- GPDs and their physics content
- Dual representation for GPDs
- Which piece of GPDs physics content can be extracted in principle from DVCS amplitude?
- Abel tomography
- GPD quintessence function and its physics content
- Modeling of the quintessence function

# Main idea

Aim to understand $B^*$ in terms of $q, \bar{q}$ & gluons

AVAILABLE PROBES & THEIR QCD structure

$$\left\{\begin{matrix}\gamma \\ W^\pm, Z^0\end{matrix}\right\} \Longleftrightarrow \begin{matrix}\langle B^* | \bar{q}\hat{Q}\gamma_\mu q | N\rangle \\ \langle B^* | \bar{q}\hat{Q}_W \gamma_\mu (1-\gamma_5) q | B\rangle\end{matrix}$$


$$\pi \Longleftrightarrow \langle B^* | \,???\, | B\rangle$$

QCD operator unknown!


- Have only $C=-1$
- Local in space & time
- Doesnot contain gluon d.o.f.

- Structure in terms of QCD d.o.f. is unknow

In gedanken experiment one can excite $B^*$ by a graviton.


$$\langle B^* | \overbrace{\bar{q}\gamma_\mu(\partial_\nu - \hat{A}_\nu)q + \frac{1}{4} G^a_{\mu\alpha} G^a_{\alpha\nu}}^{\{\text{energy-momentum tensor in QCD}\}} | B \rangle$$

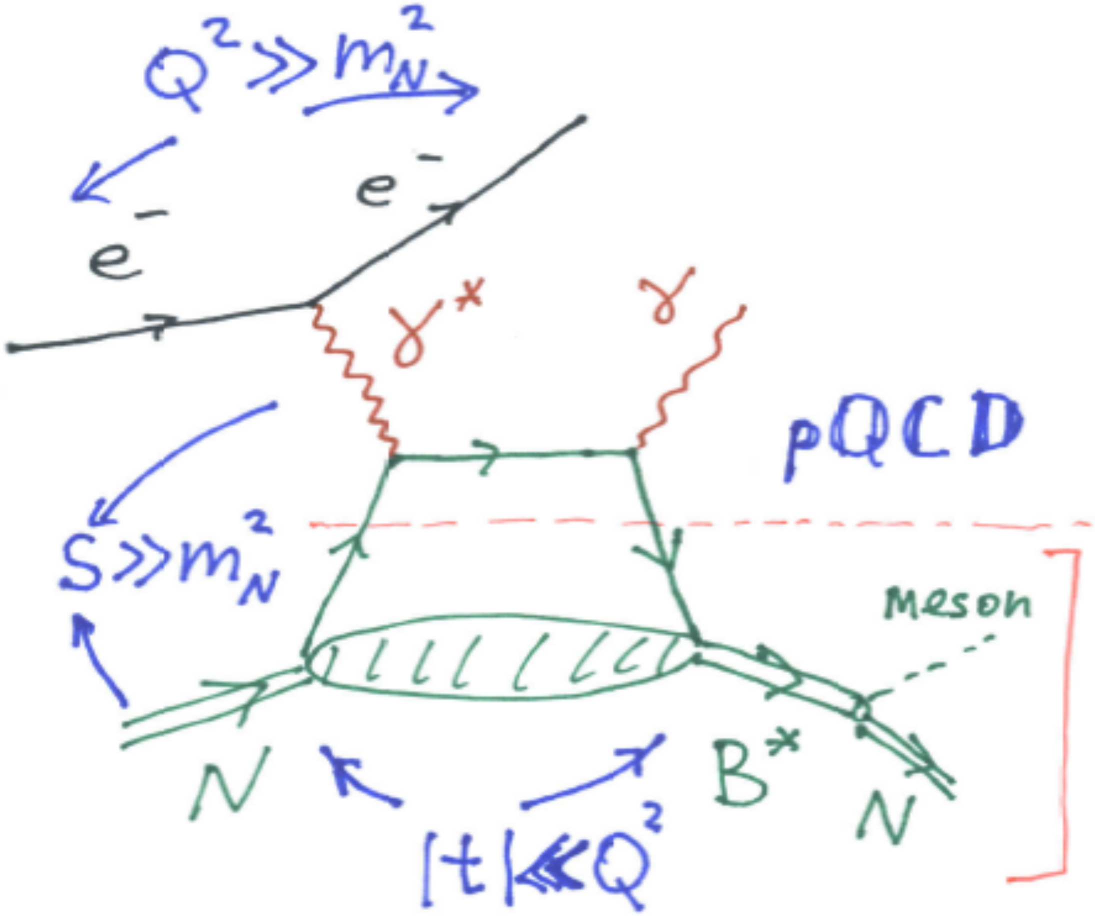
gluon operators!

In such experiment one can learn a lot of new things about $B^*$, BUT:

$$\frac{\text{rate of } B^* \text{ graviproduction}}{\text{rate of } B^* \text{ photoproduction}} \simeq \frac{m_N}{M_{Pl}} \cdot \frac{1}{\alpha} \simeq 10^{-17}$$

Hopeless !? No!

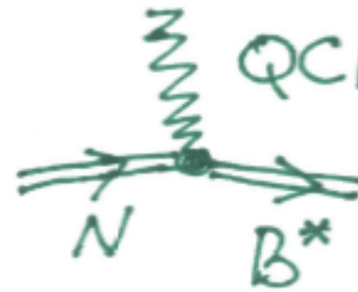
We can design probes a'la graviton with HELP OF HARD EXCLUSIVE processes LIKE DVCS!




QCD factorization theorem

/Collins, Frankfurt Strikman '97/

Soft interaction



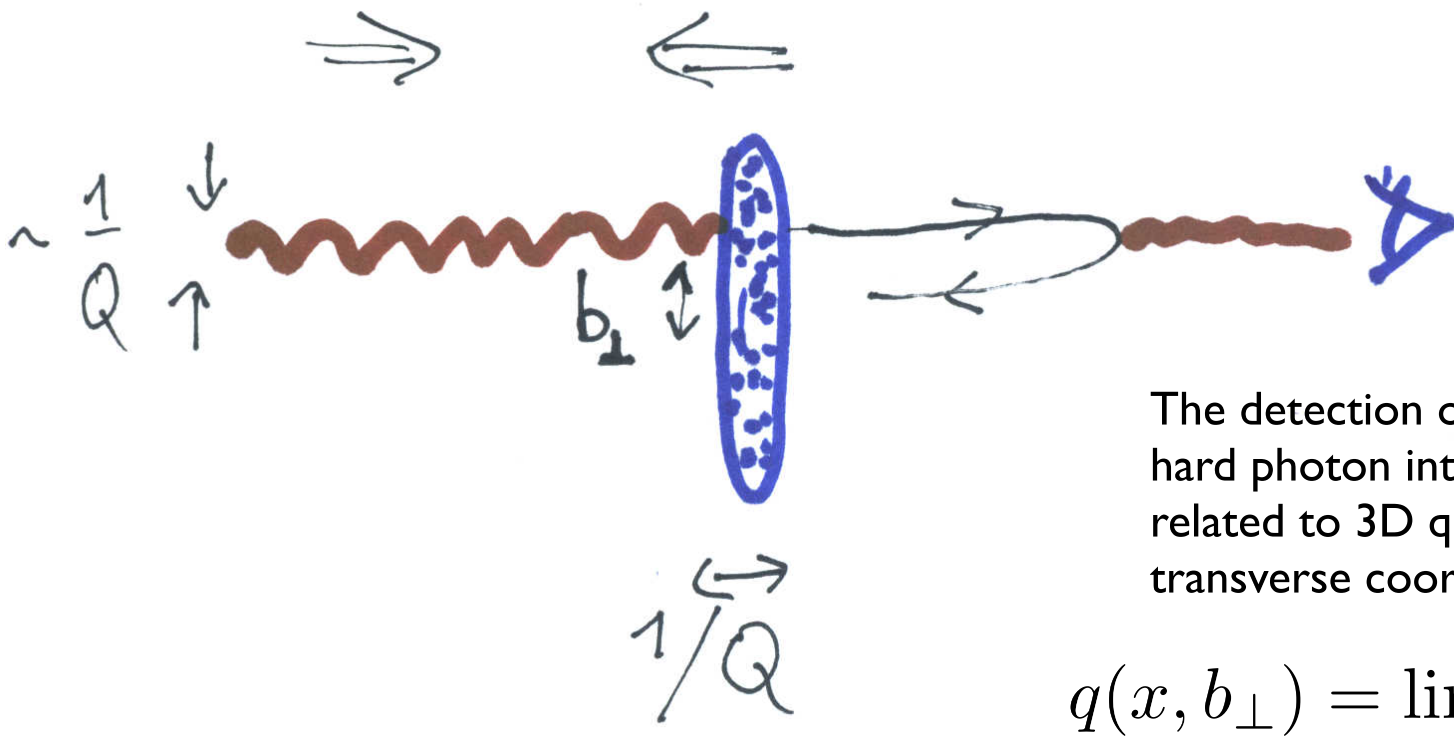

QCD string

$$\langle B^* | \bar{q}(z) \gamma_\mu \, P\exp\left[ i \int_0^z dx \, n_\mu A_\mu(x) \right] q(0) | B \rangle$$

gluons


NB. Non-digonal DVCS = excitation of $B^*$ by soft QCD string

# Relation of GPDs to 3D image of hadrons




The detection of the final photon allows to find 3D location of the hard photon interaction with an (anti) quark. Therefore GPDs are related to 3D quark density (in longitudinal momentum and in transverse coordinate spaces)

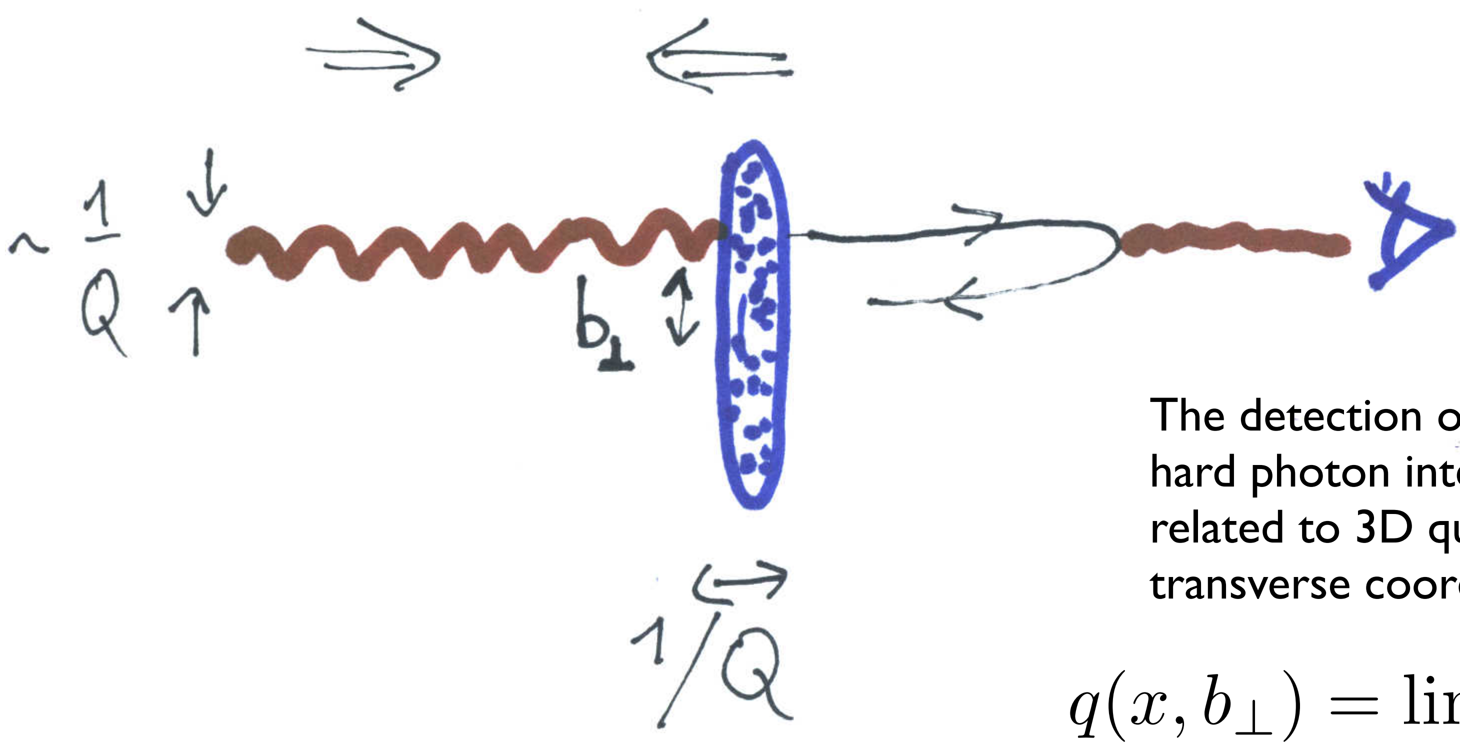
$$q(x, b_\perp) = \lim_{\xi \to 0} \int \frac{d^2\Delta_\perp}{(2\pi)^2} \; e^{i\Delta_\perp \cdot b_\perp} \; H(x, \xi, -\Delta_\perp^2)$$

$$H(x, \xi, t)$$

these are external kinematical variables

$$A(\xi, t) = \int_{-1}^{1} dx \; \frac{H(x, \xi, t)}{x - \xi + i0}$$

# Relation of GPDs to 3D image of hadrons

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Nice, but there are problems!

- This limit is not achievable in an experiment
- The variable x is integrated out in the amplitude.

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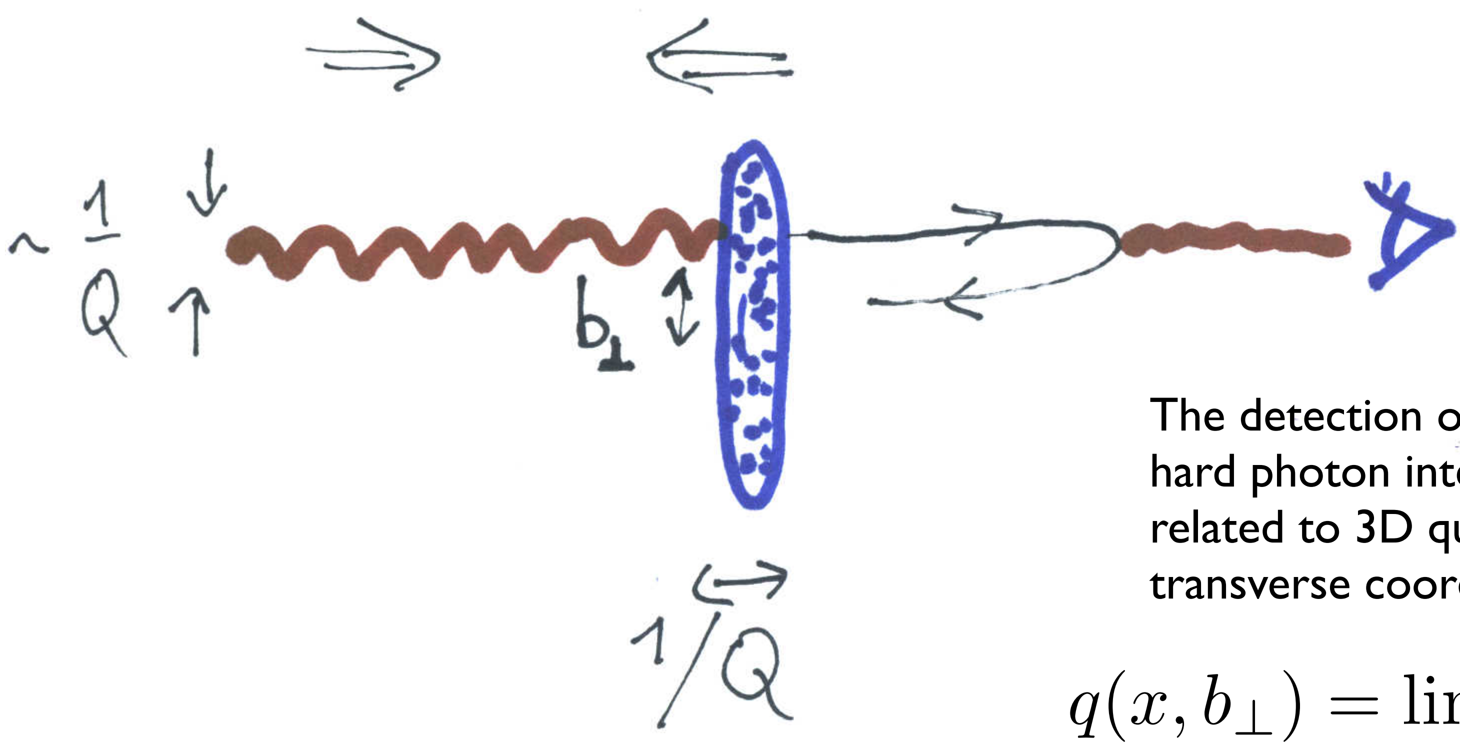


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Nice, but there are problems!

- This limit is not achievable in an experiment
- The variable x is integrated out in the amplitude.

What gives us a hope to get nevertheless 3D image of the nucleon?

- The amplitude is obtained as the "sectional image" of GPD (integration out x depends on $\xi$ ). That is typical problem for tomography! What kind of tomography?
- $x$ and $\xi$ dependences in GPD are interrelated due to polynomiality property of GPDs.

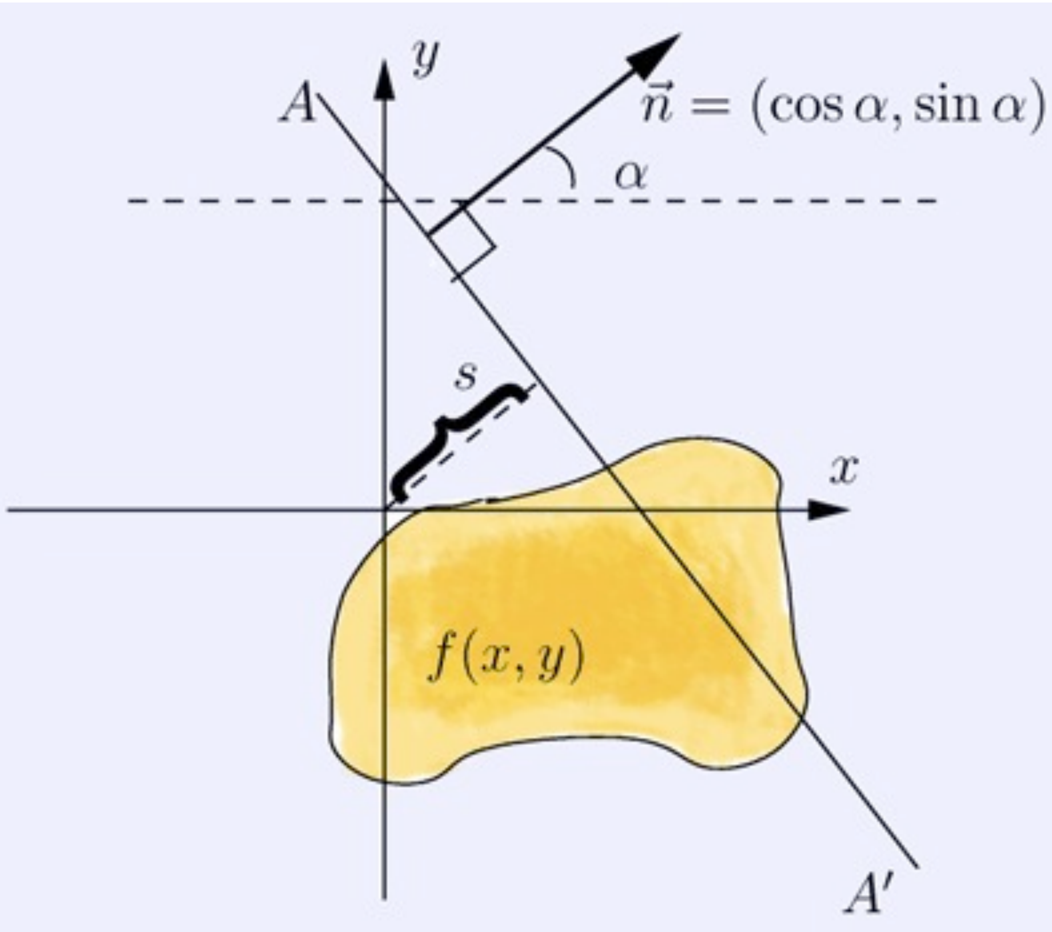
# Polynomiality of GPDs

$$\int_{-1}^{1} dx\ x^N\ H(x,\xi,t) = h_0^{(N)}(t) + h_2^{(N)}(t)\ \xi^2 + \ldots + h_{N+1}^{(N)}(t)\ \xi^{N+1}$$

Very nontrivial property!!! The x and xi dependences are interrelated!
The solution in terms of Radyushkin's double distributions:

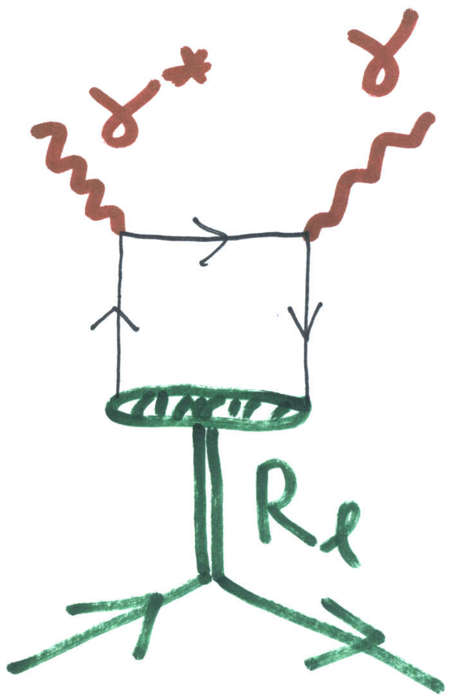
$$H(x,\xi) = \int_{-1}^{1} d\beta\ \int_{-1+|\beta|}^{1-|\beta|} d\alpha\ \delta(x-\beta-\alpha\xi)\ F(\beta,\alpha) + \theta\left[1-\frac{x^2}{\xi^2}\right]\ D\left(\frac{x}{\xi}\right)$$

Looks like the typical tomography problem! Unfortunately, to restore DD one needs GPD in cross channel $\xi \geq 1$, i.e. one needs an analytical continuation, which is almost impossible.




Another possibility to implement the polynomiality property of GPD is to use dual representation for GPD

# Dual representation of GPDs




Idea: to write down the GPD as the sum of t-channel exchanges:

$$H(x,\xi,t) = \sum_{\substack{n=1 \\ \text{odd}}}^{\infty} \sum_{\substack{l=0 \\ \text{even}}}^{n+1} B_{nl}(t)\, \theta\left(1-\frac{x^2}{\xi^2}\right)\left(1-\frac{x^2}{\xi^2}\right) C_n^{3/2}\left(\frac{x}{\xi}\right) P_l\left(\frac{1}{\xi}\right)$$

Conformal spin

Partial wave in the t-channel

QCD scale dependence of $B_{nl}(t)$ is simply multiplicative!

$$B_{nl}(t;\mu) = \left(\frac{\alpha_s(\mu)}{\alpha_s(\mu_0)}\right)^{\gamma_n/2b_0} B_{nl}(t;\mu_0)$$

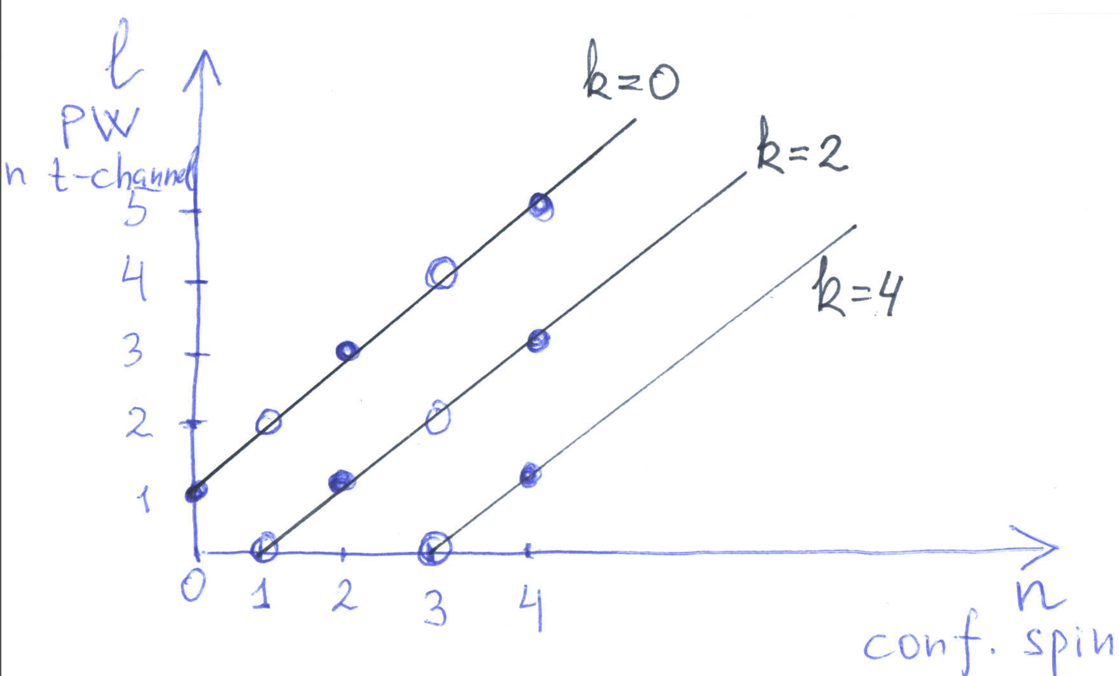
Problems:

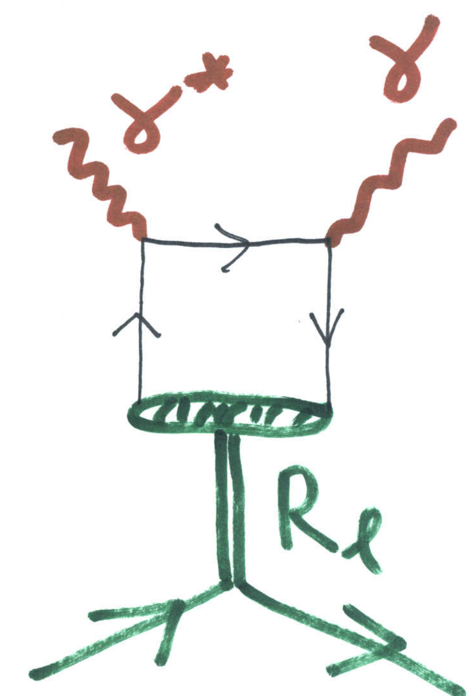
- Each term of the sum of the t-channel exchanges has the support $|x| \leq \xi$
- The sum is divergent for large partial waves, but the Mellin moments of the sum are finite.
- The situation is similar to the sum of t-channel exchanges in hadron hadron interactions
  Solution is the analytical continuation.

Pay attention that for the dual sum representation:

$$H(x,\xi,t) = \sum_{\substack{n=1 \\ \text{odd}}}^{\infty} \sum_{\substack{l=0 \\ \text{even}}}^{n+1} B_{nl}(t)\, \theta\left(1-\frac{x^2}{\xi^2}\right)\left(1-\frac{x^2}{\xi^2}\right) C_n^{3/2}\left(\frac{x}{\xi}\right) P_l\left(\frac{1}{\xi}\right)$$

the polynomiality property of GPD is automatic! The reason is that upper limit. The physics meaning is very simple: the angular momentum in the t-channel can not be larger than the Lorentz spin of the local QCD operator (Wigner-Eckart theorem!).



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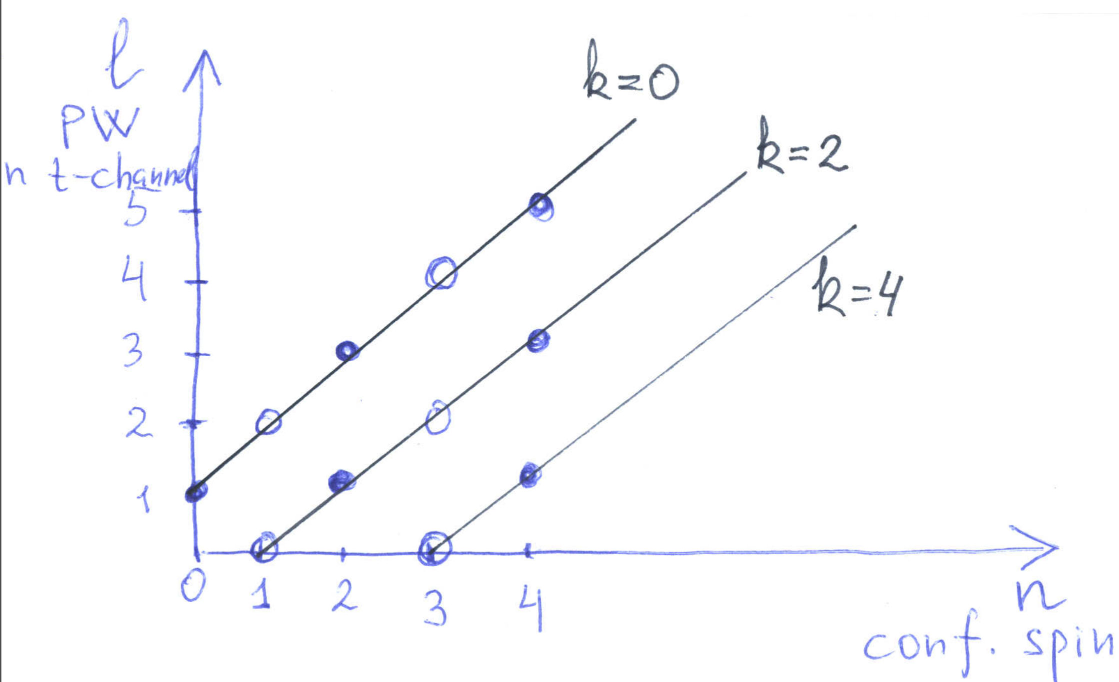


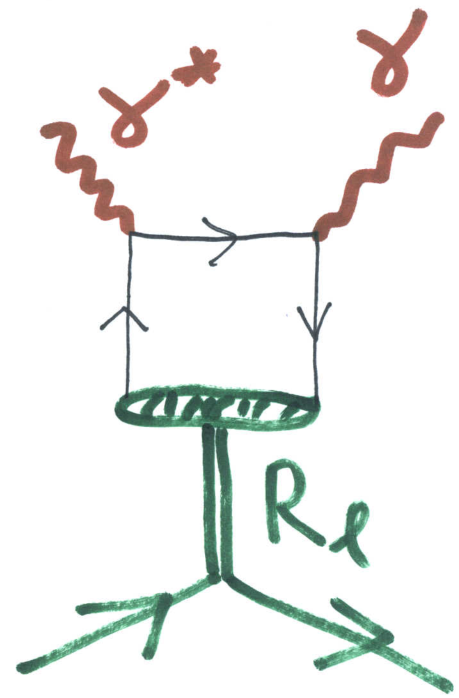


Great! But how to sum up such "bad sum"?
Let me illustrate on "toy example"

# The summation of "bad sums". Toy example.

We consider the following "toy" sum

$$H(x,\xi,t)=\sum_{\substack{n=1\\ \text{odd}}}^{\infty}\sum_{\substack{l=0\\ \text{even}}}^{n+1} B_{nl}(t)\,\theta\left(1-\frac{x^2}{\xi^2}\right)\left(1-\frac{x^2}{\xi^2}\right)\,C_n^{3/2}\left(\frac{x}{\xi}\right)\,P_l\left(\frac{1}{\xi}\right)$$

It has similar problems:

- Each term of the sum has the support at x=0 only
- The sum is divergent for large partial waves, but the Mellin moments of the sum are finite.

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and obtain:

$$h(x,\xi) = \frac{1}{\pi}\mathrm{Im}\int_0^1 dy\; Q(y)\frac{x}{\sqrt{x^2+y^2-2xy/\xi}}$$

Now compute Im part differently! Just by inspection of the region where one gets negative expression under square root!

we obtain that:

$$h(x,\xi) = \sum_{n=0}^{\infty} \frac{(-1)^n b_n}{(n+1)!}\ \delta^{(n)}(x)\ P_n\left(\frac{1}{\xi}\right)$$

$$= \frac{1}{\pi}\int_{\frac{x-x\sqrt{1-\xi^2}}{\xi}}^{1} dy\ Q(y)\frac{x}{\sqrt{x^2+y^2-2xy/\xi}}$$

The resulting summation gave us the function that:

- has the support not only at x=0 (remember that each term of the "toy sum" lives only at x=0)
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The same steps we can do for GPD representation in terms of t-channel exchanges

$$H(x,\xi,t) = \sum_{\substack{n=1 \\ \text{odd}}}^{\infty} \sum_{\substack{l=0 \\ \text{even}}}^{n+1} B_{nl}(t)\ \theta\left(1-\frac{x^2}{\xi^2}\right)\left(1-\frac{x^2}{\xi^2}\right)\ C_n^{3/2}\left(\frac{x}{\xi}\right)\ P_l\left(\frac{1}{\xi}\right)$$

$$h(x,\xi) = \sum_{n=0}^{\infty} \frac{(-1)^n b_n}{(n-1)!} \; \delta^{(n)}(x) \; P_n\left(\frac{1}{\xi}\right)$$

1st step - introduce generating function: $b_n = \int_0^1 dy\; y^n Q(y)$

$$H(x,\xi,t) = \sum_{\substack{n=1\\ \text{odd}}}^{\infty} \sum_{\substack{l=0\\ \text{even}}}^{n+1} B_{nl}(t)\; \theta\left(1-\frac{x^2}{\xi^2}\right)\left(1-\frac{x^2}{\xi^2}\right)\; C_n^{3/2}\left(\frac{x}{\xi}\right)\; P_l\left(\frac{1}{\xi}\right)$$

1st step - introduce generating functions: $B_{n\;n+1-k}(t) = \int_0^1 dy\; y^n\; Q_k(y,t)$

Now we have to introduce a set of functions $Q_k(y,t)$ because we have an additional index that counts the partial waves.

We call this set of functions as forward-like functions because:

- Their evolution is usual DGLAP evolution (the same as for usual forward PDFs)

$$B_{nl}(t;\mu) = \left(\frac{\alpha_s(\mu)}{\alpha_s(\mu_0)}\right)^{\gamma_n/2b_0} B_{nl}(t;\mu_0)$$

- $Q_0(y,t)$ is directly related to 3D quark distribution

$$Q_0(y,t) = q(y,t) - \frac{y}{2}\int_y^1 \frac{dz}{z^2}\; q(z,t)$$

# Essence of the dual representation of GPDs

The GPD $H(x,\xi,t)$ is equivalent to a (infinite) set of forward-like functions $Q_k(y,t)$

$$H(x,\xi,t) = \sum_{k=0}^{\infty} \int_0^1 dy\; M_k(x,\xi|y)\; Q_k(y,t)$$

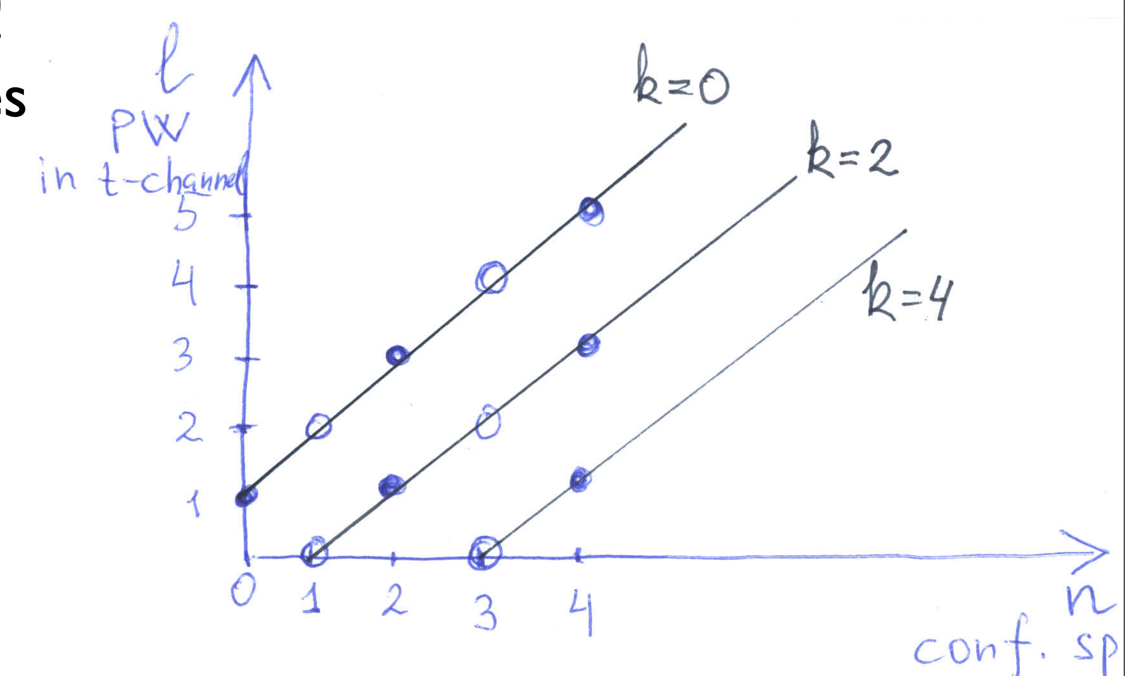
Known (very nice, related to elliptic functions) kernel!

What all these efforts for? What we achieved under guidance of simple physics picture?

(I) We reduced the continuous variable $\xi$ to a discrete index k
(II)The polynomiality is guaranteed.
(III) We know that k=0 corresponds to 3D parton densities!
(IV) We know that k=2 contains FFs of EMT (Jq, shear forces

Can we obtain all forward-like functions from the amplitude?

$$A(\xi,t) = \int_{-1}^{1} dx\; \frac{H(x,\xi,t)}{x-\xi+i0}$$



# The amplitude in terms of forward-like functions

The GPD $H(x,\xi,t)$ is equivalent to a (infinite) set of forward-like functions $Q_k(y,t)$
The amplitude:

$$A(\xi,t) = \int_{-1}^{1} dx\, \frac{H(x,\xi,t)}{x-\xi+i0}$$

$$\mathrm{Im}\; A(\xi,t) = \int_{\frac{1-\sqrt{1-\xi^2}}{\xi}}^{1} \frac{dx}{x} N(x,t) \left[\frac{1}{\sqrt{\frac{2x}{\xi}-x^2-1}}\right]$$

$$\mathrm{Re}\; A(\xi,t) = \int_0^{\frac{1-\sqrt{1-\xi^2}}{\xi}} \frac{dx}{x} N(x,t) \left[\frac{1}{\sqrt{1-\frac{2x}{\xi}+x^2}} + \frac{1}{\sqrt{1+\frac{2x}{\xi}+x^2}} - \frac{2}{\sqrt{1+x^2}}\right]$$

$$+ \int_{\frac{1-\sqrt{1-\xi^2}}{\xi}}^{1} \frac{dx}{x} N(x,t) \left[\frac{1}{\sqrt{1+\frac{2x}{\xi}+x^2}} - \frac{2}{\sqrt{1+x^2}}\right] + 2D(t)$$

The amplitude is expressed in terms of unique combination of forward-like functions!
The information about full GPD is lost in observables!

$$N(x,t) = \sum_{k=0}^{\infty} x^k\, Q_k(x,t)$$

# Properties of the amplitude in terms of forward-like functions

$$\mathrm{Im}\ A(\xi,t)=\int_{\frac{1-\sqrt{1-\xi^2}}{\xi}}^{1}\frac{dx}{x}N(x,t)\left[\frac{1}{\sqrt{\frac{2x}{\xi}-x^2-1}}\right]$$

$$\mathrm{Re}\ A(\xi,t)=\int_0^{\frac{1-\sqrt{1-\xi^2}}{\xi}}\frac{dx}{x}N(x,t)\left[\frac{1}{\sqrt{1-\frac{2x}{\xi}+x^2}}+\frac{1}{\sqrt{1+\frac{2x}{\xi}+x^2}}-\frac{2}{\sqrt{1+x^2}}\right]$$

$$+\int_{\frac{1-\sqrt{1-\xi^2}}{\xi}}^{1}\frac{dx}{x}N(x,t)\left[\frac{1}{\sqrt{1+\frac{2x}{\xi}+x^2}}-\frac{2}{\sqrt{1+x^2}}\right]+2D(t)$$

The amplitude in this form:

- satisfies automatically the dispersion relations in which

$$D(t)=\sum_{n=1}^{\infty}d_n(t)=\frac{1}{2}\int_{-1}^{1}dz\ \frac{D(z,t)}{1-z}$$

is the corresponding subtraction constant. It is related to the D-form factor.

$$D(t)=\int_0^1\frac{dz}{z}\,Q_0(z,t)\left(\frac{1}{\sqrt{1+z^2}}-1\right)+\int_0^1\frac{dz}{z}\,\left[N(z,t)-Q_0(z,t)\right]\,\frac{1}{\sqrt{1+z^2}}$$

- it is very easy to work with the amplitude (no singular integrals)

# The GPD quintessence function

The GPD $H(x,\xi,t)$ is equivalent to a (infinite) set of forward-like functions $Q_k(y,t)$ but the amplitude:

$$\mathrm{Im}\ A(\xi,t) = \int_{\frac{1-\sqrt{1-\xi^2}}{\xi}}^{1} \frac{dx}{x} N(x,t) \left[ \frac{1}{\sqrt{\frac{2x}{\xi} - x^2 - 1}} \right]$$

depends only on one particular combination of them.

$$N(x,t) = \sum_{k=0}^{\infty} x^k\ Q_k(x,t)$$

From that we conclude that in observables we definitely loose information about full GPD! It is very difficult to separate out $Q_0(x,t)$, which is equivalent to 3D parton densities.

Question: can we restore N(x,t) from knowledge of the amplitude?
Answer: YES! Therefore we call N(x,t) as GPD quintessence function, as it contains the maximal information which we can obtain about GPD from the amplitude.

Note that now the relation between the amplitude and N(x,t) provide us with new type of tomographic problem (amplitude is obtained as a "sectional imaging " of N(x,t)). What kind of tomography we have now ?

# The Abel tomography

$$\mathrm{Im}\; A(\xi,t) = \int_{\frac{1-\sqrt{1-\xi^2}}{\xi}}^{1} \frac{dx}{x} N(x,t) \left[ \frac{1}{\sqrt{\frac{2x}{\xi} - x^2 - 1}} \right]$$

After Zhukovsky transformation (used in aerodynamics) of the variable x

$$\frac{1}{w} = \frac{1}{2}\left(x + \frac{1}{x}\right)$$

The amplitude gets very simple form:

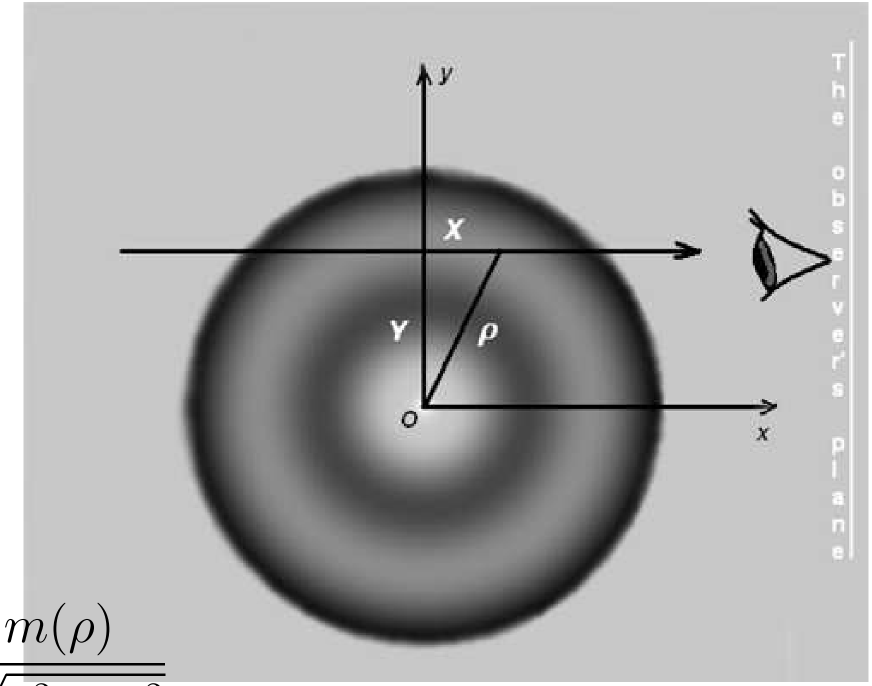
$$\mathrm{Im}\; A^{\mathrm{tw2}}(\xi,t) \;=\; \int_{\xi}^{1} \frac{dw}{w}\, M(w,t)\, \frac{\sqrt{\xi}}{\sqrt{w-\xi}}$$

with

$$M(w,t) = N\left(\frac{1-\sqrt{1-w^2}}{w}, t\right) \frac{w}{\sqrt{2(1-w^2)}\sqrt{1-\sqrt{1-w^2}}}$$

In "aerodynamics variable" w, the integral for the amplitude has the form of Abel integral. Typical for Abel tomography! What is this tomography?

# The Abel tomography




Suppose we make a photograph of a spherically symmetric body. And we want to derive a 3D distribution of density in the body.

The "photograph" is given by: $$a(y) = \int_{-\infty}^{\infty} dx\; m(\rho)$$

Using spherical symmetry of the body we write: $$a(y) = \int_{y^2}^{\infty} d\rho^2\; \frac{m(\rho)}{\sqrt{\rho^2 - y^2}}$$

which with obvious renaming of of variables is equivalent to our expression for the amplitude

$$\mathrm{Im}\; A^{\mathrm{tw2}}(\xi, t) \;=\; \int_{\xi}^{1} \frac{dw}{w}\; M(w,t)\; \frac{\sqrt{\xi}}{\sqrt{w-\xi}}$$

The integral equation: $a(\xi,t) = \int_{\xi}^{1} dw\; \frac{m(w,t)}{\sqrt{w-\xi}}$, can be easily solved!

$$m(w,t) = -\frac{1}{\pi}\frac{d}{dw}\int_{w}^{1} d\xi\; \frac{a(\xi,t)}{\sqrt{\xi - w}}.$$

Applying this technique to the expression for the amplitude in terms of GPD quintessence function N(x,t) we obtain:

# The tomography for GPD quintessence function N(x,t)

$$\mathrm{Im}\ A(\xi,t) = \int_{\frac{1-\sqrt{1-\xi^2}}{\xi}}^{1} \frac{dx}{x} N(x,t) \left[ \frac{1}{\sqrt{\frac{2x}{\xi} - x^2 - 1}} \right]$$

Inversion of this relation:

$$N(x,t) = \frac{2}{\pi} \frac{x(1-x^2)}{(1+x^2)^{3/2}} \int_{\frac{2x}{1+x^2}}^{1} \frac{d\xi}{\xi^{3/2}} \frac{1}{\sqrt{\xi - \frac{2x}{1+x^2}}} \left\{ \frac{1}{2} \mathrm{Im}\ A(\xi,t) - \xi \frac{d}{d\xi} \mathrm{Im}\ A(\xi,t) \right\}$$

We see that N(x,t) is indeed GPD quintessence function! It is completely restored from the amplitude.

Remember that N(x,t) contains only part of the information about full GPD and that is the part of info about GPDs which we can maximally obtain by measurements!

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What is the physics content of N(x,t) ?

# Physics content of GPD quintessence N(x,t)

$$N(x,t) = Q_0(x,t) + x^2 Q_2(x,t) + x^4 Q_4(x,t) + \ldots$$

Contains 3D quark densities

Contains FFs of energy-momentum tensor (Jq, shear forces, etc.)

Even if we know complete amplitude, we are not able to separate these contributions :(
However, there is a principle possibility to make the separation via logarithmic scaling violation. (Very difficult to implement in near future experiments.)

What to do?

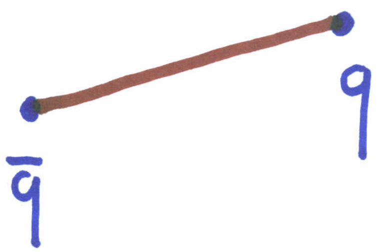
- Look for new physics motivations! Study in more details the physics content of N(x,t).
- Model building for forward-like functions.

# Physics content of GPD quintessence N(x,t)

What are the Mellin moments of N(x,t) $\int_0^1 dx\, x^{J-1} N(x,t)$ ?

Note that in contrast to the Mellin moments of GPDs these integrals are direct observables: they are expressed via the amplitude !

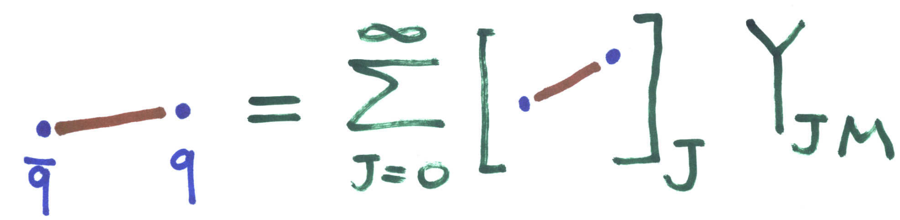
The hard pQCD interaction creates for us QCD string operator.




$$\bar{q}(n)\, P\exp\left(i\int_{-n}^{n} dx^\mu A_\mu\right) q(-n)$$

That softly interacts with the target.

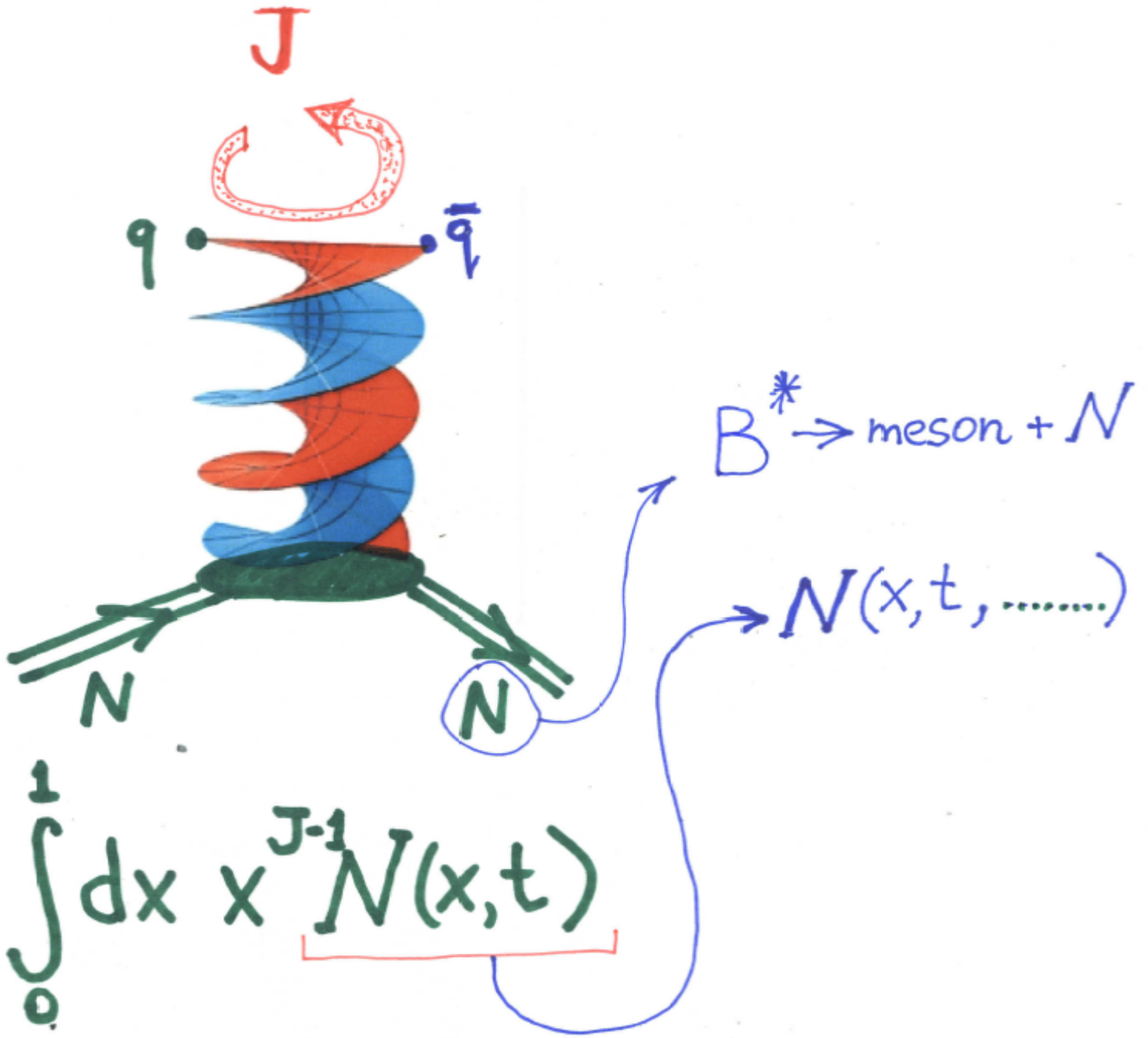
Can we decompose the QCD string into states with fixed angular momentum?




Very simple calculations shows that with fixed angular momentum J! $\int_0^1 dx\, x^{J-1} N(x,t) = F_J(t)$ gives FF of QCD string

It seems that quintessence function N(x,t) provides us with new tool to study QCD strings. Also it opens a new possibilities for studies of nucleon excitations.

# Possible applications of GPD quintessence N(x,t)



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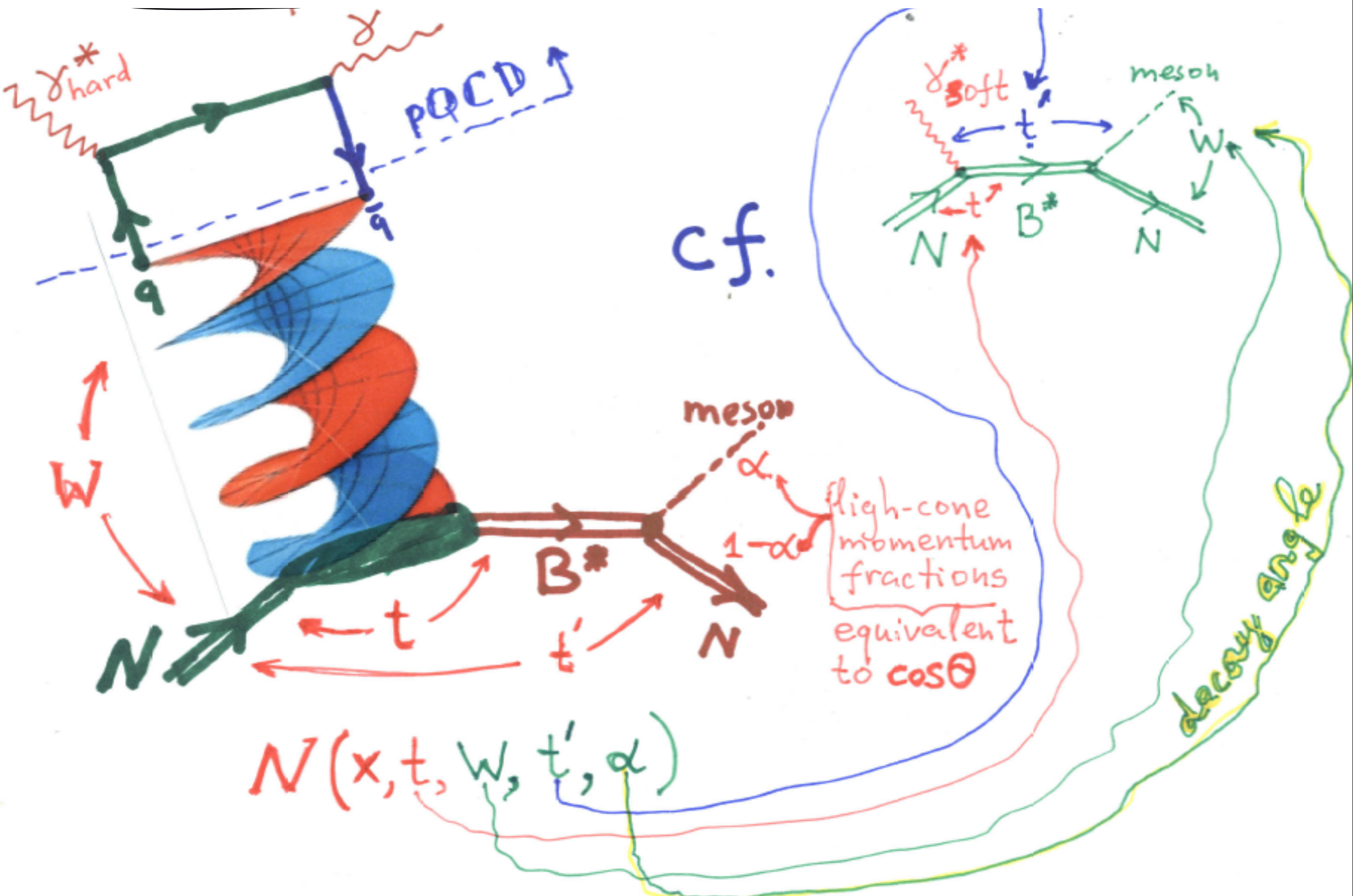

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**There is also possibility to make the separation via twist-3 effects. However this separation requires Wandzura-Wilczek approximation for twist-3 GPDs.**

The WW approximation consists in neglecting the operators which contain gluon field strength. The theory of instanton vacuum predicts that contribution of such operators is parametrically small in the instanton packing fraction.

# Quintessence N(x,t) in twist-3 DVCS amplitude

Leading twist-2 DVCS amplitude is in one-to-one correspondence with the quintessence N(x,t) via the Abel tomography

$$N(x,t) = Q_0(x,t) + x^2 Q_2(x,t) + x^4 Q_4(x,t) + \dots$$

Equivalent to the tw-2 amplitude

Contains 3D quark densities

Contains FFs of energy-momentum tensor (Jq, shear forces, etc.)

For the twist-3 amplitude one can also perform the Abel tomography. In WW approximation we obtain:

$$N_S(x,t) = \frac{1}{\pi}\frac{(1-x^2)}{\sqrt{1+x^2}}\int_{\frac{2x}{1+x^2}}^{1}\frac{d\xi}{\sqrt{\xi}}\frac{1}{\sqrt{\xi-\frac{2x}{1+x^2}}}\left\{\mathrm{Im}A^{\mathrm{tw2}}(\xi,t)-\mathrm{Im}A^{\mathrm{tw3}}(\xi,t)\right\}$$

$$N_S(x,t) = \int_x^1 \frac{dz}{z}\left(1-\frac{x}{z}\right)\left(Q_0(z,t)+x^2Q_2(z,t)+\dots\right)$$

# Cross process $\gamma^* + \gamma \to h + \bar{h}.$

The amplitude of cross process can expressed in terms of the SAME quintessence function N(x,t)

$$A^{\rm cross}(\eta,t) = \int_0^1 \frac{dx}{x} N(x,t) \left[\frac{1}{\sqrt{1-2x\eta+x^2}} + \frac{1}{\sqrt{1+2x\eta+x^2}} - \frac{2}{\sqrt{1+x^2}}\right] + 2D(t)$$

$\eta = \cos\theta_{\rm cm}$

timelike

A possibility to "touch" N(x,t) at BABAR, BELLE or PANDA ?! Or at EIC through "generalized Primakoff process"

If one uses the Abel tomography formula, one gets the relation between DVCS amplitude and $\gamma^* + \gamma \to h + \bar{h}.$

$$A^{\rm cross}(\eta,t) = \frac{2}{\pi}\int_0^{|\eta|} d\xi \frac{\xi}{1-\xi^2}\ {\rm Im} A\left(\frac{\xi}{|\eta|}, t\right) + 2\ D(t)$$

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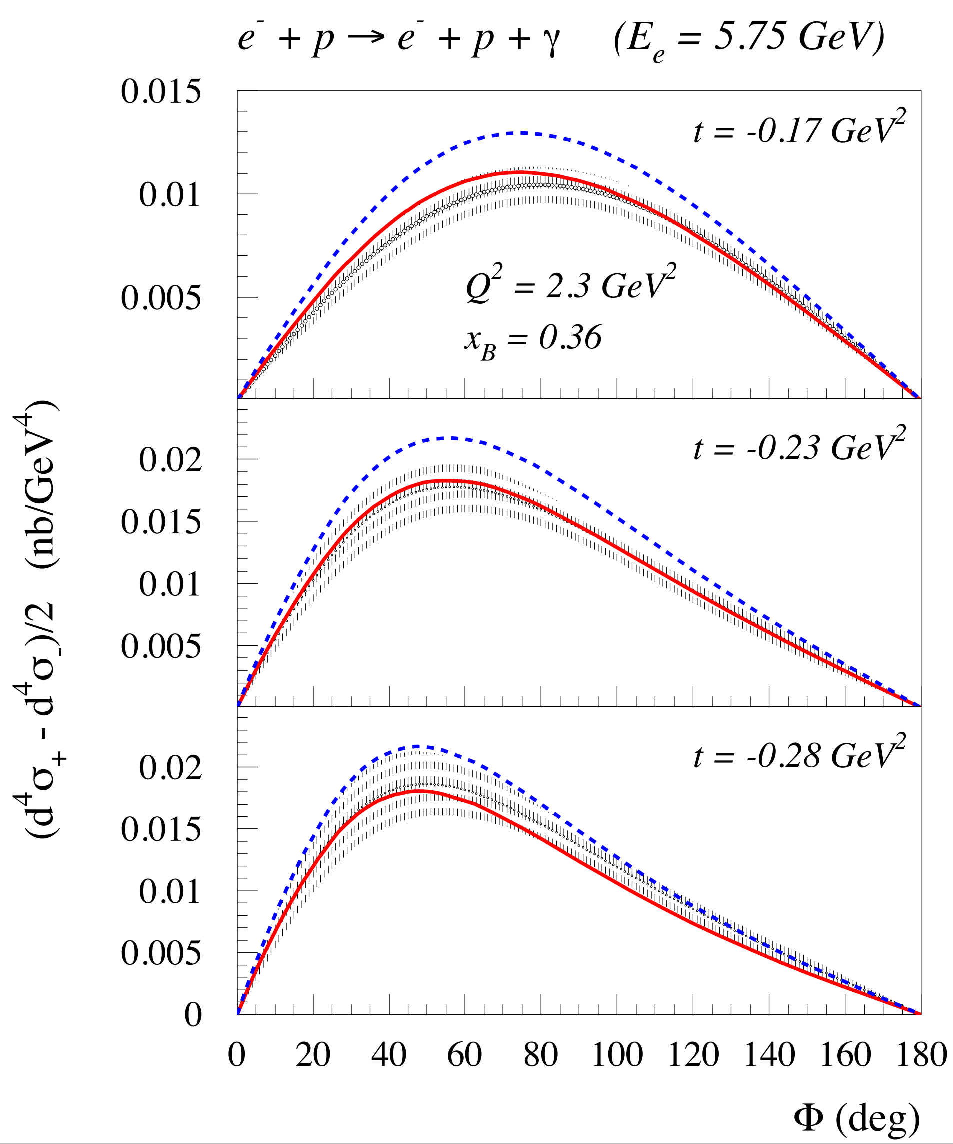

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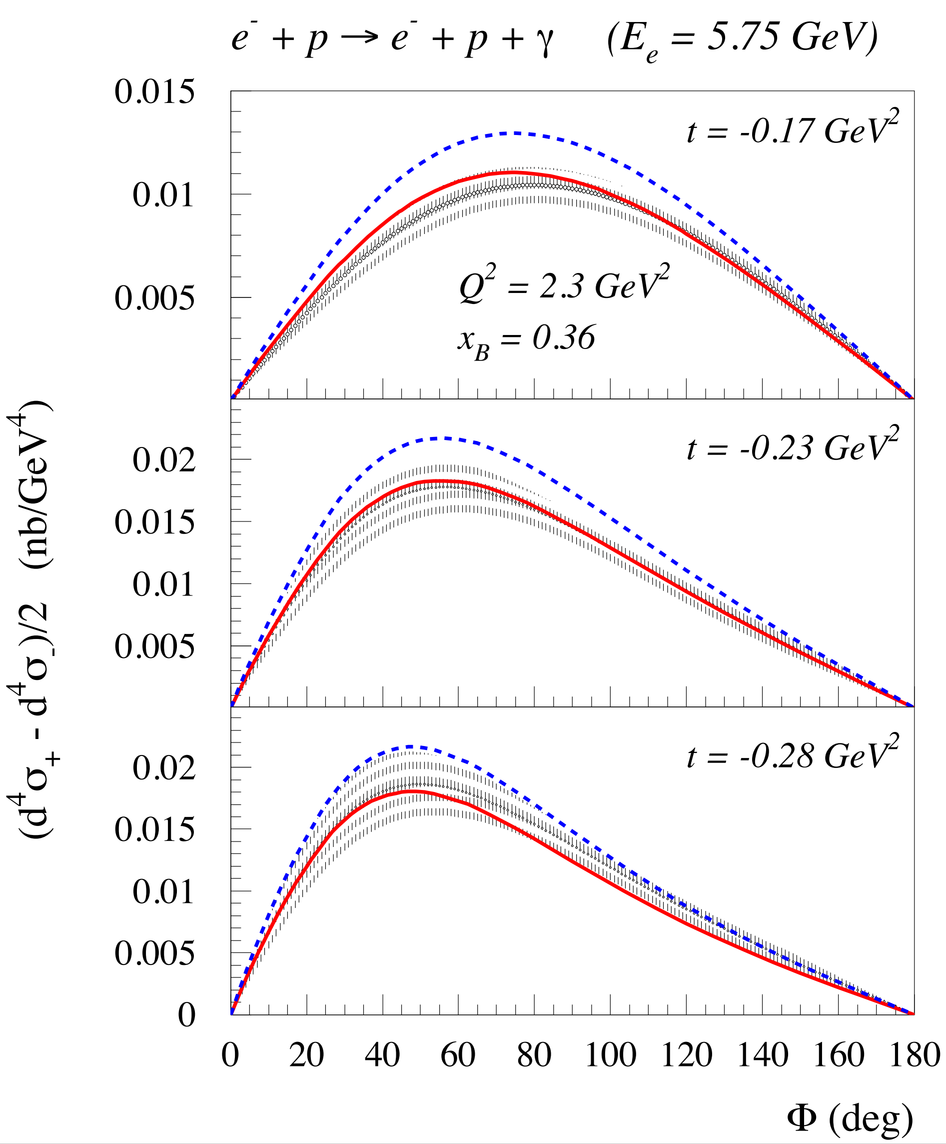


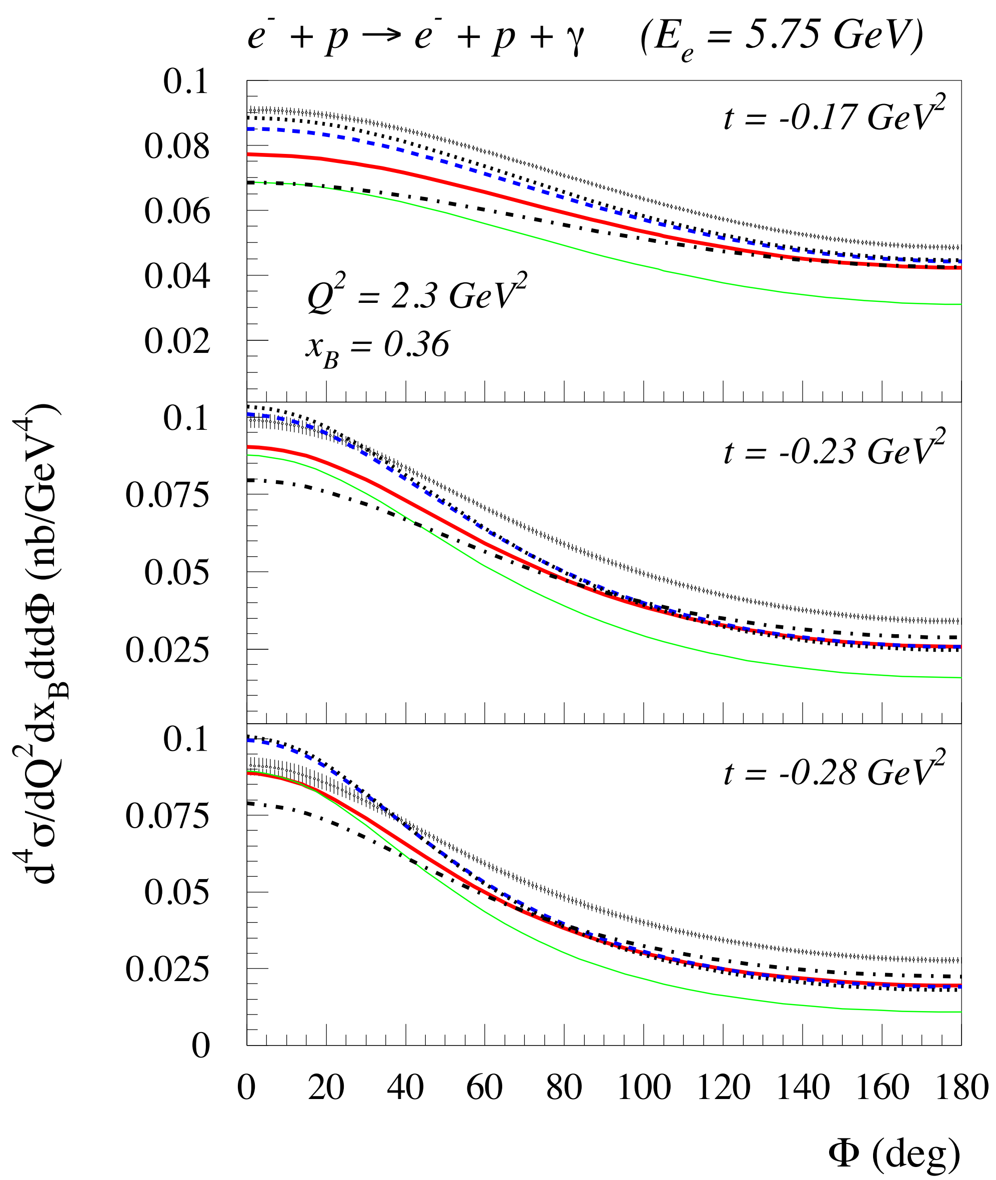

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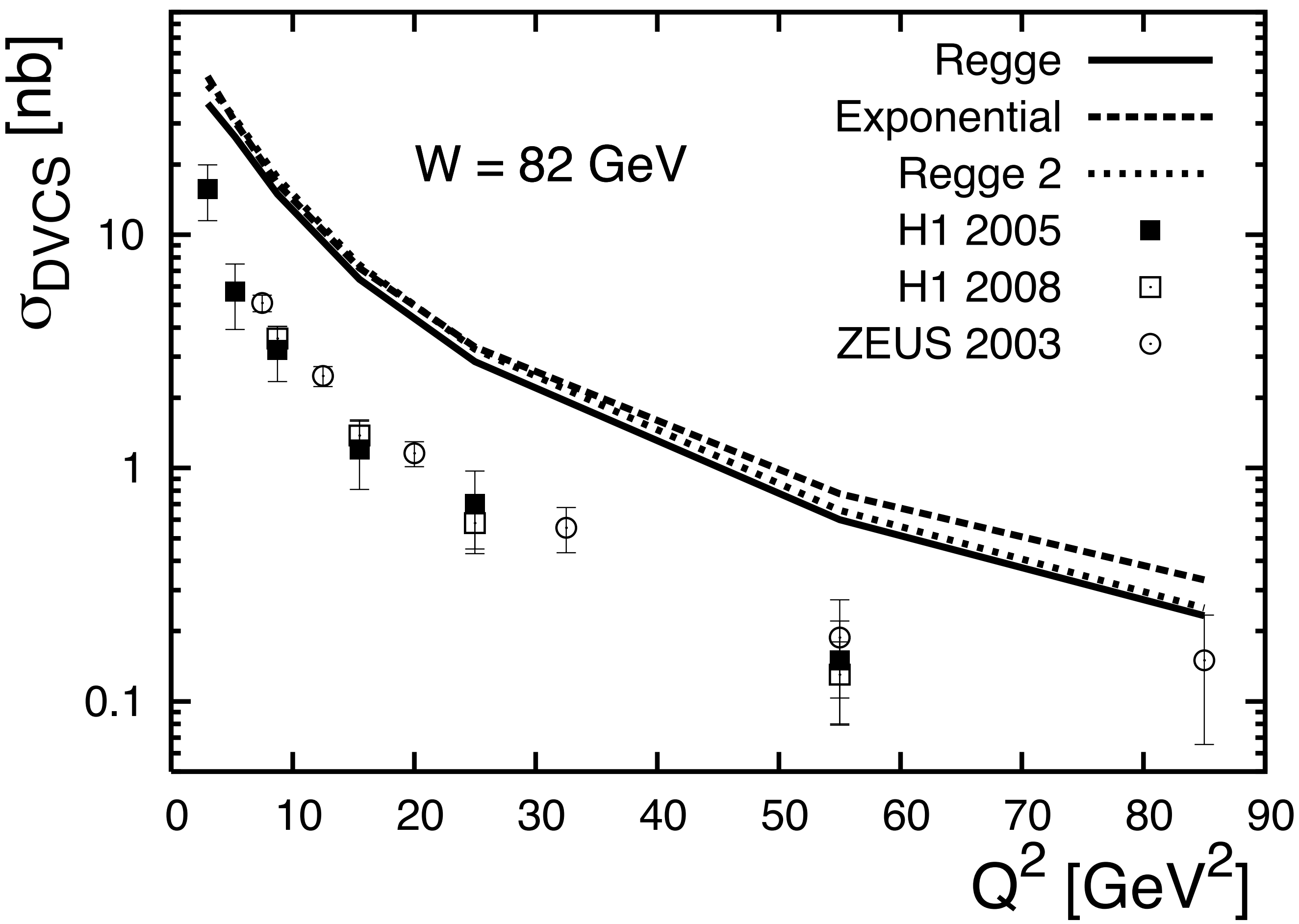

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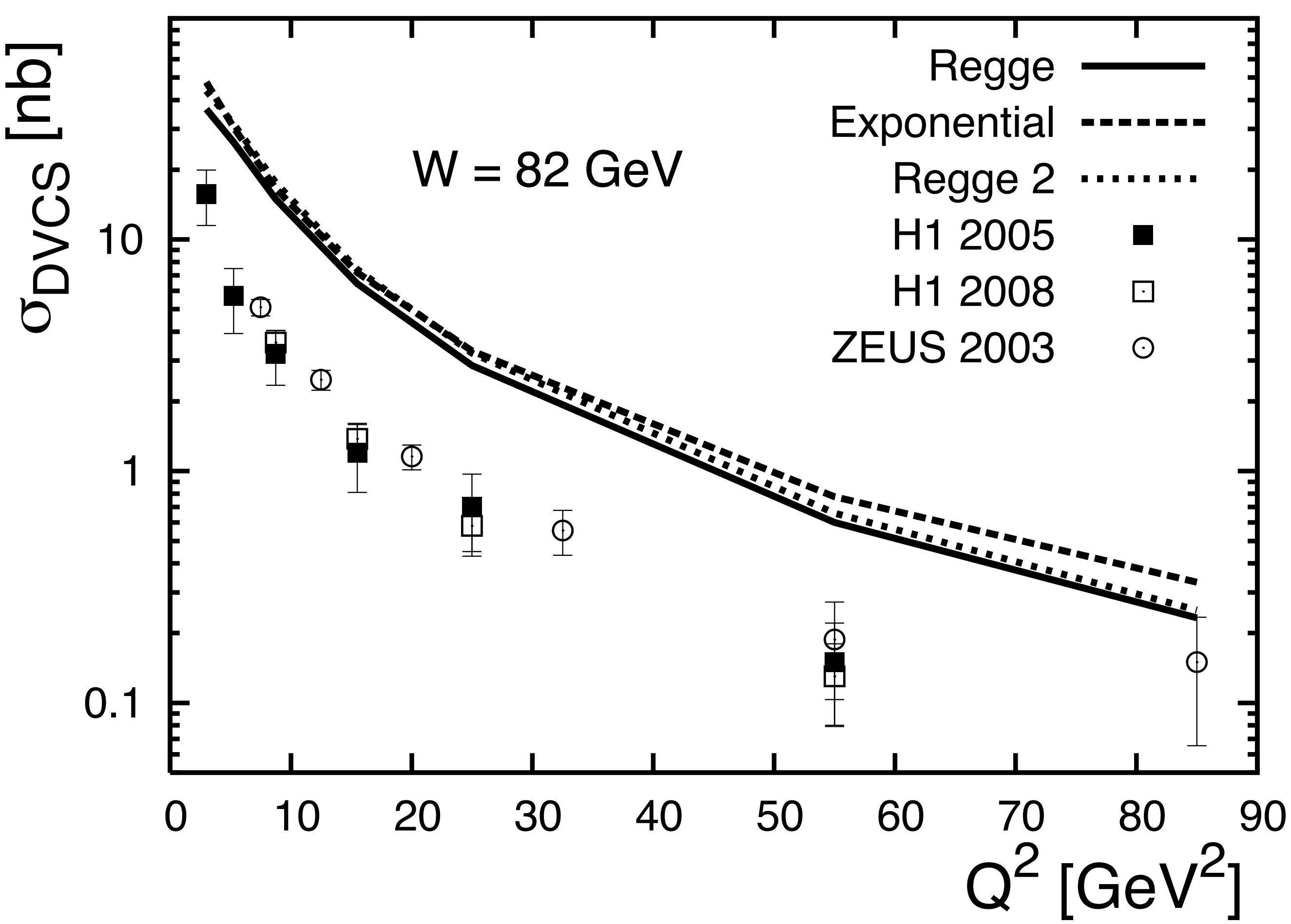


That shows that $Q_{2,4,\ldots}(x,t)$ are large in the small x region! One should expect that: $$\frac{Q_2(x,t)}{Q_0(x,t)} \sim \frac{1}{x^2}$$

That makes our life complicated, but more interesting!

The presence of strong small x singularity can bring new insight into structure of GPDs! Possibility for a holography (in progress).

## Conclusions

- We can not restore full GPDs from the amplitudes. (Different GPDs can give the same amplitudes)

- The maximally restorable info about GPDs is contained in quintessence function.

- The quintessence function can be restored from the amplitude via the Abel tomography

- Mellin moments of N(x,t) have nice interpretation in terms of QCD string operator of fixed angular momentum.

Guide to references

- relations of GPDs to 3D parton densities are derived by M. Burhardt (2001)
- Radon tomography for GPDs and DDs: O. Teryaev (2001)
- the summation of "bad sums" is based on Shuvaev transformation /A. Shuvaev(1999)/
- the dual representation of GPDs: /MVP (1998), A. Shuvaev, MVP (2002)/
- dispersion relations for DVCS: /O. Teryaev (2005), O. Teryaev, I. Anikin (2007), M. Diehl, D.Ivanov (2008)/
- Abel tomography: /MVP (2007), A. Moiseeva, MVP (2009)/
- detailed theoretical studies of dual representation for GPDs: K. Semenov-Tian-Shansky (2007-2010)
- phenomenological application of minimalist dual model: /V. Guzey, T. Teckentrup (2006-2008), M. Vanderhaeghen, MVP (2008)/
- analytical properties, conformal wave expansion, and holography for GPDs /D. Mueller (2007-2010)/