Axial Anomaly and chiral (magnetic and vortical) effects

Helmholtz International School
Lattice QCD, Hadron Structure and Hadronic Matter

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Outline

Lecture 1 (on the blackboard)

Physical interpretation

Anomaly as Landau levels flow

Triangle diagram

Lecture 2

Anomaly and hadronic spectrum: t'Hooft principle

Abelian vs non-Abelian Anomalies

Chiral Magnetic Effect

Chiral Vortical Effect

Observation of chiral effects

Symmetries and conserved operators

- (Global) Symmetry -> conserved current ($\partial^\mu J_\mu = 0$)
- Exact:
- U(1) symmetry – charge conservation - electromagnetic (vector) current
- Translational symmetry – energy momentum tensor $\partial^\mu T_{\mu\nu} = 0$

Massless fermions (quarks) – approximate symmetries

- Chiral symmetry (mass flips the helicity)

$$\partial^{\mu} J^{5}{}_{\mu} = 0$$

- Dilatational invariance (mass introduce dimensional scale – c.f. energy-momentum tensor of electromagnetic radiation)

$$T_{\mu\mu} = 0$$

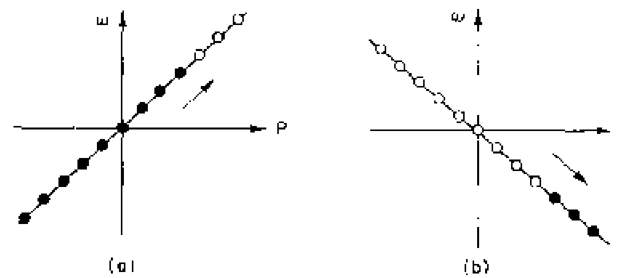
Quantum theory

- Currents -> operators
- Not all the classical symmetries can be preserved -> anomalies
- Enter in pairs (triples?...)
- Vector current conservation <-> chiral invariance
- Translational invariance <-> dilatational invariance

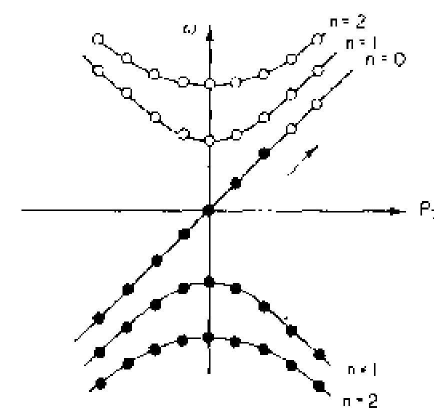
Calculation of anomalies

- Many various ways
- All lead to the same operator equation

$$\partial^\mu j^{(0)}_{5\mu} = 2i\sum_q m_q \bar{q}\gamma_5 q - \left(\frac{N_f\alpha_s}{4\pi}\right) G^a_{\mu\nu}\tilde{G}^{\mu\nu,a}$$



- UV vs IR languages- understood in physical picture (Gribov, Feynman, Nielsen and Ninomiya) of Landau levels flow (E||H)



Anomaly and virtual photons

- Often assumed that only manifested in real photon amplitudes
- Not true – appears at any Q^2
- Natural way – dispersive approach to anomaly (Dolgov, Zakharov'70) - anomaly sum rules
- One real and one virtual photon – Horejsi,OT'95
- where

$$\int_{4m^2}^{\infty} A_3(t;q^2,m^2)dt = \frac{1}{2\pi}$$

$$F_j(p^2) = \frac{1}{\pi}\int_{4m^2}^{\infty}\frac{A_j(t)}{t-p^2}dt, \qquad j=3,4$$

$$\begin{aligned} T_{\alpha\mu\nu}(k,q) &= F_1\,\varepsilon_{\alpha\mu\nu\rho}k^\rho + F_2\,\varepsilon_{\alpha\mu\nu\rho}q^\rho \\ &+ F_3\,q_\nu\varepsilon_{\alpha\mu\rho\sigma}k^\rho q^\sigma + F_4\,q_\nu\varepsilon_{\alpha\mu\rho\sigma}k^\rho q^\sigma \\ &+ F_5\,k_\mu\varepsilon_{\alpha\nu\rho\sigma}k^\rho q^\sigma + F_6\,q_\mu\varepsilon_{\alpha\nu\rho\sigma}k^\rho q^\sigma \end{aligned}$$

Dispersive derivation

- Axial WI $F_2 - F_1 = 2mG + \frac{1}{2\pi^2}$
- GI $F_2 - F_1 = (q^2 - p^2)F_3 - q^2 F_4$
- No anomaly for imaginary parts

$$(q^2 - t)A_3(t) - q^2 A_4(t) = 2mB(t) \qquad F_j(p^2) = \frac{1}{\pi}\int_{4m^2}^{\infty} \frac{A_j(t)}{t - p^2}dt, \qquad j = 3, 4$$

- Anomaly as a finite subtraction

$$F_2 - F_1 - 2mG = \frac{1}{\pi}\int_{4m^2}^{\infty} A_3(t)dt \qquad \int_{4m^2}^{\infty} A_3(t; q^2, m^2)dt = \frac{1}{2\pi}$$

Properties of anomaly sum rules

- Valid for any Q^2 (and quark mass)
- No perturbative QCD corrections (Adler-Bardeen theorem)
- No non-perturbative QCD correctioons (t'Hooft consistency principle)
- Exact – powerful tool

Mesons contributions (Klopot, Oganesian, OT)

Phys.Lett.B695:130-135,2011 (1009.1120) and 1106.3855

- Pion – saturates sum rule for real photons $ImF_3 = \sqrt{2} f_\pi \pi F_{\pi\gamma\gamma*}(Q^2)\delta(s - m_\pi^2)$ $F_{\pi\gamma^*\gamma}(0) = \frac{1}{2\sqrt{2}\pi^2 f_\pi}$
- For virtual photons – pion contribution is rapidly decreasing $F_{\pi\gamma\gamma^*}^{\mathrm{asymp}}(Q^2) = \frac{\sqrt{2} f_\pi}{Q^2} + \mathcal{O}(1/Q^4)$
- This is also true also for axial and higher spin mesons (longitudianl components are dominant)
- Heavy PS decouple in a chiral limit

Anomaly as a collective effect

- One can never get constant summing finite number of decreasing function
- Anomaly at finite Q^2 is a collective effect of meson spectrum
- General situation –occurs for any scale parameter (playing the role of regulator for massless pole)
- For quantitative analysis – quark-hadron duality

Mesons contributions within quark hadron duality – transition FF (talks of P. Kroll, S. Mikhailov, A. Pimikov)

- Pion: $$F_{\pi\gamma\gamma*}(Q^2) = \frac{1}{2\sqrt{2}\pi^2 f_\pi} \frac{s_0}{s_0 + Q^2}$$
- Cf Brodsky&Lepage, Radyushkin – comes now from anomaly!
- Axial meson contribtion to ASR

$$\int_0^\infty A_3(s;Q^2)ds = \frac{1}{2\pi} = I_\pi + I_{a_1} + I_{cont}. \qquad I_{a_1} = \frac{1}{2\pi} Q^2 \frac{s_1 - s_0}{(s_1 + Q^2)(s_0 + Q^2)}$$

Content of Anomaly Sum Rule ("triple point")

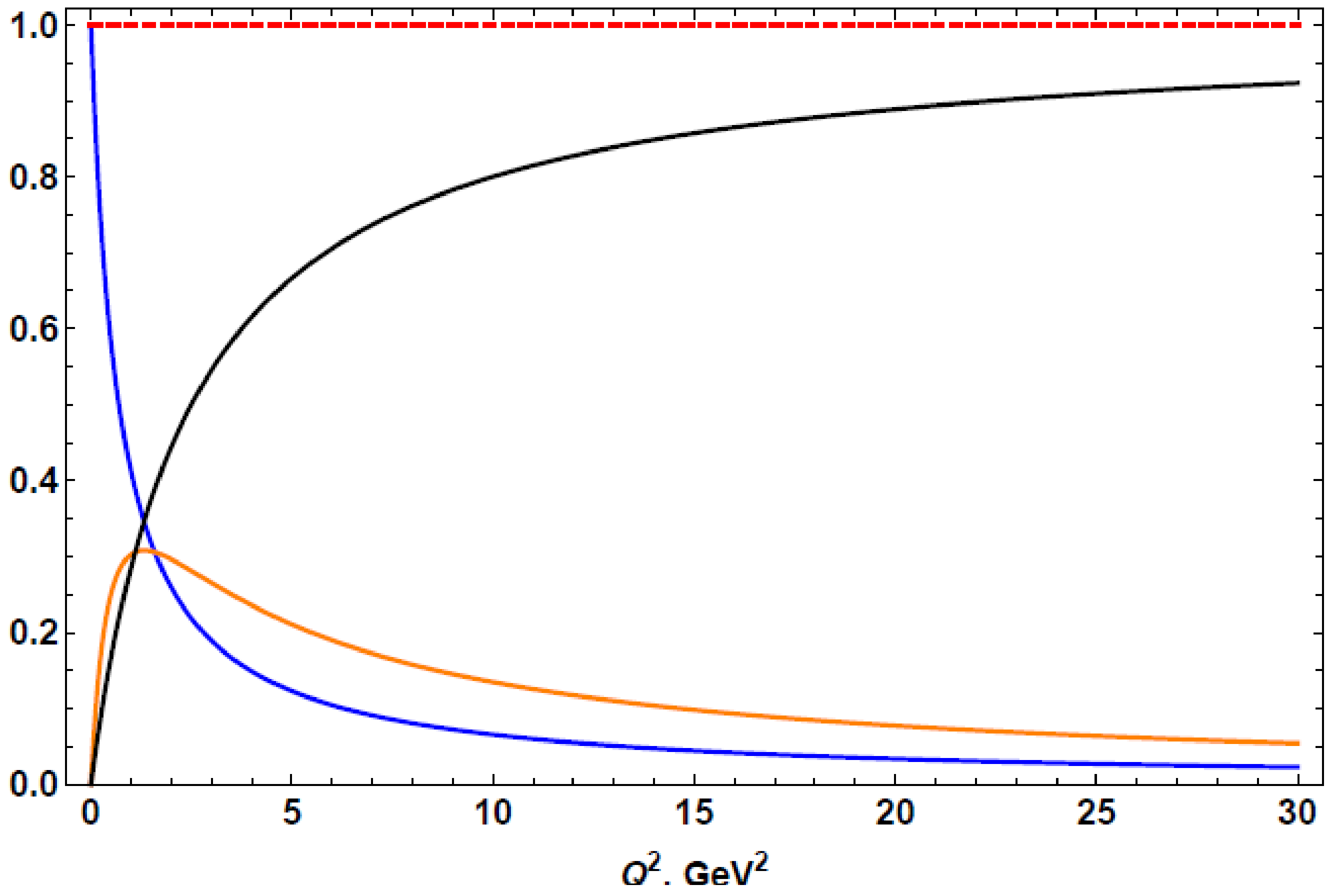


Figure 1: Relative contributions of π (blue line) and a_1 (orange line) mesons, intervals of duality are $s_0 = 0.7\ GeV^2$ and $s_1 - s_0 = 1.8\ GeV^2$ respectively, and continuum (black line), continuum threshold is $s_1 = 2.5\ GeV^2$

ASR and BaBar data

- In the BaBar(2009) region – main contribution comes from the continuum
- Small relative correction to continuum –due to exactness of ASR must be compensated by large relative contributions to lower states!
- Amplification of corrections

$$\frac{\delta I_{cont}/I^0_{cont}}{\delta I_\pi/I^0_\pi} = \frac{s_0}{Q^2} \simeq \frac{1}{30} \quad Q^2 = 20\ GeV^2,\ s_0 = 0.7\ GeV^2$$

- Smaller for eta because of larger duality interval (supported by BaBar)

Corrections to Continuum

- Perturbative – zero at 2 loops level (massive-Pasechnik&OT – however cf Melnikov; massless-Jegerlehner&Tarasov)
- Non-perturbative (e.g. instantons)
- The general properties of ASR require decrease at asymptotically large Q^2 (and $Q^2=0$)
- Corresponds to logarithmic contribution (cf Radyushkin, Polyakov, Dorokhov).

$$I_{cont} = \frac{1}{2\pi}\frac{Q^2}{s_0+Q^2} - cs_0\frac{\ln(Q^2/s_0)+b}{Q^2},$$

$$I_\pi = \frac{1}{2\pi}\frac{s_0}{s_0+Q^2} + cs_0\frac{\ln(Q^2/s_0)+b}{Q^2}.$$

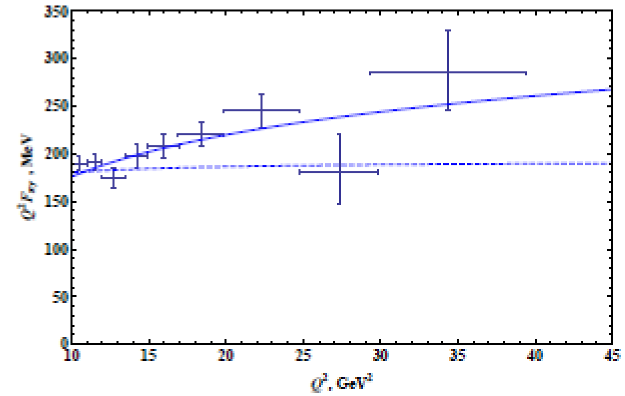
Modelling of corrections

- Continuum vs pion

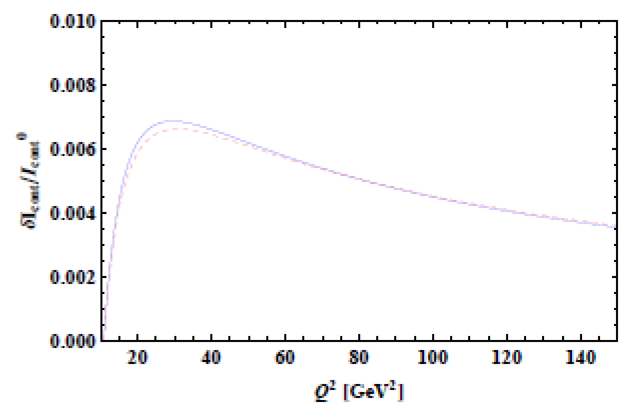
$$I_{cont} = \frac{1}{2\pi}\frac{Q^2}{s_0+Q^2} - cs_0\frac{\ln(Q^2/s_0)+b}{Q^2},$$

$$I_{\pi} = \frac{1}{2\pi}\frac{s_0}{s_0+Q^2} + cs_0\frac{\ln(Q^2/s_0)+b}{Q^2}.$$

- Fit $b = -2.74, \quad c = 0.045.$



- Continuum contribution similar for Radyushkin's approach

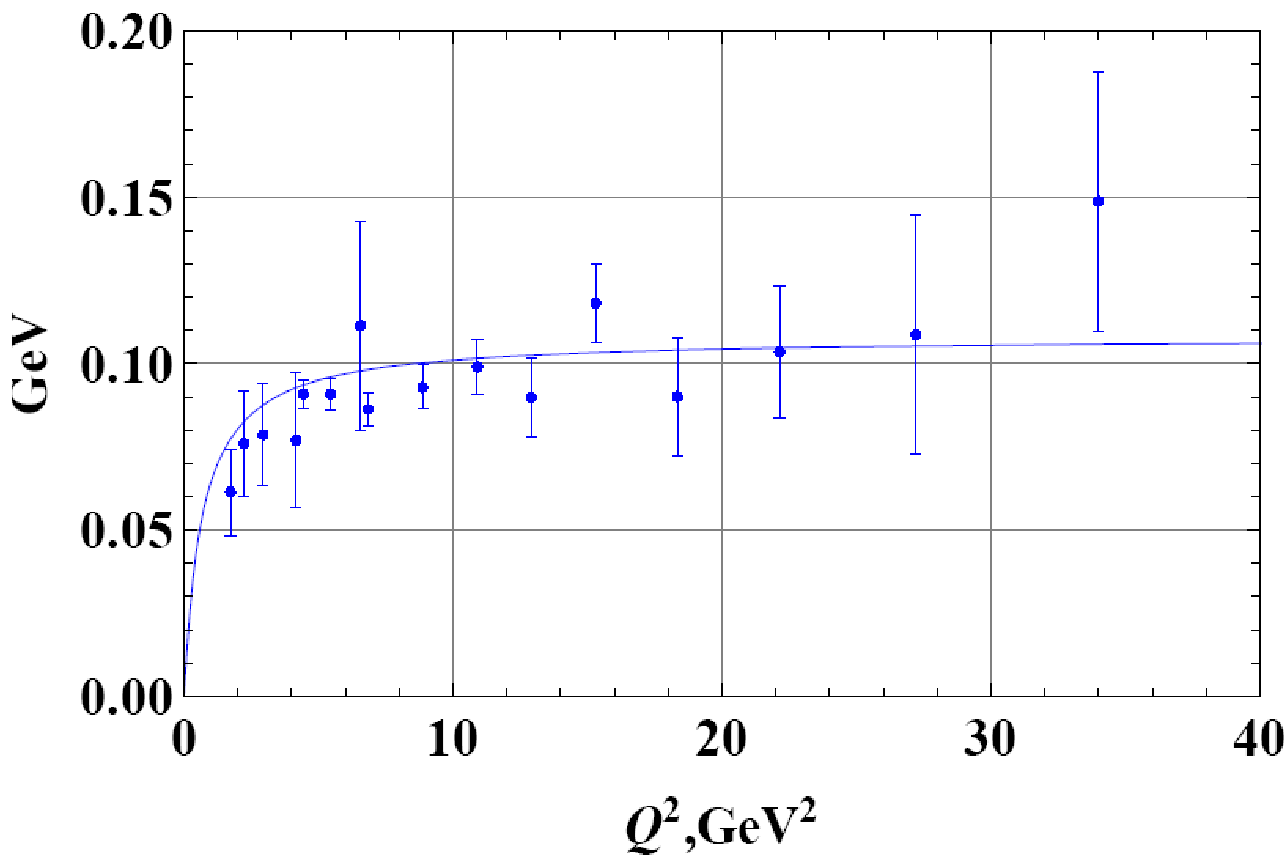


Interplay of pion with lower resonances

- Small (NP) corrections to continuum – interplay of pion with higher states
- A1 – decouples for real photons
- Relation between transition FF's of pion and A1 (testable!)

Generalization for eta(')

- Octet channel sum rule (gluon anomaly free)



Conclusions/Discussion-I

- New manifetsation of Axial Anomaly - Anomaly Sum Rule – exact NPQCD tool- do not require QCD factorization
- Anomaly for virtual photons – collective effect (with fast excitation of collective mode)

- Exactness of ASR – very unusual situation when small pion contribution can be studied on the top of large continuum – amplification of corrections to continuum
- BaBar data – small negative correction to continuum
- If continuum is precisely described by Born term– interplay with A1 (TO BE STUDIED THEORETICALLY AND EXPERIMENTALLY)
- Similar collective effect is expected for finite temperature and/or chemical potential

Anomaly in Heavy Ion Collisions - Chiral Magnetic Effect (D. Kharzeev)

From QCD back to electrodynamics: Maxwell-Chern-Simons theory

$$\mathcal{L}_{\rm MCS} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} - A_\mu J^\mu + \frac{c}{4}\,P_\mu J^\mu_{CS}.$$

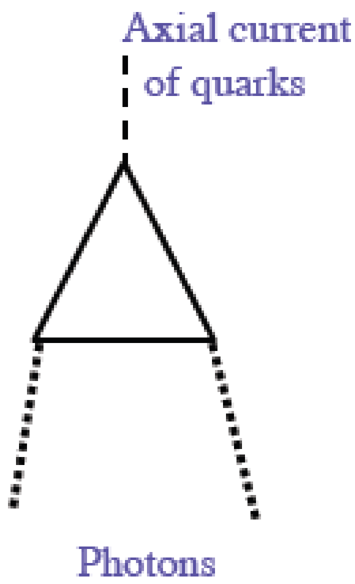
$$J^\mu_{CS} = \epsilon^{\mu\nu\rho\sigma}A_\nu F_{\rho\sigma} \qquad P_\mu = \partial_\mu\theta = (M, \vec{P})$$

$$\vec{\nabla}\times\vec{B} - \frac{\partial\vec{E}}{\partial t} = \vec{J} + c\left(M\vec{B} - \vec{P}\times\vec{E}\right),$$

$$\vec{\nabla}\cdot\vec{E} = \rho + c\vec{P}\cdot\vec{B},$$

$$\vec{\nabla}\times\vec{E} + \frac{\partial\vec{B}}{\partial t} = 0,$$

$$\vec{\nabla}\cdot\vec{B} = 0,$$



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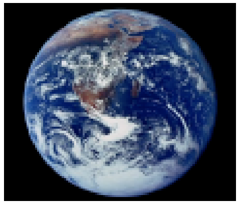
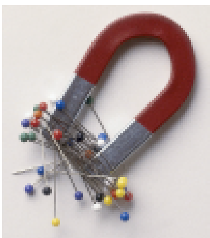
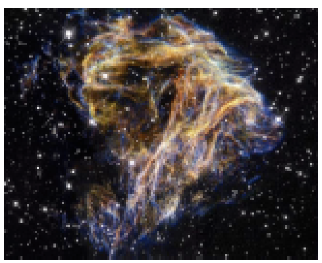
Comparison of magnetic fields

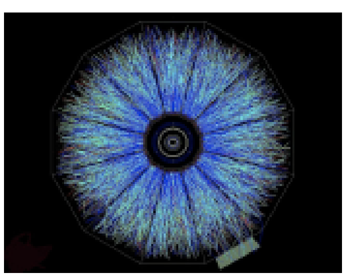






The Earths magnetic field	0.6 Gauss
A common, hand-held magnet	100 Gauss
The strongest steady magnetic fields achieved so far in the laboratory	4.5×10^5 Gauss
The strongest man-made fields ever achieved, if only briefly	10^7 Gauss
Typical surface, polar magnetic fields of radio pulsars	10^{13} Gauss
Surface field of Magnetars	10^{15} Gauss

http://solomon.as.utexas.edu/~duncan/magnetar.html



At BNL we beat them all!

Off central Gold-Gold Collisions at 100 GeV per nucleon

$$e\,B(\tau = 0.2\,\mathrm{fm}) \;=\; 10^3 \sim 10^4\ \mathrm{MeV}^2 \sim 10^{17}\,\mathrm{Gauss}$$

Induced current for (heavy - with respect to magnetic field strength) strange quarks

- Effective Lagrangian

$$L = c(F\tilde{F})(G\tilde{G})/m^4 + d(FF)(GG)/m^4$$

- Current and charge density from c (~7/45) – term $j^\mu = 2c\tilde{F}^{\mu\nu}\partial_\nu(G\tilde{G})/m^4$
- $\rho \sim \vec{H}\vec{\nabla}\theta$ (multiscale medium!)

  $\theta \sim (G\tilde{G})/m^4 \to \int d^4x G\tilde{G}$

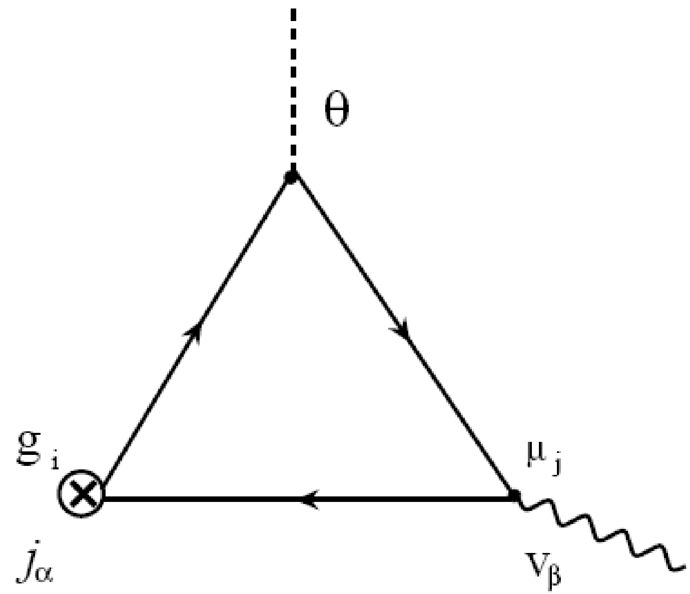
- Light quarks -> matching with D. Kharzeev et al' -> correlation of density of electric charge with a gradient of topological one (Lattice ?)

Properties of perturbative charge separation

- Current carriers are obvious - strange quarks -> matching -> light quarks?
- No relation to topology (also pure QED effect exists)
- Effect for strange quarks is of the same order as for the light ones if topological charge is localized on the distances ~ $1/m_s$, strongly (4^{th} power!) depends on the numerical factor : Ratio of strange/light – sensitive probe of correlation length
- Universality of strange and charm quarks separation - charm separation suppressed as $(m_s /m_c)^4$ ~ 0.0001
- Charm production is also suppressed – relative effects may be comparable at moderate energies (NICA?) – but low statistics

Anomaly in medium – new external lines in VVA graph

- Gauge field -> velocity
- CME -> CV(ortical)E
- Kharzeev, Zhitnitsky (07) – EM current
- Straightforward generalization: any (e.g. baryonic) current – neutron asymmeries@NICA - Rogachevsky, Sorin, OT - **Phys.Rev.C82:054910,2010**.



Baryon charge with neutrons – (Generalized) Chiral Vortical Effect

- Coupling: $e_j A_\alpha J^\alpha \Rightarrow \mu_j V_\alpha J^\alpha$
- Current: $J_e^\gamma = \frac{N_c}{4\pi^2 N_f} \varepsilon^{\gamma\beta\alpha\rho} \partial_\alpha V_\rho \partial_\beta (\theta \sum_j e_j \mu_j)$
- - Uniform chemical potentials: $J_i^\nu = \frac{\sum_j g_{i(j)} \mu_j}{\sum_j e_j \mu_j} J_e^\nu$
- - Rapidly (and similarly) changing chemical potentials:

$$J_i^0 = \frac{|\vec{\nabla} \sum_j g_{i(j)} \mu_j|}{|\vec{\nabla} \sum_j e_j \mu_j|} J_e^0$$

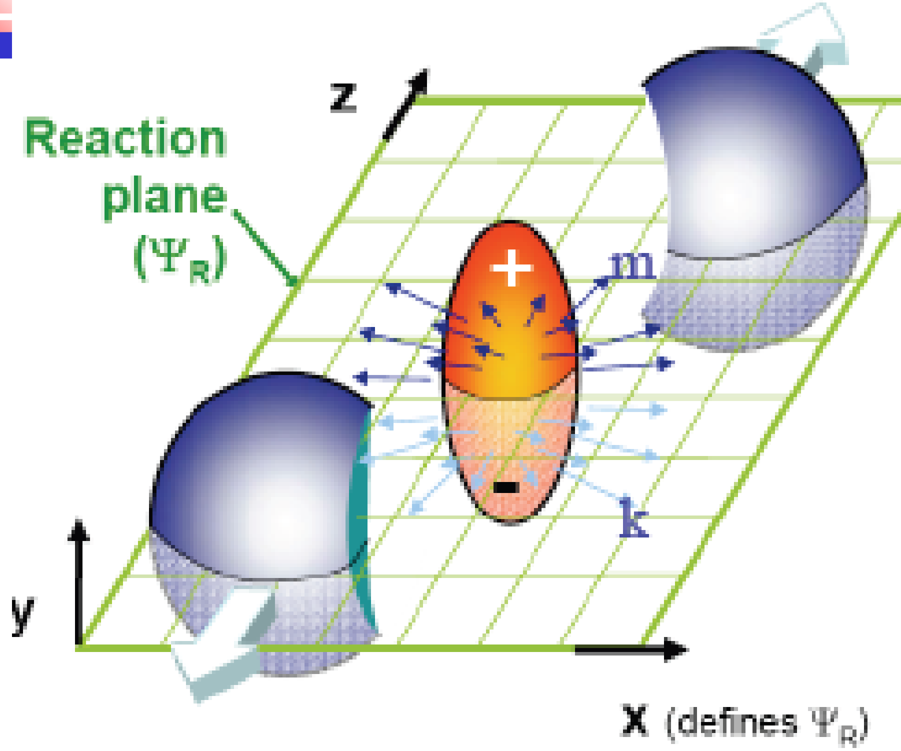
Comparing CME and CVE

- Orbital Angular Momentum and magnetic moment are proportional – Larmor theorem
- CME for 3 flavours – no baryon charge separation (2/3-1/3-1/3=0!) (Kharzeev, Son) - but strange mass!
- Same scale as magnetic field

Observation of chiral effects

- Sign of topological field fluctuations unknown – need quadratic (in induced current) effects
- CME – like-sign and opposite-sign correlations – S. Voloshin
- No antineutrons, but like-sign baryonic charge correlations possible
- Look for neutron pairs correlations!
- MPD@NICA (lecture of A. Sorin) may be well suited for neutrons!

Charge asymmetry w.r.t. reaction plane: how to detect it?



$$\langle \cos(\phi_\alpha + \phi_\beta - 2\Psi_{RP}) \rangle =$$
$$= \langle \cos \Delta\phi_\alpha \cos \Delta\phi_\beta \rangle - \langle \sin \Delta\phi_\alpha \sin \Delta\phi_\beta \rangle$$
$$= [\langle v_{1,\alpha} v_{1,\beta} \rangle + B^{in}] - [\langle a_\alpha a_\beta \rangle + B^{out}].$$

S.Voloshin, hep-ph/0406311

A sensitive measure of the asymmetry:

$$a^k a^m = \langle \sum_{ij} \sin(\varphi_i^k - \Psi_R) \ \sin(\varphi_j^m - \Psi_R) \rangle$$

Expect $a^+ a^+ = a^- a^- > 0; \quad a^+ a^- < 0$

RHIC data for CME

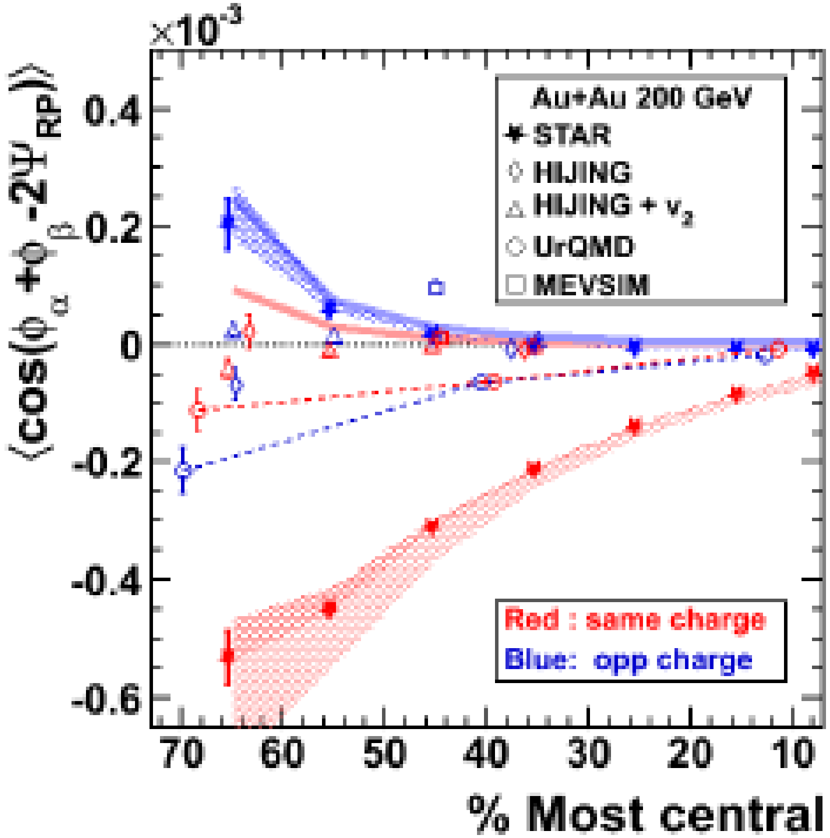
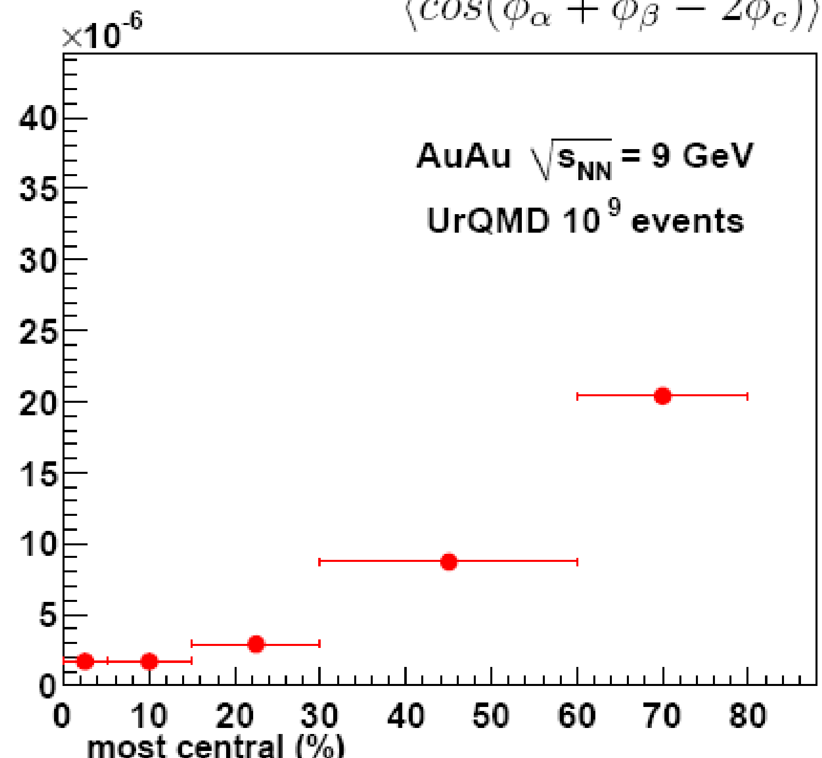


Figure 2. (Taken from [17]) STAR results compared to simulations for 200 GeV Au+Au. Blue symbols mark opposite-charge correlations, and red are same-charge. The shaded bands show the systematic error due to uncertainty in v_2 measurements. In simulations the true reaction plane from the generated event was used. Thick solid lighter colored lines represent non reaction-plane dependent contribution as estimated by HIJING. Corresponding estimates from UrQMD are about factor of two smaller.

Estimates of statistical accuracy at NICA MPD (months of running)

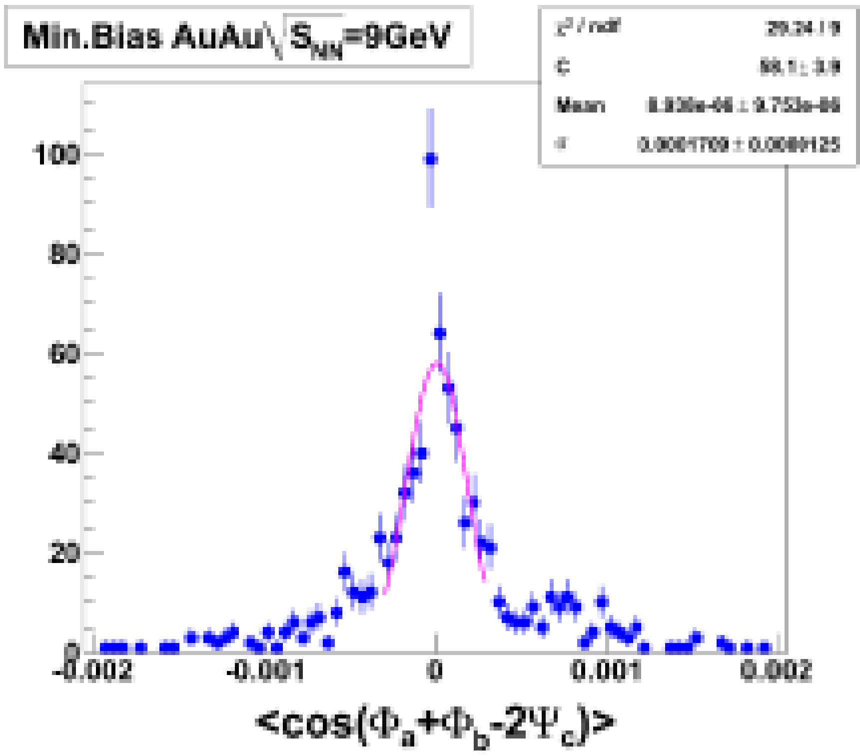
- UrQMD model : $Au + Au$ at $\sqrt{s_{NN}} = 9$ GeV
- 2-particles -> 3-particles correlations
  no necessity to fix the event plane
- 2 neutrons from mid-rapidity $(|\eta| < 1)$
- +1 from ZDC $(|\eta| > 3)$

$\langle cos(\phi_\alpha + \phi_\beta - 2\phi_c) \rangle$



Background effects

- Can correlations be simulated by UrQMD generator?



Other sources of quadratic effects

- Quadratic effect of induced currents – not necessary involve (C)P-violation
- May emerge also as C&P even quantity
- Complementary probes of two-current correlators desirable
- Natural probe – dilepton angular distributions

Observational effects of current correlators in medium

- McLerran Toimela'85 $W^{\mu\nu} = \int d^4x \, e^{-iq\cdot x} \langle J^\mu(x) J^\nu(0) \rangle$
- Dileptons production rate

$$\frac{d(R/V)}{d^4q\, d^3p\, d^3p'} = -\frac{1}{E_p E_{p'}} e^4 \frac{1}{(2\pi)^6} \times \delta^{(4)}(p+p'-q) L^{\mu\nu}(p,p') \times (1/q^4) W_{\mu\nu}(q) \,.$$

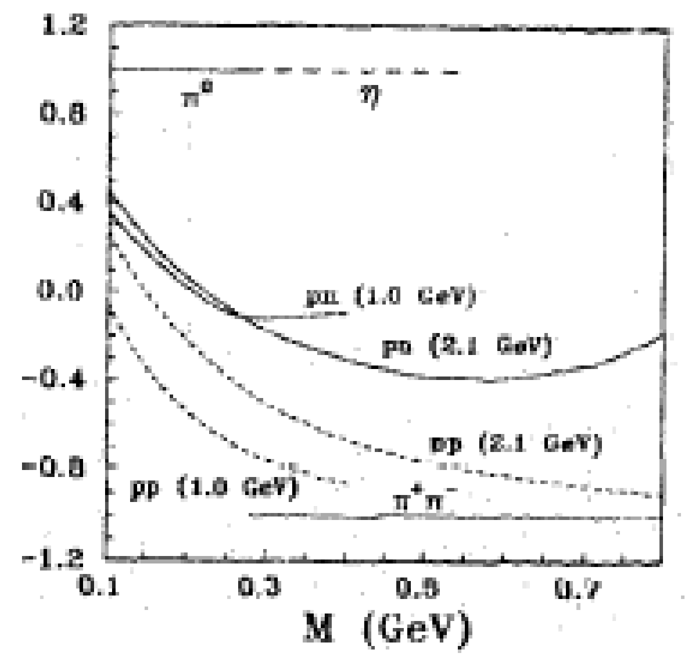
- Structures –similar to DIS F1, F2 (p ->v)

Tensor polarization of in-medium vector mesons (Bratkovskaya, Toneev, OT'95)

- Hadronic in-medium tensor – analogs of spin-averaged structure functions: p -> v
- Only polar angle dependence
- Tests for production mechanisms - recently performed by HADES in Ar+KCl at 1.75 A GeV !

$$W^{\mu\nu} = W_1(q^2, vq)\left(g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2}\right) + W_2(q^2, vq)\left(v^\mu - q^\mu \frac{vq}{q^2}\right)\left(v^\nu - q^\nu \frac{vq}{q^2}\right)$$

$$\frac{d\sigma}{d\cos\theta} \sim 1 + \frac{|v|^2}{2W_1/W_2 + 1 - (vq)^2/q^2}\cos^2\theta$$



General hadronic tensor and dilepton angular distribution

- Angular distribution

$$d\sigma \propto 1 + \lambda \cos^2\theta + \mu \sin 2\theta \cos\phi + \frac{\nu}{2}\sin^2\theta \cos 2\phi + \rho \sin 2\theta \sin\phi + \sigma \sin^2\theta \sin 2\phi$$

- Positivity of the matrix (= hadronic tensor in dilepton rest frame)

$$\begin{pmatrix} \frac{1-\lambda}{2} & \mu & \rho \\ \mu & \frac{1+\lambda+\nu}{2} & \sigma \\ \rho & \sigma & \frac{1+\lambda-\nu}{2} \end{pmatrix}$$

$$|\lambda| \le 1,\ |\nu| \le 1+\lambda,\ \mu^2 \le \frac{(1-\lambda)(1+\lambda-\nu)}{4}$$

$$\rho^2 \le \frac{(1-\lambda)(1+\lambda+\nu)}{4},\ \sigma^2 \le \frac{(1-\lambda)^2-\nu^2}{4}$$

- + cubic – det M> 0
- 1st line – Lam&Tung by SF method

Magnetic field conductivity and asymmetries

- zz-component of conductivity (~hadronic) tensor dominates
- $\lambda = -1$
- Longitudinal polarization with respect to magnetic field axis
- Effects of dilepton motion – work in progress

Other signals of rotation

- Hyperons (in particular, Λ) polarization (self-analyzing in weak decay)
- Searched at RHIC (S. Voloshin et al.) – oriented plane (slow neutrons) - no signal observed
- No tensor polarizations as well

Why rotation is not seen?

- Possible origin – distributed orbital angular momentum and local spin-orbit coupling
- Only small amount of collective OAM is coupled to polarization
- The same should affect lepton polarization
- Global (pions) momenta correlations (handedness)

New sources of Λ polarization coupling to rotation

- Bilinear effect of vorticity – generates quark axial current (Son, Surowka)
- Strange quarks - should lead to Λ polarization
- Proportional to square of chemical potential – small at RHIC – may be probed at FAIR & NICA

$$j_A^\mu \sim \mu^2 \left(1 - \frac{2\,\mu\, n}{3\,(\epsilon + P)}\right) \epsilon^{\mu\nu\lambda\rho}\, V_\nu\, \partial_\lambda V_\rho$$

Conclusions/Discussion - II

- Anomalous coupling to fluid vorticity – new source of neutron asymmetries
- Related to the new notion of relativistic chaotic flows
- Two-current effects – dilepton tensor polarization
- New source of hyperon polarization in heavy ions collisions