Minimal Supersymmetric Standard Model (MSSM)

- SUSY: # of fermions = # of bosons   N=1 SUSY: (φ,ψ) (λ, A_μ)
- SM: 28 bosonic d.o.f. & 90 (96) fermionic d.o.f.

There are no particles in the SM that can be superpartners

SUSY associates known bosons with new fermions
and known fermions with new bosons

- Even number of the Higgs doublets – min = 2

Cancellation of axial anomalies (in each generation)

$$Tr\ Y^3 = 3(\tfrac{1}{27}+\tfrac{1}{27}-\tfrac{64}{27}+\tfrac{8}{27})-1-1+8=0$$

colour u_L d_L u_R d_R ν_L e_L e_R

Higgsinos

-1+1=0

Particle Content of the MSSM

Superfield	Bosons	Fermions	$SU_c(3)$	$SU_L(2)$	$U_Y(1)$
Gauge					
G^a	*gluon* g^a	*gluino* $\tilde{\mathrm{g}}^a$	8	1	0
V^k	*Weak* $\mathrm{W}^k (W^{\pm}, Z)$	*wino, zino* $\tilde{w}^k (\tilde{w}^{\pm}, \tilde{z})$	1	3	0
V'	*Hypercharge* $\mathrm{B}(\gamma)$	*bino* $\tilde{b}(\tilde{\gamma})$	1	1	0
Matter					
L_i	sleptons $\tilde{L}_i = (\tilde{\nu}, \tilde{e})_L$	leptons $L_i = (\nu, e)_L$	1	2	-1
E_i	sleptons $\tilde{E}_i = \tilde{e}_R$	leptons $E_i = e_R$	1	1	2
Q_i	squarks $\tilde{Q}_i = (\tilde{u}, \tilde{d})_L$	quarks $Q_i = (u, d)_L$	3	2	$1/3$
U_i	squarks $\tilde{U}_i = \tilde{u}_R$	quarks $U_i = u_R^c$	3^*	1	$-4/3$
D_i	squarks $\tilde{D}_i = \tilde{d}_R$	quarks $D_i = d_R^c$	3^*	1	$2/3$
Higgs					
H_1	Higgses H_1	higgsinos $\tilde{H}_1$	1	2	-1
H_2	Higgses H_2	higgsinos $\tilde{H}_2$	1	2	1

The MSSM Lagrangian

$$L = L_{gauge} + L_{Yukawa} + L_{SoftBreaking}$$

The Yukawa Superpotential

Superfields

$$W_R = y_U Q_L H_2 U_R + y_D Q_L H_1 D_R + y_L L_L H_1 E_R + \mu H_1 H_2$$

Yukawa couplings

Higgs mixing term

$$W_{NR} = \lambda_L L_L L_L E_R + \lambda_L' L_L Q_L D_R + \mu' L_L H_2 + \lambda_B U_R D_R D_R$$

Violate: Lepton number Baryon number

These terms are forbidden in the SM

$$\lambda_L, \lambda_L' < 10^{-6}, \quad \lambda_B < 10^{-9}$$

R-parity

$$R = (-)^{3(B-L)+2S}$$

The Usual Particle : R = + 1
SUSY Particle : R = - 1

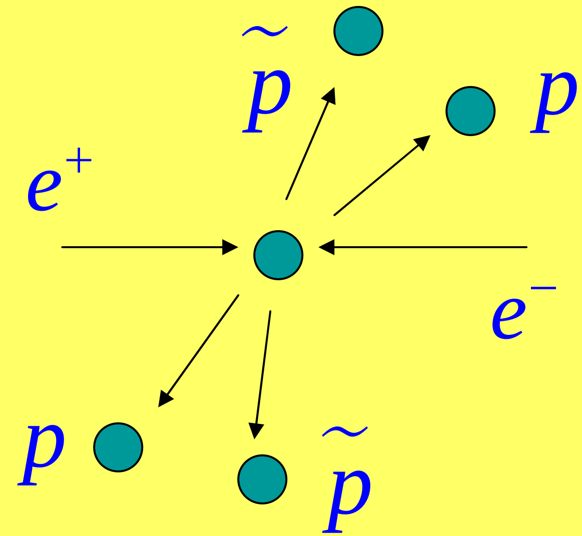
B - Baryon Number
L - Lepton Number
S - Spin

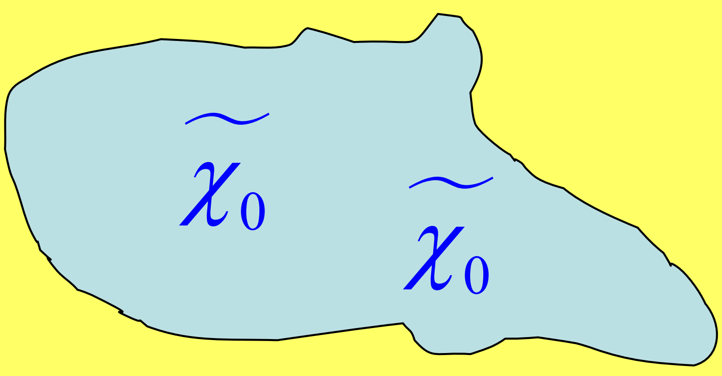
The consequences:

- The superpartners are created in pairs
- The lightest superparticle is stable

⇓

- The lightest superparticle (LSP) should be neutral - the best candidate is neutralino (photino or higgsino) $\tilde{\chi}_0$
- It can survive from the Big Bang and form the Dark matter in the Universe

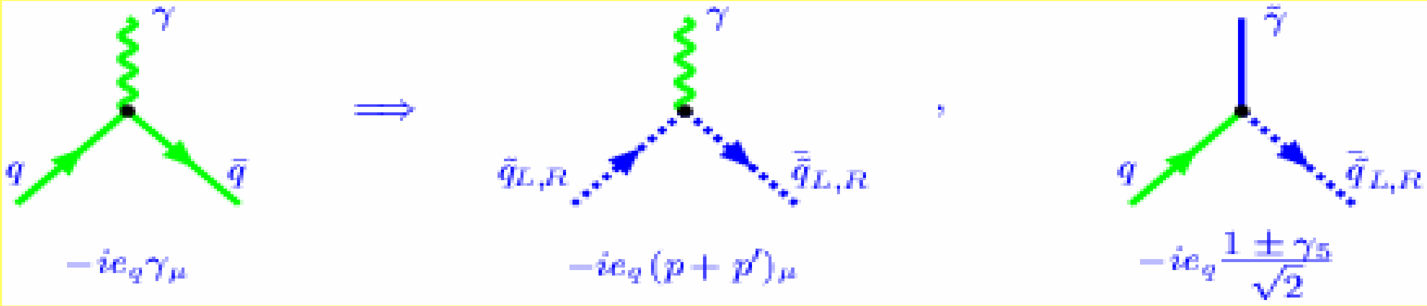


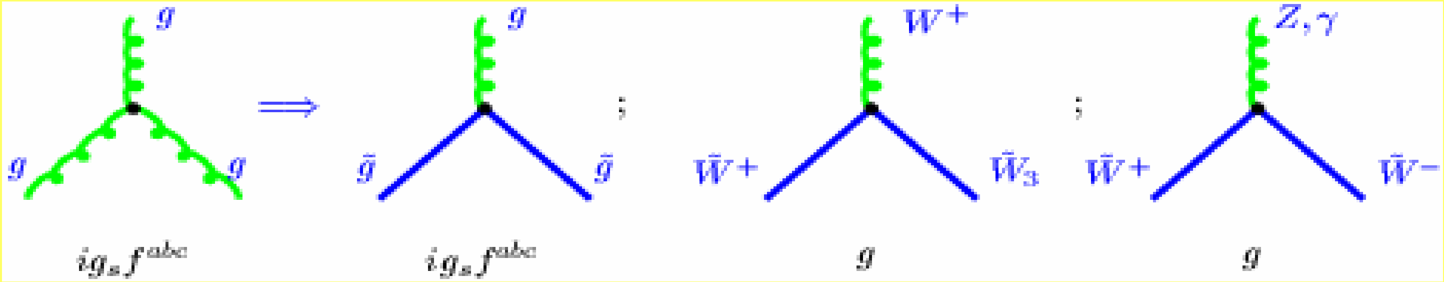


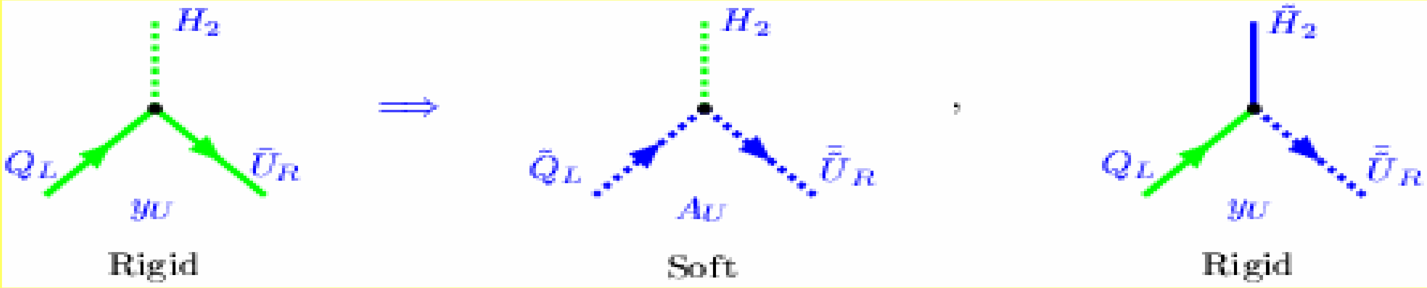
Interactions in the MSSM

SM → MSSM

Vertices

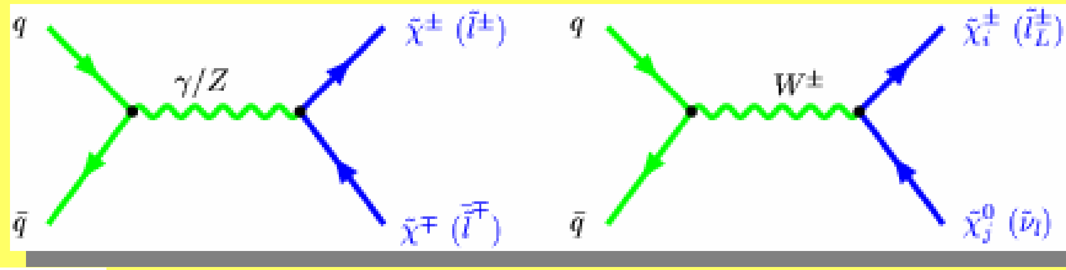


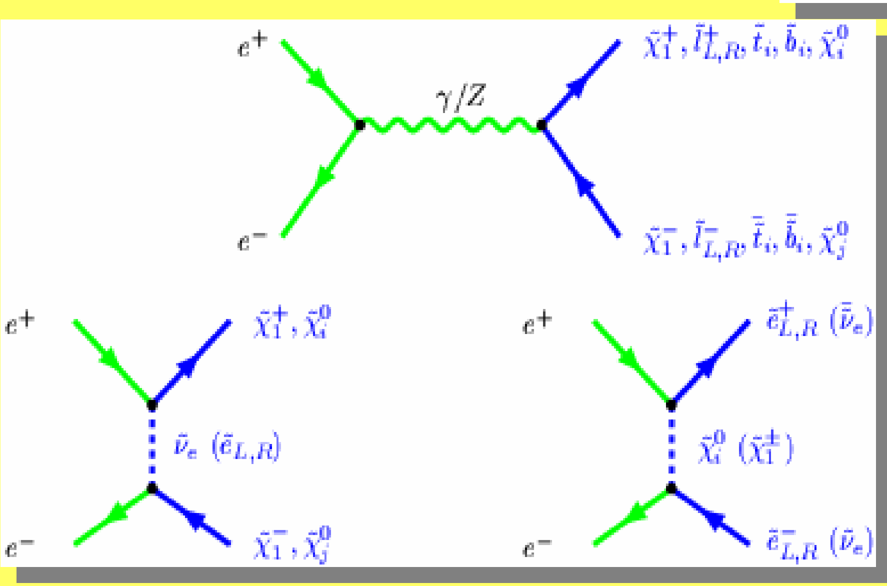


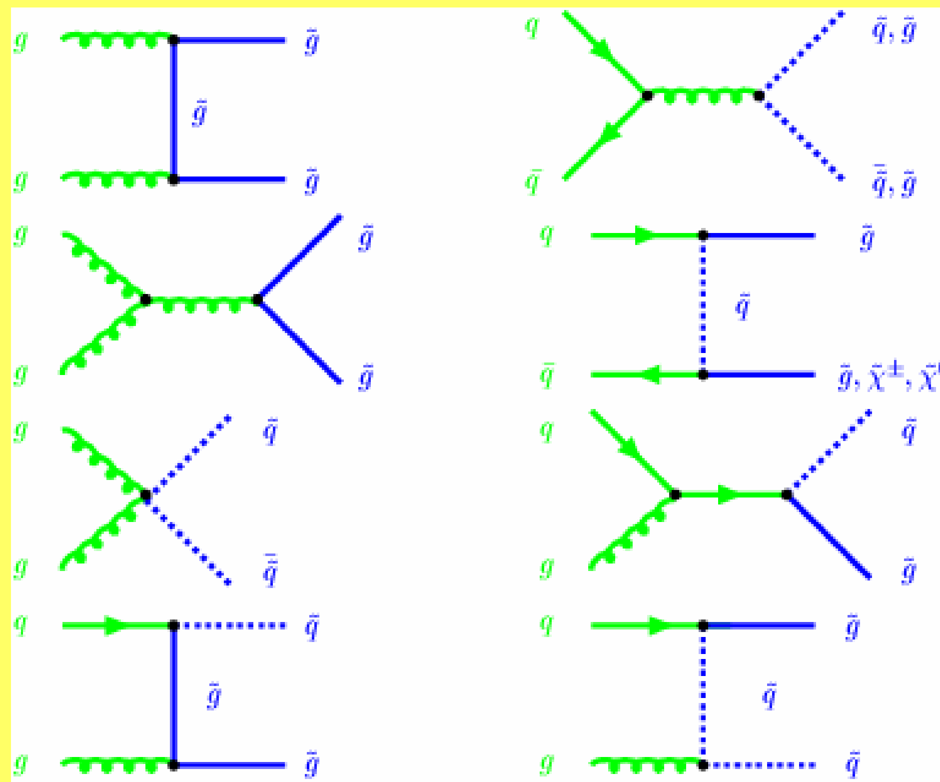


Creation of Superpartners at colliders

Annihilation channel

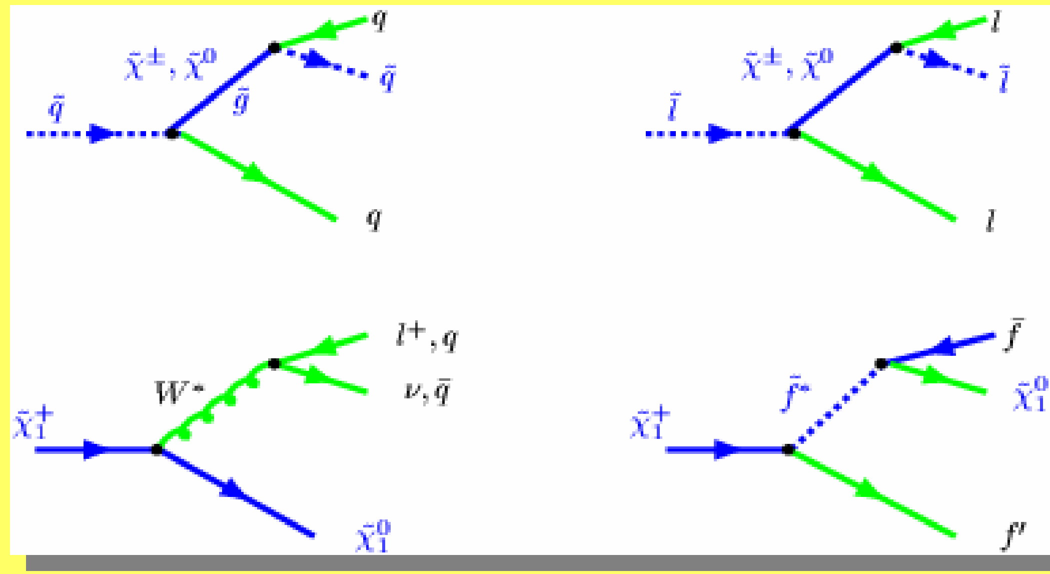






Gluon fusion, ee, qq scattering and qg scattering channels

Decay of Superpartners



squarks

$$\tilde{q}_{L,R} \to q + \tilde{\chi}_i^0$$

$$\tilde{q}_L \to q' + \tilde{\chi}_i^\pm$$

$$\tilde{q}_{L,R} \to q + \tilde{g}$$

sleptons

$$\tilde{l} \to l + \tilde{\chi}_i^0$$

$$\tilde{l}_L \to \nu_l + \tilde{\chi}_i^\pm$$

chargino

$$\tilde{\chi}_i^\pm \to e + \nu_e + \tilde{\chi}_i^0$$

$$\tilde{\chi}_i^\pm \to q + \bar{q}' + \tilde{\chi}_i^0$$

gluino

$$\tilde{g} \to q + \bar{q} + \tilde{\gamma}$$

$$\tilde{g} \to g + \tilde{\gamma}$$

neutralino

$$\tilde{\chi}_i^0 \to \tilde{\chi}_1^0 + l^+ + l^-$$

$$\tilde{\chi}_i^0 \to \tilde{\chi}_1^0 + q + \bar{q}'$$

$$\tilde{\chi}_i^0 \to \tilde{\chi}_1^\pm + l^\pm + \nu_l$$

$$\tilde{\chi}_i^0 \to \tilde{\chi}_1^0 + \nu_l + \bar{\nu}_l$$

Final sates

$$l^+ l^- + \not{E}_T$$

$$2 \text{ jets} + \not{E}_T$$

$$\gamma + \not{E}_T$$

$$\not{E}_T$$

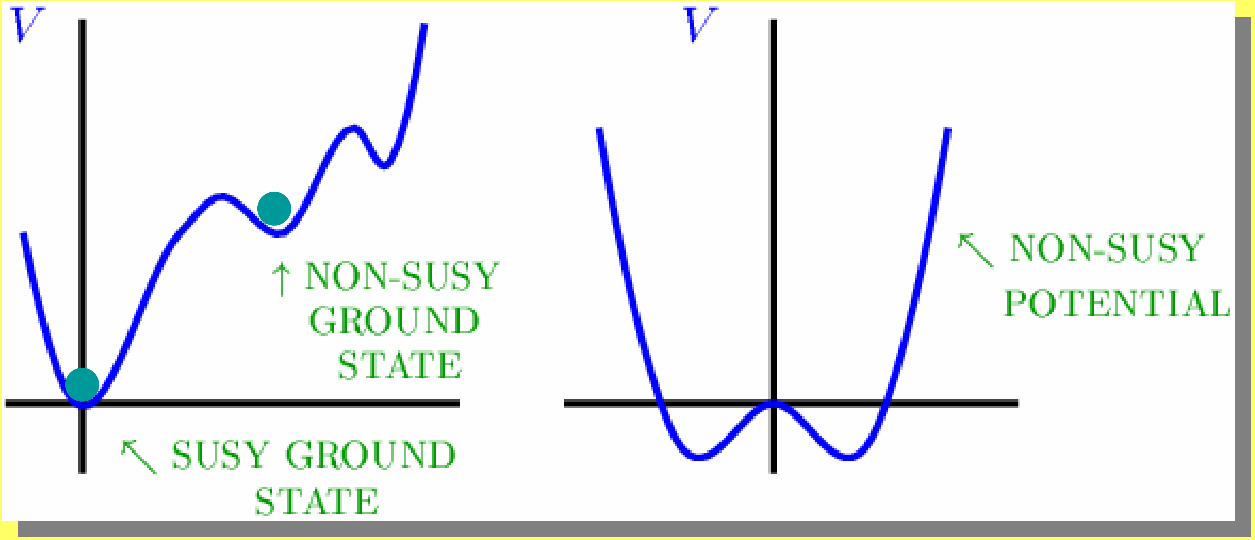
Spontaneous Breaking of SUSY

Energy $E = <0|H|0>$

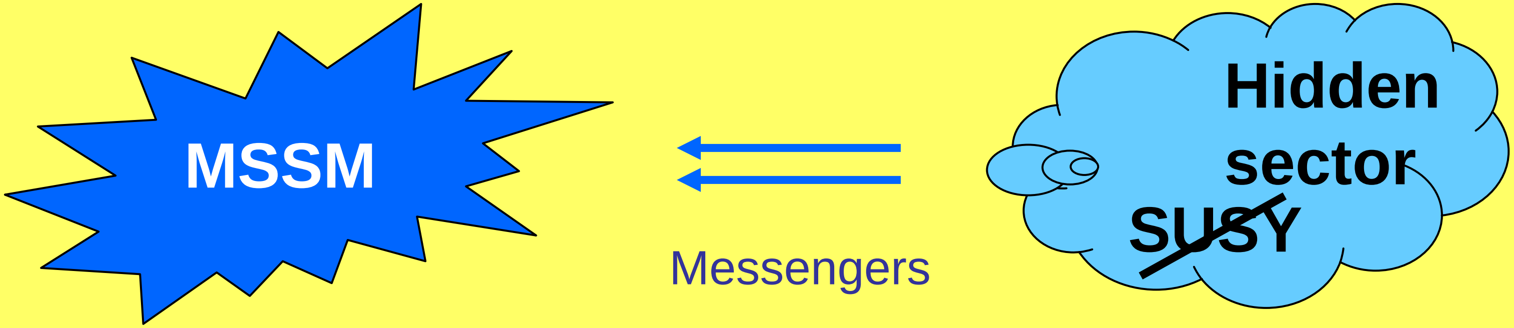
$$\{Q^i_\alpha, \overline{Q}^j_{\dot\beta}\} = 2\delta^{ij}(\sigma^\mu)_{\alpha\dot\beta}P_\mu$$

$$E = \frac{1}{4}\sum_{\alpha=1,2} <0|\{Q^i_\alpha, \overline{Q}^j_\alpha\}|0> = \frac{1}{4}\sum_\alpha |Q_\alpha|0>|^2 \geq 0$$

$E = <0|H|0> \neq 0$ if and only if $Q_\alpha|0> \neq 0$



Soft SUSY Breaking



Gravitons, gauge, gauginos, etc

Breaking via F and D terms in a hidden sector

$$-L_{Soft} = \sum_{\alpha} M_i \tilde{\lambda}_i \tilde{\lambda}_i + \sum_i m_{0i}^2 \, | A_i |^2 + + \sum_{ijk} \mathrm{A}_{ijk} A_i A_j A_k + \sum_{ij} \mathrm{B}_{ij} A_i A_j$$

gauginos — scalar fields

Over 100 of free parameters !

Soft SUSY Breaking

SUGRA S-dilaton, T-moduli $< F_T > \neq 0, < F_S > \neq 0$

$$M_{S\not{U}SY} \sim \frac{< F_T >}{M_{PL}} + \frac{< F_S >}{M_{PL}} \sim m_{3/2}$$

gravitino mass ~ 1 TeV

$$m_i^2 \sim \mathrm{B} \sim m_{3/2}^2, \; M_i \sim \mathrm{A} \sim m_{3/2}$$

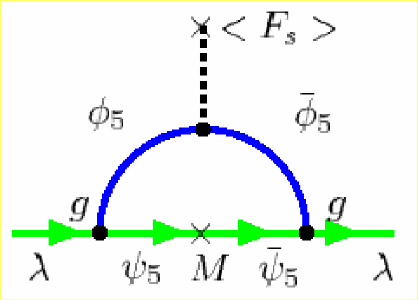
$$L_{soft} = -\sum_i m_i^2 \, | A_i |^2 - \sum_i M_i (\lambda_i \lambda_i + \bar{\lambda}_i \bar{\lambda}_i) - \mathrm{B} W^{(2)}(A) - \mathrm{A} W^{(3)}(A)$$

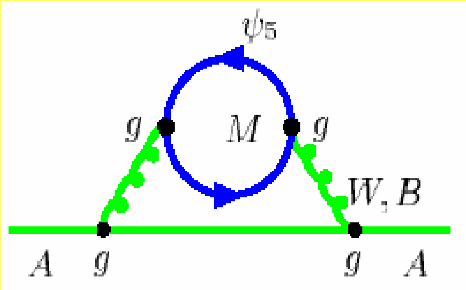
Gauge mediation Scalar singlet S $< F_S > \neq 0$

Messenger Φ $W \sim S \Phi^{+} \Phi$

$$m_{\tilde{G}} \sim \frac{< F_S >}{M_{PL}} \frac{M}{M_{PL}} \sim 10^{-14} \frac{M}{[GeV]}$$

gravitino mass





$$M_i \sim c_i N \frac{\alpha_i}{4\pi} \frac{< F_S >}{M}$$

gaugino

$$m_i^2 \sim \left(\frac{< F_S >}{M_{PL}} \right)^2 N \left(\frac{\alpha_i}{4\pi} \right)^2$$

squark

MSSM Parameter Space

- Three gauge couplings
- Three (four) Yukawa matrices
- The Higgs mixing parameter
- Soft SUSY breaking terms

mSUGRA Universality hypothesis (gravity is colour and flavour blind): Soft parameters are equal at Planck (GUT) scale

$$-L_{Soft} = A\{y_t Q_L H_2 U_R + y_b Q_L H_1 D_R + y_L L_L H_1 E_R\} + B\mu H_1 H_2 + m_0^2 \sum_i |\varphi_i|^2 + \tfrac{1}{2} M_{1/2} \sum_\alpha \tilde{\lambda}_\alpha \tilde{\lambda}_\alpha$$

Five universal soft parameters: $A,\ m_0,\ M_{1/2},\ B \leftrightarrow\ \tan\beta = v_2 / v_1$ and μ

versus m and λ in the SM

Mass Spectrum

$$L_{gaugino-Higgsino} = -\tfrac{1}{2} M_3 \bar{\lambda}_a \lambda_a - \tfrac{1}{2} \bar{\chi} M^{(0)} \chi - (\bar{\psi} M^{(c)} \psi + h.c.)$$

Chargino

$$\psi = \begin{pmatrix} \tilde{W}^+ \\ \tilde{H}^+ \end{pmatrix} \qquad M^{(c)} = \begin{pmatrix} M_2 & \sqrt{2} M_W \sin\beta \\ \sqrt{2} M_W \cos\beta & \mu \end{pmatrix} \Rightarrow \begin{pmatrix} \chi_1^+ \\ \chi_2^+ \end{pmatrix}$$

Neutralino

$$(\chi_1^0, \chi_2^0, \chi_3^0, \chi_4^0)$$

$$\chi = \begin{pmatrix} \tilde{B}^0 \\ \tilde{W}^3 \\ \tilde{H}_1^0 \\ \tilde{H}_2^0 \end{pmatrix} \qquad M^{(0)} = \begin{pmatrix} M_1 & 0 & -M_Z \cos\beta \sin W & M_Z \sin\beta \sin W \\ 0 & M_2 & M_Z \cos\beta \cos W & -M_Z \sin\beta \cos W \\ -M_Z \cos\beta \sin W & M_Z \cos\beta \cos W & 0 & -\mu \\ M_Z \sin\beta \sin W & -M_Z \sin\beta \cos W & -\mu & 0 \end{pmatrix}$$

Squarks & Sleptons

Mass Spectrum

$$\tilde m_t^2 = \begin{pmatrix} \tilde m_{tL}^2 & m_t(A_t - \mu\cot\beta) \\ m_t(A_t - \mu\cot\beta) & \tilde m_{tR}^2 \end{pmatrix} \Rightarrow \begin{pmatrix} \tilde t_1 \\ \tilde t_2 \end{pmatrix}$$

$$\tilde m_b^2 = \begin{pmatrix} \tilde m_{bL}^2 & m_b(A_b - \mu\tan\beta) \\ m_b(A_b - \mu\tan\beta) & \tilde m_{bR}^2 \end{pmatrix} \Rightarrow \begin{pmatrix} \tilde b_1 \\ \tilde b_2 \end{pmatrix}$$

$$\tilde m_{tL}^2 = \tilde m_Q^2 + m_t^2 + \tfrac{1}{6}(4M_W^2 - M_Z^2)\cos 2\beta,$$

$$\tilde m_{tR}^2 = \tilde m_U^2 + m_t^2 - \tfrac{2}{3}(M_W^2 - M_Z^2)\cos 2\beta,$$

$$\tilde m_{bL}^2 = \tilde m_Q^2 + m_b^2 - \tfrac{1}{6}(2M_W^2 + M_Z^2)\cos 2\beta,$$

$$\tilde m_{bR}^2 = \tilde m_D^2 + m_b^2 + \tfrac{1}{3}(M_W^2 - M_Z^2)\cos 2\beta,$$

$$\tilde m_{\tau L}^2 = \tilde m_L^2 + m_\tau^2 - \tfrac{1}{2}(2M_W^2 - M_Z^2)\cos 2\beta,$$

$$\tilde m_{\tau R}^2 = \tilde m_E^2 + m_\tau^2 + (M_W^2 - M_Z^2)\cos 2\beta.$$

$$\tilde m_\tau^2 = \begin{pmatrix} \tilde m_{\tau L}^2 & m_\tau(A_\tau - \mu\tan\beta) \\ m_\tau(A_\tau - \mu\tan\beta) & \tilde m_{\tau R}^2 \end{pmatrix} \Rightarrow \begin{pmatrix} \tilde \tau_1 \\ \tilde \tau_2 \end{pmatrix}$$

SUSY Higgs Bosons

SM

$$H = \begin{pmatrix} H^0 \\ H^- \end{pmatrix} = \begin{pmatrix} \mathrm{v} + \frac{S + iP}{\sqrt{2}} \\ H^- \end{pmatrix} = \exp\left(i \frac{\vec{\xi}\vec{\sigma}}{2}\right) \begin{pmatrix} \mathrm{v} + \frac{S}{\sqrt{2}} \\ 0 \end{pmatrix}$$

4=2+2=3+1

$$H \to H' = \exp\left(i \frac{\vec{\alpha}\vec{\sigma}}{2}\right) H \xrightarrow{(\vec{\alpha} = -\vec{\xi})} H' = \begin{pmatrix} \mathrm{v} + \frac{S}{\sqrt{2}} \\ 0 \end{pmatrix}$$

MSSM

8=4+4=3+5

$$H_1 = \begin{pmatrix} H_1^0 \\ H_1^- \end{pmatrix} = \begin{pmatrix} \mathrm{v}_1 + \frac{S_1 + iP_1}{\sqrt{2}} \\ H_1^- \end{pmatrix}, \quad H_2 = \begin{pmatrix} H_2^+ \\ H_2^0 \end{pmatrix} = \begin{pmatrix} H_2^+ \\ \mathrm{v}_2 + \frac{S_2 + iP_2}{\sqrt{2}} \end{pmatrix},$$

$$\mathrm{v}_1^2 + \mathrm{v}_2^2 = \mathrm{v}^2, \quad \mathrm{v}_2/\mathrm{v}_1 \equiv \tan\beta$$

The Higgs Potential

$$V_{tree}(H_1, H_2) = m_1^2 |H_1|^2 + m_2^2 |H_2|^2 - m_3^2 (H_1 H_2 + h.c.) + \frac{g^2 + g'^2}{8}(|H_1|^2 - |H_2|^2)^2 + \frac{g^2}{2}|H_1^+ H_2|^2$$

At the GUT scale: $m_1^2 = m_2^2 = \mu_0^2 + m_0^2, \; m_3^2 = -B\mu_0$

Minimization

$$\frac{1}{2}\frac{\delta V}{\delta H_1} = m_1^2 v_1 - m_3^2 v_2 + \frac{g^2 + g'^2}{4}(v_1^2 - v_2^2) v_1 = 0,$$

$$\frac{1}{2}\frac{\delta V}{\delta H_2} = m_2^2 v_2 - m_3^2 v_1 - \frac{g^2 + g'^2}{4}(v_1^2 - v_2^2) v_2 = 0.$$

$< H_1 > \equiv v_1 = v \cos\beta, \; < H_2 > \equiv v_2 = v \sin\beta,$

Solution

$$v^2 = \frac{4(m_1^2 - m_2^2 \tan^2\beta)}{(g^2 + g'^2)(\tan^2\beta - 1)},$$

$$\sin 2\beta = \frac{2m_3^2}{m_1^2 + m_2^2}$$

At the GUT scale

$$v^2 = -\frac{4}{g^2 + g'^2} m^2 < 0$$

No SSB in SUSY theory !

Higgs Boson's Masses

$$M^{odd} = \left.\frac{\partial^2 V}{\partial P_i \partial P_j}\right|_{H_i=v_i} = \begin{pmatrix} \tan\beta & 1 \\ 1 & \cot\beta \end{pmatrix} m_3^2$$

$$M^{even} = \left.\frac{\partial^2 V}{\partial S_i \partial S_j}\right|_{H_i=v_i} = \begin{pmatrix} \tan\beta & -1 \\ -1 & \cot\beta \end{pmatrix} m_3^2 + \begin{pmatrix} \cot\beta & -1 \\ -1 & \tan\beta \end{pmatrix} M_Z^2 \cos\beta \sin\beta$$

$$M^{ch} = \left.\frac{\partial^2 V}{\partial H_i^+ \partial H_j^-}\right|_{H_i=v_i} = \begin{pmatrix} \tan\beta & 1 \\ 1 & \cot\beta \end{pmatrix} (m_3^2 + M_W^2 \cos\beta \sin\beta)$$

$G^0 = -\cos\beta P_1 + \sin\beta P_2$	*Goldstone boson* $\rightarrow Z_0$
$A = \sin\beta P_1 + \cos\beta P_2$	*Neutral* $CP = -1$ *Higgs*
$G^+ = -\cos\beta (H_1^-)^* + \sin\beta H_2^+$	*Goldstone boson* $\rightarrow W^+$
$H^+ = \sin\beta (H_1^-)^* + \cos\beta H_2^+$	*Charged Higgs*
$h = -\sin\alpha S_1 + \cos\alpha S_2$	SM *Higgs boson* $CP = 1$
$H = \cos\alpha S_1 + \sin\alpha S_2$	*Extra heavy Higgs boson*

$$\tan 2\alpha = \tan 2\beta \frac{m_A^2 + m_Z^2}{m_A^2 - m_Z^2}$$

The Higgs Bosons Masses

CP-odd neutral Higgs A

CP-even charged Higgses H$^{\pm}$

CP-even neutral Higgses h,H

$$m_A^2 = m_1^2 + m_2^2$$

$$m_{H^\pm}^2 = m_A^2 + M_W^2$$

$$M_W^2 = \frac{g^2}{2} v^2$$

$$M_Z^2 = \frac{g^2 + g'^2}{2} v^2$$

$$m_{h,H}^2 = \frac{1}{2}[m_A^2 + M_Z^2 \pm \sqrt{(m_A^2 + M_Z^2)^2 - 4 m_A^2 M_Z^2 \cos^2 2\beta}]$$

$$m_h \approx M_Z \, |\cos 2\beta| \; < \; M_Z \; !$$

→ Radiative corrections

$$m_h^2 \approx M_Z^2 \cos^2 2\beta + \frac{3 g^2 m_t^4}{16 \pi^2 M_W^2} \log \frac{\tilde{m}_{t_1}^2 \tilde{m}_{t_2}^2}{m_t^4} + 2 \; loops$$

Renormalization Group Eqns

$$\tilde{\alpha}_i \equiv \frac{g_i^2}{16\pi^2} = \frac{\alpha_i}{4\pi}, \quad Y_k \equiv \frac{y_k^2}{16\pi^2}, \quad t = \log(M_{GUT}^2/Q^2)$$

$$i = 1, 2, 3 \qquad k = U, D, L$$

The Couplings

$$\dot{\tilde{\alpha}}_i = -b_i \tilde{\alpha}_i^2, \qquad b_i^{MSSM} = (\tfrac{33}{5}, 1, -3)$$

$$\dot{Y}_U = Y_U(\tfrac{16}{3}\tilde{\alpha}_3 + 3\tilde{\alpha}_2 + \tfrac{13}{15}\tilde{\alpha}_1 - 6Y_U - Y_D),$$

$$\dot{Y}_D = Y_D(\tfrac{16}{3}\tilde{\alpha}_3 + 3\tilde{\alpha}_2 + \tfrac{7}{15}\tilde{\alpha}_1 - Y_U - 6Y_D - Y_L),$$

$$\dot{Y}_L = Y_L(3\tilde{\alpha}_2 + \tfrac{9}{5}\tilde{\alpha}_1 - 3Y_D - 4Y_L),$$

Soft terms

$$\dot{M}_i = b_i \tilde{\alpha}_i M_i,$$

$$\dot{A}_U = -(\tfrac{16}{3}\tilde{\alpha}_3 M_3 + 3\tilde{\alpha}_2 M_2 + \tfrac{13}{15}\tilde{\alpha}_1 M_1) - 6Y_U A_U - Y_D A_D,$$

$$\dot{A}_D = -(\tfrac{16}{3}\tilde{\alpha}_3 M_3 + 3\tilde{\alpha}_2 M_2 + \tfrac{7}{15}\tilde{\alpha}_1 M_1) - Y_U A_U - 6Y_D A_D - Y_L A_L,$$

$$\dot{A}_L = -(3\tilde{\alpha}_2 M_2 + \tfrac{9}{5}\tilde{\alpha}_1 M_1) - 3Y_D A_D - 4Y_L A_L,$$

$$\dot{B} = -3(\tilde{\alpha}_2 M_2 + \tfrac{1}{5}\tilde{\alpha}_1 M_1) - 3Y_U A_U - 3Y_D A_D - Y_L A_L,$$

$$\dot{\mu} = -\mu^2(3\tilde{\alpha}_2 + \tfrac{3}{5}\tilde{\alpha}_1 - 3Y_U - 3Y_D - Y_L)$$

RG Eqns for the Soft Masses

$$\dot{\tilde{m}}_Q^2 = -[\tfrac{16}{3}\tilde{\alpha}_3 M_3^2 + 3\tilde{\alpha}_2 M_2^2 + \tfrac{1}{15}\tilde{\alpha}_1 M_1^2 - Y_t(\Sigma_t + A_t^2) - Y_b(\Sigma_b + A_b^2)]$$

$$\dot{\tilde{m}}_U^2 = -[\tfrac{16}{3}\tilde{\alpha}_3 M_3^2 + \tfrac{16}{15}\tilde{\alpha}_1 M_1^2 - 2Y_t(\Sigma_t + A_t^2)]$$

$$\dot{\tilde{m}}_D^2 = -[\tfrac{16}{3}\tilde{\alpha}_3 M_3^2 + \tfrac{4}{15}\tilde{\alpha}_1 M_1^2 - 2Y_b(\Sigma_b + A_b^2)]$$

$$\dot{\tilde{m}}_L^2 = -[3\tilde{\alpha}_2 M_2^2 + \tfrac{3}{5}\tilde{\alpha}_1 M_1^2 - Y_\tau(\Sigma_\tau + A_\tau^2)]$$

$$\dot{\tilde{m}}_E^2 = -[\tfrac{12}{5}\tilde{\alpha}_1 M_1^2 - 2Y_\tau(\Sigma_\tau + A_\tau^2)]$$

$$\dot{\tilde{m}}_{H_1}^2 = -[3\tilde{\alpha}_2 M_2^2 + \tfrac{3}{5}\tilde{\alpha}_1 M_1^2 - 3Y_b(\Sigma_b + A_b^2) - Y_\tau(\Sigma_\tau + A_\tau^2)]$$

$$\dot{\tilde{m}}_{H_2}^2 = -[3\tilde{\alpha}_2 M_2^2 + \tfrac{3}{5}\tilde{\alpha}_1 M_1^2 - 3Y_t(\Sigma_t + A_t^2)]$$

$$\Sigma_t = m_Q^2 + m_U^2 + m_{H_2}^2,\ \Sigma_b = m_Q^2 + m_D^2 + m_{H_1}^2,\ \Sigma_\tau = m_L^2 + m_E^2 + m_{H_1}^2$$

Radiative EW Symmetry Breaking

Due to RG controlled running of the mass terms from the Higgs potential they may <u>change sign</u> and trigger the appearance of non-trivial minimum leading to spontaneous breaking of EW symmetry - this is called <u>Radiative EWSB</u>