

# CHIRAL QUARK MODELS OF HADRONIC MATTER (IV)

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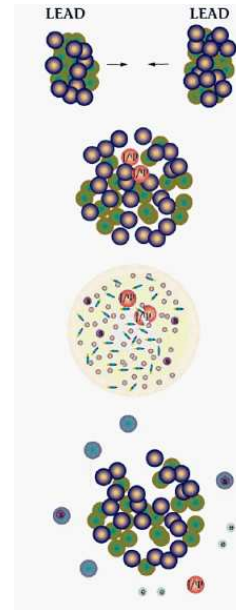
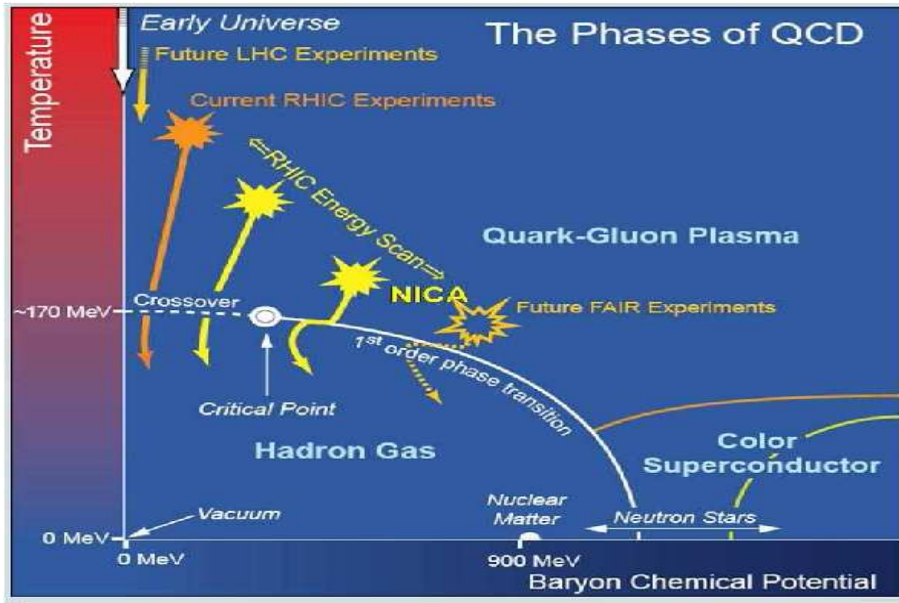
## Contents:

- Particle production in HIC: statistical model and freeze-out in the PhD
- The idea: self-induced hadron delocalization! Mott-Anderson freeze-out!
- Chiral condensate beyond meanfield and freeze-out curve
- Mott-Hagedorn resonance gas: Beth-Uhlenbeck EoS and phase diagram

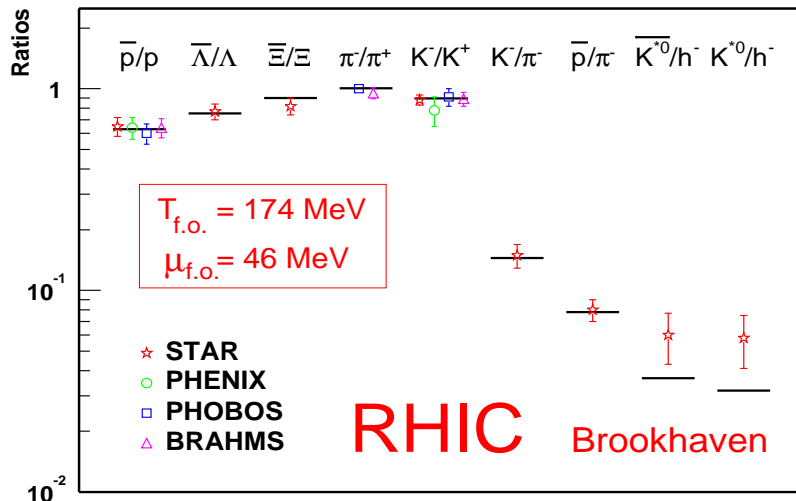
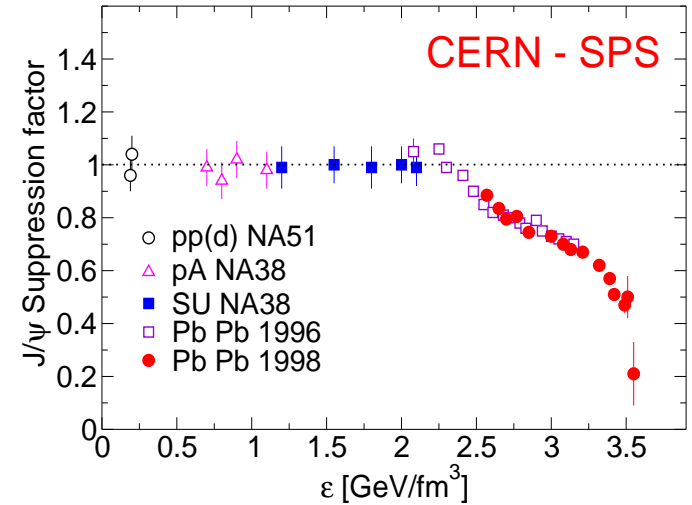


Lattice QCD, Hadron Structure and Hadronic Matter, Dubna, 05.-16.09.2011

# PHASEDIAGRAM: FREEZE-OUT IN HEAVY-ION COLLISIONS



QGP Signal: Anomalous  $J/\psi$  suppression



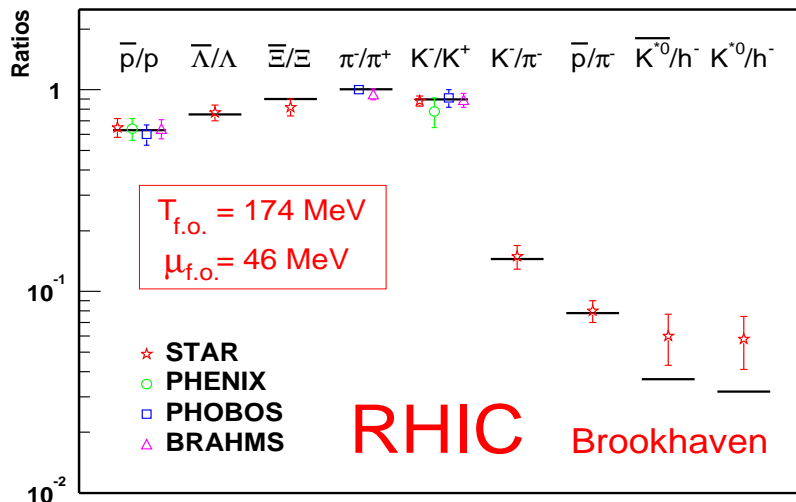
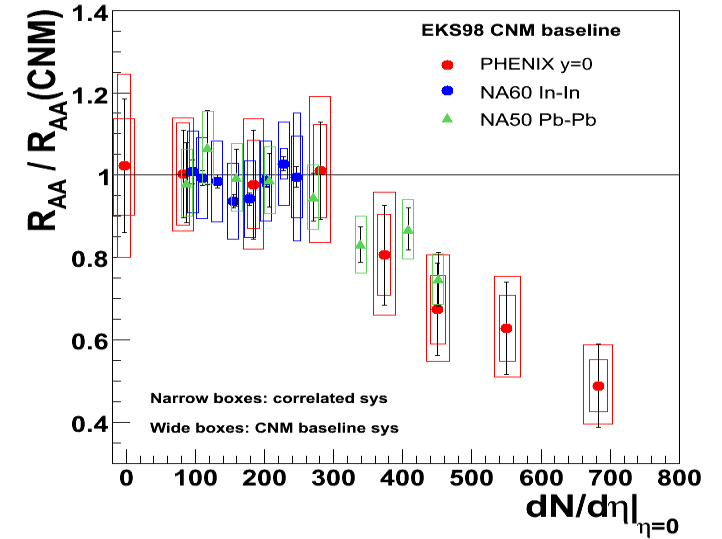
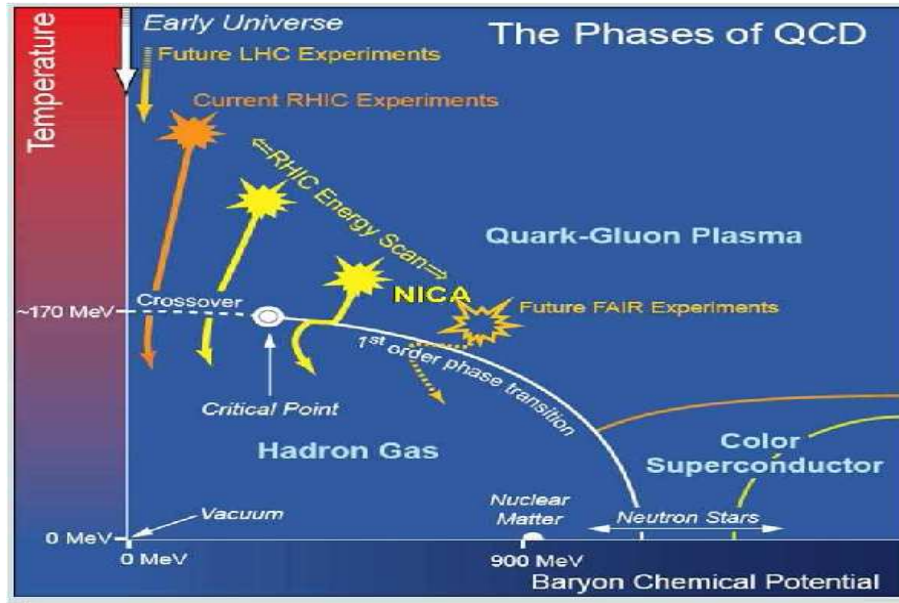
Statistical model describes composition of hadron yields in Heavy-Ion Collisions with few freeze-out parameters.

$$\ln Z[T, V, \{\mu\}] = \pm V \sum_i \frac{g_i}{2\pi^2} \int_0^\infty dp p^2 \ln[1 \pm \lambda_i \exp(-\beta \epsilon_i(p))]$$

$$\lambda_i(T, \{\mu\}) = \exp[\beta(\mu_B B_i + \mu_S S_i + \mu_Q Q_i)]$$

Braun-Munzinger, Redlich, Stachel, in *QGP III* (2003)

# PHASEDIAGRAM: FREEZE-OUT IN HEAVY-ION COLLISIONS



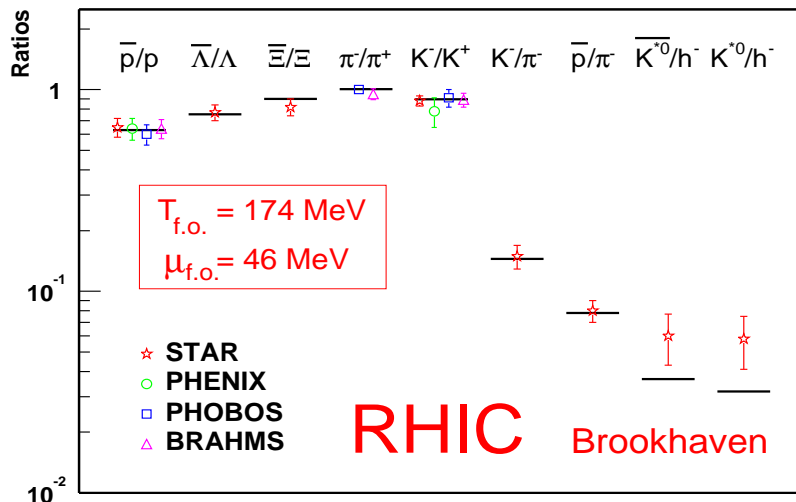
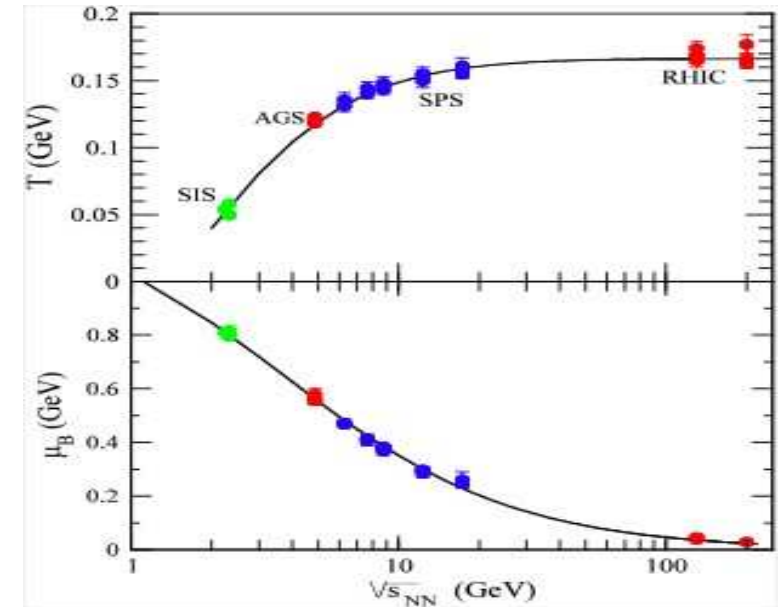
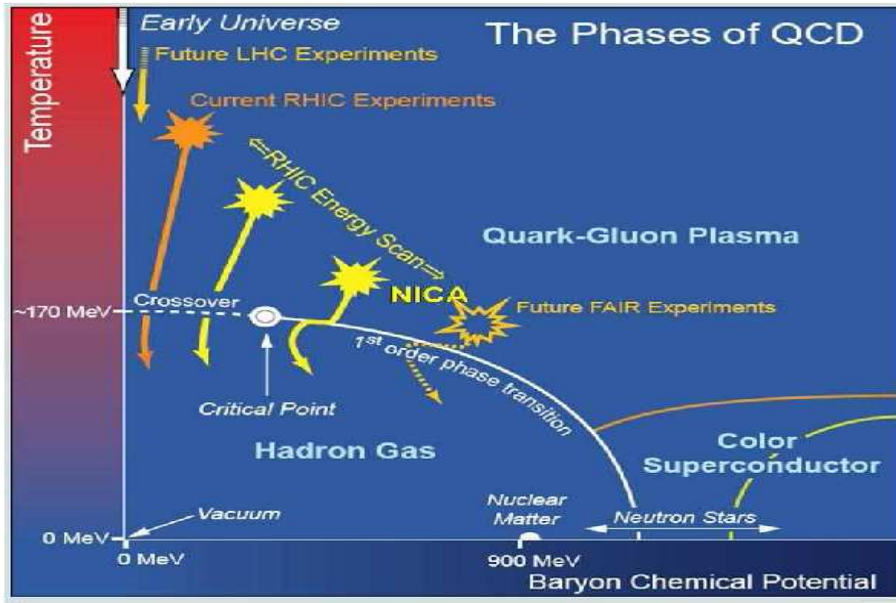
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Braun-Munzinger, Redlich, Stachel, in *QGP III* (2003)

# PHASEDIAGRAM: FREEZE-OUT IN HEAVY-ION COLLISIONS (II)



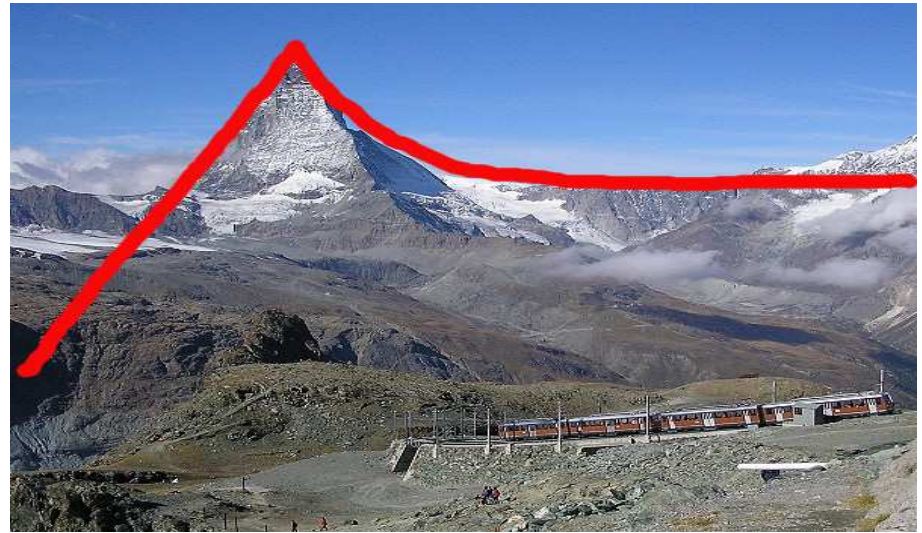
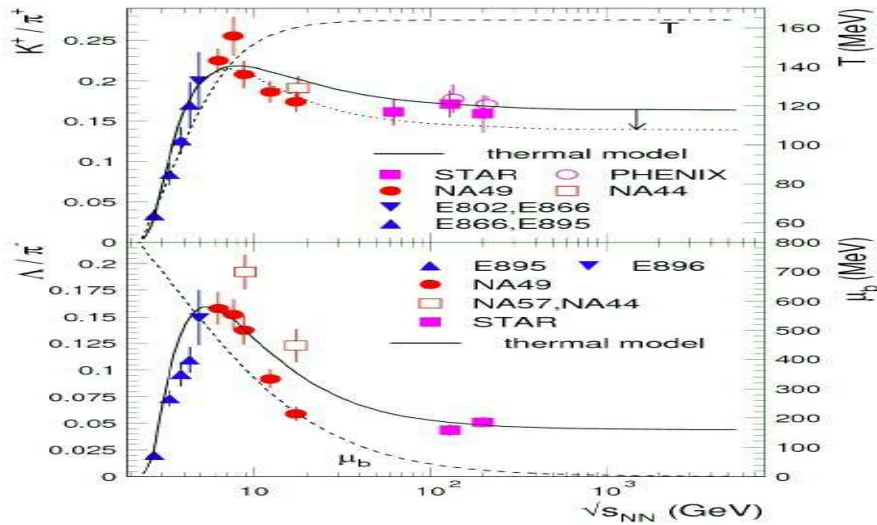
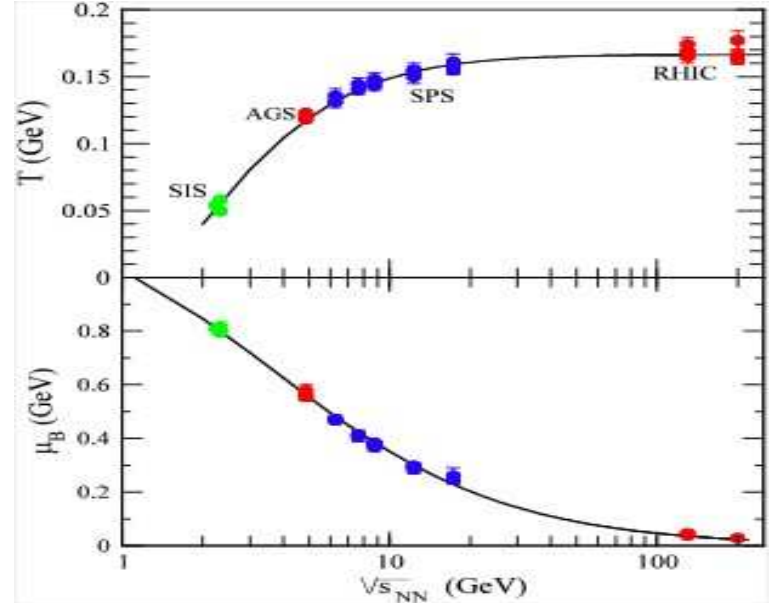
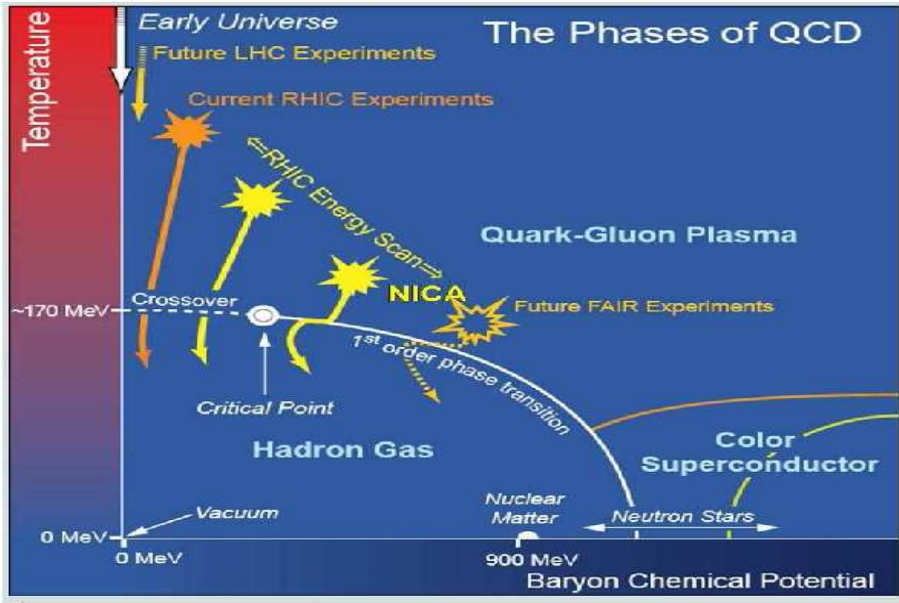
**Statistical model** describes composition of hadron yields in Heavy-Ion Collisions with few **freeze-out** parameters.

$$\ln Z[T, V, \{\mu\}] = \pm V \sum_i \frac{g_i}{2\pi^2} \int_0^\infty dp p^2 \ln[1 \pm \lambda_i \exp(-\beta \varepsilon_i(p))]$$

$$\lambda_i(T, \{\mu\}) = \exp[\beta(\mu_B B_i + \mu_S S_i + \mu_Q Q_i)]$$

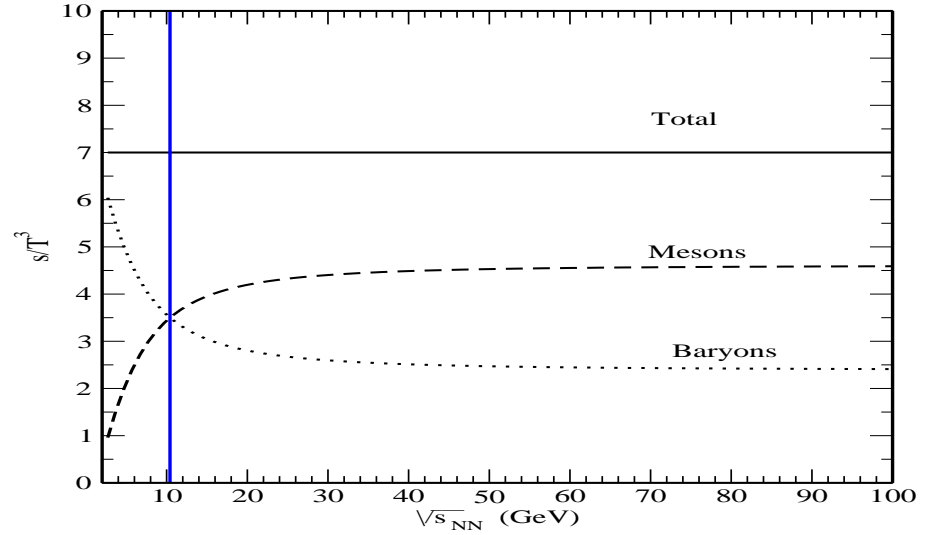
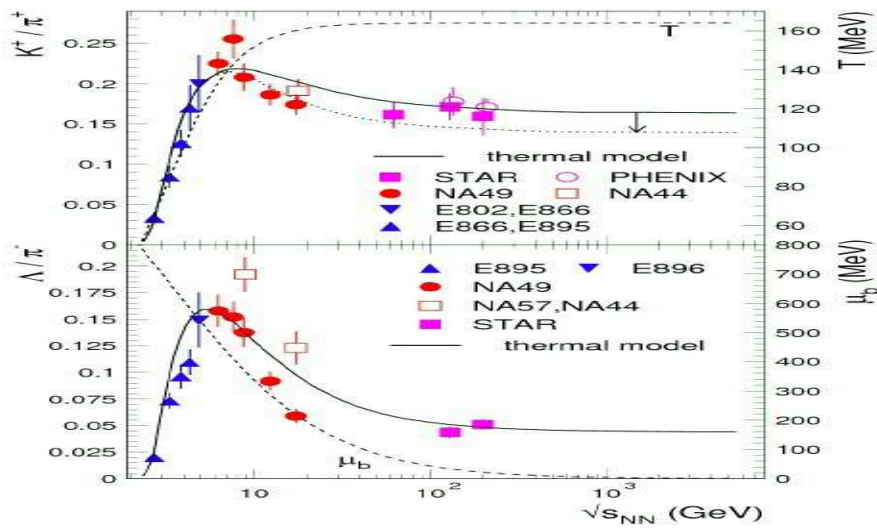
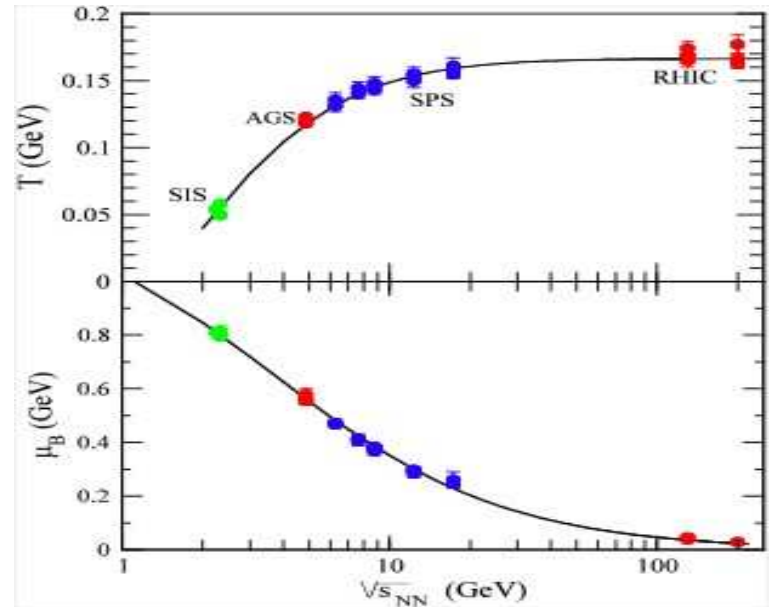
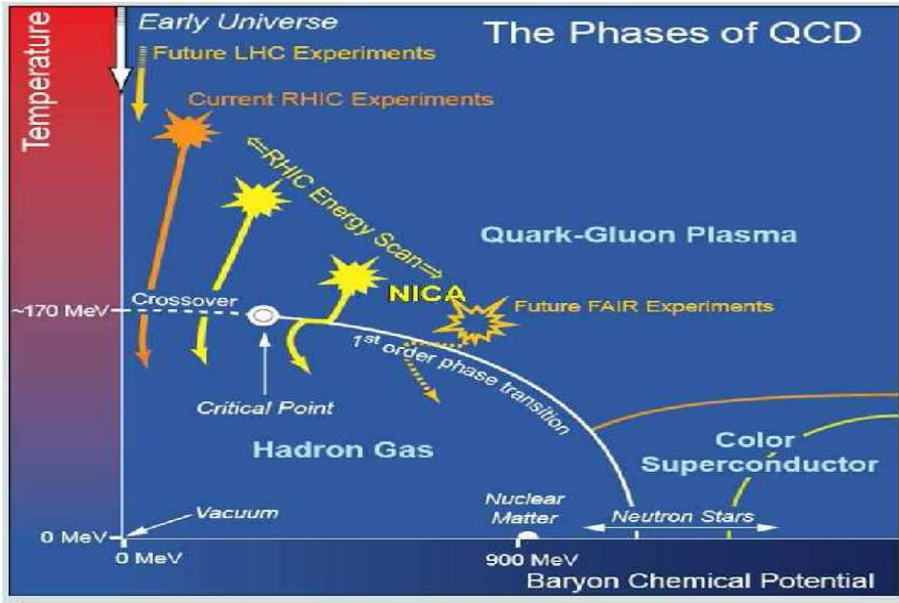
Braun-Munzinger, Redlich, Stachel, in *QGP III* (2003)

# PHASEDIAGRAM: FREEZE-OUT IN HEAVY-ION COLLISIONS (III)



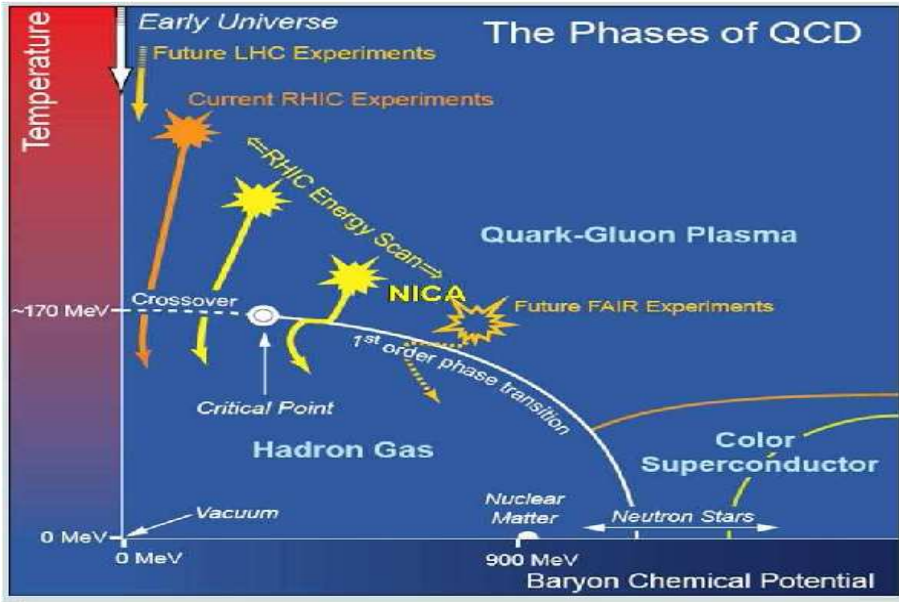
Strange MatterHorn (Pisarski)

# PHASEDIAGRAM: FREEZE-OUT IN HEAVY-ION COLLISIONS (III)

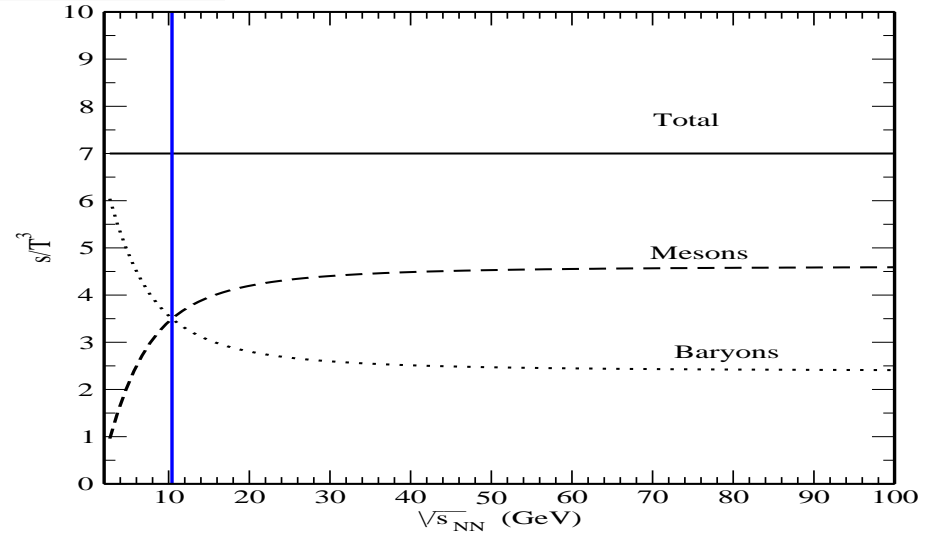
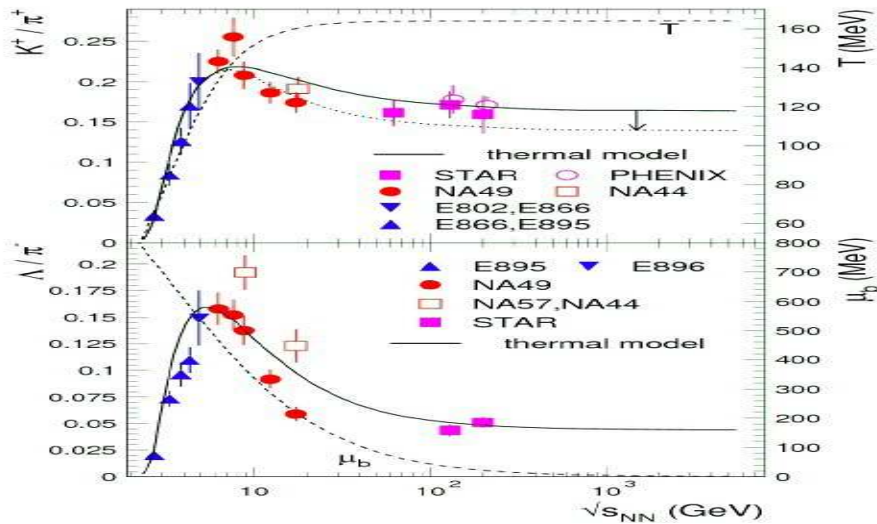
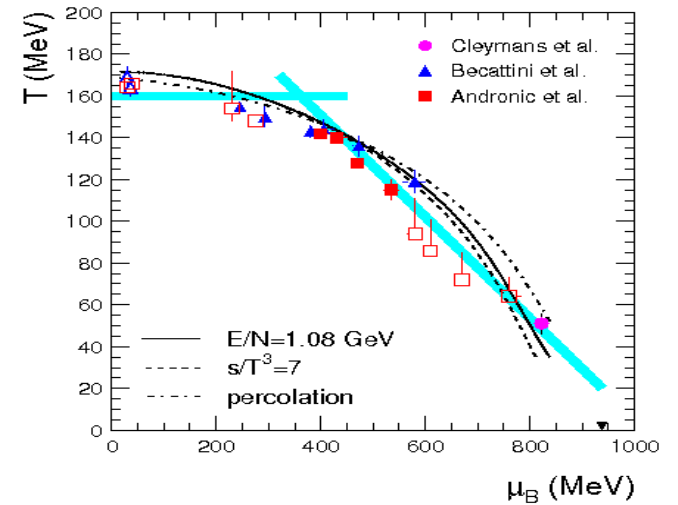


Baryon  $\rightarrow$  Meson Dominance

# PHASEDIAGRAM: FREEZE-OUT IN HEAVY-ION COLLISIONS (IV)

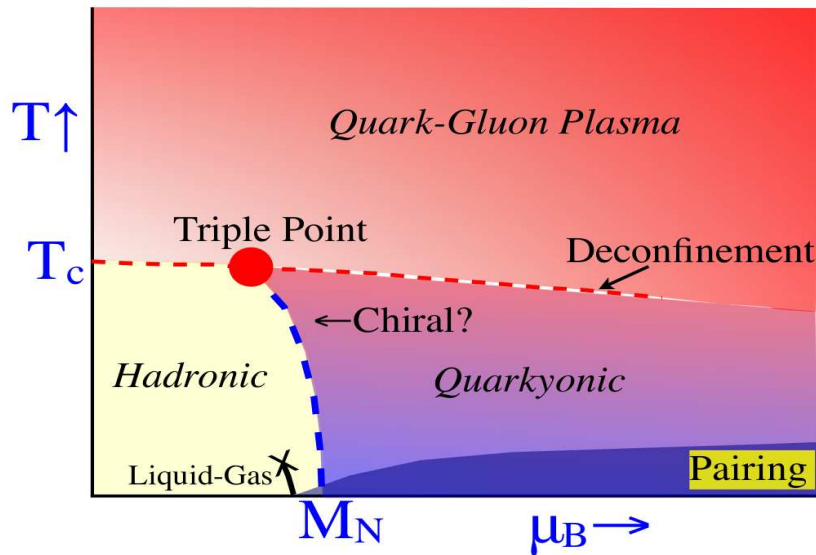


Andronic et al., arxiv:0911.4806

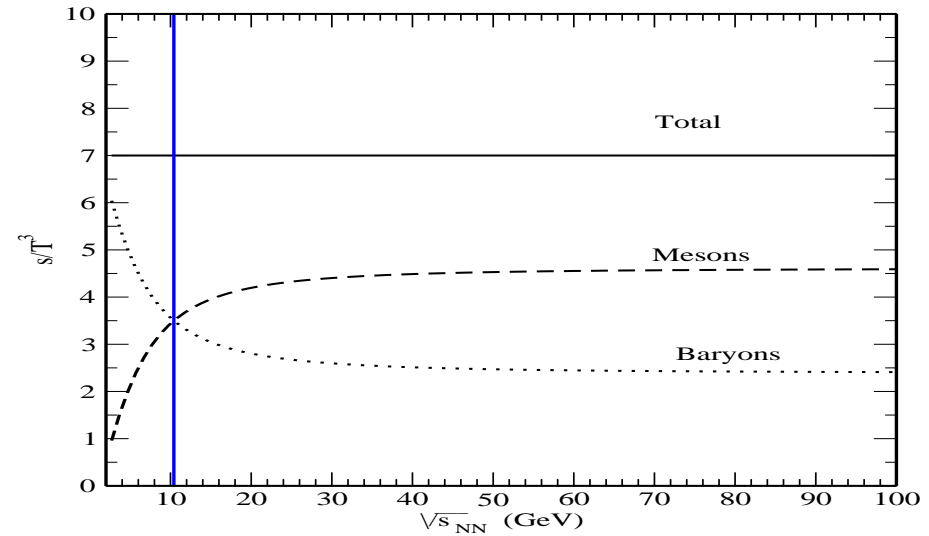
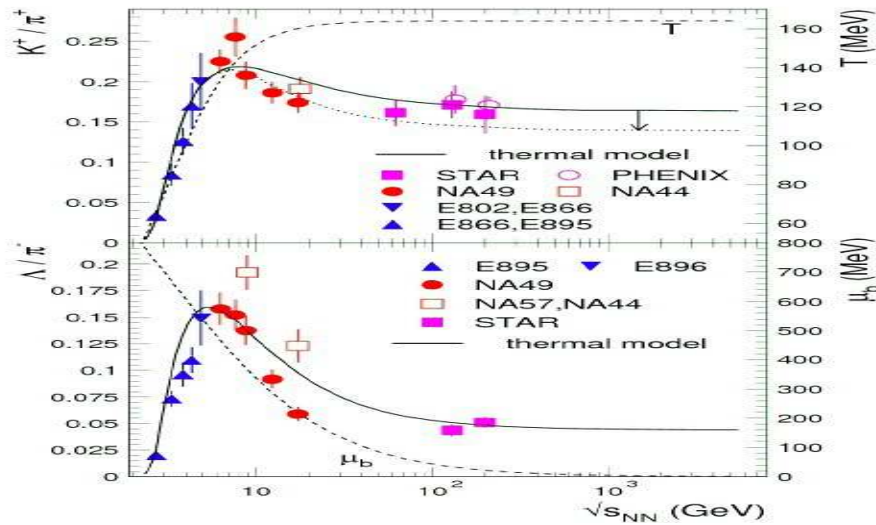
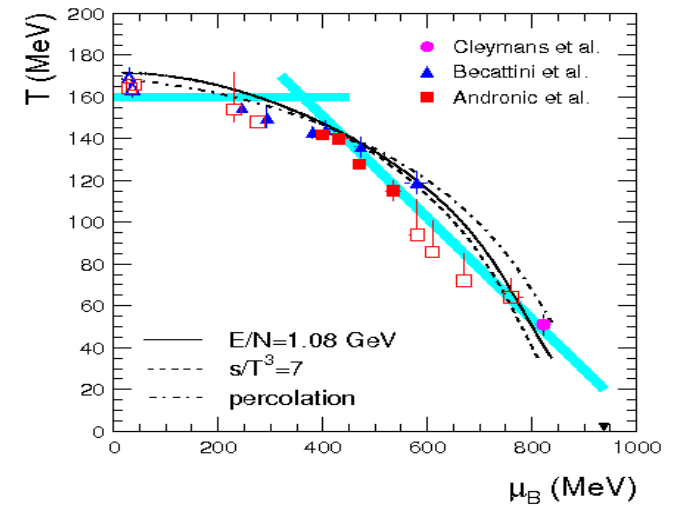


Baryon → Meson Dominance

# PHASEDIAGRAM: FREEZE-OUT IN HEAVY-ION COLLISIONS (V)



Andronic et al., arxiv:0911.4806



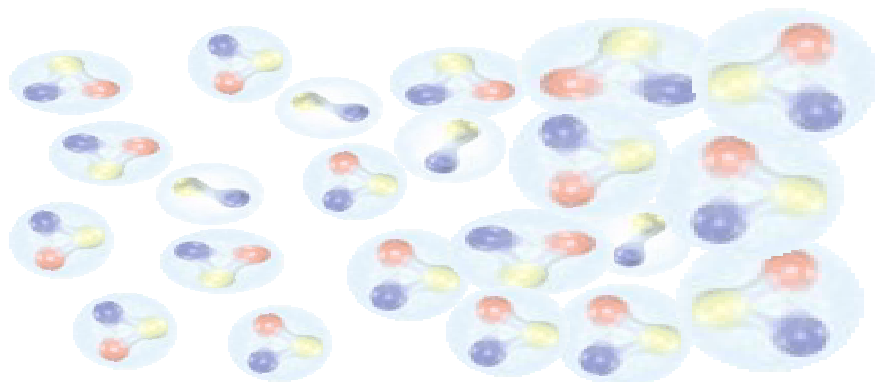
Andronic et al., arxiv:0911.4806; NPA (2010)



QUARKYONIC PHASE = CHIRAL SYMMETRY + CONFINEMENT



## WHAT HAPPENS ON “HAPPY ISLAND”?

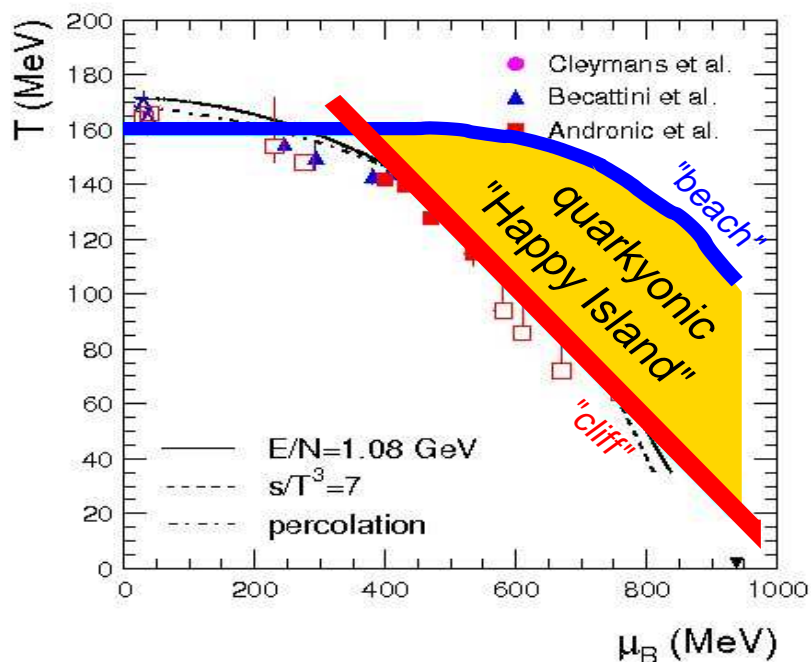


“beach”: hadron resonances  $\rightarrow$  QGP

“cliff”:

- (unmodified) vacuum bound state energies
- fast chemical equilibration

Andronic et al., arxiv:0911.4806



Explanation:

Strong medium dependence of rates for flavor (quark) exchange processes

Reason:

- lowering of thresholds
- increase of hadron size (Pauli principle)  $\rightarrow$  geometrical overlap (percolation)

## IDEA: FREEZE-OUT BY HADRON LOCALIZATION (INVERSE MOTT-ANDERSON MECHANISM)

**Kinetic freeze-out:**  $\tau_{\text{exp}}(T, \mu) = \tau_{\text{coll}}(T, \mu)$

**Reactive collisions:**  $\tau_{\text{coll}}^{-1}(T, \mu) = \sum_{i,j} \sigma_{ij} n_j$

**Povh-Hüfner law:**  $\sigma_{ij} = \lambda \langle r_i^2 \rangle \langle r_j^2 \rangle$ ,  $\lambda \sim 1 \text{ GeV/fm} = 5 \text{ fm}^{-2}$

Also for quark-exchange in hadron-hadron scatt. [Martins et al., PRC 51, 2723 (1995)]

**Pion swelling at  $\chi$ SR:**  $r_\pi^2(T, \mu) = \frac{3}{4\pi^2} f_\pi^{-2}(T, \mu)$ , [Hippe & Klevansky, PRC 52, 2173 (1995)]

Use GMOR relation  $f_\pi^2(T, \mu) = -m_0 \langle \bar{q}q \rangle_{T,\mu} / M_\pi^2$  to connect hadron radii and chiral restoration!

$$r_\pi^2(T, \mu) = \frac{3M_\pi^2}{4\pi^2 m_q} |\langle \bar{q}q \rangle_{T,\mu}|^{-1}, \quad r_N^2(T, \mu) = r_0^2 + r_\pi^2(T, \mu); \quad r_\pi = 0.59 \text{ fm}, \quad r_N = 0.74 \text{ fm}, \quad r_0 = 0.45 \text{ fm}$$

**Expansion time scale:**  $\tau_{\text{exp}}(T, \mu) = a s^{-1/3}(T, \mu)$ ,

follows from  $S = s(T, \mu) V(\tau_{\text{exp}}) = \text{const}$  and  $\tau_{\text{exp}}(T, \mu) = a s^{-1/3}(T, \mu)$ .

D.B., J. Berdermann, J. Cleymans, K. Redlich, arxiv:1102.2908 (2011)

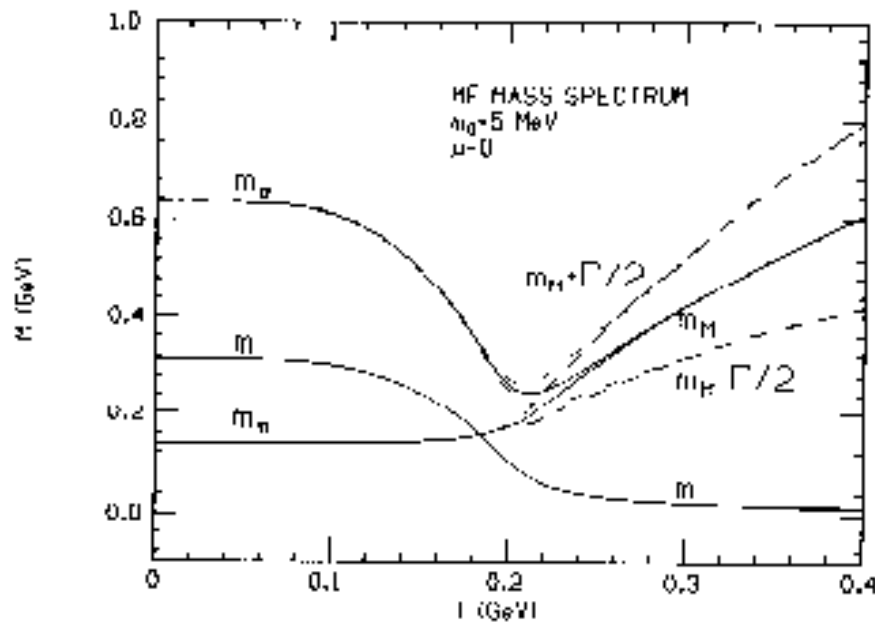
# GENERALIZED BETH-UHLENBECK EoS: NJL MODEL RESULTS

Generalized Beth-Uhlenbeck approach:

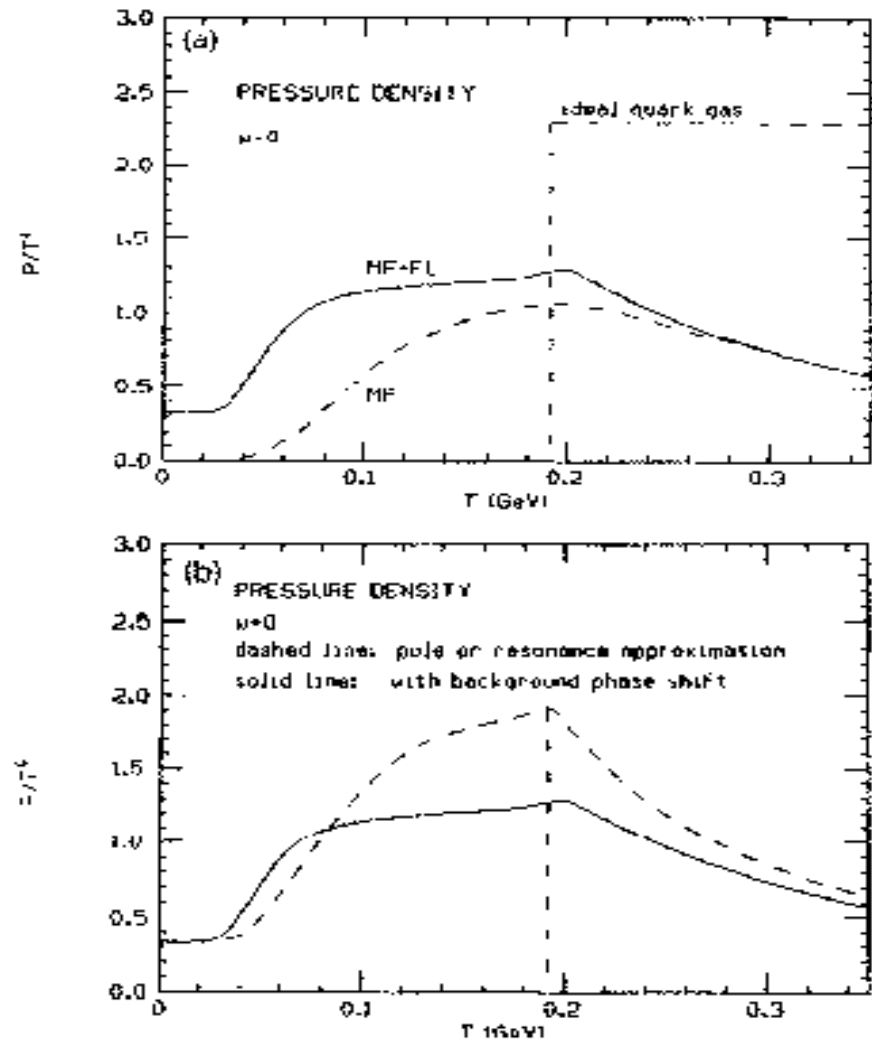
Schmidt, Röpke, Schulz, *Ann. Phys.* **202** (1990) 57

Hüfner, Klevansky et al., *Ann. Phys.* **234** (1994) 225

*P. Zhuang et al. / Nuclear Physics A 576 (1994) 525-552*



*P. Zhuang et al. / Nuclear Physics A 576 (1994) 525-552*

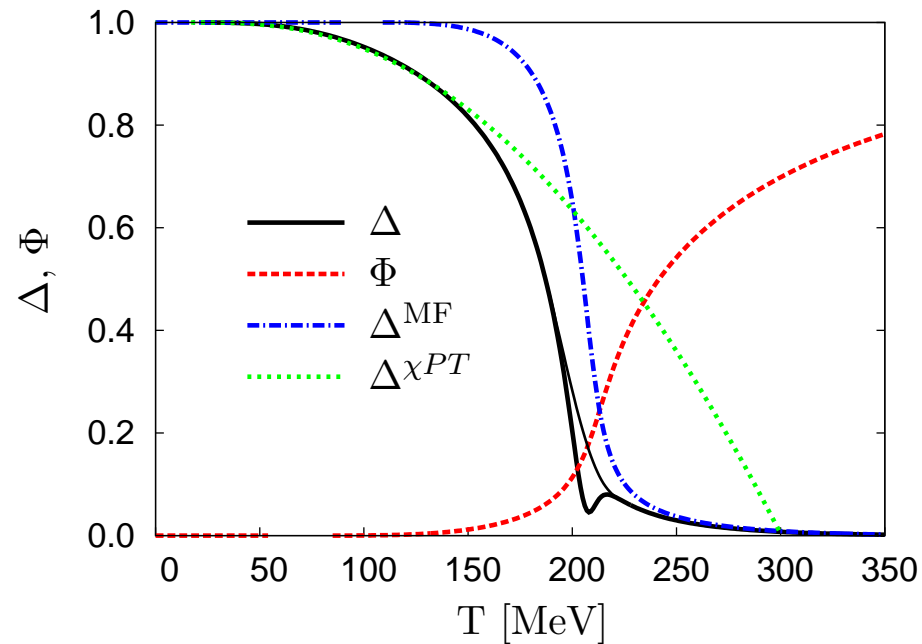


# GEN. BETH-UHLENBECK EOS: NONLOCAL PNJL RESULTS

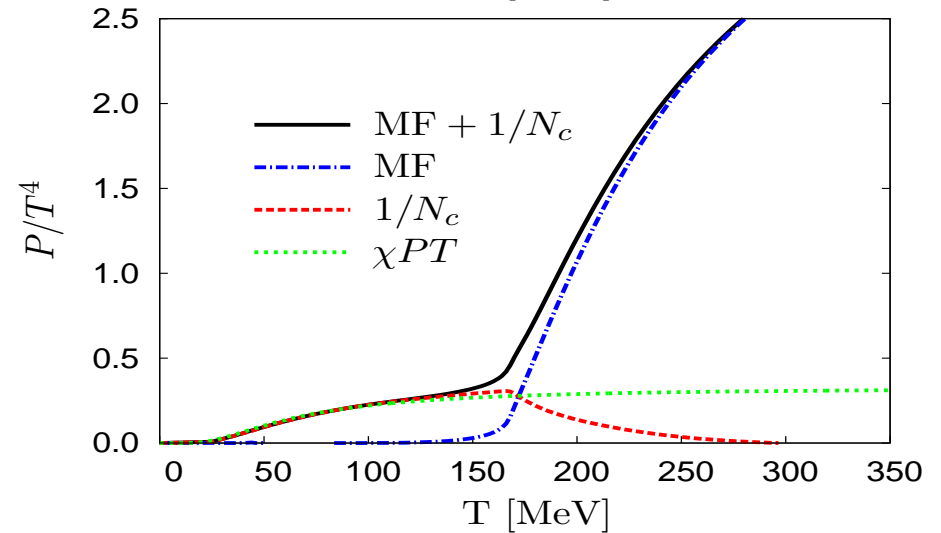
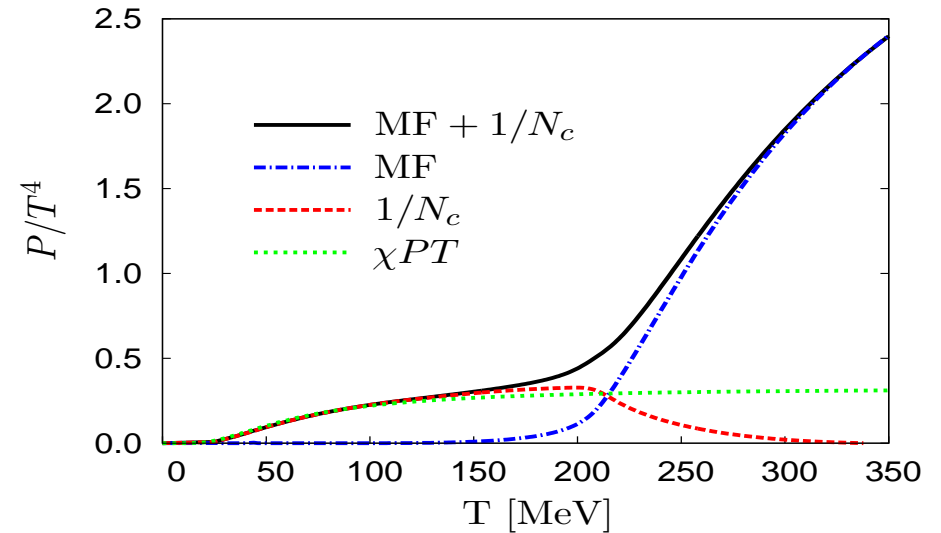
Nonlocal PNJL model beyond meanfield:

Blaschke et al., *Yad. Fiz.* 71 (2008)

Radzhabov et al., *PRD* 83 (2011) 116004



PL-Potential with  $T_0 = 270$  MeV (upper panel),  
and  $T_0 = 208$  MeV (lower panel)  $\implies$



## PNJL BEYOND MF: PION ( $q\bar{q}$ ) AND NUCLEON ( $qqq$ ) MEDIUM

**Idea:** melting  $\langle \bar{q}q \rangle \rightarrow$  swelling hadrons  $\rightarrow$  flavor kinetics = quark percolation  $\rightarrow$  freeze-out

$$\langle \bar{q}q \rangle(T, \mu) = \frac{\partial}{\partial m_0} \Omega(T, \mu), \quad \Omega(T, \mu) = \Omega_{\text{PNJL, MF}}(T, \mu) + \Omega_{\text{meson}}(T, \mu) + \Omega_{\text{baryon}}(T, \mu)$$

$$\Omega_{\text{meson}}(T, \mu) = \sum_{M=\pi, \dots} d_M \int \frac{d\omega}{\pi} \int \frac{d^3k}{(2\pi)^3} \left\{ \frac{\omega}{2} + T \ln [1 - e^{-\beta\omega}] \right\} A_M(\omega, k),$$

$$\Omega_{\text{baryon}}(T, \mu) = - \sum_{B=N, \dots} d_B \int \frac{d\omega}{\pi} \int \frac{d^3k}{(2\pi)^3} \left\{ \frac{\omega}{2} + T \ln [1 + e^{-\beta(\omega - \mu_B)}] + (\mu_B \leftrightarrow -\mu_B) \right\} A_B(\omega, k),$$

$$A_M(\omega, k) = \pi \delta(\omega - E_M(k)) + \text{continuum}, \quad A_B(\omega, k) \dots \text{analogous}$$

Remove vacuum terms; neglect continuum (for the freeze-out);

use GMOR:  $M_\pi^2 f_\pi^2 = -m_0 \langle \bar{q}q \rangle$  and  $\sigma_N = m_0 (\partial m_N / \partial m_0) = 45 \text{ MeV}$ ,

Enforce  $M_\pi(T, \mu) = \text{const}$  by setting  $f_\pi^2(T, \mu) = -m_0 \langle \bar{q}q \rangle(T, \mu) / M_\pi^2$ , (“BRST”, arxiv:1005.4610)

$$-\langle \bar{q}q \rangle(T, \mu) = -\langle \bar{q}q \rangle_{\text{PNJL, MF}}(T, \mu) + \frac{M_\pi^2 T^2}{8m_0} + \frac{\sigma_N}{m_0} n_{s,N}(T, \mu)$$

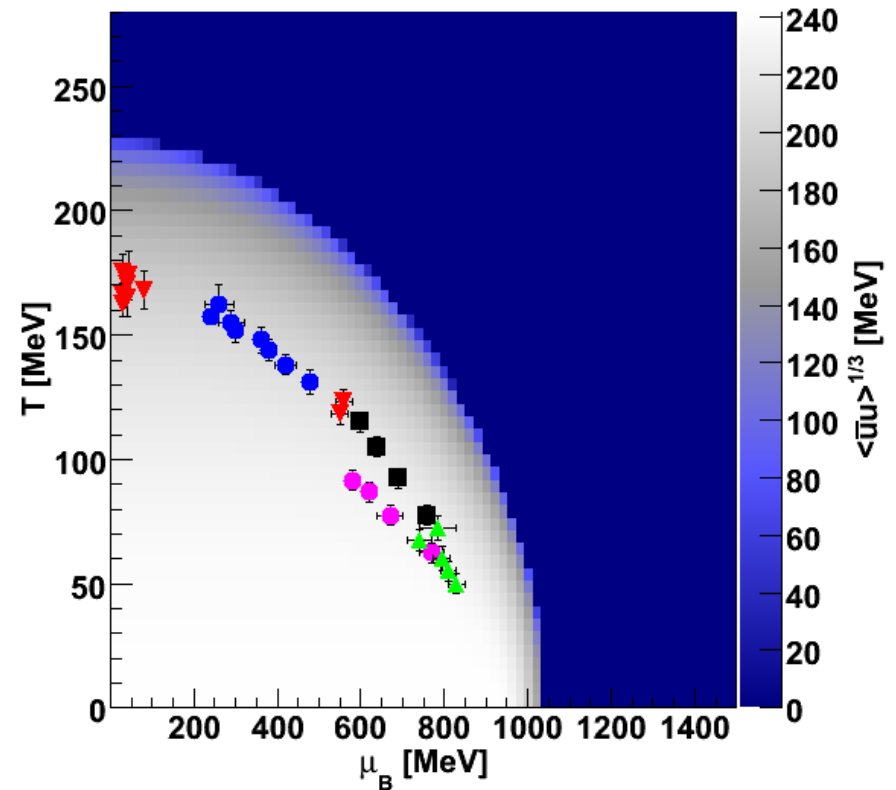
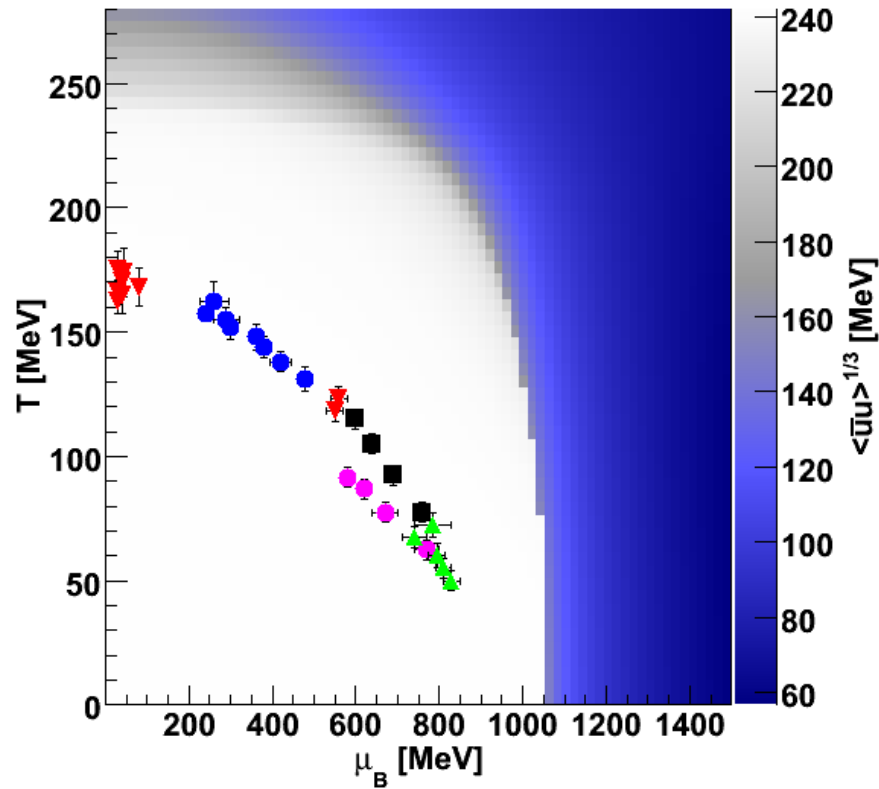
with the scalar nucleon density  $n_{s,N}(T, \mu) = \frac{2}{\pi^2} \int_0^\infty dp p^2 \frac{m_N}{E_N(p)} \{ f_N(T, \mu) + f_N(T, -\mu) \}$

**D.B., J. Berdermann, J. Cleymans, K. Redlich, arxiv:1102.2908 (2011)**

# PNJL MODEL BEYOND MF - RESULTS

$$-\langle \bar{q}q \rangle = -\langle \bar{q}q \rangle_{\text{PNJL, MF}}$$

$$-\langle \bar{q}q \rangle = -\langle \bar{q}q \rangle_{\text{PNJL, MF}} + \frac{M_\pi^2 T^2}{8m_0} + \frac{\sigma_N}{m_0} n_{s,N}(T, \mu) + \dots$$

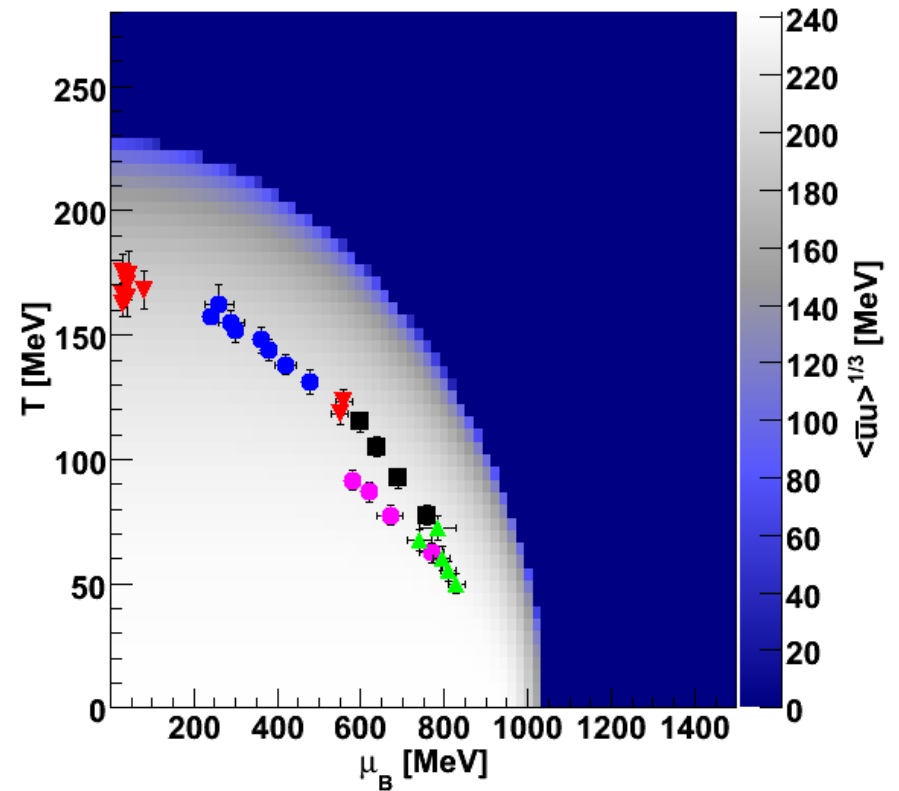
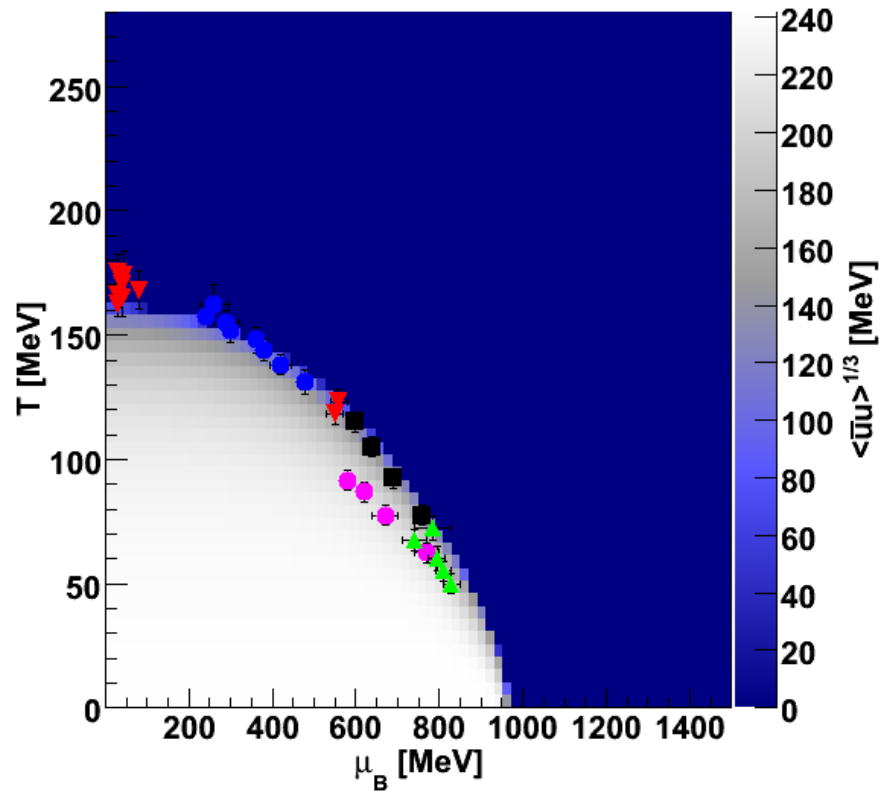


D.B., J. Berdermann, J. Cleymans, K. Redlich, arxiv:1102.2908 (2011)

# PNJL MODEL BEYOND MF - RESULTS

$$-\langle \bar{q}q \rangle = -\langle \bar{q}q \rangle_{\text{PNJL, MF}} + \kappa_M \frac{M_\pi^2 T^2}{8m_0} + \kappa_B \frac{\sigma_N}{m_0} n_{s,N}(T, \mu)$$

$$-\langle \bar{q}q \rangle = -\langle \bar{q}q \rangle_{\text{PNJL, MF}} + \frac{M_\pi^2 T^2}{8m_0} + \frac{\sigma_N}{m_0} n_{s,N}(T, \mu) + \dots$$

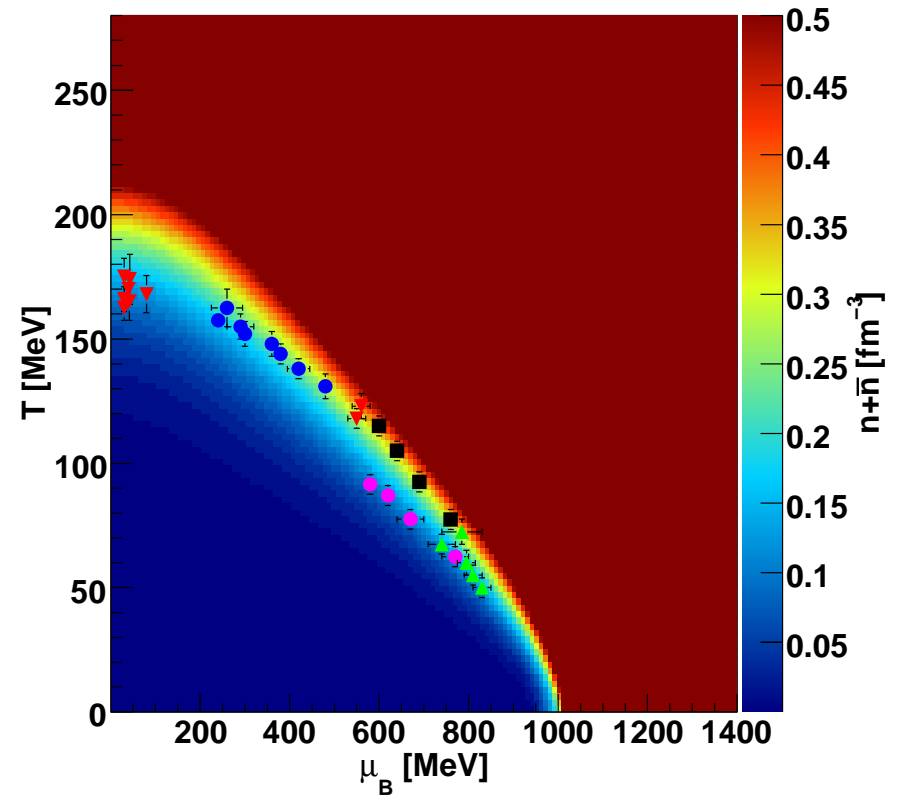
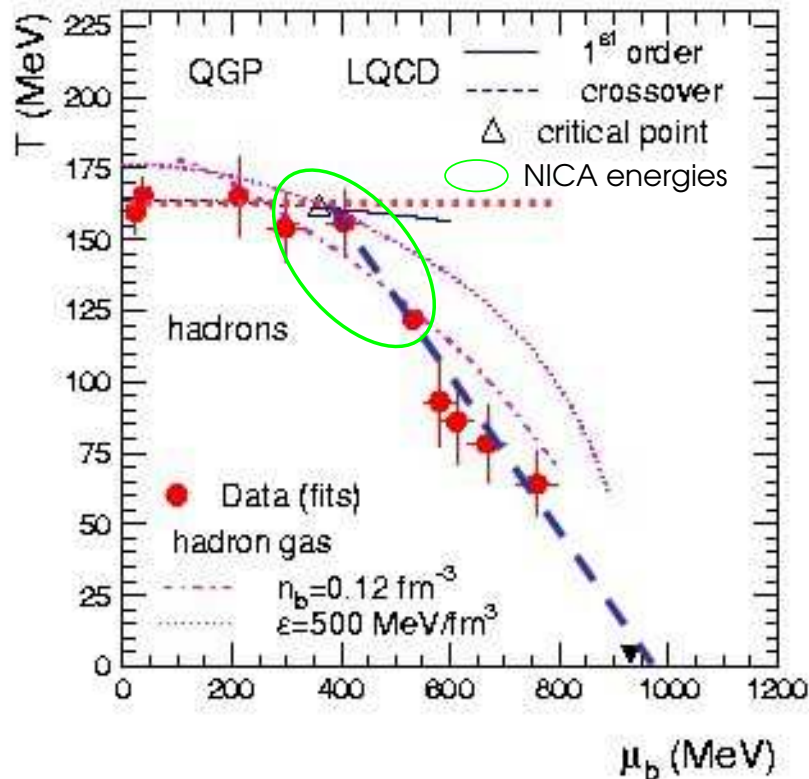


D.B., J. Berdermann, J. Cleymans, K. Redlich, arxiv:1102.2908 (2011)



# PNJL MODEL BEYOND MF vs. PHENOMENOLOGICAL FIT

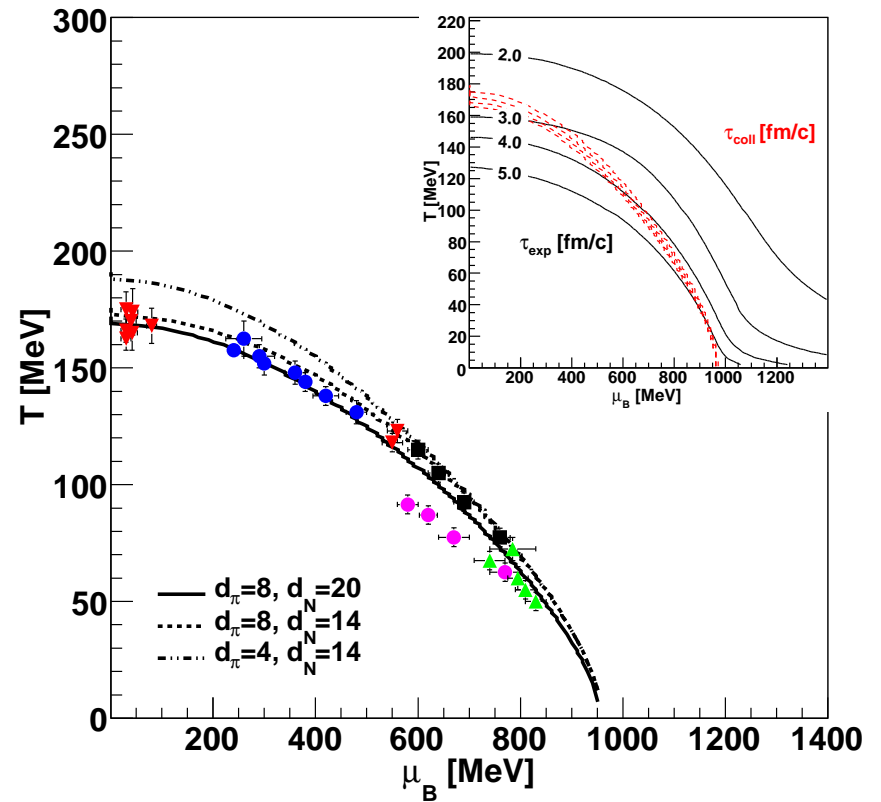
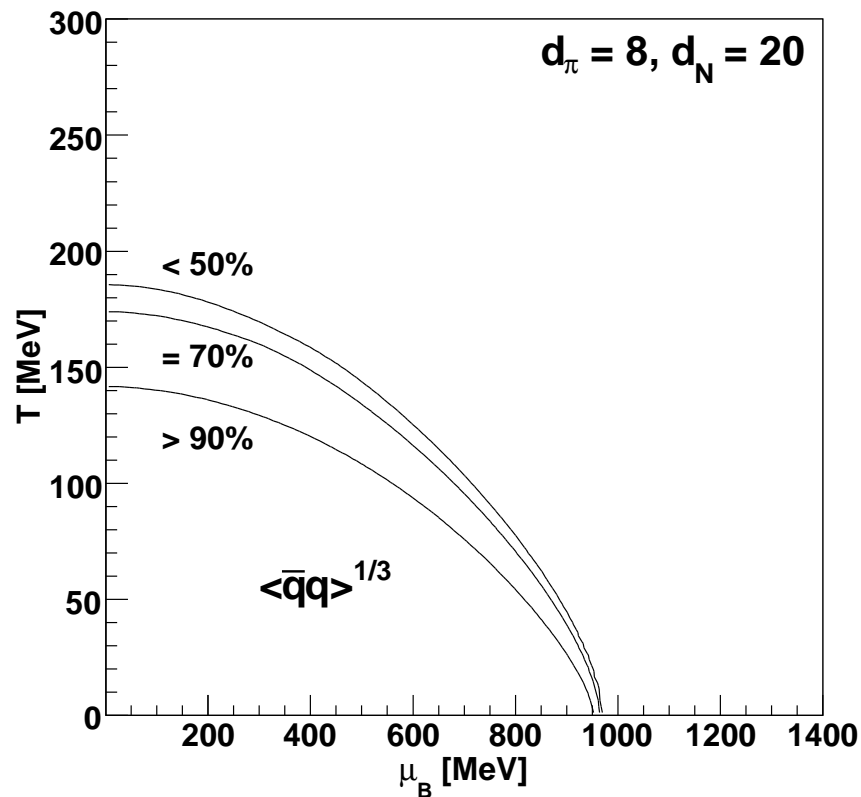
$$n_b = n(T, \mu) + \bar{n}(T, \mu) = \sum_{i=N,\Delta,xB} d_i \int \frac{dp p^2}{2\pi^2} \left[ \frac{1}{\exp(\beta[E_i(p) - \mu]) + 1} + (\mu \leftrightarrow -\mu) \right]$$



D.B., J. Berdermann, J. Cleymans, K. Redlich, arxiv:1102.2908 (2011)

# PNJL MODEL BEYOND MF CONDENSATE VS. FREEZE-OUT COND.

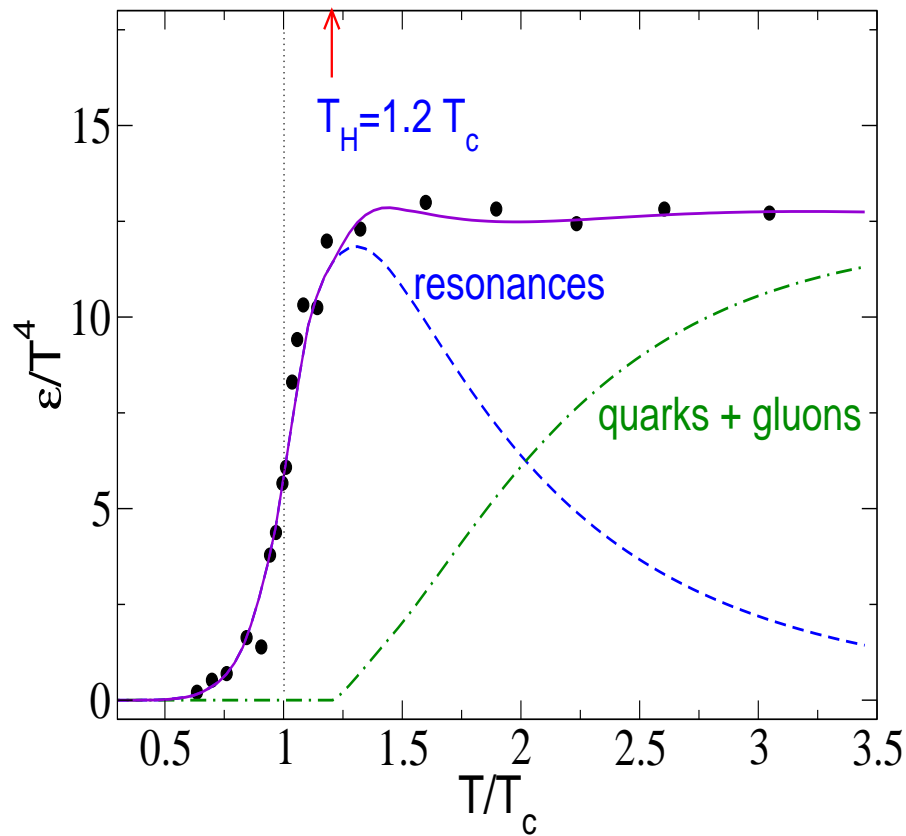
$$\langle \bar{q}q \rangle = \langle \bar{q}q \rangle_{\text{MF}} \left[ 1 - \frac{T^2}{8f_\pi^2(T, \mu)} - \frac{\sigma_N n_{s,N}(T, \mu)}{M_\pi^2 f_\pi^2(T, \mu)} \right], \quad n_{s,\pi} = d_\pi M_\pi T^2 / 12$$



D.B., J. Berdermann, J. Cleymans, K. Redlich, arxiv:1102.2908 (2011);  
 Few Body Systems (2011) in press.

# LATTICE QCD EoS AND MOTT-HAGEDORN GAS

$$\varepsilon_R(T, \{\mu_j\}) = \sum_{i=\pi, K, \dots} \varepsilon_i(T, \{\mu_i\}) + \sum_{r=M, B} g_r \int_{m_r} dm \int ds \rho(m) A(s, m; T) \int \frac{d^3p}{(2\pi)^3} \frac{\sqrt{p^2 + s}}{\exp\left(\frac{\sqrt{p^2 + s} - \mu_r}{T}\right) + \delta_r}$$



Hagedorn mass spectrum:  $\rho(m)$

Spectral function for heavy resonances:

$$A(s, m; T) = N_s \frac{m\Gamma(T)}{(s - m^2)^2 + m^2\Gamma^2(T)}$$

Ansatz with **Mott effect** at  $T = T_H = 192$  MeV:

$$\Gamma(T) = B\Theta(T - T_H) \left(\frac{m}{T_H}\right)^{2.5} \left(\frac{T}{T_H}\right)^6 \exp\left(\frac{m}{T_H}\right)$$

No width below  $T_H$ : Hagedorn resonance gas  
Apparent phase transition at  $T_c \sim 160$  MeV

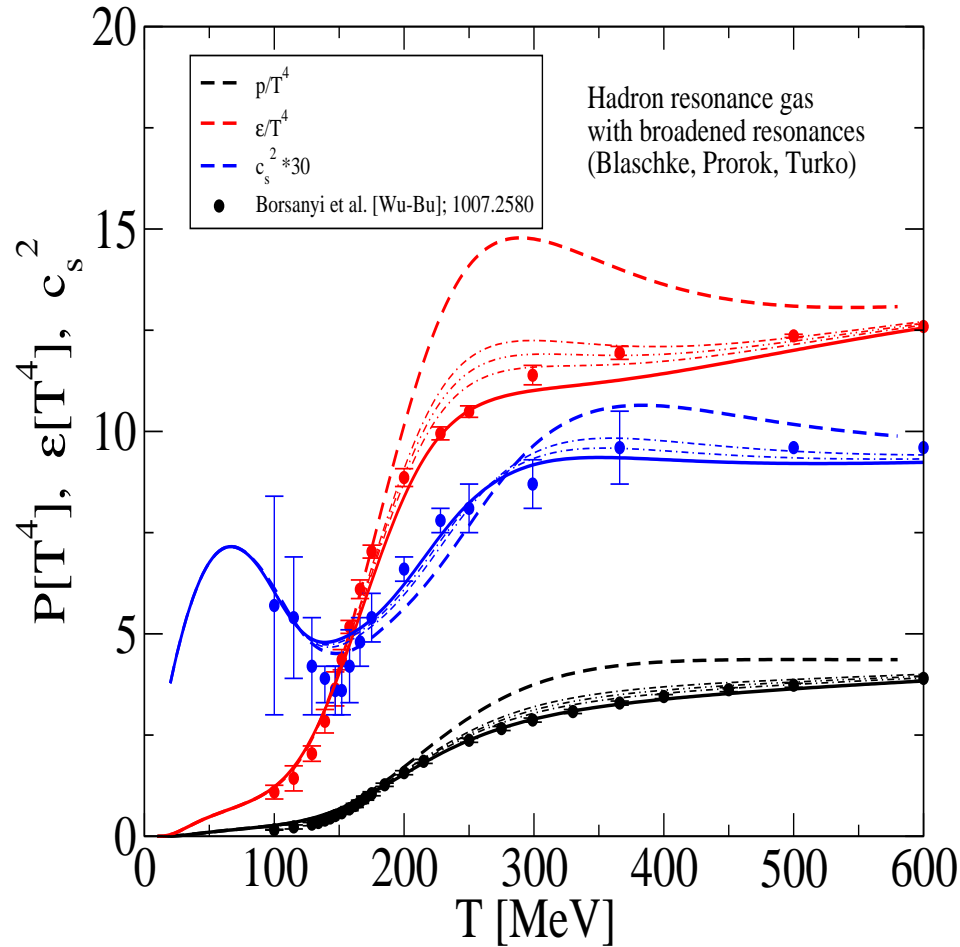
**Blaschke & Bugaev, Fizika B13, 491 (2004)**

**Prog. Part. Nucl. Phys. 53, 197 (2004)**

**Blaschke & Yudichev (2006)**

# HYBRID APPROACH: PNJL & MOTT-HAGEDORN RESONANCE GAS

$$\varepsilon_{\text{hybrid}}(T, \{\mu_j\}) = \varepsilon_{\text{PNJL}}(T, \{\mu_i\}) + \sum_{r=M,B} g_r \int ds A(s, m_r; T) \int \frac{d^3p}{(2\pi)^3} \frac{\sqrt{p^2 + s}}{\exp\left(\frac{\sqrt{p^2 + s} - \mu_r}{T}\right) + \delta_r}$$



**Spectral function** for heavy resonances:

$$A(s, m; T) = N_s \frac{m\Gamma(T)}{(s - m^2)^2 + m^2\Gamma^2(T)}$$

Ansatz with **Mott effect** at  $T = T_H = 198$  MeV:

$$\Gamma(T) = B\Theta(T - T_H) \left(\frac{m}{T_H}\right)^{2.5} \left(\frac{T}{T_H}\right)^6 \exp\left(\frac{m}{T_H}\right)$$

Apparent phase transition at  $T_c \sim 165$  MeV

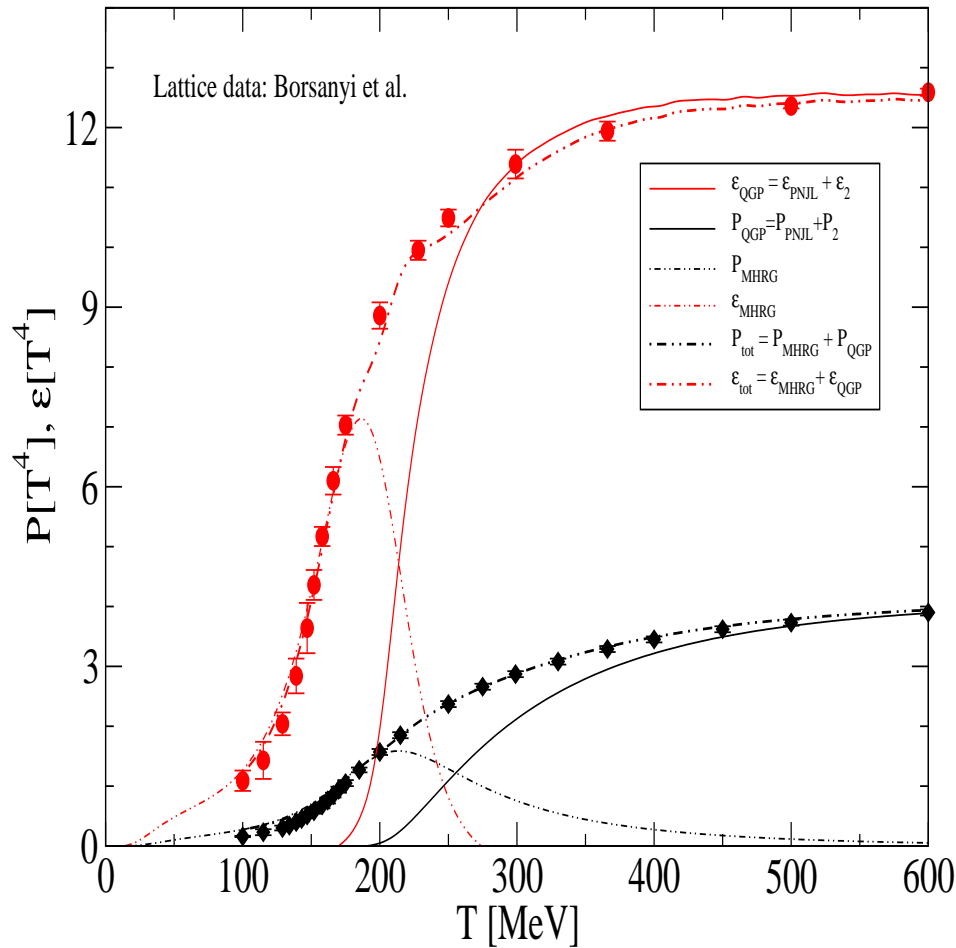
**Blaschke & Bugaev, Fizika B13, 491 (2004)**

**Prog. Part. Nucl. Phys. 53, 197 (2004)**

**Blaschke, Prorok & Turko, in preparation**

# HYBRID APPROACH: PNJL & MOTT-HAGEDORN RESONANCE GAS

$$\varepsilon_{\text{hybrid}}(T, \{\mu_j\}) = \varepsilon_{\text{PNJL}}(T, \{\mu_i\}) + \sum_{r=M,B} g_r \int ds A_r(s, m_r; T) \int \frac{d^3p}{(2\pi)^3} \frac{\sqrt{p^2 + s}}{\exp\left(\frac{\sqrt{p^2 + s} - \mu_r}{T}\right) + \delta_r}$$



Spectral function for heavy resonances:

$$A_r(s, m; T) = N_s \frac{m \Gamma_r(T)}{(s - m^2)^2 + m^2 \Gamma_r^2(T)}$$

Ansatz motivated by chemical freeze-out model:

$$\Gamma_r(T) = \tau_r^{-1}(T) = \sum_h \lambda \langle r_r^2 \rangle_T \langle r_h^2 \rangle_T n_h(T)$$

Apparent phase transition at  $T_c \sim 165$  MeV

Blaschke & Bugaev, *Fizika B13*, 491 (2004)

*Prog. Part. Nucl. Phys.* 53, 197 (2004)

Blaschke, Prorok & Turko, in preparation

# ROADMAP FOR PHASE DIAGRAM AND EoS RESEARCH

