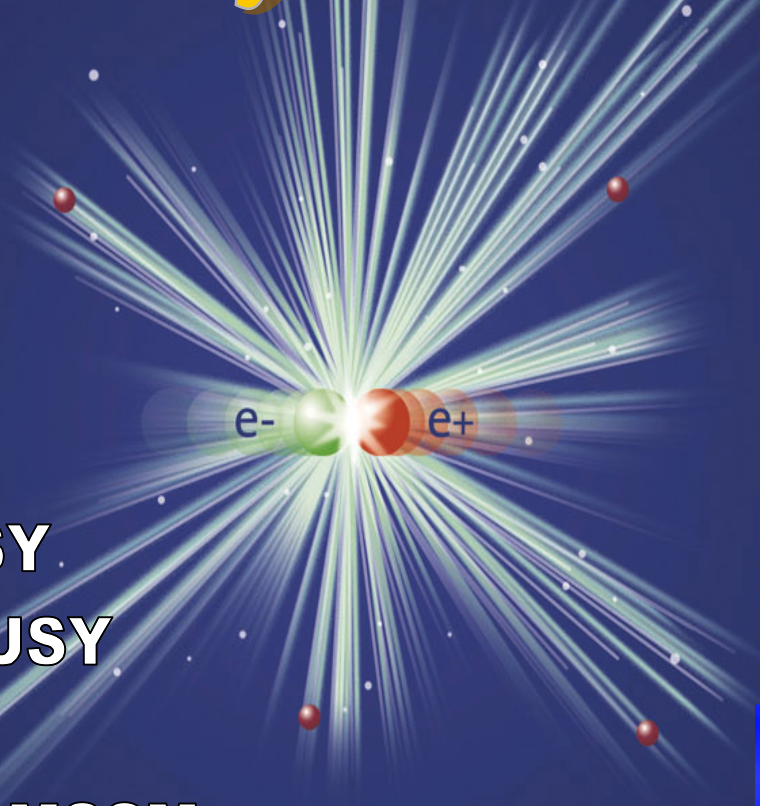
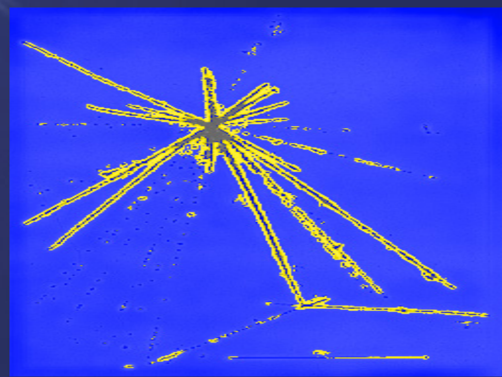
Supersymmetry in Particle Physics

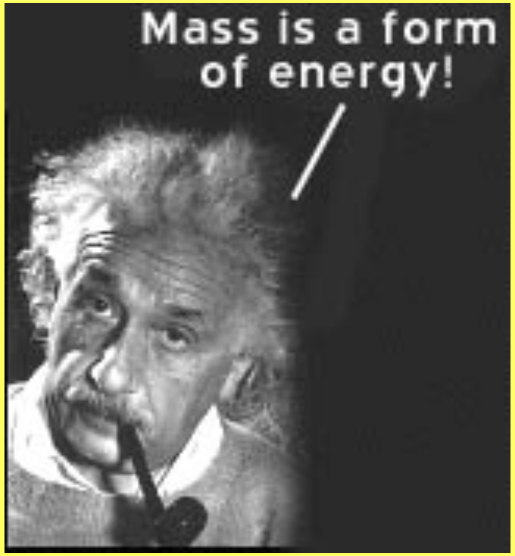


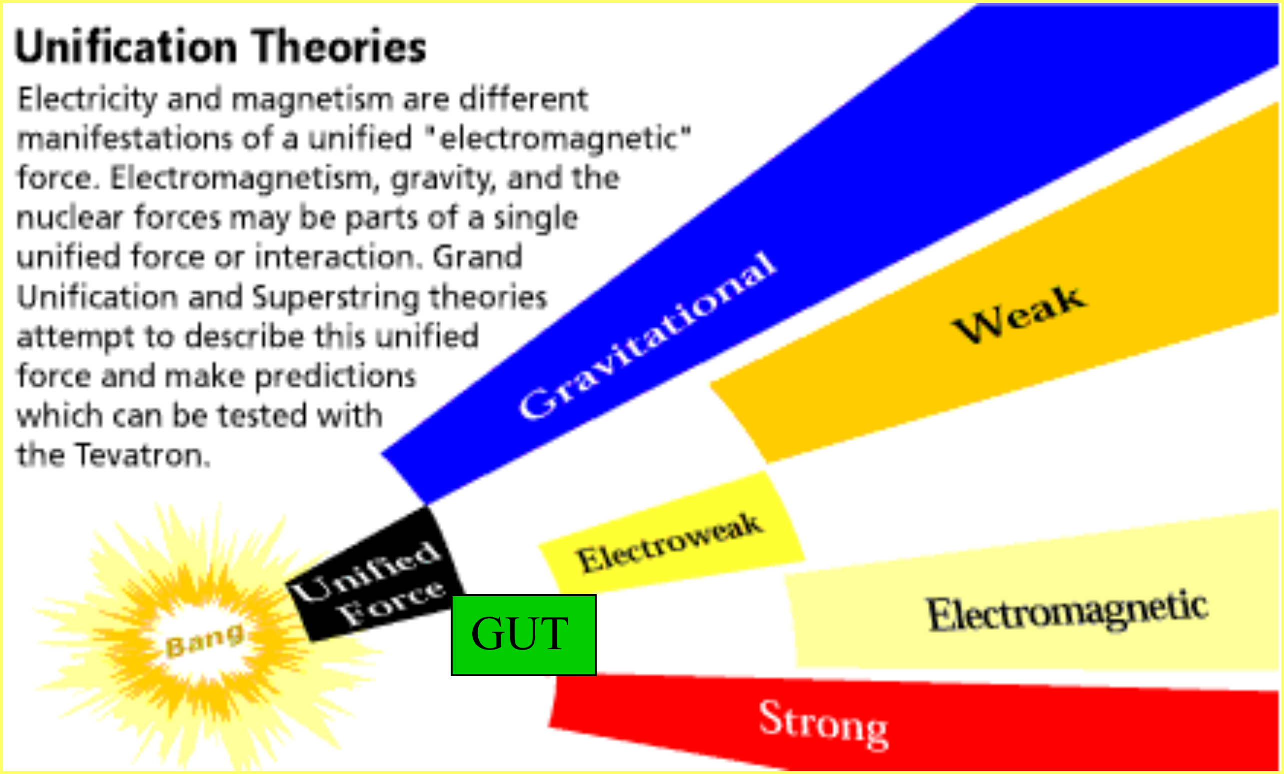
Outline

1. What is SUSY
2. Basics of SUSY
3. The MSSM
4. Constrained MSSM
5. SUSY Searches
6. SUSY DM

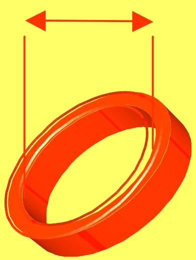


Unification Paradigm



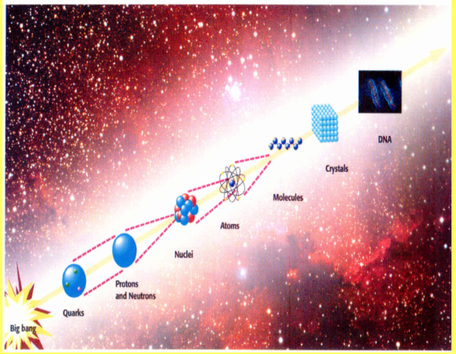


$10^{-34}\,m$

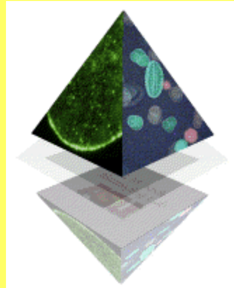


D=10

- Unification of strong, weak and electromagnetic interactions within Grand Unified Theories is the new step in unification of all forces of Nature
- Creation of a unified theory of everything based on string paradigm seems to be possible



What is SUSY?



• **Supersymmetry** is a boson-fermion sy[illegible] that is aimed to unify all forces in N[illegible] gravity within a singe fram[illegible]

$Q|boson>=|f$ [illegible] $n>$

$[b,b]$ [illegible] $\delta^{ij}(\sigma^{\mu})_{\alpha\dot{\beta}}P_{\mu}$

First papers in 1971-1972

No evidence in particle physics yet

• Modern [illegible] ry in particle physics are based o[illegible] gin, though low energy manifestatio[illegible] SY can be found (?) at modern colliders and in non-accelerator experiments

Motivation of SUSY in Particle Physics

- **Unification with Gravity**
- Unification of gauge couplings
- Solution of the hierarchy problem
- Dark matter in the Universe
- Superstrings

$$Q\,|\,boson> = |\,fermion> \qquad Q\,|\,fermion> = |\,boson>$$

$$spin\ 2 \rightarrow\ spin\ 3/2\ \rightarrow\ spin\ 1\ \rightarrow\ spin\ 1/2\ \rightarrow\ spin\ 0$$

Unification of matter (fermions) with forces (bosons) naturally arises
from an attempt to unify gravity with the other interactions

$$\{Q^i_\alpha, \bar{Q}^j_{\dot\beta}\} = 2\delta^{ij}(\sigma^\mu)_{\alpha\dot\beta}P_\mu \quad \Rightarrow \quad \{\delta_\varepsilon, \bar{\delta}_{\bar\varepsilon}\} = 2(\varepsilon\sigma^\mu\bar{\varepsilon})P_\mu$$

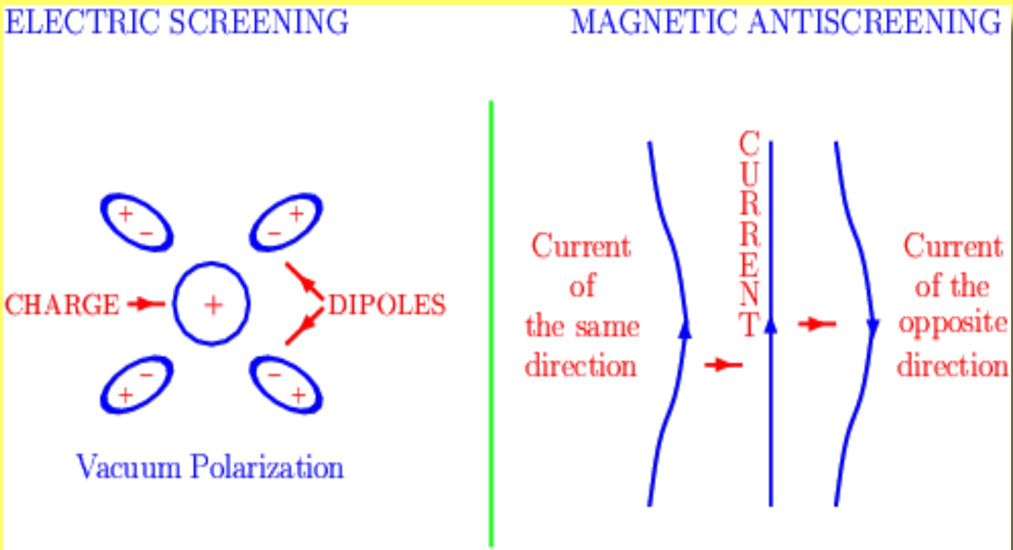
$$\varepsilon = \varepsilon(x)\ \text{ local coordinate transf. } \Rightarrow\ \text{(super)gravity}$$

Local supersymmetry = general relativity !

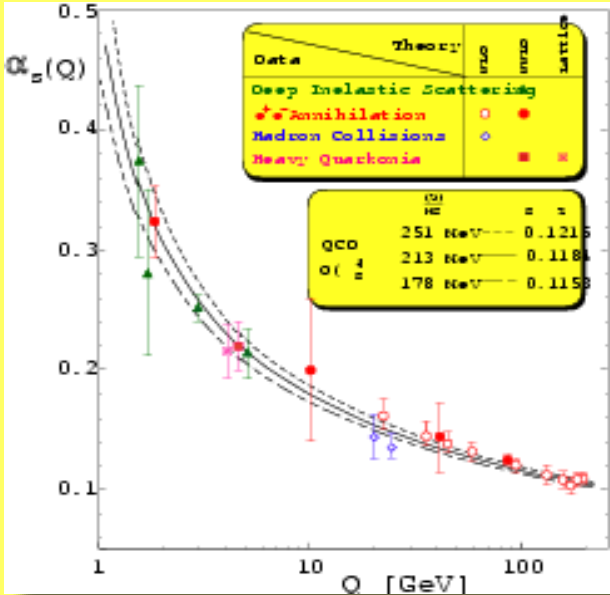
Motivation of SUSY in Particle Physics

- Unification of gauge couplings

	Low Energy		$\Rightarrow$	*High Energy*
$SU_c(3)$	$SU_L(2)$	$U_Y(1)$	$\Rightarrow$	G_{GUT} (or G^n + symm)
gluons	W, Z	*photon*	$\Rightarrow$	*gauge bosons*
quarks	*leptons*		$\Rightarrow$	*fermions*
g_3	g_2	g_1	$\Rightarrow$	g_{GUT}



$$\alpha_i = \alpha_i\left(\frac{Q^2}{\Lambda^2}\right) = \alpha_i(\text{distance})$$



Running of the strong coupling

Motivation of SUSY

RG Equations $\frac{d\tilde{\alpha}_i}{dt} = b_i \tilde{\alpha}_i^2, \quad \tilde{\alpha}_i = \alpha_i / 4\pi = g_i^2 / 16\pi^2, \quad t = \log(Q^2/\mu^2)$

$$SM: \quad b_i = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} = \begin{pmatrix} 0 \\ -22/3 \\ -11 \end{pmatrix} + N_{Fam} \begin{pmatrix} 4/3 \\ 4/3 \\ 4/3 \end{pmatrix} + N_{Higgs} \begin{pmatrix} 1/10 \\ 1/6 \\ 0 \end{pmatrix}$$

$$MSSM: \quad b_i = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} = \begin{pmatrix} 0 \\ -6 \\ -9 \end{pmatrix} + N_{Fam} \begin{pmatrix} 2 \\ 2 \\ 2 \end{pmatrix} + N_{Higgs} \begin{pmatrix} 3/10 \\ 1/2 \\ 0 \end{pmatrix}$$

Unification of the Coupling Constants
in the SM and in the MSSM

Input

$\alpha^{-1}(M_Z) = 128.978 \pm 0.027$

$\sin^2\theta_{\overline{MS}} = 0.23146 \pm 0.00017$

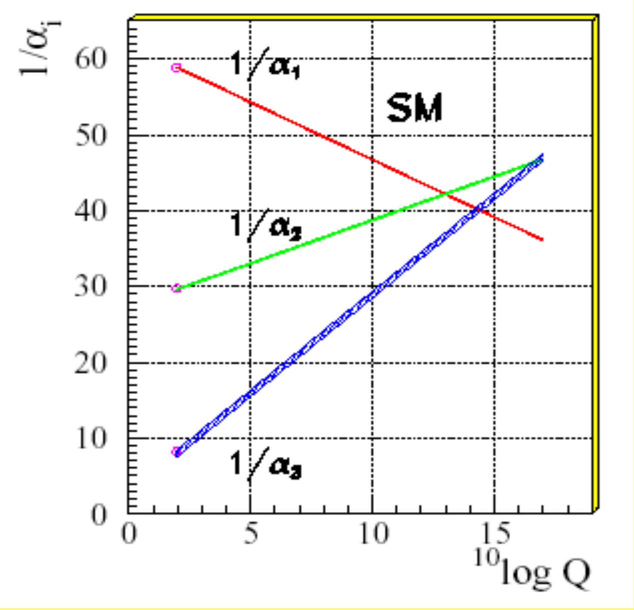
$\alpha_s(M_Z) = 0.1184 \pm 0.0031$

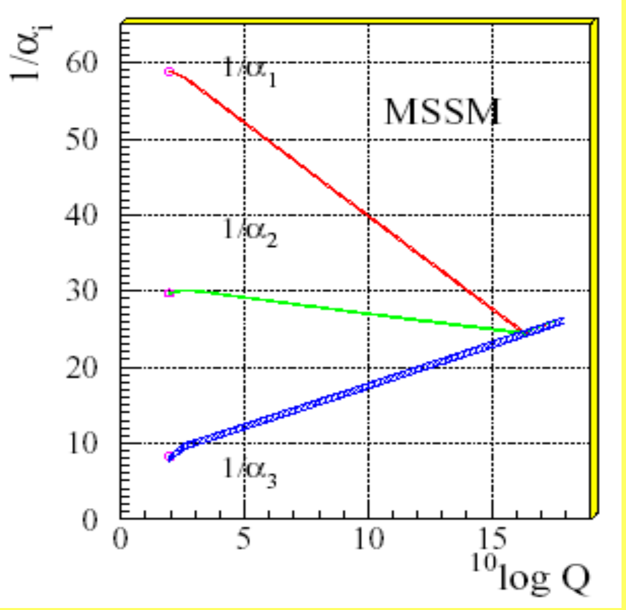
Output

$M_{SUSY} = 10^{3.4\pm0.9\pm0.4}$ GeV

$M_{GUT} = 10^{15.8\pm0.3\pm0.1}$ GeV

$\alpha_{GUT}^{-1} = 26.3 \pm 1.9 \pm 1.0$





SUSY yields unification!

Motivation of SUSY

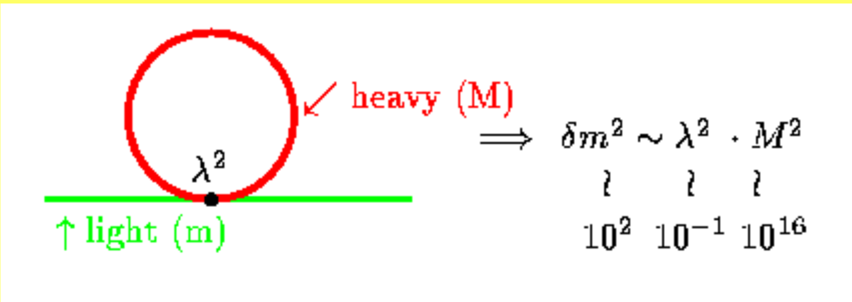
- Solution of the Hierarchy Problem

$$m_H \sim v \sim 10^2 \text{ GeV}$$

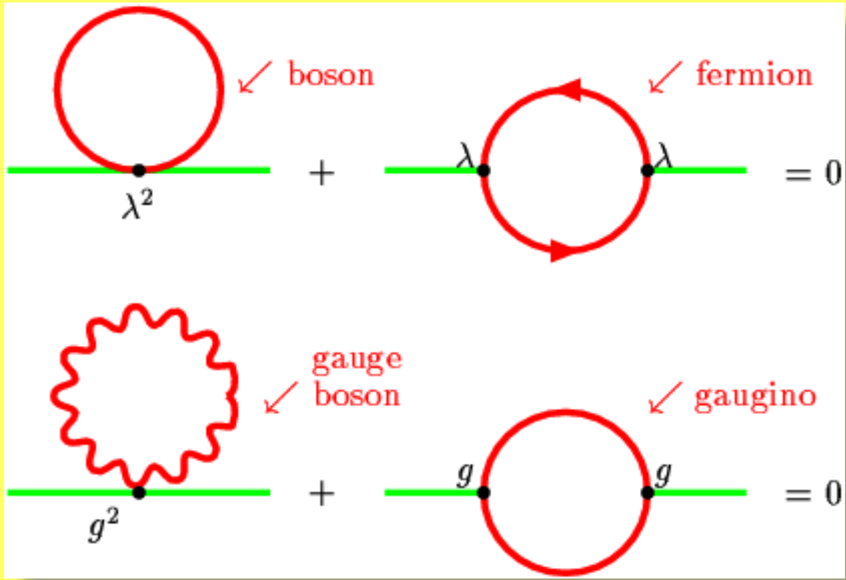
$$m_\Sigma \sim V \sim 10^{16} \text{ GeV}$$

$$\frac{m_H}{m_\Sigma} \sim 10^{-14} \ll 1$$

Destruction of the hierarchy by radiative corrections



Cancellation of quadratic terms

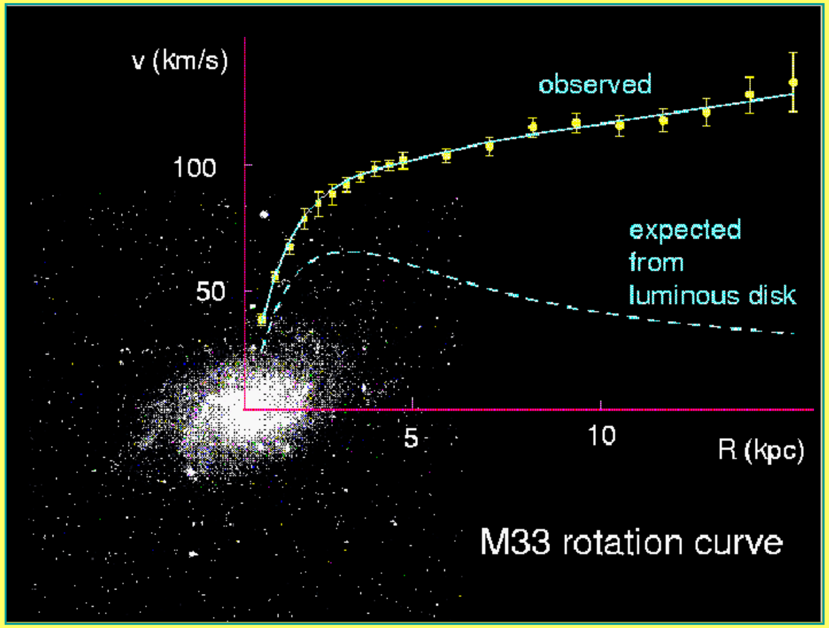


$$\sum_{bosons} m^2 = \sum_{fermions} m^2$$

SUSY may also explain the origin of the hierarchy due to radiative mechanism

Motivation of SUSY

- Dark Matter in the Universe



The flat rotation curves of spiral galaxies provide the most direct evidence for the existence of large amount of the dark matter.

Spiral galaxies consist of a central bulge and a very thin disc, and surrounded by an approximately spherical halo of dark matter

SUSY provides a candidate for the Dark matter – a stable neutral particle

Cosmological Constraints

New precise cosmological data

$$\Omega h^2 = 1 \quad \Longleftrightarrow \quad \rho = \rho_{crit}$$

$$\Omega_{vacuum} \approx 73\%$$

$$\Omega_{DarkMatter} \approx 23 \pm 4\%$$

$$\Omega_{Baryon} \approx 4\%$$

- Supernova Ia explosion
- CMBR thermal fluctuations

(recent news from WMAP)

Dark Matter in the Universe:

→ Hot DM (not favoured by galaxy formation)

→ Cold DM (rotation curves of Galaxies) ← SUSY

Superalgebra

$(Super)\ Algebra$

Lorentz Algebra

$$[P_\mu, P_\nu] = 0,\ [P_\mu, M_{\rho\sigma}] = i(g_{\mu\rho}P_\sigma - g_{\mu\sigma}P_\rho),$$

$$[M_{\mu\nu}, M_{\rho\sigma}] = i(g_{\nu\rho}M_{\mu\sigma} - g_{\nu\sigma}M_{\mu\rho} - g_{\mu\rho}M_{\nu\sigma} + g_{\mu\sigma}M_{\nu\rho}),$$

SUSY Algebra

$$[Q^i_\alpha, P_\mu] = [\bar{Q}^i_{\dot\alpha}, P_\mu] = 0,$$

$$[Q^i_\alpha, M_{\mu\nu}] = \tfrac{1}{2}(\sigma_{\mu\nu})^\beta_\alpha Q^i_\beta,\ [\bar{Q}^i_{\dot\alpha}, M_{\mu\nu}] = -\tfrac{1}{2}\bar{Q}^i_{\dot\beta}(\bar\sigma_{\mu\nu})^{\dot\beta}_{\dot\alpha},$$

$$\{Q^i_\alpha, \bar{Q}^j_{\dot\beta}\} = 2\delta^{ij}(\sigma^\mu)_{\alpha\dot\beta}P_\mu$$

$$\alpha, \dot\alpha, \beta, \dot\beta = 1,2;\ i,j = 1,2,\ldots,N.$$

The only possible graded Lie algebra that mixes integer and half-integer spins and changes statistics

Superspace

$$x_\mu \rightarrow x_\mu, \theta_\alpha, \bar\theta_{\dot\alpha}$$

Grassmannian parameters

$$\alpha, \dot\alpha = 1,2$$

$$\vartheta^2_\alpha = 0,\ \bar\vartheta^2_{\dot\alpha} = 0$$

SUSY Generators

$$Q_\alpha = \frac{\partial}{\partial\vartheta_\alpha} - i\sigma^\mu_{\alpha\dot\alpha}\bar\theta^{\dot\alpha}\partial_\mu$$

$$\bar{Q}_{\dot\alpha} = -\frac{\partial}{\partial\bar\vartheta_{\dot\alpha}} + i\theta_\alpha\sigma^\mu_{\alpha\dot\alpha}\partial_\mu$$

$$Q^2_\alpha = 0,\ \bar{Q}^2_{\dot\alpha} = 0$$

Supertranslation

$$x_\mu \rightarrow x_\mu + i\theta\sigma_\mu\bar\xi - i\xi\sigma_\mu\bar\theta,$$

$$\theta \rightarrow \theta + \xi,$$

$$\bar\theta \rightarrow \bar\theta + \bar\xi$$

Quantum States

Quantum states: Vacuum = $\vert E,\lambda >$ $\quad Q\vert E,\lambda >= 0$

Energy helicity

$$[Q^i_\alpha, P_\mu] = [\bar{Q}^i_{\dot\alpha}, P_\mu] = 0$$

State	Expression	# of states
vacuum	$\vert E,\lambda >$	1
1-particle	$\bar{Q}_i \vert E,\lambda >=\vert E,\lambda + 1/2 >$	$\binom{N}{1} = N$
2-particle	$\bar{Q}_i \bar{Q}_j \vert E,\lambda >=\vert E,\lambda + 1 >$	$\binom{N}{2} = \frac{N(N-1)}{2}$
...	...	...
N-particle	$\bar{Q}_1 \bar{Q}_2 ... \bar{Q}_N \vert E,\lambda >=\vert E,\lambda + N/2 >$	$\binom{N}{N} = 1$

Total # of states: $\sum_{k=0}^{N} \binom{N}{k} = 2^N = 2^{N-1} bosons + 2^{N-1} fermions$

SUSY Multiplets

Chiral multiplet $N=1,\ \lambda=0$

helicity	-1/2	0	1/2
# of states	1	2	1

(φ, ψ) — scalar, spinor

Vector multiplet $N=1,\ \lambda=1/2$

helicity	-1	-1/2	1/2	1
# of states	1	1	1	1

(λ, A_μ) — spinor, vector

Members of a supermultiplet are called **superpartners**

N=4	SUSY YM	helicity			-1	–1/2	0	1/2	1		
	λ = -1	# of states			1	4	6	4	1		
N=8	SUGRA	helicity	-2	–3/2	–1	–1/2	0	1/2	1	3/2	2
	λ = -2	# of states	1	8	28	56	70	56	28	8	1

$N \leq 4S$ ← spin

$N \leq 4$ For renormalizable theories (YM)

$N \leq 8$ For (super)gravity

Simplest (N=1) SUSY Multiplets

Bosons and Fermions come in pairs

$$(\varphi, \psi) \qquad (\lambda, A_\mu) \qquad (\tilde{g}, g)$$

Spin 0 Spin 1/2 Spin 1/2 Spin 1 Spin 3/2 Spin 2

scalar chiral fermion majorana fermion vector gravitino graviton

SUSY Transformation

N=1 SUSY Chiral supermultiplet:

parameter of SUSY transformation (spinor)

spin=0 → $\delta_\varepsilon A = \sqrt{2}\varepsilon\psi,$

spin=1/2 → $\delta_\varepsilon \psi = i\sqrt{2}\sigma^\mu \bar{\varepsilon}\partial_\mu A + \sqrt{2}\varepsilon F,$

$\delta_\varepsilon F = i\sqrt{2}\bar{\varepsilon}\sigma^\mu \partial_\mu \psi$

Auxiliary field

(unphysical d.o.f. needed to close SYSY algebra)

SUSY multiplets ➡ Superfiled in Superspace

$(y = x + i\theta\sigma\bar{\theta})$

superfield

$$\Phi(y,\theta) = A(y) + \sqrt{2}\theta\psi(y) + \theta\theta F(y)$$

$$= A(x) + i\theta\sigma^\mu\bar{\theta}\partial_\mu A(x) + \tfrac{1}{4}\theta\theta\bar{\theta}\bar{\theta}\Box A(x)$$

$$+ \sqrt{2}\theta\psi(x) - i/\sqrt{2}\theta\theta\partial_\mu\psi(x)\sigma^\mu\bar{\theta} + \theta\theta F(x)$$

Expansion over grassmannian parameter

component fields

$$\theta^2 = \theta_1\theta_2, \quad \theta_1^2 = \theta_2^2 = 0!$$

Gauge superfields

Gauge superfield

$$V(x,\theta,\bar\theta) = C(x) + i\theta\chi(x) - i\bar\theta\bar\chi(x) + i\theta\theta M(x) - i\bar\theta\bar\theta M^+(x)$$
$$-\theta\sigma^\mu\bar\theta v_\mu(x) + i\theta\theta\bar\theta[\bar\lambda(x) + i\bar\sigma^\mu\partial_\mu\chi(x)] - i\bar\theta\bar\theta\theta[\lambda(x) + i\sigma^\mu\partial_\mu\bar\chi(x)]$$
$$+\tfrac{1}{2}\theta\theta\bar\theta\bar\theta[D(x) + \tfrac{1}{2}\Box C(x)]$$

Gauge transformation

$$V \to V + \Phi + \bar\Phi$$

$$C \to C + A + A^*$$
$$\chi \to \chi - i\sqrt{2}\psi$$
$$M \to M - 2iF$$
$$v_\mu \to v_\mu - i\partial_\mu(A - A^*)$$
$$\lambda \to \lambda$$
$$D \to D$$

Wess-Zumino gauge

$$C = \chi = M = 0$$

physical fields

Field strength tensor

$$W_\alpha = -\tfrac{1}{4}\bar D^2 e^V D_\alpha e^{-V}$$

Covariant derivatives

$$D_\alpha = \frac{\partial}{\partial\vartheta_\alpha} + i\sigma^\mu_{\alpha\dot\alpha}\bar\vartheta^{\dot\alpha}\partial_\mu$$
$$\bar D_{\dot\alpha} = -\frac{\partial}{\partial\bar\theta_\alpha} - i\theta_{\dot\alpha}\sigma^\mu_{\alpha\dot\alpha}\partial_\mu$$

$$W_\alpha = -i\lambda_\alpha + \theta_\alpha D - \tfrac{i}{2}(\sigma^\mu\bar\sigma^\nu\theta)_\alpha F_{\mu\nu} + \theta^2\sigma^\mu D_\mu\bar\lambda$$

How to write SUSY Lagrangians

1st step

Take your favorite Lagrangian written in terms of fields

2nd step

Replace $$Field\ (\varphi,\psi,A_\mu)\ \Rightarrow\ Superfield\ (\Phi,V)$$

3rd step

Replace

$$Action = \int d^4x\, L(x) \quad \Longrightarrow \quad \int d^4x\, d^4\theta\, L(x,\theta,\bar{\theta})$$

Grassmannian integration in superspace $\int d\theta_\alpha = 0,\ \int \theta_\beta d\theta_\alpha = \delta_{\alpha\beta}$

Superfield Lagrangians

$$Action = \int d^4x\, L \quad \Longrightarrow \quad \int d^4x\, d^4\theta\, L$$

Grassmannian integration in superspace $\int d\theta_\alpha = 0, \ \int \theta_\beta d\theta_\alpha = \delta_{\alpha\beta}$

Matter fields

$$L = \int d^2\theta d^2\overline{\theta}\ \Phi_i^+\Phi_i + \int d^2\theta\ \underbrace{(\lambda_i\Phi_i + \tfrac{1}{2} m_{ij}\Phi_i\Phi_j + \tfrac{1}{3} y_{ijk}\Phi_i\Phi_j\Phi_k)}_{\text{Superpotential}} + h.c.]$$

Gauge fields

$$L = \tfrac{1}{4}\int d^2\theta\ W^\alpha W_\alpha + \int d^2\overline{\theta}\ \overline{W}^{\dot{\alpha}}\overline{W}_{\dot{\alpha}} = \tfrac{1}{2}D^2 - \tfrac{1}{4}F_{\mu\nu}F^{\mu\nu} - i\lambda\sigma^\mu D_\mu\overline{\lambda}$$

Gauge transformation $\Phi \to e^{-ig\Lambda}\Phi,\ \Phi^+ \to \Phi^+ e^{ig\Lambda^+},\ V \to V + i(\Lambda - \Lambda^+)$

Gauge invariant interaction $\Phi^+\Phi \to \Phi^+ e^{gV}\Phi$

Gauge Invariant SUSY Lagrangian

Superfields

$$L_{SUSY\ YM} = \tfrac{1}{4}\int d^2\theta\ \mathrm{Tr}(W^\alpha W_\alpha) + \tfrac{1}{4}\int d^2\bar\theta\ \mathrm{Tr}(\overline{W}^\alpha \overline{W}_\alpha)$$
$$+\int d^2\theta d^2\bar\theta\ \overline{\Phi}_{ia}(e^{gV})^a_b\Phi^b_i + \int d^2\theta\ \mathcal{W}(\Phi_i) + \int d^2\bar\theta\ \overline{\mathcal{W}}(\overline{\Phi}_i)$$

Components

$$L_{SUSY\ YM} = -\tfrac{1}{4}F^a_{\mu\nu}F^{a\mu\nu} - i\lambda^a\sigma^\mu D_\mu\bar\lambda^a + \tfrac{1}{2}\underline{D^aD^a}$$
$$+(\partial_\mu A_i - igv^a_\mu T^a A_i)^\dagger(\partial_\mu A_i - igv^a_\mu T^a A_i) - i\bar\psi_i\bar\sigma^\mu(\partial_\mu\psi_i - igv^a_\mu T^a\psi_i)$$
$$-\underline{D^a}gA^\dagger_i T^a A_i - i\sqrt{2}gA^\dagger_i T^a\lambda^a\psi_i + i\sqrt{2}g\bar\psi_i T^a\bar\lambda^a A_i + \underline{F^\dagger_i F_i}$$
$$+\frac{\partial\mathcal{W}}{\partial A_i}\underline{F_i} + \frac{\partial\overline{\mathcal{W}}}{\partial A^\dagger_i}\underline{F^\dagger_i} - \tfrac{1}{2}\frac{\partial^2\mathcal{W}}{\partial A_i\partial A_j}\psi_i\psi_j - \tfrac{1}{2}\frac{\partial^2\overline{\mathcal{W}}}{\partial A^\dagger_i\partial A^\dagger_j}\bar\psi_i\bar\psi_j$$

Potential

$$D^a = -gA^\dagger_i T^a A_i,\quad F_i = -\frac{\partial\mathcal{W}}{\partial A_i} \quad\rightarrow\quad \mathrm{V} = \tfrac{1}{2}D^aD^a + F^\dagger_i F_i$$