HADRONS & HADRONIC MATTER IN CHIRAL QUARK MODELS (III)

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Main ideas for this lecture (punchline):

- How to understand the statement: "A system of interacting elementary particles can be reformulated as a system of noninteracting resonances" ?
- Bound states can be treated as a new species $\Longrightarrow$ "chemical picture"
- Physical picture: there are also scattering states!
  EoS with bound and scattering states: Beth-Uhlenbeck EoS (1936/37)
- Generalized Beth-Uhlenbeck EoS includes the Mott transition:
  bound states $\Longrightarrow$ resonances in the scattering continuum

Chiral Model Field Theory for Quark Matter

- Partition function as a Path Integral (imaginary time $\tau = i\ t$)

$$Z[T,V,\mu] = \int \mathcal{D}\bar{q}\mathcal{D}q \exp\left\{-\int^{\beta} d\tau \int_V d^3x [\bar{q}(i\gamma^\mu\partial_\mu - m_0 - \gamma^0\mu)q + \sum_{M=\pi,\sigma} G_M(\bar{q}\Gamma_M q)^2]\right\}$$

- Couplings: $G_\pi = G_\sigma = G_S$ (chiral symmetry)
- Vertices: $\Gamma_\sigma = \mathbf{1}_D \otimes \mathbf{1}_f \otimes \mathbf{1}_c$; $\Gamma_\pi = i\gamma_5 \otimes \vec{\tau} \otimes \mathbf{1}_c$
- Bosonization (Hubbard-Stratonovich Transformation)

$$\exp\left[G_S(\bar{q}\Gamma_\sigma q)^2\right] = \text{const.} \int \mathcal{D}\sigma \exp\left[\frac{\sigma^2}{4G_S} + \bar{q}\Gamma_\sigma q\sigma\right]$$

- Integrate out quark fields $\longrightarrow$ bosonized partition function

$$Z[T,V,\mu] = \int \mathcal{D}\sigma\mathcal{D}\pi \exp\left\{-\frac{\sigma^2+\pi^2}{4G_S} + \frac{1}{2}\text{Tr } \ln S^{-1}[\sigma,\pi]\right\}$$

- Systematic evaluation: Mean fields + Fluctuations
  - Mean-field approximation: order parameters for phase transitions (gap equations)
  - Lowest order fluctuations: hadronic correlations (bound & scattering states)

Mean Field plus (Gaussian) Fluctuations

- Separate the mean-field part of the quark determinant

$$\mathrm{Tr}\,\ln S^{-1}[\sigma,\pi] = \mathrm{Tr}\,\ln S^{-1}_{\mathrm{MF}}[m] + \mathrm{Tr}\,\ln[1 + (\sigma + i\gamma_5\vec{\tau}\vec{\pi})S_{\mathrm{MF}}[m]]$$

- Mean-field quark propagator

$$S_{\mathrm{MF}}(\vec{p}, i\omega_n; m) = \frac{\gamma_0(i\omega_n+\mu) - \vec{\gamma}\cdot\vec{p} + m}{(i\omega_n+\mu)^2 - E_p^2}$$

- Expand the logarithm: $\ln(1+x) = -\sum_{n=1}^{\infty}(-1)^n x^n/n = x - x^2/2 + ...$
- Thermodynamic potential in Gaussian approximation

$$\Omega(T,\mu) = -T\ln Z(T,\mu) = \Omega_{\mathrm{MF}}(T,\mu) + \Omega^{(}_{\mathrm{M}}2)(T,\mu) + \ldots$$

$$\Omega(T,\mu) = \frac{N_M}{2}\int\frac{d^2p}{(2\pi)^3}\frac{1}{\beta}\sum_n \mathrm{e}^{i\nu_n\eta}\ln\left[1 - 2G_S\Pi_M(\vec{p}, i\nu_n)\right]\ ,\quad N_\sigma = 1,\ N_\pi = 3$$

- Mesonic polarization loop

$$\Pi_M(\vec{p}, i\nu_n) = -\frac{1}{\beta}\sum_{n'}\mathrm{e}^{i\nu_{n'}\eta}\int\frac{d^2k}{(2\pi)^3}\mathrm{Tr}\left[\Gamma_M S_{\mathrm{MF}}(-\vec{k}, -i\omega_{n'})\Gamma_M S_{\mathrm{MF}}(\vec{k}+\vec{p}, i\omega_{n'} + i\nu_n)\right]$$