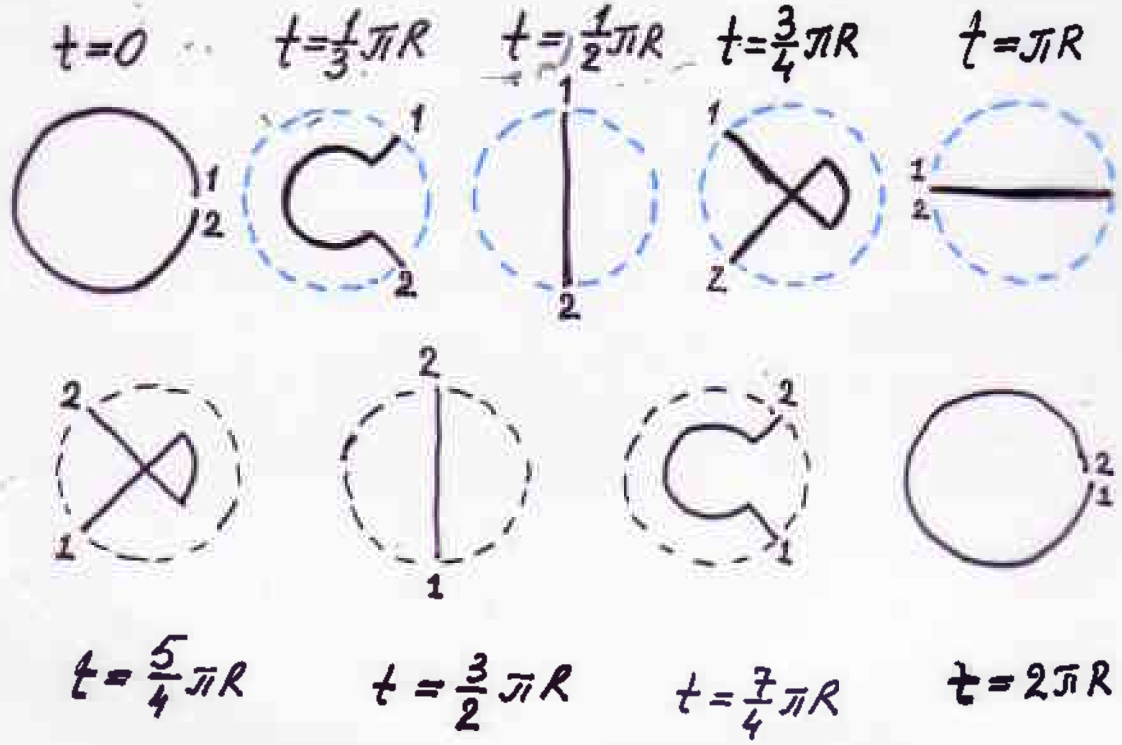
Примеры движения свободной струны



Струна в форме кольца

$$R = R(t) = R_0 \cdot \cos\frac{t}{R}.$$

Струна в форме кольца, разрезанного в одной точке



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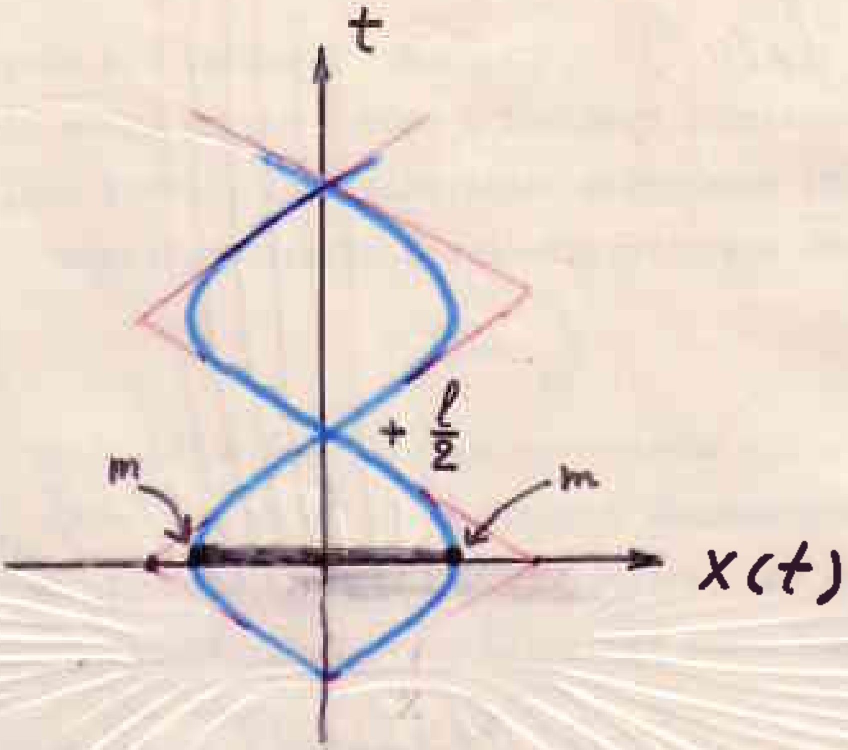
Движение струны с массами на концах в 2-мерном пространстве-времени



траектории концов струны с массами

—"—"— свободной струны

$$\mathcal{L} = \varkappa^2 \sqrt{1 + \frac{\varphi_x^2 - \varphi_t^2}{\varkappa^2}}$$

Covariant consideration.

$$\ddot{x}^\mu - x''^\mu = 0, \quad \mu = 0, 1, \ldots, D-1$$ eqs. mot.

$$(\dot{x} \pm x')^2 = 0$$ gauge con.

$$x'^\mu(\tau, 0) = x'^\mu(\tau, \pi) = 0$$ boundary conditions

$$x_\mu(\tau, \sigma) = \frac{i}{\sqrt{\pi\gamma}} \sum_{n \neq 0} e^{-in\tau} \frac{\alpha_{n\mu}}{n} \cos(n\sigma) + Q_\mu + P_\mu \frac{\tau}{\pi\gamma}$$

$$\alpha_{n\mu} = \alpha^*_{-n\mu}, \quad n \neq 0$$

$$P_\mu = \gamma \int_0^\pi \dot{x}_\mu(\tau, \sigma)\, d\sigma$$

$$Q_\mu = \frac{1}{\pi} \int_0^\pi x_\mu(0, \sigma)\, d\sigma$$

$$(\dot{x} \pm x')^2 = -\frac{2}{\pi\gamma} \sum_{n=-\infty}^{+\infty} e^{-in(\tau \pm \sigma)} L_n = 0,$$

where

$$L_n = -\frac{1}{2} \sum_{m=-\infty}^{+\infty} \alpha^\mu_{n-m} \alpha_{m\mu} = 0,$$

$$n = 0, \pm 1, \pm 2,$$

$$L^*_n = L_{-n}$$

Spin of the string

If $D \neq 4$ then spin is defined by the formula

$$J^2 = \frac{W}{M^2}, \quad W = \frac{1}{2} M_{\mu\nu} M^{\mu\nu} M^2 - (M_{\mu\sigma} P^\sigma)^2$$

where

$$M_{\mu\nu} = Q_\mu P_\nu - Q_\nu P_\mu - \frac{i}{2} \sum_{n \neq 0} \frac{1}{n} (\alpha_{-n\mu} \alpha_{n\nu} - \alpha_{-n\nu} \alpha_{n\mu})$$

If we put $\tau \sim x^0$ then $\alpha_n^0 = 0$, $n \neq 0$

$$J^2 = \sum_{n,m} \frac{1}{nm} \left\{ |(\vec{\alpha}_n^* \vec{\alpha}_m)|^2 - |(\vec{\alpha}_n \vec{\alpha}_m)|^2 \right\} \Rightarrow$$

$$\rightarrow J^2 \leq \sum_{n,m} (\vec{\alpha}_m^* \vec{\alpha}_m)(\vec{\alpha}_n^* \alpha_n) = \frac{M^2}{(2\pi\gamma)^2}$$

$$M^2 = P_\mu^2 = -\pi\gamma \sum_{m \neq 0} \alpha_{m\mu} \alpha_{-m}^\mu \rightarrow$$

$$\rightarrow 2\pi\gamma \sum_{n>0} \vec{\alpha}_n^* \vec{\alpha}_n .$$

$$J \leq \alpha' M^2, \quad \alpha' = (2\pi\gamma)^{-1}$$

Noncovariant consideration

After imposing the orthonormal gauge conditions

$$(\dot{x} \pm x')^2 = 0$$

we have still the residual gauge freedom

$$\bar{\tau} \pm \bar{\sigma} = f_{\pm}(\tau \pm \sigma)$$

$$u^{\pm} = \tau \pm \sigma$$

$$\ddot{x}^{\mu} - x''^{\mu} = 0 \qquad \frac{\partial^2 x^{\mu}}{\partial u^{+} \partial u^{-}} = 0$$

$$(\dot{x} \pm x')^2 = \left(\frac{\partial x}{\partial u^{\pm}}\right)^2 = 0$$

All this enables one to introduce the lightcone gauge conditions

$$n_{\mu} x^{\mu} = \frac{nP}{\gamma \pi} \tau + nQ,$$

where n_{μ} is a constant isotropic vector

$$n^2 = 0$$

$$n^{\mu} = \{1, 1, 0, 0, \ldots, 0\}$$

The light cone variables

$$x^\mu = \{x^+, x^-, \vec{x}_\perp\}, \quad x^\pm = \frac{1}{\sqrt{2}}(x^0 \pm x^1)$$

$$(xy) = x^\mu y_\mu = x^+ y^- + x^- y^+ - \vec{x}_\perp \vec{y}_\perp$$

$$x^2 = 2x^+ y^- - \vec{x}_\perp^{\,2}$$

After imposing the light cone gauge we can treat the transverse components $\vec{x}_\perp(\tau, \sigma)$ as independent variables and $x^\pm(\tau,\sigma)$ components as dependent ones

$$\dot{x}^+ = \pi\gamma(\dot{\vec{x}}_\perp^{\,2} + \vec{x}_\perp'^{\,2})/2P^-, \quad x'^+ = \pi\gamma\dot{\vec{x}}_\perp \vec{x}'/P^-,$$

$$\dot{x}^- = P^-/\pi\gamma, \quad x'^- = 0.$$

The same seperation can be made in terms of the Fourier amplitudes

$$\alpha_n^+ = \frac{\sqrt{\pi\gamma}}{P^-} L_{n\perp}, \quad n = 0, \pm 1, \pm 2, \ldots, \quad \alpha_k^- = 0, \ k \neq 0,$$

where

$$L_{n\perp} = \frac{1}{2} \sum_{m=-\infty}^{+\infty} \sum_{i=2}^{D-1} \alpha_{n-m}^i \alpha_m^i, \quad \alpha_0^i = \frac{P^i}{\sqrt{\pi\gamma}}, \quad \alpha_0^\pm = \frac{P^\pm}{\sqrt{\pi\gamma}},$$

$$i = 2, \ldots, D-2$$

These simple formulas are obtained only due to the light cone gauge with $\underline{n^2 = 0}$. !

Equation

$$\alpha_0^+ = \frac{\sqrt{\pi\gamma}}{p^-} L_{0\perp}$$

gives the string mass squared

$$M^2 = P^2 = 2P^+P^- - \vec{P}_\perp^{\,2} = \pi\gamma \sum_{n\neq 0} \sum_{i=2}^{D-1} \alpha_{-n}^i \alpha_n^i$$

M^2 is positive definite at the classical level.

Lecture 2.

Hamiltonian description and quantization

$$P_\mu(\tau,\sigma) = -\frac{\partial \mathcal{L}}{\partial \dot{x}^\mu} = \gamma \frac{(\dot{x}x')x'_\mu - x'^2 \dot{x}_\mu}{\sqrt{(\dot{x}x')^2 - \dot{x}^2 x'^2}}$$

There is two primary first class constraints

$$\left.\begin{aligned} \varphi_1 &= \gamma^2 x'^2 + p^2 \approx 0, \\ \varphi_2 &= x'p \approx 0 \end{aligned}\right\} (\gamma x'^\mu \pm p^\mu)^2 \approx 0$$

$$\mathcal{H}_c = -\dot{x}_\mu p^\mu - \mathcal{L} \equiv 0$$

Dynamics in phase space is determined by H_T

$$H_T = \int_0^\pi d\sigma\, \mathcal{H}_T = \int_0^\pi d\sigma \left[\lambda_1(\tau,\sigma)\varphi_1(\tau,\sigma) + \lambda_2(\tau,\sigma)\varphi_2(\tau,\sigma)\right]$$