Heavy quarks at T>0 and thermal dileptons

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1) Quantum statistical mechanics, scalar and fermion fields at *T>0*, high temperature QCD, color screening and quarkonium suppression

2) Meson correlation functions for *T>0*, spectral functions, thermal dileptons

3) Static quark correlation functions at *T>0*, effective field theory approach, potential models for quarkonium spectral functions

Dubna, September 5-17, 2011

Path integral formulation of quantum statistical mechanics

Transition amplitude in QM and its path integral represenation

$$F(q',t';q,t) = \langle q'|e^{-i\hat{H}(t'-t)}|q\rangle$$

$t \to -i\tau,\ t' \to -i\tau$ (imaginary time)

$$F(q'-i\tau';q,-i\tau) = \langle q'|e^{-\hat{H}(\tau'-\tau)}|q\rangle$$

$$\hat{H} = \frac{1}{2}p^2 + V(q)$$

$$F(q,-i\tau';q,-i\tau) = \int \mathcal{D}q \, \mathrm{exp}\left[-\int_{\tau}^{\tau'} d\tau'' \left(\frac{1}{2}\dot{q}^2(\tau'') + V(q(\tau''))\right)\right]$$

$$q(\tau) = q,\ q(\tau') = q'$$

Partition function in statistical mechanics:

$$Z(\beta) = \mathrm{Tr} e^{-\beta\hat{H}},\ \beta = 1/T$$

$$Z(\beta) = \sum e^{-\beta E_n}, \quad \hat{H}|n\rangle = E_n|n\rangle$$

$$Z(\beta) = \int dq \langle q | e^{-\beta \widehat{H}} | q \rangle$$

$$Z(\beta) = \int dq F(q, -i\beta; q, 0)$$

$$\Downarrow$$

$$Z(\beta) = \int \mathcal{D}q(\tau) \exp\left[- \int_0^\beta d\tau \left(\frac{1}{2}\dot{q}^2(\tau) + V(q(\tau)) \right) \right] = \int \mathcal{D}q(\tau) e^{-S_E(\beta)},$$

$$q(\beta) = q(0)$$

Euclidean action $S_E(\beta) = \int_0^\beta d\tau \left(\frac{1}{2}\dot{q}^2(\tau) + V(q(\tau)) \right)$

We can also calculate the generating functional

$$Z(\beta; j) = \int \mathcal{D}q \exp\left[-S_E(\beta) + \int_0^\beta j(\tau) q(\tau) d\tau \right]$$

$$\Delta(\tau_1, \tau_2) = \frac{1}{Z(\beta)} \frac{\delta^2 Z(\beta; j)}{\delta j(\tau_1) \delta j(\tau_2)} |_{j=0} = \frac{1}{Z(\beta)} \int \mathcal{D}q q(\tau_1) q(\tau_2) e^{-S_E(\beta)}$$

Correlation function in real and imaginary time in the operator formalism:

$$\widehat{q}(-i\tau) = e^{\widehat{H}\tau}\widehat{q}e^{-\widehat{H}\tau}$$

$$\widehat{q}(t) = e^{i\widehat{H}t}\widehat{q}e^{-i\widehat{H}t}$$

$$\Delta(\tau_1, \tau_2) = \langle T\widehat{q}(-i\tau_1)\widehat{q}(-i\tau_2)\rangle_\beta = \frac{1}{Z(\beta)}\mathrm{Tr}\left[T\widehat{q}(-i\tau_1)q(-i\tau_2)\right]$$

$$\Delta(\tau) = \Delta(\tau, 0) = \Delta(\tau - \beta)$$

$$D^>(t, t') = \langle\widehat{q}(t)\widehat{q}(t')\rangle_\beta$$

$$D^<(t, t') = \langle\widehat{q}(t')\widehat{q}(t)\rangle_\beta$$

$$D_R(t, t') = \langle\theta(t - t')[\widehat{q}(t), \widehat{q}(t')]\rangle_\beta$$

$$e^{-\beta\widehat{H}}\widehat{q}(t)e^{\beta\widehat{H}} = \widehat{q}(t + i\beta) \Rightarrow D^>(t, t') = D^<(t + i\beta, t')$$

Kubo-Martin-Schwinger (KMS) condition

$$\boxed{\Delta(\tau) = D^>(-i\tau, 0)}$$

Different correlation functions $\Delta(\tau)$, $D^{>}(t)$, $D^{<}(t)$ and $D_R(t)$ are related to the spectral function $\sigma(k_0)$

$$D^{>}(k_0) = \int_{-\infty}^{\infty} dt e^{ik_0 t} D^{>}(t)$$

$$D^{<}(k_0) = \int_{-\infty}^{\infty} dt e^{ik_0 t} D^{<}(t) = \int_{-\infty}^{\infty} dt e^{ik_0 t} D^{>}(t - i\beta) = e^{-\beta k_0} D^{>}(k_0)$$

$$\boxed{\sigma(k_0) = \frac{D^{>}(k_0) - D^{<}(k_0)}{2\pi} = \frac{1}{\pi} Im D_R(k_0)}$$

$$\Downarrow$$

$$D^{>}(k_0) = (1 + f(k_0))\sigma(k_0), \;\; f(k_0) = (e^{-\beta k_0} - 1)^{-1}$$

$$\Downarrow$$

$$\boxed{\Delta(\tau) = \int_0^{\infty} dk_0 \sigma(k_0) \frac{\cosh(k_0 \cdot (\tau - \beta/2))}{\sinh(\beta k_0/2)}}$$

$$\sigma(k_0) = \frac{1}{Z(\beta)} \sum_{n,m} e^{-\beta E_n} \left[\delta(k_0 + E_n - E_m) - \delta(k_0 + E_m - E_n)\right] |\langle n|\widehat{q}|m\rangle|^2$$

$$\sigma(k_0) = -\sigma(-k_0), \quad sgn(k_0)\sigma(k_0) > 0$$

Thermodynamics of scalar field theory

Straightforward generalization to infinite number of degrees of freedom $q(t) \to \phi_x(t) \equiv \phi(t,x)$

$$L = \frac{1}{2}(\partial_\mu \phi)(\partial^\mu \phi) - \frac{1}{2} m^2 \phi^2 - \frac{\lambda}{4!}\phi^4$$

$$\Downarrow$$

$$S_E(\beta) = \int_0^\beta d\tau \int d^3x \left(\frac{1}{2}(\partial_\tau \phi)^2 + \frac{1}{2}(\partial_i \phi)^2 + \frac{1}{2} m^2 \phi^2 + \frac{\lambda}{4!}\phi^4 \right)$$

$$Z(\beta; j) = \int \mathcal{D}\phi \, \mathrm{exp}(-S_E(\beta) + \int_0^\beta d\tau \int d^3x j(\tau,x)\phi(\tau,x))$$

$$\phi(0,x) = \phi(\beta,x)$$

Free field limit ($\lambda = 0$):

$$Z(\beta; j) = \int \mathcal{D}\phi \, \mathrm{exp} \left[- \int d^4x_E \frac{1}{2} \phi(-\partial_\tau^2 - \nabla^2 + m^2)\phi + \int_0^\beta d^4x_E j(x_E)\phi(x_E) \right]$$

$$x_E = (\tau, x)$$

Gaussian integration:

$$Z_0(\beta;j) = Z(\beta)\,\mathsf{exp}\left[\int_0^\beta d^4x_E dy_E\ j(x_E)\Delta_0(x_E-y_E)j(y_E)\right]$$

$$Z(\beta) = (det\Delta_0)^{1/2} = \mathrm{Tr}\ln\Delta_0$$

$$\left[-\partial_\tau^2 - \nabla^2 + m^2\right]\Delta_0(x_E-y_E) = \delta(\tau_x-\tau_y)\delta(x-y)$$

$$\Downarrow$$

$$(\omega_n^2+k^2+m^2)\Delta_0(i\omega_n,k) = (\omega_n^2+\omega_k^2)\Delta_0(i\omega_n,k) = 1$$

$$\omega_n = 2\pi Tn,\ \ \omega_k^2 = k^2+m^2$$

$$\Downarrow$$

$$\Delta_0(i\omega_n,k) = \frac{1}{\omega_n^2+\omega_k^2}$$ -Matsubara propagator

Mixed (Saclay) representation:

$$\Delta_0(\tau,k) = T\sum_n e^{-i\omega_n\tau}\Delta_0(i\omega_n,k)$$

$$[-\partial_\tau^2+\omega_k^2]\Delta_0(\tau,k) = \delta(\tau_x-\tau_y),\ \ \Delta_0(\tau-\beta) = \Delta(\tau)$$

$$\to \Delta_0(\tau) = \tfrac{1}{2\omega_k}((1+f(\omega_k))e^{-\omega_k\tau}+f(\omega_k)e^{\omega_k\tau}), f(\omega_k) = (e^{\beta\omega_k}+1)^{-1}$$

$$\ln Z(\beta) = \frac{1}{2}\mathrm{Tr}\ln\Delta_0 = \frac{1}{2}\sum_{n=-\infty}^{\infty}\sum_k \ln\beta^2\Delta(i\omega_n, k) =$$

$$-\frac{1}{2}\sum_{n=-\infty}^{\infty} V\int\frac{d^3k}{(2\pi)^3}\ln\beta^2[\omega_n^2+\omega_k^2] =$$

$$\sum_n \frac{d}{d\omega_k^2}\ln[\omega_n^2+\omega_k^2] = \sum_n \frac{1}{\omega_n^2+\omega_k^2} = \beta\Delta_0(\tau=0,k) = \frac{\beta}{2\omega_k}(1+2f(\omega_k))$$

$$\Downarrow$$

$$\sum_n \ln\beta^2(\omega_n^2+\omega_k^2) = \beta\omega + 2\ln(1+e^{-\beta\omega_k}) + const$$

$$\ln Z(\beta) = -V\int\frac{d^3k}{(2\pi)^3}\left[\frac{1}{2}\beta\omega_k + \ln(1-e^{-\beta\omega_k})\right]$$

$$F(T,V) = T\ln Z(\beta),\ p = -\partial F(T,V)/\partial V,\ S = -\frac{\partial F(T,V)}{\partial T}, U = F - TS$$

Massless case $(m=0 \rightarrow \omega_k = k)$:

$$p = \int\frac{d^3k}{(2\pi)^3}\left[\frac{1}{2}k + T\ln(1-e^{-\beta k}))\right] = \frac{\pi^2 T^4}{90}$$

$$\epsilon(T) = U(T,V)/V = 3p,\ \ s(T) = S(T,V)/V = 4/3\epsilon(T)$$

Dirac fields at finite temperature

Free Dirac Hamiltonian

$$\hat{H} = \int d^3x \psi^\dagger \gamma_0 (-i\gamma \cdot \nabla + m)\psi(x)$$

$\hat{Q} = \int d^3x \psi^\dagger \gamma^0 \psi$ -conserved charge

Canonical and grand canonical partition functions

$$Z_{can} = \mathrm{Tr} e^{-\beta \hat{H}}, \quad Z = \mathrm{Tr} e^{-\beta \hat{H} + \mu \hat{Q}}$$

$$Z = \int \mathcal{D}(\psi^*_\alpha,\ \psi_\alpha) \exp\left(-\int_0^\beta d\tau \left[\psi_\alpha(\partial_\tau - \mu)\psi_\alpha + H(\psi^*_\alpha, \psi_\alpha)\right]\right)$$

fermion fields anticommute $\Rightarrow \psi_\alpha(\beta) = -\psi_\alpha(0)$

$\Rightarrow \quad \omega_n = (2n+1)\pi T, n = 0, \ \pm 1, \pm 2 ...$

$$\begin{aligned} Z &= \mathrm{Tr} \ln \left[-i\beta((-i\omega_n + \mu) - \gamma^0 \gamma \cdot k - m\gamma_0)\right] \\ &= 2 \sum_n \sum_k \ln \left[\beta^2 \left((\omega_n + i\mu)^2 + \omega_k\right)\right] \end{aligned}$$

$$2V \int \frac{d^3k}{(2\pi)^3} \left[\beta\omega_k + \ln(1 + e^{-\beta(\omega_k - \mu)}) + \ln(1 + e^{-\beta(\omega_k + \mu)})\right]$$

Gauge fields at finite temperature

$$Z(\beta) = \int \mathcal{D}(A^a_\mu, \eta_b, \eta_c) \exp\left[-\int_0^\beta d^4x_E \mathcal{L}_{eff}(x)\right]$$

$$\mathcal{L}_{eff}(x) = \frac{1}{4}F^a_{\mu\nu}(x)F^a_{\mu\nu}(x)+\frac{1}{2\xi}(\partial_{\mu A^a_\mu})^2+\bar{\eta}_a(x)\left[\partial^2\delta_{ab} + f_{abc}A^c_\mu\partial_\mu\right]\eta_b(x)$$

$$A_\mu(0,x) = A_\mu(\beta,x), \quad \eta_a(0,x) = \eta_a(\beta,x)$$

$$\ln Z(\beta) = -\frac{1}{2} \times 4(N_c^2-1)\sum_n\sum_k \ln[\beta^2(\omega_n^2+k^2)] +$$

4 gluons

$$\frac{1}{2} \times 2(N_c^2-1)\sum_n\sum_k \ln[\beta^2(\omega_n^2+k^2)]$$

ghosts

$$p(T) = 2(N_c^2-1)\frac{\pi^2T^4}{90}$$

QCD thermodynamics at low and high temperatures

high-T (T>>Λ), weak coupling expansion should work due to asymptotic freeedom
=> thermodynamics can be described in terms of quarks and gluons => QGP

low-T : hadrons are “good” degrees of freedom and weakly interacting for T<<Λ
(use chPT, Gerber, Leutwyler, NPB 321 (89) 387)

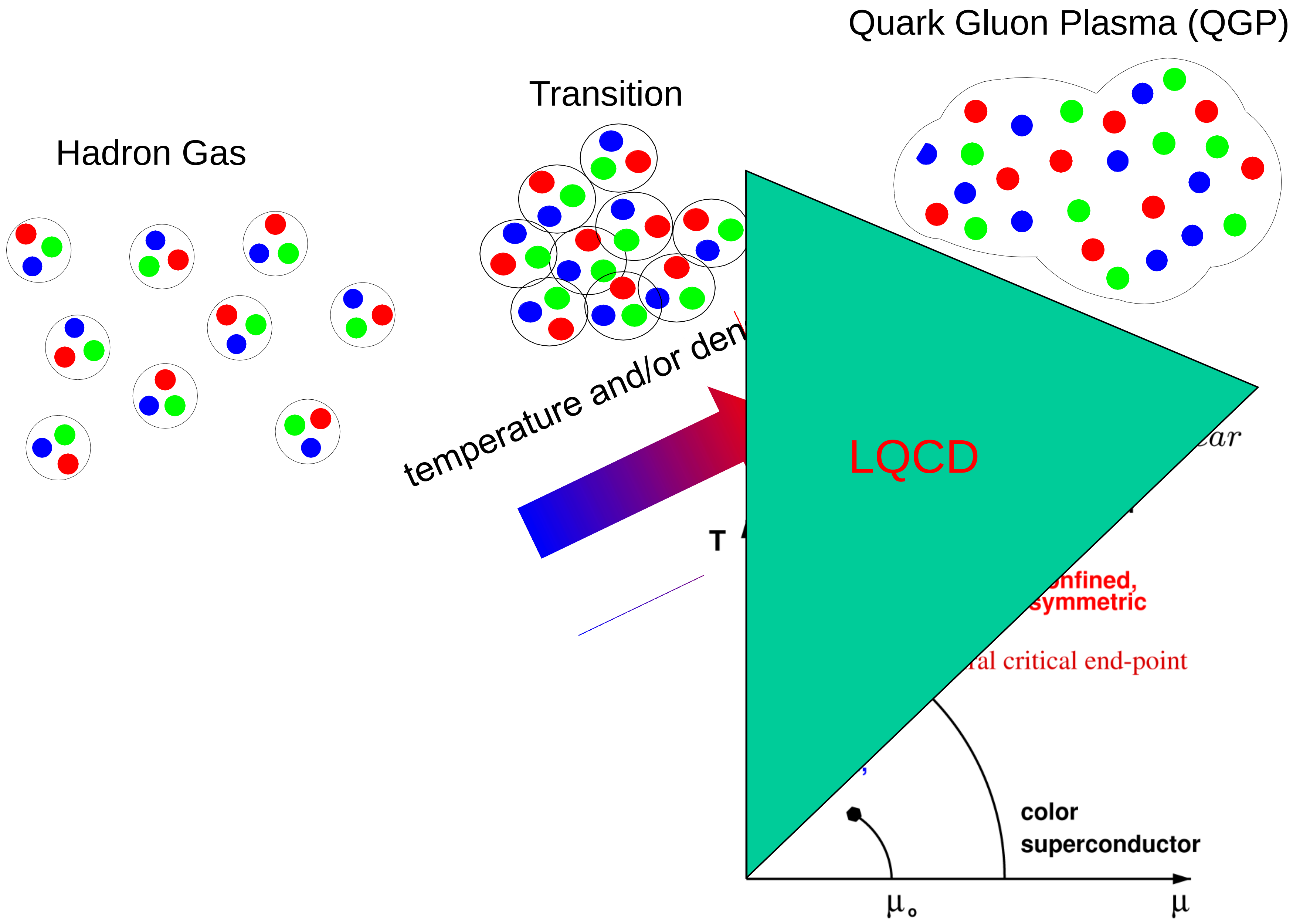
The simplest approach : consider gas of non-interacting hadrons
too naïve ? Not necessarily many hadronic interactions dominated by
resonance exchange in the s-channel , e.g. $\pi\pi \to \rho$

⇓

interacting hardon gas → non-interacting resonance gas

Hagedorn, Nouvo Cim. 35 (65) 395
Chapline et al, PRD 8 (73) 4302
Karsch et al, Eur.Phys.J.C29 (03)549

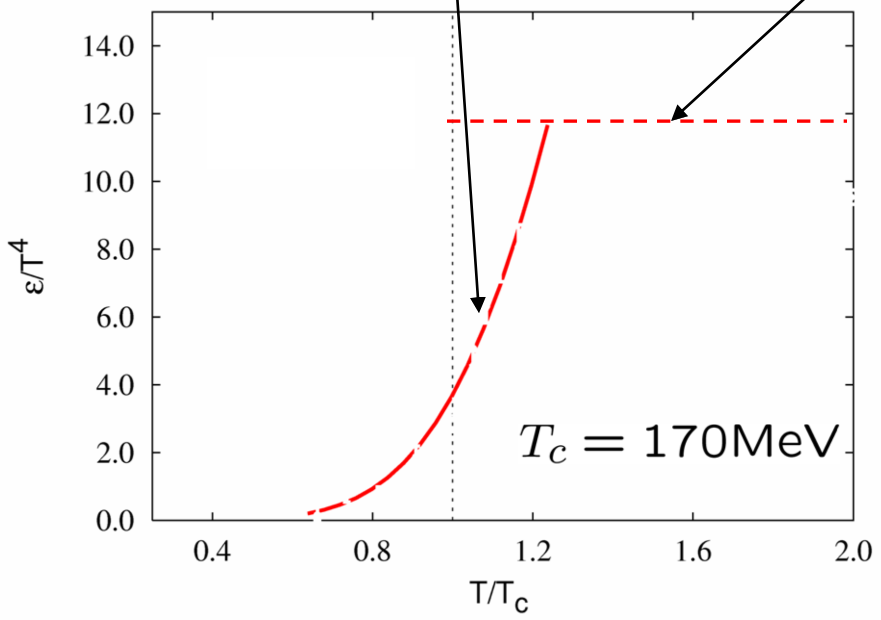
Deconfinement at high temperature and density



$$\ln Z(T,V) = \sum_i \frac{V d_i}{2\pi^2} \int_0^\infty dk k^2 \eta \ln(1 + \eta e^{-\beta\sqrt{p^2+m_i^2}})$$

$\eta = -1$ -boson, $\eta = +1$ -fermion Calculate $\ln Z$ using the masses of about 1000 experimentally known non-strange resonances

ideal gas of quarks and gluons

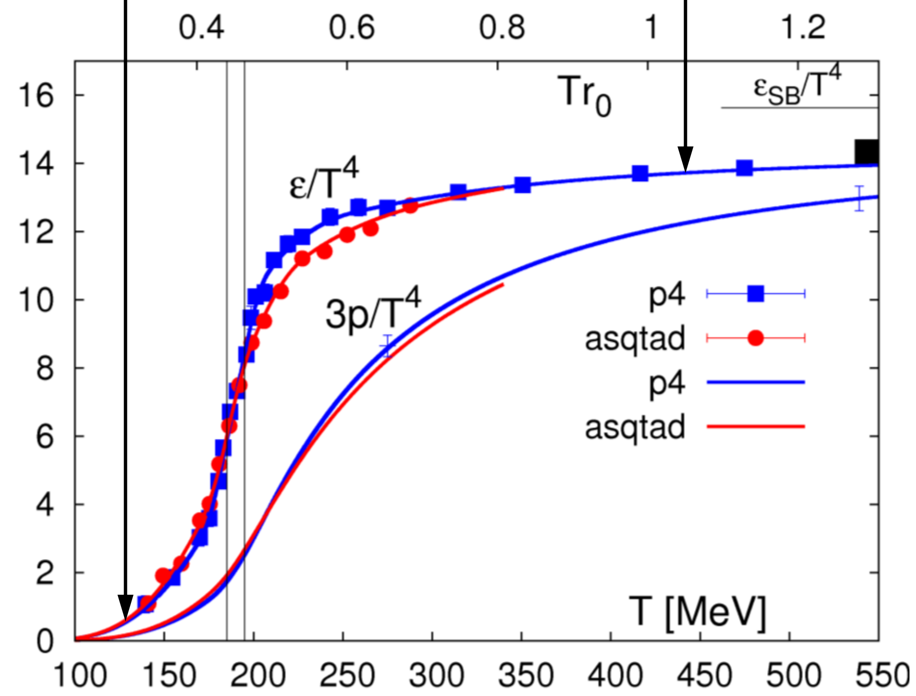


Deconfinement transition : rapid increase of the pressure, energy denisty, entropy density (liberation of many new degrees of freedom ?) Cabbibo, Parisi, PLB 59 (75) 67

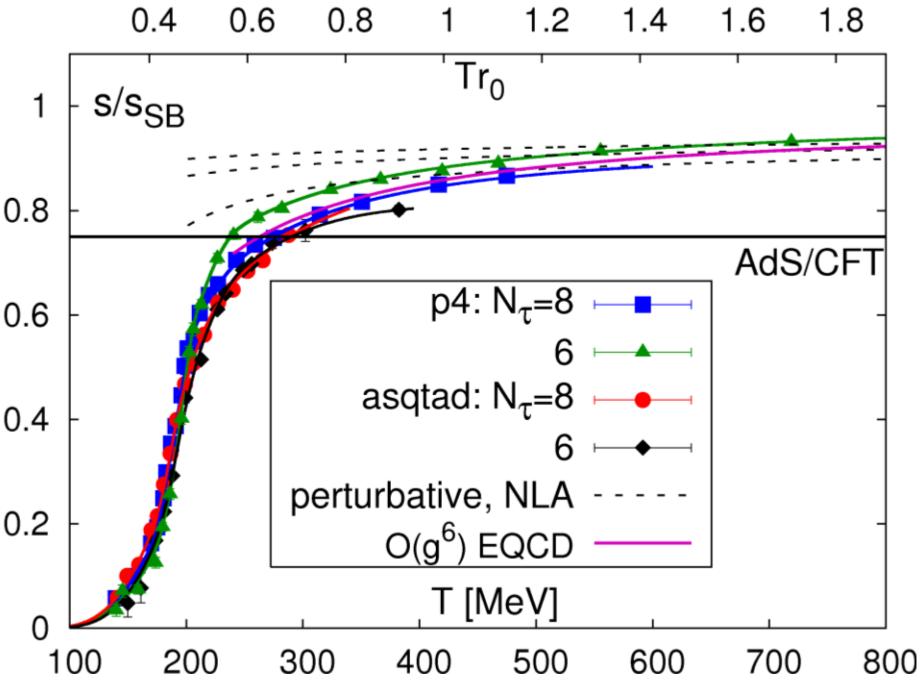
Deconfinement : entropy, pressure and energy density

pion gas = 3 light d.o.f.

free gas of quarks and gluons = 18 quark+18 anti-quarks +16 gluons =52 mass-less d.o.f



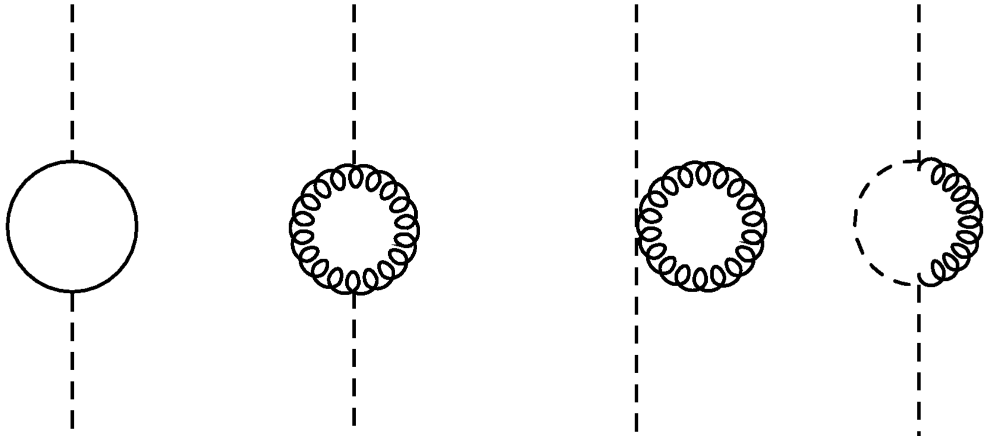
Bazavov et al (HotQCD), PRD 80 (09) 14504



Petreczky, NPA 830 (10) 11c

- rapid change in the number of degrees of freedom at *T=160-200*MeV: deconfinement
- deviation from ideal gas limit is about 10% at high *T* consistent with the perturbative result
- no large discretization errors in the pressure and energy density at high *T*
- no continuum limit yet !

Color screening in perturbation theory



Gluon self energy in the static limit:

$$\Pi_{00}(\omega_n = 0, k \to 0) = m_D^2 = (\frac{N_c}{3} + \frac{N_f}{6})g^2T^2$$

$$\Pi_{ii}(\omega_n = 0, k \to 0) = 0$$

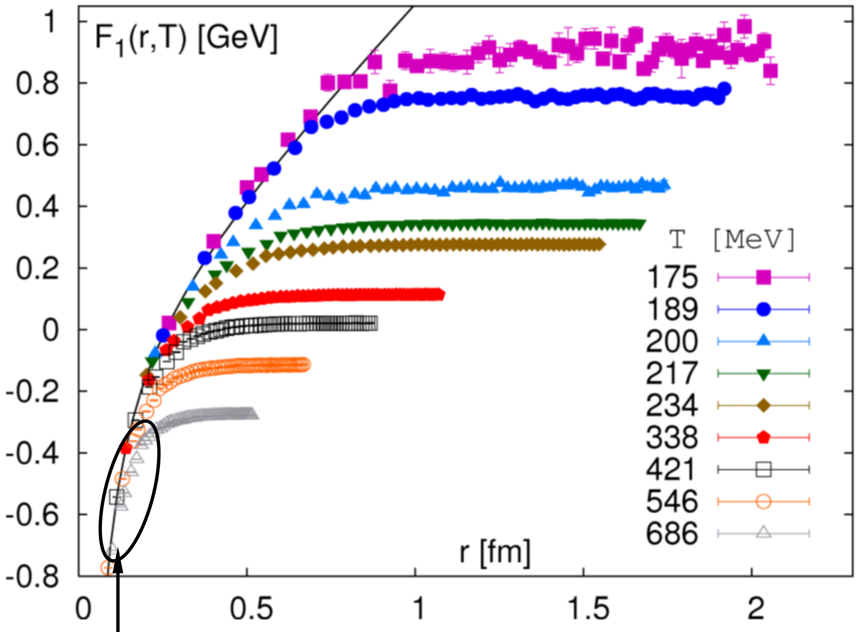
$$V(r) \simeq -\frac{N_c^2 - 1}{2N_c} g^2 \int \frac{d^3x}{(2\pi)^3} e^{i\vec{k}\cdot\vec{r}} \frac{1}{k^2 + \Pi_{00}(k)} = -\frac{N_c^2 - 1}{2N_c} \alpha_s \frac{e^{-m_D r}}{r}$$

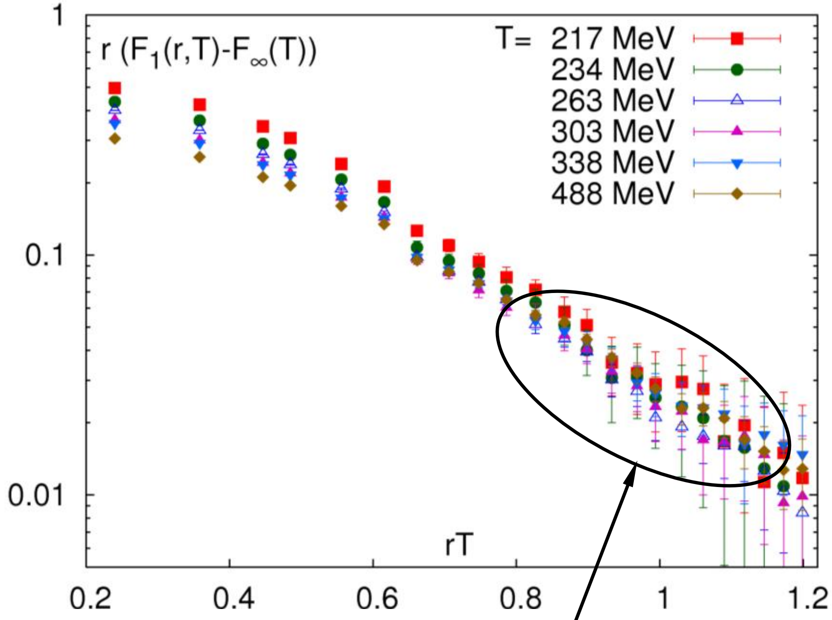
chromo-electric fields are screened but chromo-magnetic fields are not screened (at least in perturbation theory)

Color screening in lattice QCD

p4 action, $(2+1)-$flavor QCD, $16^3 \times 4$ lattices, $m_\pi \simeq 220$ MeV

P.P., JPG 37 (10) 094009 ; arXiv:1009.5935





$F_1(r,T)$ T-independent at short distances

$F_1(r,T)$ scales with T and is exponentially screened for $r > 0.8/T$

$$F_1(r) = -\frac{4}{3}\frac{\alpha_s}{r}\exp(-m_D r) + 2F_Q(T), m_D \sim T$$

Significant temperature dependence of the static quark anti-quark free energy for $r \simeq 0.3 - 0.5$ fm.

charmonium melting @ RHIC Digal, P.P., Satz, PRD 64 (01) 094015

Heavy Quarkonium and QCD

Bound states of heavy quark and anti-quark: $c\bar{c},\ b\bar{b}$

$m_c \simeq 1.3\text{GeV},\ m_b \simeq 4.5\text{GeV}\ \Lambda_{\text{QCD}} \simeq 200 MeV$ → non-relativistic treatment

Heavy quark velocity $v \ll 1$ is the small $\Rightarrow$ distinct energy scales :

$$m \gg mv \gg mv^2$$ → EFT: pNRQCD

quark mass

inverse size ~ typical momenta *k* of quarks inside the bound state

binding energy

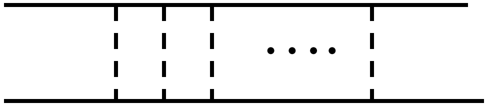
For gluons mediating the interactions $k \gg k_0 \Rightarrow$ inteactions can be considered instantaneous

Many gluon exchanges are possible in the bound state $\Rightarrow$ ladder resummation

NR reduction of BS equation:

$$(-\frac{\nabla_r^2}{m} + V(r))\psi(r) = E_n\psi(r)$$

⟷



Cornell potential :

$$V(r) = -\frac{4}{3}\frac{\alpha_s}{r} + \sigma r$$

1-gluon exchange

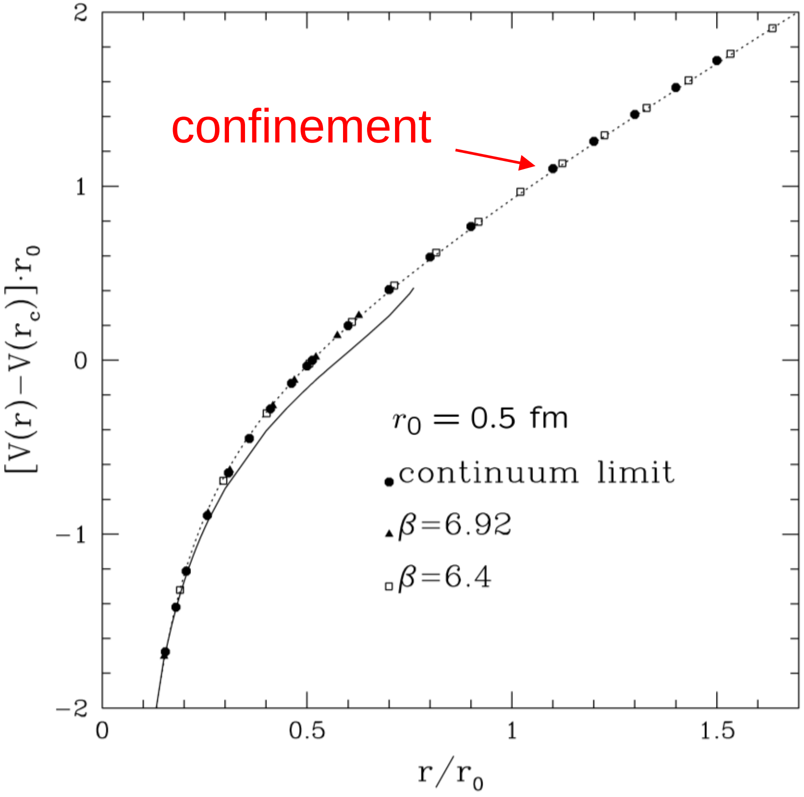
confinement

→ Quarkonia mass spectra $M_n = 2m + E_n$

Static energy on the lattice and quarkonium spectrum

Input from the lattice : approximate the potential by the energy of the static $Q\bar{Q}$

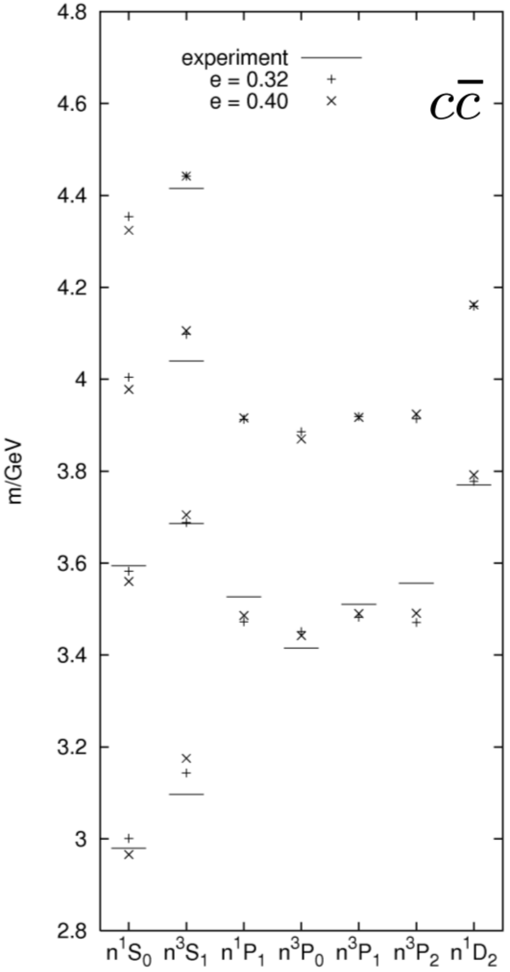
Necco, Sommer, NPB 623 (02) 271

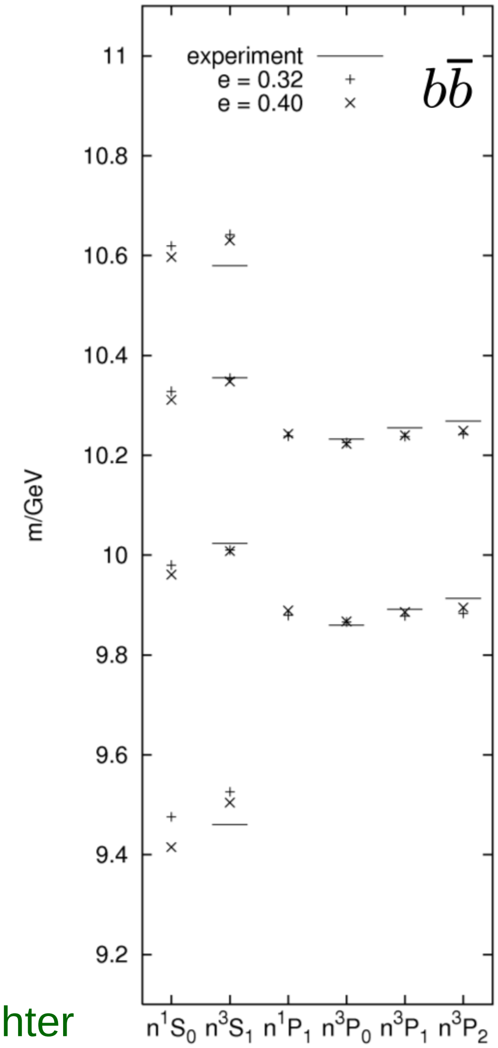


confinement

dotted line : $V(r)= -0.26/r+\sigma\, r$

solid line: 3-loop resummed perturbation theory





Bali, Schilling, Wachter
PRD56 (1997) 2566

Interactions in the octet channel and hybrid static energies

$$3 \otimes \bar{3} = 1 \oplus 8$$

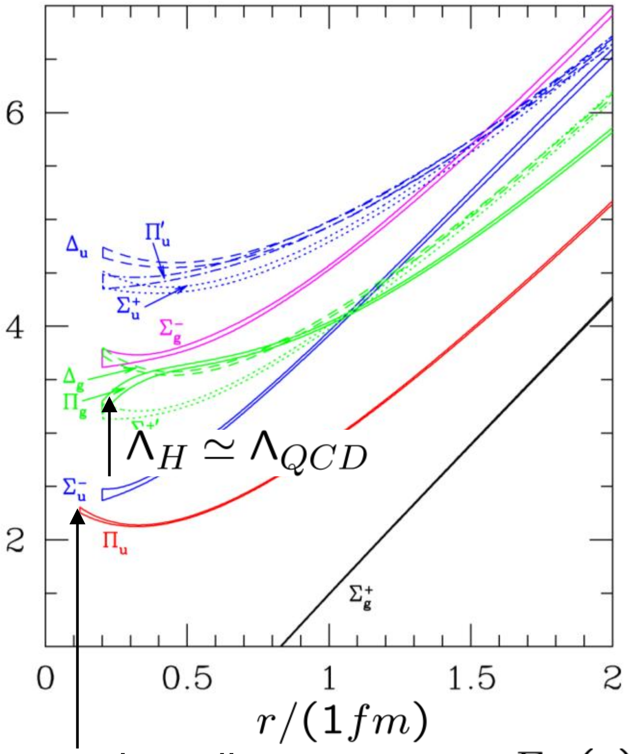
singlet potential

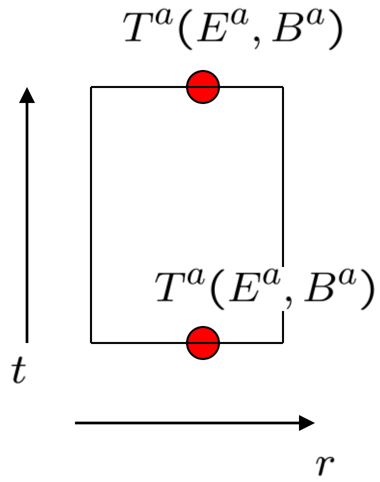
$$V_s(r) = -\frac{N_c^2 - 1}{2N_c}\frac{\alpha_s}{r}$$

octet potential

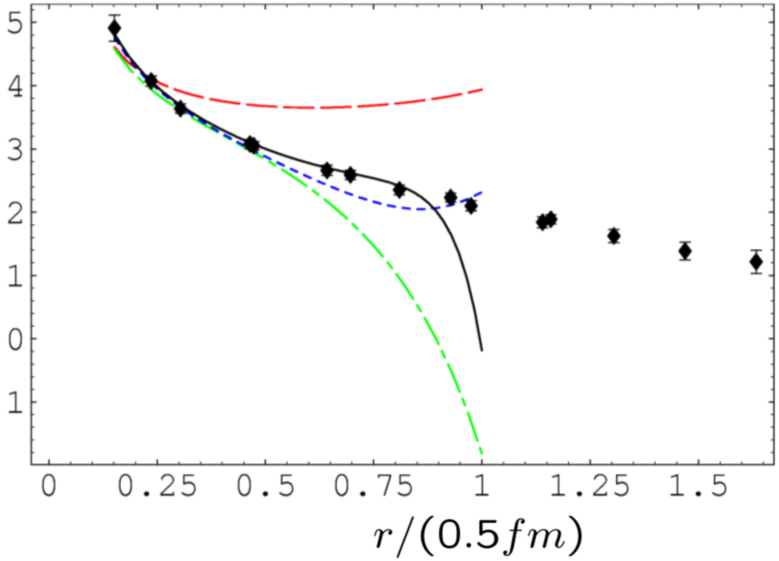
$$V_o(r) = +\frac{1}{2N_c}\frac{\alpha_s}{r}$$

Excited energy levels of static $Q\bar{Q}$ pairs (hybrid potentials) are classified according to the symmetry group of 2-atom molecule





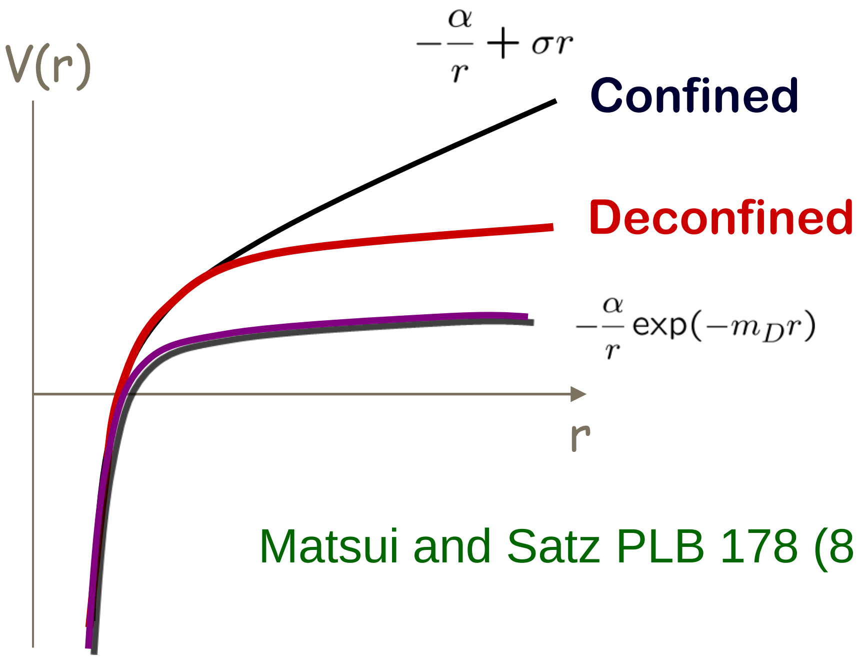
$$E_{\Sigma_g^+}(r) - E_{\Pi_u}(r) = V_o(r) - V_s(r) + \Lambda^{RS}$$



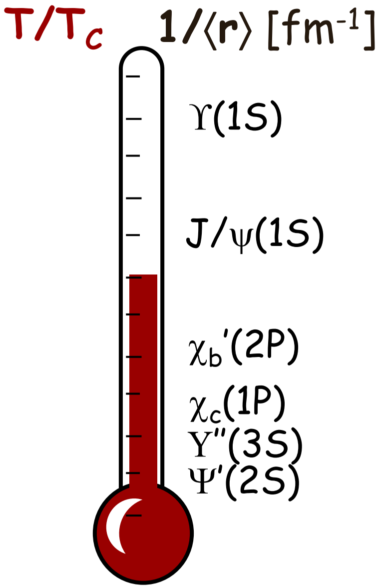
Bali, Pineda, PRDD69 (04) 094001

short distance QCD $E_H(r) = V_o(r) + \Lambda_H$

Color screening in QCD and quarkonia melting



Matsui and Satz PLB 178 (86) 416



Implicit assumptions :

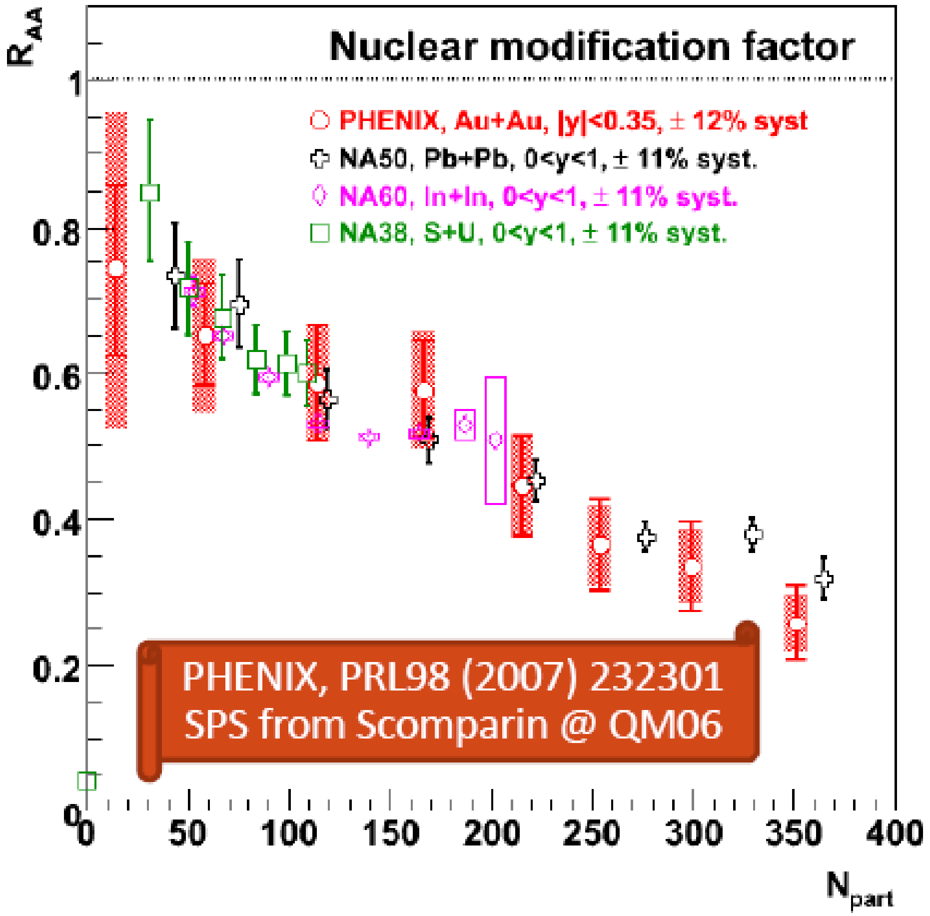
- strong color screening above deconfinement
- validity of potential models with T-dep. potentials
- formation time for quarkonia << formation time of QGP
- very short time scale for decorrelating un-bound quark anti-quark pair

use quarkonia
as thermometer
of the matter created in
RHIC

Quarkonium suppression in heavy ion collisions

Vector quarkonium (J/ψ, Υ) can be easily measured in heavy ion collisions through the dilepton channel

R_{AA}=(J/ψ yield in AA collisions)/(J/ψ yield in pp collisions x # of collisions)



PHENIX, PRL98 (2007) 232301
SPS from Scomparin @ QM06

possible signal for formation of deconfined medium in heavy ion collisions

Euclidean correlators and spectral functions

Lattice QCD is formulated in imaginary time

$$G(\tau,\vec{p},T)=\int d^3x\,e^{i\vec{p}\cdot\vec{x}}\left\langle J_H(\tau,\vec{x})\,J_H{}^{+}(0,0)\right\rangle,$$

$$J_H(\tau,\vec{x})=\overline{\psi}(\tau,\vec{x})\Gamma_H\psi(\tau,\vec{x})$$

$$\Gamma_H=1,\ \gamma_{5},\ \gamma_{\mu},\ \gamma_5\cdot\gamma_\mu$$

Physical processes take place in real time

$$D^{>}(t,\vec{p},T)=\int d^3x\,e^{i\vec{p}\cdot\vec{x}}\left\langle J_H(t,\vec{x})\,J_H\ \ (0,0)\right\rangle,$$

$$D^{<}(t,\vec{p},T)=\int d^3x\,e^{i\vec{p}\cdot\vec{x}}\left\langle J_H(0,\vec{0})\,J_H\ \ (t,\vec{x})\right\rangle$$

$$\frac{D^{>}(\omega)-D^{<}(\omega)}{2\pi}=\frac{1}{\pi}\operatorname{Im}D_R(\omega)=\sigma(\omega)$$

$$\boxed{G(\tau,T)=D^{>}(-i\tau)}$$

$$\boxed{G(\tau,T)=\int_0^\infty d\omega\,\sigma(\omega,T)\frac{\cosh(\omega(\tau-1/(2T))}{\sinh(\omega/(2T))}}$$

if $T=0$ and $\sigma(\omega)=\sum_n A_n\delta(\omega-E_n)$ $\Rightarrow$ $G(\tau)=A_0e^{-E_0\tau}+A_1e^{-E_1\tau}+\ldots$

fit the large distance behavior of the lattice correlation functions

This is not possible for $T>0,\ \tau_{max}=1/T$

and in the case of resonances, e.g. $R(\omega)=\frac{\sigma_{e^+e^-\to hadrons}}{\sigma_{e^+e^-\to\mu^+\mu^-}}=\sigma(\omega)/\omega^2$

Spectral functions at T>0 and physical observables

Heavy meson spectral functions:

$$J_H = \bar{\psi}\,\Gamma_H\,\psi$$

quarkonia properties at T>0

heavy quark diffusion in QGP: *D*

Quarkonium suppression (R_{AA})

Open charm/beauty suppression (R_{AA})

thermal dilepton production rate functions :

Light vector meson spectral functions:

$$J_\mu = \bar{\psi}\,\gamma_\mu\,\psi$$

$$\frac{dW}{d\omega d^3p} = \frac{5\alpha_{em}^2}{27\pi^2}\frac{1}{e^{\omega/T}-1}\sigma_{\mu\mu}(\omega,p,T)$$

thermal photon production rate

$$p\frac{dW}{d^3p} = \frac{5\alpha_{em}}{9\pi}\frac{1}{e^{p/T}-1}\sigma_{\mu\mu}(\omega=p,p,T)$$

Thermal photons and dileptons provide information about the temperature of the medium produced in heavy ion collisions

Low mass dileptons are sensitive probes of chiral symmetry restoration at *T>0*

electric conductivity ζ :

Homework:

1) Prove the integral equation :

$$\Delta(\tau) = \int_0^\infty dk_0 \sigma(k_0) \frac{\cosh(k_0 \cdot (\tau - \beta/2))}{\sinh(\beta k_0/2)}$$

Show that:

$$\sigma(k_0) = \frac{1}{Z(\beta)} \sum_{n,m} e^{-\beta E_n} \left[\delta(k_0 + E_n - E_m) - \delta(k_0 + E_m - E_n)\right] |\langle n|\hat{q}|m\rangle|^2$$

Hint : use relation between $\sigma(k_0)$ and $D^{>,<}(k_0)$ and insert a complete set of energy eigenstates into $D^{>,<}(t)$)

3) Prove the sum rule

$$\int_{-\infty}^\infty k_0 \sigma(k_0) dk_0 = 1$$