Meson Form Factors in AdS/QCD

Lecture 1: ρ meson form factors

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Based on papers written in collaboration with H.R. Grigoryan

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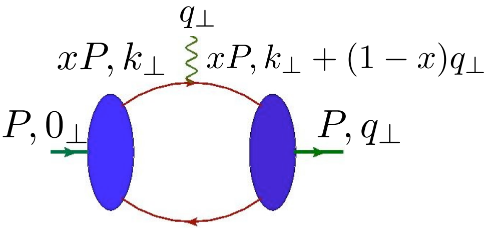
Hadronic form factors

- Hadronic form factors: $(1/Q^2)^{n_q-1}$ counting rules for a hadron made of n_q quarks
- Exclusive-inclusive connection: Parton distributions behave like $(1-x)^{2n_q-3}$
- Expectation: some fundamental/easily visible reason

Soft mechanism

- Early idea: Feynman mechanism/Drell-Yan formula [PRL 70]

$$F(Q^2) = \int_0^1 dx \int d^2\mathbf{k}_\perp \, \Psi^*(x, \mathbf{k}_\perp + \bar{x}\mathbf{q}_\perp)\Psi(x, \mathbf{k}_\perp)$$



Take region where both $\Psi_M(x, \mathbf{k}_\perp)$ and $\Psi^*_M(x, \mathbf{k}_\perp + \bar{x}\mathbf{q}_\perp)$ are maximal

Soft mechanism (cont'd)

- Drell-Yan formula

$$F(Q^2) = \int_0^1 dx \int d^2\mathbf{k}_\perp \, \Psi^*(x, \mathbf{k}_\perp + \bar{x}\mathbf{q}_\perp)\Psi(x, \mathbf{k}_\perp)$$

Take region where both $\Psi_M(x, \mathbf{k}_\perp)$ and $\Psi_M^*(x, \mathbf{k}_\perp + \bar{x}\mathbf{q}_\perp)$ are maximal:
• $|\mathbf{k}_\perp| \sim \Lambda$ is small and
• $\bar{x} \equiv 1 - x$ is close to 0, so that $|\bar{x}\mathbf{q}_\perp| \sim \Lambda$
If $|\Psi(x, \Lambda)|^2 \sim (1-x)^{2n-3}$ then

$$F(Q^2) \sim \int_0^{\Lambda/Q} \bar{x}^{2n-3} \, d\bar{x} \sim (1/Q^2)^{n-1}$$

$\Rightarrow$ Causal relation: Form of $f(x)$ determines $F(Q^2)$

Hard mechanism

- Another region in DY formula

$$F(Q^2) = \int_0^1 dx \int d^2\mathbf{k}_\perp \, \Psi^*(x, \mathbf{k}_\perp + \bar{x}\mathbf{q}_\perp)\Psi(x, \mathbf{k}_\perp)$$

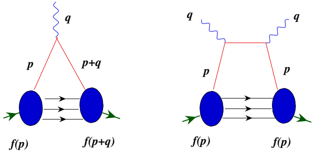
• finite x and small $|\mathbf{k}_\perp|$, e.g., region $|\mathbf{k}_\perp| \ll \bar{x}|\mathbf{q}_\perp|$, where $\Psi(x, \mathbf{k}_\perp)$ is maximal. Then

$$F_M(Q^2) \sim 2\int_0^1 dx \, |\Psi^*(x, \bar{x}\mathbf{q}_\perp)\, \varphi(x)|$$

$\Rightarrow$ form factor repeats large-$\mathbf{k}_\perp$ behavior of WF

- Mechanism was proposed by G.B. West [PRL 70] (in covariant BS-type formalism)

West's model



$$F(Q^2) \sim \int d^4p f(p) f(p+q)$$

- $f(p)$ is a function of $t \equiv p^2$ and spectator mass M^2
- If $f(t, M^2) \sim t^{-n} g(M^2)$, then $F(Q^2) \sim (1/Q^2)^n$

$$\nu W_2(x) \sim \int_{t_{\min}}^{t_{\max} \sim -2\nu} dt f^2(t, M^2) \sim (t_{\min})^{2n-1}$$

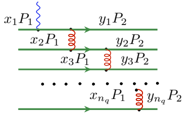
where $t_{\min} = \left(\frac{-x}{1-x}\right) \left[M^2 - (1-x) M_N^2\right]$

$$\Rightarrow \nu W_2(x) \sim (1-x)^{2n-1}$$

DY vs West model

- DY: Active parton is "on-shell" $p^2 \sim \Lambda^2$
- $F(Q^2)$ reflects the size of phase space in which $1 - x \sim \Lambda/Q$
- West model: Active parton is highly virtual
- $F(Q^2)$ reflects shape of WF for large virtualities
  $\Rightarrow$ Two mechanisms are completely different
  Surpise: $(1/Q^2)^n \Leftrightarrow (1 - x)^{2n-1}$ holds in both models!
- NB: In DY model, n is not necessarily integer
- NB: In West's model, $(1/Q^2)^n$ and $(1 - x)^{2n-1}$ have the same cause, but not "causing" each other

Hard mechanism & pQCD



- Integer n naturally appear in hard model: reflect number of hard propagators
- Hard exchange in a theory with dimensionless coupling constant gives $n = n_q - 1$ [BF 73]
- Consequence of scale invariance [MMT 73]
- QCD: $(\alpha_s/Q^2)^{n_q-1}$
- Suppression: $F_\pi(Q^2) \to (2\alpha_s/\pi)s_0/Q^2$
  $\left[s_0 = 4\pi^2 f_\pi^2 \approx 0.7\,\mathrm{GeV}^2\right]$
- Known: $\alpha_s/\pi \sim 0.1$ is penalty for an extra loop
- AdS/QCD model: $F_\pi(Q^2) \to s_0/Q^2$ [Grigoryan, AR]

AdS/QCD

AdS/QCD claims nonperturbative explanation of quark counting rules

Reason: conformal invariance & short-distance behavior of normalizable modes $\Phi(\zeta)$

Form factor in AdS/CFT [Polchinsky,Strassler]

$$F(Q^2) = \int_0^{1/\Lambda} \frac{d\zeta}{\zeta^3}\, \Phi_{P'}(\zeta) J(Q,\zeta) \Phi_P(\zeta)$$

Nonnormalizable mode: $J(Q,\zeta) = \zeta Q K_1(\zeta Q) \equiv \mathcal{K}_1(\zeta Q)$

Normalizable modes for mesons: $\Phi(\zeta) = C\zeta^2 J_{L+1}(\beta_{L,k}\zeta\Lambda)$

For large Q: $\mathcal{K}_1(\zeta Q) \sim e^{-\zeta Q} \Rightarrow$ only small $\zeta \lesssim 1/Q$ work

$\Rightarrow F_{L=0}(Q^2) \to 1/Q^4$

Wrong power?

Hard-Wall AdS/QCD

- 5-dimensional space: $\{x^\mu, z\} \equiv X^M$
- AdS_5 metric with hard wall

$$ds^2 = \frac{1}{z^2}\left(\eta_{\mu\nu}dx^\mu dx^\nu - dz^2\right), \qquad 0 \le z \le z_0 = 1/\Lambda ,$$

- 5-dimensional vector gauge field $A_M(X)$ with $M = \mu, z$
- AdS/QCD correspondence with 4D field $A_\mu(x)$

$$A_\mu(x, z=0) = A_\mu(x)$$

- 5D gauge action for vector field

$$S_{\text{AdS}} = -\frac{1}{4g_5^2}\int d^4x\, dz\, \sqrt{g}\, \text{Tr}\left(F_{MN}F^{MN}\right)$$

- Field-strength tensor $F_{MN} = \partial_M A_N - \partial_N A_M - i[A_M, A_N]$
- Coupling constant $g_5^2 = 6\pi^2/N_c$ is small in large-N_c limit

Bulk-to-boundary Propagator

- Free-field satisfies $\Box_5 A(X) = 0$ or

$$\Box_4 A(x,z) + z\partial_z \left(\frac{1}{z}\, \partial_z A(x,z) \right) = 0$$

- In momentum 4D representation

$$z\partial_z \left(\frac{1}{z}\, \partial_z \tilde{A}(p,z) \right) + p^2 \tilde{A}(p,z) = 0 \qquad (*)$$

- AdS/QCD correspondence

$$\tilde{A}_\mu(p,z) = \tilde{A}_\mu(p)\, \frac{V(p,z)}{V(p,0)}$$

- Bulk-to-boundary propagator $V(p,z)$ satisfies $(*)$
- Gauge invariant boundary condition $F_{\mu z}(x,z_0) = 0$ on IR wall $\Rightarrow$ Neumann b.c. $\partial_z V(p,z_0) = 0$

Bound state expansion

- Solution for $V(p,z)$ with Neumann b.c. $(P = \sqrt{p^2})$

$$V(p,z) = Pz\left[Y_0(Pz_0)J_1(Pz) - J_0(Pz_0)Y_1(Pz)\right]$$

- Bound state expansion (uses Kneser-Sommerfeld formula)

$$\frac{V(p,z)}{V(p,0)} \equiv \mathcal{V}(p,z) = -\sum_{n=1}^{\infty} \frac{g_5 f_n}{p^2 - M_n^2}\,\psi_n(z)$$

- Masses: $M_n = \gamma_{0,n}/z_0$ (Bessel zeros: $J_0(\gamma_{0,n}) = 0$))
- "Decay constants"

$$f_n = \frac{\sqrt{2}M_n}{g_5 z_0 J_1(\gamma_{0,n})}$$

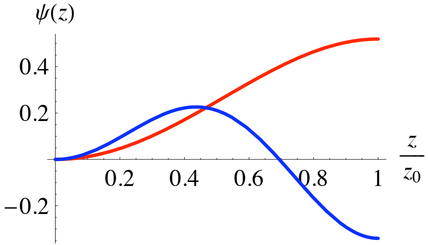
- "ψ" wave functions

$$\psi_n(z) = \frac{\sqrt{2}}{z_0 J_1(\gamma_{0,n})}\, zJ_1(M_n z)$$

Wave functions of ψ type

- Obey equation of motion with $p^2 = M_n^2$
- Satisfy $\psi_n(0) = 0$ at UV and $\partial_z \psi_n(z_0) = 0$ at IR boundary
- Normalized according to

$$\int_0^{z_0} \frac{dz}{z} \, |\psi_n(z)|^2 = 1$$



- Do not look like bound state w.f. in quantum mechanics

Wave functions of ϕ type

- Introducing ϕ wave functions

$$\phi_n(z) \equiv \frac{1}{M_n z}\,\partial_z \psi_n(z) = \frac{\sqrt{2}}{z_0 J_1(\gamma_{0,n})}\,J_0(M_n z)$$

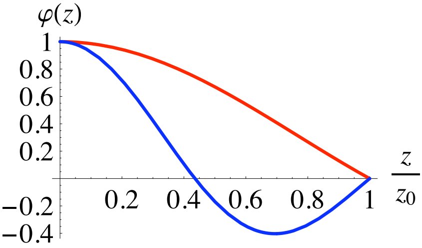
- Reciprocity:

$$\psi_n(z) = -\frac{z}{M_n}\,\partial_z \phi_n(z)$$

- Give couplings $g_5 f_n/M_n$ as their values at the origin
- Satisfy Dirichlet b. c. $\phi_n(z_0) = 0$ at confinement radius
- Are normalized by

$$\int_0^{z_0} dz\, z\, |\phi_n(z)|^2 = 1$$

Wave functions of ϕ type

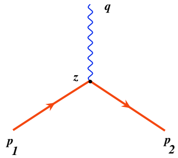


- Are analogous to bound state wave functions in quantum mechanics
- ψ w.f. correspond to vector-potential
- ϕ w.f. correspond to field-strength

Three-Point Function

- "Mercedes-Benz" form

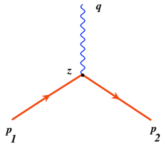
$$W(p_1, p_2, q) = \int_0^{z_0} \frac{dz}{z} \, \mathcal{V}(p_1, z) \, \mathcal{V}(p_2, z) \, \mathcal{V}(q, z)$$



- For spacelike q (with $q^2 = -Q^2$)

$$\mathcal{V}(iQ, z) \equiv \mathcal{J}(Q, z) = Qz \left[K_1(Qz) + I_1(Qz) \frac{K_0(Qz_0)}{I_0(Qz_0)} \right]$$

Form Factors



- Bound-state expansion

$$\mathcal{J}(Q,z) = \sum_{m=1}^{\infty} \frac{g_5 f_m}{Q^2 + M_m^2} \psi_m(z)$$

- Infinite tower of vector mesons [Son,Stephanov,Strassler]
- Transition form factors

$$F_{nk}(Q^2) = \int_0^{z_0} \frac{dz}{z} \mathcal{J}(Q,z) \psi_n(z) \psi_k(z)$$

Diagonal form factors

- In terms of ψ functions

$$F_{nn}(Q^2) = \int_0^{z_0} \frac{dz}{z}\, \mathcal{J}(Q,z)\, |\psi_n(z)|^2$$

- In terms of ϕ functions

$$F_{nn}(Q^2) = \frac{1}{1+Q^2/2M_n^2} \int_0^{z_0} dz\, z\, \mathcal{J}(Q,z)\, |\phi_n(z)|^2$$

- Define

$$\mathcal{F}_{nn}(Q^2) = \int_0^{z_0} dz\, z\, \mathcal{J}(Q,z)\, |\phi_n(z)|^2$$

- Direct analogue of diagonal bound state form factors in quantum mechanics

Form Factors

- Three form factors for vector mesons

$$\langle\rho^+(p_2,\epsilon')|J^\mu_{\rm EM}(0)|\rho^+(p_1,\epsilon)\rangle$$
$$= -\epsilon'_\beta\epsilon_\alpha\Big[\,\eta^{\alpha\beta}(p_1^\mu+p_2^\mu)\,G_1(Q^2)$$
$$+(\eta^{\mu\alpha}q^\beta-\eta^{\mu\beta}q^\alpha)(G_1(Q^2)+G_2(Q^2))$$
$$-\frac{1}{M^2}q^\alpha q^\beta(p_1^\mu+p_2^\mu)\,G_3(Q^2)\,\Big]$$

- Hard-wall model gives

$$-\epsilon'_\beta\epsilon_\alpha\left[\eta_{\alpha\beta}(p_1+p_2)_\mu+2(\eta_{\alpha\mu}q_\beta-\eta_{\beta\mu}q_\alpha)\right]F_{nn}(Q^2)$$

- Prediction: $G_1(Q^2)=G_2(Q^2)=F_{nn}(Q^2); G_3(Q^2)=0$ [SS]
- Moments: magnetic $\mu=2$, quadrupole $D=-1/M^2$, same result as for pointlike meson (Brodsky & Hiller)

+++ Form Factor

- +++ component of 3-point correlator gives combination

$$\mathcal{F}(Q^2) = G_1(Q^2) + \frac{Q^2}{2M^2} G_2(Q^2) - \left(\frac{Q^2}{2M^2}\right)^2 G_3(Q^2)$$

- For ρ-meson, $\mathcal{F}(Q^2)$ coincides with IMF LL transition that has leading $\sim 1/Q^2$ behavior in pQCD

Large-Q^2 behavior of $\mathcal{F}(Q^2)$

- Hard-wall model prediction

$$\mathcal{F}(Q^2) = \int_0^{z_0} dz\, z\, \mathcal{J}(Q,z)\, |\phi(z)|^2$$

- For large Q:

$$\mathcal{J}(Q,z) \to zQK_1(Qz) \sim e^{-Qz}$$

- Only $z \sim 1/Q$ contribute $\Rightarrow$ $\phi(z)$ may be substituted by $\phi(0)$
- Asymptotic normalization of $\mathcal{F}(Q^2)$ is given by

$$\frac{|\phi(0)|^2}{Q^2} \int_0^{\infty} d\chi\, \chi^2\, K_1(\chi) = 2\, \frac{|\phi(0)|^2}{Q^2}$$

- Same power of $1/Q^2$ as in pQCD, but no α_s/π factor

Soft-Wall model

- Take model with z^2 barrier (Karch et al.)
- Equation for bulk-to-boundary propagator $V(p,z)$

$$z\partial_z \left[\frac{1}{z}\, e^{-\kappa^2 z^2} \partial_z V \right] + p^2\, e^{-\kappa^2 z^2} V = 0$$

- Solution normalized to 1 for $z = 0$ ($a = -p^2/4\kappa^2$)

$$\mathcal{V}(p,z) = a \int_0^1 dx\, x^{a-1} \, \exp\left[-\frac{x}{1-x}\, \kappa^2 z^2 \right] \; ,$$

- Propagator has poles at locations $p^2 = 4(n+1)\kappa^2 \equiv M_n^2$

$$\mathcal{V}(p,z) = \kappa^2 z^2 \sum_{n=0}^{\infty} \frac{L_n^1(\kappa^2 z^2)}{a+n+1} = \sum_{n=0}^{\infty} \frac{g_5 f_n}{M_n^2 - p^2}\, \psi_n(z)$$

Wave Functions

- ψ wave functions

$$\psi_n(z) = z^2 \sqrt{\frac{2}{n+1}}\, L_n^1(\kappa^2 z^2)$$

- Coupling constants

$$g_5 f_n = \frac{1}{z}\, e^{-\kappa^2 z^2}\, \partial_z \psi_n(z)\Big|_{z=\epsilon\to 0} = \sqrt{8(n+1)}\kappa^2$$

- ϕ wave functions

$$\phi_n(z) = \frac{1}{M_n z} e^{-\kappa^2 z^2} \partial_z \psi_n(z) = \frac{2}{M_n} e^{-\kappa^2 z^2} L_n^0(\kappa^2 z^2)$$

$$\phi_0(z) = \sqrt{2} e^{-\kappa^2 z^2} \quad , \quad \phi_1(z) = \sqrt{2} e^{-\kappa^2 z^2}(1 - \kappa^2 z^2)$$

Form Factors & ρ-Meson Dominance

- Form factor of the lowest state

$$\mathcal{F}_{00}(Q^2) = 2\int_0^\infty dz\, z\, e^{-\kappa^2 z^2}\, \mathcal{J}(Q,z)$$

- Using representation for $\mathcal{J}(Q,z)$ gives

$$\mathcal{F}_{00}(Q^2) = \frac{1}{1+Q^2/M_0^2}$$

- Exact vector dominance is due to overlap integral

$$\mathcal{F}_{m,00} \equiv 2\int_0^\infty dz\, z^3\, e^{-z^2}\, L_m^1(z^2) = \delta_{m0}$$

Large-Q^2 behavior

- Large-Q^2 behavior of $\mathcal{F}$ form factor

$$\mathcal{F}_{nn}(Q^2) \to \frac{\Phi_n^2(0)}{Q^2} \int_0^\infty d\chi \, \chi^2 \, K_1(\chi) = \frac{2\,\Phi_n^2(0)}{Q^2}$$

- In hard-wall model:

$$\Phi_0^{\rm H}(0) = \frac{\sqrt{2} m_\rho}{\gamma_{0,1} J_1(\gamma_{0,1})} \Rightarrow \mathcal{F}_\rho^{\rm H}(Q^2) \to \frac{2.56 m_\rho^2}{Q^2}$$

- In soft-wall model:

$$\Phi_0^{\rm S}(0) = \frac{m_\rho}{\sqrt{2}} \Rightarrow \mathcal{F}_\rho^{\rm S}(Q^2) \to \frac{m_\rho^2}{Q^2}$$

Summary

- Form Factors in AdS/QCD are given by QM-like formulas
- Only one mechanism $z \sim 1/Q$ for large Q
- IMF (LL) form factor of vector meson indeed behaves like $1/Q^2$ for large Q^2
- Exact ρ-dominance for $\mathcal{F}(Q^2)$ in soft-wall model
- Large-Q^2 asymptotics is $\mathcal{O}(1/Q^2)$ vs. pQCD $\mathcal{O}(\alpha_s/\pi)\,\mathcal{O}(1/Q^2)$

Conclusion

- AdS/QCD provides instructive model
  for what may happen
  with form factors in QCD