Homework:

1) Prove the integral equation :

$$\Delta(\tau) = \int_0^\infty dk_0 \sigma(k_0) \frac{\cosh(k_0 \cdot (\tau - \beta/2))}{\sinh(\beta k_0/2)}$$

Show that:

$$\sigma(k_0) = \frac{1}{Z(\beta)} \sum_{n,m} e^{-\beta E_n} \left[\delta(k_0 + E_n - E_m) - \delta(k_0 + E_m - E_n)\right] |\langle n|\hat{q}|m\rangle|^2$$

Hint : use relation between $\sigma(k_0)$ and $D^{>,<}(k_0)$ and insert a complete set of energy eigenstates into $D^{>,<}(t)$)

3) Prove the sum rule

$$\int_{-\infty}^{\infty} k_0 \sigma(k_0) dk_0 = 1$$

Euclidean correlators and spectral functions

Lattice QCD is formulated in imaginary time

$$G(\tau,\overrightarrow{p},T)=\int d^3x\, e^{i\vec{p}\cdot\vec{x}}\left\langle J_H(\tau,\vec{x})\, J_H{}^{+}(0,0)\right\rangle,$$

$$J_H(\tau,\vec{x})=\overline{\psi}(\tau,\vec{x})\Gamma_H\psi(\tau,\vec{x})$$

Physical processes take place in real time

$$D^{>}(t,\overrightarrow{p},T)=\int d^3x\, e^{i\vec{p}\cdot\vec{x}}\left\langle J_H(t,\vec{x})\, J_H\;\;(0,0)\right\rangle,$$

$$D^{<}(t,\overrightarrow{p},T)=\int d^3x\, e^{i\vec{p}\cdot\vec{x}}\left\langle J_H(0,\vec{0})\, J_H\;\;(t,\vec{x})\right\rangle$$

$$\frac{D^{>}(\omega)-D^{<}(\omega)}{2\pi}=\frac{1}{\pi}\operatorname{Im}D_R(\omega)=\sigma(\omega)$$

$\Gamma_H=1,\ \gamma_5,\ \gamma_\mu,\ \gamma_5\cdot\gamma_\mu$

$$R(\omega)=\frac{\sigma_{e^+e^-\to hadrons}}{\sigma_{e^+e^-\to\mu^+\mu^-}}=\frac{\sigma(\omega)}{\omega^2}$$

$$\boxed{G(\tau,T)=D^{>}(-i\tau)}$$

⇓

$$\boxed{G(\tau,T)=\int_0^\infty d\omega\,\sigma(\omega,T)\frac{\cosh(\omega(\tau-1/(2T))}{\sinh(\omega/(2T))}}$$

if $T=0$ and $\sigma(\omega)=\sum_n A_n\delta(\omega-E_n)$ ⇒ $G(\tau)=A_0e^{-E_0\tau}+A_1e^{-E_1\tau}+\ldots$

fit the large distance behavior of the lattice correlation functions

This is not possible for $T>0,\ \tau_{max}=1/T$ ⇒ Maximum Entropy Method (MEM)

Spectral functions at T>0 and physical observables

Heavy meson spectral functions:

$$J_H = \overline{\psi}\,\Gamma_H\,\psi$$

quarkonia properties at T>0
heavy quark diffusion in QGP: D

Quarkonium suppression (R_{AA})
Open charm/beauty suppression (R_{AA})

thermal dilepton production rate
(# of dileptons/photons per unit 4-volume)

Light vector meson spectral functions:

$$J_\mu = \overline{\psi}\,\gamma_\mu\,\psi$$

$$\frac{dW}{d\omega d^3p} = \frac{5\alpha_{em}^2}{27\pi^2}\frac{1}{e^{\omega/T}-1}\frac{\sigma_{\mu\mu}(\omega,p,T)}{\omega^2-p^2}$$

thermal photon production rate :

$$p\frac{dW}{d^3p} = \frac{5\alpha_{em}}{9\pi}\frac{1}{e^{p/T}-1}\sigma_{\mu\mu}(\omega=p,p,T)$$

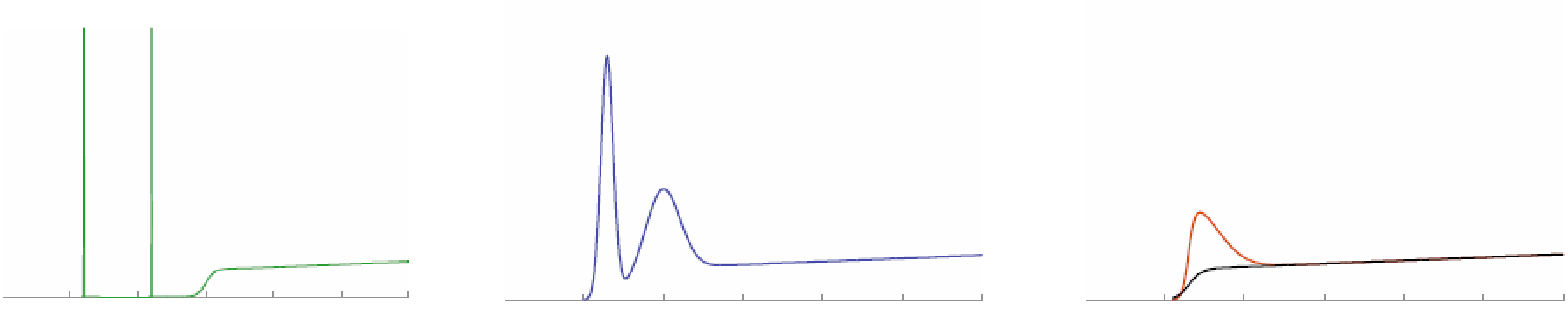
2 massless quark (u and d) flavors are
assumed; for arbitrary number of flavors
$5/9 \longrightarrow \Sigma_f Q_f^2$

Thermal photons and dileptons provide
information about the temperature of
the medium produced in heavy ion collisions
Low mass dileptons are sensitive probes
of chiral symmetry restoration at $T>0$

electric conductivity ζ :

Meson spectral functions and lattice QCD

In-medium properties and/or dissolution of quarkonium states are encoded in the spectral functions

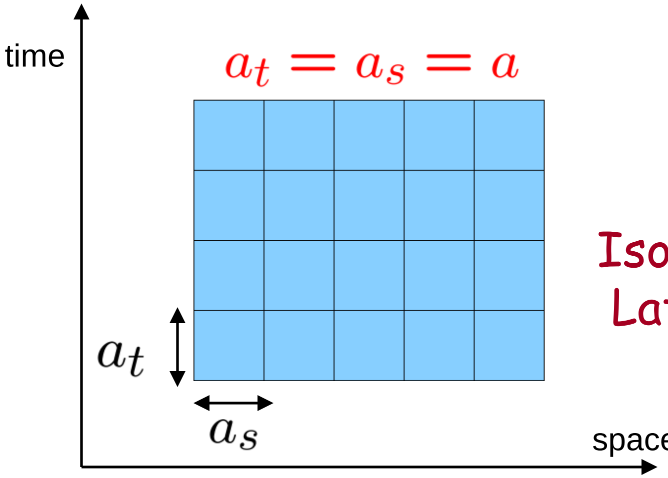


Melting is seen as progressive broadening and disappearance of the bound state peaks

Need to have detailed information on meson correlation functions → large temporal extent N_τ

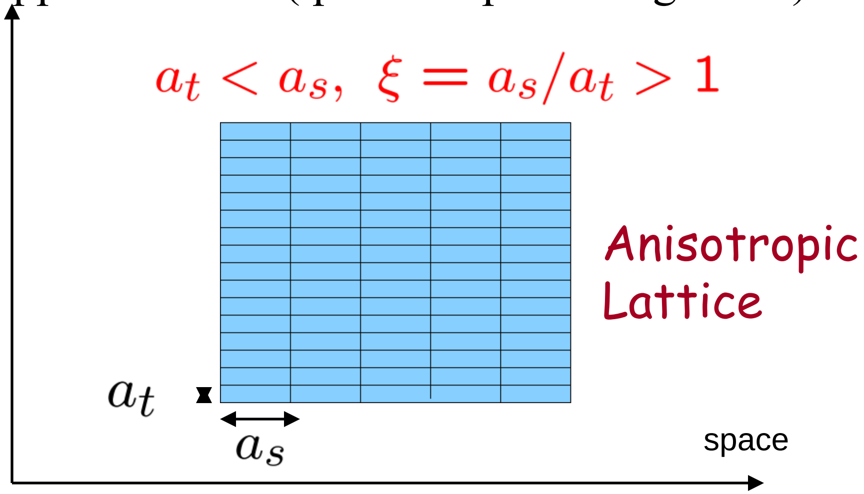
Good control of discretization effects → small lattice spacing a

Computationally very demanding → use quenched approximation (quark loops are neglected)



$$T = \frac{1}{N_t a_t}$$

Isotropic Lattice



An example: charmonium spectral functions in lattice QCD

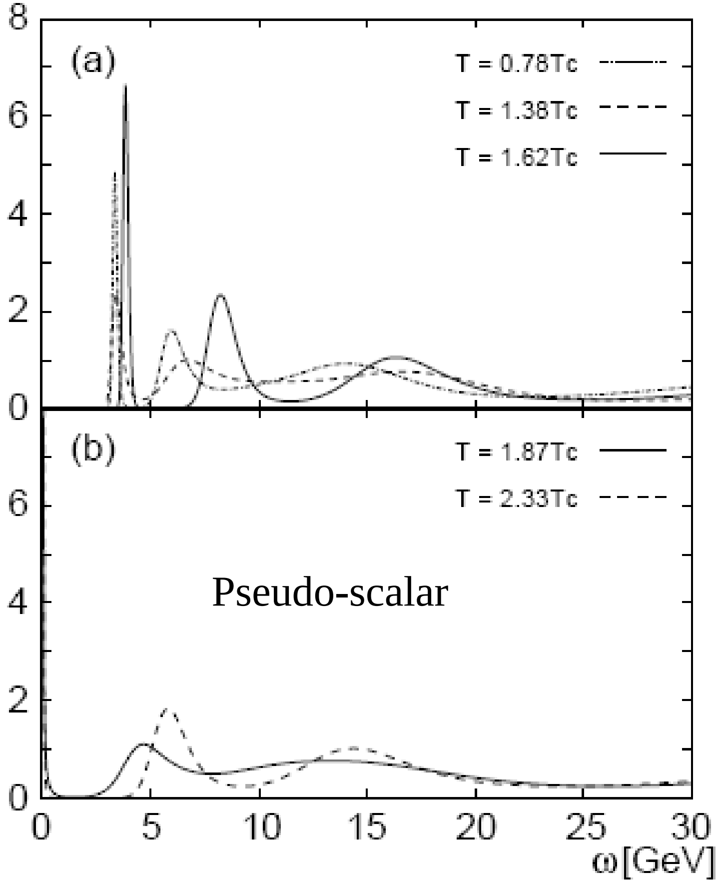
γ_5 : Pseudo – scalar(PS) $\rightarrow \eta_c\ (^1S_0)$

1 : Scalar(SC) $\rightarrow \chi_{c0}\ (^3P_0)$

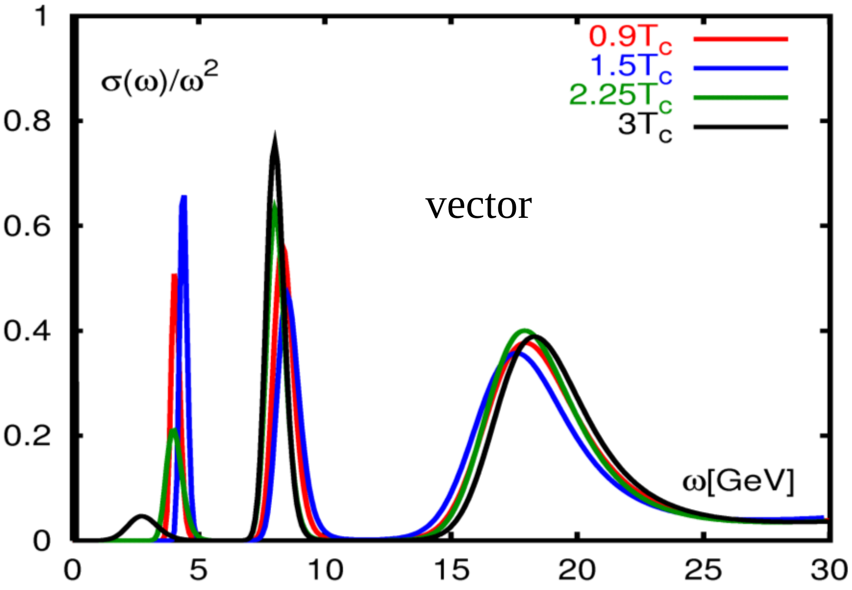
γ_μ : Vector(VC) $\rightarrow J/\psi\ (^3S_1)$

$\gamma_5\gamma_\mu$: Axial – Vector(AX) $\rightarrow \chi_{c1}\ (^3P_1)$

Asakawa, Hatsuda, PRL 92 (2004) 01200



Datta, et al, PRD 69 (04) 094507



1S state charmonia may survive at lease up to *1.6*T_c *??*

see also

Umeda et al, EPJ C39S1 (05) 9, Iida et al, PRD 74 (2006) 074502

Meson spectral functions in the free theory

At high energy ω and high T the meson spectral functions can be calculated using perturbation theory

LO (free theory) :

$$\sigma_i(\omega) = \theta(\omega^2 - 4M^2)\frac{1}{4\pi^2}\omega^2\sqrt{1-\frac{4M^2}{\omega^2}}\left(A_i + B_i\frac{4M^2}{\omega^2}\right)\tanh(\omega/4T) + \chi_i\omega\delta(\omega)$$

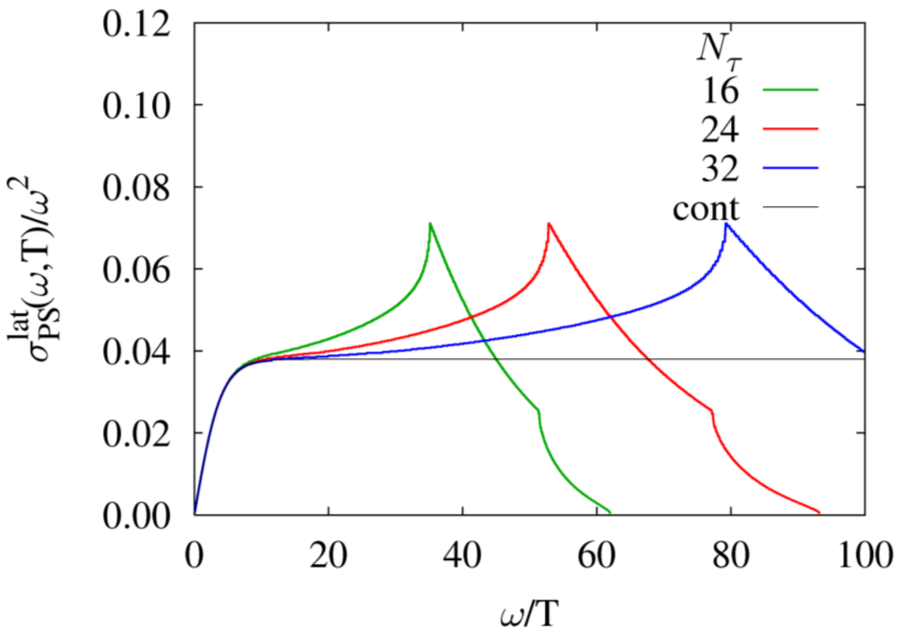
zero mode contribution
→ transport coefficients

$$\chi^i(T) = \frac{6}{\pi^2}\int_0^\infty dp p^2\left(a_i + b_i\frac{M^2}{E_p^2} + c_i\frac{p^2}{E_p^2}\right)\left(-\frac{\partial n_F}{\partial E_p}\right)$$

$$a_{sc} = 0,\ a_{ax} = 1,\ a_{vc} = 0,\ a_{ps} = 0;$$
$$b_{sc} = 1,\ b_{ax} = 2,\ b_{vc} = 0,\ b_{ps} = 0;$$
$$c_{sc} = 0,\ c_{ax} = 0,\ c_{vc} = 1,\ c_{ps} = 0;$$

Karsch et al, PRD 68 (03) 014504
Aarts, Martinez Resco NPB 726 (05) 93

The free spectral functions can also be calculated on the lattice using Wilson type fermions



Heavy quark diffusion, linear response and Euclidean correlatots

Linear response :

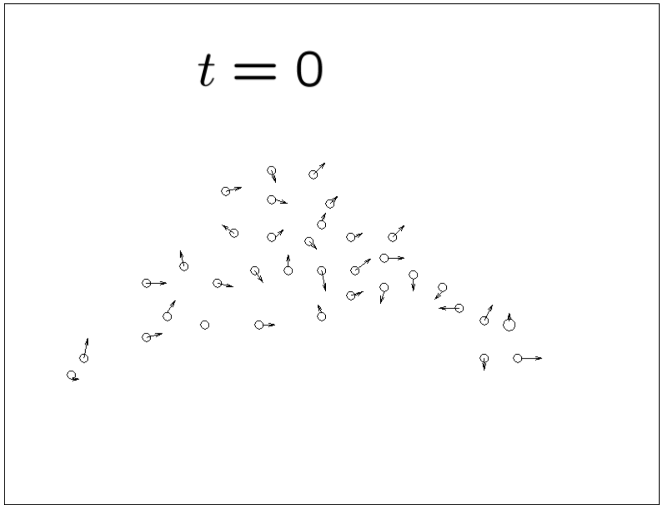
$$H = H_0 - \int d^3x \mu(x,t) N(x,t), \;\; N(x,t) = \bar{q}(x,t)\gamma_0 q(x,t), \; \mu(x,t) = e^{\epsilon t}\theta(-t)\mu(x)$$

$$\langle \delta N(x,t) \rangle = \int_{-\infty}^{\infty} dt' D^R_{NN}(x, t'-t)\mu(x,t')$$

$$\sigma_{NN}(k,\omega) = \frac{1}{\pi} \mathrm{Im} D^R_{NN}(k,\omega)$$

$$D^{Rij}_{JJ}(k,\omega) = (\delta_{ij} - \frac{k_i k_j}{k^2}) D^R_{JJ,T}(k,\omega) + \frac{k_i k_j}{k^2} D^R_{JJ,L}(k,\omega)$$

$$\frac{\omega^2}{k^2} D^R_{NN}(k,\omega) = D^R_{JJ,L}(k,\omega)$$



Euclidean correlators:

P.P., Teaney, PRD 73 (06) 014508

$$G^{00}(k,\tau) = \int d^3x e^{i\mathbf{kx}} \langle J^0_E(x,\tau) J^0_E(0,0)\rangle = -D_{NN}(k,-i\tau) = -\int_0^{\infty} d\omega \sigma_{NN}(k,\omega) K(\tau,\omega,T)$$

$$G^{ij}(k,\tau) = \int d^3x e^{i\mathbf{kx}} \langle J^i_E(x,\tau) J^j_E(0,0)\rangle = D^{ij}_{JJ}(k,-i\tau) = \int_0^{\infty} d\omega \sigma^{ij}_{JJ}(k,\omega) K(\tau,\omega,T)$$

$$K(\tau,\omega,T) = \frac{\cosh(\omega(\tau - 1/(2T)))}{\sinh(\omega/(2T))}$$

Correlators and diffusion

P.P., Teaney, PRD 73 (06) 014508

$$\sigma_{NN,JJ}(k,\omega) = \sigma^{\text{high}}_{NN,JJ}(k,\omega) + \sigma^{\text{low}}_{NN,JJ}(k,\omega)$$

$$G^{L,\text{low}}_{JJ}(k,\tau) \simeq 2T\int_0^\infty \frac{d\omega}{\omega}\,\sigma^{L,\text{low}}_{JJ}(k,\omega)\left[1-\frac{1}{6}\left(\frac{\omega}{2T}\right)^2+\omega^2\frac{1}{2}(\tau-\beta/2)^2+\dots\right]$$

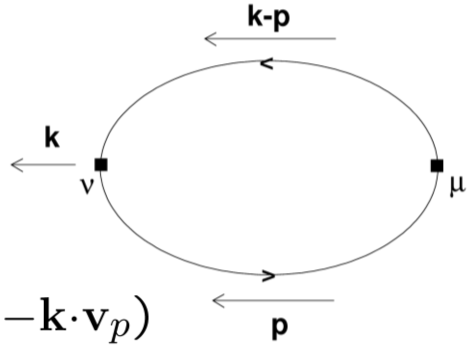
$$G^{L,\text{low}}_{JJ}(k,\tau) = \frac{T}{k^2}\left[\partial_t^{(1)}D^R_{NN}(k,t)+\frac{1}{24\,T^2}\partial_t^{(3)}D^R_{NN}(k,t) - \partial_t^{(3)}D^R_{NN}(k,t)\,\frac{1}{2}(\tau-\beta/2)^2+\dots\right]_{t=0}$$

$$\beta = 1/T$$

Collisionless Boltzmann equation :

$$\left(\frac{\partial}{\partial t}+v^i_p\frac{\partial}{\partial x^i}\right)f(x,p,t)=0$$

Bare 1-loop :



$$\sigma^{\text{low}}_{NN}(k,\omega) = \frac{1}{T}\int\frac{d^3p}{(2\pi)^3}f_p(1\pm f_p)\,\mathbf{k}\cdot\mathbf{v}_p\,\delta(\omega-\mathbf{k}\cdot\mathbf{v}_p)$$

$$\sigma^{\text{low}}_{JJ}(k,\omega) = \chi\frac{\omega^3}{k^2}\frac{1}{\sqrt{2\pi k^2\frac{T}{M}}}\exp\left(-\frac{\omega^2}{2k^2\frac{T}{M}}\right)$$

$\mathbf{k}\to 0$

$$\sigma^{\text{low}}_{JJ}(k,\omega) = \chi\frac{T}{M}\omega\delta(\omega)$$

$$\sigma^{L,\text{low}}_{NN}(k,\omega) = \chi\,\omega\delta(\omega)$$

$$G^{\text{low}}_{JJ}(k,\omega) = T\chi\frac{T}{M}$$

$$G^{\text{low}}_{NN}(k,\omega) = T\chi$$

Correlators and diffusion

$t_{tran} \sim M/T^2 \gg 1$

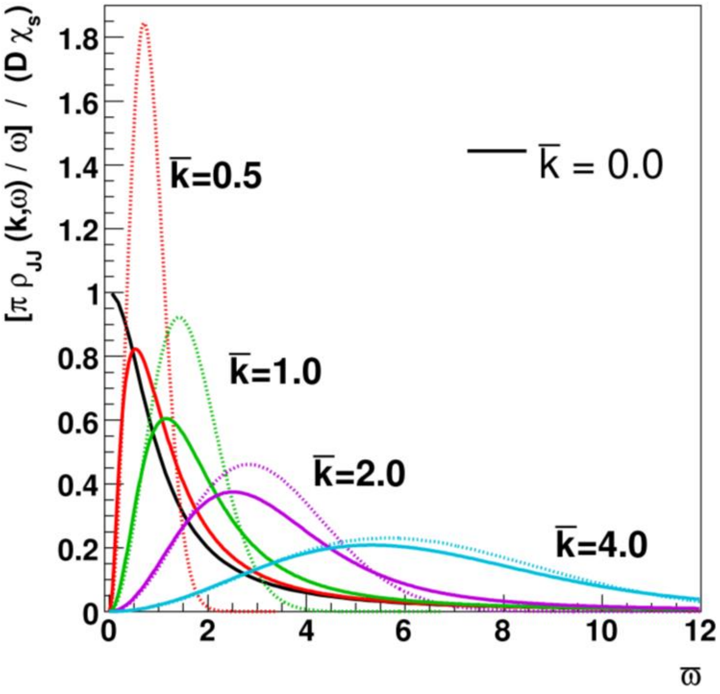
Moore, Teaney, PRC 71 (05) 064904

$$\frac{dx^i}{dt} = \frac{p^i}{M}, \frac{dp^i}{dt} = \xi^i(t) - \eta p^i,$$

$$\langle \xi^i(t) \xi^j(t') \rangle = \kappa \delta^{ij} \delta(t - t')$$

$$\eta = \frac{\kappa}{2MT}, \quad D = \frac{T}{M\eta}$$

$\bar{k} = kD\sqrt{M/T}, \; \bar{\omega} = \omega D(M/T)$



$$t \gg 1/\eta : \partial_t N(x,t) + D\nabla^2 N(x,t) = 0$$

$k \ll \eta\sqrt{M/T}$:

$$D^R_{NN}(k,\omega) = \frac{\chi D k^2}{-i\omega + k^2 D} - \frac{\chi D k^2}{-i\omega + \eta}$$

$$D^R_{NN}(k=0,\omega) = \chi\omega\delta(\omega)$$

$$D^R_{JJ}(k=0,\omega) = \chi\omega\frac{1}{\pi}\frac{T}{M}\frac{\eta}{\omega^2 + \eta^2}$$

P.P., Teaney, PRD 73 (06) 014508

Transport contribution to the Euclidean correlators

Interactions smear out the

$\chi\omega\delta(\omega)$ term

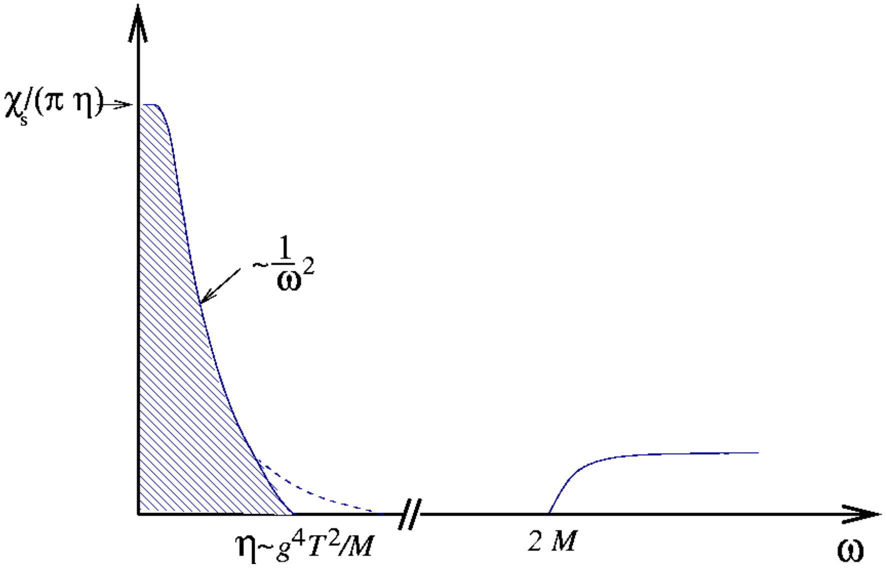
$width \sim \eta$

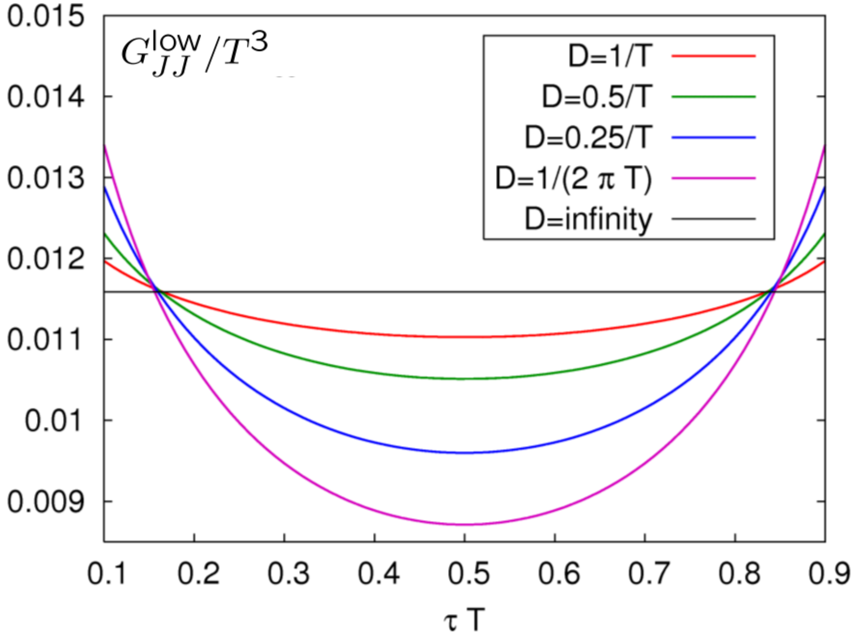
valid only for $\omega < \eta \ll T$ but

$K(\omega,\tau,T) - \frac{2T}{\omega}$ has support for $\omega \sim T$

$G^{\text{low}}_{JJ}(\tau) \neq const$ but has a small curvature around $\tau = 1/(2T)$

$$\sigma^{\text{low}}_{JJ}(\omega) = \chi\omega \frac{1}{\pi}\frac{T}{M}\frac{\eta}{\omega^2+\eta^2}$$





P.P., Teaney, PRD 73 (06) 014508

Reconstruction of the spectral functions : MEM

$$G(\tau,T)=\int_0^\infty d\omega\sigma(\omega,T)\cdot K(\omega,T)$$

$\mathcal{O}(10)$ data and $\mathcal{O}(100)$ degrees of freedom to reconstruct

Bayes' theorem :
P[X|Y]=P[Y|X]P[X]/P[Y]

P[XY]=P[X|Y]P[Y]
=P[Y|X]P[X]

Bayesian techniques: find $\sigma(\omega,T)$ which maximizes

$$P[\sigma|DH]\sim P[D|\sigma H]P[\sigma|H]$$

data

Prior knowledge

prior probability

$H\,:\;\sigma(\omega,T)>0$ ➡ Maximum Entropy Method (MEM): $P[\sigma|H]=e^{\alpha S}$

Asakawa, Hatsuda, Nakahara, PRD 60 (99) 091503, Prog. Part. Nucl. Phys. 46 (01) 459

$$P[\sigma|DH]=P[\sigma|D\alpha m]=\exp(-\frac{1}{2}\chi^2+\alpha S)$$

Likelihood function

Shannon-Janes entropy:

$$S=\int_0^\infty d\omega[\sigma(\omega)-m(\omega)-\sigma(\omega)\ln\frac{\sigma(\omega)}{m(\omega)}]$$

$m(\omega)$ - default model $m(\omega\gg\Lambda_{QCD})=m_0\omega^2$ -perturbation theory

Procedure for calculating the spectral functions

How to find numerically a global maximum in the parameters space of $O(100)$ dimenisons for fixed α ?

$$\sigma(\omega) = m(\omega)\,\text{exp}\left[\sum_{i=1}^{N} s_i u_i(\omega)\right], \quad N \leq N_\tau/2$$

Find the basis $u_i(\omega)$ through

SVD of $K = U\Sigma V,\ u_i(\omega_j) = U_{ji}$

or use $u_i(\omega) = K(\omega, \tau_i)$

Bryan, Europ. Biophys. J. 18 (1990) 165
(Bryan algorithm)

Jakovác, P.P. , Petrov, Velytsky,
PRD 75 (2007) 014506 (J-algorithm)

⇩

maximization of $P[\sigma|D\alpha m]$ reduces to minimization of

$$U = \frac{\alpha}{2}\sum_{i,j=1}^{N} s_i C_{ij} s_j + \sum_{l=0}^{N_\omega} \sigma(\omega_l)\Delta\omega - \sum_{i=1}^{N} \bar{G}(\tau_i) s_i$$

↑ covariance matrix

↑ ensemble average

which can be done using Levenberg-Marquardt algorithm ⇨ $\hat{\sigma}_\alpha$

Procedure for calculating the spectral functions (cont'd)

How to deal with the α-dependence of the result ?

$$\sigma(\omega) = \int d\alpha \ \hat{\sigma}_\alpha(\omega) \ P[\alpha|Dm]$$

For good data $P[\sigma|D\alpha m]$ is sharply peak around $\sigma(\omega) = \hat{\sigma}_\alpha(\omega)$ and using Bayes' theorem

$$\begin{aligned} P[\alpha|Dm] &\sim \int [d\sigma] P[D|\sigma\alpha m] P[\sigma|\alpha m] P[\alpha|m] \\ &\sim P[\alpha|m] \int [d\sigma] \exp\left[-\frac{1}{2}\chi^2 + \alpha S\right] \\ &\sim P[\alpha|m] \exp\left[\frac{1}{2}\sum_k \frac{\alpha}{\alpha+\lambda_k} + \alpha S(\hat{\sigma}_\alpha) - \frac{1}{2}\chi^2(\hat{\sigma}_\alpha)\right] \end{aligned}$$

λ_k are the eigenvalues of $\Lambda_{ll'} = \frac{1}{2}\sqrt{\sigma_l}\frac{\partial\chi^2}{\partial\sigma_l\partial\sigma_{l'}}\sqrt{\sigma_{l'}}|_{\sigma=\hat{\sigma}_\alpha}$ and common choices for $P[\alpha|m]$ are $P[\alpha|m] = const$ and $P[\alpha|m] = 1/\alpha$.

In practice $P[\alpha|Dm]$ is peaked at some α_{max}.

Charmonium spectral functions at T=0

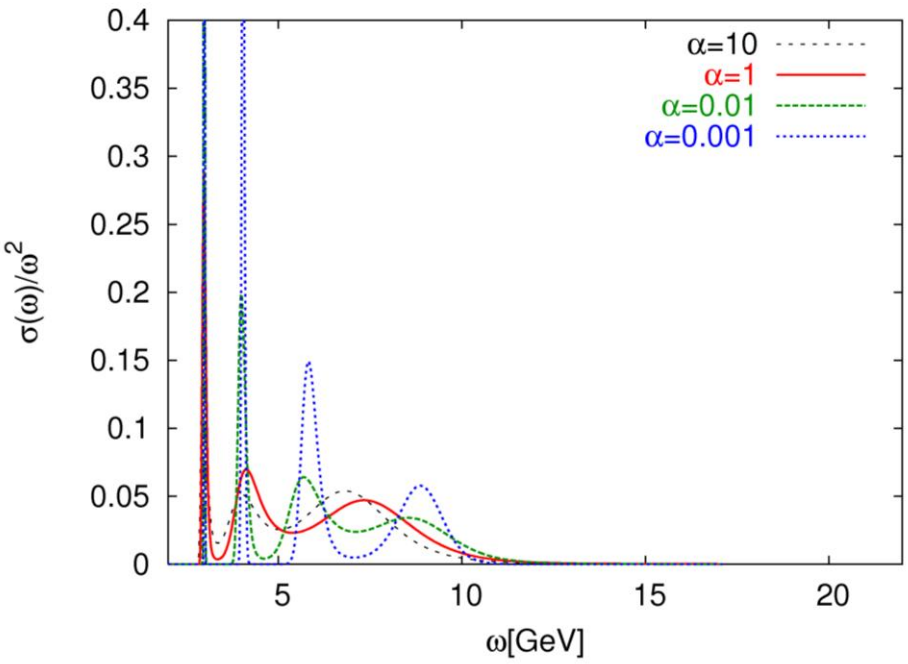
Anisotropic lattices: $16^3 \times 64, \xi = 2\ 16^3 \times 96, \xi = 4,\ 24^3 \times 160, \xi = 4$

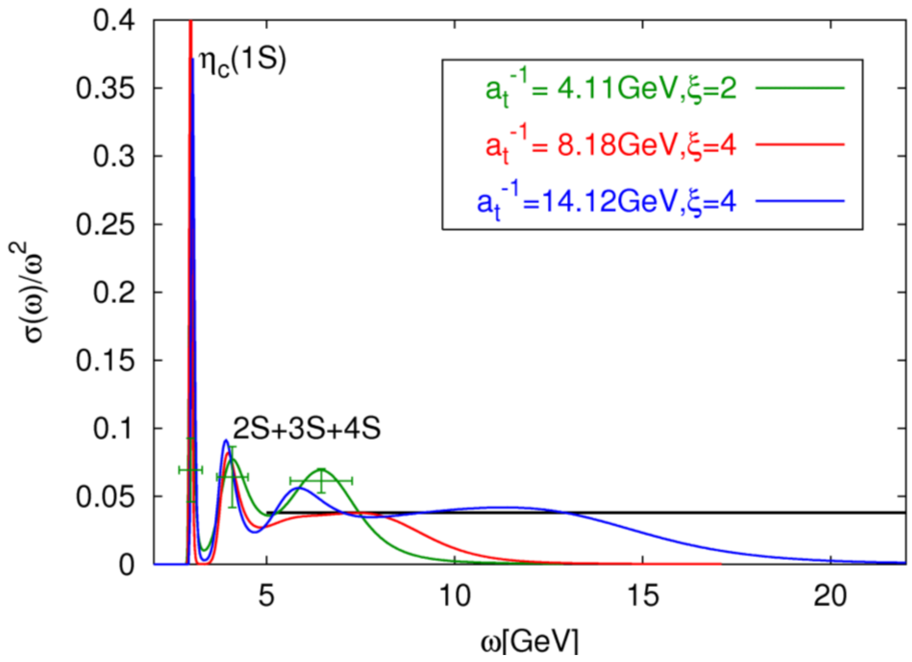
$L_s = 1.35 - 1.54$fm, #configs=500-930;

Wilson gauge action and Fermilab heavy quark action

Pseudo-scalar (PS) → S-states

Jakovác, P.P. , Petrov, Velytsky, PRD 75 (2007) 014506



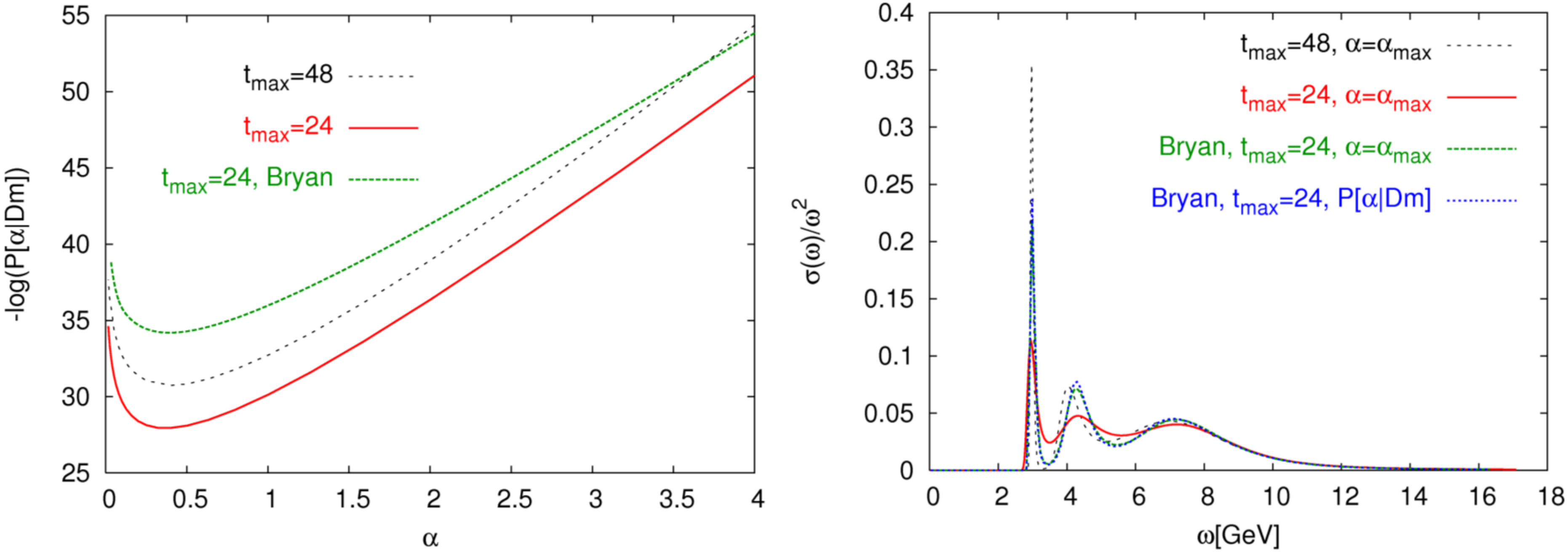


For $\omega > 5$ GeV the spectral function is sensitive to lattice cut-off ;

good agreement with 2-exponential fit for peak position and amplitude

Charmonium spectral functions at T=0 (cont'd)

PS, $16^3 \times 96$, $a_t^{-1} = 8.18$ GeV, $\xi = 4$



$P[\alpha|Dm]$ has a well defined maximum at some α_{max}

Bryan algorithm and the J-algorithm give similar results, but the use of the former is limited $\tau_{max} = 24$.

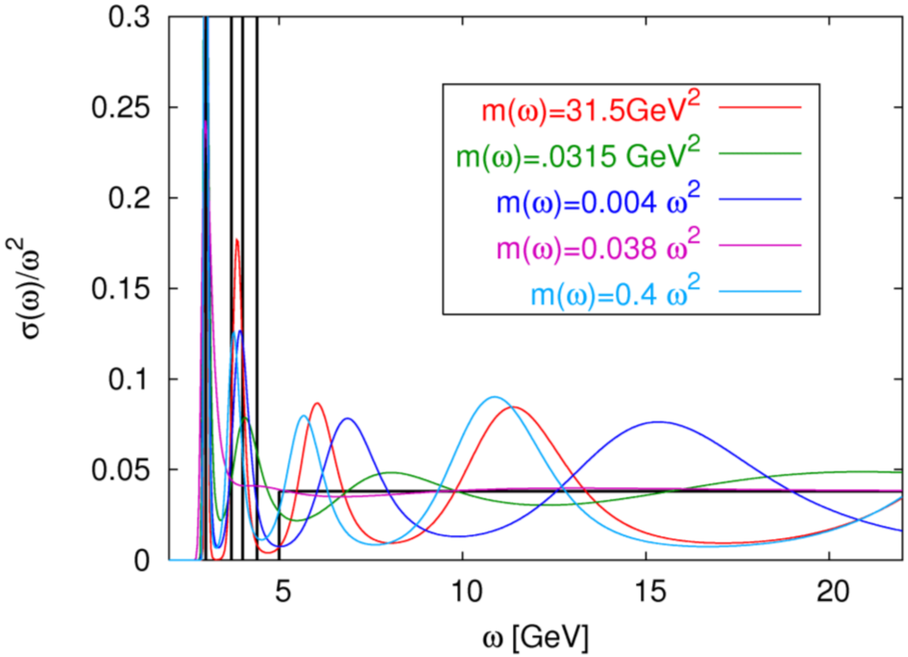
Charmonium spectral functions at T=0 (cont'd)

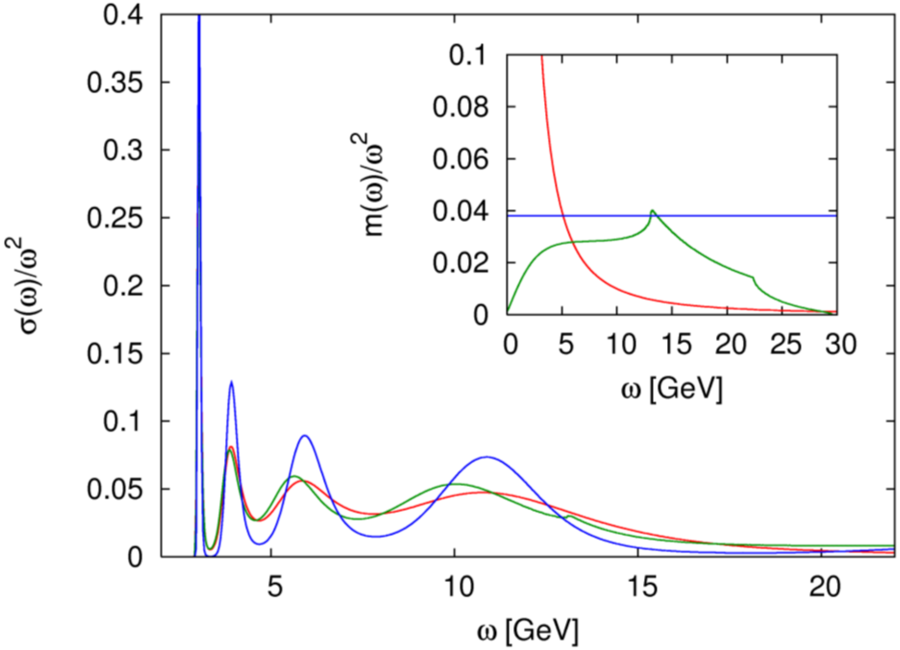
What states can be resolved and what is the dependence on the default model ?

Reconstruction of an input spectral function :

Lattice data in PS channel for:

$$a_t^{-1} = 14.12\text{GeV},\ N_t = 160$$





Ground states is well resolved, no default model dependence;
Excited states are not resolved individually, moderate dependence on the default model;
Strong default model dependence in the continuum region, $\omega > 5$ GeV

Charmonia correlators at T>0

temperature dependene of $G(\tau,T)$

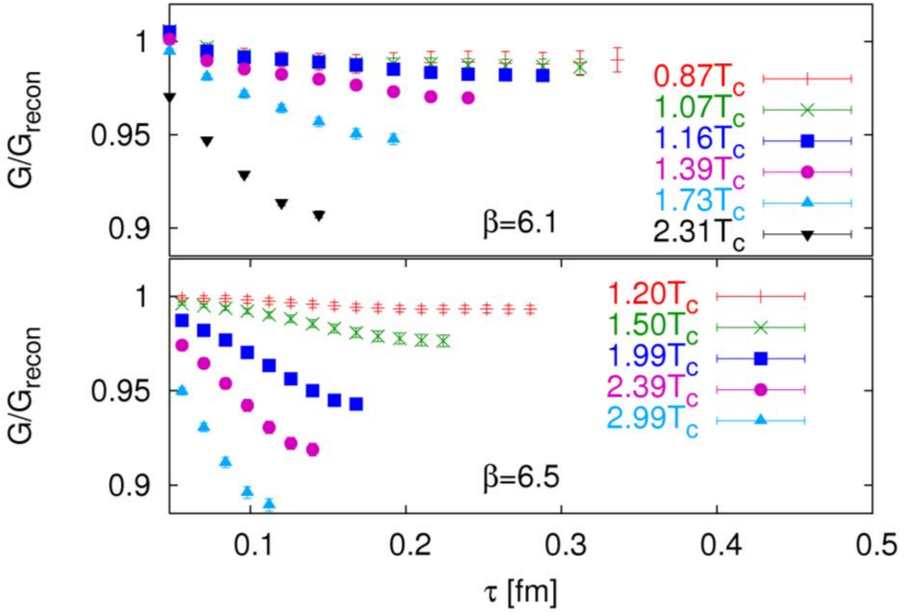
$$G(\tau,T)=\int_0^\infty d\omega\sigma(\omega,T)\frac{\cosh(\omega(\tau-1/(2T))}{\sinh(\omega/(2T))}$$

If there is no T-dependence in the spectral function, $G(\tau,T)/G_{recon}(\tau,T)=1$

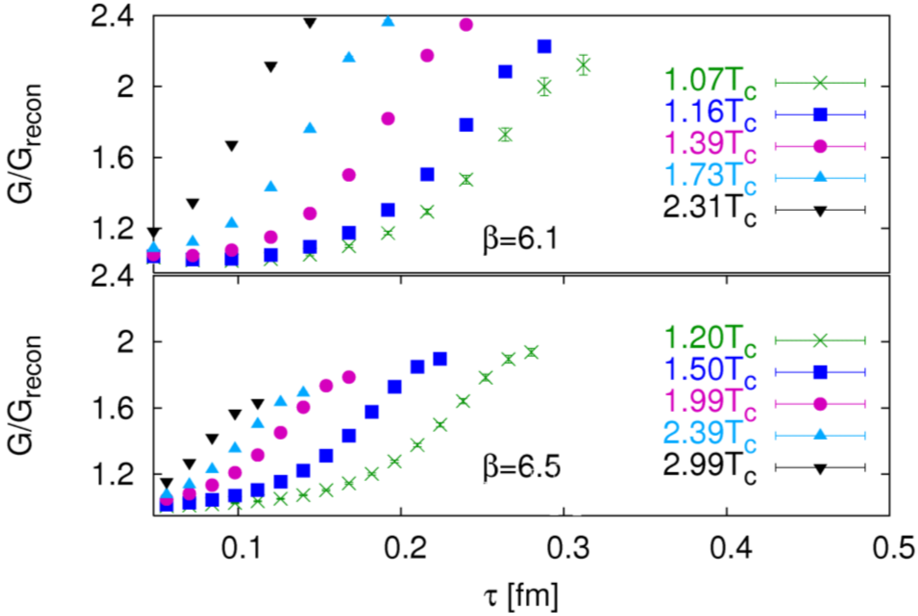
$$G_{recon}(\tau,T)=\int_0^\infty d\omega\sigma(\omega,T=0)\frac{\cosh(\omega\cdot(\tau-\frac{1}{2T}))}{\sinh(\omega/(2T))}$$

Jakovác, P.P. , Petrov, Velytsky, PRD 75 (2007) 014506

PS, $\Gamma_H=\gamma_5$

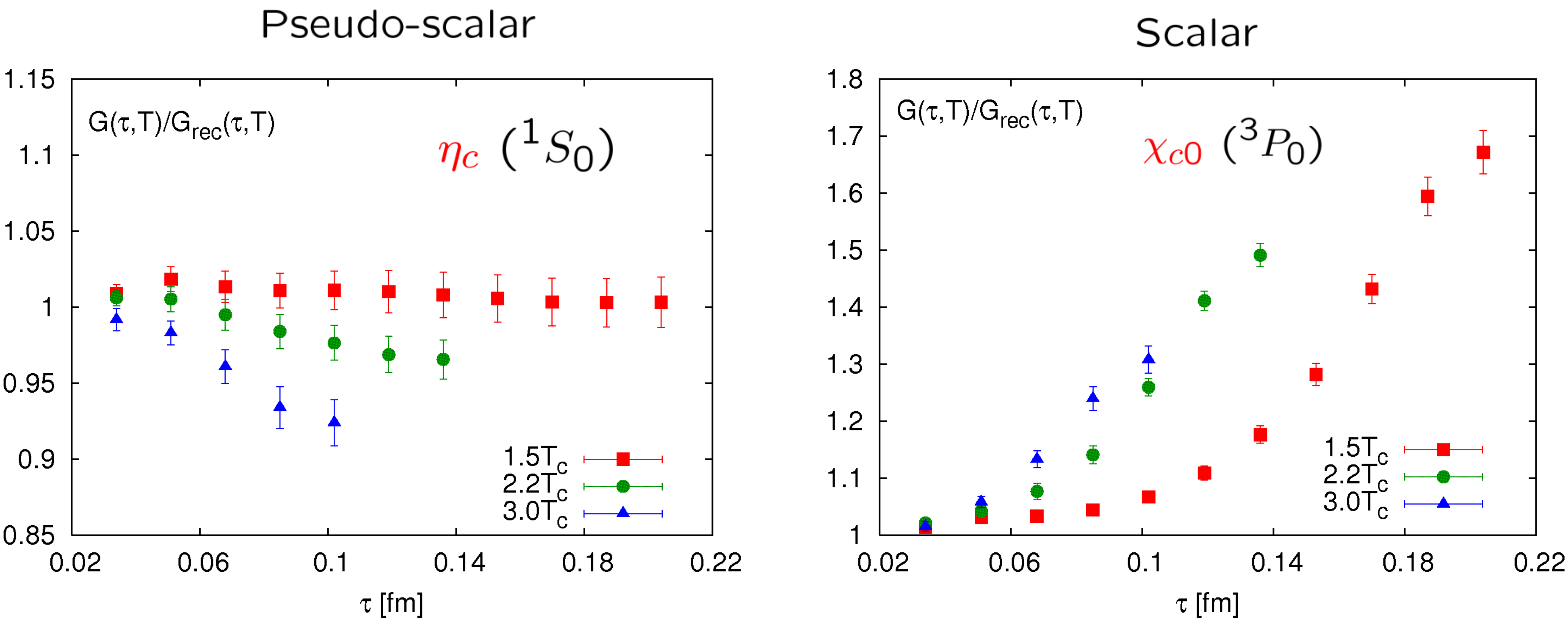


SC, $\Gamma_H=1$



in agreement with calculations on isotropic lattice: Datta et al, PRD 69 (04) 094507

Temperature dependence of quarkonium

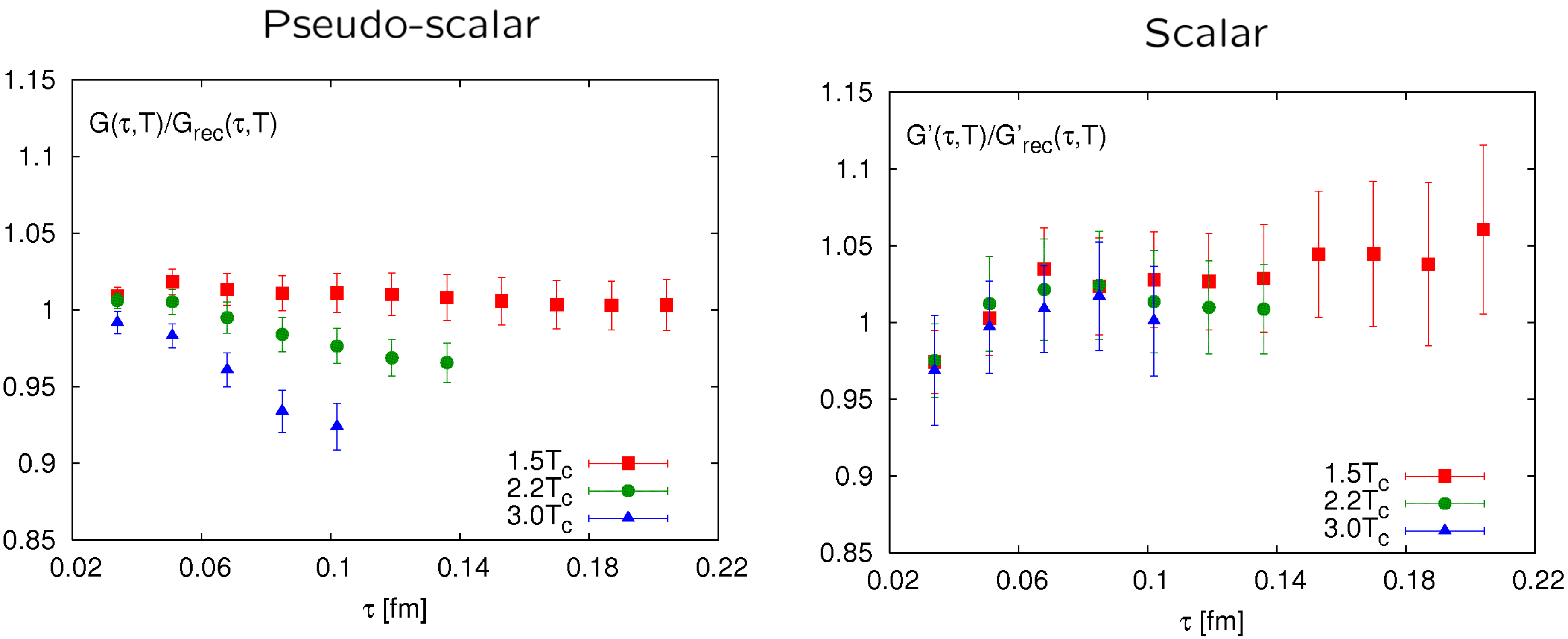


Datta, Karsch, P.P , Wetzorke, PRD 69 (2004) 094507

zero mode contribution is not present in the time derivative of the correlator

Umeda, PRD 75 (2007) 094502

Temperature dependence of quarkonium

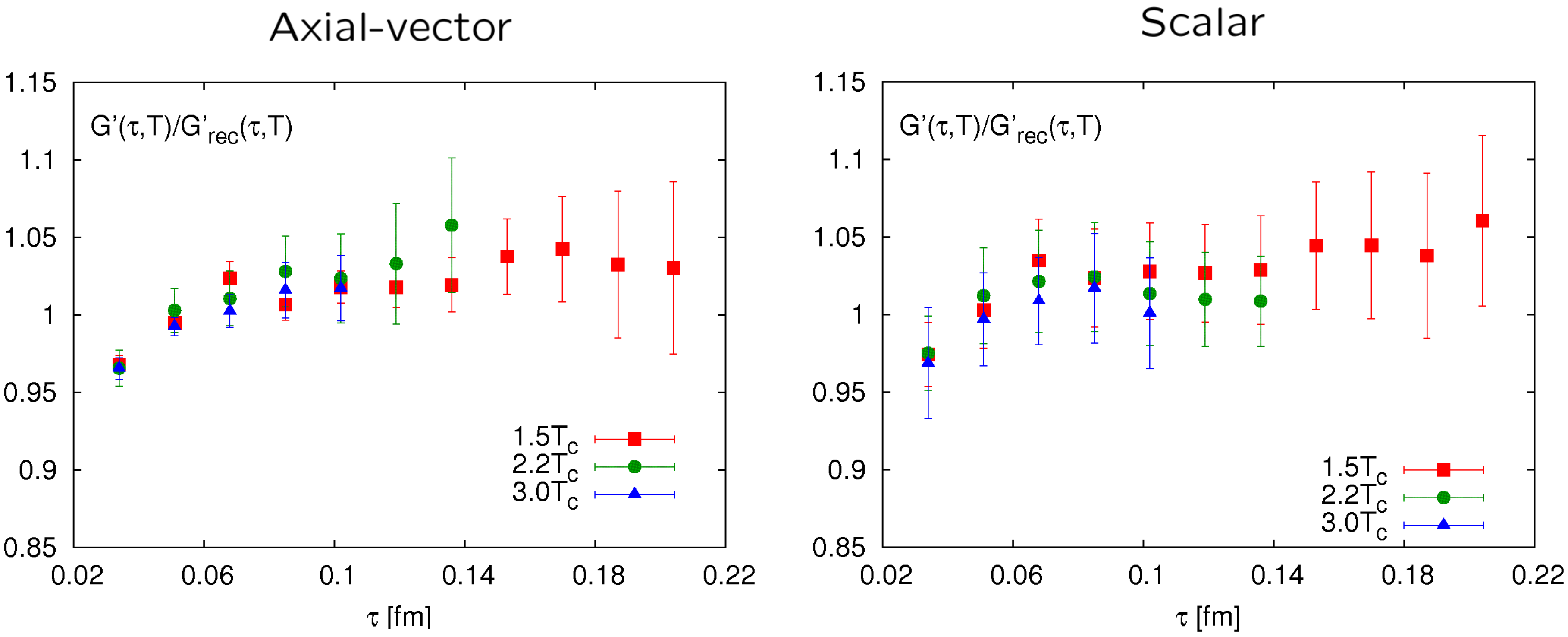


No change in the derivative of the scalar quarkonium correlator up to $3T_c$!

Almost the entire temperature dependence of the scalar correlators is given by the zero mode contribution !

In agreement with previous findings: Mócsy, P.P, PRD 77 (2008) 014501
Umeda, PRD 75 (2007) 094502

Temperature dependence of quarkonium



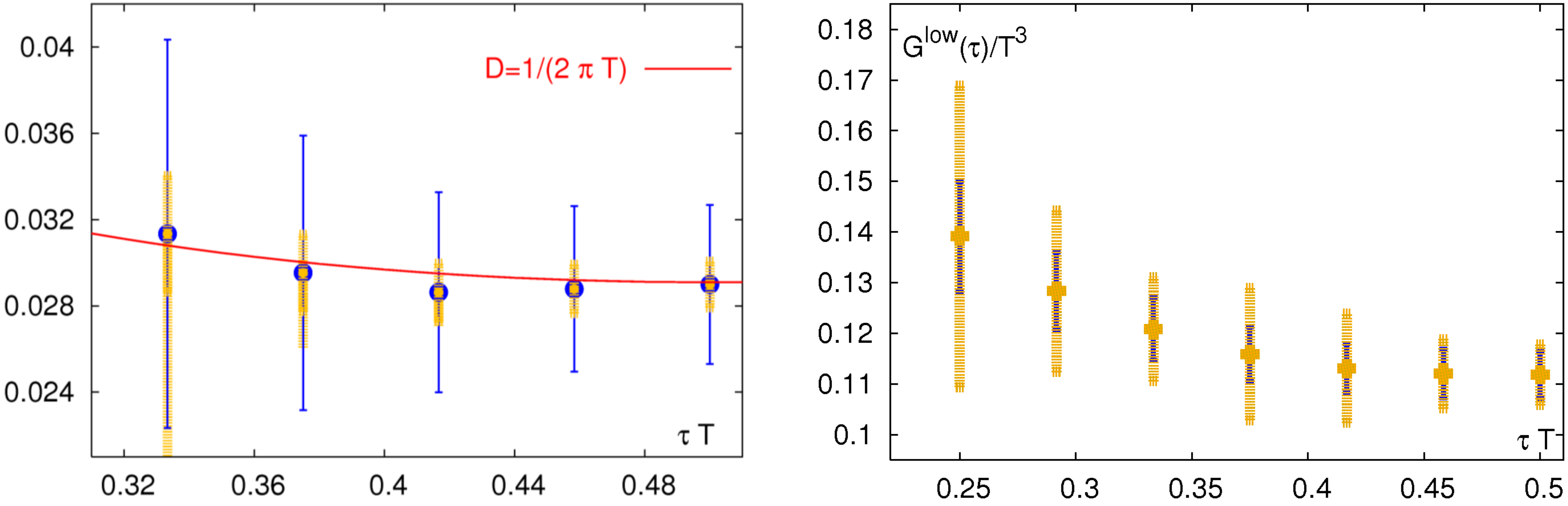
The situation is the same in the axial-vector correlator

In agreement with previous findings: Mócsy, P.P, PRD 77 (2008) 014501
Umeda, PRD 75 (2007) 094502

Estimating the zero mode contribution

$$G^{\text{low}}(\tau,T) = G(\tau,T) - G_{rec}(\tau,T)$$

vector $1.5T_c$ axial-vector



The curvature of $G_i^{\text{low}}(\tau,T)$ is governed by heavy quark diffusion
No diffusion $(D=\infty \leftrightarrow \eta=0)$: $G_i^{\text{low}} = const = T\chi_i(T)$

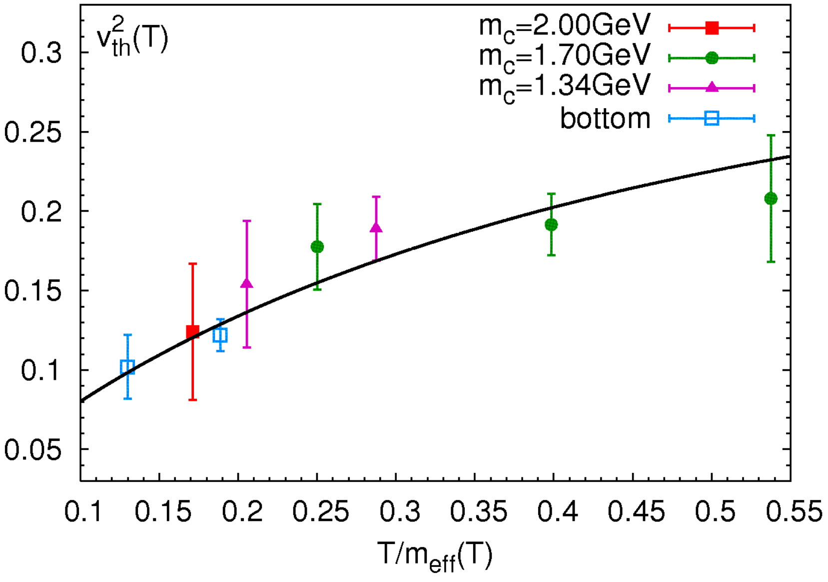
$G_i^{\text{low}}(\tau,T)$ is τ-idependent within errors and
$\chi_i(T) \simeq G_i^{\text{low}}(\tau=1/(2T),T)$

Numerical results on the zero mode contribution

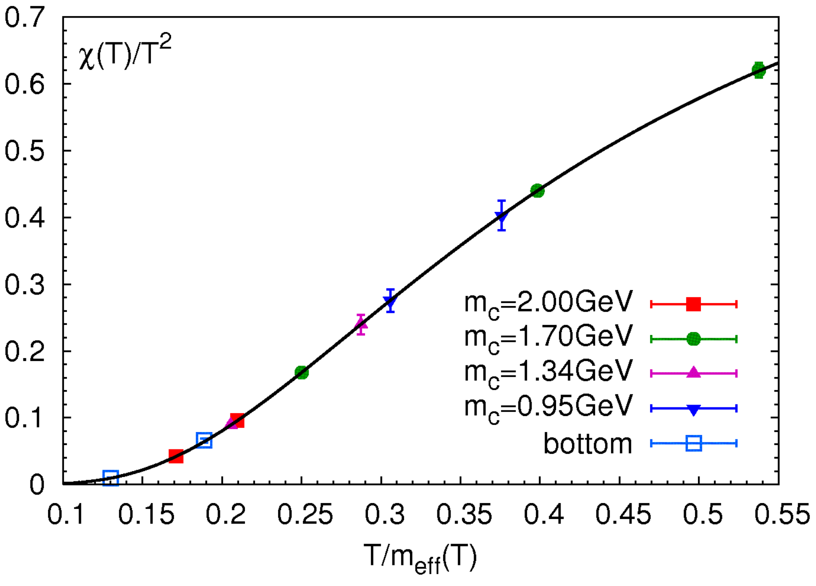
Fit $\chi(T)$ using quasi-particle model with T-dependent effective quark mass

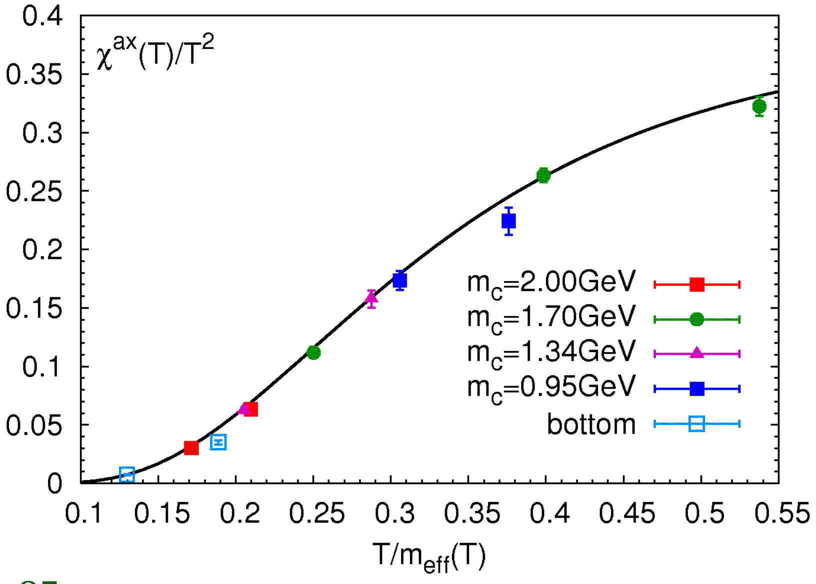
$$\chi(T) = \frac{6}{\pi^2}\int_0^\infty dp p^2 \left(-\frac{\partial n_F}{\partial E_p}\right)$$

$$E_p^2 = p^2 + m_{eff}^2(T)$$



$$G^{low}_{Vii}/G_{00} \simeq \frac{\int d^3p \frac{p_i^2}{E_p^2} e^{-E_p/T}}{\int d^3p e^{-E_p/T}} \simeq v_{th}^2 \simeq T/M$$





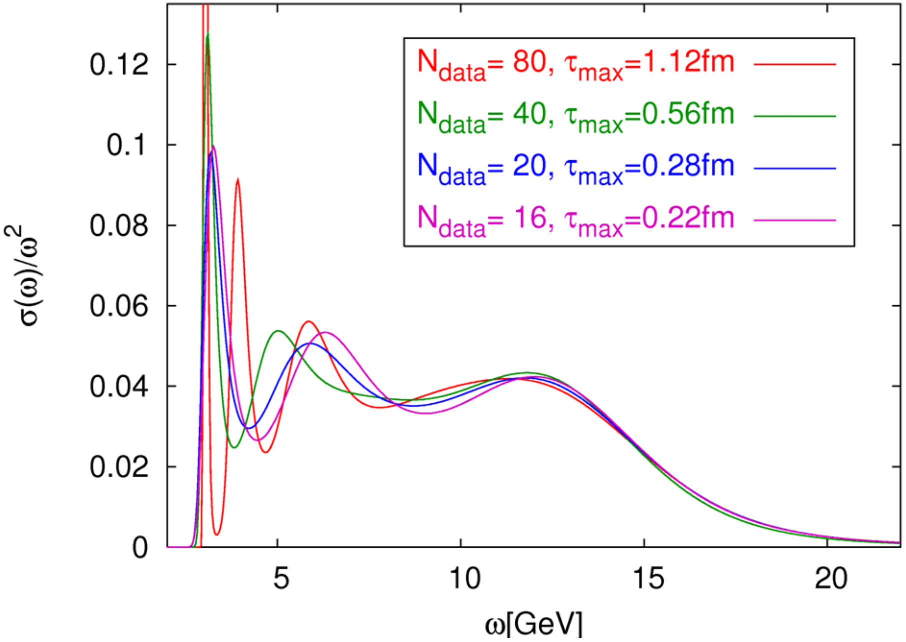
P.P. EPJC 62 (09) 85

Charmonia spectral functions at T>0

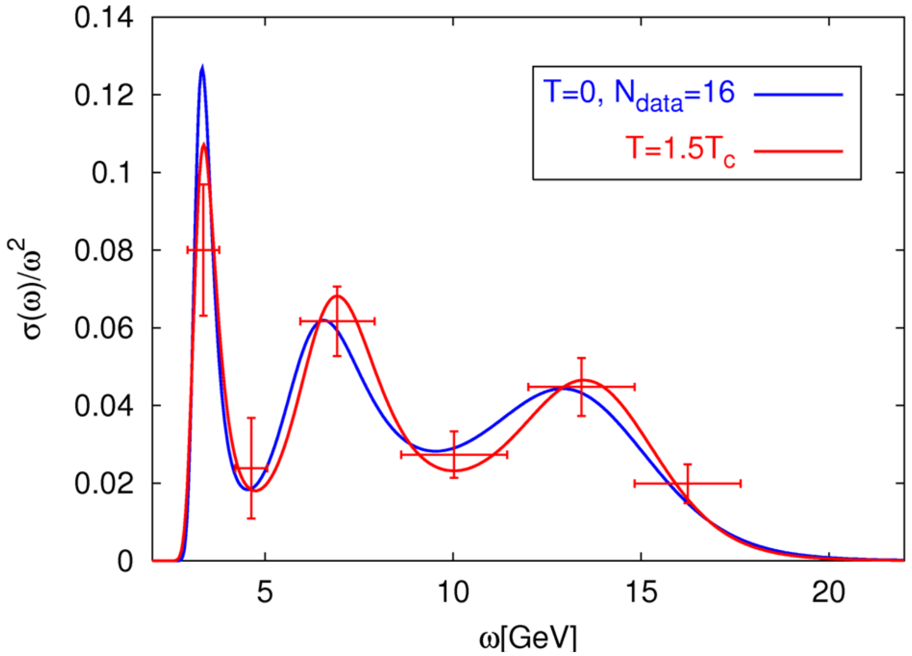
$$T = 1/(N_t a) \quad \leftrightarrow \quad \tau_{max} = 1/(2T)$$

$$\text{PS},\ 24^3 \times N_t,\ a_t^{-1} = 14.12\ \text{GeV},\ \xi = 4$$

$T = 0, \quad N_t = 160$



$T = 1.5 T_c, \quad N_t = 32$

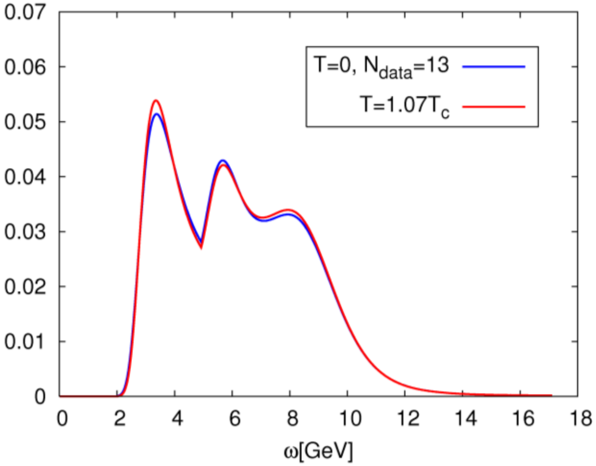


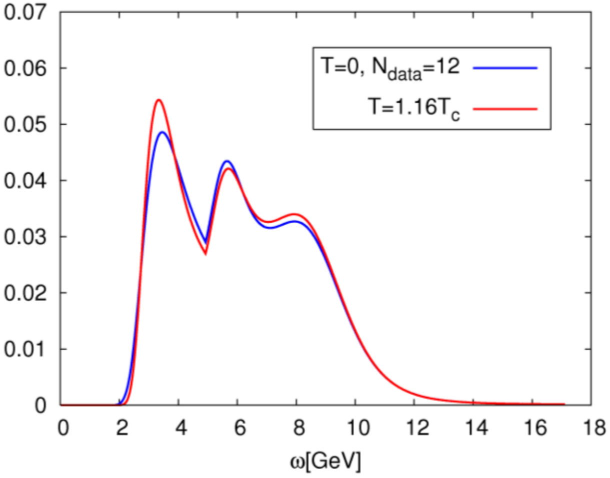
ground state peak is shifted, excited states are not resolved when $\tau_{max}, \quad N_{data}$ become small

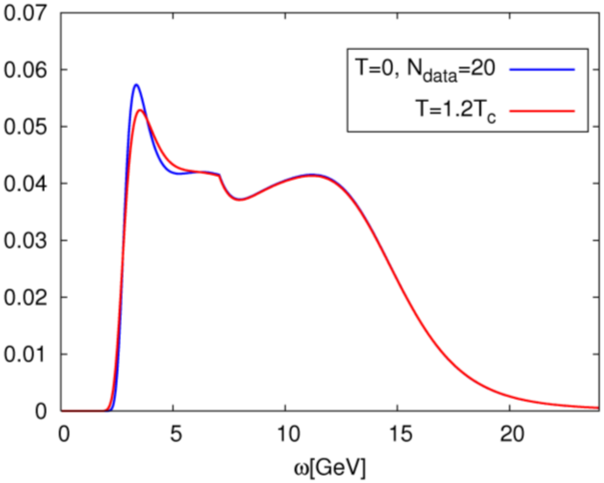
no temperature dependence in the PS spectral functions within errors

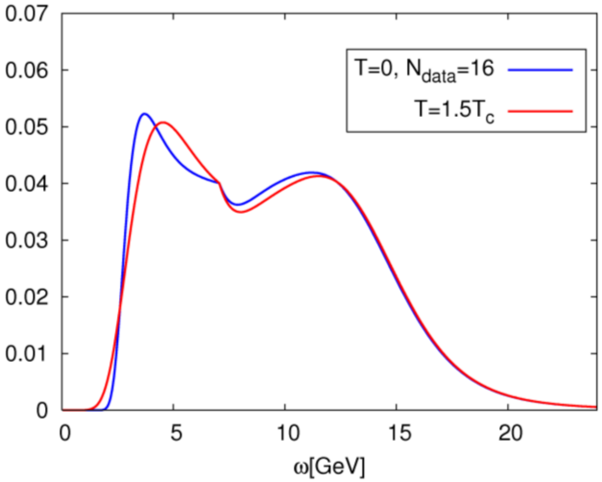
Jakovác, P.P. , Petrov, Velytsky, PRD 75 (2007) 014506

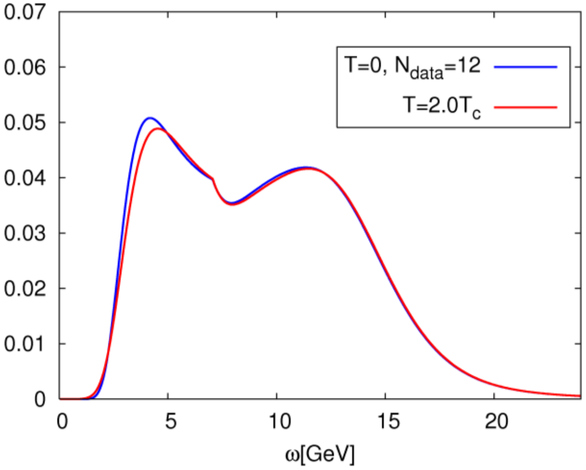
Using default model from the high energy part of the T=0 spectral functions :
resonances appears as small structures on top of the continuum,
little *T*-dependence in the PS spectral functions till $T \simeq 2.4T_c$
but no clear peak structrure

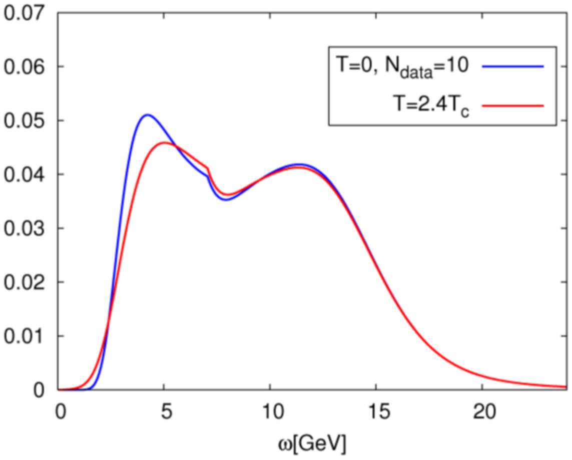








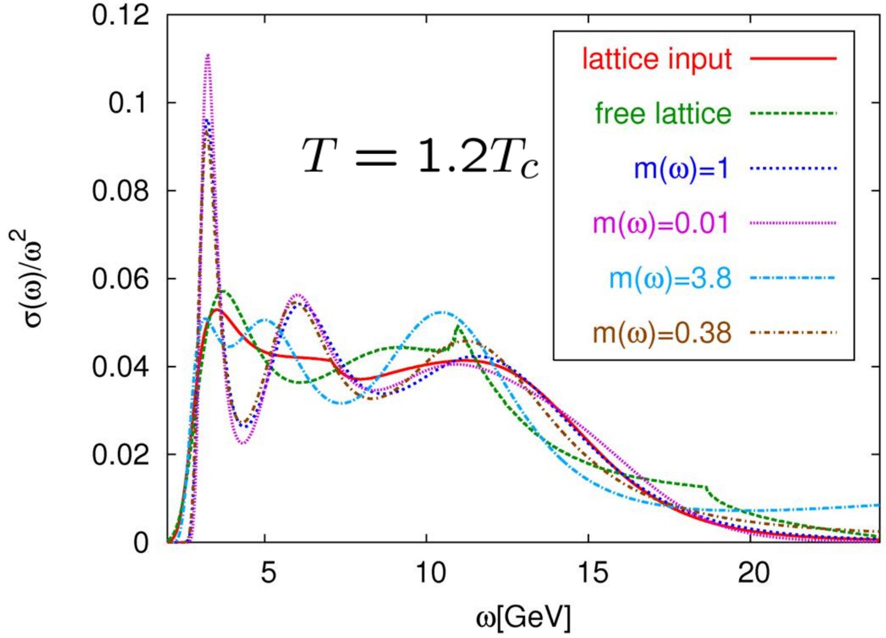




Charmonia spectral functions at T>0 (cont'd)

Jakovác, P.P. , Petrov, Velytsky, PRD 75 (2007) 014506

PS, $24^3 \times 40$, $a_t^{-1} = 14.12$ GeV, $\xi = 4$,

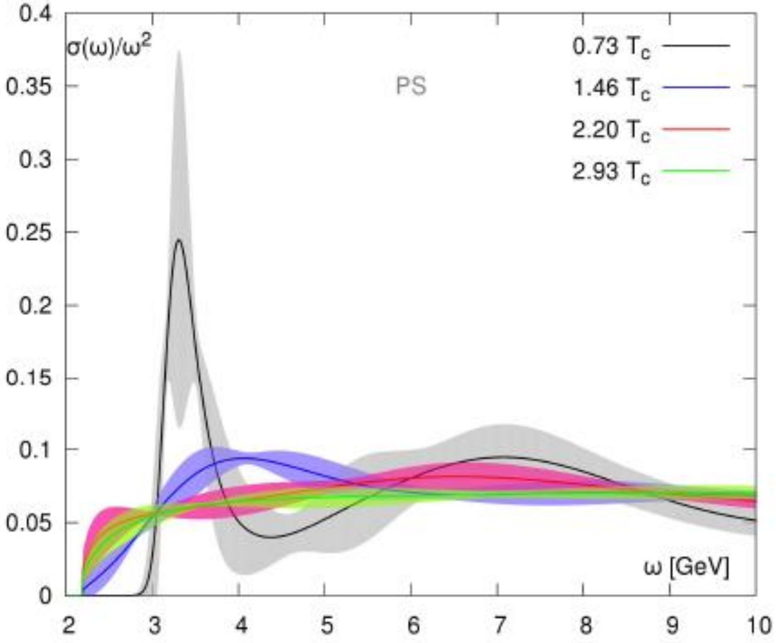


there is a strong dependence on the default model $m(\omega)$ at finite temperature

Ding et al arXiv:1011.0695 [hep-lat]

$128^3 \times N_\tau$, $N_\tau = 96 - 24$

$a_t^{-1} = a_s^{-1} = a^{-1} = 18.97$ GeV



$m(\omega)$:

free lattice spectral functions

with realistic choices of the default model no peaks can be seen in the spectral functions in the deconfined phase !

Spatial charmonium correlators

Spatial correlation functions can be calculated for arbitrarily large separations $z \to \infty$

$$G(z,T) = \int_0^{1/T} d\tau \int dxdy \langle J(\mathbf{x},-i\tau), J(\mathbf{x},0)\rangle_T, \quad G(z \to \infty, T) \simeq A e^{-m_{scr}(T) z}$$

but related to the same spectral functions $G(z,T) = \int_{-\infty}^{\infty} e^{ipz} \int_0^{\infty} d\omega \frac{\sigma(\omega,p,T)}{\omega}$

Low *T* limit :

$$\sigma(\omega,p,T) \simeq A_{mes}\delta(\omega^2 - p^2 - M_{mes}^2)$$

$$A_{mes} \sim |\psi(0)|^2 \to m_{scr}(T) = M_{mes}$$

$$G(z,T) \simeq |\psi(0)|^2 e^{-M_{mes}(T)z}$$

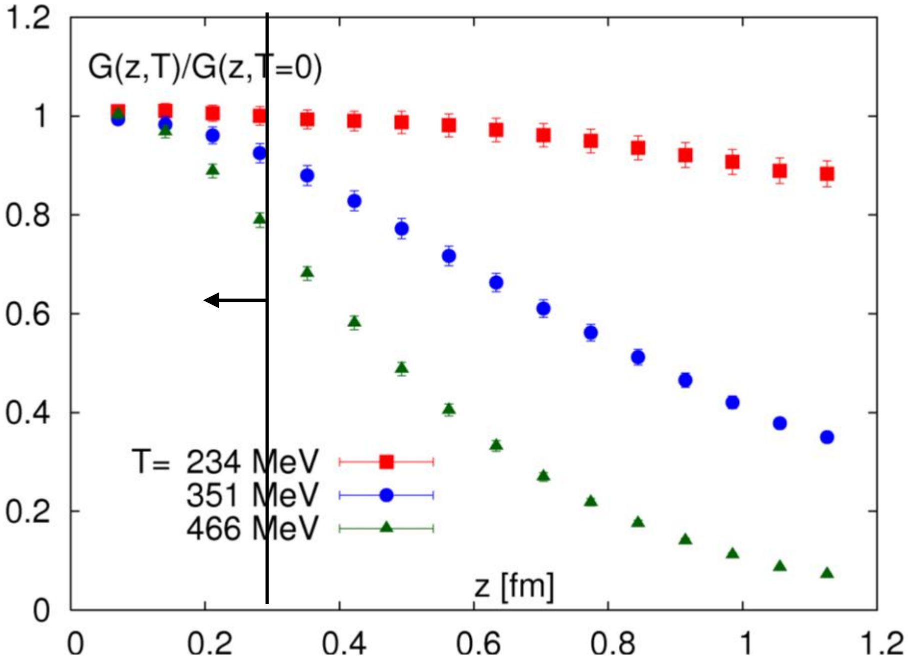
High *T* limit :

$$m_{scr}(T) \simeq 2\sqrt{m_c^2 + (\pi T)^2}$$

p4 action, dynamical *(2+1)-f* *32*^{3}x*8* and *32*^{3}x*12* lattices

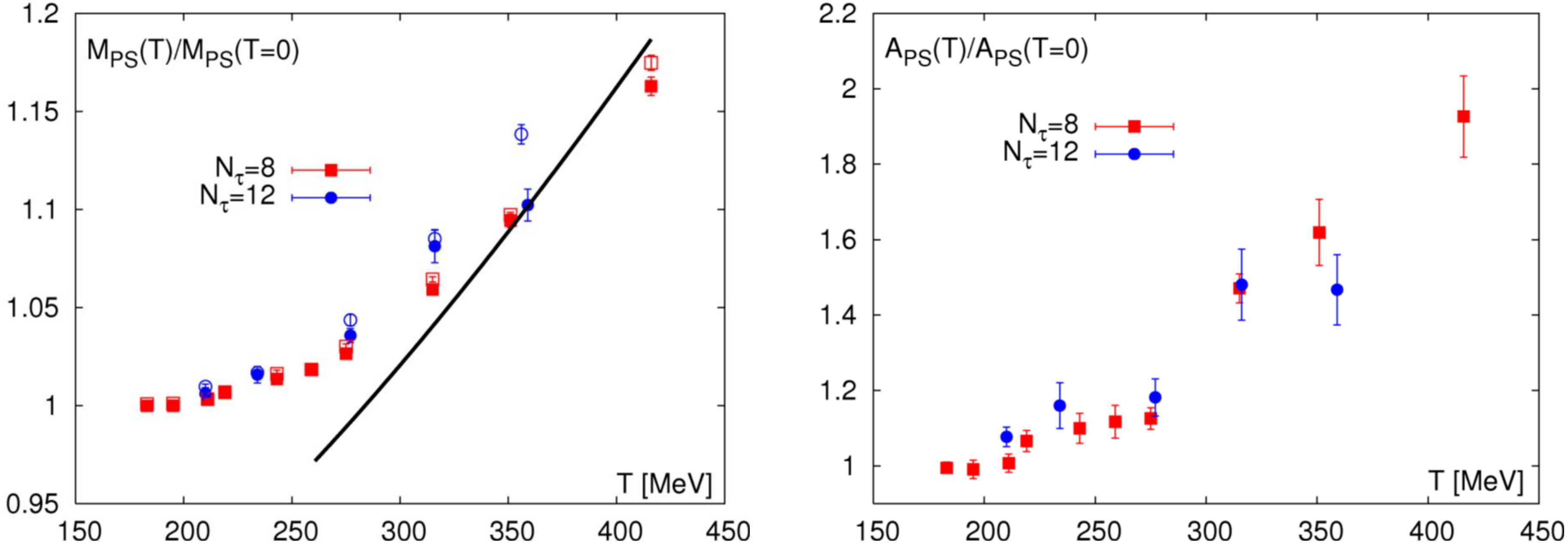
Significant temperature dependence already for *T=234* MeV, large *T*-dependence in the deconfined phase

For small separations *(z T<1/2)* significant *T*-dependence is seen



Spatial charmonium correlators

pseudo-scalar channel => 1S state , point sources: filled; wall sources: open



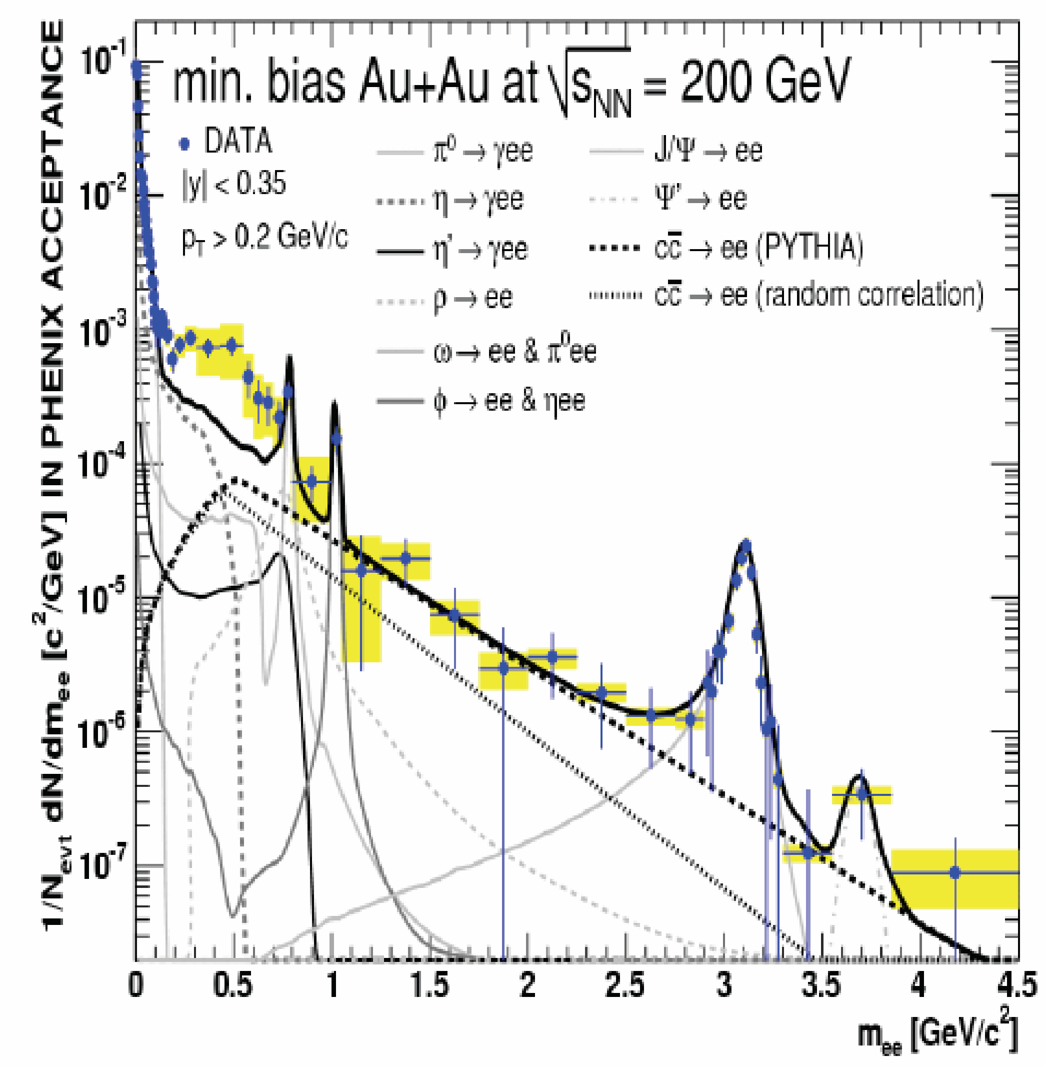
- no T-dependence in the screening masses and amplitudes (wave functions) for **$T<200$ MeV**

- moderate T-dependence for **$200<T<275$ MeV** => medium modification of the ground state

- Strong T-dependence of the screening masses and amplitudes for **$T>300$ MeV**, compatible with free quark behavior assuming **$m_c=1.2$ GeV** => dissolution of 1S charmonium !

Thermal dileptons and light vector meson correlators

PHENIX



Thermal dileptons :

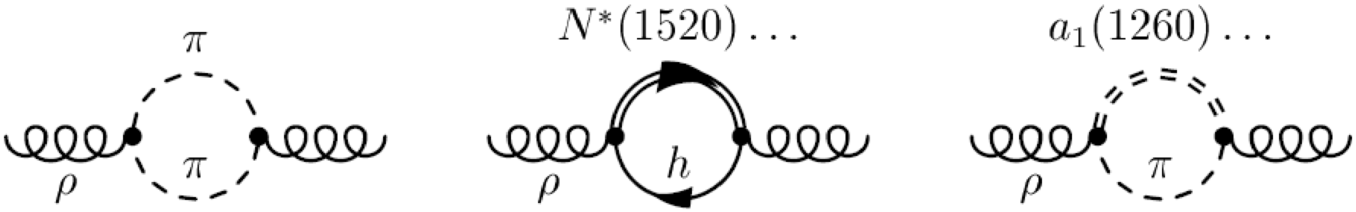
direct measurement of the temperature of the produced matter, test consequences of chiral symmetry restoration

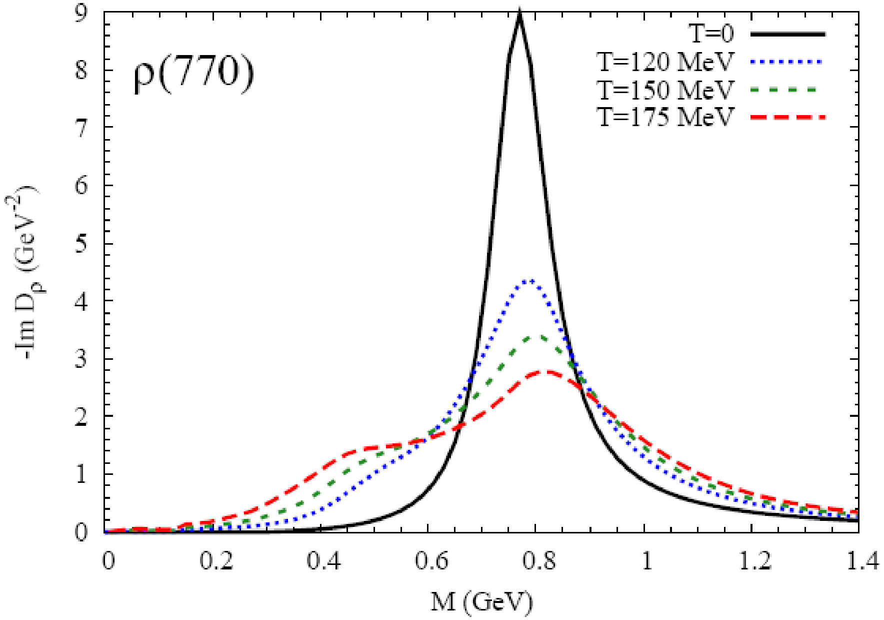
Modifications of the vector spectral functions in hot hadronic matter

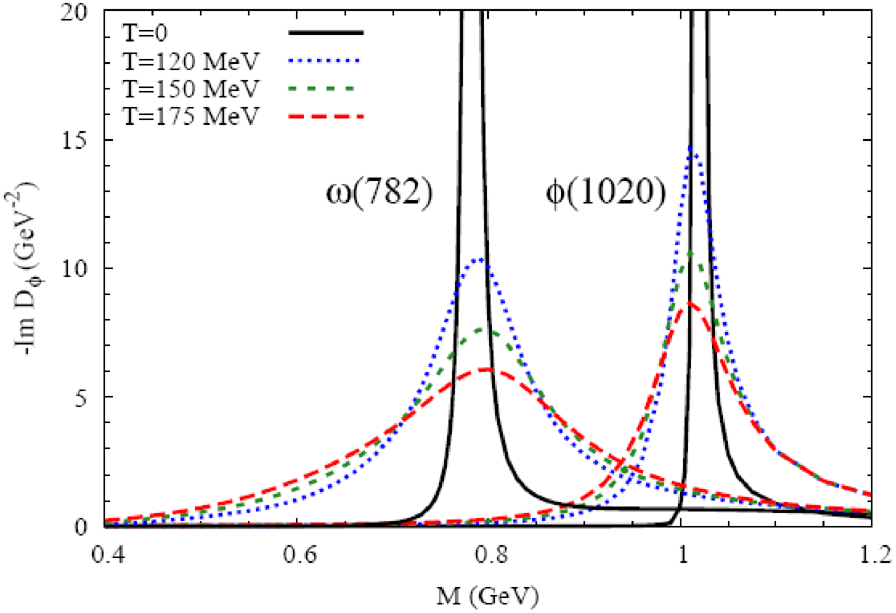
R. Rapp and J. Wambach, Eur. Phys. J. A **6**, 415 (1999).

R. Rapp, M. Urban, M. Buballa, and J. Wambach, Phys. Lett. B **417**, 1 (1998).

R. Rapp, Phys. Rev. C **63**, 054907 (2001).







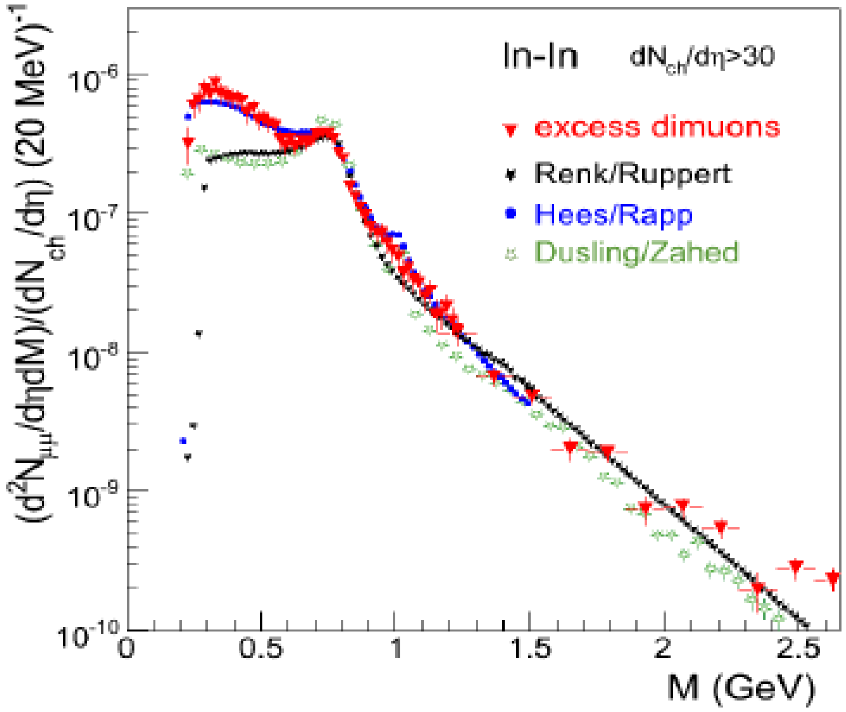
Thermal dileptons at SPS

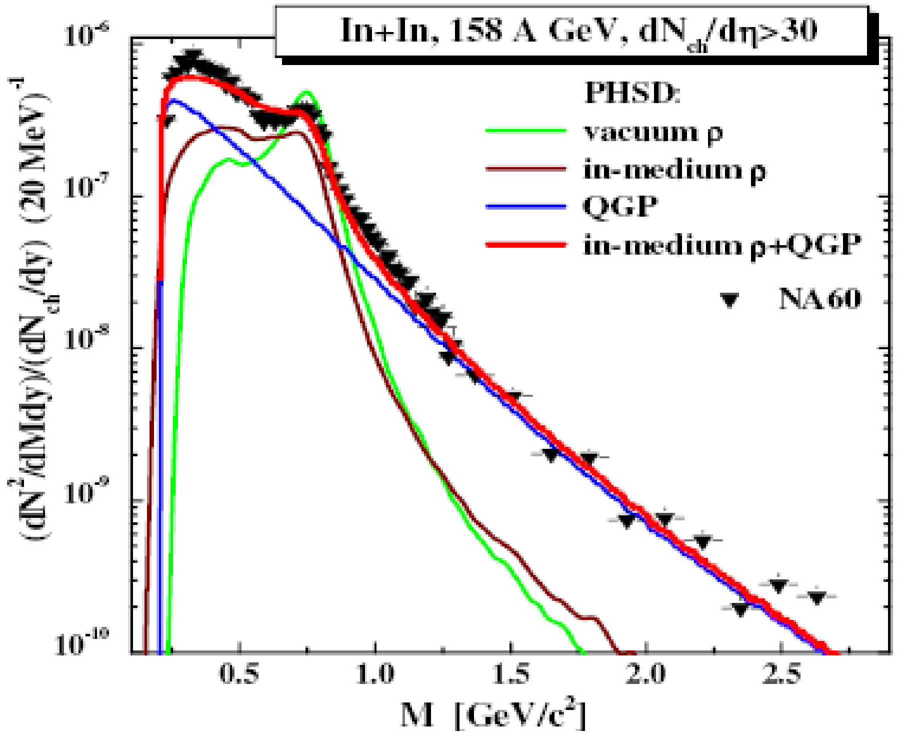
In the low mass region (LMR) excess dileptons are due to the in-medium modivications of the ρ-meson melting induced by baryon interactions

Models which incorporate this (Hess/Rapp and PHSD) can well describe the NA60 data !

NA60 : Eur. Phys. J 59 (09) 607
CERN Courier. 11/2009
fireball models and hydro model (Dusling/Zahed)

Linnyk, Cassing, microscopic transport
PHSD model, talk at Hard Probes 2010



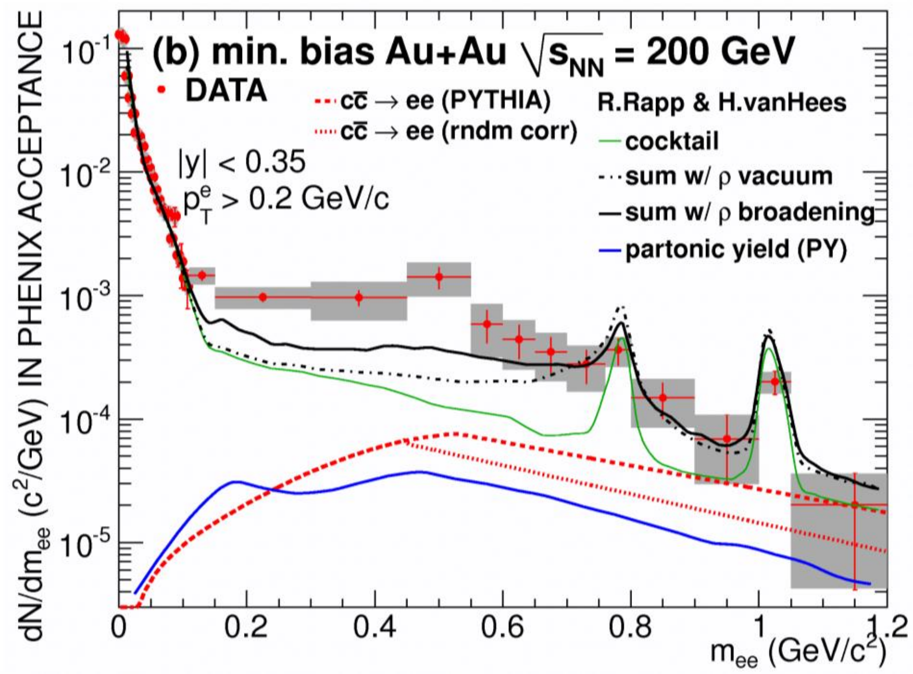


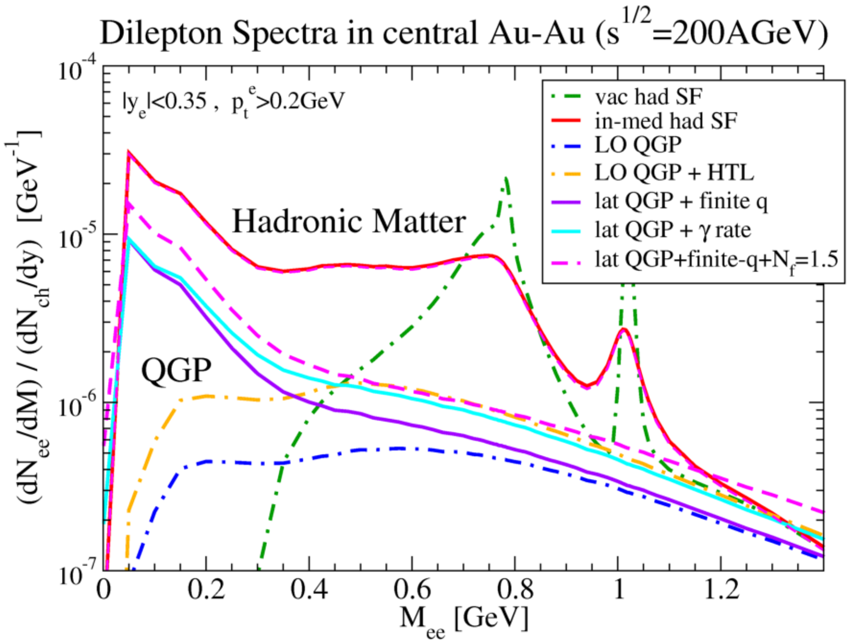
There is also an excess in the intermediate mass region (IMR) which could have partonic origin (D/Z, R/R, PHSD) or hadronic (H/R, $\pi a_1 \rightarrow \mu^+\mu^-$)

Thermal dileptons at RHIC and LMR puzzle

Models that described the SPS dilepton data fails for RHIC in low mass region !

Rapp, arXiV:1010.1719





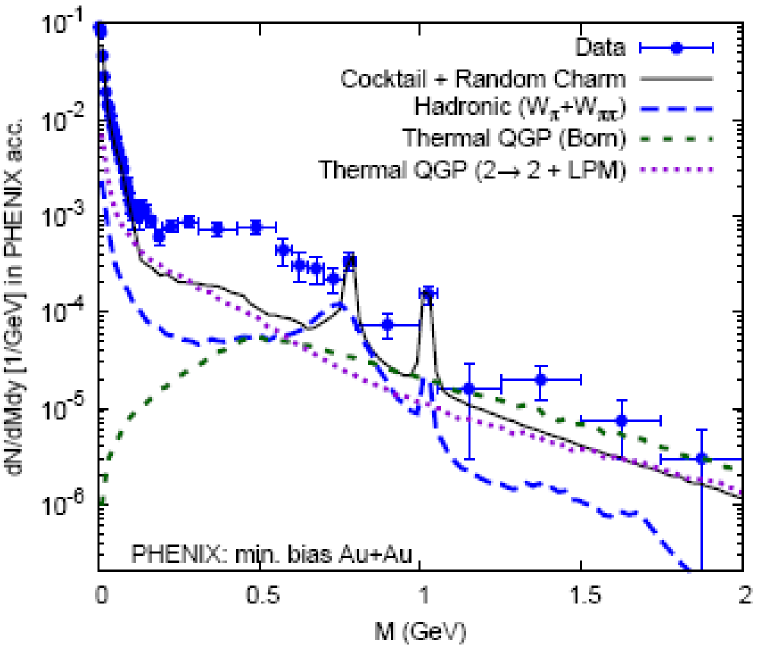
In the low mass region hadronic contribution dominates because of the larger 4-volume but there is large uncertainty in the QGP rate

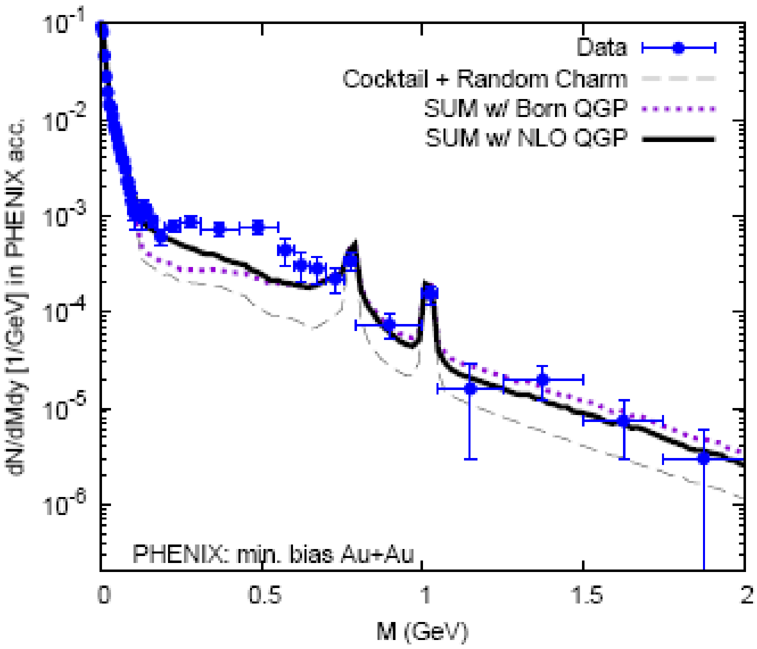
new lattice QCD based estimates are much larger than the perturbative QGP rates but it is not yet clear if this solves the LMR dilepton puzzle

➡ more is going on in the broad transition region (~50MeV from the new lQCD results)

Thermal dileptons at RHIC and unceratinties in the QGP rates

Dusling, Zahed, arXiv:0911.2426





Kinematic effects are important
in the low mass region
NLO QGP rate >> LO (Born) QGP rate
One needs, however, at least an order
of magnitude larger QGP rate to
explain the data

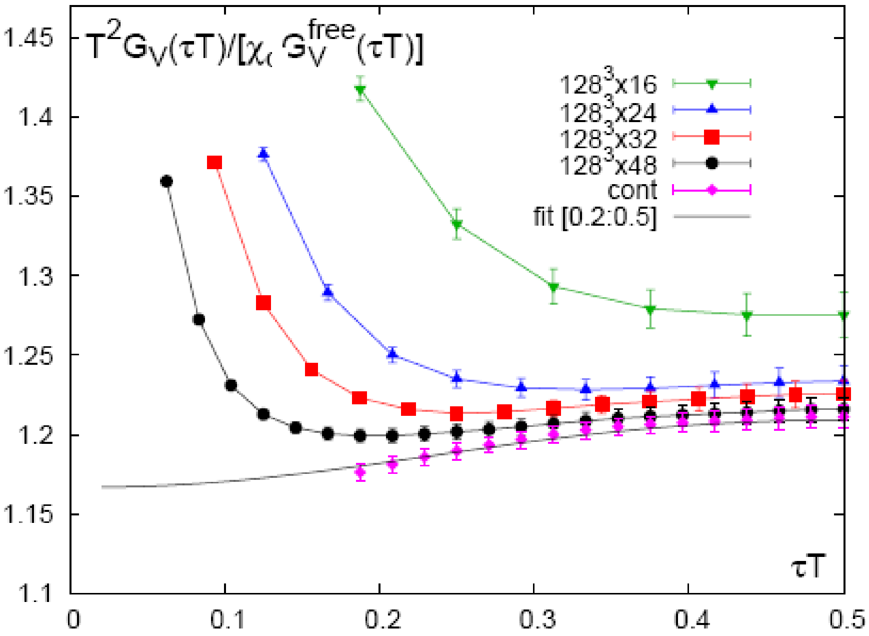
Also in the IMR there is potentially
a factor 2 uncertainty in the QGP rate
Born rate ~ 2x NLO rate

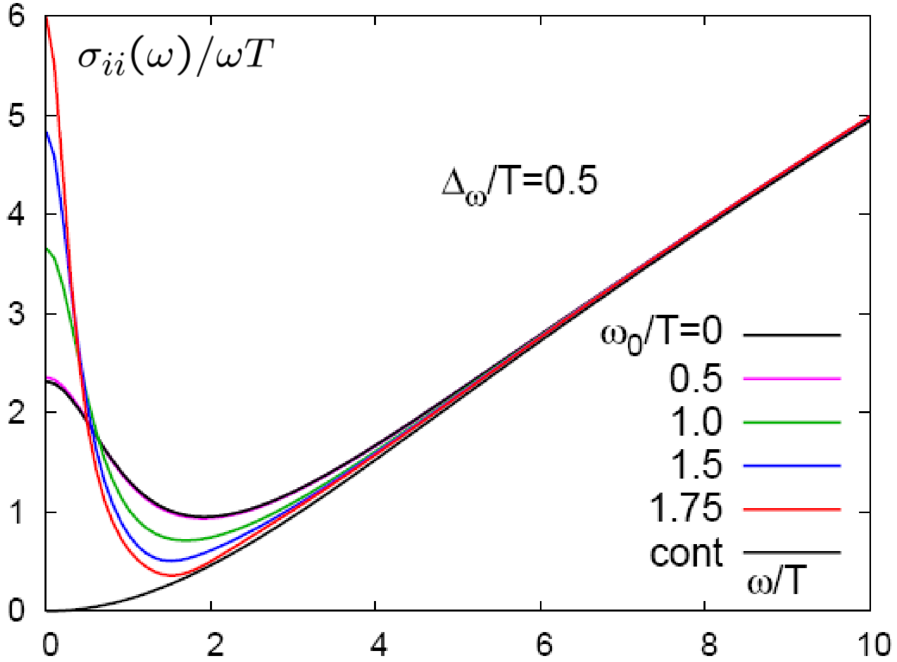
Need to constrain the QGP yield
by lattice QCD

Lattice calculations of the vector spectral functions

Ding et al, PRD 83 (11) 034504

Isotropic Wilson gauge action, quenched non-perturbatively improved clover fermion action on $128^3 \times N_\tau$ lattices, $T = 1.45 T_c$, $m_q^{\overline{MS}}(2\text{GeV}) = 0.1/T$, $N_\tau = 24,\ 32, 48\ (a^{-1} = 9.4 - 18.8\text{GeV})$





$$\sigma_{ii}(\omega) = \chi c_{BW} \frac{1}{\pi} \frac{\omega \Gamma/2}{\omega^2 + (\Gamma/2)^2} + \frac{3}{4\pi^2}(1+k)\omega^2 \tanh(\omega/4T)\Theta(\omega_0, \Delta_\omega),$$

$$\Theta(\omega_0, \Delta_\omega) = (1 + e^{(\omega_0^2 - \omega^2)/\omega\Delta_\omega})^{-1}$$

Fit parameters : $c_{BW},\ \Gamma,\ k$

Different choices of : ω_0, Δ_ω

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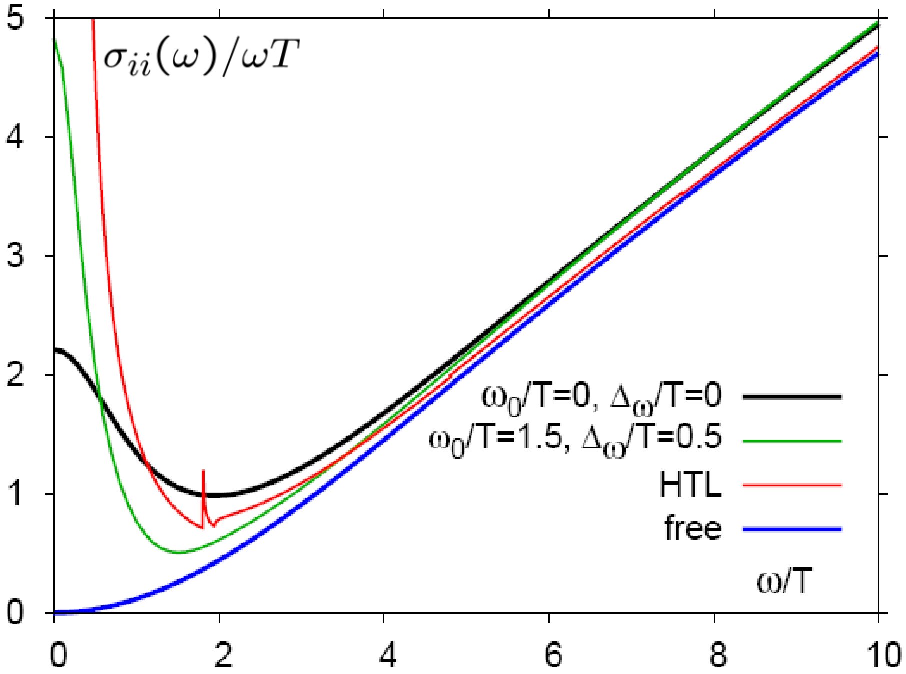
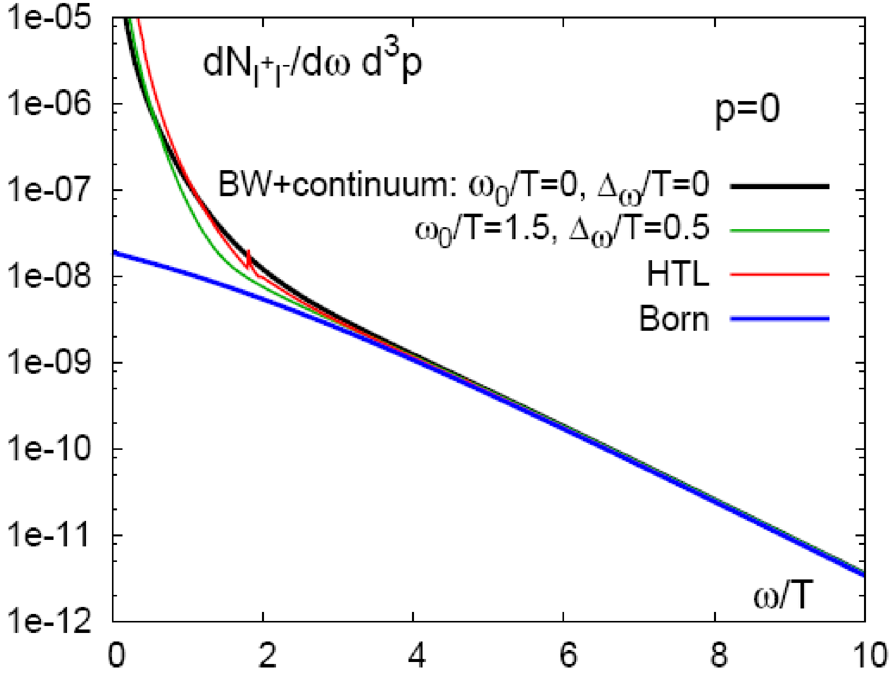
$N_\tau = 24,\ 32, 48$ $(a^{-1} = 9.4 - 18.8\mathrm{GeV})$



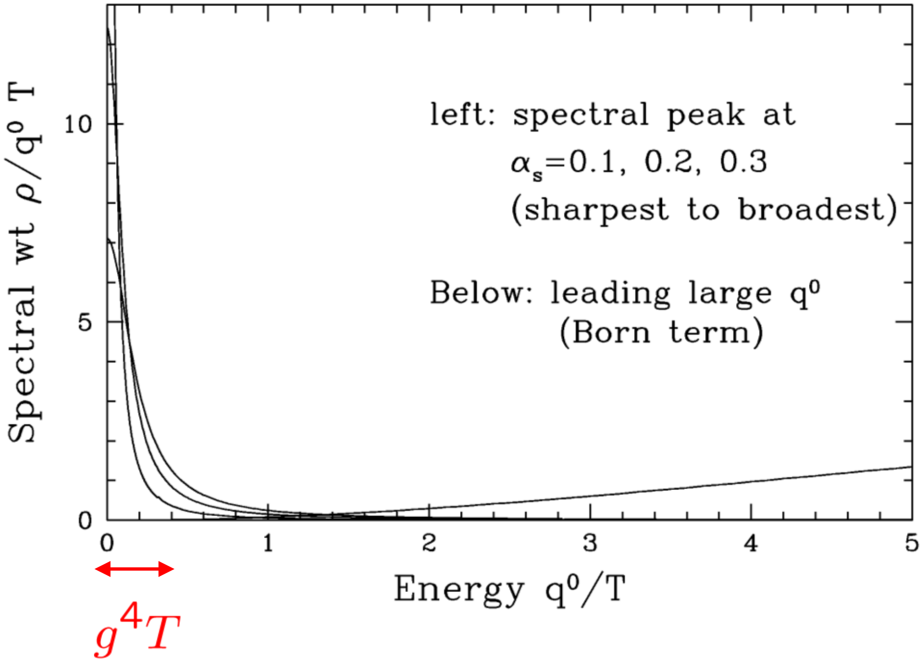


- The HTL resummed perturbative result diverges for $\omega \to 0$ limit
- The lattice results show significant enhancement over the LO (Born) result for small ω
- The lattice result is HTL result for $2<\omega/T<4$ but is much smaller for $\omega/T<2$

Strongly coupled or weakly coupled QGP ?

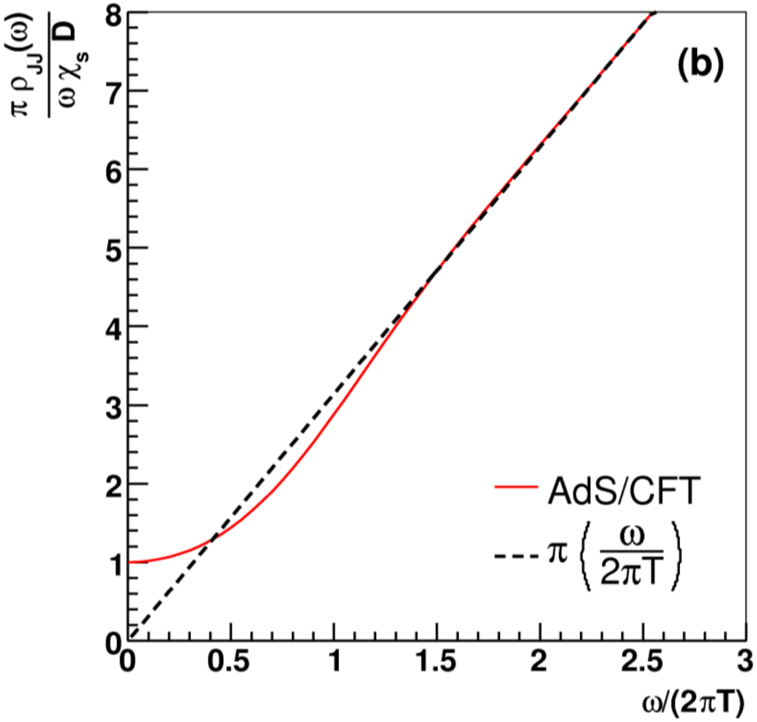
Weak coupling caculation of the
vector current spectral function in QCD

Moore, Robert, hep-ph/0607172



vector current correlator in
N=4 SUSY at strong coupling

Teaney, PRD74 (06) 045025



lattice results are closer to the weakly coupled QGP