Summary lec 4

- Discussed theoretical formulation of $\mathcal{Q} = 4$ SYM in 2D and $\mathcal{Q} = 16$ SYM in 4D
- Outstanding theoretical issue: lattice actions preserve 1 SUSY what fine tuning needed for restoration of full SUSY in continuum limit ?
  - Info from (lattice) p theory - renormalization ..
  - Non-perturbative checks
- Simulations:
  - Pfaffian complex – sign/phase problem ?
  - Scalar infrared divergences ?

Quantum corrections

- Lattice theory we have described certainly yields $\mathcal{N} = 4$ in *naive* continuum limit
- But what about quantum corrections ? 15 SUSYs broken at O(a). Naively they are restored $a \to 0$. Seen that loop effects can change this ....
- What are possible dangerous counter terms that can arise ?
- Lattice symmetries tell us what may happen *in principle*. Here:
  - Gauge invariance
  - $\mathcal{Q}$-symmetry.
  - Point group symmetry - eg. S^5 for A_4^*
  - Exact fermionic shift symmetry $\eta \to \eta + \epsilon I$

Analysis of counter terms

- Care only about *relevant* counter terms – use dimensional analysis to rank terms $[\Psi] = 3/2$, $[\mathcal{U}] = 1$. Only care about $[O] \leq 4$
- $\mathcal{Q}$ symmetry implies all terns are $\mathcal{Q}$-exact.
- So $O \sim \mathcal{Q}\left(\Psi\mathcal{U}\mathcal{U}\right)$ ($[\mathcal{Q}] = 1/2$) (no $\mathcal{Q}(\Psi\mathcal{U})$ ops)
- O must correspond to (trace of) closed loop for G.I
- S^5 symmetry – operator should be invariant under permutation of indices.

Find:

$$S = \sum \mathrm{Tr}\left[\alpha_1 \chi_{ab}\mathcal{F}_{ab} + \alpha_2 \eta \overline{\mathcal{D}}_a^{(-)}\mathcal{U}_a - \frac{1}{2}\alpha_3 \eta d - \alpha_4 S_{\mathrm{closed}}\right]$$

Observations

- Only marginal ops *already in classical action* possible.
- Spreading fermions over links has a bonus: generic fermion bilinears not allowed by gauge invariance - fermions remain massless.
- Log fine tuning needed if $\alpha_1 \neq \alpha_2 \neq \alpha_3 \neq \alpha_4$
- S^5 PGS guarantees twisted $SO(4)'$ restored as $a \to 0$. Any relevant SO(4) breaking counter term would break S^5.
- Absence of scalar masses can be confirmed by computing effective potential $\Gamma_{\rm eff}(\mathcal{U}^c)$

Sketch of 1-loop $\Gamma_{\rm eff} = 0$ - bosons

- Classical vacua
  constant commuting complex matrices $\mathcal{U}_\mu^{\rm classical}$
- Expand to quadratic order $\mathcal{U}_\mu(x) = I + \mathcal{A}_\mu^{\rm classical} + a_\mu(x)$.
- Fix gauge using function $\overline{\mathcal{D}}_\mu^{(-)} a_\mu + h.c = 0$. Choose Feynman parameter so that bosonic action looks like

$$S_B = \overline{a}_\nu \overline{\mathcal{D}}_\mu^{(-)} \mathcal{D}_\mu^{(+)} a_\nu$$

- Integrate - results in factor $\det^{-5}\left(\overline{\mathcal{D}}_\mu^{(-)} \mathcal{D}_\mu^{(+)}\right)$
- Here $\mathcal{D}$ dnotes covariant difference operator in background $\mathcal{A}^{\rm classical}$

Fermionic contribution

- Ghosts: Faddev-Popov term is just

$$\det\left(\overline{\mathcal{D}}_{\mu}^{(-)}\mathcal{D}_{\mu}^{(+)}\right)$$

- Fermion operator takes the same form as in bare action with the rule that all discrete covariant difference ops are too be taken in the background field.
- One can also show (use fact that all covariant difference ops in general background commute)

$$\left(Pf(M_F)\right) \stackrel{Maple}{=} \det{}^4\left(\left(\overline{\mathcal{D}}_{\mu}^{(-)}\mathcal{D}_{\mu}^{(+)}\right)\right)$$

- Thus $Z_{\rm pbc} = 1$ and $\Gamma[A^{\rm classical}] = 0$ at 1-loop.

Consequences

- Actually this result remains true to all orders! Since $S \sim \beta \mathcal{Q}\Lambda$ can show that $\frac{\partial \ln Z}{\partial \beta} =< \mathcal{Q}\Lambda >= 0$ and hence Z has no dependence on β. Can be computed exactly as $\beta \to \infty$ - semiclassical limit (1-loop)
- Thus classical moduli space not lifted due to quantum corrections! So scalars naturally massless in lattice theory - exact SUSY indeed protects theory and reduces fine tuning.

Going further

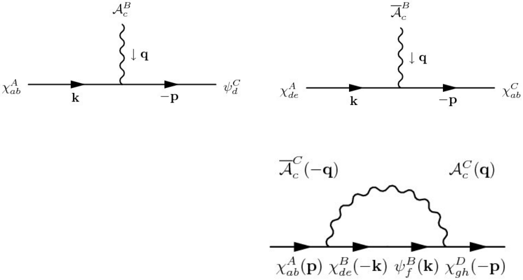
- Thus we learn on the basis of lattice symmetries and the topological nature of $\mathcal{Q}$ that quantum corrections can *at most* log shift the coefficients of the separate $\mathcal{Q}$ invariant kinetic terms that make up the classical action.
- In principle restoration of additional 15 susys can depend on differential flows in these wave function renormalizations due to quantum effects.
- To proceed further we must do an explicit 1-loop calc using lattice p theory

Ingredients for perturbation theory

Lattice rules for A_4^* lattice (Feynman gauge):

- Boson propagator $<\overline{\mathcal{A}}_a^C(k)\mathcal{A}_b^D(-k)>=\frac{1}{\hat{k}^2}\delta_{ab}\delta^{CD}$ with $\hat{k}^2=4\sum_a\sin^2(k_a/2)$
- Fermion propagator $M_{\rm KD}^{-1}(k)=\frac{1}{\hat{k}^2}M_{\rm KD}(k)$ with $M(k)$ a 16×16 block matrix acting on (η,ψ_a,χ_{ab})
- Vertices: $\psi\eta$, $\psi\chi$ and $\chi\chi$.
- Only 4 Feynman graphs needed to find 1-loop contributions to fermion self-energies (determines 3 out of 4 coeffs)
- One additional bosonic propagator for remaining coeff.

Example: chi-chi propagator



Compute all amputated fermion self energies. Final result:

- Fermion self-energies vanish for $p \to 0$. Zero mass.
- $\Sigma_i^{(2)} = 1 + Ag^2 \ln \mu a$. Single A for all fermion Σ_i.

Why so simple ?

- The lattice diagrams are just log divergent - like their continuum counterparts.
- Furthermore, these divergences are *the same as the continuum theory* since they originate in regions of k space where the lattice vertices and propagators approach their continuum counterparts.
- But in the continuum theory the coefficients of the different self -energies must be same - since that theory has full supersymmetry which would be violated if the different twisted components picked up different corrections.
- Remember that twisting (in the continuum) is just a change of variables....
- Conclusion: the lattice theory must also (at 1 loop) inherit this structure and hence also require no fine tuning to approach a continuum limit with full susy.

Summary

- Perturbative studies indicate that scalars remain massless to all orders in g
- Potential log tuning needed to handle wave function renormalization but at 1-loop even this is not present. So no tuning needed aty weak coupling
- Actually same args indicate that $\beta(g) = 0$ at 1-loop !in lattice theory!
- To understand situation for strong coupling need simulations. Measure Ward identities corresponding to broken SUSYs as $\beta \to \infty$. Are they restored ? If not can I tune bare α_i by hand to zero them out ?

Numerical Implementation: problems ...

1. After integration over the twisted fermions we find a Pfaffian $\mathrm{Pf}(M(\mathcal{U}))$. In general this is *complex*. Monte Carlo requires a positive definite weight. Thus we perform simulations using $(M^\dagger M)^{\frac{1}{4}}$. Fold phase into observables using reweighting. If fluctuations in phase large - fails - famous sign problem.
2. Non-trivial moduli space: (scalar) fields can run off to infinite values as long as $[B_\mu, B_\nu] = 0$. Survives quantum corrections. Stability of simulations ?

Simulations

- RHMC algorithm (as for lattice QCD)
- Need C++ objects for twisted fermions, (complexified) gauge fields.
- A_4^* can be deformed to hypercubic lattice plus body diagonal - index fields on latter (simple)
- Can parallelize with e.g. MPI or use GPUs

Scalars

- Simulate $U(N)$ theories. Observe that trace mode of scalars is indeed unstable. Regulate using mass term

$$\Delta S = \mu^2 \sum \left[\frac{1}{N} \mathrm{Tr} \left(U_a^\dagger U_a \right) - 1 \right]^2$$

  As $a \to 0$ $U^\dagger U \to e^{2B} \sim I + 2B$ so gives mass to trace mode only

- Since sector decouples susy is naively ok as $a \to 0$ ...
- Exact 0 mode of fermions removed by using apbc.
- Test in 2D. Looked at $\mathcal{Q} = 4$ and $\mathcal{Q} = 16$ theories

Scalar eigenvalues vs lattice spacing

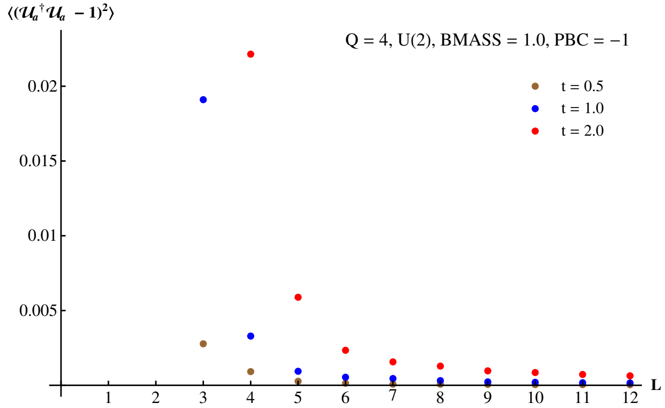


Figure: $\mathcal{Q} = 4$ model $\beta = 1$, $U(2)$

Sending $L \to \infty$ holding $t = g^2\beta^2$ fixed. See $aB \to 0$ as required.

Pfaffian phase vs lattice spacing

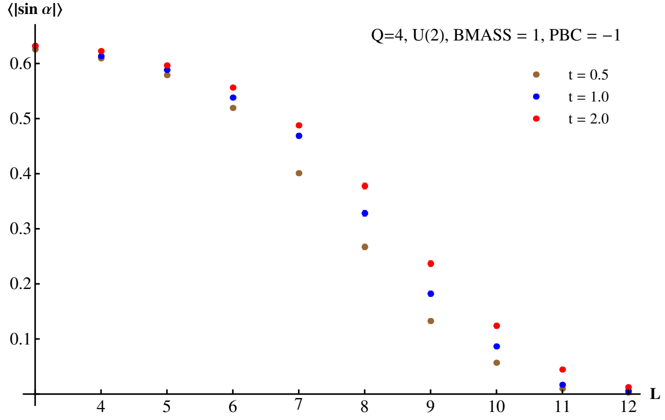


Figure: $\mathcal{Q} = 4$ model $\beta = 1,\ U(2)$

Phase fluctuations go to zero in continuum limit

$\mathcal{Q}$-supersymmetry

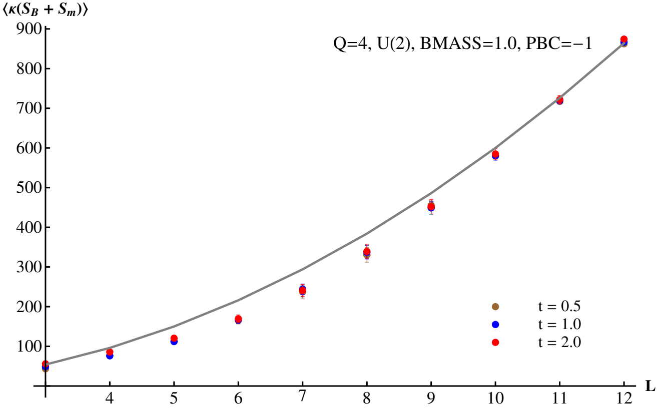


Figure: $\mathcal{Q} = 4$ model $\beta = 1.0$, $U(2)$

Exact $\mathcal{Q}$ Ward identity. SUSY OK as $a \to 0$

Gauge-gravity dualities

Original AdS/CFT correspondence:

Quantum $\mathcal{N} = 4$ YM dual Semiclassical strings +D3-branes

In practice most tests/applications: YM taken at large (N, λ) - classical solutions SUGRA in AdS_5

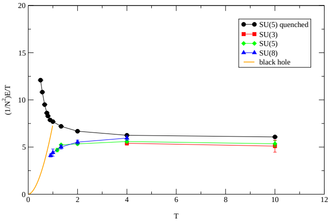
Many other examples eg. Low temperature thermodyamics of dimensionally reduced theory

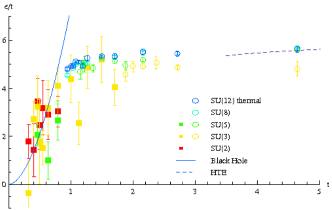
$\mathcal{N} = 4$ SYM in $D = (p + 1)$ and Semiclassical black Dp-branes

Explore using lattice actions ...

Black holes from YM

$p = 0$ case: black holes in type II SUGRA – dual to large N low T $\mathcal{N} = 4$ on circle (with T .Wiseman, Imperial)





Energy vs temperature for SYMQM system+BH prediction using semiclassical Bekenstein-Hawking.

Single deconfined phase.

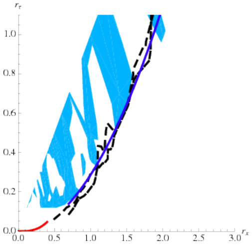
Black Strings

$p = 1$ case: black string in type II SUGRA – dual to large N $\mathcal{N} = 4$ on 2D torus (sizes r_x and r_τ)

- Depending on r_x, r_τ black string solution may become less stable than black hole. Supergravity analysis predicts $r_\tau < cr_x^2,\ r_x, r_\tau \to \infty$ (c unknown)
  Gregory-LaFlamme transition in gravity
- In dual gauge theory see thermal phase transition associated with breaking of center symmetry - order parameter spatial Polyakov line.

work with A. Joseph and T. Wiseman

Black hole-black string phase transition



Boundary between confined/deconfined phases corresponds to $\frac{1}{N}|P_s| = 0.5$

Good agreement with supergravity - blue curve - $r_\tau = c r_x^2$ with fitted $\boxed{c \sim 3.5}$.

Good agreement with high T dim reduction - red curve

Conclusions

- Lattice SUSY is a fascinating field with potential to play a role both in LHC physics and string theory.
- In some cases a fraction of SUSY can be preserved on lattice - using discretizations of topologically twisted theories.
- Renormalization of these theories strongly constrained by exact lattice symmetries including exact SUSY.
- Simulation of these theories feasible at strong coupling using same tools as for lattice QCD.
- New lattice theories may offer new insight into gauge-gravity dualities and problems in quantum gravity.

References

See these recent reviews and references therein.

- Exact lattice supersymmetry, S. Catterall, D. B. Kaplan and M . Ünsal, Phys. Rep 2009 (arXiv:0903:4881)
- J. Giedt, Int. J. Mod. Phys.A21:3039-3094,2006.
- Very many others ... (Damgaard, Sugino, d'Adda, Kawamoto, Matsuura,Wipf,...) More than 50 papers in last few years
- Didn't talk about: sigma models, theories with fundamentals, susy breaking