Summary of lecture II

1) Using MEM we can determine the ground state and general features of the quarkonium spectral functions at $T = 0$

2) In the deconfined phase $G(\tau, T) = G^{\text{high}}(\tau, T) + G_i^{\text{low}}(\tau, T)$, $G^{\text{low}}(\tau, T)$ carries information about the trasnport and dominates the T-dependence of the correlator

3) $\sigma^{\text{low}}(\omega, T) = \frac{1}{\pi}\chi(T)\eta\omega/(\omega^2 + \eta^2)$, $\eta = T/M/D \ll T \Rightarrow G^{\text{low}}(\tau, T) \simeq const \Rightarrow$ it is very difficult to extract heavy quark diffusion constant D

4) $G_i^{\text{high}}(\tau, T)$ shows only very small T-dependence and the ratio $G(\tau, T)/G_{rec}(\tau) \simeq 1$ or $G'(\tau, T)/G'_{rec}(\tau) \simeq 1$

5) Quarkonium spectral functions cannot be reliably calculated using MEM in the deconfined phase

6) Spatial quarkonium correlation functions do show significant T-dependence and suggest charmonium dissolution above 300MeV

Summary of lecture III

Correlation functions of static quarks at $T>0$ and color screening

pNRQCD at $T>0$ and potentials
$Im\, V(r,T) \neq 0$

potential models for quarkonium at $T>0$

quarkonium spectral functions

light vector meson correlation functions : thermal dilepton rate, electric conductivity

Free energy of static quark anti-quark pair and other correlators

McLerran, Svetitsky, PRD 24 (81) 450

$\psi_a^\dagger(\tau,x)$, $\psi_a(\tau,x)$-creation annihilation operators for static quarks at time τ and position x

$\psi_a^{\dagger c}(\tau,x)$, $\psi_a^c(\tau,x)$-creation annihilation operators for static anti-quarks at time τ and position x

$$[\psi_a(\tau,x),\psi_b^\dagger(\tau,y)]_+ = \delta(x-y)\delta_{ab}$$

$$(-i\partial_\tau - gA_0(\tau,x))\,\psi(\tau,x) = 0$$

formal solution $\psi(\tau,x) = \mathcal{P}\exp\left(ig\int_0^\tau d\tau' A_0(\tau',x)\right)\psi(0,x) = W(x)\psi(0,x)$

lattice : $W(x) = \prod_{x_0=0}^{N_\tau-1} U_0(x,\tau)$

Free energy of static quark anti-quark pair

$$Z(\beta)e^{-\beta F(x,y)} = \sum_s < s|e^{-\beta H}|s >$$

$|s>$ denotes any state with a static quark at position x and static anti-quark at position y;

Let us denote by $|s'>$ states with no static quarks

$$e^{-\beta F(x,y)} = \sum_{s'} \frac{1}{N_c^2} \sum_{a=a',b=b'} < s'|\psi_a(0,x)\psi_b^c(0,y)e^{-\beta H}\psi_{a'}^{\dagger}(0,x)\psi_{b'}^{\dagger c}(0,y)|s'> \quad (1)$$

$$e^{-\beta H}e^{\beta H}$$

$$e^{-\beta H}O(\tau)e^{\beta H} = O(\tau+\beta)$$

$$= \sum_{s'} \frac{1}{N_c^2} \sum_{a=a',b=b'} < s'|e^{-\beta H}\psi_a(\beta,x)\psi_b^c(\beta,y)\psi_{a'}^{\dagger}(0,x)\psi_{b'}^{\dagger c}(0,y)|s'>$$

$$= Z(\beta)\frac{1}{N_c^2} < \mathrm{Tr}W(x)\mathrm{Tr}W^{\dagger}(y) >= Z(\beta)G(r,T), \;\; r = |x-y|$$

$L(x) = \mathrm{Tr}W(x)$- Polyakov loop

Consider more general correlation function:

$$\mathcal{G}_{aa'bb'}(x,y;\beta,0) = \sum_{s'} < s'|e^{-\beta H}\psi_a(\beta,x)\psi_b^c(\beta,y)\psi_{a'}^{\dagger}(0,x)\psi_{b'}^{\dagger c}(0,y)|s'>$$

$$N_c = 3, \;\; 3\otimes\bar{3} = 1\oplus 8: \qquad P_1 = \frac{1}{9}I\otimes I - \frac{2}{3}t^{\alpha}\bar{t}^{\alpha}, P_8 = \frac{8}{9}I\otimes I + \frac{2}{3}t^{\alpha}\bar{t}^{\alpha}$$

$$G_1(r,T) = \mathrm{Tr}(P_1\mathcal{G})/(\mathrm{TrP}_1) = \frac{1}{3}\mathrm{Tr}\langle W(x)W^{\dagger}(y)\rangle$$ analog of the Wilson loop at $T=0$

$$G_8(r,T) = \mathrm{Tr}(P_8\mathcal{G})/(\mathrm{TrP}_8) = \frac{1}{8}\langle \mathrm{Tr}W(x)\mathrm{Tr}W^{\dagger}(y)\rangle - \frac{1}{8}\frac{1}{3}\mathrm{Tr}\langle W(x)W^{\dagger}(y)\rangle$$

$$G(r,T)=\frac{1}{9}G_1(r,T)+\frac{8}{9}G_8(r,T)\equiv\frac{1}{9}e^{-F_1(r,T)/T}+\frac{8}{9}e^{-F_8(r,T)/T}$$

Perturbation theory ($T \gg T_c$):

$$F_1(r,T)=-\frac{4}{3}\frac{\alpha_s}{r}e^{-m_D r}-\frac{4}{3}\alpha_s m_D,$$
$$F_8(r,T)=+\frac{1}{6}\frac{\alpha_s}{r}e^{-m_D r}-\frac{4}{3}\alpha_s m_D,$$

The work to separate $Q\bar{Q}$ from distance r_1 to r_2

$$A=F(r_2)-F(r_1)$$

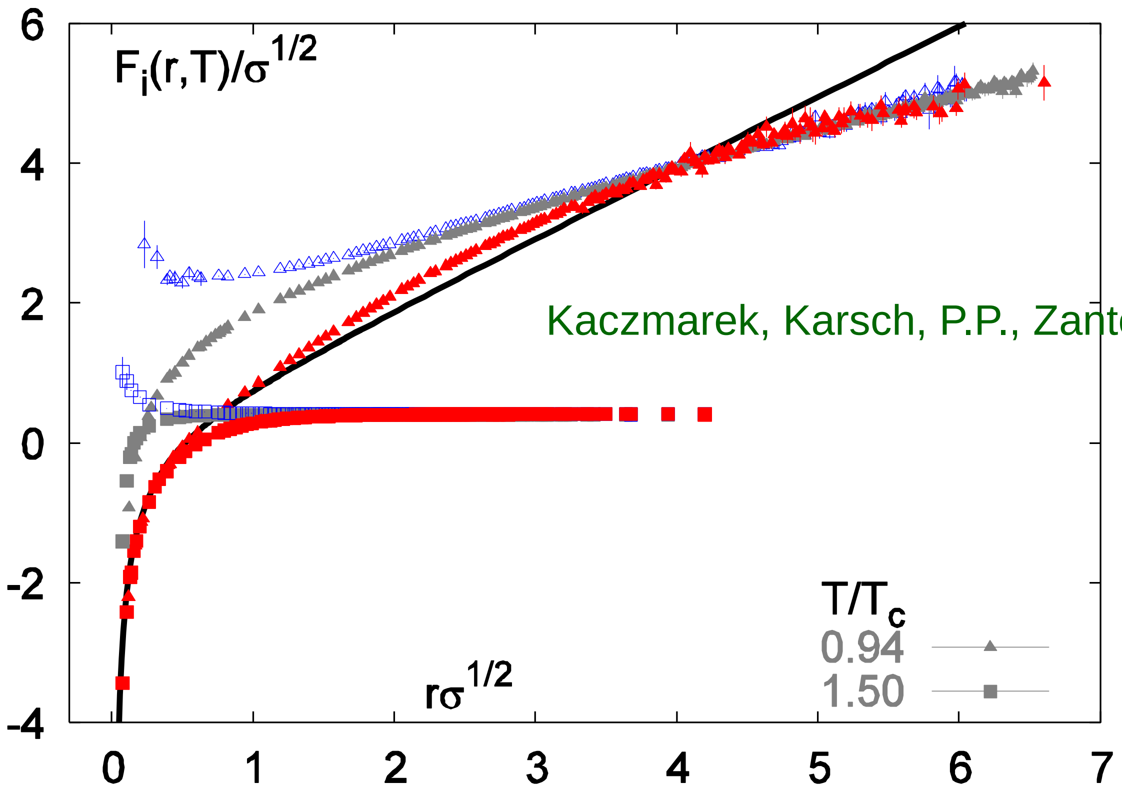
In leading order perturbation theory:

$$F(r,T)=-\frac{1}{9}\frac{\alpha_s^2}{r^2 T}\exp(-2m_D r)$$

In QED

$$F(r,T)=-\frac{\alpha}{r}\exp(-m_D r)$$

In QCD the work is reduced due to cancelation between color singlet and octet contribution



Kaczmarek, Karsch, P.P., Zantow, hep-lat/0309121

The spectral represenation of singlet and averaged correlators $(T < T_c)$:

$$G_1(r,\beta) = \sum_{n=1}^{\infty} c_n(r) e^{-\beta E_n(r)}, \qquad G(r,\beta) = \frac{1}{9}\sum_{n=1}^{\infty} e^{-\beta E_n(r)}$$

Jahn, Philipsen,
PRD 70 (04) 0074504

can be generelized to arbitrary τ and $T > T_c$:

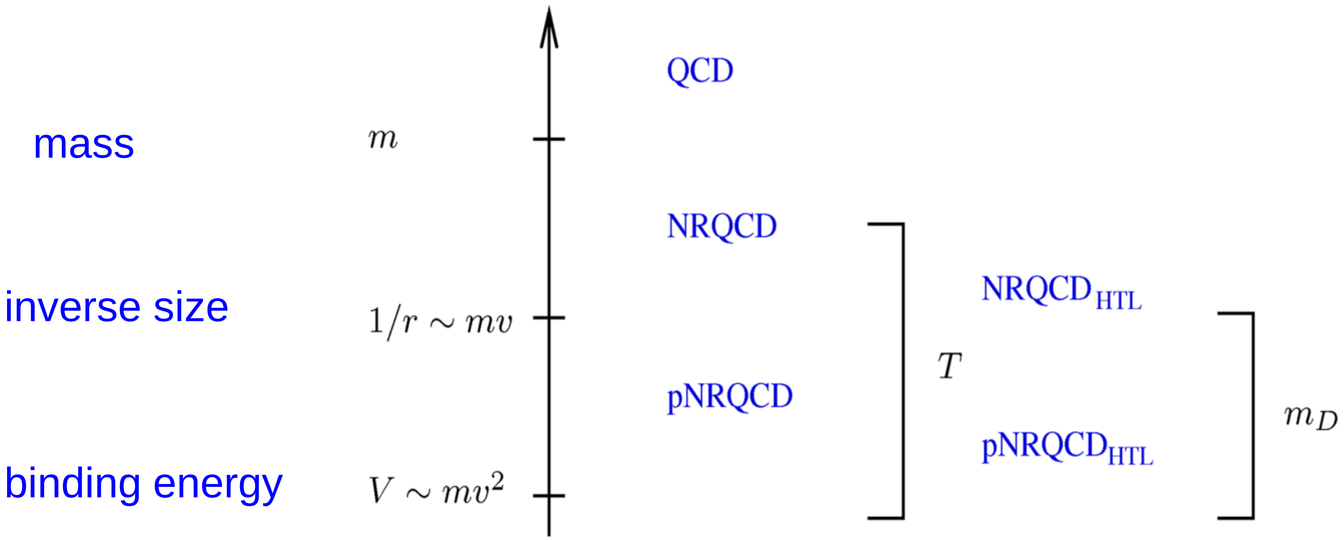
$$G_1(r,T) = \int_0^{\infty} d\omega \sigma^1(\omega,T) e^{-\tau\omega},$$

Rothkopf, Hatsuda, Sasaki
arXiv:1108.1579 [hep-lat]

if $\sigma^1(\omega) \sim \delta_\Gamma(\omega - E_1(r))$ static energies can be defined and extracted Im$E_1(r) \sim \Gamma$

Effective field theory approach for heavy quark bound states and potential models

The heavy quark mass provides a hierarchy of different energy scales



The scale separation allows to construct sequence of effective field theories: NRQCD, pNRQCD

Potential model appears as the tree level approximation of the EFT
and can be systematically improved

Brambilla, Ghiglieri, P.P., Vairo, PRD 78 (08) 014017

pNRQCD at finite temperature for static quarks

EFT for energy scale : $E_{bind} \sim \Delta V = (V_o - V_s) \sim mv^2$

Ultrasoft quark and gluons

$$\mathcal{L} = -\frac{1}{4} F^a_{\mu\nu} F^{a\,\mu\nu} + \sum_{i=1}^{n_f} \bar{q}_i \, i D\!\!\!/ q_i$$

Singlet $Q\bar{Q}$ field　　　　Octet $Q\bar{Q}$ field

$$+ \int d^3r \, \mathrm{Tr} \left\{ \mathrm{S}^\dagger \left[i\partial_0 - \frac{-\nabla^2}{m} - V_s(r,T) \right] \mathrm{S} + \mathrm{O}^\dagger \left[iD_0 - \frac{-\nabla^2}{m} - V_o(r,T) \right] \mathrm{O} \right\}$$

$$+ V_A \, \mathrm{Tr} \left\{ \mathrm{O}^\dagger \vec{r} \cdot g\vec{E} \, \mathrm{S} + \mathrm{S}^\dagger \vec{r} \cdot g\vec{E} \, \mathrm{O} \right\} + \frac{V_B}{2} \mathrm{Tr} \left\{ \mathrm{O}^\dagger \vec{r} \cdot g\vec{E} \, \mathrm{O} + \mathrm{O}^\dagger \mathrm{O} \vec{r} \cdot g\vec{E} \right\} + \ldots$$

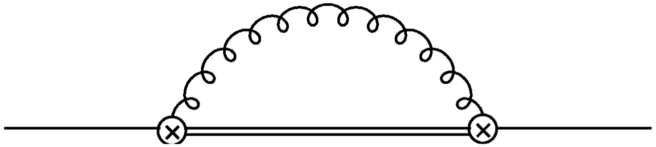
potential is the matching parameter of EFT !

Free field limit => Schrödinger equation

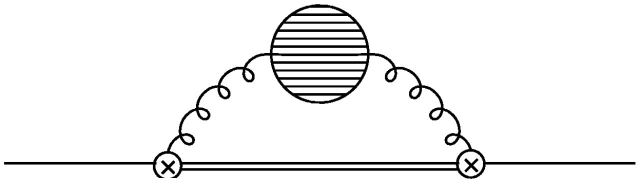
$$\left[i\partial_0 - \frac{-\nabla^2}{m} - V_s(r,T) \right] S(r,t) = 0$$

$E_{bind} \sim \Delta V \sim \alpha_s / r \ll T, \; m_D$ there are thermal contribution to the potentials

Singlet-octet transition :



Landau damping :



Brambilla, Ghiglieri, P.P., Vairo, PRD 78 (08) 014017

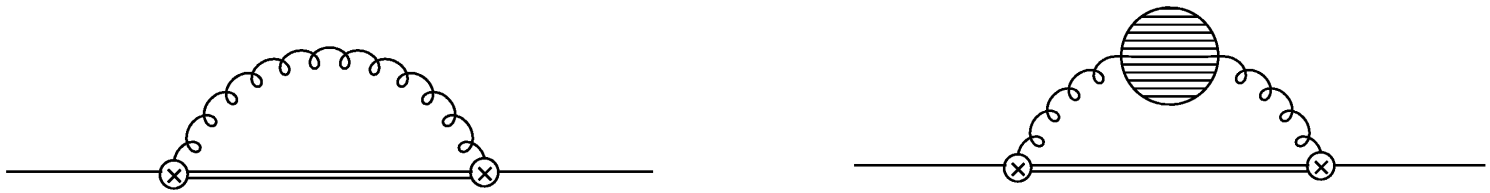
Thermal pNRQCD in the small distance regime

$$r \ll 1/T \ll 1/m_D$$

The heavy quarks do not feel the medium and the quark anti-quark pair interacts with the medium as a dipole

$$NRQCD \xrightarrow{1/r} pNRQCD \xrightarrow{T,\ m_D} pNRQCD_{therm}$$

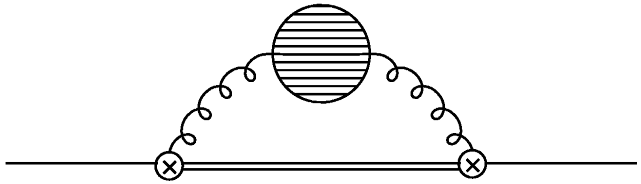
Contribution from scale *T*:



$$\delta V_s(r,T) = \frac{\pi}{9} N_c C_F \alpha_s^2 r T^2 - \frac{i}{6} N_c^2 C_F \alpha_s^3 T$$
$$-\frac{3}{2}\zeta(3) C_F \frac{\alpha_s}{\pi} r^2 T m_D^2 + \frac{2}{3}\zeta(3) N_c C_F \alpha_s^2 r^2 T^3$$
$$+i\left[\frac{C_F}{6}\alpha_s r^2 T m_D^2 \left(\frac{1}{\epsilon} - \ln\frac{T^2}{\mu_1^2}\right) + \frac{4\pi}{9}\ln 2 N_c C_F \alpha_s^2 r^2 T^3\right]$$

The $1/\varepsilon$ pole is of IR origin and will cancel against UV poles from lower scales

Contribution from scale m_D :



$$\delta V_s(r,T) = -\frac{C_F}{6}\alpha_s r^2 m_D^3 + i\frac{C_F}{6}\alpha_s r^2 T m_D^2 \left(-\frac{1}{\epsilon} - \ln\frac{\mu_2^2}{m_D^2}\right)$$

The $1/\varepsilon$ pole is of UV origin and will cancel against IR poles from scale T giving a finite imaginary part that contains a term :

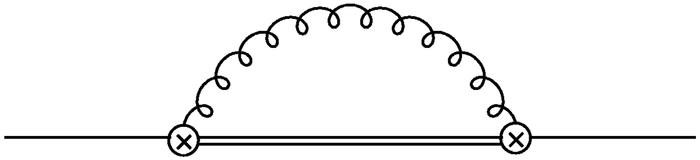
$$-i\frac{C_F}{6}\alpha_s r^2 T m_D^2 \left(\ln\frac{T^2}{m_D^2} + const.\right)$$

The logarithm ensures that the imaginary part is always negative in the weak coupling regime ($m_D << T$)

The potential for $r \ll 1/T \ll 1/m_D$:

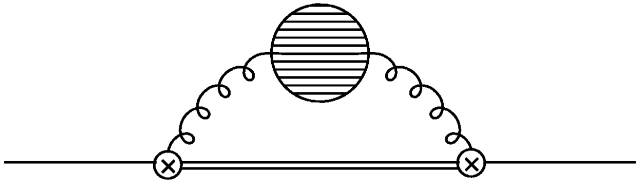
ReV$_s(r,T)$ | ImV$_s(r,T)$

$-C_F \frac{\alpha_s}{r}$ | 0



$\Delta V = V_o - V_s$

T: $g^2T^3r^2 \times \frac{\Delta V}{T} \sim \alpha_s^2 T^2 r$ | $g^2T^3r^2 \times \left(\frac{\Delta V}{T}\right)^2 \sim \alpha_s^3 T$



T: $g^2T^3r^2 \times \left(\frac{m_D}{T}\right)^2$ | $g^2T^3r^2 \times \left(\frac{m_D}{T}\right)^2$

m_D: $g^2T^3r^2 \times \left(\frac{m_D}{T}\right)^3$ | $g^2T^3r^2 \times \left(\frac{m_D}{T}\right)^2$

Thermal pNRQCD in the large distance regime

$$1/T \ll r$$

Heavy quarks interact with the medium which generates thermal mass and thermal width

$$NRQCD \;\; {}^{T} \rightarrow NRQCD_{HTL} \;\; {}^{1/r, m_D} \rightarrow pNQRCD_{HTL}$$

$$\mathcal{L} \rightarrow -\frac{1}{4} F^a_{\mu\nu} F^{a\,\mu\nu} + \sum_{i=1}^{n_f} \bar{q}_i \, i \not{D} q_i + \delta\mathcal{L}_{HTL}$$

no modification of the heavy quark sector at LO

1) *1/r >> m_D* => scales *1/r* and m_D are integrated out subsequently

1/ε poles of IR and UV origin appear in ImV_s when scales *1/r* and m_D are integrated out, but these poles cancel in the sum as this happened in the short distance regime

2) *1/r ~ m_D* => scales *1/r* and m_D are integrated out simultaneously

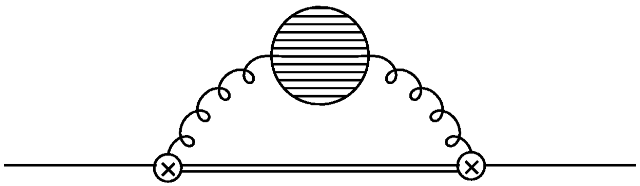
Singlet part of the lagrangian becomes :

$$\int d^3 r \, \mathrm{Tr}\left\{ \mathrm{S}^\dagger \left[i\partial_0 + V_s(r,T) + 2\delta m \right] \mathrm{S} \right.$$

$$\delta m = -\frac{C_F}{2} \alpha_s (m_D + iT)$$

thermal mass and width of the heavy quark

The potential for $r \ll 1/m_D$:



	$\mathrm{Re}V_s(r,T)$	$\mathrm{Im}V_s(r,T)$
$1/r$:	$-C_F\frac{\alpha_s}{r} - C_F\alpha_s T^2 r \left(\frac{m_D}{T}\right)^2$	$g^2T^3r^2 \times \left(\frac{m_D}{T}\right)^2$
m_D:	$g^2T^3r^2 \times \left(\frac{m_D}{T}\right)^3$	$g^2T^3r^2 \times \left(\frac{m_D}{T}\right)^2$

The potential for $r \sim 1/m_D$:

$$V_s(r,T) = -C_F\frac{\alpha_s}{r}\exp(-m_D r) + iC_F\alpha_s T\frac{2}{rm_D}\int_0^\infty dx\frac{\sin(rm_D x)}{(x^2+1)^2}$$

Laine, Philipsen, Romatschke, Tassler, JHEP 073 (2007) 054

$\mathrm{Re}(V_s(r,T) + 2\delta m)$ is identical to the LO singlet free energy $F_1(r,T)$

$$r \sim 1/m_D \leftrightarrow T \sim gm \Rightarrow \mathrm{Re}V_s \sim g^2/r \sim g^4 m \ll \mathrm{Im}V_s \sim g^2 T \sim g^3 m$$

The imaginary part of the potential is larger than the real part => quarkonium melting is determined by Landau damping and not by screening as originally suggested by Matusi and Satz

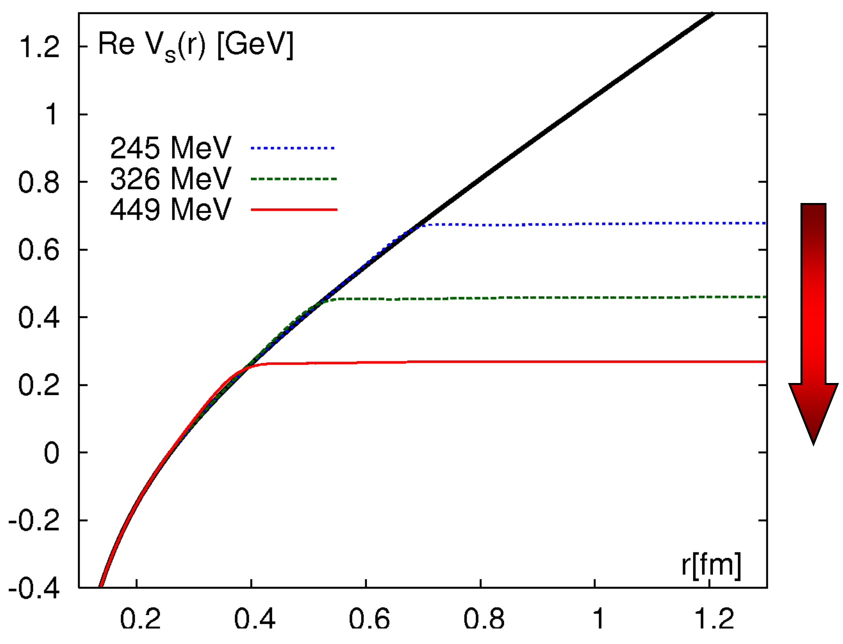
pNRQCD beyond weak coupling and potential models

Above deconfinement the binding energy is reduced and eventually $E_{bind} \sim mv^2$ is the smallest scale in the problem (zero binding) $mv^2 >> \Lambda_{QCD}, 2\pi T, m_D$ => most of medium effects can be described by a T-dependent potential

Determine the potential by non-perturbative matching to static quark anti-quark potential calculated on the lattice

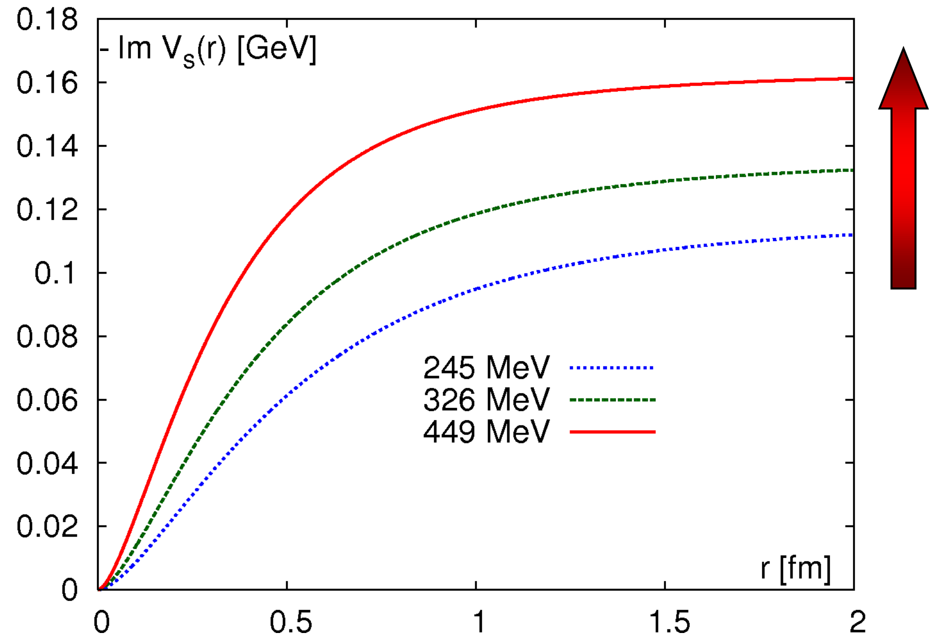
Caveat : it is difficult to extract static quark anti-quark energies from lattice correlators => constrain Re$V_s(r)$ by lattice QCD data on the singlet free energy, take Im$V_s(r)$ from pQCD calculations

"Maximal" value for the real part



Mócsy, P.P., PRL 99 (07) 211602

Minimal (perturbative) value for imaginary part



Laine et al, JHEP0703 (07) 054,
Beraudo, arXiv:0812.1130

Lattice QCD based potential model

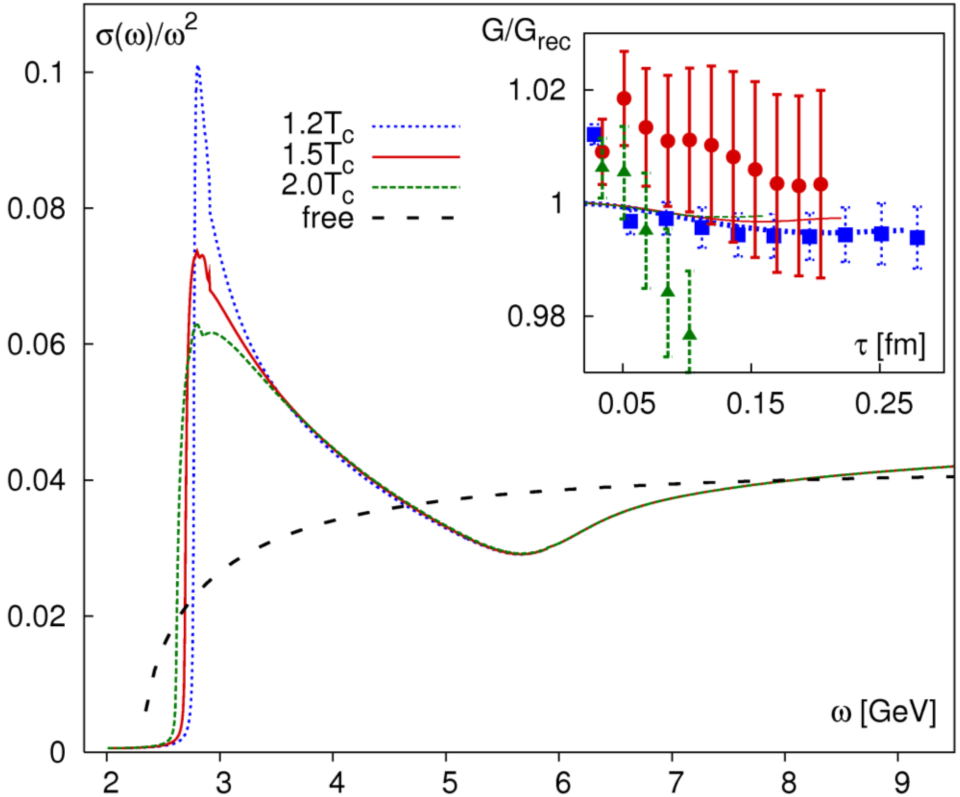
If the octet-singlet interactions due to ultra-soft gluons are neglected :

$$\left[i\partial_0 - \frac{-\nabla^2}{m} - V_s(r,T)\right] S(r,t) = 0 \quad \Rightarrow \quad \sigma(\omega, T)$$

potential model is not a model but the tree level approximation of corresponding EFT that can be systematically improved

Test the approach vs. LQCD : quenched approximation, $F_1(r,T) < \mathrm{Re}V_s(r,T) < U_1(r,T)$, $\mathrm{Im}V(r,T) \approx 0$

Mócsy, P.P., PRL 99 (07) 211602, PRD77 (08) 014501, EPJC ST 155 (08) 101



- resonance-like structures disappear already by *1.2T$_c$*
- strong threshold enhancement above free case

  => indication of correlations

- height of bump in lattice and model are similar

- The correlators do not change significantly despite the melting of the bound states => it is difficult to distinguish bound state from threshold enhancement in lattice QCD

The role of the imaginary part for charmonium

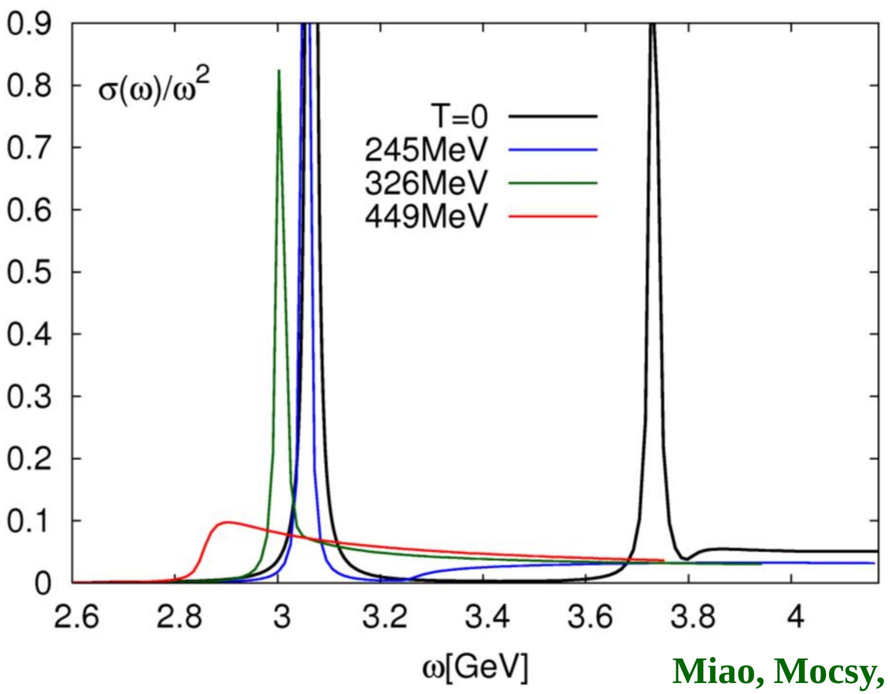
Take the upper limit for the real part of the potential allowed by lattice calculations

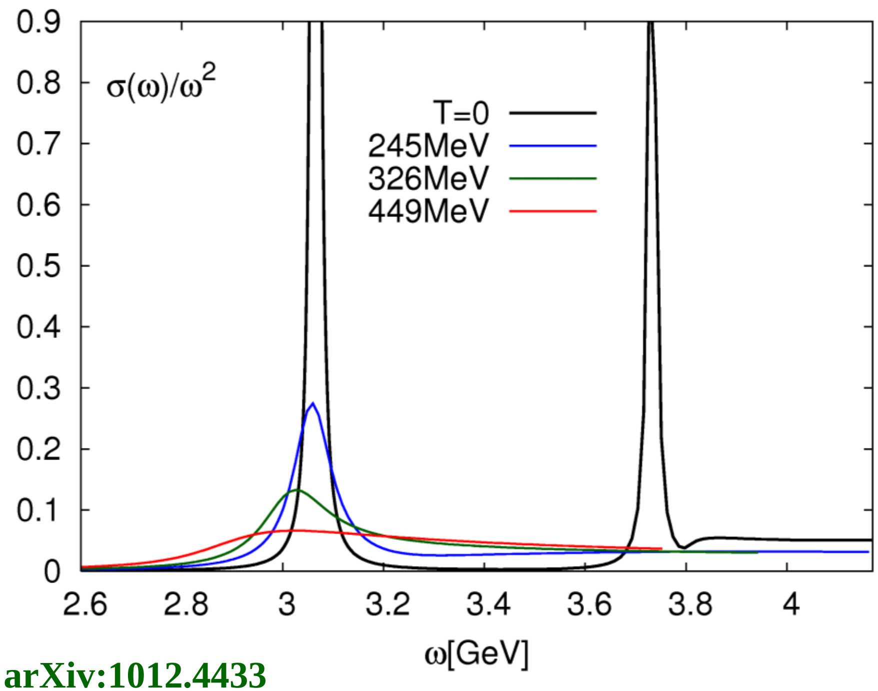
Mócsy, P.P., PRL 99 (07) 211602,

Take the perturbative imaginary part
Burnier, Laine, Vepsalainen JHEP 0801 (08) 043

Im $V_s(r) = 0$:
1S state survives for $T = 330$ MeV

imaginary part of $V_s(r)$ is included :
all states dissolves for $T > 250$ MeV





Miao, Mocsy, P.P., arXiv:1012.4433

no charmonium state could survive for $T > 250$ MeV
this is consistent with our earlier analysis of **Mócsy, P.P., PRL 99 (07) 211602** ($T_{dec} \sim 204 MeV$)
as well as with **Riek and Rapp, arXiv:1012.0019 [nucl-th]**

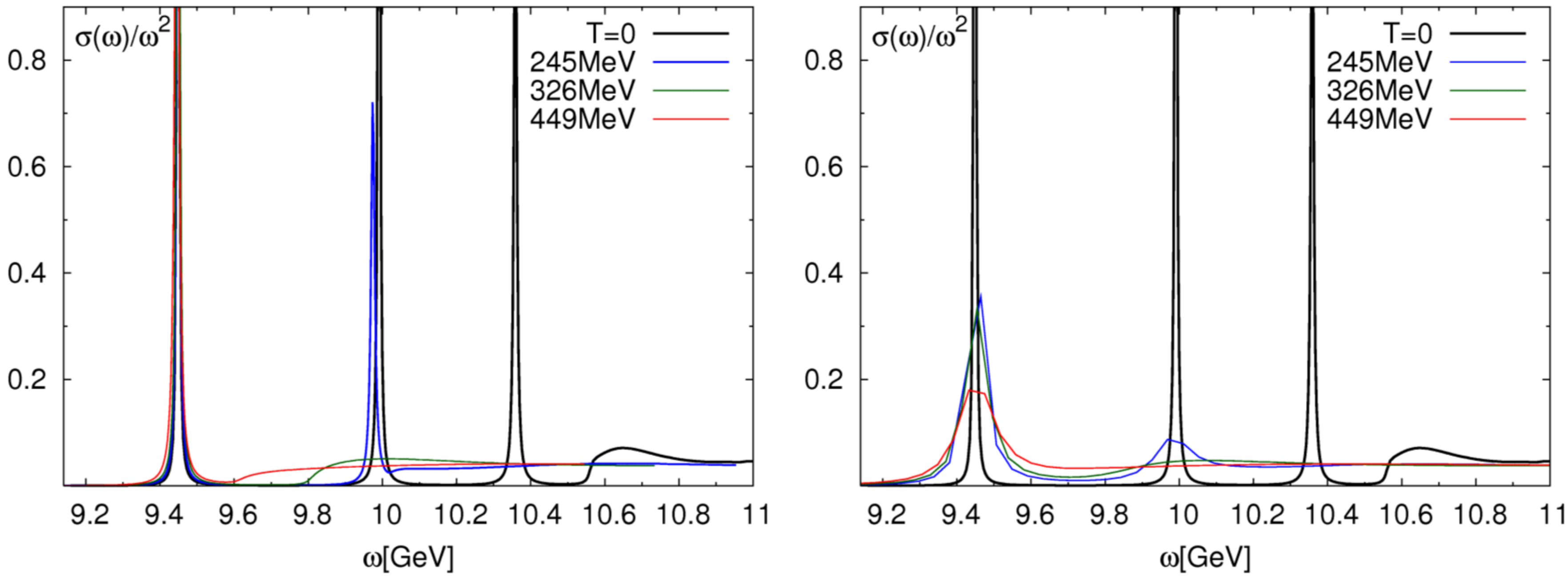
The role of the imaginary part for bottomonium

Take the upper limit for the real part of the potential allowed by lattice calculations
Mócsy, P.P., PRL 99 (07) 211602,

Take the perturbative imaginary part
Burnier, Laine, Vepsalainen JHEP 0801 (08) 043

Im $V_s(r)$ =0:
2S state survives for $T > 250$ MeV
1S state could survive for $T>450$ MeV

with imaginary part:
2S state dissolves for $T>250$ MeV
1S states dissolves for $T>450$ MeV

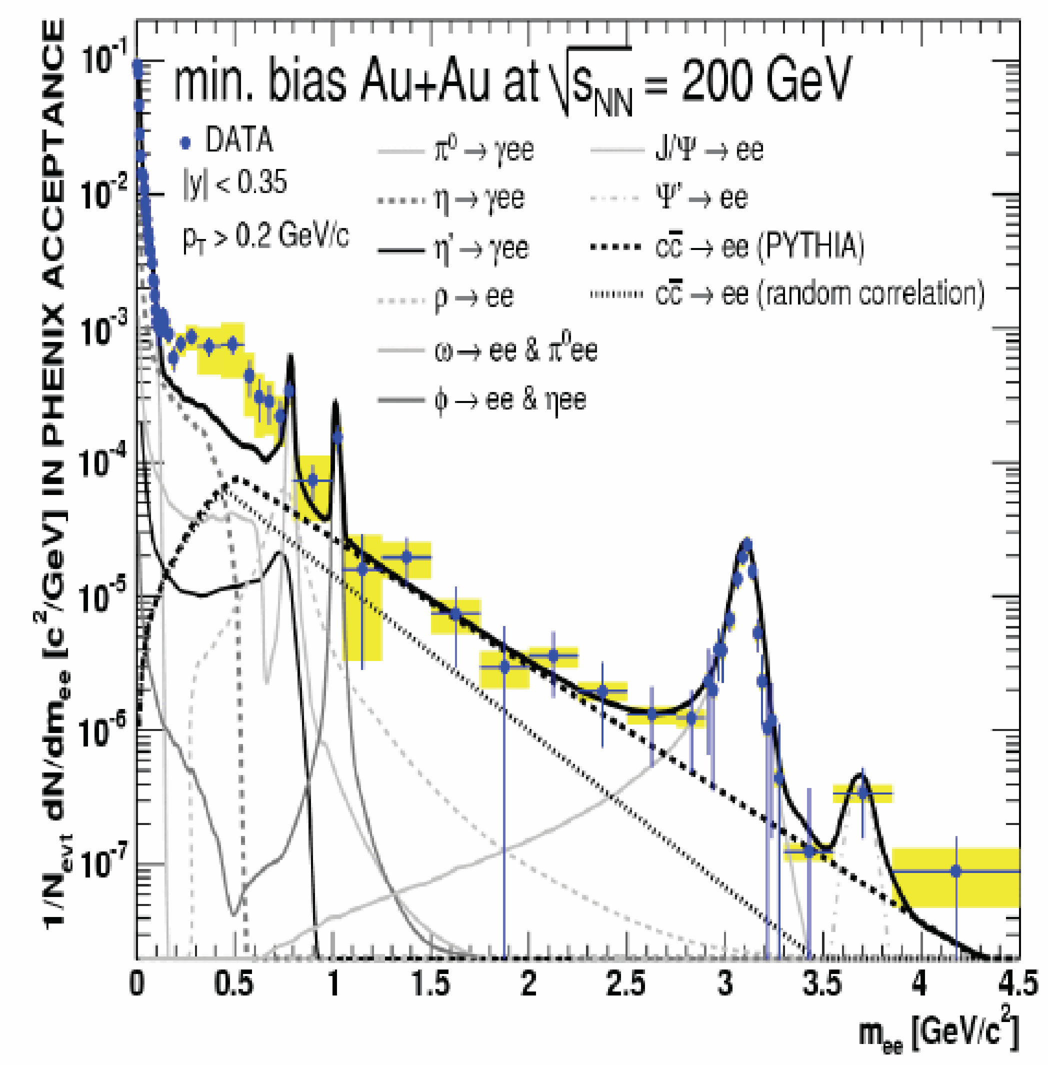


Miao, Mocsy, P.P., arXiv:1012.4433

Excited bottomonium states melt for $T \approx 250$ MeV ; 1S state melts for $T \approx 450$ MeV
this is consistent with our earlier analysis of **Mócsy, P.P., PRL 99 (07) 211602** ($T_{dec} \sim 204 MeV$)
as well as with **Riek and Rapp, arXiv:1012.0019 [nucl-th]**

Thermal dileptons and light vector meson correlators

PHENIX



Thermal dileptons :

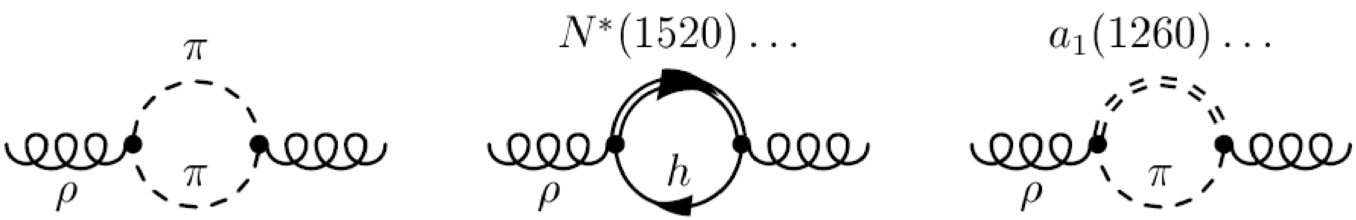
direct measurement of the temperature of the produced matter, test consequences of chiral symmetry restoration

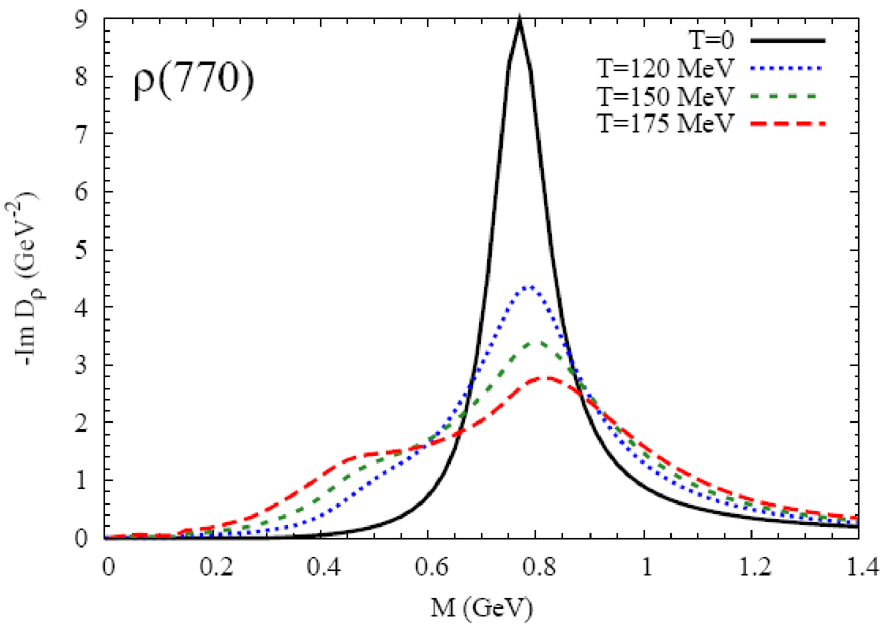
Modifications of the vector spectral functions in hot hadronic matter

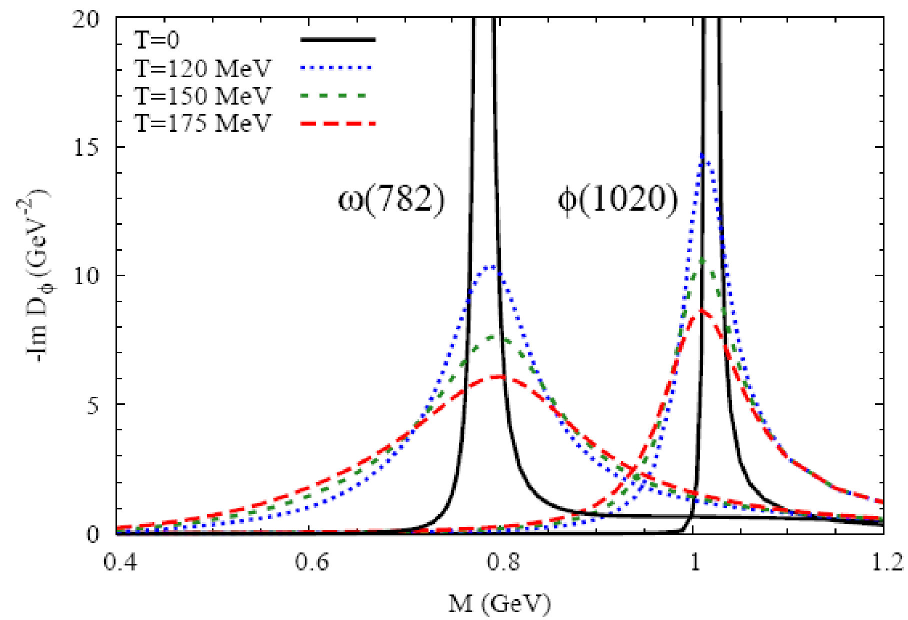
R. Rapp and J. Wambach, Eur. Phys. J. A **6**, 415 (1999).

R. Rapp, M. Urban, M. Buballa, and J. Wambach, Phys. Lett. B **417**, 1 (1998).

R. Rapp, Phys. Rev. C **63**, 054907 (2001).







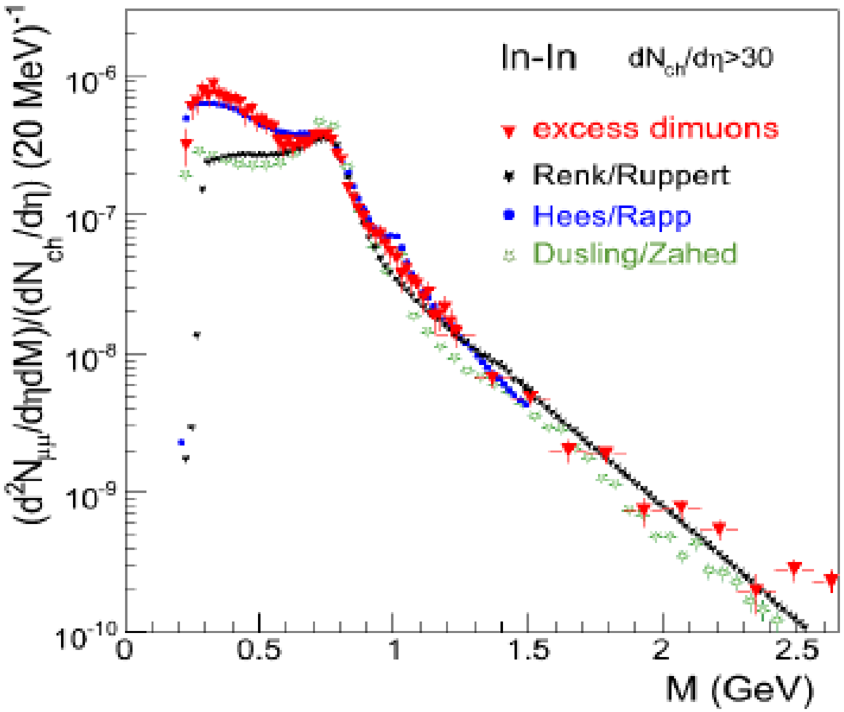
Thermal dileptons at SPS

In the low mass region (LMR) excess dileptons are due to the in-medium modivications of the ρ-meson melting induced by baryon interactions

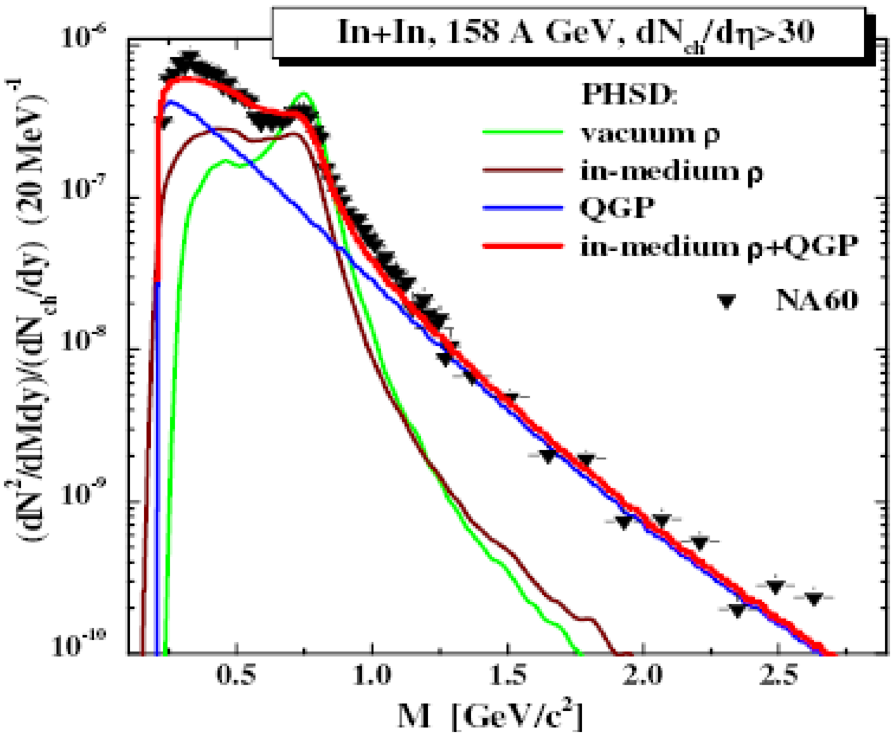
Models which incorporate this (Hess/Rapp and PHSD) can well describe the NA60 data !

NA60 : Eur. Phys. J 59 (09) 607
CERN Courier. 11/2009

fireball models and hydro model (Dusling/Zahed)



Linnyk, Cassing, microscopic transport
PHSD model, talk at Hard Probes 2010

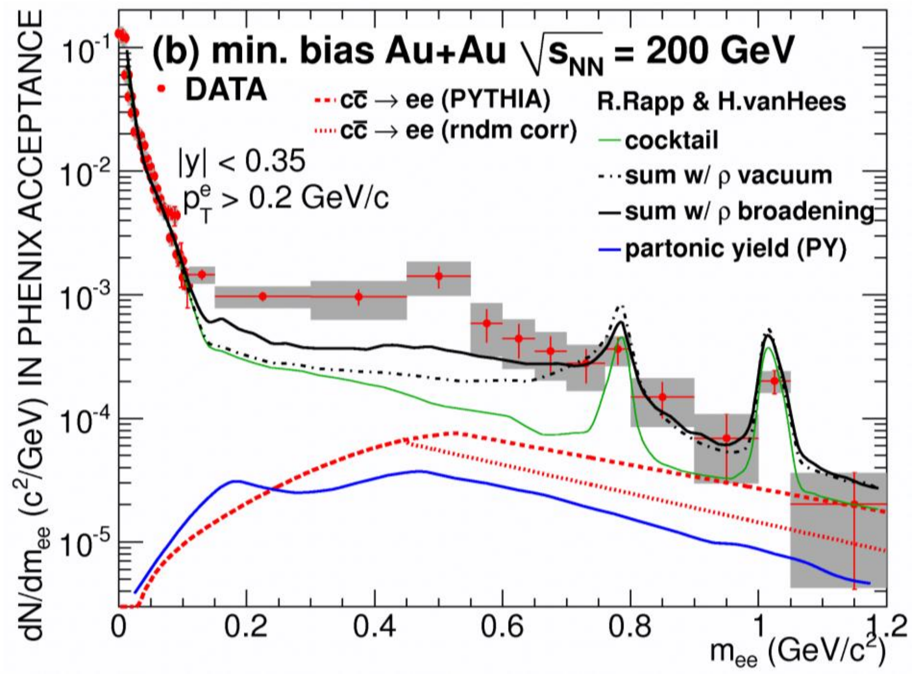


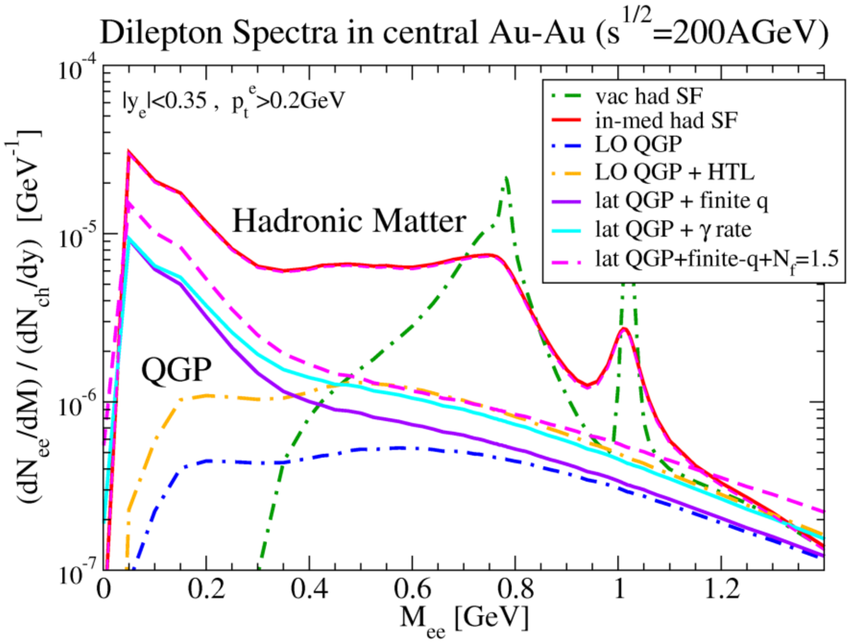
There is also an excess in the intermediate mass region (IMR) which could have partonic origin (D/Z, R/R, PHSD) or hadronic (H/R, $\pi a_1 \rightarrow \mu^+\mu^-$)

Thermal dileptons at RHIC and LMR puzzle

Models that described the SPS dilepton data fails for RHIC in low mass region !

Rapp, arXiV:1010.1719





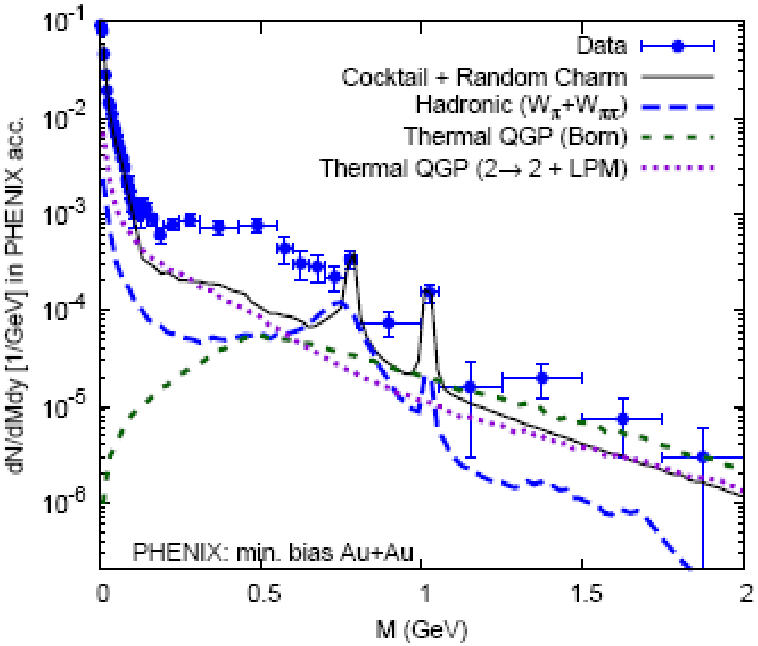
In the low mass region hadronic contribution dominates because of the larger 4-volume but there is large uncertainty in the QGP rate

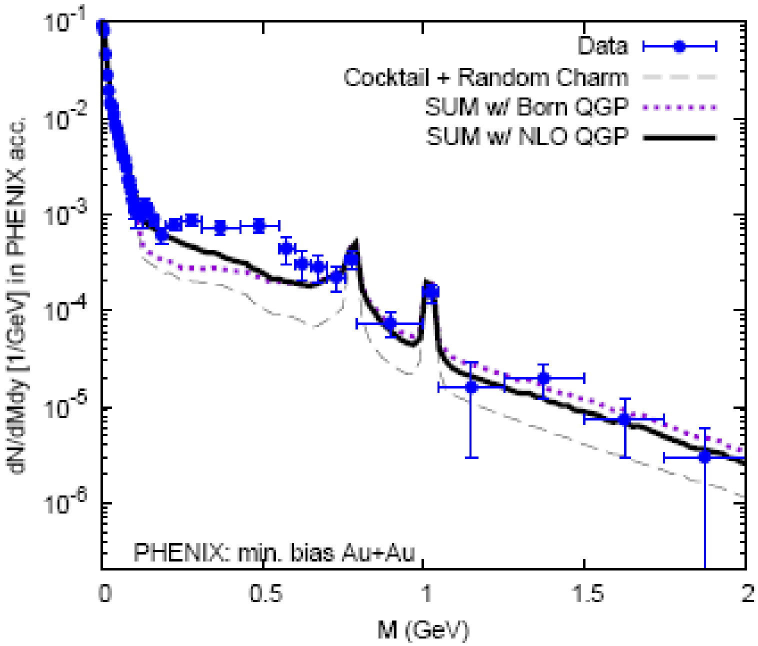
new lattice QCD based estimates are much larger than the perturbative QGP rates but it is not yet clear if this solves the LMR dilepton puzzle

⇨ more is going on in the broad transition region (~50MeV from the new lQCD results)

Thermal dileptons at RHIC and unceratinties in the QGP rates

Dusling, Zahed, arXiv:0911.2426





Kinematic effects are important
in the low mass region
NLO QGP rate >> LO (Born) QGP rate
One needs, however, at least an order
of magnitude larger QGP rate to
explain the data

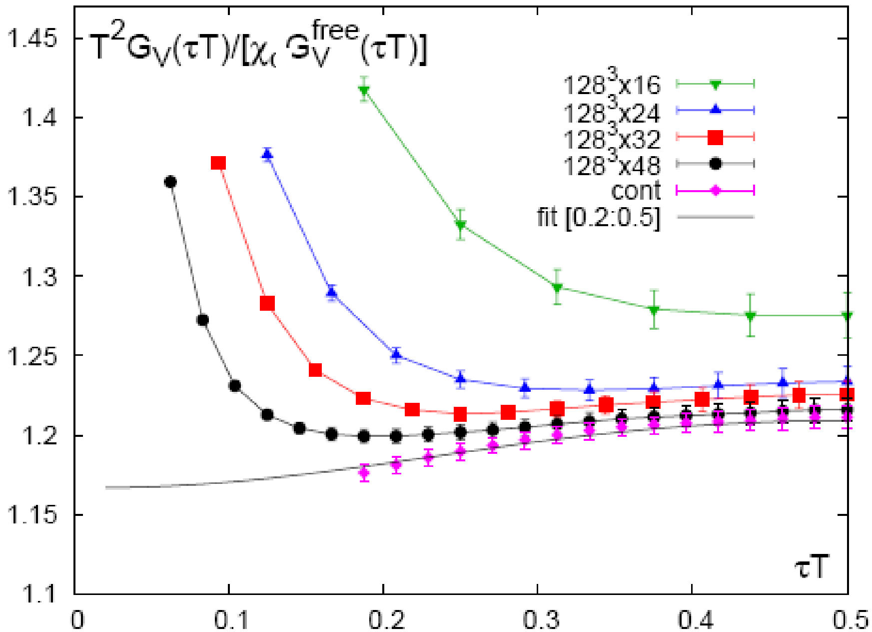
Also in the IMR there is potentially
a factor 2 uncertainty in the QGP rate
Born rate ~ 2x NLO rate

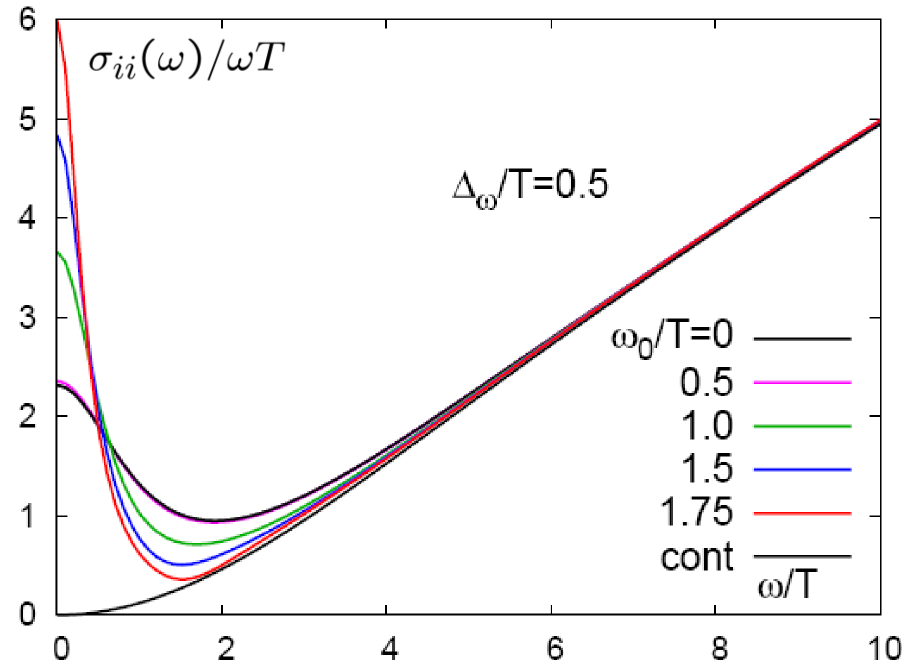
→ Need to constrain the QGP yield
by lattice QCD

Lattice calculations of the vector spectral functions

Ding et al, PRD 83 (11) 034504

Isotropic Wilson gauge action, quenched non-perturbatively improved clover fermion action on $128^3 \times N_\tau$ lattices, $T = 1.45T_c$, $m_q^{\overline{MS}}(2\text{GeV}) = 0.1/T$, $N_\tau = 24,\ 32, 48$ $(a^{-1} = 9.4 - 18.8\text{GeV})$





$$\sigma_{ii}(\omega) = \chi c_{BW} \frac{1}{\pi} \frac{\omega\Gamma/2}{\omega^2 + (\Gamma/2)^2} + \frac{3}{4\pi^2}(1+k)\omega^2 \tanh(\omega/4T)\Theta(\omega_0, \Delta_\omega),$$

$$\Theta(\omega_0, \Delta_\omega) = (1 + e^{(\omega_0^2 - \omega^2)/\omega\Delta_\omega})^{-1}$$

Fit parameters : $c_{BW},\ \Gamma,\ k$

Different choices of : ω_0, Δ_ω

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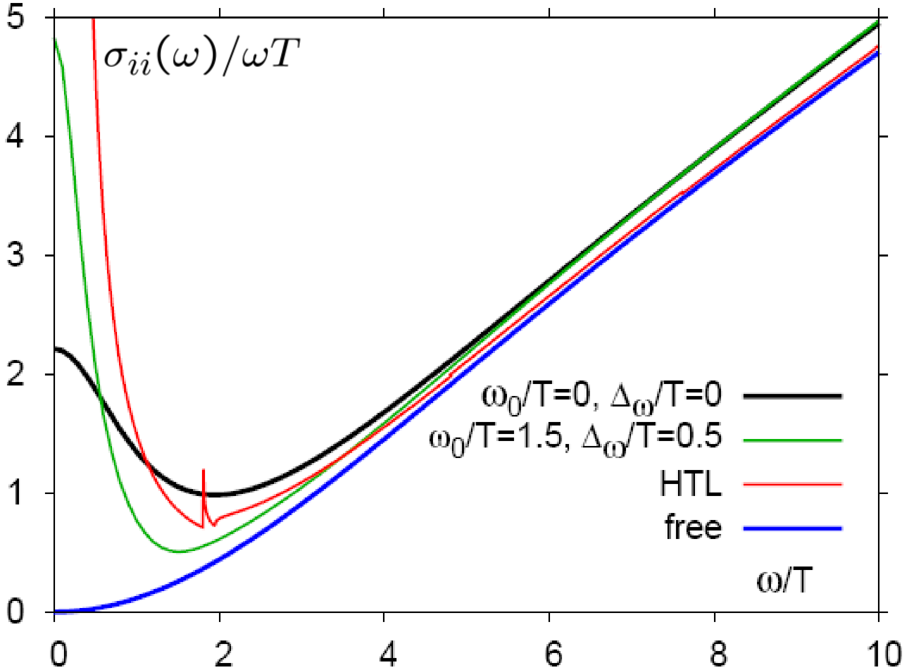
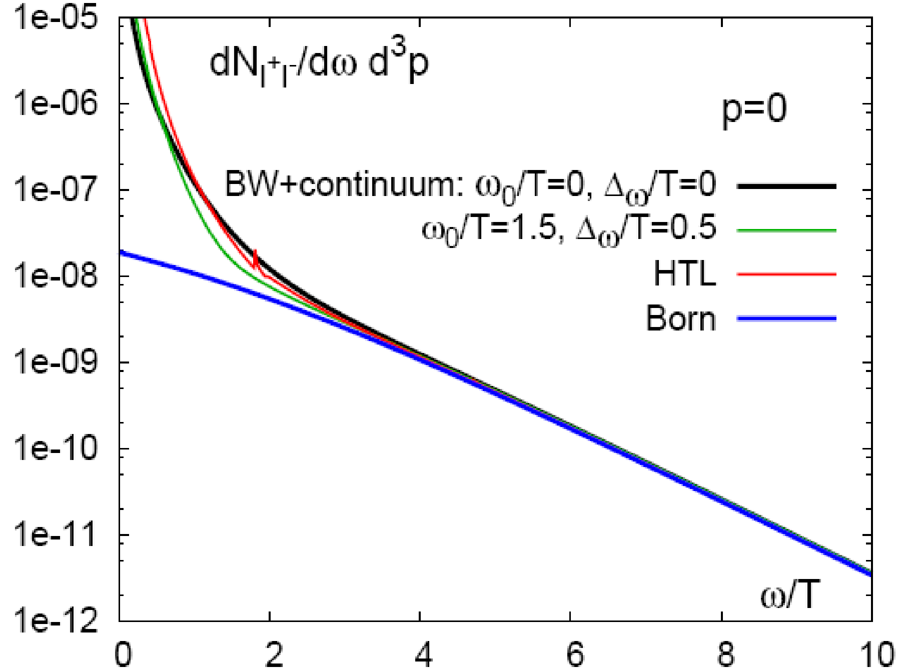
Ding et al, PRD 83 (11) 034504

Electric conductivity: $\zeta = \frac{C_{em}}{6} \lim_{\omega\to 0} \frac{\sigma_{ii}}{\omega}$ ⟹ $1/3 < \frac{1}{C_{em}} \frac{\zeta}{T} < 1$



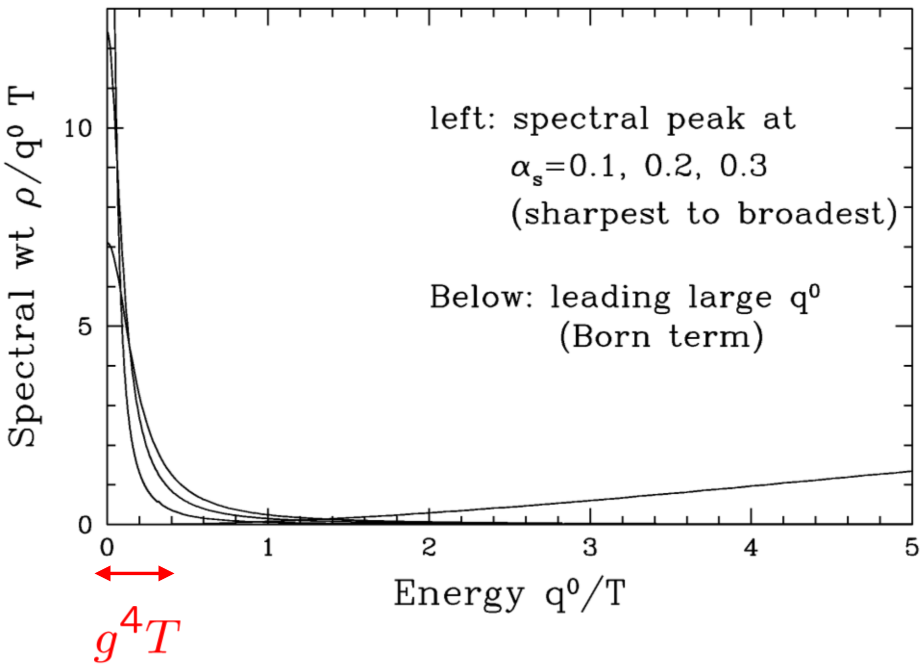


- The HTL resummed perturbative result diverges for $\omega \to 0$ limit
- The lattice results show significant enhancement over the LO (Born) result for small ω
- The lattice result is HTL result for $2<\omega/T<4$ but is much smaller for $\omega/T<2$

Strongly coupled or weakly coupled QGP ?

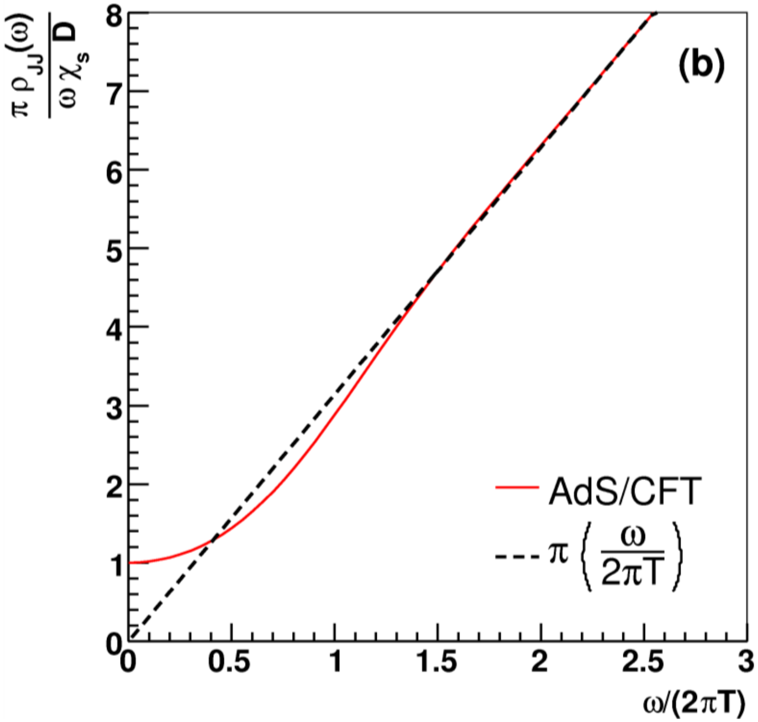
Weak coupling caculation of the
vector current spectral function in QCD

Moore, Robert, hep-ph/0607172



vector current correlator in
N=4 SUSY at strong coupling

Teaney, PRD74 (06) 045025



lattice results are closer to the weakly coupled QGP

Homework:

Using the definition

$$\exp(-F_1(r,T)/T) = \frac{1}{3}\mathrm{Tr}\langle W(\vec{r})W^\dagger(0)\rangle, \quad \text{and} \quad W \simeq 1 + igA_0/T$$

show that $F_1 = -\frac{4}{3}\frac{\alpha_s}{r}\exp(-m_D r)$ at leading order
(hint : use a gauge where A_0 is time independent and the re-summed gluon propagator $D_{00}^{ab}(k) = \delta_{ab}/(\vec{k}^2 + m_D^2)$)

Using the above result for F_1 and the leading order relation $F_8/F_1 = -1/8$ as well as the relation

$$\exp(-F/T) = \frac{1}{9}\exp(-F_1/T) + \frac{8}{9}\exp(-F_8/T)$$

show that $F = -\frac{1}{9}\frac{\alpha_s^2}{r^2 T}\exp(-2m_D r)$ (hint : expand the exponent to second order)

Arrive at the above result for F_{av} starting from its definition $\exp(-F/T) = \frac{1}{9} < \mathrm{Tr}W(r)\mathrm{Tr}W^\dagger(0) >$ and using the perturbative expansion for the Wilson line W. (hint : use the same steps as in the redivation of F_1 above)

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