Lattice QCD, Hadron Structure
and Hadronic Matter

Lower-dimensional defects in lattice gluodynamics

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Motivation

Experiments at LHC
do not see the "Standard Higgs"yet
(masses from 145 to 450 GeV excluded)

Psychological pressure to consider alternatives to SM

Also, SM = pert. th, and perturbatively

$$\delta M_H^2 \sim \alpha \Lambda_{UV}^2 \gg M_H^2$$

Generically,
non-perturbative alternatives to the SM are encouraged

Higgses and lattice

Often overlooked:
Lattice is practically the only source of knowledge on non-perturbative, Higgs-like scalars

Alternative title of the lecture:
scalar fields in the lattice gluodynamics

Conclusions:
there are a few highly non-trivial observations on scalars on the lattice meeting theorists' desires
but much more questions left open

Outline

Confinement and Scalar Fields

- Abelian case
- non-Abelian case

Confinement and strings

- Magnetic strings
- Scalars living on the strings

Deconfinement and scalars

- From 4d to 3d
- From magnetic monopoles to dyons

References

The lecture is rather theoretical.
We will try to summarize some theoretical ideas closely related to interpretation of the lattice data but in fact independent

The theoretical developments took many years and the presentation is sketchy.

No particular lattice data are quoted. However, close collaboration with the ITEP Lattice Group is gratefully acknowledged.

No references here (turn to reviews for references)

Continuum theory

Abrikosov-Nielsen-Olesen vortex

Charged scalar coupled to electromagnetic field:

$$S = \int d^4x \left(\frac{1}{4e^2} F_{\mu\nu}^2 + |D_\mu \phi|^2 + V(\phi) \right)$$

where the potential

$$V(\phi) = \left(|\phi|^2 - v^2 \right)^2$$

There exists a static solution
with finite energy per unit of length
called vortex
The pert. vacuum $(\phi = 0)$ is restored along the axis of the vortex

Quantization of magnetic flux

The solution is:

$$\phi \;=\; \phi(\rho)e^{i\theta} \quad , \qquad A_\theta \;=\; a(\rho)$$

with boundary conditions:

$$\phi(\rho\to\infty) \;=\; v \;\; , \phi(\rho=0) \;=0, \qquad a(\infty) \;=\; \frac{1}{\rho}$$

The magnetic flux gets quantized:

$$\int_0^{2\pi} d\theta \; A_\theta \cdot \rho \;=\; 2\pi$$

The flux is the same as for the Dirac magnetic monopole

Confinement of the magnetic monopoles

The vortex can be closed or
end up with magnetic monopoles
For a large separation R of the monopoles

$$\lim_{R\to\infty} V_{M\bar{M}} \;=\; \sigma_{abrikosov} \cdot R$$

where $\sigma_{Abrikosov}$ is energy of the vortex per unit length

$$\sigma_{Abrikosov} \;\sim\; v^2$$

in fact, physics of superconductors, well understood and checked

Magnetic charges (monopoles) are confined
if a charged scalar is condensed

Dual superconductor model

To explain the confinement of quarks one interchanges electric and magnetic fields assuming

- In the physical vacuum, there is magnetically charged scalar condensed

$$v_{magn} \neq 0$$

Then

$$\lim_{R\to\infty} V_{Q\bar{Q}} \sim v^2_{magn}$$

and there is confinement of quarks

This idea dominates theory of confinement for 35 years
The whole issue is what are the magnetic degree of freedom, symmetry and so on

Mini-conclusion

So far we understood how confinement is related to scalars, in the Abelian case and classically

However, in YM case there are no elementary scalars. Scalars should be made out of the glue.

Quantum, or lattice dual superconductor

Start with pure Abelian gauge field. In the continuum,

$$S = \frac{1}{4e^2}\int d^4x (F_{\mu\nu})^2$$

The vacuum is probed by external heavy electric charges
The definition of the confinement is:

$$< W > = < P\exp\ \left(\int_C idx_\mu A_\mu\right) > \sim\ exp\ (-\sigma \cdot R \cdot T)$$

In the lattice version there is confinement of charges at

$$e^2\ >\ e^2_{crit}\ \sim\ 1$$

although there is no elementary scalar in the original action

To understand need elements of quantum geometry

Polymer representation of field theory

A touch of quantum geometry:
Start with classical action for a particle of mass M

$$S_{cl} = M \cdot L$$

Quantum propagator is given by the path integral

$$D(x, x') = \Sigma_{paths} exp(-S_{cl}(path))$$

To enumerate the paths, discretize space-time
that is introduce lattice spacing a for pure theoretical needs

Polymer representation, cnt'd

The probablity to observe a particular path
is equal to the product of the action factor for each link,

$$exp(-S_{link}) \;=\; exp(-M \cdot a)$$

Since the number of steps is L/a

$$W_{particular\ \ trajectory} \;=\; exp\big(-(M \cdot a)L/a\big)$$

where a is the lattice spacing
Next, account for the fact that there different trajectories of the same length L

Action vs entropy

At each step of constructing trajectory
we can choose one of 7 directions
without changing the whole length.
Total number is:

$$N(L) \ \approx \ exp(\ ln7 \cdot L/a)$$

As a result,

$$W(L) \ \sim \ exp(-M \cdot a + ln7)L/a$$

Two exponential factors pushing in opposite directions
Analogy:

$$(free\ energy) \ = \ (energy) \ - \ (entropy)$$

Both are divergent in the limit $a \to 0$

Mass renormalization

One can rewrite the equations above as statement on mass renormalization.
Namely, actual (renormalized) mass is

$$m_{phys}^2 = \frac{const}{a}\left(M(a) - \frac{ln7}{a}\right)$$

Thus, we come back to the quadratic divergence in mass of a scalar particla

Mini conclusion

Quantum geometry deals with geometrical objects changing directions on the lattice spacing scale and provides with an alternative to the standard field theory.
The problem of the quadratic divergence in the mass is reproduced.

Abelian magnetic degree of freedom

How to match quantum geometry to physics.
Start with free Abelian gauge field

$$S = \frac{1}{4e^2}\int d^4x (F_{\mu\nu})^2$$

There is a singular solution, magnetic monopole

$$\mathbf{H} = \frac{\mathbf{r}}{r^3} Q_M$$

where the magnetic charge Q_M is fixed in terms of charge e by the condition that the Dirac string is invisible

Fine tuning in language of quantum geometry

The monopole mass

$$M_{monopole} = \frac{1}{8\pi}\int_0^{\infty} \mathbf{H}^2 d^3 r$$

$$M_{monopole} = \frac{const}{e^2 a}$$

If

$$\frac{const}{e^2} = ln7 + O(m^2_{phys} a^2)$$

then $m^2_{monopole} \approx 0$

Can get negative m^2_{phys} once entropy enhancement prevails over suppression due to the action (singular monopole bare mass)

Symmetry breaking in geometrical terms

For $m^2_{phys} \; > \; 0$ probability to have length L trajectory

$$W(L) \; \sim \; exp(-m^2_{phys} L \cdot a)$$

and there exist only finite clusters of trajectories
If $m^2_{phys} < 0$ then $L \rightarrow \infty$ is not suppressed
there appears an infinite cluster
Most beautiful, infinite cluster is dilute first:
Probability θ of a link to belong to the infinite cluster:

$$\theta(link) \; \sim \; |m^2_{phys} a^2|^{\gamma} \quad , \quad \gamma \; > \; 0$$

Emergence of the infinite cluster is
the spontaneous symmetry breaking

Relation to field theory

On the lattice, monopole trajectories are defined as violations of the Bianchi identity:

$$\partial_\mu \tilde{F}_{\mu\nu} \equiv j_\nu^{monopole}$$

One can measure length of monopole trajectories for each configuration

$$< L_{monopole} > \equiv \rho_{monopole} V_{tot}$$

$$< \phi^2_{magn} > = (const)\rho_{monopole} \cdot a$$

If we count only trajectories in infinite cluster

$$< \phi_{magn} >^2 = (const)\rho^{infinite}_{monopole} \cdot a$$

Trading singularity for a scalar field

Classically (Higgs model) we have both electromagnetic field and charged scalar

Quantum mechanically, we start with free electromagnetic field, keep singular (monopole) solutions and see that these solutions can be traded for scalar field. Classically this is the same Higgs model

Mini- conclusion

On the lattice, the monopole trajectories are uniquely determined in terms of quantum fields $\{A_\mu(x)\}$. All the theory predictions turn to be true.
The role of the condensate v_{magn} is played by the infinite cluster of the monopole trajectories.

Jump from Abelian to non-Abelian case?

Quark confinement is non-Abelian $\left(L = 1/4g^2(G^a_{\mu\nu})^2\right)$
Would be nice to trade singularities of non-Abelian fields for a physical scalar field again

However, there is no direct generalization to non-Abelian case:

- no constant to tune: strong coupling runs, it is not a constant
- Monopoles are Abelian construct
- quite a few further reasons

We will come back to this later

Naive generalization

The idea is to replace the original, non-Abelian field configurations with the "closest"Abelian configurations. First, use gauge-fixing to minimize the functional

$$R = \sum_{lattice} [(A^1_\mu)^2 + (A^2_\mu)^2]$$

where indices $1, 2$ are color indices
Second, project non-Abelian component out:

$$\bar{A}^{1,2}_\mu = 0, \quad \bar{A}^3_\mu \neq 0$$

Define monopoles in projected fields:

$$\partial_\beta \tilde{F}_{\beta\alpha} \equiv (j)^{monopole}_\alpha$$

Numerical, but beautiful findings

The monopoles defined in terms of projected fields exhibit remarkable features. E.g.:

- There exist both finite and infinite clusters of monopole trajectories.
  removal of the infinite cluster destroys confinement
- Simple scaling laws:

$$\rho_{monopole}^{infinite} - = (const)\Lambda_{QCD}^{3}$$

This is highly nontrivial since Λ_{QCD} is related to the lattice spacing through 2-loop β-function

Mini conclusion

Empirically, the lattice monopoles in Yang-Mills case look physical objects (know about Λ_{QCD})

The definition of the monopoles, however, looks rather arbitrary since there are many different Abelian projections.
The definition is in pure lattice terms, non-local.
No path to continuum theory visible.

Most beautiful of all

The total monopole density has two terms:

$$\rho_{tot}^{monopole} = \frac{(const)\Lambda_{QCD}^2}{a} + (const)'\Lambda_{QCD}^3$$

and depends explicitly on the lattice spacing. The first term is due to finite (small) clusters.
In terms of matrix element:

$$\lim_{a\to 0} < \phi_{magn}^2 > \ \Lambda_{QCD}^2$$

and is exactly what we would like to have for Higgs
Lattice monopoles provide first example of dynamical solution of the hierarchy problem

Self-tuning

Matrix element $< \phi^2_{magn} >$ does not depend on the measuring procedure
We can also clarify the mechanism on the UV scale
It turns out that
non-Abelian action associated with the monopoles

$$S_{mon} \approx ln7 \frac{L}{a}$$

where L is the length of trajectory. The magic $ln7$ is hit without any fine tuning by hand
in Yang-Mills theory self-tuning of monopoles is observed

Monopoles and surfaces

An alternative way to express results

Monopoles cover densely a 2d surface

Just span smallest-area surface on the monopole trajectories and get

$$(Area)_{tot} \sim V_{tot}\Lambda^2_{QCD}$$

Actually these strings were defined and observed on the lattice as the so called center vorteces. We skip this point.

Monopoles and strings

Surfaces in quantum geometry correspond to strings in the continuum limit. Quite obvious.
Amusingly, For the string tension the resut for $(Area)_{tot}$ implies

$$T_{string} \sim \Lambda^2_{QCD}$$

Physics of monopoles, or scalars is physics of strings
Strings with physical tension in the vacuum seem to be observed

Challenging observations

- For the monopole condensate we get

$$< \phi_{magn} >^2 \sim \Lambda^3_{QCD} \cdot a$$

  that is, vanishing in the continuum limit
- at the deconfinement phase transition monopole trajectories become parallel to the (Euclidean) time and continue to ensure confinement in the 3d YM (time slice of the 4d volume)

Thoughts and doubts

- There can be no interesting singular fields because of the asymptotic freedom
- It is known that there is no fine-tuning procedure for surfaces (strings)

there are provisional answers to this particular questions (blackboard) but...

Further strategy

Imagine that by luck (!?) we did detect physical objects through particular projection. How to go further without having continuum theory definition of the object observed.

A possibility: guess which physical object it could be and check the answers for observables which can be found (calculated) in the lattice

Educated guess: magnetic strings

Magnetic string, by definition are closed in vacuum and can be open on the so called 't Hooft line
This can be checked on the lattice (blackboard).
Fits well

Educated guess next generation: holographic models

In modern language, strings living in extra dimensions are considered.

Cigar-shape geometry (blackboard)

Prediction: magnetic strings are topologically charged
Can be checked on the lattice (blackboard) and works again

Gukov-Witten operator

Integral over surface from:

$$S = \alpha G^a_{\mu\nu} d\sigma_{\mu\nu} + \beta \tilde{G}^a_{\mu\nu} d\sigma_{\mu\nu}$$

Where the color field is rotated in the Abelian directions.

This could be the right answer for the lattice strings

Mini conclusions

In recent years magnetic strings, as continuum-theory defects, were understood much better in holographic models. Some striking similarities to the lattice strings.

main problem: all oservations are rather unique, no industry yet. therefore, it is not a mainstream yet.