QCD Thermodynamics on the lattice

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Day I:

- Introduction: Dense Matter and Heavy Ion collisions
- Finite-T lattice QCD: Chiral symmetry and the hadron spectrum

Day II:

- Chiral (phase) transition: O(4) scaling and T_c
- Deconfinement: Polyakov loop and $Z(3)$ symmetry, baryon number and electric charge fluctuations, the QCD equation of state
  - thermodynamics at $\mu_B \neq 0$ (lectures by C. Schmidt)

Helmholtz Summer School

Lattice QCD, Hadron Structure and Hadronic Matter
Dubna, Russia, 5-18 September, 2011

some review articles

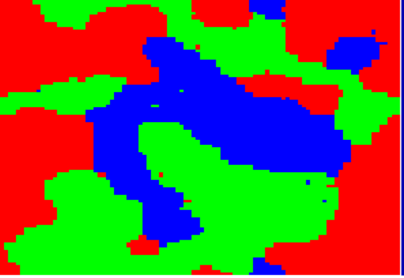
H. Meyer-Ortmanns, Phase transitions in quantum chromodynamics,
Rev. Mod. Phys. 68 (1996) 473

F. Karsch, Lattice QCD at High Temperature and Density,
Lect. Notes Phys. 583 (2002) 209

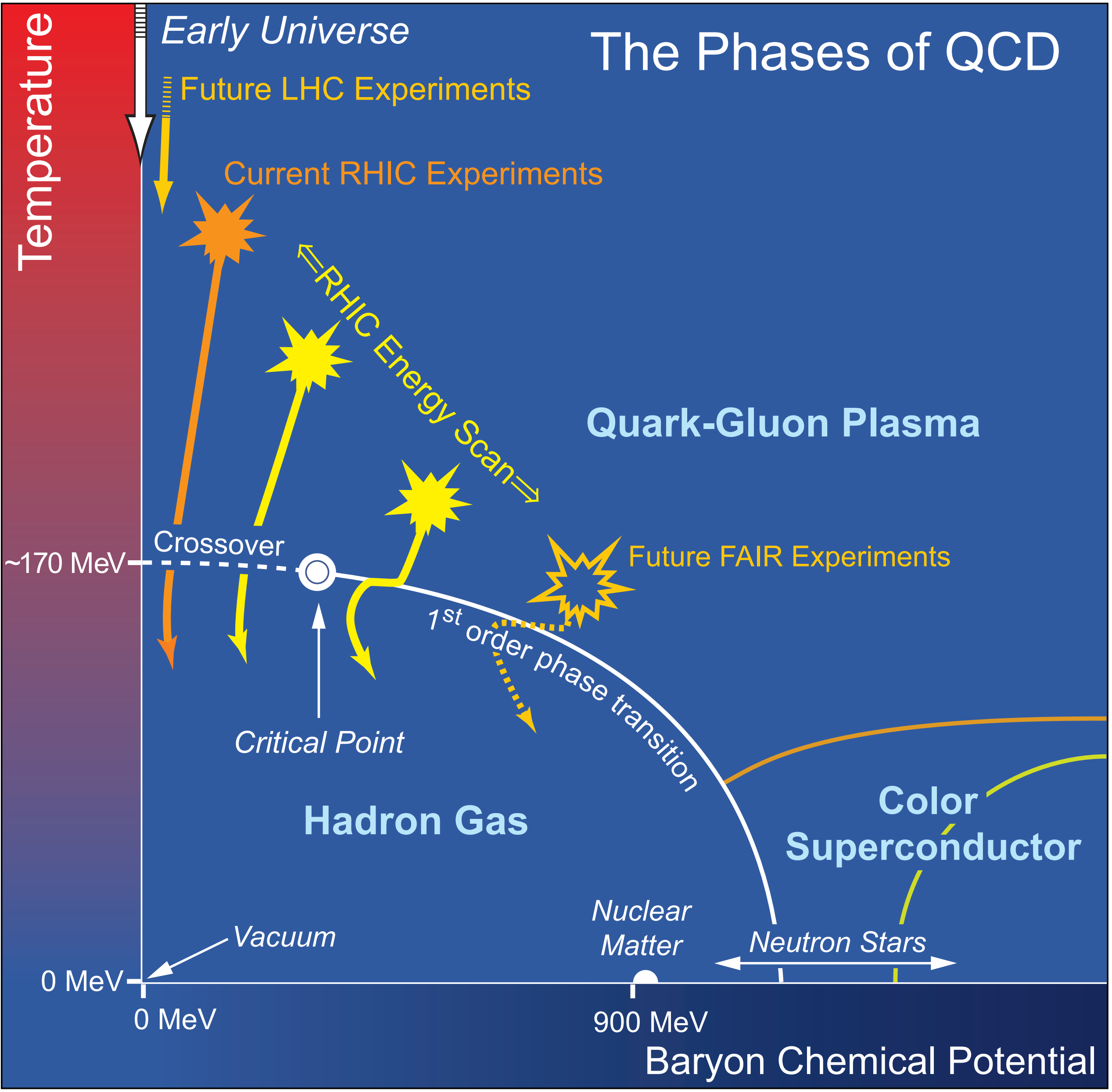
E. Laermann, O. Philipsen, Status of lattice QCD at finite T,
Ann. Rev. Nucl. Part. Sci. 53 (2003) 163

C. DeTar and U. Heller, QCD Thermodynamics from the Lattice,
Eur. Phys. J. A41, 405-437 (2009)

K. Fukushima, T. Hatsuda, The phase diagram of dense QCD,
Rept. Prog. Phys. 74, 014001 (2011)



The Phases of Nuclear Matter



physics of the early universe

hot: $T \sim 10^{12} K$

experimentally accessible

in Heavy Ion Collisions at

SPS, RHIC, LHC, FAIR

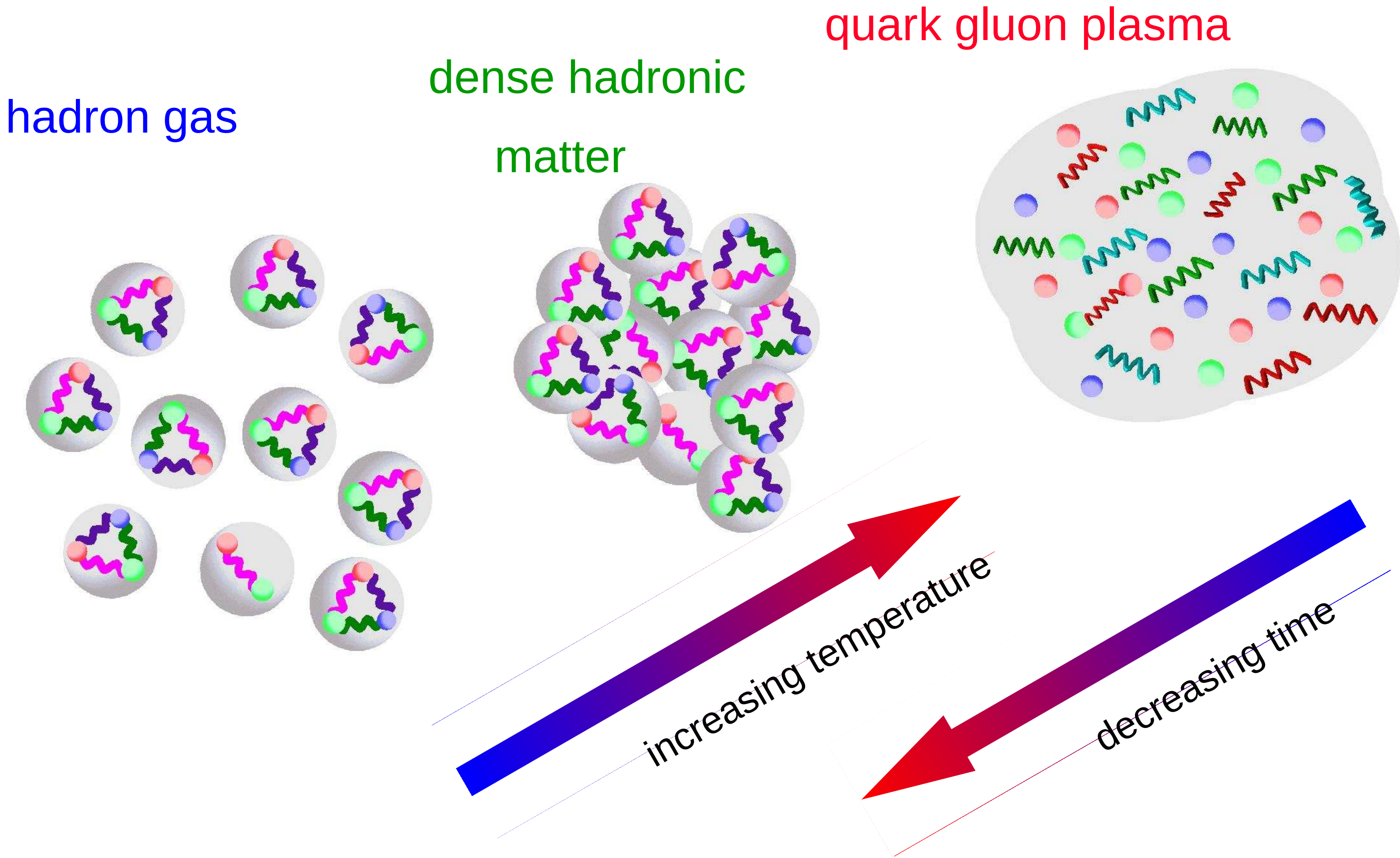
properties of compact stars

dense: $n_B \sim 10 n_{NM}$

From matter to elementary particles...
...to elementary particle matter

temperatures in the early universe after 10^{-6} sec: $\sim 10^{12}$ K

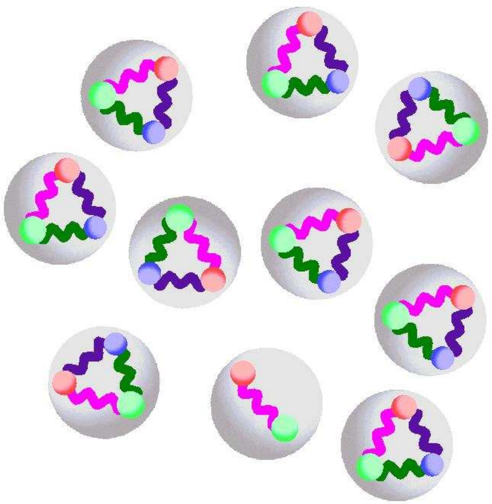
density of neutron stars: $\sim$ (3-10)-times nuclear matter density



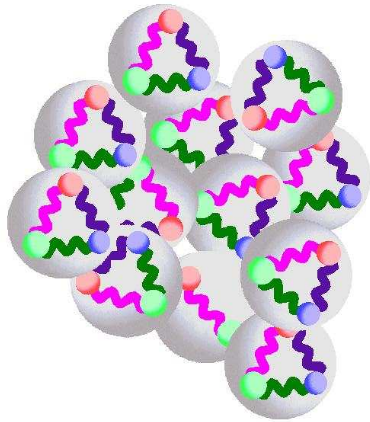
From Hadronic Matter to the Quark Gluon Plasma

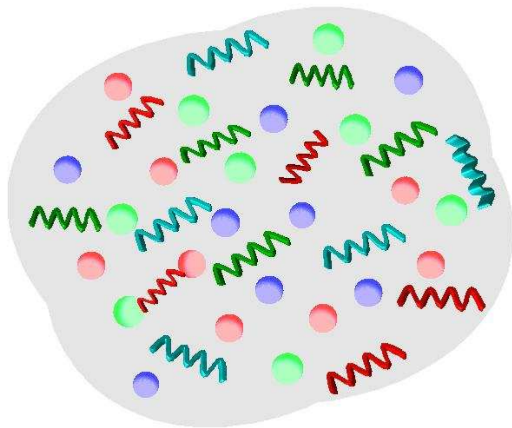
with the help of QCD

hadron gas



dense hadronic matter





J.C. Collins, M.J. Perry, Superdense Matter: Neutrons and asymptotically free quarks? PRL 34 (1975) 1353

N. Cabibbo, G. Parisi, Exponential Hadronic Spectrum and Quark Liberation, PL B59 (1975) 67

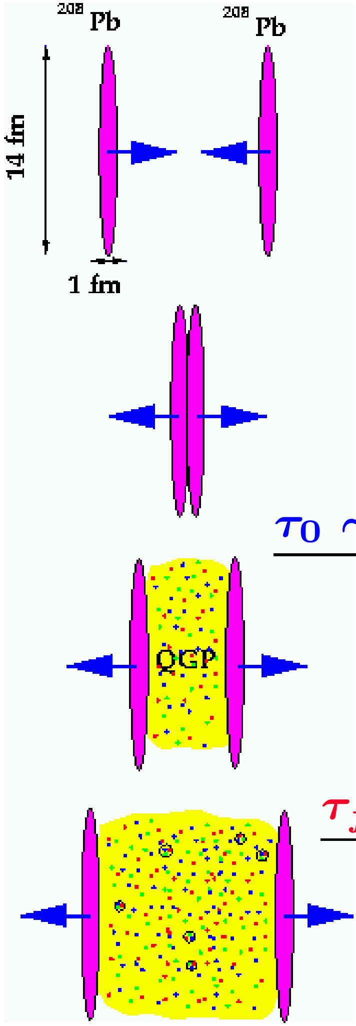
QuantumChromoDynamics
(Fritsch, Gell-Mann, 1972)

n_f quarks;
$(N_c^2 - 1)$ gluons;

confinement;
asymptotic freedom;
chiral symmetry breaking;

Creating hot and dense matter in heavy ion collisions

Creating a QGP in A-A Collisions (RHIC)



beam energy: $\mathbf{200\ GeV/A}$ (for Au)

$\sim \mathcal{O}(\mathbf{1000})$ particles/event at central rapidity

initial (thermalized) energy density

$\epsilon(\tau_0) \sim \mathbf{10\ GeV/fm^3}$

$\boldsymbol{\tau_0} \sim \mathbf{(0.5 - 1.0)}$fm

initial temperature;
$\sim \mathbf{1.5\ T_c}$;
$\sim \mathbf{250\ MeV}$

baryon density
$\mu_B \simeq \mathbf{50\ MeV}$

$\boldsymbol{\tau_f} = ?$

phase transition at $\mathbf{T_c \simeq 170\ MeV}$

back to the ordinary QCD vacuum

observable properties of QGP?

"measured" in experiment; using Bjorken formula

hydrodynamic expansion at constant $\boldsymbol{S}$, $\boldsymbol{N_B}$

need EoS: $\boldsymbol{p(\epsilon)} \Rightarrow \boldsymbol{v_s}$

(transport coefficients)

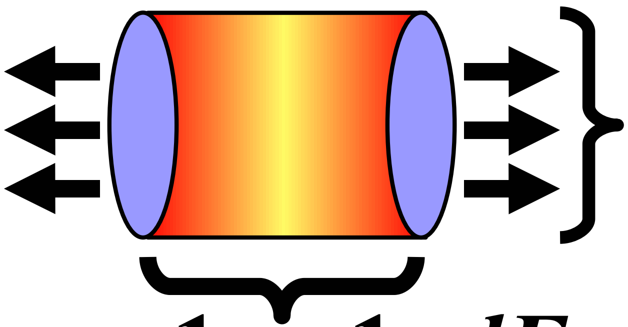
hydro: $\boldsymbol{\epsilon(\tau)}$

lattice QCD: $\boldsymbol{\epsilon(T)}$

$\Rightarrow \boldsymbol{\epsilon(\tau_0),\ T_f \equiv T_c,\ \tau_f}$

Bjorken formula:

Estimating the energy density of the dense (thermalized) matter created in an A-A collision



$$\varepsilon_{Bj} = \frac{1}{\pi R^2} \frac{1}{\tau_0} \frac{dE_T}{dy}$$

$R \simeq 1.2\ A^{1/3}$fm: transverse radius

τ_0: equilibration time $\hat{=}$ time after collision at which "some form of equilibrium" is reached

$dE_T/dy \simeq \langle E_T \rangle dN/dy$: transverse energy per unit rapidity

Bjorken formula:

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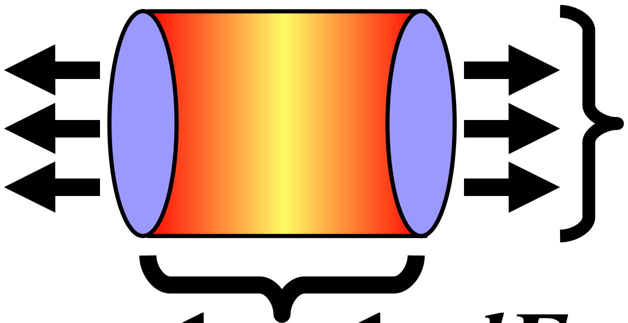


$$\varepsilon_{Bj} = \frac{1}{\pi R^2} \frac{1}{\tau_0} \frac{dE_T}{dy}$$

$R_{Au} \simeq 7$ fm;
$\tau_0 \simeq 1$ fm
$\langle E_T \rangle \simeq 1$ GeV
$dN/dy \simeq 1000$

$\Downarrow$

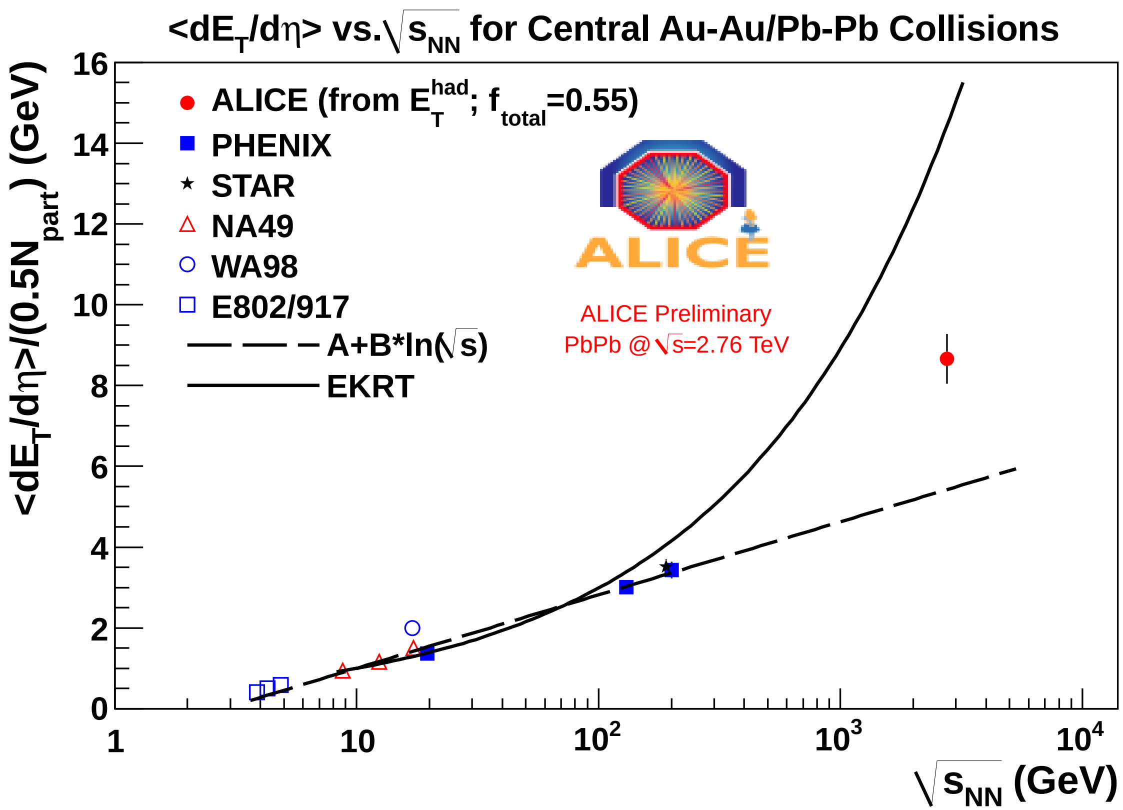
$\epsilon_{Bj} \simeq 7$ GeV/fm^3

$R \simeq 1.2\ A^{1/3}$fm: transverse radius

τ_0: equilibration time $\hat{=}$ time after collision at which "some form of equilibrium" is reached

$dE_T/dy \simeq \langle E_T \rangle dN/dy$: transverse energy per unit rapidity

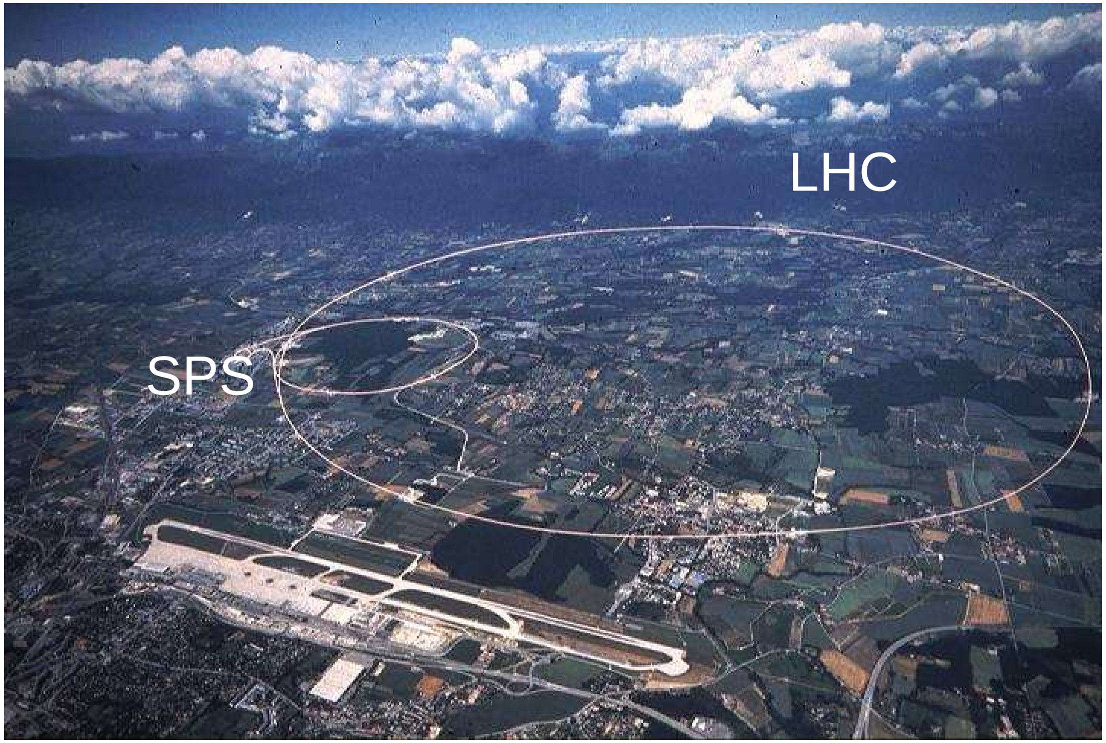
Transverse energy in A-A collisions



- RHIC $\Rightarrow$ LHC: transverse energy ($\Leftrightarrow$ energy density) doubles;
- initial temperature increase by at least 20% $\Rightarrow$ $T_0 \sim (2 - 2.5)T_c$

Heavy Ion Collisions at the SPS/LHC@CERN:

A-A collisions since 1986



Pb-Pb beams: $\sqrt{s} = 17.4$ GeV/A (SPS)

2.7 TeV/A (LHC)

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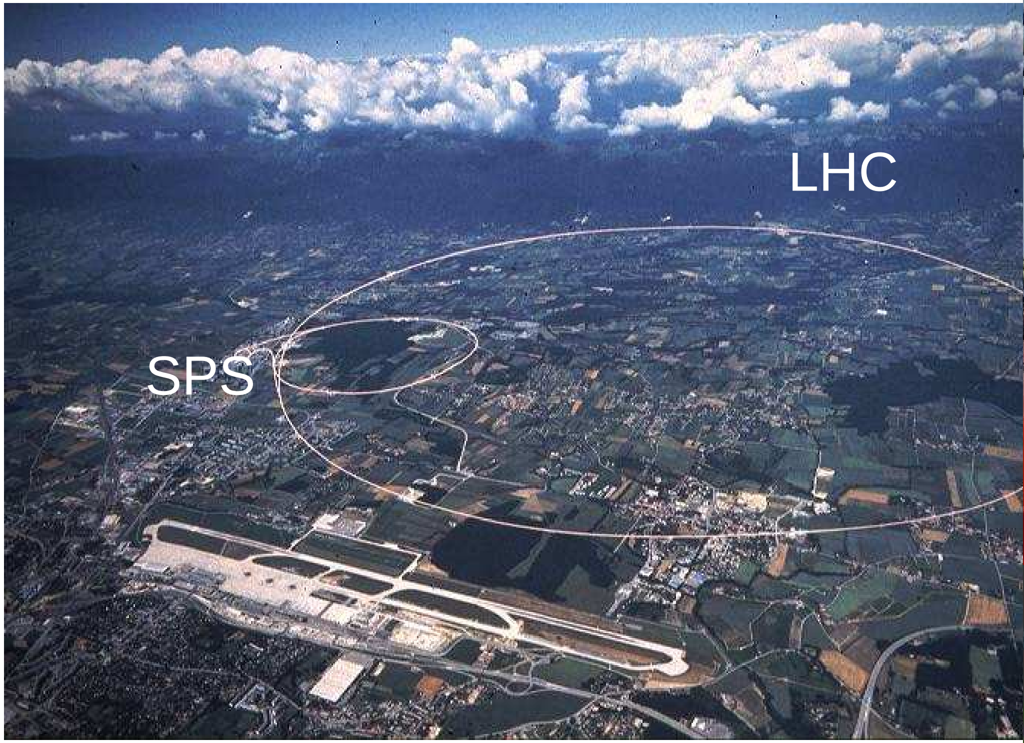
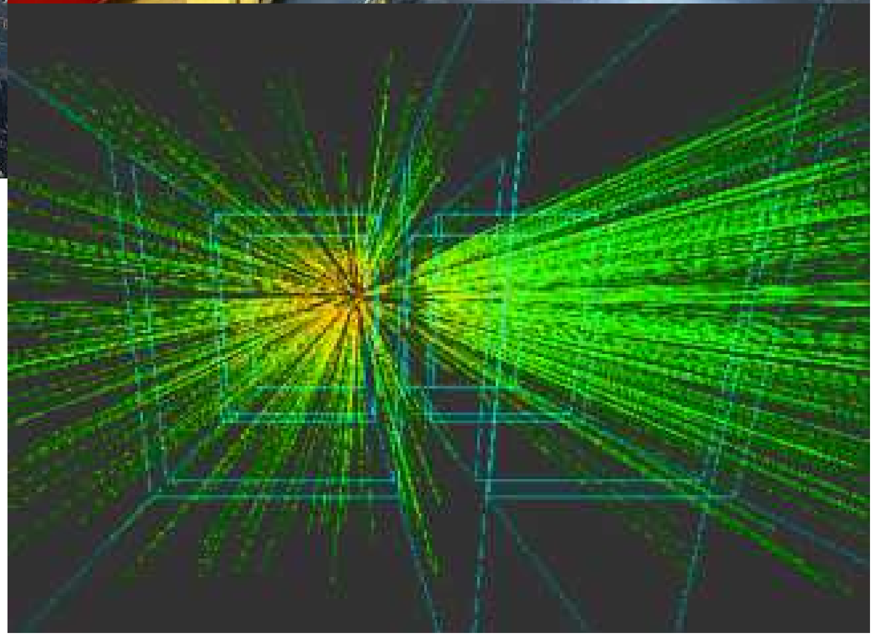


SPS tunnel



Pb-Pb beams: $\sqrt{s} = 17.4$ GeV/A (SPS)

2.7 TeV/A (LHC)

estimated temperature: $T_0 \simeq (1\text{-}1.2)\ T_c$

estimated initial energy density:

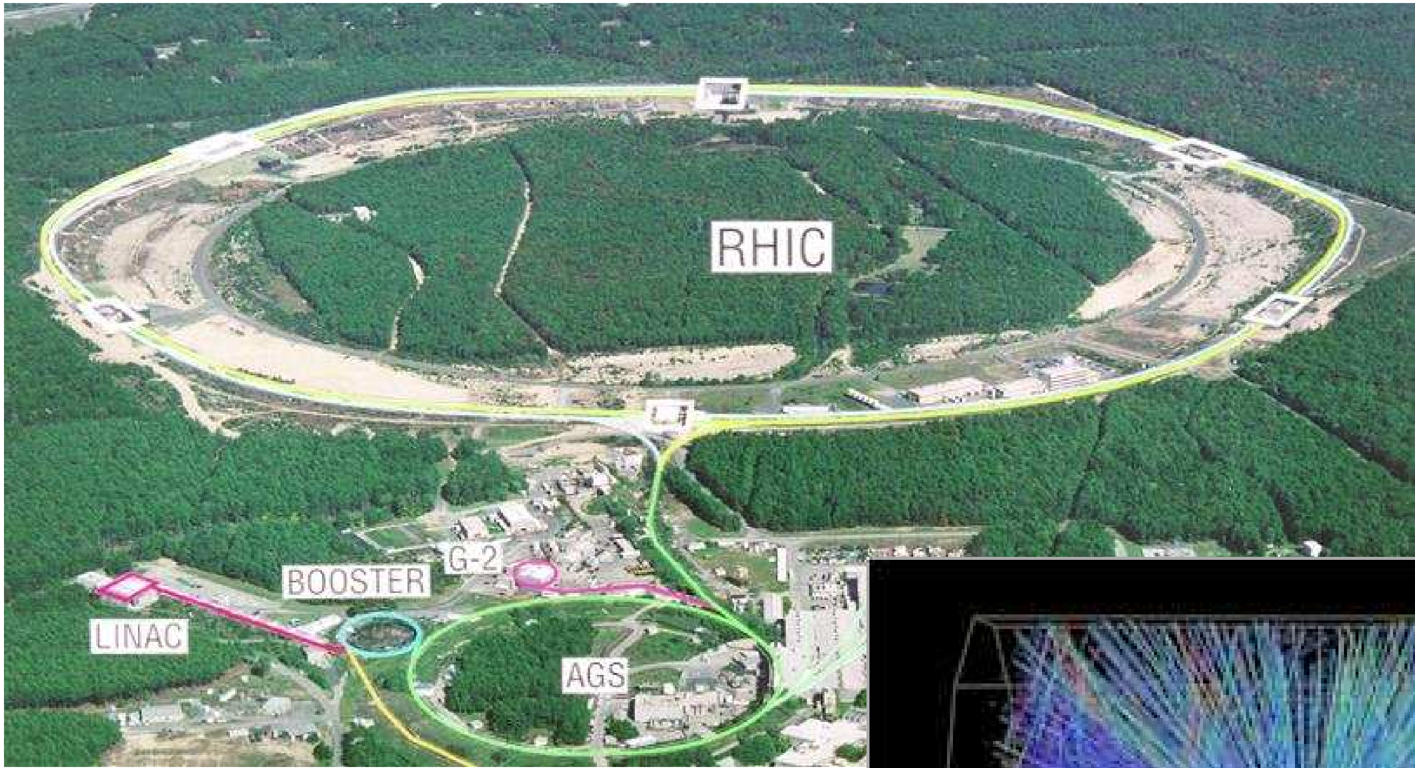
$\epsilon_0 \simeq (1-2)$ GeV/fm^3

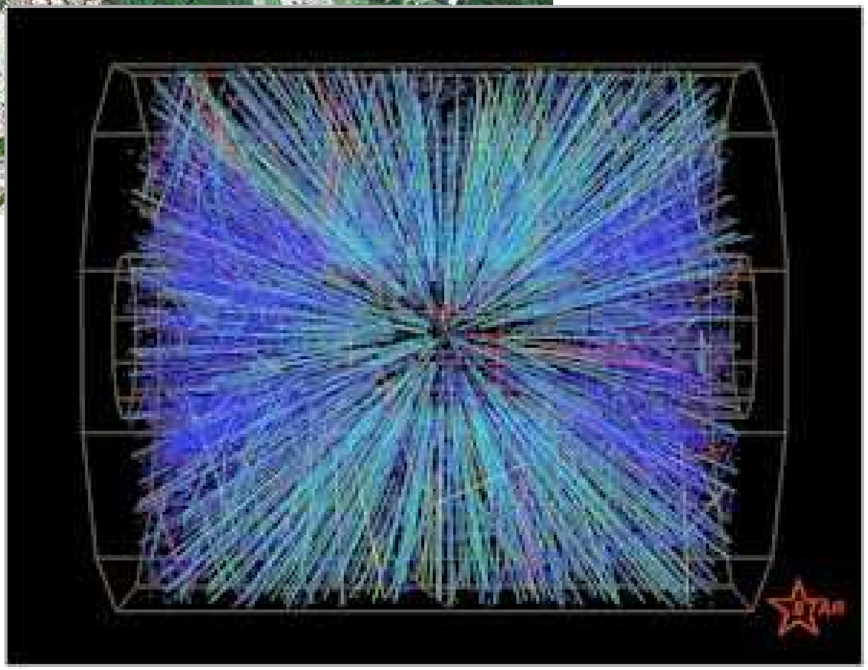
NA49 event

Heavy Ion collisions at the RHIC@BNL:

AU-AU beams: $\sqrt{s} = \mathbf{130},\ \mathbf{200}$ GeV/A

Heavy Ion collisions at the RHIC@BNL:





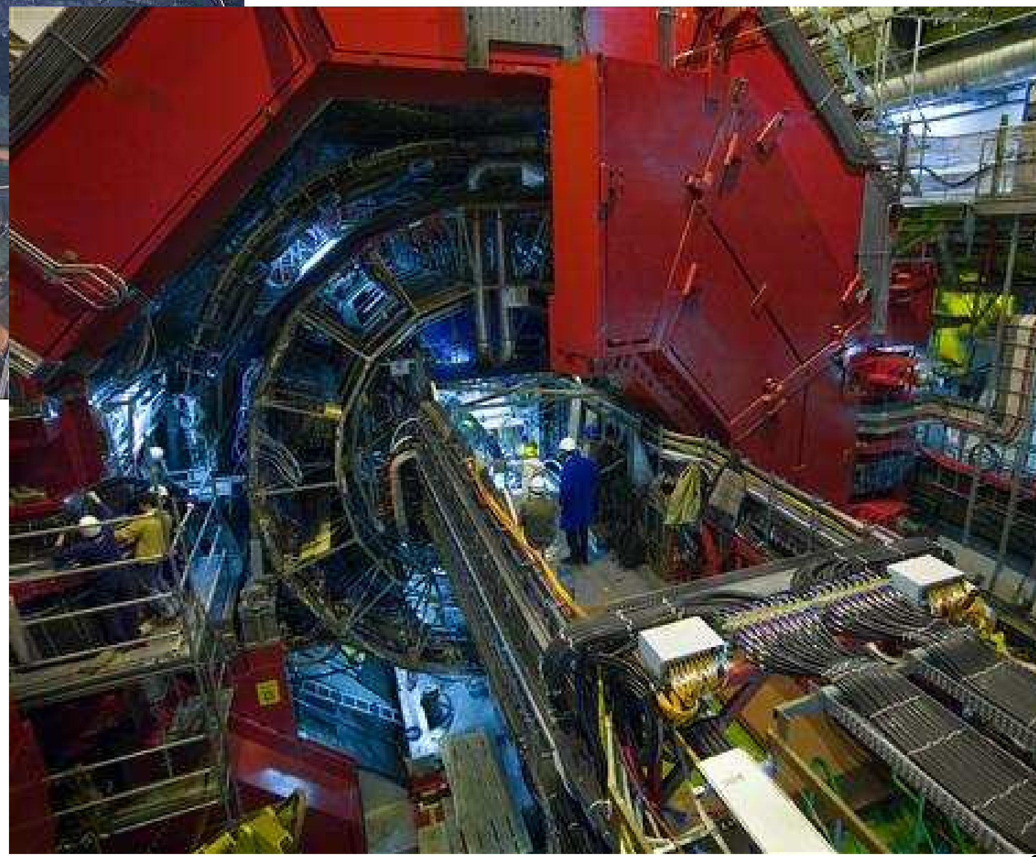
AU-AU beams: $\sqrt{s} = 130,\ 200$ GeV/A

estimated temperature: $T_0 \simeq (1.5\text{-}2)T_c$
estimated initial energy density:
$\epsilon_0 \simeq (5 - 15)$ GeV/fm^3

Heavy Ion Collisions at the SPS/LHC@CERN:

A-A collisions since 1986

ALICE@LHC

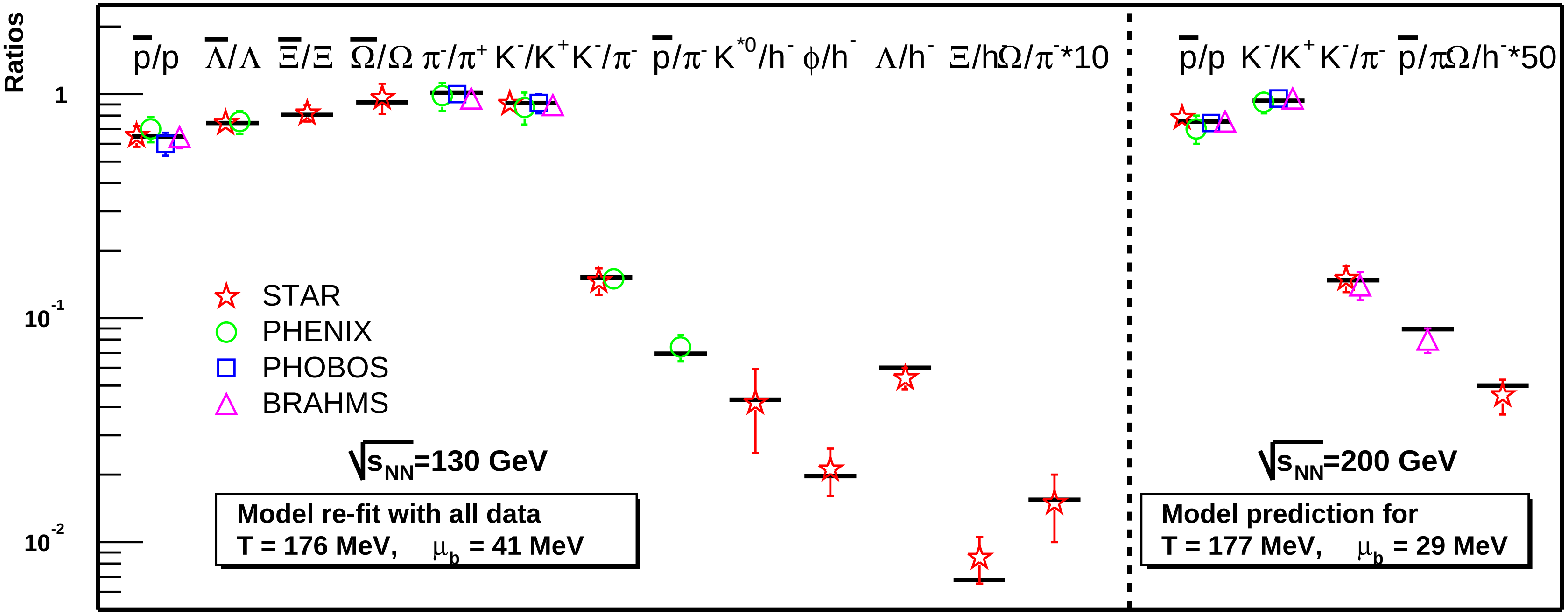


Pb-Pb beams: $\sqrt{s} = 2.7$ TeV/A (LHC)

estimated temperature:

$$T_0 \simeq (2-3)\, T_c$$

Particle ratios and freeze out conditions

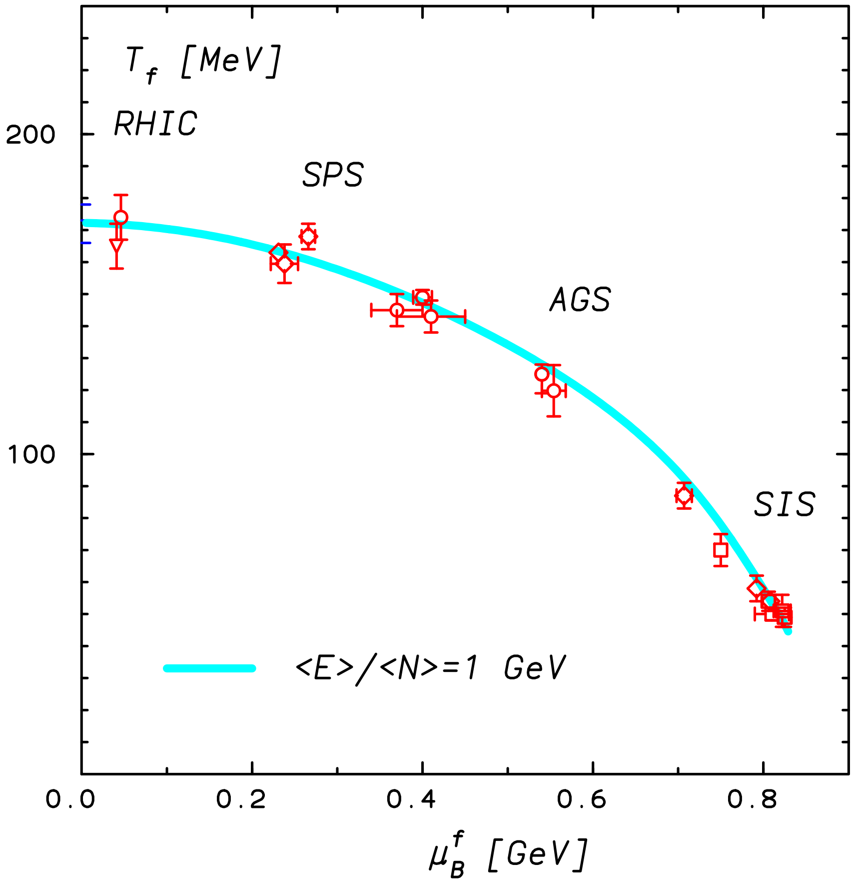


resonance gas: $Z(T,V,\mu_i) = \mathrm{Tr} e^{-\beta(H-\sum_i \mu_i Q_i)}$

describes observed particle ratios and
freeze out conditions

P. Braun-Munzinger, D. Magestro, K. Redlich,
J. Stachel, Phys. Lett. B518 (2001) 41

Particle ratios and freeze out conditions



- Is the freeze out temperature the critical temperature of the QCD transition?

- Which role do resonances play for the occurence of the transition to the QGP?

resonance gas

describes observed particle ratios and freeze out conditions

$$\ln \ Z(T,V,\mu_B,..) \ = \sum_{m_i} \ln \ Z_i(T,V,\mu_B,..)$$

QCD thermodynamics at non-zero temperature and density

THERMODYNAMICS: $Z(V,T,\mu) = \mathrm{Tr}_V \mathrm{e}^{-\frac{1}{T}(\hat{H}-\mu\hat{N})}$

Euclidean path integral: $\tau \equiv it \quad \Rightarrow \quad \tau \in [0, 1/T)$

partition function: $Z(V,T,\mu) = \int \mathcal{D}\mathcal{A}\mathcal{D}\psi\mathcal{D}\bar{\psi}\ \mathrm{e}^{-S_E}$

$$S_E = \int_0^{1/T} dx_0 \int_V d^3x\ \mathcal{L}_E(\mathcal{A}, \psi, \bar{\psi}, \mu)$$

temperature volume chemical potential

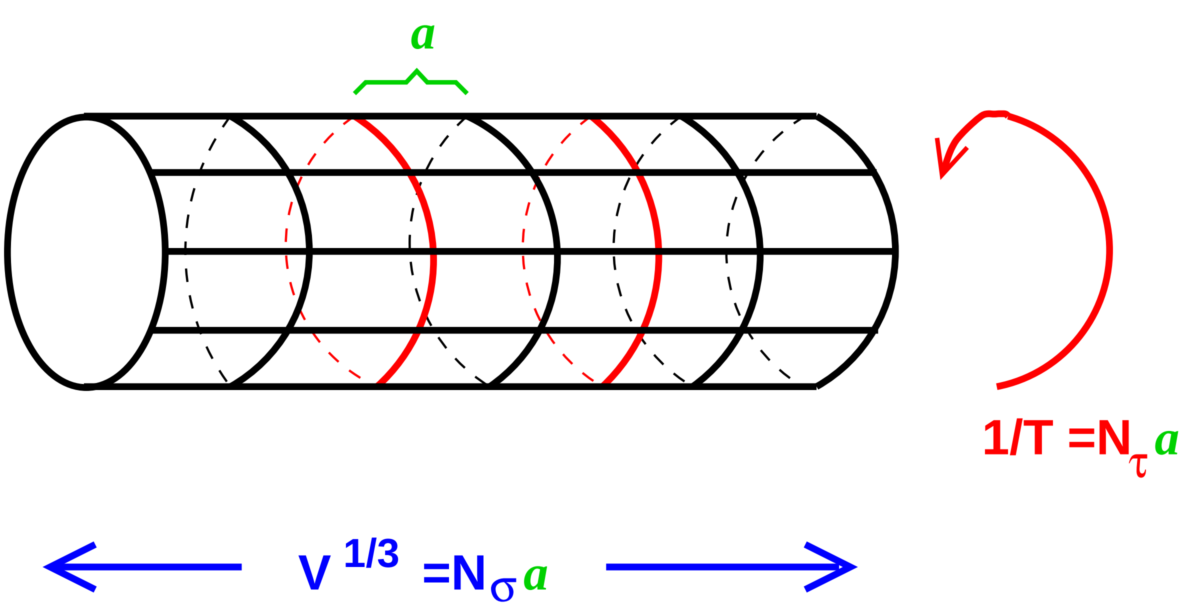
QCD:

$$\mathcal{L}_E = \frac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu} + \bar{\psi}_{j,a} \left(\sum_{\nu=0}^{3} \gamma_\nu \left(i\partial_\nu + \frac{g}{2}\mathcal{A}_\nu - i\mu\delta_{0,\nu} \right) - m_j \right)^{a,b} \psi_{j,b}$$

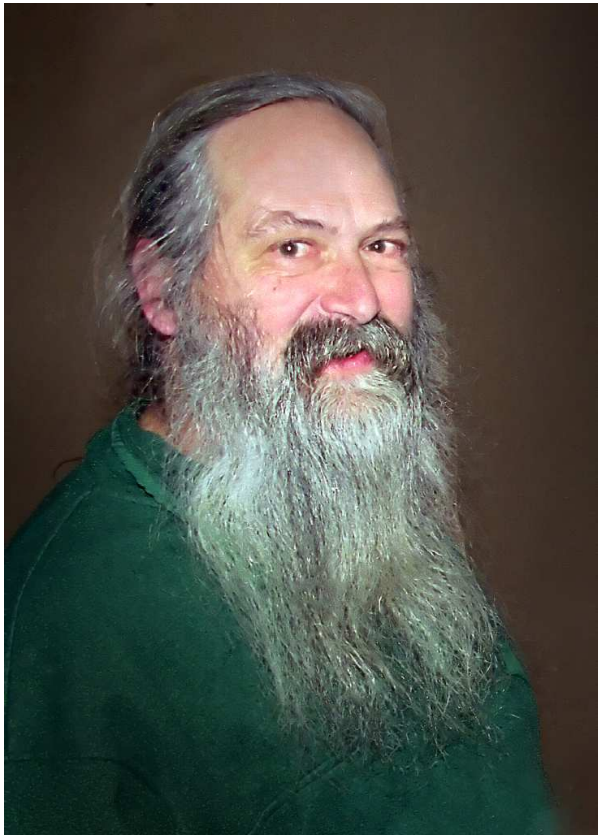
$a, b = 1, ..., N_c^2 - 1$, colour

$j = 1, ..., n_f$, flavour

Analyzing hot and dense matter on the lattice: $N_\sigma^3 \times N_\tau$



Michael Creutz



Phys. Rev. D21 (1980) 2308

Quantum Chromo Dynamics

partition function: $Z(V, T, \mu) = \int \mathcal{D}\mathcal{A}\mathcal{D}\psi\mathcal{D}\bar{\psi}\ \mathrm{e}^{-S_E}$

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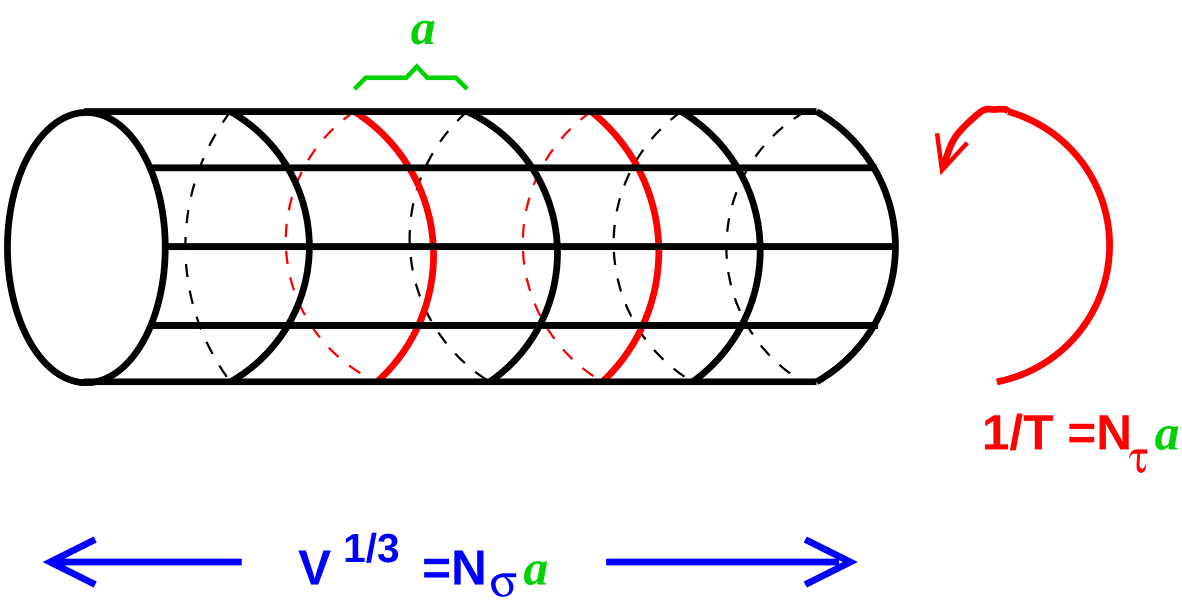

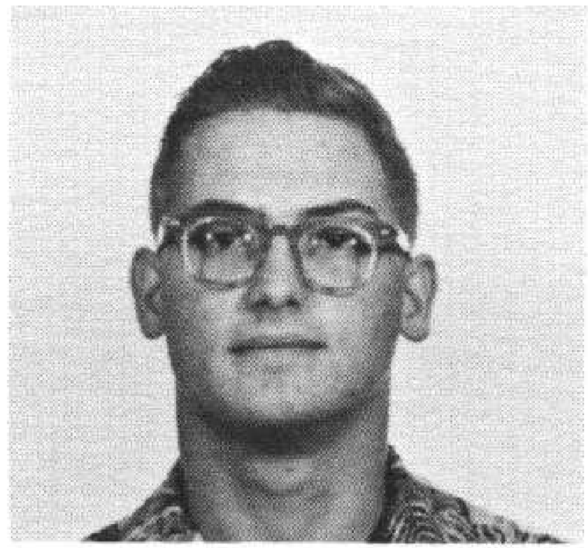

Michael J. Creutz
San Dieguito Un. H
Encinitas, Calif.

Quantum Chromo Dynamics

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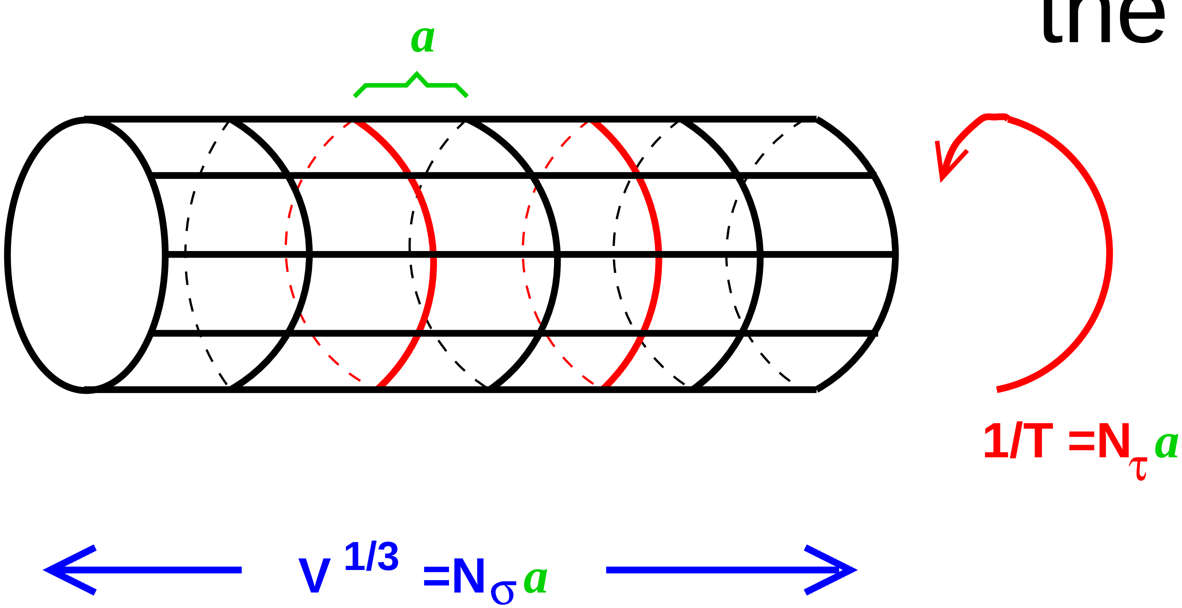
temperature volume chemical potential

QCD Thermodynamics: Simulating hot and dense matter

the lattice: $N_\sigma^3 \times N_\tau$



lattice spacing : a

gauge coupling : $\beta = 6/g^2$

the problem:

fermion determinant requires large scale computing

particularly difficult problem:

- finite density QCD (complex determinant)

$\mathcal{O}(10^6)$ grid points;
$\mathcal{O}(10^8)$ d.o.f.;

integrate eq. of motion

partition function:

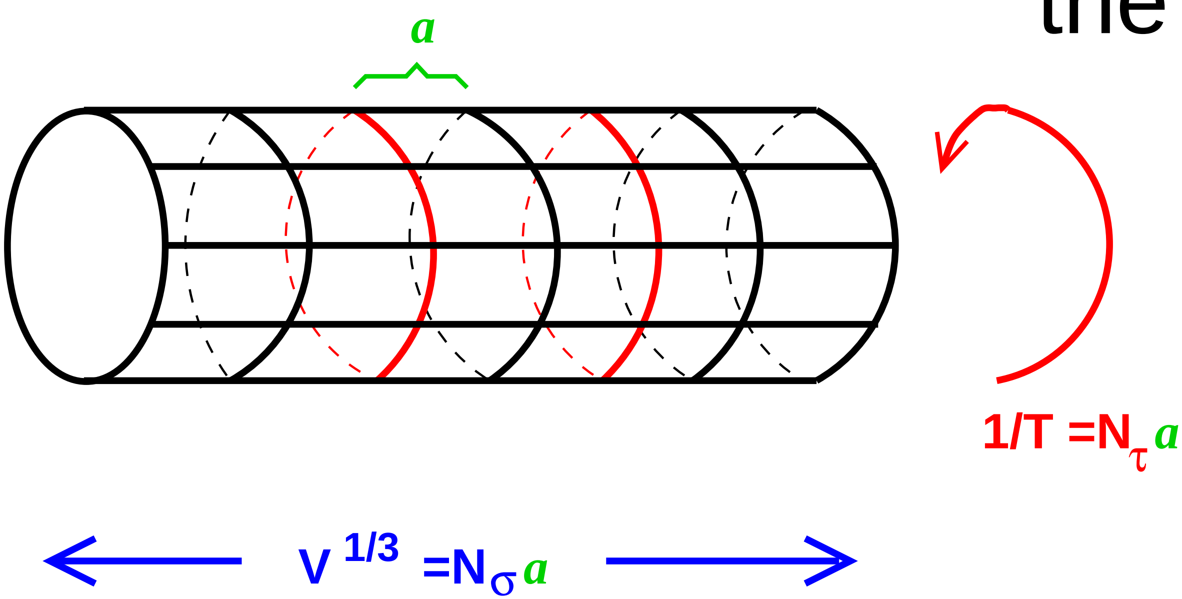
$$Z(V,T,\mu) = \int \mathcal{D}\mathcal{A}\, DetM(\mathcal{A},\mu)\, e^{-S_G}$$

$$S_E = \int_0^{1/T} dx_0 \int_V d^3x\, \mathcal{L}_E(\mathcal{A},\psi,\bar{\psi},\mu)$$

temperature volume chemical potential

QCD Thermodynamics: Simulating hot and dense matter

the lattice: $\mathbf{N_\sigma^3} \times \mathbf{N_\tau}$



lattice spacing : $\mathrm{a}_\sigma,\ \mathrm{a}_\tau$

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temperature volume chemical potential

bulk thermodynamics:

$$\frac{p}{T^4} = -\frac{1}{VT^3}\ln Z$$

$$\frac{\epsilon}{T^4} = -\frac{1}{VT^4}\frac{\partial}{\partial T^{-1}}\ln Z$$

$$\frac{n_q}{T^3} = \frac{1}{VT^3}\frac{\partial}{\partial \mu_q/T}\ln Z$$

$$\frac{\chi_q}{T^2} = \frac{1}{VT^3}\frac{\partial^2 \ln Z}{\partial(\mu_q/T)^2} = \frac{T}{V}\left(\langle N_q^2\rangle - \langle N_q\rangle^2\right)$$

Detecting the QCD phase transition on the lattice

Deconfinement vs.

chiral symmetry restoration

phase transition $\Leftrightarrow$ breaking/restoration of global symmetries

$\Updownarrow$

exist only for

$m_q = 0$ and $m_q \to \infty$

B. Svetitsky, L.G. Yaffe, NPB210, 423 (1982)

R. Pisarski, F. Wilczek, PRD29, 338 (1984)

global symmetries – suggest order of the phase transition

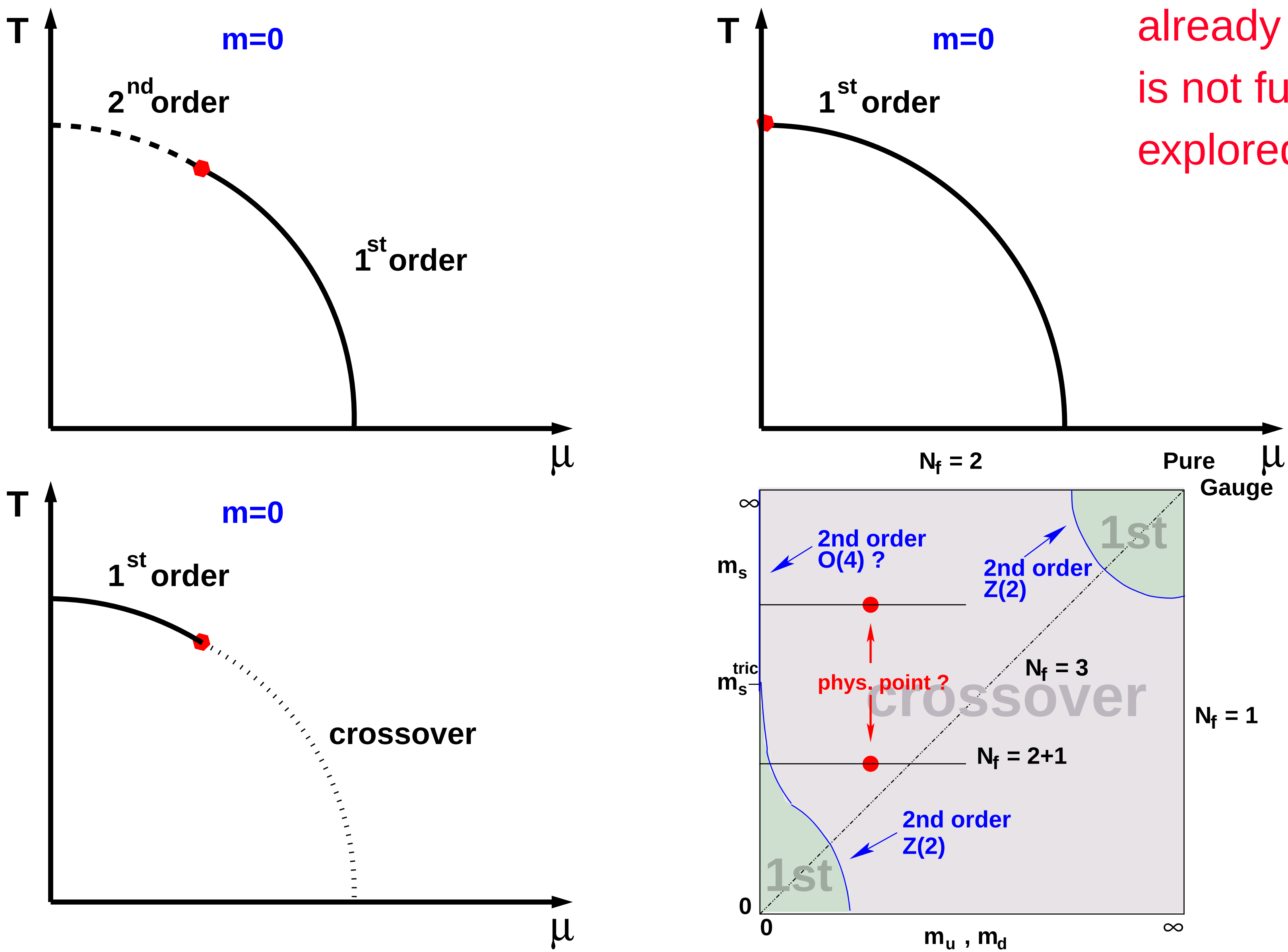
– control universal behaviour at second order transition

$m_q = \infty$: $Z(3) \;\Rightarrow\; 1^{st}$ order

$m_q = 0,\ n_f = 2$: $SU(2) \times SU(2) \simeq O(4) \;\Rightarrow\; 2^{nd}$ order (possible)

$m_q = 0,\ n_f = 3$: $SU(3) \times SU(3)$, no fixed point $\Rightarrow\; 1^{st}$ order

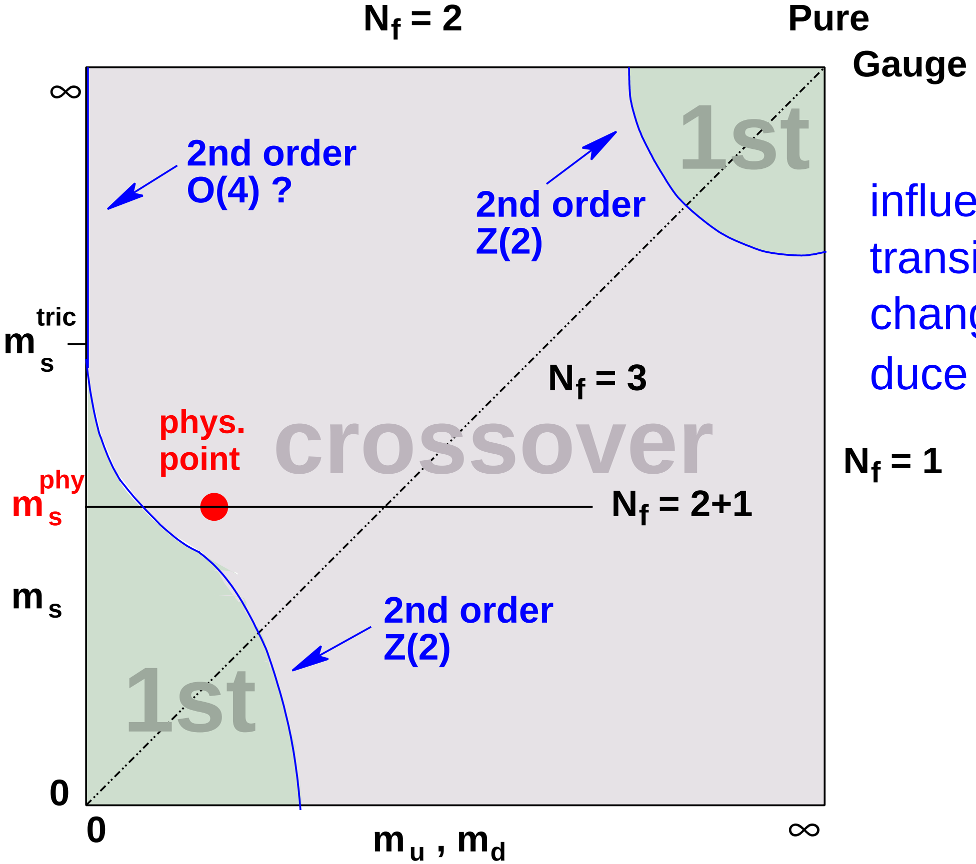
Critical behavior in hot and dense matter:
QCD phase diagram: chiral limit ($m_l = 0$)



already $\mu_B = 0$ is not fully explored

Phase diagram for $\mu_B = 0$

- already the $\mu_B = 0$ phase diagram is not fully explored
- phase boundary is known to be very sensitive to cut-off effects



influence of $U_A(1)$ breaking on QCD transition in the chiral limit; may change $O(4)$ to $O(4) \times O(2)$, can induce 1^{st} order transition

- $N_\tau = 4$, standard staggered fermions:
  $\Rightarrow m_{ps}^{crit} \simeq 300$ MeV for $n_f = 3$, *i.e.* larger than physical m_π

FK, E. Laermann, C Schmidt, PL B520 (2001) 41

Phase diagram for $\mu_B = 0$

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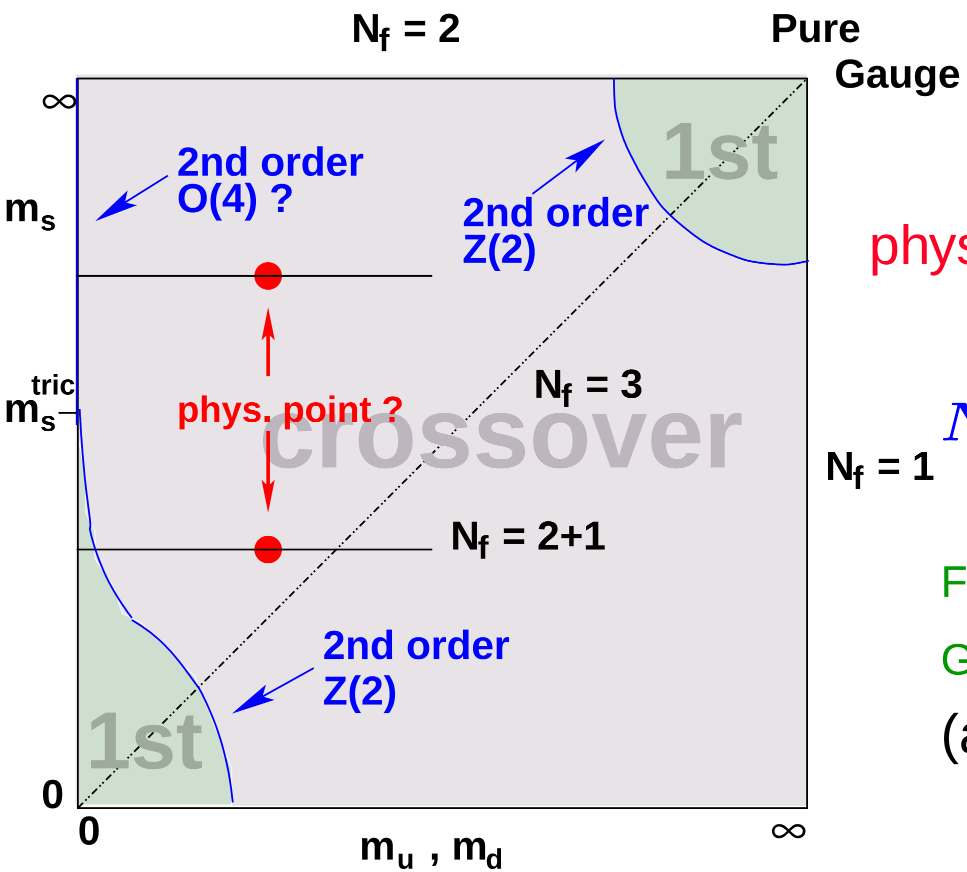
- phase boundary is known to be very sensitive to cut-off effects: $N_\tau = 4$, standard staggered fermions $\Rightarrow m_{ps}^{crit} \simeq 300$ MeV for $n_f = 3$, *i.e.* larger than physical m_π



physical point may be above m_s^{tric}

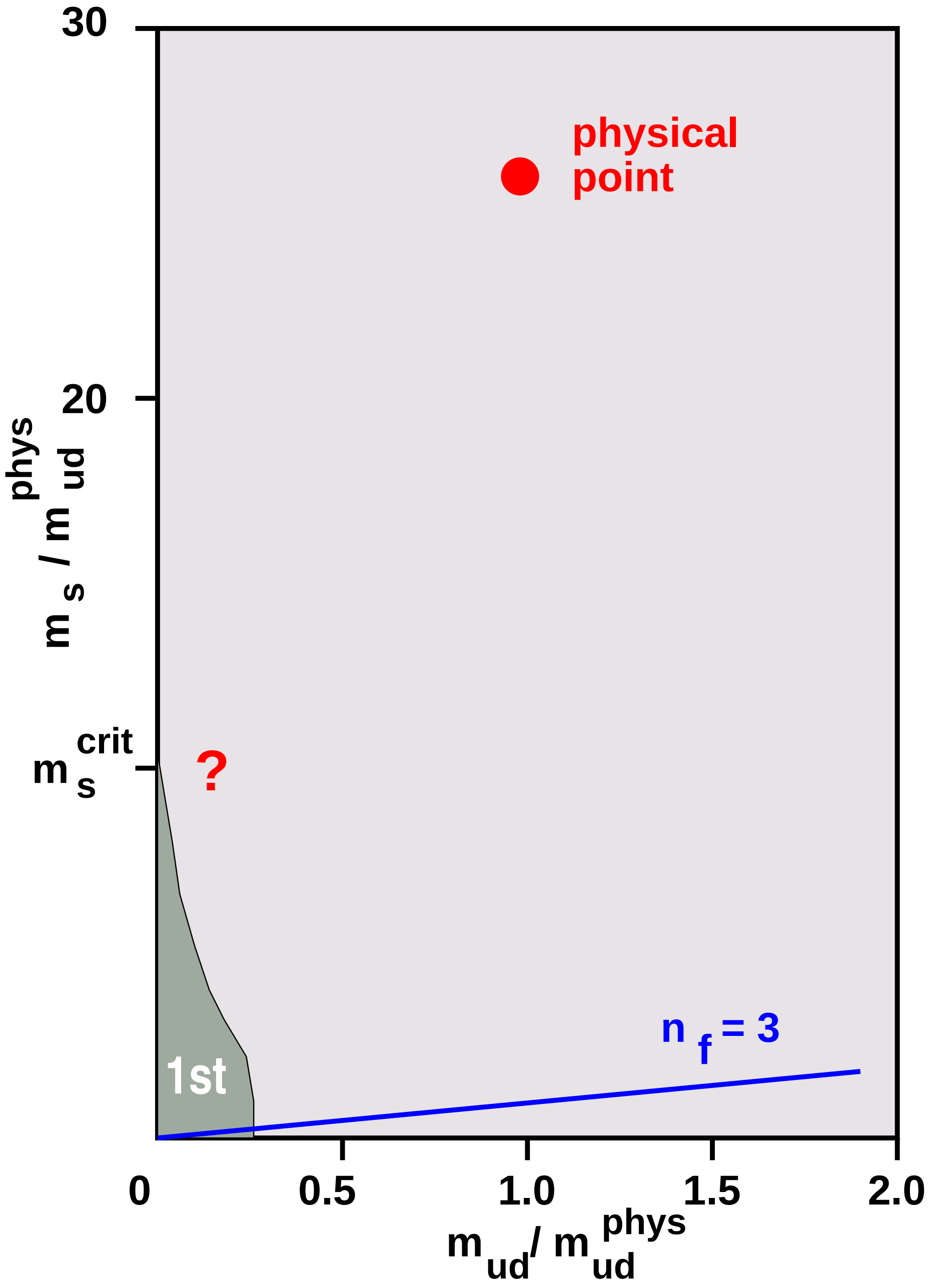
$N_\tau = 4, 6$; improved actions:
$\Rightarrow m_{ps}^{crit} \lesssim 70$ MeV

FK et al, NP(Proc.Suppl) 129 (2004) 614

G. Endrodi et al, PoS LAT 2007 (2007) 182

(also $N_\tau = 6$, unimp.)

Phase diagram for $\mu_B = 0$



 drawn to scale

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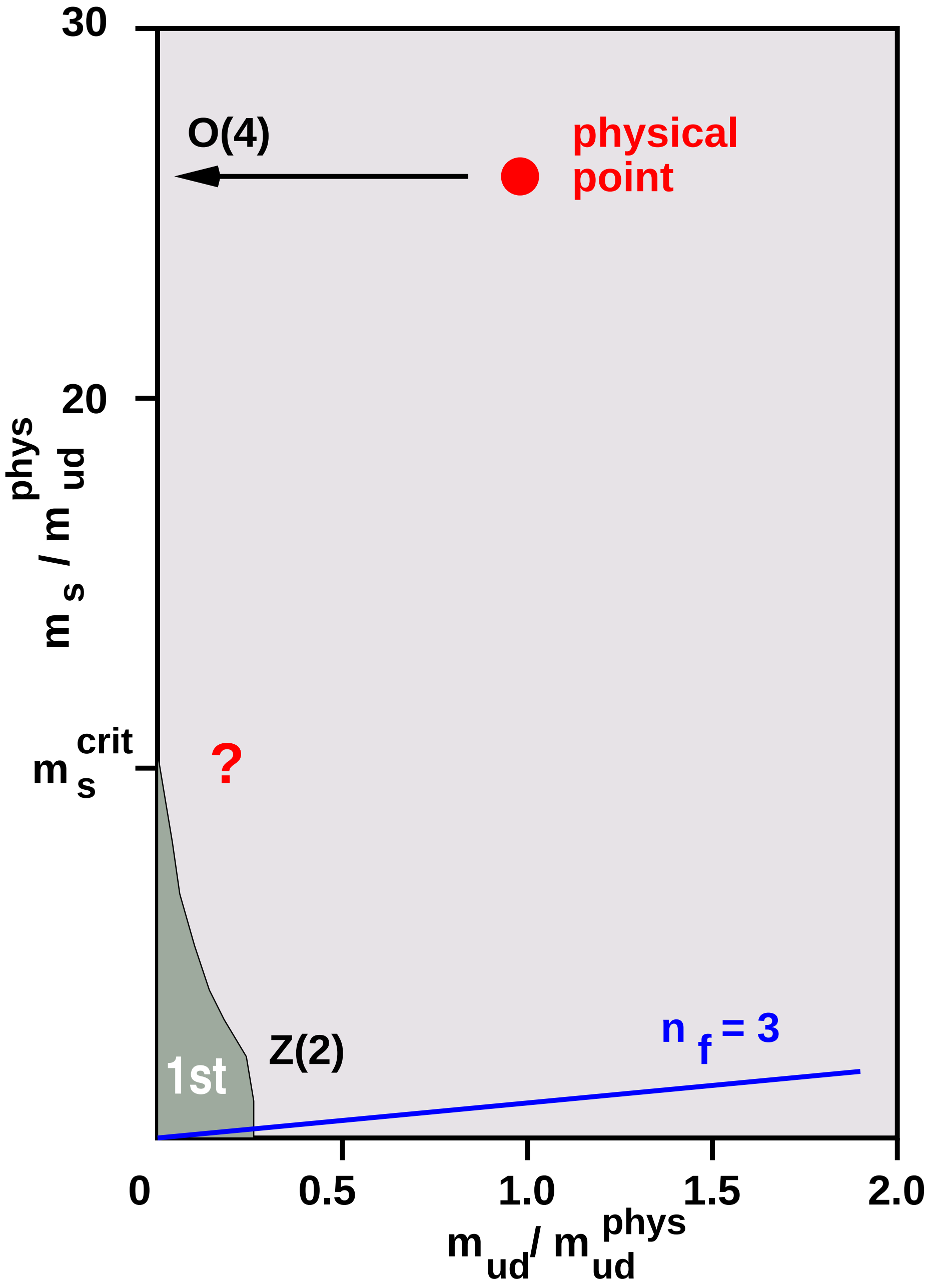

- drawn to scale

  Is physics at the physical quark mass point sensitive to (universal) properties of the chiral phase transition?

physical point may be above m_s^{tric}

$N_\tau = 4, 6$; improved actions:

$\Rightarrow m_{ps}^{crit} \lesssim 70$ MeV

FK et al, NP(Proc.Suppl) 129 (2004) 614

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(also $N_\tau = 6$, unimp.)

Symmetries of the QCD Lagrangian

$$U_V(1) \times U_A(1) \times SU_L(n_f) \times SU_R(n_f)$$

$$\mathcal{L}_F \sim \bar{\psi}_L \not{D}_\mu \psi_L + \bar{\psi}_R \not{D}_\mu \psi_R - m_q (\bar{\psi}_L \psi_R + \bar{\psi}_R \psi_L)$$

chiral projection:

$$P_\epsilon = \frac{1}{2}(1 + \epsilon\gamma_5) \ , \ \epsilon = \pm 1 \ , \quad P_\epsilon^2 = P_\epsilon \ , \ P_+ P_- = 0$$

$$\psi = \psi_L + \psi_R$$

$$\psi_L = P_+ \psi \ , \ \psi_R = P_- \psi$$

$$\bar{\psi}_L = \bar{\psi} P_- \ , \ \bar{\psi}_R = \bar{\psi} P_+$$

Symmetries of the QCD Lagrangian

$$U_V(1) \times U_A(1) \times SU_L(n_f) \times SU_R(n_f)$$

$$\mathcal{L}_F \sim \bar{\psi}_L \not{D}_\mu \psi_L + \bar{\psi}_R \not{D}_\mu \psi_R - m_q(\bar{\psi}_L \psi_R + \bar{\psi}_R \psi_L)$$

$U_V(1)$: baryon number $\psi^\Theta = \mathrm{e}^{i\Theta}\psi \; , \; \bar{\psi}^\Theta = \bar{\psi}\mathrm{e}^{-i\Theta}$

$U_A(1)$: axial symmetry $\psi^\Theta = \mathrm{e}^{i\Theta\gamma_5}\psi \; , \; \bar{\psi}^\Theta = \bar{\psi}\mathrm{e}^{i\Theta\gamma_5}$

$SU_{L,R}(n_f)$: flavour symmetry $G_\epsilon \equiv P_{-\epsilon} \cdot 1 + P_\epsilon U_\epsilon \quad , \quad U_\epsilon \in U(n_f)$

$$G \equiv G_+(U_+)G_-(U_-) :$$
$$\psi' = G\psi \quad , \quad \bar{\psi}' = \bar{\psi}G^\dagger$$

$\psi \equiv (\psi_1, ...\psi_{n_f})$

The QCD mass term

$$\bar{\psi}\psi = \bar{\psi}_L\psi_R + \bar{\psi}_R\psi_L$$

- $U_L(n_f) \times U_R(n_f)$ transformation:

$$\bar{\psi}'\psi' = \bar{\psi}_R U_+^\dagger U_- \psi_L + \bar{\psi}_L U_-^\dagger U_+ \psi_R = \bar{\psi}_R V^\dagger \psi_L + \bar{\psi}_L V \psi_R$$

$$V \equiv U_-^\dagger U_+ \equiv \mathrm{e}^{i\Theta_a T_a}, \quad a = 1, ..., n_f^2 - 1$$

- infinitesimal transformation:

$$\begin{aligned}\delta\bar{\psi}\psi &= -i\Theta_a\bar{\psi}_R T_a\psi_L + i\Theta_a\bar{\psi}_L T_a\psi_R \\ &= i\Theta_a\bar{\psi}\gamma_5 T_a\psi + \mathcal{O}(\Theta^2)\end{aligned}$$

The QCD mass term

$$\bar\psi\psi = \bar\psi_L\psi_R + \bar\psi_R\psi_L$$

- $U_L(n_f) \times U_R(n_f)$ transformation:

$$\bar\psi'\psi' = \bar\psi_R U_+^\dagger U_-\psi_L + \bar\psi_L U_-^\dagger U_+\psi_R = \bar\psi_R V^\dagger\psi_L + \bar\psi_L V\psi_R$$

$$V \equiv U_-^\dagger U_+ \equiv \mathrm{e}^{i\Theta_a T_a}, \quad a = 1, ..., n_f^2 - 1$$

- infinitesimal transformation:

$$\begin{aligned}\delta\bar\psi\psi &= -i\Theta_a\bar\psi_R T_a\psi_L + i\Theta_a\bar\psi_L T_a\psi_R \\ &= i\Theta_a\bar\psi\gamma_5 T_a\psi + \mathcal{O}(\Theta^2)\end{aligned}$$

mixes flavour components
adds pseudo-scalar component to scalar

The QCD mass term

$$\bar{\psi}\psi = \bar{\psi}_L\psi_R + \bar{\psi}_R\psi_L$$

- $U_L(n_f) \times U_R(n_f)$ transformation:

$$\bar{\psi}'\psi' = \bar{\psi}_R U_+^\dagger U_- \psi_L + \bar{\psi}_L U_-^\dagger U_+ \psi_R = \bar{\psi}_R V^\dagger \psi_L + \bar{\psi}_L V \psi_R$$

$$V \equiv U_-^\dagger U_+ \equiv \mathrm{e}^{i\Theta_a T_a}, \quad a = 1, ..., n_f^2 - 1$$

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$\Longrightarrow$ $\langle\bar{\psi}\psi\rangle = 0$, if χ-symmetry not spontaneously broken

$\Longrightarrow$ $T = 0:$ $\lim_{m_q\to 0}\langle\bar{\psi}\psi\rangle \neq 0 \;\Leftrightarrow$ Goldstone particle

Topology, $U_A(1)$: A primer
$U_A(1)$ Symmetry Restoration

- Toplogical charge:

$$Q = \frac{g^2}{32\pi^2}\int d^4x F^a_{\mu\nu}\tilde{F}^{\mu\nu}_a \quad , \quad \tilde{F}^{\mu\nu}_a = \frac{1}{2}\epsilon^{\mu\nu}_{\rho\sigma}F^{\rho\sigma}_a$$

$$Q = \int d^4x q(x) \quad , \quad q(x) = \frac{g^2}{32\pi^2}F^a_{\mu\nu}\tilde{F}^{\mu\nu}_a$$

- topological charge fluctuations

$$\chi_{top} \equiv \frac{1}{V_4}\langle Q^2\rangle = \int d^4x \langle q(x)q(0)\rangle \quad , \quad \chi^{T=0}_{top} \simeq (180\mathrm{MeV})^4$$

$$\frac{2n_f}{f_\pi^2}\chi_{top} = m^2_{\eta'} + m^2_\eta - 2m^2_K \quad , \quad \mathrm{Witten-Veneziano\ \ rel.}$$

- axial current: $J^\mu_5(x) = \bar{\psi}(x)\gamma_\mu\gamma_5\psi(x)$

$$\partial_\mu J^\mu_5 = -\frac{g^2}{16\pi^2}F^a_{\mu\nu}\tilde{F}^{\mu\nu}_a \quad , \quad U_A(1)\ \mathrm{breaking} \ \Rightarrow \ m_{\eta'} \gg m_\pi$$

Meson Spectrum and Chiral Symmetry Restoration

scalar, flavor singlet operator: $O_\sigma = \bar\psi\psi = \bar\psi_L\psi_R + \bar\psi_R\psi_L$

- $U_L(n_f) \times U_R(n_f)$ transformation:

$$\bar\psi'\psi' = \bar\psi_R U_+^\dagger U_- \psi_L + \bar\psi_L U_-^\dagger U_+ \psi_R = \bar\psi_R V^\dagger \psi_L + \bar\psi_L V \psi_R$$

$$V \equiv U_-^\dagger U_+ \equiv \mathrm{e}^{i\Theta_a T_a}, \quad a = 1, ..., n_f^2 - 1$$

- choose transformation: $\Theta_a = \pi/2$

$$\bar\psi'\psi' = -i\frac{\pi}{2}\Theta_a\left(\bar\psi_R T_a \psi_L + \bar\psi_L T_a \psi_R\right) \sim \bar\psi\gamma_5 T_a \psi \equiv O_\pi$$

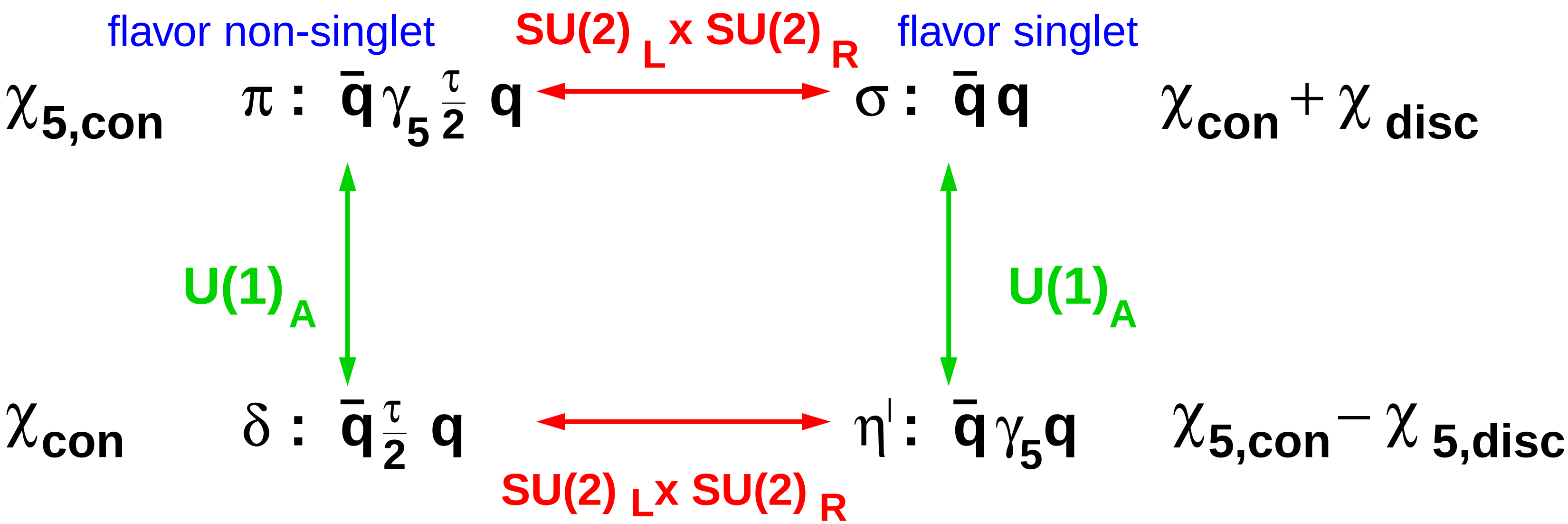
pseudo-scalar, flavor non-singlet

$$\Longrightarrow \qquad G_\pi(x,T) = \langle O_\pi(0) O_\pi^\dagger(x)\rangle \sim \mathrm{e}^{-m_\pi(T)x}$$

$$\Longrightarrow \qquad G_\sigma(x,T) = \langle O_\sigma(0) O_\sigma^\dagger(x)\rangle \sim \mathrm{e}^{-m_\sigma(T)x}$$

χ-symmetry restoration: $G_\pi(x,T) \equiv G_\sigma(x,T)$

Meson Spectrum and Chiral Symmetry Restoration



correlation functions:

$$G_\delta(x) = -\mathrm{tr}\langle\, M_l^{-1}(x,0) M_l^{-1}(0,x)\,\rangle$$

$$G_\sigma(x) = G_\delta(x) + \langle \mathrm{tr} M_l^{-1}(x,x) \mathrm{tr} M_l^{-1}(0,0)\rangle - \langle \mathrm{tr} M_l^{-1}(x,x)\rangle\, \langle \mathrm{tr} M_l^{-1}(0,0)\rangle$$

susceptibilities:

$$\frac{\chi_\sigma}{T^2} = \frac{\chi_{con}}{T^2} + \frac{\chi_{disc}}{T^2} = N_\tau^2 \sum_x G_\sigma(x,T)$$

$$\frac{\chi_\delta}{T^2} = \frac{\chi_{con}}{T^2}$$

Vector Meson Spectrum and Chiral Symmetry Restoration

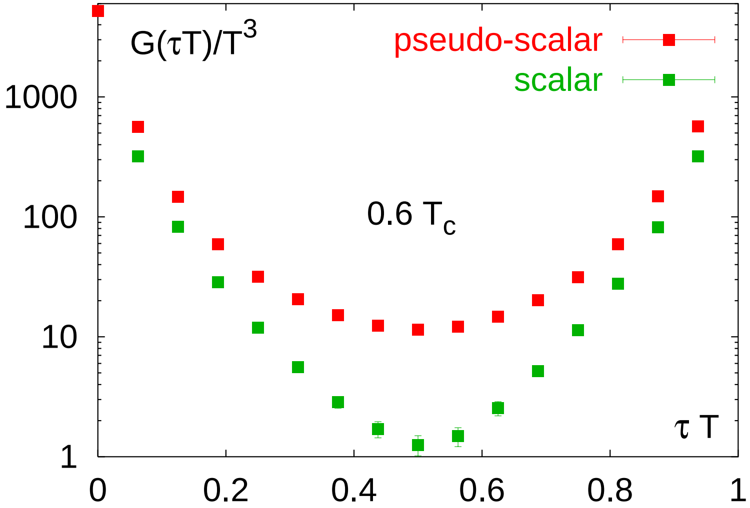
- testing $SU(2)_L \times SU(2)_R$ restoration with correlation functions is difficult as the calculation of ”disconnected correlation functions” is difficult (noisy)
- test $U(1)_A$ is more straightforward as only connected correlation functions are involved
- $\Rightarrow$ I) test $SU(2)_L \times SU(2)_R$ restoration in the vector/axial-vector channels

$$O_{\rho,\mu}(x) = \bar{u}\gamma_\mu d(x), \quad O_{a1,\mu}(x) = \bar{u}\gamma_5\gamma_\mu d(x)$$

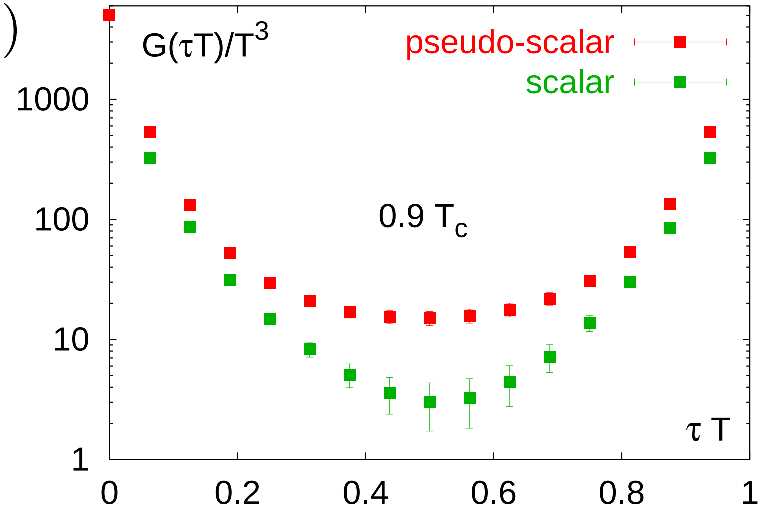
- $\Rightarrow$ II) test $SU(2)_L \times SU(2)_R$ restoration using susceptibilities; disconnected contributions much easier to handle
χ_σ is related to chiral susceptibility $\chi_m = \mathrm{d}\langle\bar{\psi}\psi\rangle/\mathrm{d}m$

Effective $U_A(1)$ symmetry restoration above T_c

$\pi: J_{PS} \sim \bar{q}\gamma_5\tau q \qquad \Leftrightarrow \qquad \delta: J_S \sim \bar{q}\tau q$



$U_A(1)$



chiral symmetry breaking below T_c $\Rightarrow$
light pseudo-scalar pion, heavy scalar (δ);

discrepancy decreases with increasing temperature

Effective $U_A(1)$ symmetry restoration above T_c

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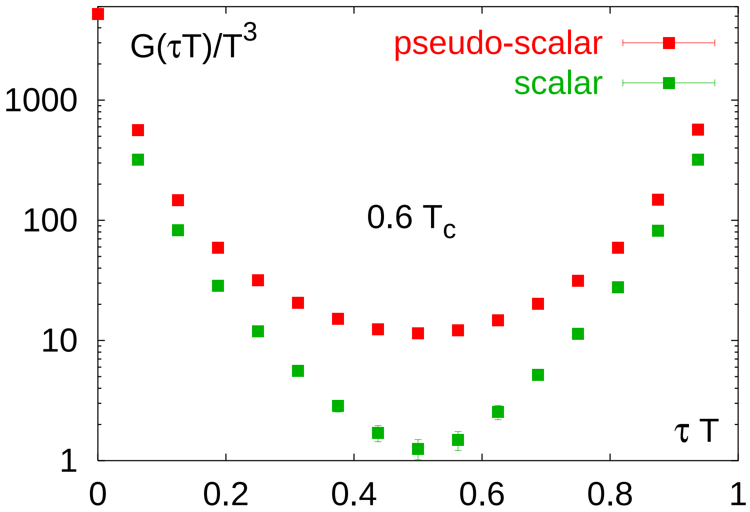
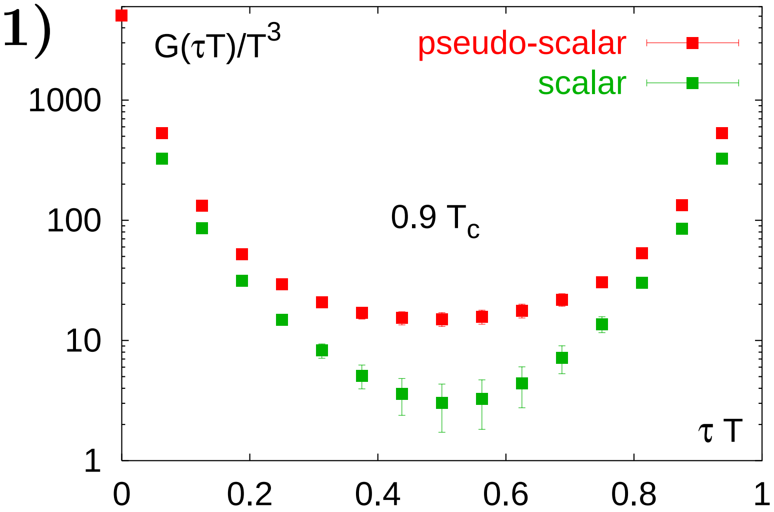
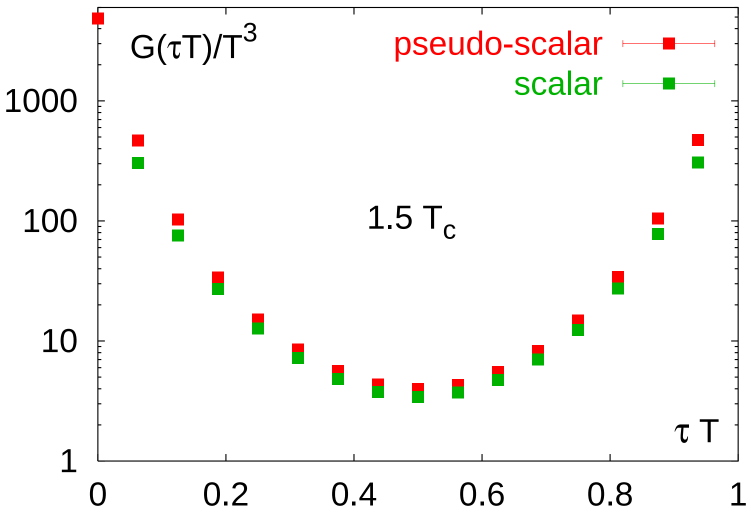


$U_A(1)$





chiral symmetry restoration $\Leftrightarrow$
degeneracy of correlation functions

effective $U_A(1)$ restoration

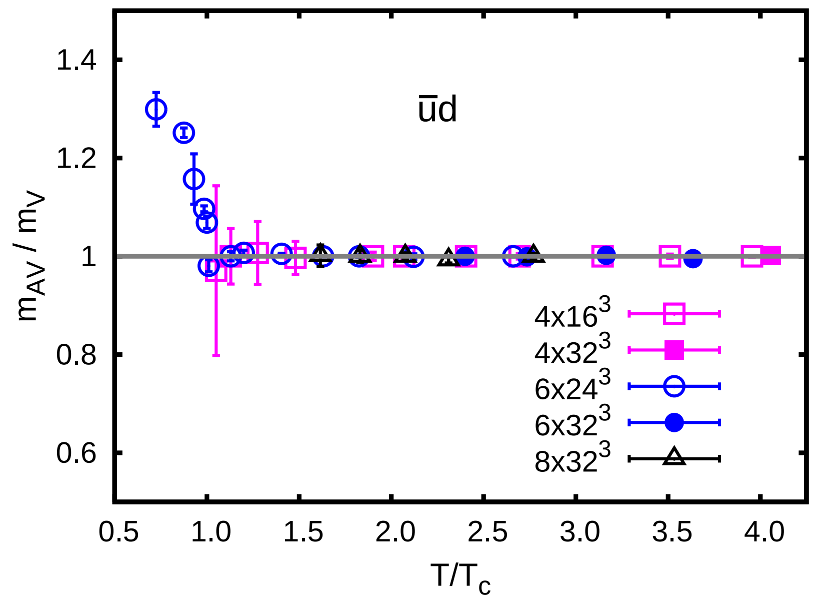
$$m_\delta(T) \;\rightarrow\; m_\pi(T)$$

Meson Spectrum and Chiral Symmetry Restoration

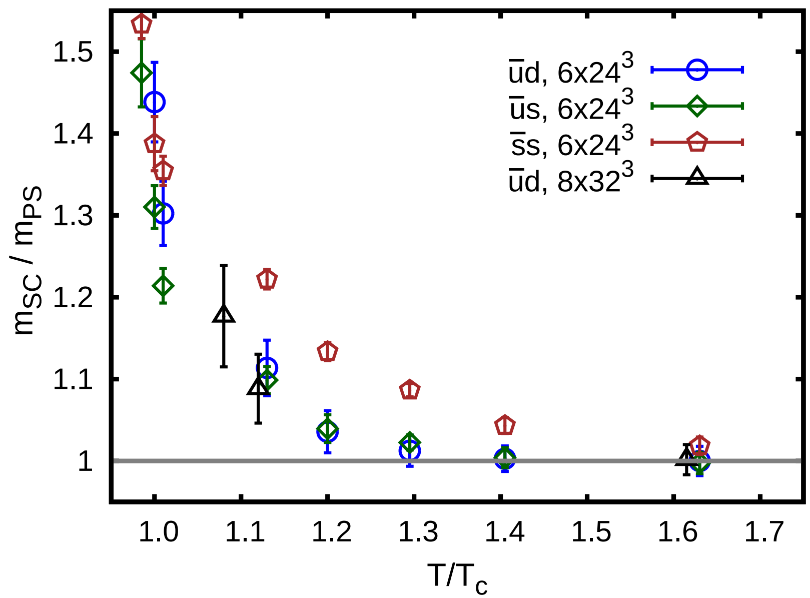
calculations with $\mathcal{O}(a^2)$ improved staggered fermions (p4-action):

screening masses: $G_H(z) \sim \mathrm{e}^{-m_H z}$

$SU(2)_L \times SU(2)_R$ restoration



$U(1)_A$ "effective restoration"



$m_{SC} \neq m_{PS} \;\Leftrightarrow\; U(1)_A$ not restored at T_c for chiral symmetry restoration

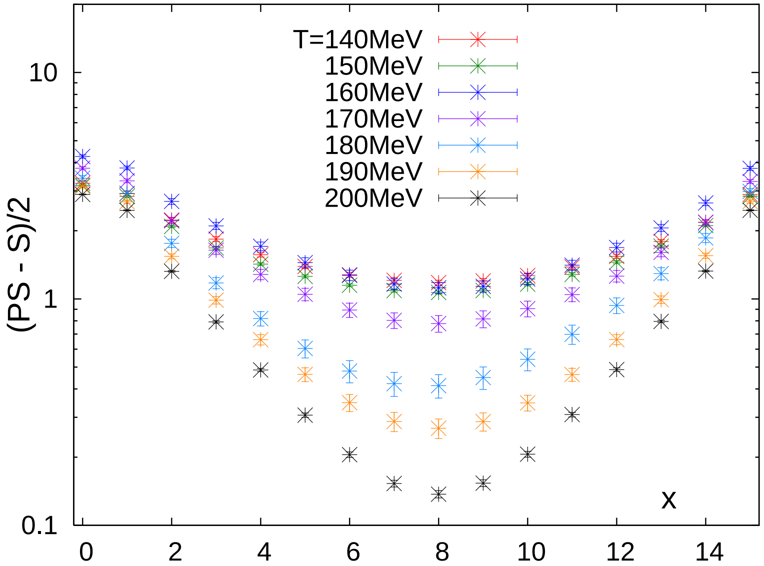
M. Cheng et al., Eur. Phys. J. C71, 1564 (2011)

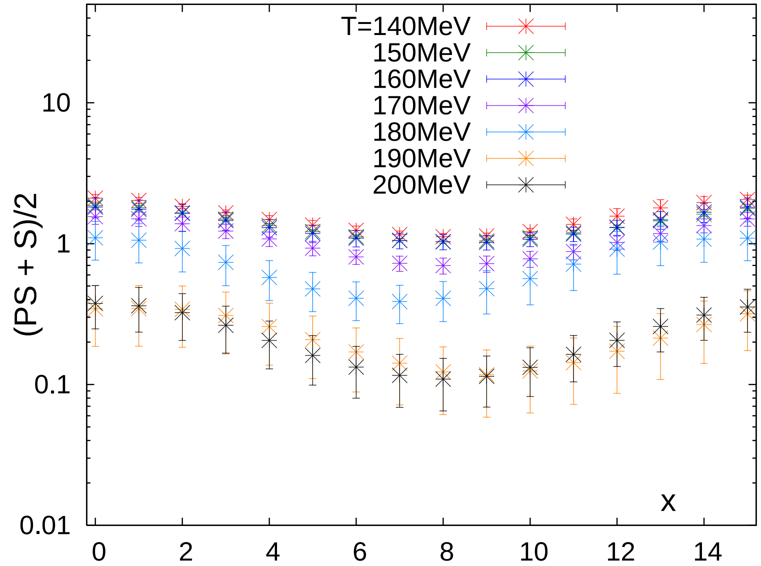
The flavor non-singlet correlation functions pseudo-scalar (π) versus scalar (δ)

- What generates the differences between $G_\delta(x,T)$ and $G_\pi(x,T)$ in the $SU(2)_L \times SU(2)_R$ symmetric phase?

$$G_{\delta(\pi)}(x) = \langle \bar{u}_L d_R(x) \bar{d}_L u_R(0) + \bar{u}_R d_L(x) \bar{d}_R u_L(0) \rangle$$
$$\pm \langle \bar{u}_L d_R(x) \bar{d}_R u_L(0) + \bar{u}_R d_L(x) \bar{d}_L u_R(0) \rangle$$

under $U(1)_A$ transformation terms are variant / invariant

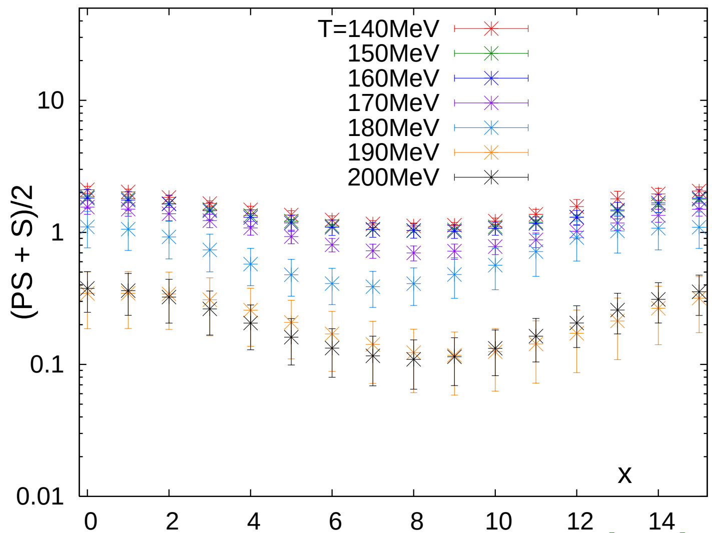




DWF calculation; HotQCD preliminary

The flavor non-singlet correlation functions pseudo-scalar (π) versus scalar (δ)

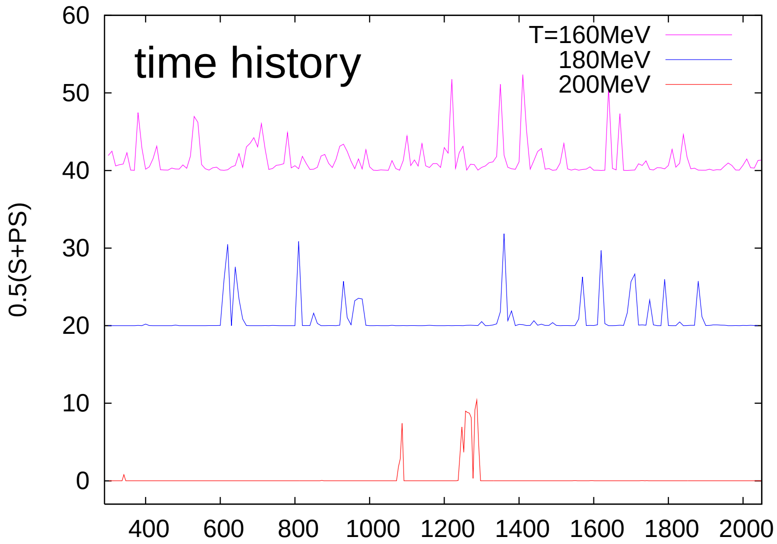
$U(1)_A$ variant contribution: $\Delta(x) = (G_\pi(x) + G_\delta(x))/2T^3$

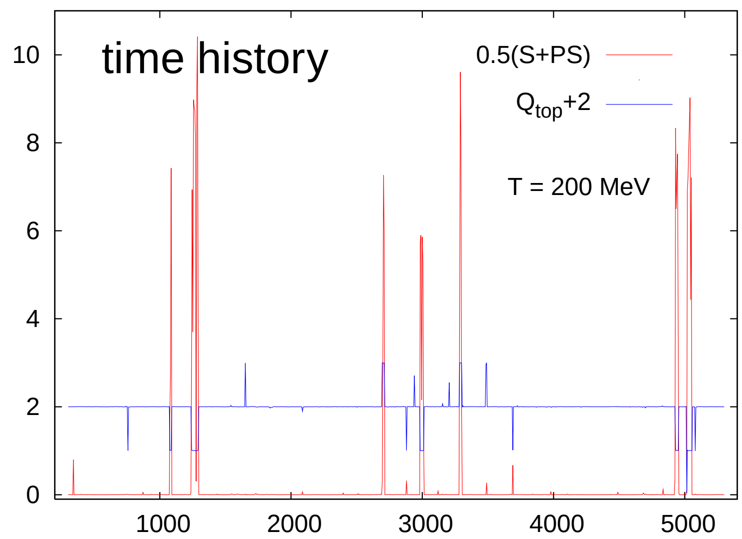


$\Delta(x) \neq 0$ 'only' on configurations

with non-trivial topology: $|Q_{top}| \neq 0$

DWF calculation; HotQCD preliminary





Eigenvalue spectrum of the fermion matrix

chiral condensate

$$\langle\bar{\psi}\psi\rangle_l = \frac{n_f}{4}\frac{1}{N_\sigma^3 N_\tau}\mathrm{Tr}\langle M_l^{-1}\rangle = \frac{n_f}{4}\frac{1}{N_\sigma^3 N_\tau}\sum_j \frac{1}{m_l + i\lambda_j}$$

$$= \int \mathrm{d}\lambda\, \rho_V(\lambda)\frac{2m_l}{m_l^2+\lambda^2}$$

chiral limit: $\langle\bar{\psi}\psi\rangle_l = \pi \lim_{m_l\to 0}\lim_{V\to\infty}\rho_V(0) \equiv \pi\rho(0)$

(Banks-Casher relation)

$$\Delta_{\pi-\delta} \equiv (\chi_\pi - \chi_\delta)/T^2 = \int \mathrm{d}\lambda\, \rho_V(\lambda)\frac{4m_l^2}{(m_l^2+\lambda^2)^2}$$

$U(1)_A$ remains broken, if $\rho(\lambda) \sim \lambda$ (problematic) or $\rho(\lambda) \sim m^2\delta(\lambda)$...??

$U(1)_A$ restored, if (for instance) $\rho(\lambda)$ has a gap or $\rho(\lambda) \sim \lambda^a,\ a > 1$

Eigenvalue spectrum of the fermion matrix

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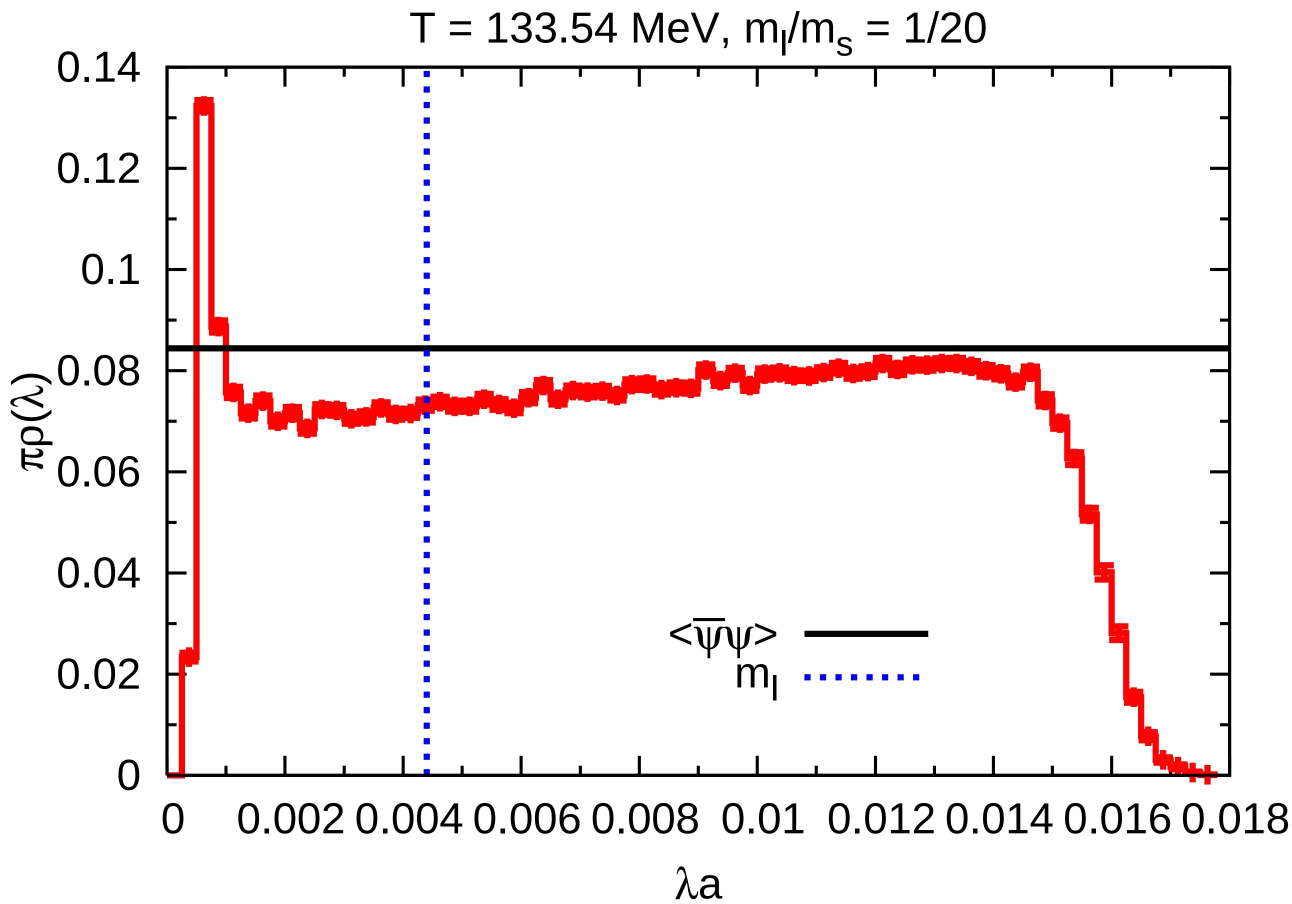

1^{st} 100 eigen-values

Eigenvalue spectrum of the fermion matrix

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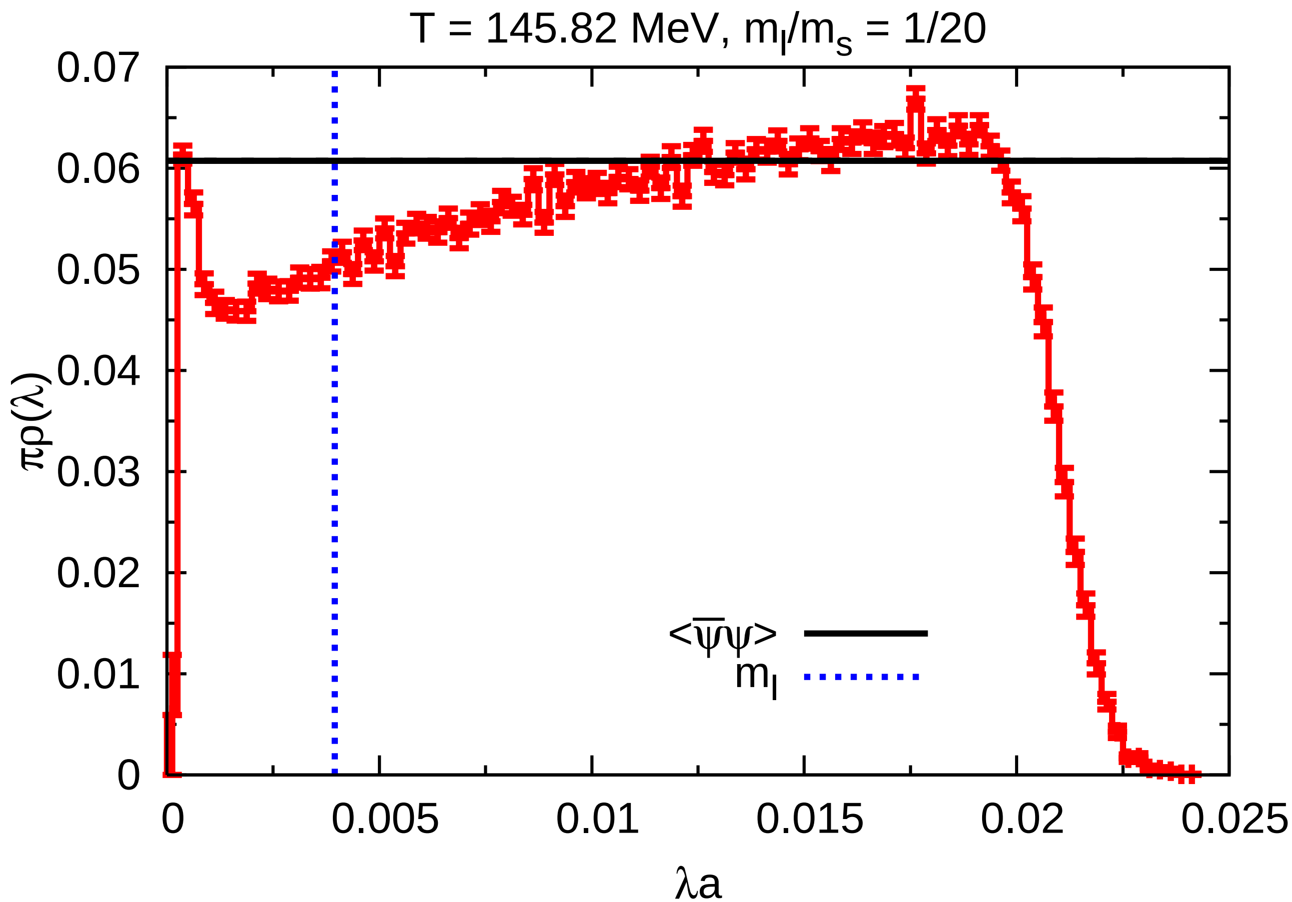

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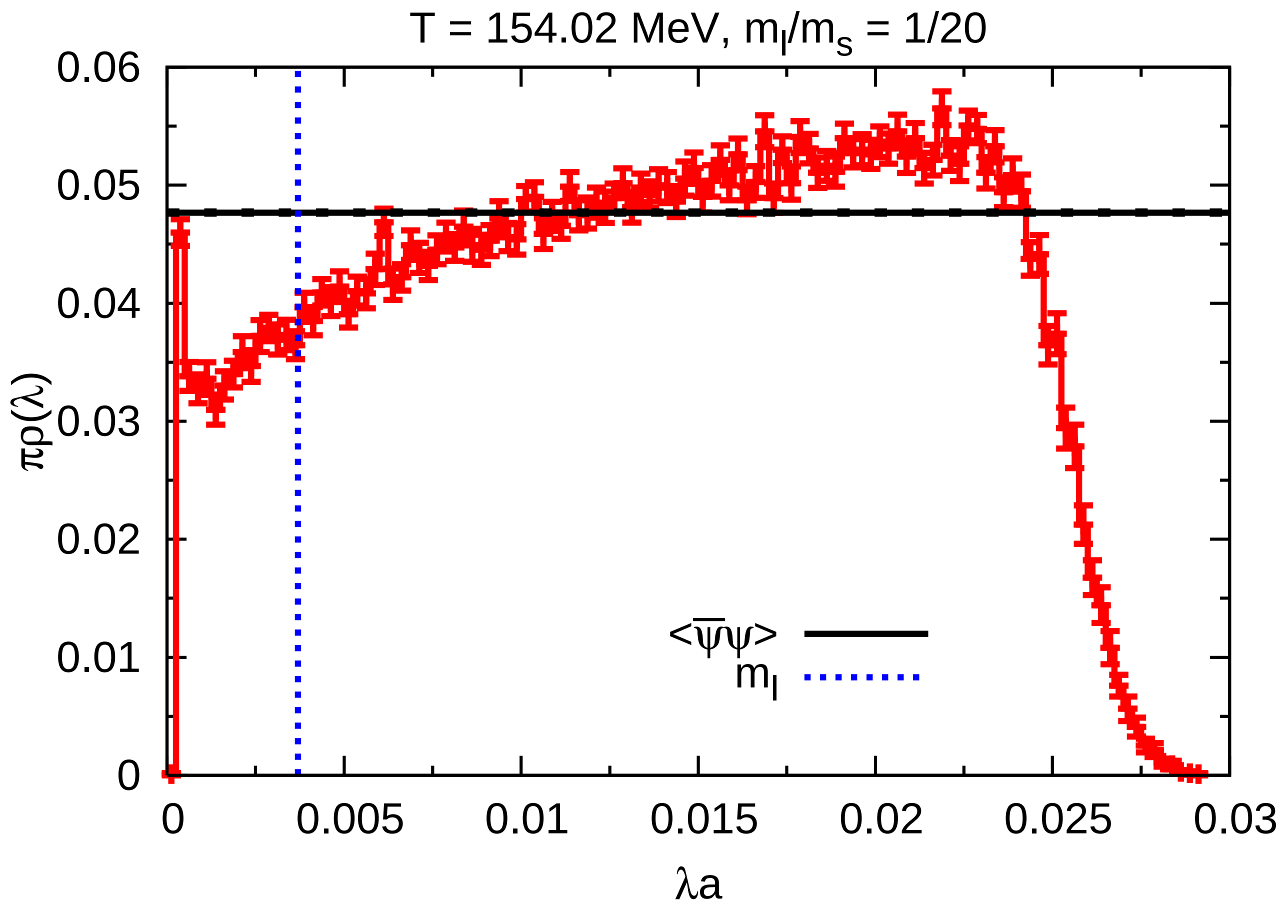

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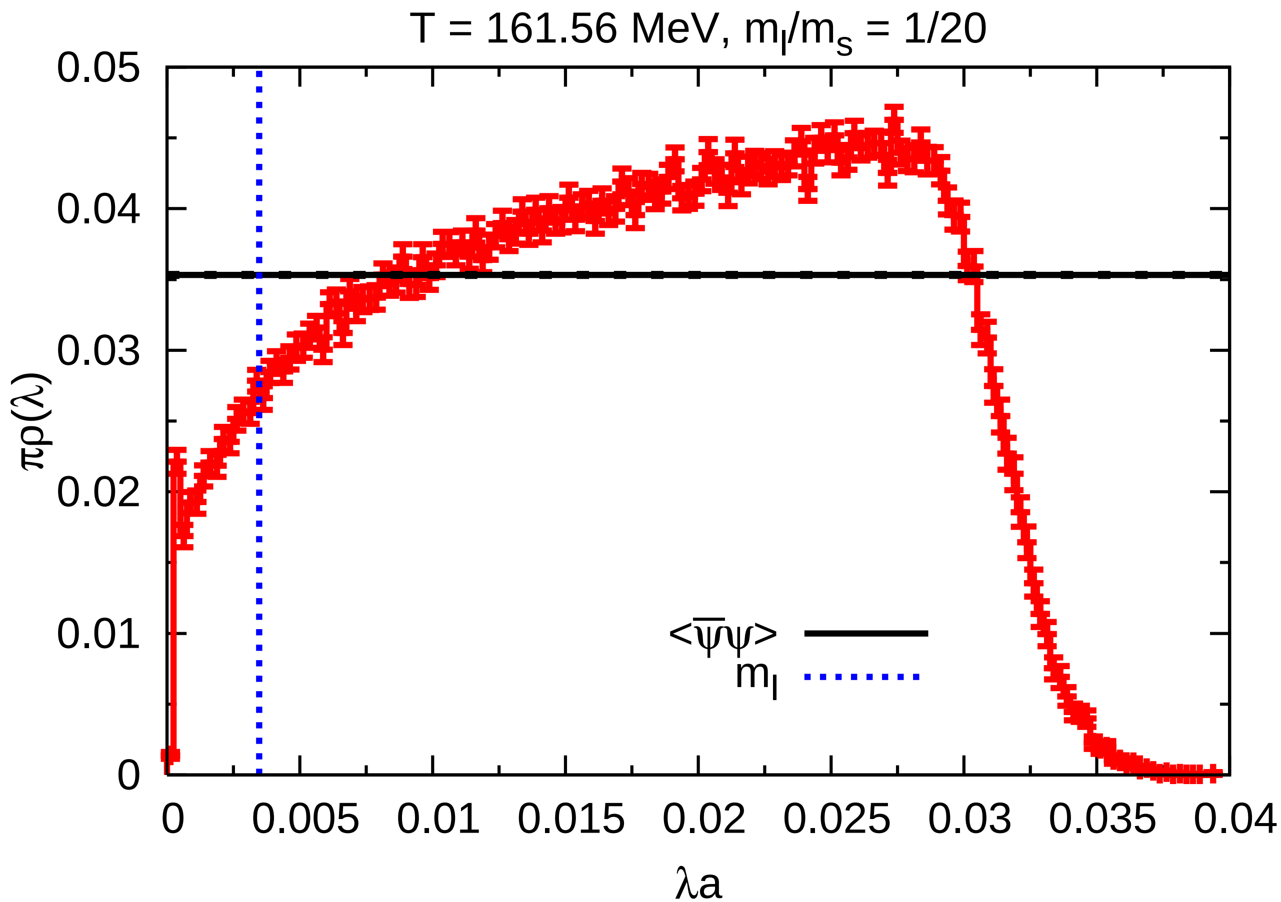

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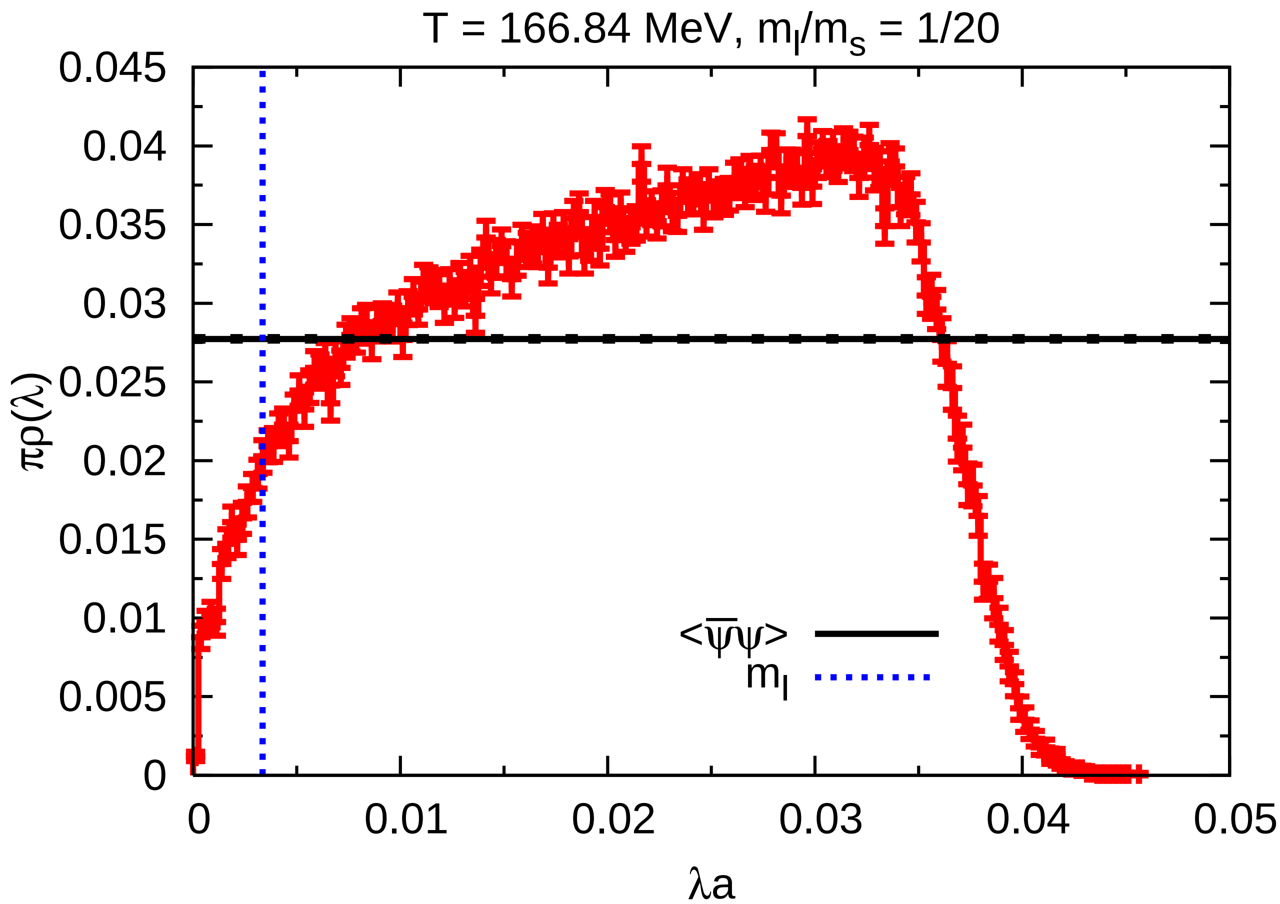

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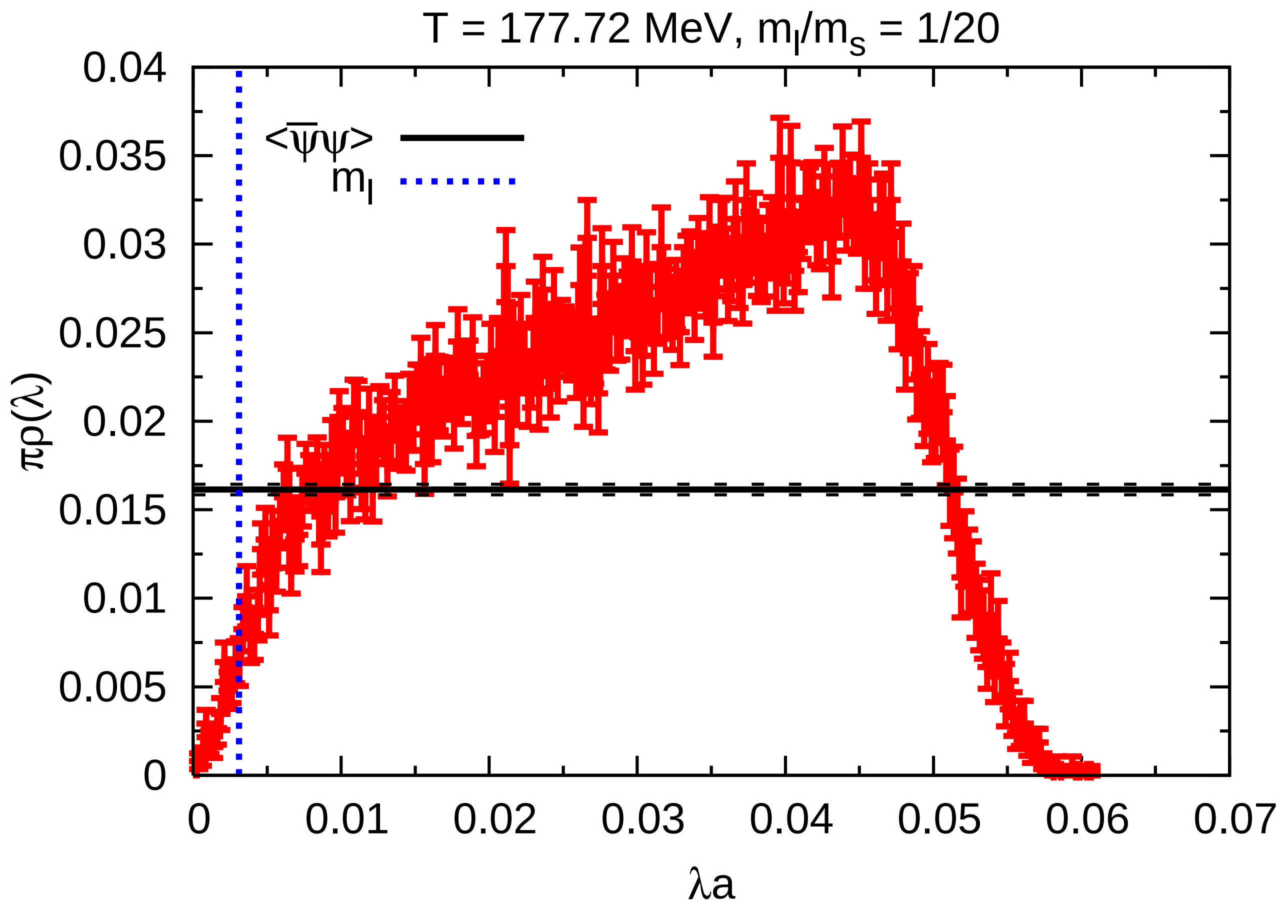

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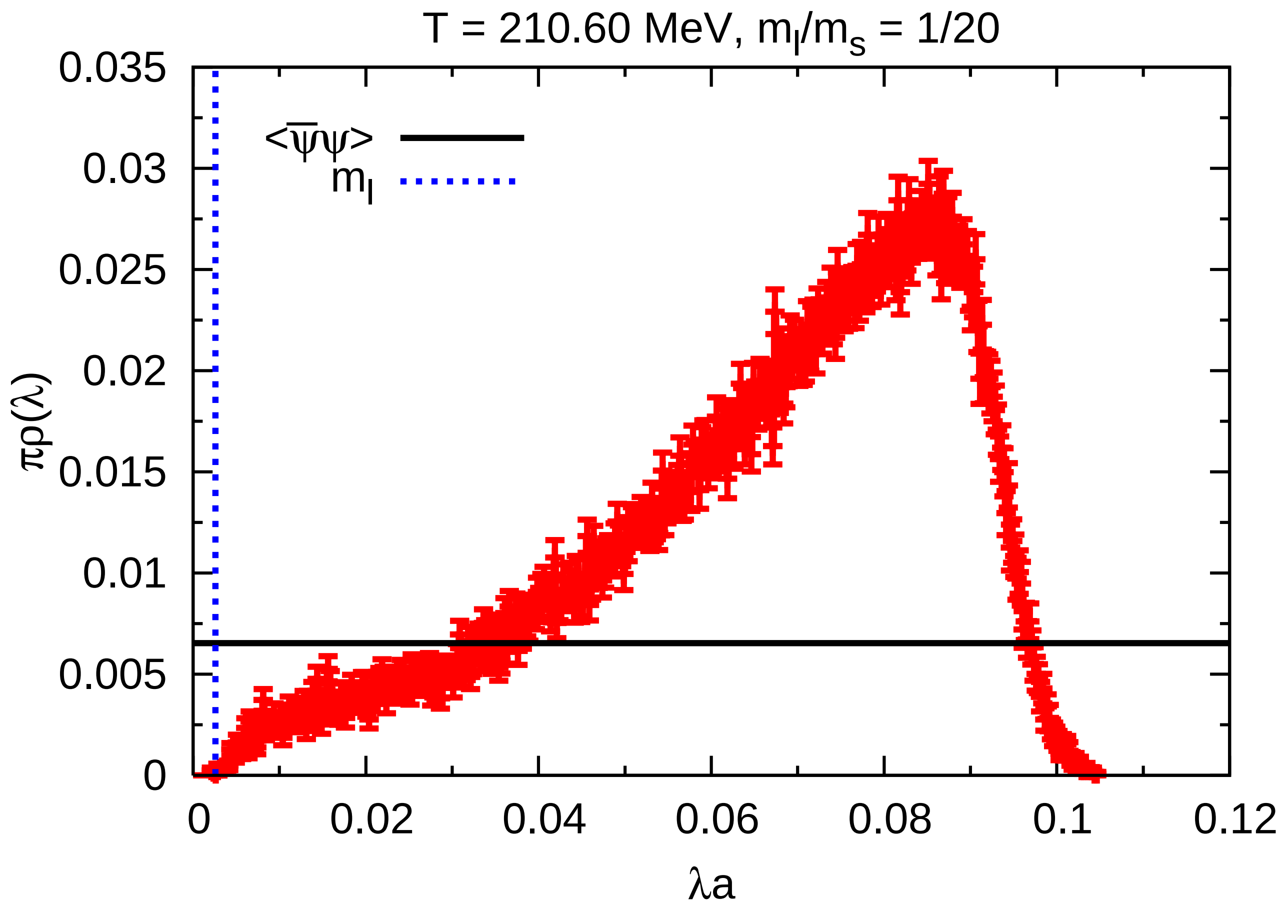

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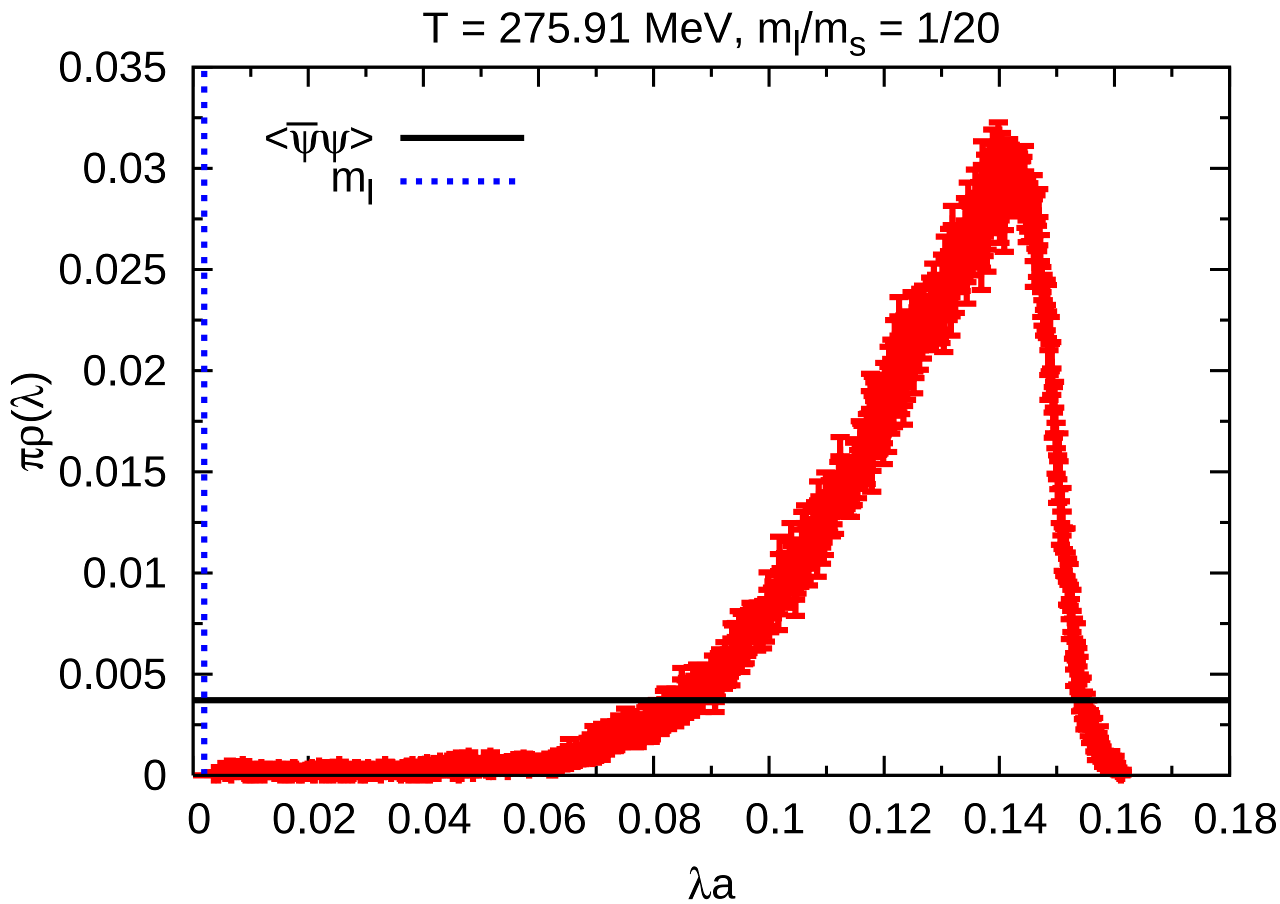

Summary: T-dependence of correlation functions

- analysis of correlation functions with meson quantum numbers gives insight in the T-dependence of $SU(2)_L \times SU(2)_R$ as well as $U_A(1)$ symmetry breaking
- current studies suggest that a 'significant' $U_A(1)$ symmetry breaking persists at the time of $SU(2)_L \times SU(2)_R$ restoration
- HOWEVER: The $V \to \infty$, $m \to 0$ limits in the $U_A(1)$ sector are subtle. No final conclusion on the effective $U_A(1)$ restoration can be drawn at present; studies with chiral fermion formulations may be crucial....