Exotic mesons from lattice QCD

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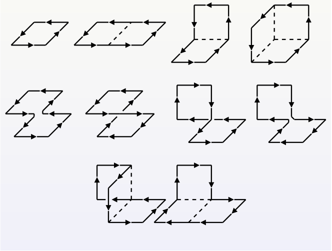
Review

- Exotic mesons are states that do not fit naturally into the simplest quark model
- Considerable experimental interest in these states
- Statistical precision has been difficult to obtain on the lattice
- Using better and new techniques might help...

Glueball spectroscopy

- Creation operator for glueball should excite gluon fields alone.
- Smooth, gauge-invariant operator on gluon field = smeared Wilson loops
- Variational method easy to implement, since operators involve only gluons (no linear solvers!)

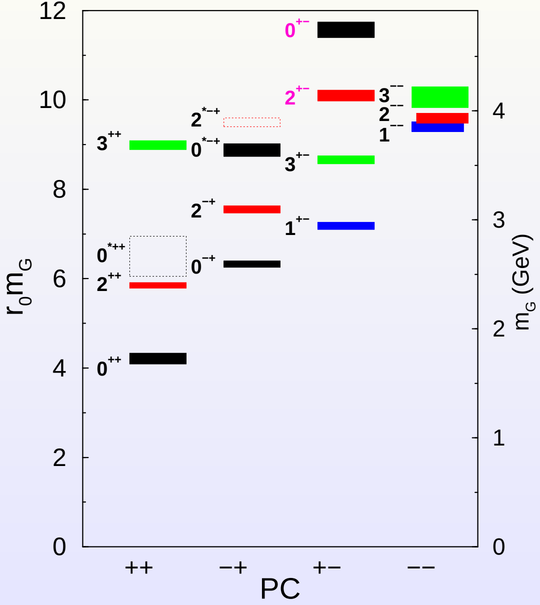


- Correlation function is

$$C(t) = \langle \, \mathrm{Tr}\, U_{\mathcal{A}}(t) \quad \mathrm{Tr}\, U_{\mathcal{B}}(0) \, \rangle$$

- **Problem:** variance in measurements very large and states are intrinsically heavy

The SU(3) Yang-Mills glueball spectrum



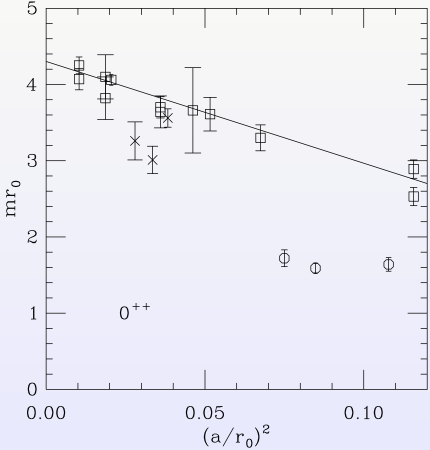
- No quarks - Yang-Mills theory only
- Ensembles of more than 10,000 configurations
- Anisotropic discretisation: $a_s > a_t$
- No light spin-exotic glueballs

C. Morningstar and MP
PRD60:034509 (1999)

Glueballs on dynamical lattices

- Studies to date focus on the lightest (scalar) mode

  [C. Michael and C. McNeile, PRD63 114503 (2001)]



- Newer study with staggered quarks finds YM result for 0^{++}, 0^{-+} and 2^{++} [UKQCD, PRD82 034501 (2010)]

Isovector and isoscalar mesons

Creating mesons in QFT

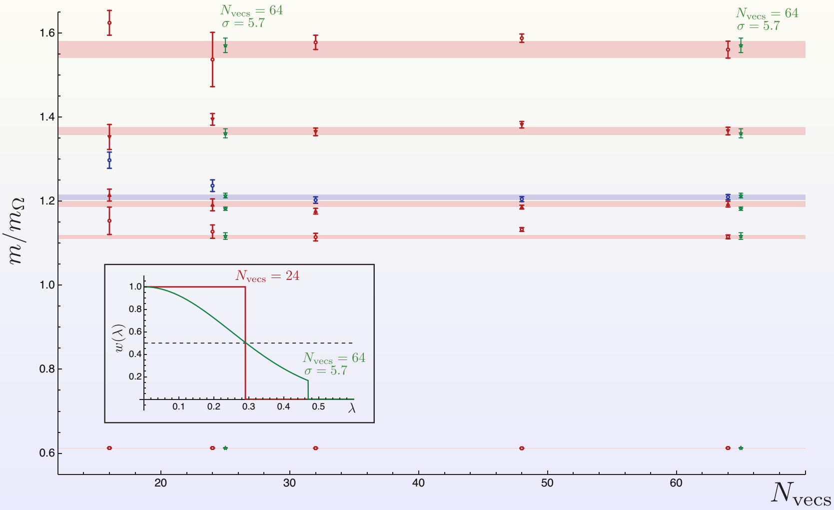
- Add powers of D_i to increase L
- $N=2$ - reduce D_iD_j to get $L=0,1,2$.
- $L=1$ from $\epsilon_{ijk}D_jD_k = B_i$
- Chromomagnetic operator - intrinsic gluon excitation

	Singlet	Triplet
$N=0$	$\bar{\psi}\gamma_5\psi$	$\bar{\psi}\gamma_i\psi$
	0^{-+}	1^{--}
$N=1$	$\bar{\psi}\gamma_5 D_i\psi$	$\bar{\psi}\gamma_i D_j\psi$
	1^{+-}	$\{0,1,2\}^{++}$
$N=2$	$\bar{\psi}\gamma_5\{D_i,D_j\}\psi$	$\bar{\psi}\gamma_i\{D_j,D_k\}\psi$
	2^{-+}	$\{1,2,3\}^{--}$
	$\bar{\psi}\gamma_5[D_i,D_j]\psi$	$\bar{\psi}\gamma_i[D_j,D_k]\psi$
	1^{--}	$\{0,\mathbf{1},2\}^{-+}$

- Replace continuum derivatives with finite differences and then reduce the resulting representations of O_h

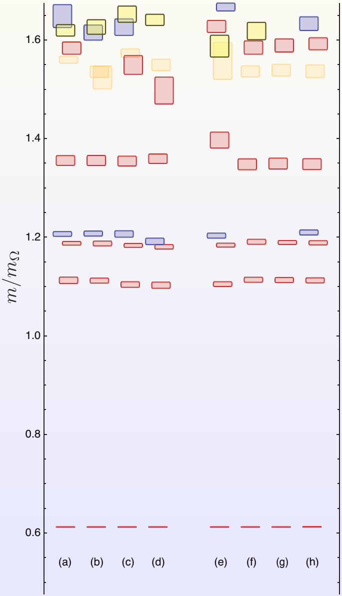
[J. Dudek *et.al.* PRD77:034501 (2008)]

Distillation: Varying the size of the space



- 16^3 spatial volume, $I = 1\ T_1^{--}$ - excited states unstable below $N \approx 40$

The variational basis



- Can check sensitivity of spectrum to operator basis:

(a) dim-26 op basis, up to D^3

(b) Full basis, two noisest removed

(c) Full basis, four noisest removed

(d) No D^3 except continuum $J = 4$

(e) No $[D, D]$ ops

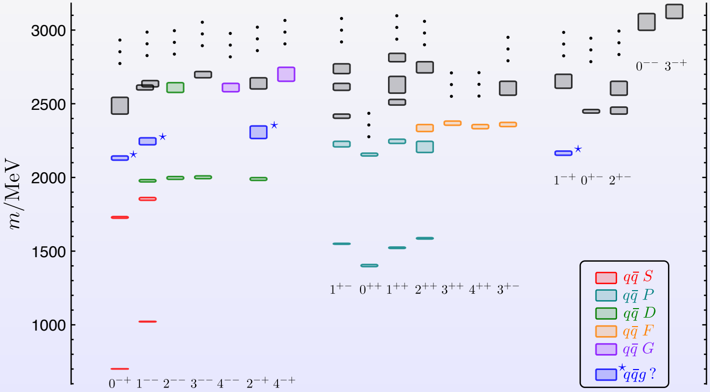
(f) No continuum $J = 3$

(g) No continuum $J = 3, 4$

(h) No continuum $J = 4$

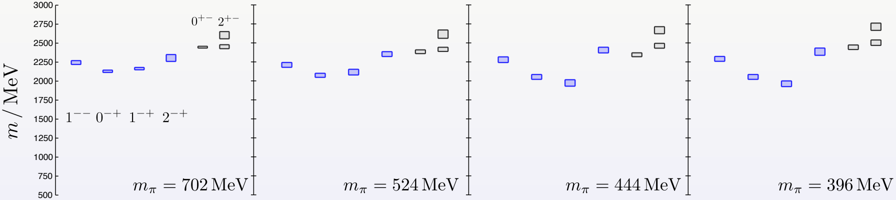
Isovector meson spectrum ($m_\pi = ?MeV$

- Below $2GeV$, quark model explains all data
- First identification of the hybrid singlet/triplet?
- Still at unphysical m_π (and not in continuum limit)



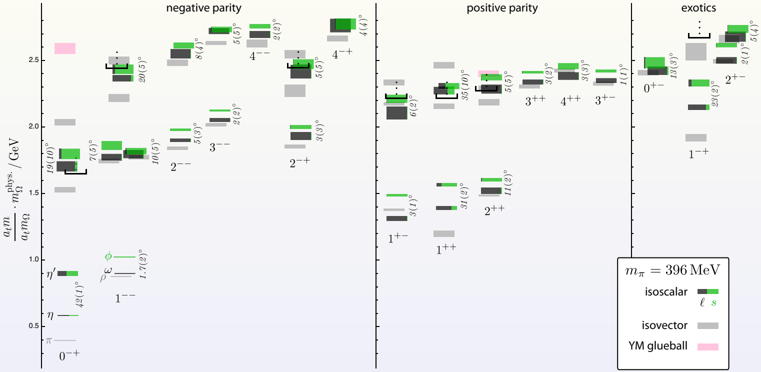
Isovector hybrid mesons - the “supermultiplet?”

[J.Dudek arXiv:1106.5515]



- Substantial mass dependence not seen.
- Possible to test model predictions (bag, string, constituent gluons ...)?
- PDG lists two 1^{-+} states $\pi_1(1400)$ and $\pi_1(1600)$

Isoscalar meson spectrum

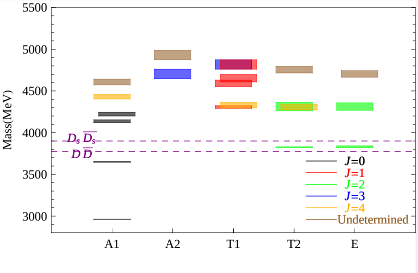


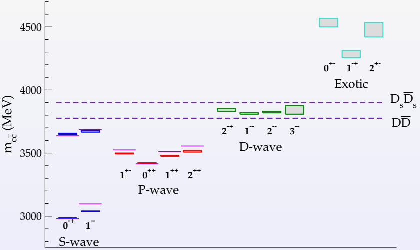
- $V = 16^3$ using GPUs to compute all-t propagators
- Percent-level statistical precision possible
- light-strange mixing computed
- **BUT** - 0^{++} not shown here!

[J. Dudek *et.al.* PRD83:111502 (2011)]

Charmonium spectroscopy

- Significant experimental interest in charmed hybrids - do they explain the narrow states above $D - \bar{D}$ threshold?





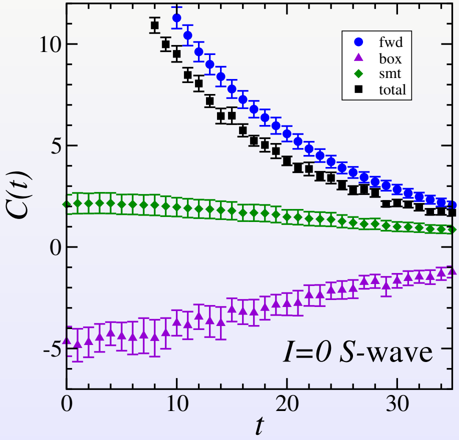
PRELIMINARY [G.Moir, L.Liu, P.Vilaseca, MP, S.Ryan]

- Distilled charm quarks - good statistical precision again
- Statistical error on 1^{-+} hybrid $\approx 17\text{MeV}$

Scattering on
the Euclidean lattice

I=0 $\pi - \pi$ scattering - measuring $\langle \pi\pi | \pi\pi \rangle$

- Stochastic insertion into distillation space works well



[C. Morningstar *et.al.*: PRD83:114505 (2011)]

Particle(s) in a box

- Spatial lattice of extent L with periodic boundary conditions
- Allowed momenta are quantized: $p = \frac{2\pi}{L}(n_x, n_y, n_z)$ with $n_i \in \{0, 1, 2, \ldots L-1\}$
- Energy spectrum is a set of **discrete** levels, classified by p: Allowed energies of a particle of mass m

$$E = \sqrt{m^2 + \left(\frac{2\pi}{L}\right)^2 N^2} \quad \text{with } N^2 = n_x^2 + n_y^2 + n_z^2$$

- Can make states with **zero total momentum** from pairs of hadrons with momenta $p, -p$.
- "Density of states" **increases** with energy since there are more ways to make a particular value of N^2 e.g. $\{3, 0, 0\}$ and $\{2, 2, 1\} \rightarrow N^2 = 9$

Avoided level crossings

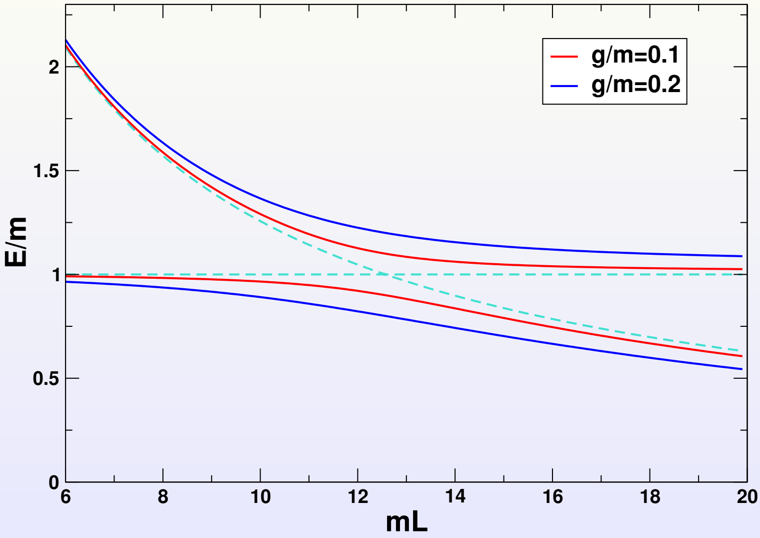
- Consider a toy model with two states (a resonance and a two-particle decay mode) in a box of side-length L
- Write a mixing hamiltonian:

$$H = \begin{pmatrix} m & g \\ g & \frac{4\pi}{L} \end{pmatrix}$$

- Now the energy eigenvalues of this hamiltonian are given by

$$E_{\pm} = \frac{(m + \frac{4\pi}{L}) \pm \sqrt{(m - \frac{4\pi}{L})^2 + 4g^2}}{2}$$

Avoided level crossings



Avoided level crossings

- **Ground-state** smoothly changes from resonance to two-particle state
- Need a large box. This example, levels cross at $mL = 4\pi \approx 12.6$
- Example: $m = 1$ GeV state, decaying to two massless pions - avoided level crossing is at $L = 2.5\text{fm}$.
- If the decay product pions have $m_\pi = 300$ MeV, this increases to $L = 3.1\text{fm}$

Lüscher's method

- Relates the spectrum in a finite box to the scattering phase shift (and so resonance properties)

Lüscher's formula

$$\delta(p) = -\phi(\kappa) + \pi n$$

$$\tan\phi(\kappa) = \frac{\pi^{3/2}\kappa}{Z_{00}(1;\kappa^2)}$$

$$\kappa = \frac{pL}{2\pi}$$

- p_n is defined for level n with energy E_n from the dispersion relation:

$$E_n = 2\sqrt{m^2 + p_n^2}$$

Lüscher's method

- Z_{00} is a generalised Zeta function:

$$Z_{js}(1,q^2) = \sum_{n \in Z^3} \frac{r^j Y_{js}(\theta,\phi)}{(n^2 - q^2)^s}$$

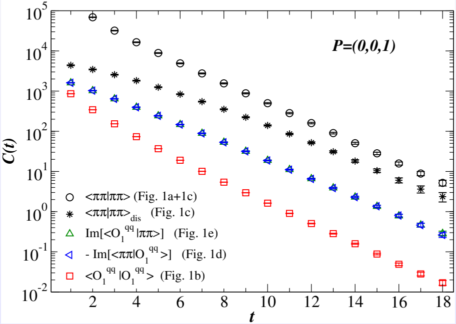
[M.Lüscher, Commun.Math.Phys.105:153-188,1986.]

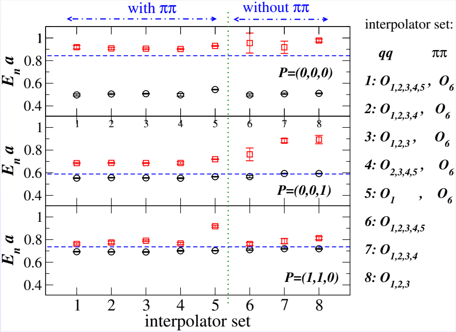
- With the phase shift, and for a well-defined resonance, can fit a Breit-Wigner to extract the **resonance width** and **mass**.

$$\delta(p) \approx \tan^{-1}\left(\frac{4p^2 + 4m_\pi^2 - m_\sigma^2}{m_\sigma \Gamma \sigma}\right)$$

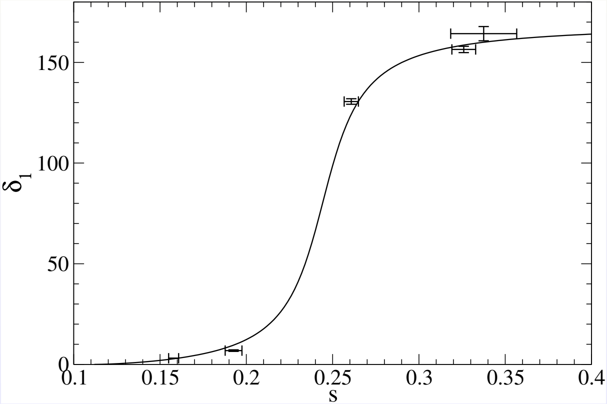
$I = 1$ scattering using distillation

- A number of groups have investigated Γ_ρ on the lattice.
- Need non-zero relative momentum of pions in final state (P-wave decay)
- New calculation using distillation: [C.Lang *et.al.* `arXiv:1105.5636`]



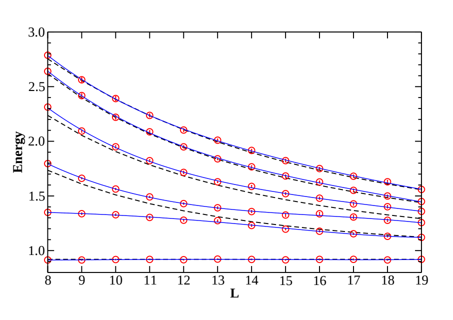


$I = 1 \pi\pi$ phase shift



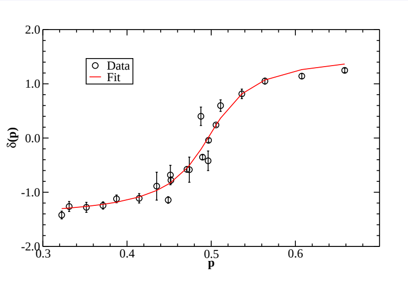
Test: O(4) Sigma model

[D. McManus, P. Giudice and MP]

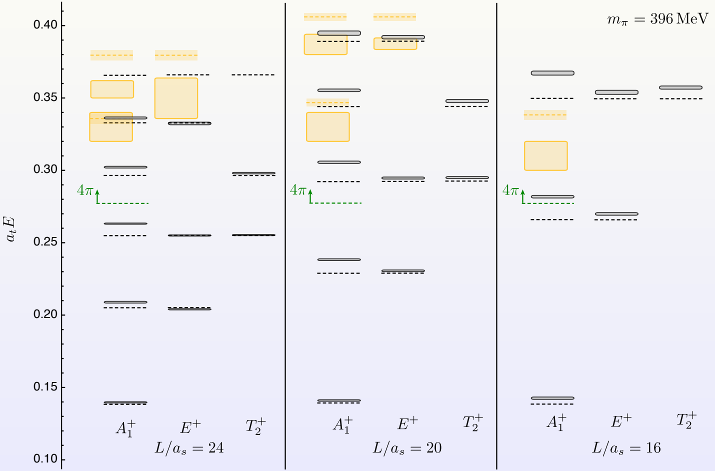


Spectrum of $O(4)$ sigma model in broken phase

Phase shift inferred from Lüscher’s method

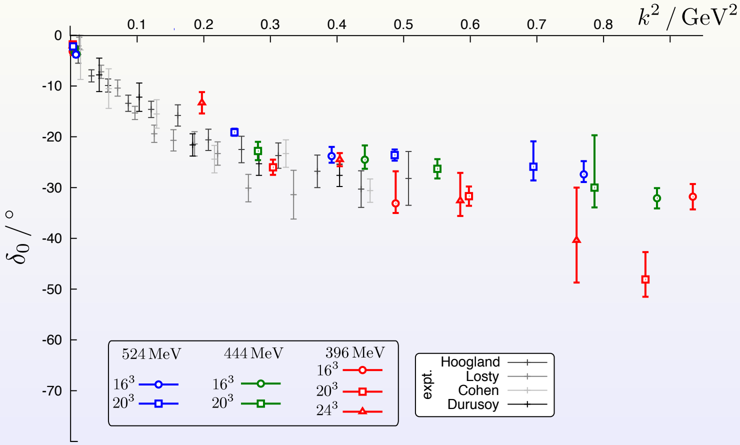


I=2 $\pi\pi$ scattering



Resolve shifts in masses away from non-interacting values

I=2 $\pi\pi$ scattering



- Non-resonant scattering in S-wave - compares well with experimental data

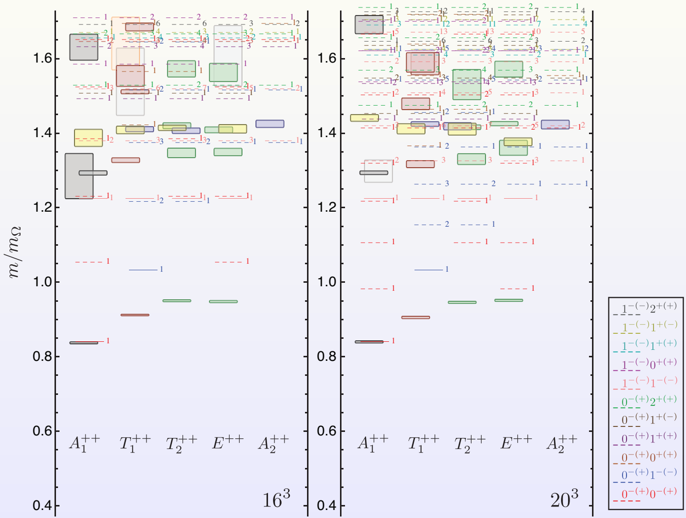
Group theory of two particles in a box

- Consider two identical particles, with momentum p and $-p$ (so zero total momentum).
- $\Omega(p)$, set of all momentum directions related by rotations in O_h
- Can make a set of operators, $\{\phi(p)\}$ from Ω and these form a (reducible) representation of O_h.
- Example: $\Phi = \{\phi(1,0,0), \phi(0,1,0), \phi(0,0,1)\}$ contains the A_1 and E irreps
- Different particles: $+p$ and $-p$ are not equivalent

p	irreducible content
(0,0,0)	A_1^g
(1,0,0)	$A_1^g \oplus E^g$
(1,1,0)	$A_1^g \oplus E^g \oplus T_2^g$
(1,1,1)	$A_1^g \oplus T_2^g$

- More complicated if mesons have internal spin

Multi-meson states in QCD



- Multi-hadron states not seen in this calculation

Summary

- **Exotic** mesons are states that do not naturally fit into the simplest quark model
- **Considerable experimental interest** in understanding these statest (so far, picture is confusing)
- New techniques in lattice spectroscopy are helping, but still could benefit from improvements - new ideas please!
- Scattering calculations on the lattice are **developing quickly**. Much more to learn about this big topic (in particular, about states above inelastic threshold)
- **No results** yet on widths of exotic resonances - are they coming soon?