preamble

Black holes in modern gravity theories

7 сентября 2010 г.

Intro

History of 'Black holes'

1. Black body
1783 — Light can't escape Big object with normal density.
2. Some strange solution in GR
1915-1933 — Schwarzschild surface.
3. Black hole
1958-1974 — The term 'Black hole', new solutions (rotating, charged), black hole mechanics.
4. Nowadays
Resolving paradoxes (thermodynamical interpretation, information paradox, initial singularity...)

Experimental evidence

1. Middle-size BH
Matter accretion, Binary systems.
Several candidates.
2. Small BH
Hawking radiation. Evaporation of primordial BHs.
Yet no results.
3. Large BH
Stellar orbits near the center of our galaxy. Large dark object is found.

BH thermodynamics

0 — Surface gravity is constant on the event horizon (temperature).
1 — Dynamics:

$$\delta M = \frac{\kappa_S}{8\pi}\,\delta A + \omega\,\delta J + \phi\,\delta q,$$

2 — Evolution: Area of event horizon can't decrease (entropy).
3 — Surface gravity can't reach zero
Yet it is always zero for extremal BHs.
(temperature)

BH thermodynamics

Entropy-area correspondence

$$\mathcal{S} = A/(4\pi).$$

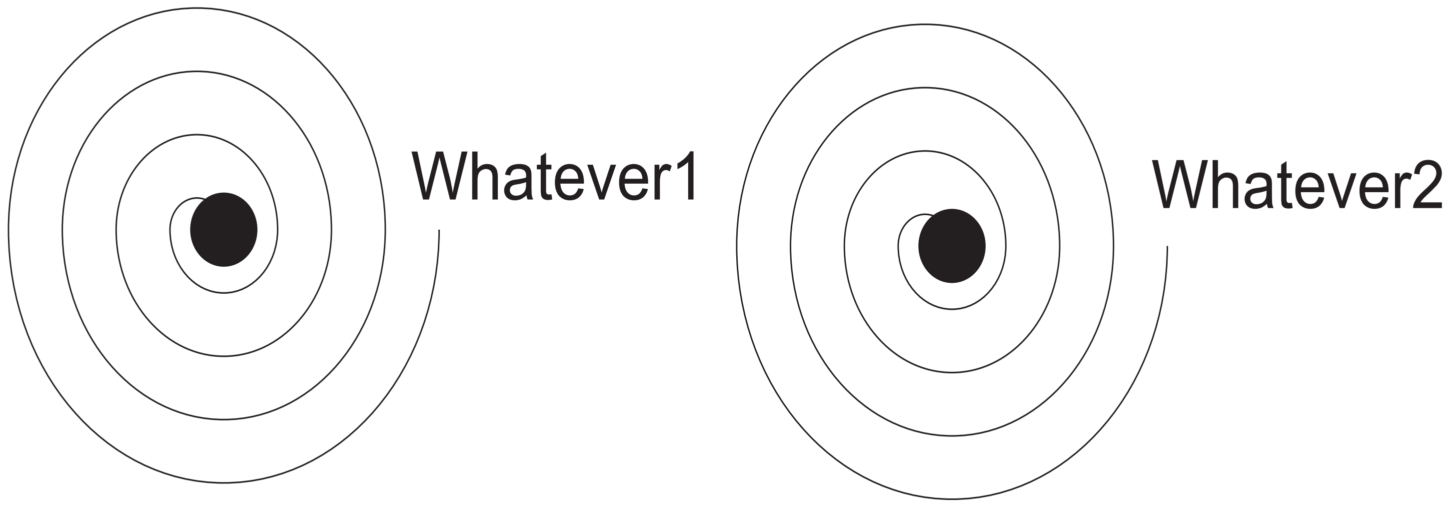
In case of radiation:
Area+Radiation entropy can't decrease.
Surface gravity-temperature correspondence

$$\theta = \kappa_S/(2\pi).$$

Thermodynamical systems have statistical interpretation.
What is the statistical entropy of the event horizon?

Information paradox

During accretion:



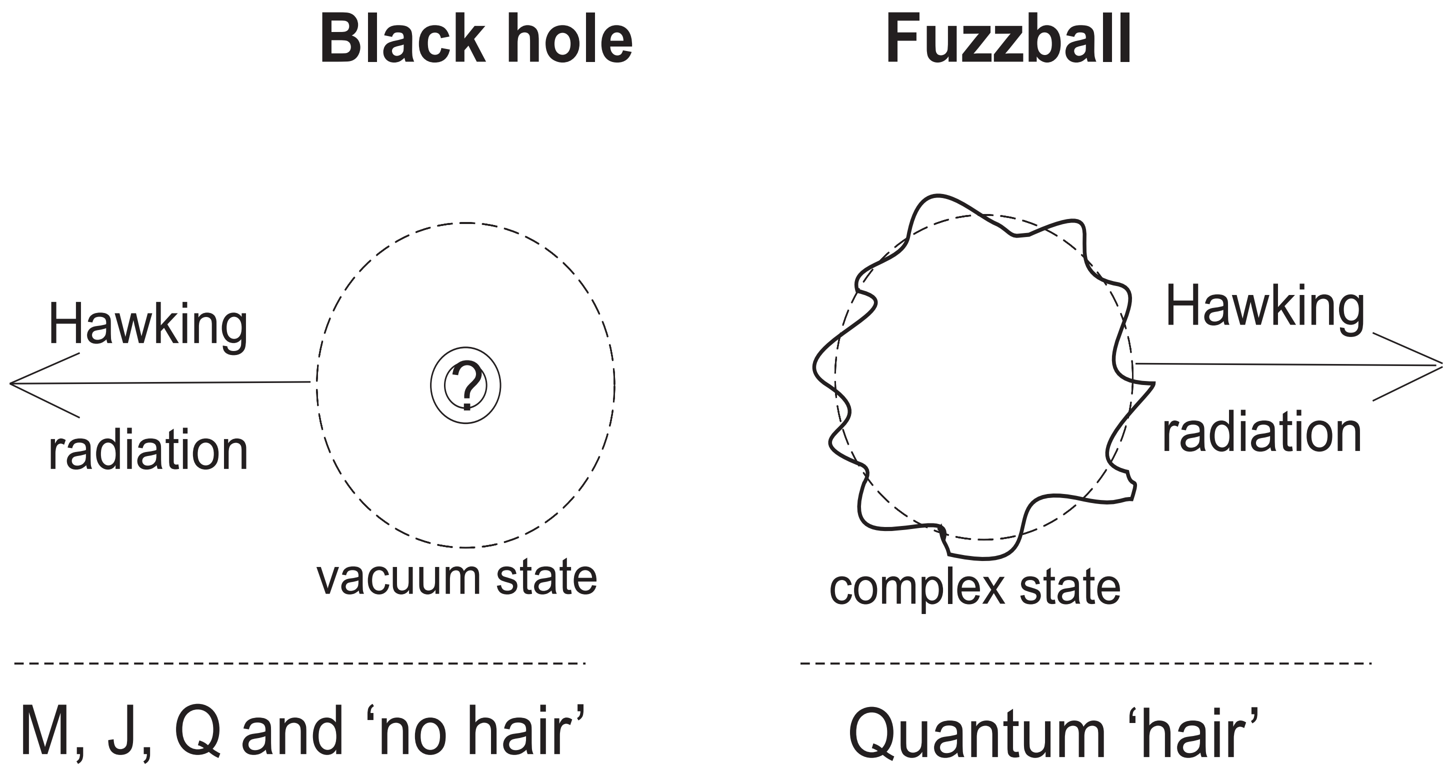
Distant observer: M,J,Q coincide
Information is lost (Whatever1-Whatever2)

Fuzzball paradigm

History

1931 — Chandrasekhar: plasma collapses to BH.
Others: something should stop the collapse.
White dwarf collapses to a neutron star.
1939 — Oppenheimer: neutron star collapses to BH.
Can something else stop the collapse?
2002 — Mathur, Lunin: collapse stops exactly at the event horizon.
Stringy fuzzball.

Fuzzball


Black hole
Fuzzball
Hawking
radiation
?
vacuum state
Hawking
radiation
complex state
M, J, Q and 'no hair'
Quantum 'hair'

Advantages

Solves information paradox,
solves singularity paradox.
Small density of large BHs:

$$R_{Sch} \sim M,\ V \sim R_{Sch}^3,\ \rho = M/V \sim R_{Sch}^{-2}.$$

Supermassive BHs have density of water or air!
During matter accretion strings recombine into very looooong strings. Their tension (and density) decreases exactly as for classical BHs!

Open questions

Collapse into BH happens at the scale

$$t_{cross} = R_{Sch}/c.$$

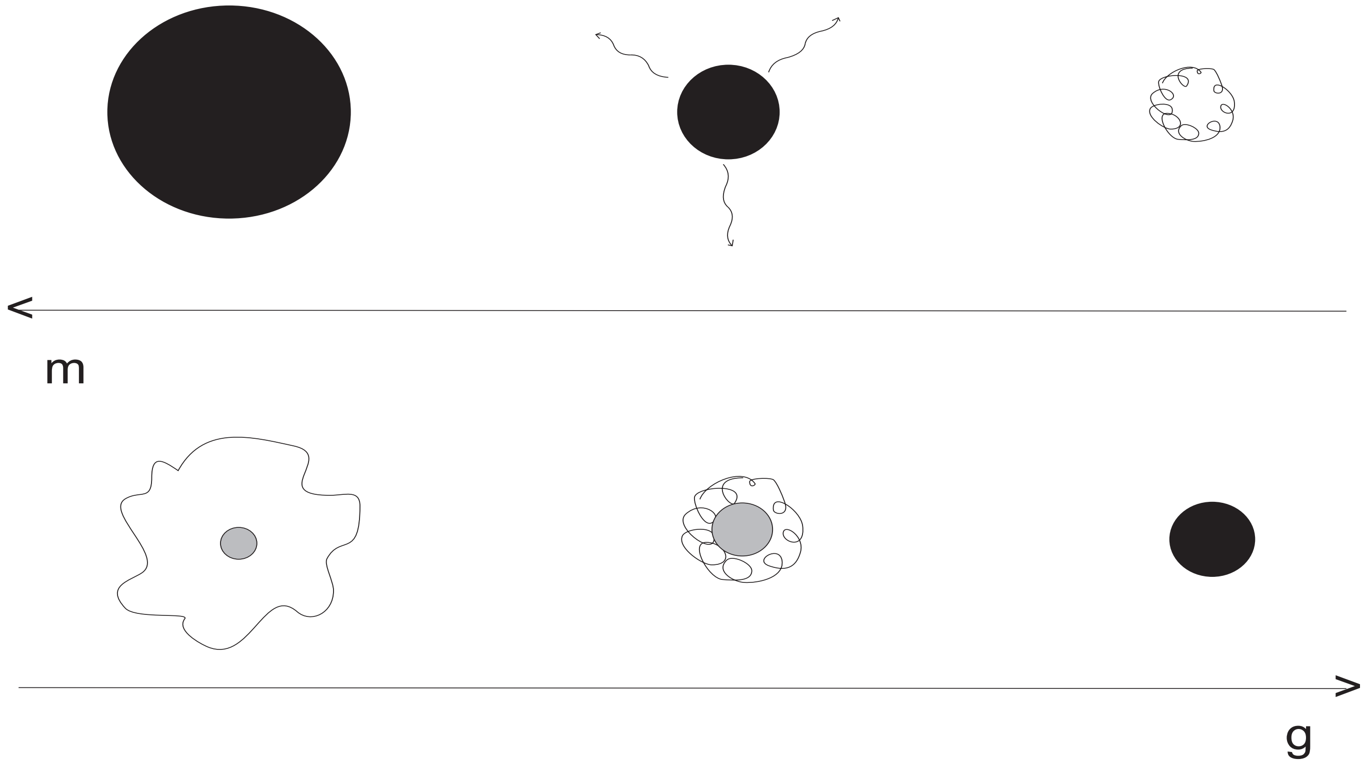
Formation of the fuzzball happens at the scale

$$t_{evap} = t_{cross}(M/m_{pl})^2 \gg t_{cross}.$$

How can they do it?

String—Black hole correspondence

BH evaporation



Beautiful coincidences
Resolves 'singularity evaporation'.
Stores information in the stringy state.

Effective bosonic action for the heterotic string:

$$\frac{1}{16\pi G^{(10)}}\int d^{10}x\sqrt{-g^{(10)}}e^{-2\Phi}\left(R+4(\partial\Phi)^2-\frac{\mathbb{H}^2}{12}\right),$$

where $\mathbb{H}=d\mathbb{B}$ is a field strength of the NS 2-form gauge potential $\mathbb{B}$. The ansatz for the compactification on $S^1\times T^5$ reads:

$$\begin{aligned} ds_{10}^2 &= ds^2+e^{2\lambda}(dx^4+A_\mu dx^\mu)+e^{2\nu}d\ell^2(T^5), \\ 2\Phi &= 2\phi+\lambda+5\nu, \qquad \mathbb{B}=B_\mu dx^\mu\wedge dx^4. \end{aligned}$$

Classical action

Dilatonic black hole can be described by the action

$$\mathcal{S} = \frac{1}{16\pi G}\int \left(R + S^{-2}(\partial S)^2 - S^2F^2\right)\ d^4x\sqrt{-g}$$

with the dilatonic exponent $S = e^{-2\phi}$ from the string theory and with the metrics

$$ds^2 = wdt^2 - w^{-1}dr + \rho^2 d\Omega^2.$$

Classical solution

The well-known Gibbons–Maeda solution can be written as

$$\rho = \sqrt{r^2 - D^2}, \; S = \frac{Q(r+D)}{P(r-D)},$$

$$w = \frac{(r-M)^2 - (M^2 + D^2 - Q^2 - P^2)}{\rho^2},$$

It has two horizons and singular dilaton.
Extremal limit: $S = Q/P \Rightarrow$ constant dilaton and Reissner–Nordström solution. Non-extremal BH: diverging dilaton (no-hair theorem).

Semi-classical action

Four-dimensional action with stringy corrections:

$$\mathcal{L} \sim S\left(R + S^{-2}(\partial S)^2 - F^2\right)\sqrt{-g} + \Delta\mathcal{L}\sqrt{-g},$$

where the correction term is second-order by curvature:

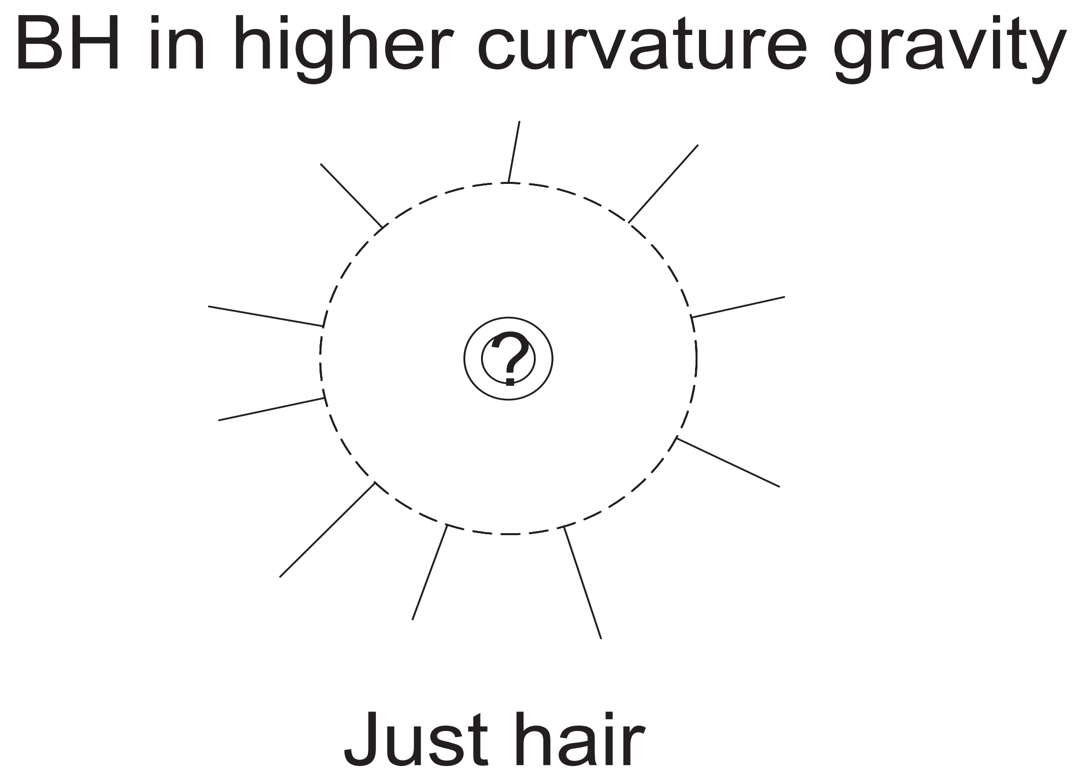
$$\Delta\mathcal{L} = \frac{\alpha}{16\pi}\psi(S)\,L_{GB},$$

where

$$L_{GB} = R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\alpha\beta\mu\nu}R^{\alpha\beta\mu\nu}.$$

Hair

In the extremal limit the dilaton is not diverging and not vanishes: dilatonic hair.



Different corrections

What is the correction function $\psi(S)$ here?

- From the S-duality symmetry:

$$\psi(S) = -\frac{3}{\pi}\ln(2S|\eta(iS)|^4)$$

with the Dedekind $\eta-$function:

$$\eta(\tau) \equiv e^{2\pi i\tau/24}\prod_{n=1}^{\infty}(1 - e^{2\pi i n\tau}).$$

- Simple choice $\psi(S) = S$.

Decouple Maxwell field

The Maxwell field is given by

$$A = -f(r)dt - m\cos\theta d\varphi$$

with the only function f easily obtained from the equations of motion:

$$f' = \frac{g}{\rho^2 S}.$$

Here g and m are charges 'on horizon' and the real value of the electric charge depends on the dilatonic asymptotic.

Find symmetries

The shift of the dilaton (only the EMD part)

$$S \to \beta S \quad w \to \beta^4 w, \quad \rho \to \frac{\rho}{\beta}, \quad r \to \beta r,$$

$$g \to g, \quad m \to \frac{m}{\beta}.$$

The charge rescaling

$$g \to \gamma g, \quad m \to \gamma m, \quad w \to \frac{w}{\gamma^2},$$

$$\rho \to \gamma \rho, \quad \alpha \to \gamma^2 \alpha.$$

Formulate ICs

Looking for the extremal black hole solutions:

$$w(r_0) = w'(r_0) = 0, \qquad \rho(r_0) = \rho_0 > 0,$$

the asymptotic of the metrics must be of the flat space

$$w(r) = const, \quad \rho'(r) = const \quad \text{as} \quad r \to \infty.$$

Non-singular series expansion on horizon:

$$w = \sum_{n=2}^{\infty} w_n x^n, \ \rho = \sum_{n=0}^{\infty} \rho_n x^n, S = \sum_{n=0}^{\infty} S_n x^n.$$

Find asymptotics

The $AdS_2 \times S^2$ metrics on horizon:

$$ds_H^2 = -w_2 x^2 dt^2 + \frac{dx^2}{w_2 x^2} + w_2^2 d\Omega_2^2.$$

The flat asymptotic (Einstein frame):

$$w_E = 1 - \frac{2M}{\hat{r}} + O(\hat{r}^{-2}),$$

$$S = S_\infty + \frac{2S_\infty D}{\hat{r}} + O(\hat{r}^{-2}).$$

Find entropy

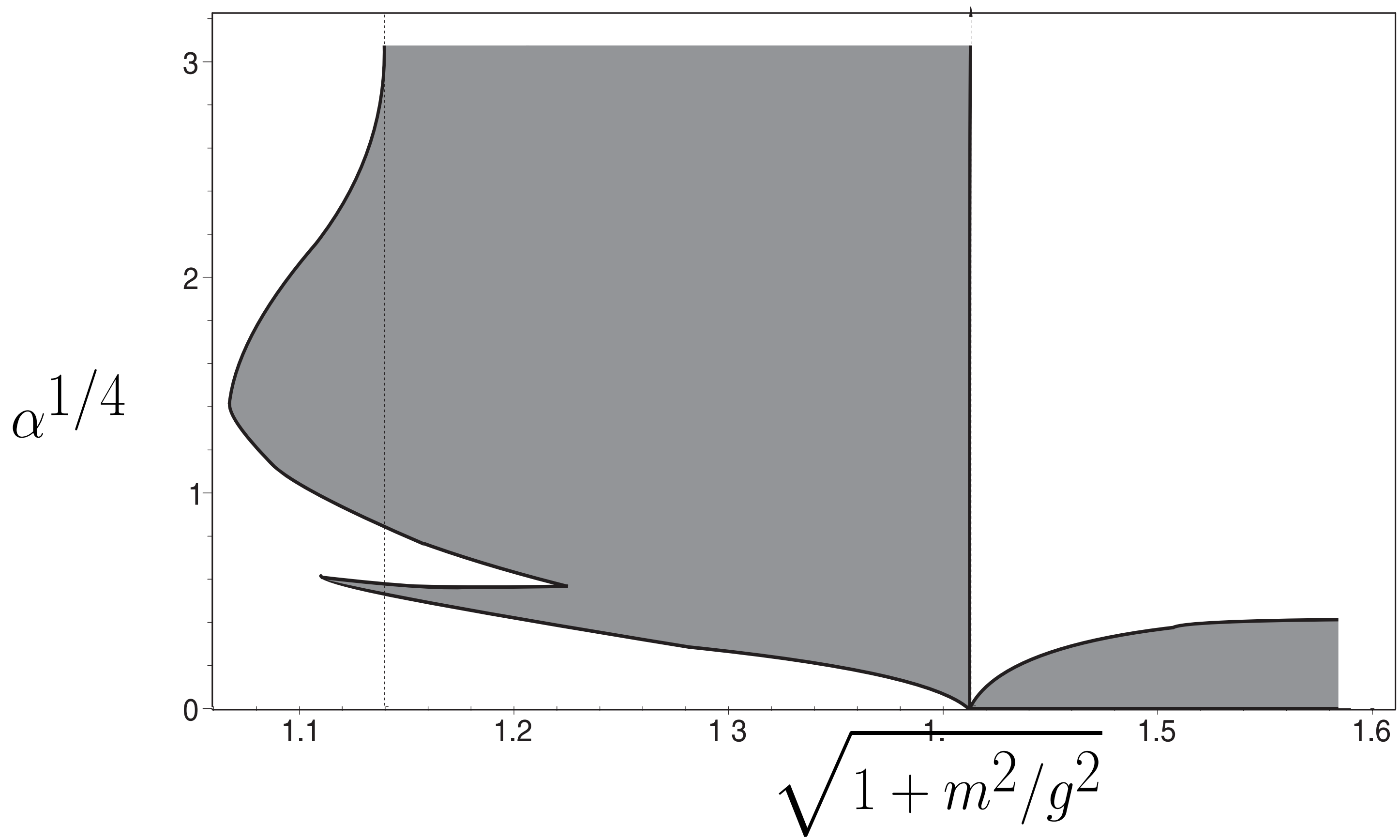
For the metrics exactly of $AdS_2 \times S^2$ type write the Sen's 'entropy function':

$$f(g, m, S_0) \equiv \int d\theta d\varphi \sqrt{-g}\mathcal{L}.$$

The next step is a Legendre transformation of $f(g, m, S_0)$ to $F(\partial_g f, \partial_m f, \partial_S f)$.
Entropy is the extremal value of F:

$$\mathrm{S} = \pi\rho_E^2 + 4\pi\alpha S_0.$$

Obtain results



Growing string corrections produce the BH, or
Evaporating BH will produce free string.

Not a Black hole

Motivation

'Naked' singularities are unpopular — at least shielded by horizon. Appears for classical BHs:

$$Q^2 + (J/M)^2 \leq M^2.$$

Modifications of the EH theory are popular. But what is a corrected gravity?

- □ Corrected gravity = First order of corrections
- □ Singular solutions => Smoothed by 'full theory' (quantum gravity, string theory).

GB Cusp

Gauss–Bonnet gravity in 4D:

$$(\text{EH action}) + (\text{GB term}) * \text{dilaton}$$

Why GB-corrections?

- □ Simple R^2 correction
- □ Comes from the string theory

Cusp is:

- □ Finite non-vanishing metric components
- □ Diverging second derivatives of metrics

Different corrections

Simple R^2 correction (EDGB):

$$\mathcal{L}^{(E)} = \left(R - \frac{(\partial_\mu \ln S)^2}{2a^2} + \alpha S \mathcal{R}^2_{GB} \right) \sqrt{-g}.$$

String-theory variant (SEDGB):

$$\mathcal{L}^{(str)} = \left(R + (\partial_\mu \ln S)^2 + \alpha \mathcal{R}^2_{GB} \right) S \sqrt{-g}.$$

Dilaton comes as $S = e^{2a\phi}$, $a = 1$ in string action.

What is the difference?

From SEDGB to EDGB (with $a = 1$): conformal transformation

$$g_{\mu\nu}^{(str)} = S^{-1} g_{\mu\nu}^{(E)}$$

leads to

$$\Delta\mathcal{S}_{GB}^{(E)} = \frac{1}{16\pi} \int \sum_{n=2}^{4} \Lambda_n (\ln S)'^n . \sqrt{-g}\, d^4x.$$

When $\mathcal{R}_{GB}^2$-correction is not small, $\Delta\mathcal{S}_{GB}$ is not small too!

History

EH system without GB-correction:

- □ Schwarzschild BH with constant dilaton.

EDGB system: P. Kanti et al, S.O. Alexeyev and M.V. Pomazanov

- □ BH-solution with inner $x^{1/2}$ singularity ($x = |r - r_s|$);
- □ Naked $x^{1/2}$ singularity.

SEDGB system: K. Maeda et al.

Formulate ICs

For spherically symmetric metrics

$$ds^2 = -w(r)\sigma(r)^2 dt^2 + \frac{dr^2}{w(r)} + \rho(r)^2 d\Omega_2^2,$$

in the gauge $\sigma = 1$ the cusp ansatz will be

$$w = \sum_{n=0}^{\infty} w_{n/z} x^{n/z}, \ \rho = \sum_{n=0}^{\infty} \rho_{n/z} x^{n/z},$$

$$S = \sum_{n=0}^{\infty} p_{n/z} x^{n/z}.$$

Various cusps

$x^{1/2}$ case:

- □ From cusp to Minkowski asymptotic;
- □ From cusp at x_1 to cusp at x_2.

$x^{1/3}$ case:

- □ From cusp at x_1 to cusp at x_2. Minkowski transition area $w \sim$ const, $\rho \sim x$, $S \sim x$.