HADRONS AND HADRONIC MATTER IN CHIRAL QUARK MODELS (II)

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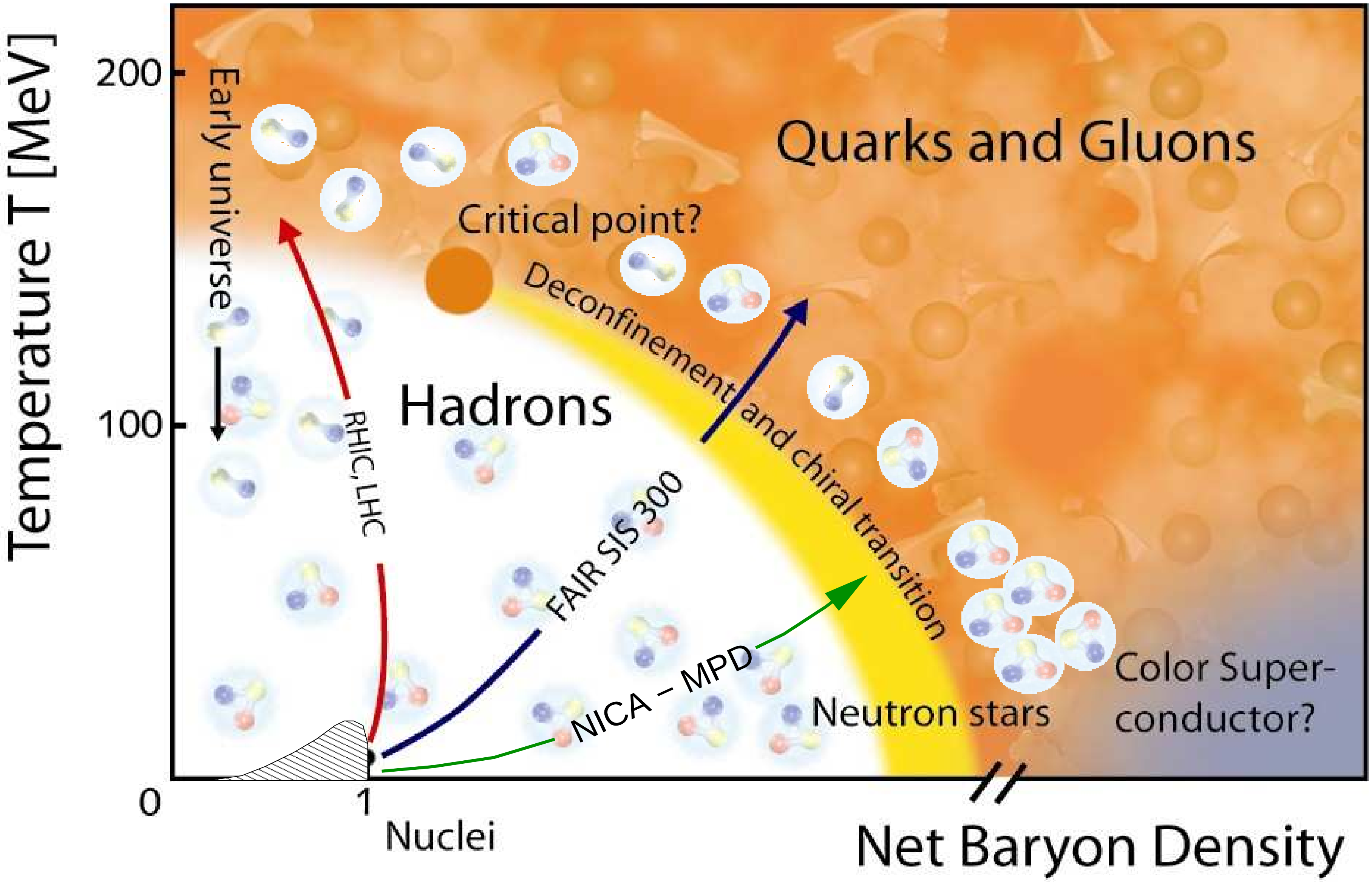
Main motivations for effective chiral quark models of QCD$|_{T,\mu}$:

- Qualitative and quantitative interpretation and understanding of lattice QCD results
- Extension from finite-T/low-μ to low-T/high-μ region of the QCD phase diagram
- Application to HIC energy scan programs (RHIC, SPS, NICA, CBM) and compact stars
- Calculation of in-medium processes, prediction of QGP signals

Lattice QCD, hadron structure and hadronic matter; Dubna, 05.-16.09.2011

Hadronic Correlations in the Phasediagram of QCD



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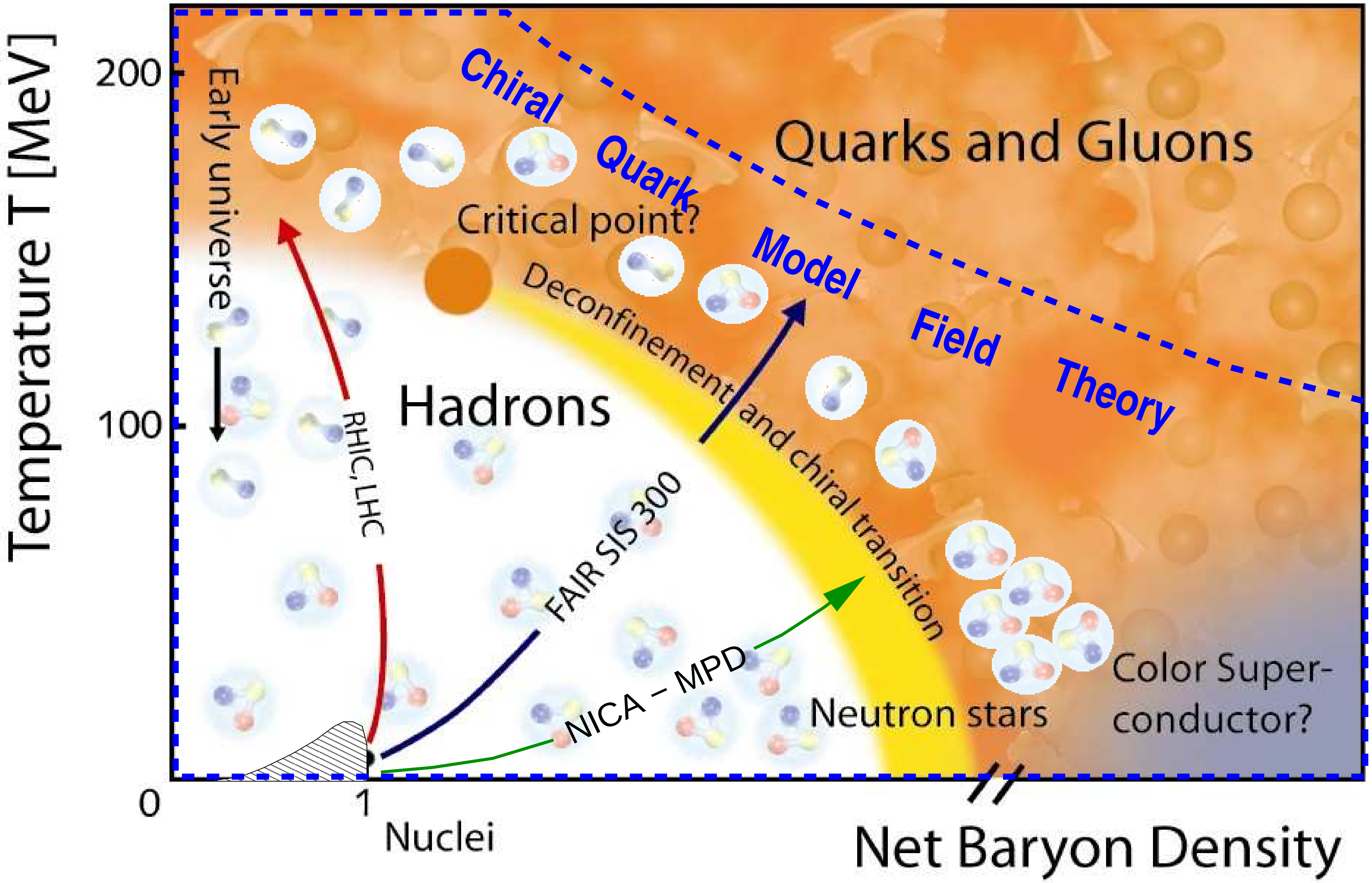


Chiral Model Field Theory for Quark Matter

- Partition function as a Path Integral (imaginary time $\tau = i\, t$)

$$Z[T,V,\mu] = \int \mathcal{D}\bar{\psi}\mathcal{D}\psi \exp\left\{-\int^{\beta} d\tau \int_V d^3x[\bar{\psi}[i\gamma^\mu\partial_\mu - m - \gamma^0(\mu + \lambda_8\mu_8 + i\lambda_3\phi_3]\psi - \mathcal{L}_{\rm int} + U(\Phi)]\right\}$$

Polyakov loop: $\Phi = N_c^{-1}{\rm Tr}_c[\exp(i\beta\lambda_3\phi_3)]$ Order parameter for deconfinement

- Current-current interaction (4-Fermion coupling) and KMT determinant interaction

$$\mathcal{L}_{\rm int} = \sum_{M=\pi,\sigma,\ldots} G_M(\bar{\psi}\Gamma_M\psi)^2 + \sum_D G_D(\bar{\psi}^C\Gamma_D\psi)^2 - K[\det_f(\bar{q}(1+\gamma_5)q) + \det_f(\bar{q}(1-\gamma_5)q)]$$

- Bosonization (Hubbard-Stratonovich Transformation)

$$Z[T,V,\mu] = \int \mathcal{D}M_M\mathcal{D}\Delta_D^\dagger\mathcal{D}\Delta_D\, {\rm e}^{-\sum_{M,D}\frac{M_M^2}{4G_M} - \frac{|\Delta_D|^2}{4G_D} + \frac{1}{2}{\rm Tr}\,\ln S^{-1}[\{M_M\},\{\Delta_D\},\Phi] + U(\Phi) + V_{\rm KMT}}$$

- Collective quark fields: Mesons (M_M) and Diquarks (Δ_D); Gluon mean field: Φ
- Systematic evaluation: Mean fields + Fluctuations
  - Mean-field approximation: order parameters for phase transitions (gap equations)
  - Lowest order fluctuations: hadronic correlations (bound & scattering states)
  - Higher order fluctuations: hadron-hadron interactions

Polyakov-loop Nambu–Jona-Lasinio Model (I)

$SU(N_c)$ pure gauge sector: Polyakov line

$$L(\vec{x}) \equiv \mathcal{P} \exp\left[i \int_0^\beta d\tau \, A_4(\vec{x},\tau) \right] \; ; \quad A_4 = iA^0 = \lambda_3\phi_3 + \lambda_8\phi_8$$

Polyakov loop

$$l(\vec{x}) = \frac{1}{N_c} \mathrm{Tr} L(\vec{x}) \, , \quad \langle l(\vec{x}) \rangle = e^{-\beta \Delta F_Q(\vec{x})} .$$

$\mathbf{Z}_{N_c}$ symmetric phase: $\langle l(\vec{x}) \rangle = 0 \implies \Delta F_Q \to \infty$: Confinement !

Polyakov loop field:

$$\Phi(\vec{x}) \equiv \langle\langle l(\vec{x}) \rangle\rangle = \frac{1}{N_c} \mathrm{Tr}_c \langle\langle L(\vec{x}) \rangle\rangle$$

Potential for the PL-meanfield $\Phi(\vec{x}) =$const., which fits quenched QCD lattice thermodynamics

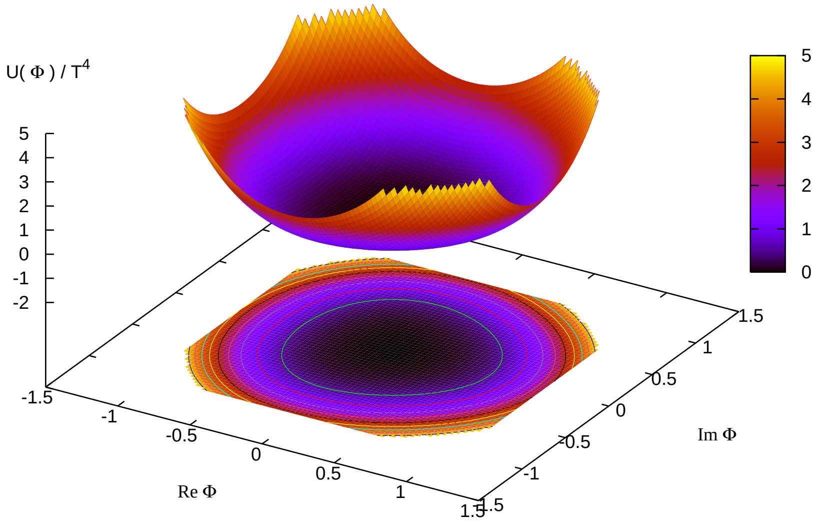
$$\frac{\mathcal{U}\left(\Phi, \bar{\Phi}; T\right)}{T^4} = -\frac{b_2(T)}{2} \bar{\Phi}\Phi - \frac{b_3}{6}\left(\Phi^3 + \bar{\Phi}^3\right) + \frac{b_4}{4}\left(\bar{\Phi}\Phi\right)^2 \; ,$$

$$b_2(T) = a_0 + a_1\left(\frac{T_0}{T}\right) + a_2\left(\frac{T_0}{T}\right)^2 + a_3\left(\frac{T_0}{T}\right)^3 .$$

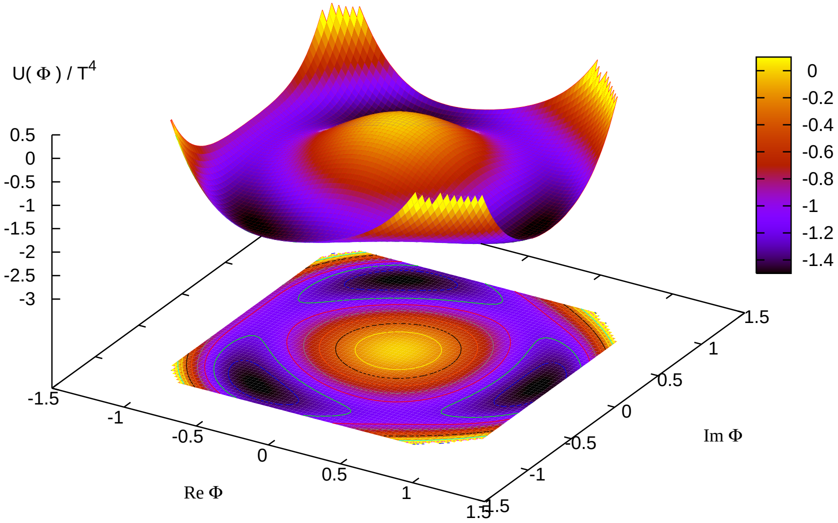
a_0	a_1	a_2	a_3	b_3	b_4
6.75	-1.95	2.625	-7.44	0.75	7.5

POLYAKOV-LOOP NAMBU–JONA-LASINIO MODEL (II)

Temperature dependence of the Polyakov-loop potential $U(\Phi, \bar{\Phi}; T)$



$T = 0.26$ GeV$< T_0$
“Color confinement”



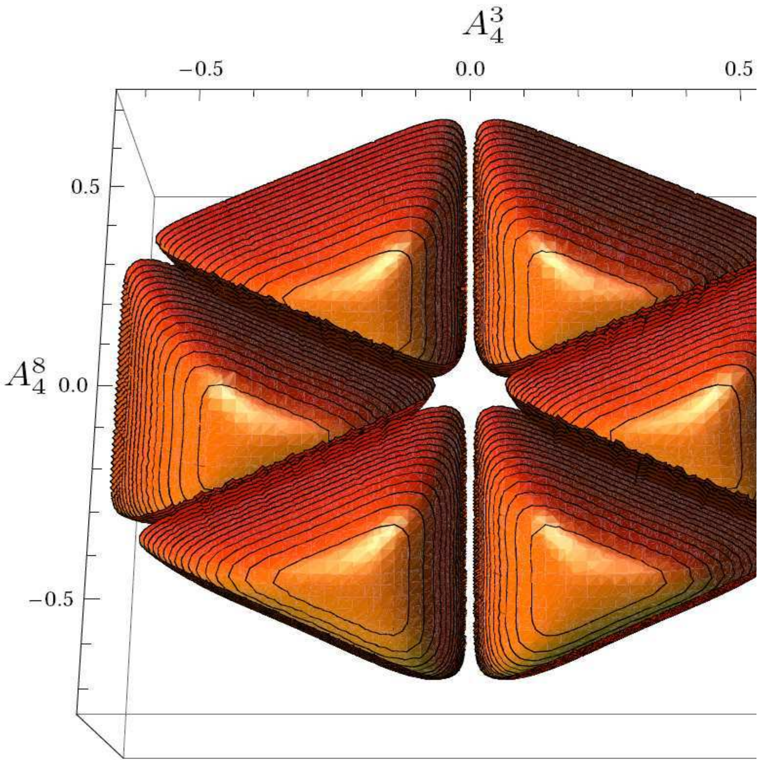
$T = 1.0$ GeV$> T_0$
“Color deconfinement”

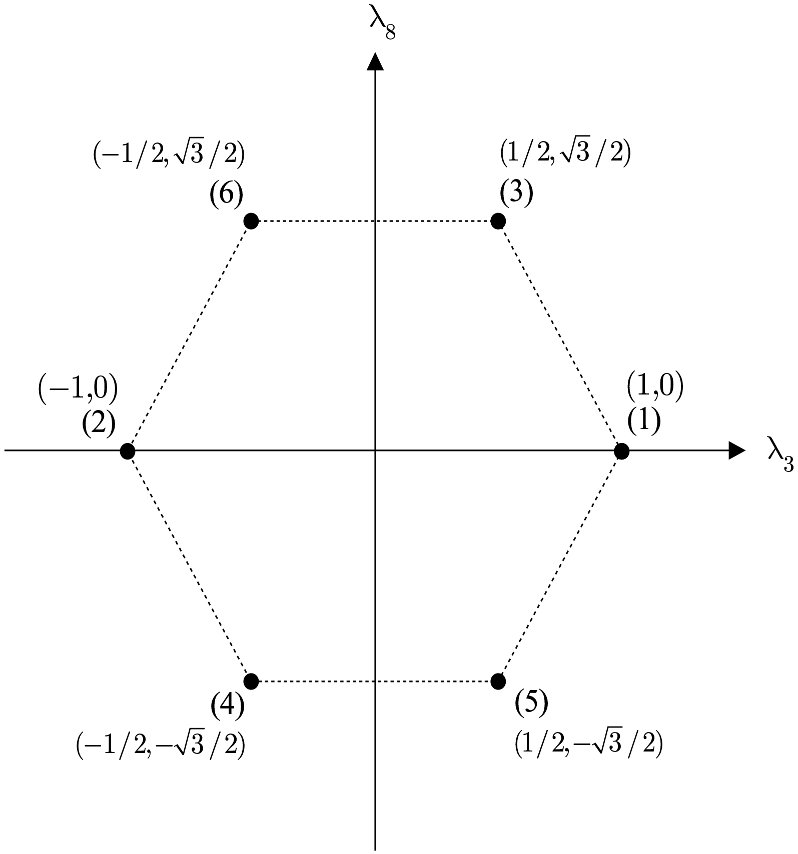
Critical temperature for pure gauge $SU_c(3)$ lattice simulations: $T_0 = 270$ MeV.

Hansen et al., Phys.Rev. D75, 065004 (2007)

POLYAKOV-LOOP VARIABLE Φ

Degeneracy in $\Phi = Tr_c\{\exp[i\beta A_4]\}/N_c$; $A_4 = \lambda_3\phi_3 + \lambda_8\phi_8$; Internal Z(3) Symmetry





Hell et al., 0810.1099 [hep-ph]

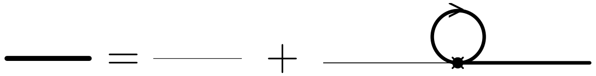
Abuki et al., 0811.1512 [hep-ph]

Polyakov-loop Nambu–Jona-Lasinio Model (III)

Lagrangian for $N_f = 2$, $N_c = 3$ quark matter, coupled to the gauge sector

$$\mathcal{L}_{PNJL} \;=\; \bar{q}(i\gamma^\mu D_\mu - \hat{m} + \gamma_0\mu)q + G_1\left[(\bar{q}q)^2 + (\bar{q}i\gamma_5\vec{\tau}q)^2\right] - \mathcal{U}\left(\Phi[A], \bar{\Phi}[A]; T\right),$$

$D^\mu = \partial^\mu - iA^\mu$; $A^\mu = \delta^\mu_0 A^0$ (Polyakov gauge), with $A^0 = -iA_4$

Diagrammatic Hartree equation: 

$$S_0(p) = \quad = -(\not{p} - m_0 + \gamma^0(\mu - iA_4))^{-1}\,; \quad S(p) = \quad = -(\not{p} - m + \gamma^0(\mu - iA_4))^{-1}$$

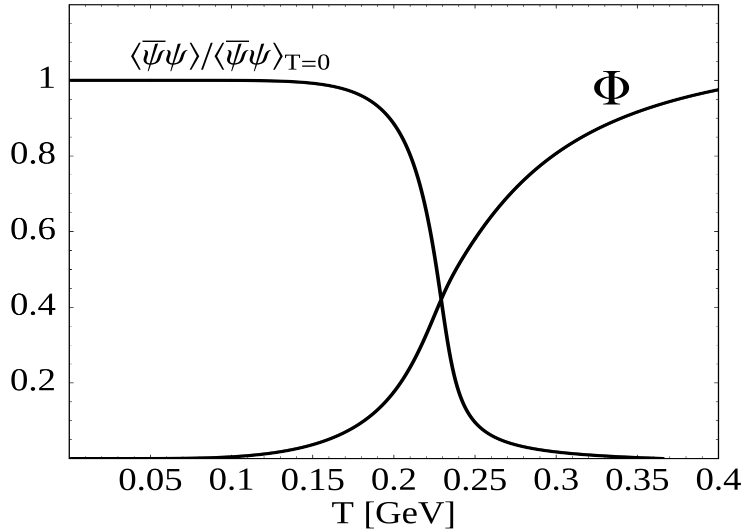
Dynamical chiral symmetry breaking $\sigma = m - m_0 \neq 0$? Solve Gap Equation! ($E = \sqrt{p^2 + m^2}$)

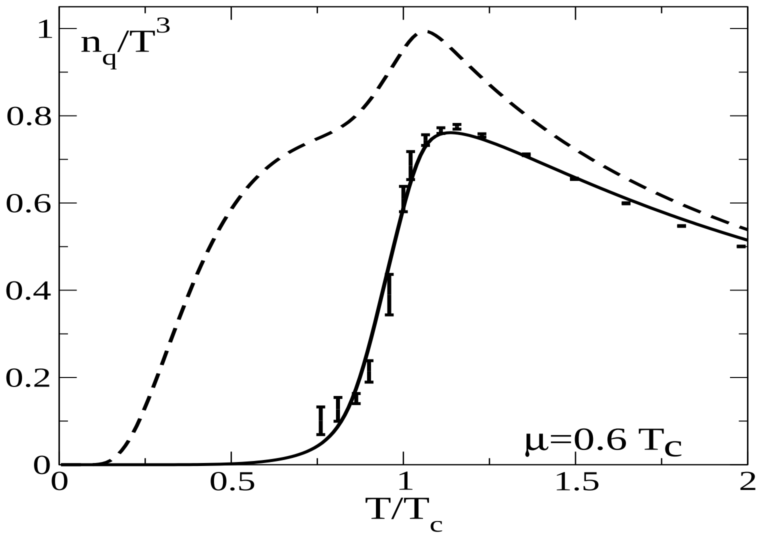
$$\begin{aligned} m - m_0 \;&=\; 2G_1 T\,\mathrm{Tr}\sum_{n=-\infty}^{+\infty}\int_\Lambda \frac{\mathrm{d}^3p}{(2\pi)^3}\frac{-1}{\not{p} - m + \gamma^0(\mu - iA_4)} \\ &=\; 2G_1 N_f N_c \int_\Lambda \frac{\mathrm{d}^3p}{(2\pi)^3}\frac{2m}{E}[1 - f^+_\Phi(E) - f^-_\Phi(E)] \end{aligned}$$

Modified quark distribution functions ($\Phi = \bar{\Phi} = 0$: "poor man's nucleon": $E_N = 3E$, $\mu_N = 3\mu$)

$$f^\pm_\Phi(E) = \frac{\left(\Phi + 2\bar{\Phi}e^{-\beta\left(E_p \mp \mu\right)}\right)e^{-\beta\left(E_p\mp\mu\right)} + e^{-3\beta\left(E_p\mp\mu\right)}}{1 + 3\left(\Phi + \bar{\Phi}e^{-\beta\left(E_p\mp\mu\right)}\right)e^{-\beta\left(E_p\mp\mu\right)} + e^{-3\beta\left(E_p\mp\mu\right)}} \longrightarrow f^\pm_0(E) = \frac{1}{1 + e^{\beta(E_N \mp \mu_N)}}$$

POLYAKOV-LOOP NAMBU–JONA-LASINIO MODEL (IV)





Grand canonical thermodynamical potential

$$\begin{aligned}\Omega(T,\mu;\Phi,m) &= \frac{\sigma^2}{2G} - 6N_f\int\frac{\mathrm{d}^3p}{(2\pi)^3}E\,\theta\left(\Lambda^2-\vec{p}^{\,2}\right)\\ &- 2N_f\,T\int\frac{\mathrm{d}^3p}{(2\pi)^3}\left\{\mathrm{Tr}_c\ln\left[1+L\,\mathrm{e}^{-(E-\mu)/T}\right]\right.\\ &+ \left.\mathrm{Tr}_c\ln\left[1+L^\dagger\,\mathrm{e}^{-(E+\mu)/T}\right]\right\}+\mathcal{U}\left(\Phi,\bar{\Phi},T\right)\end{aligned}$$

Appearance of quarks below T_c largely suppressed:

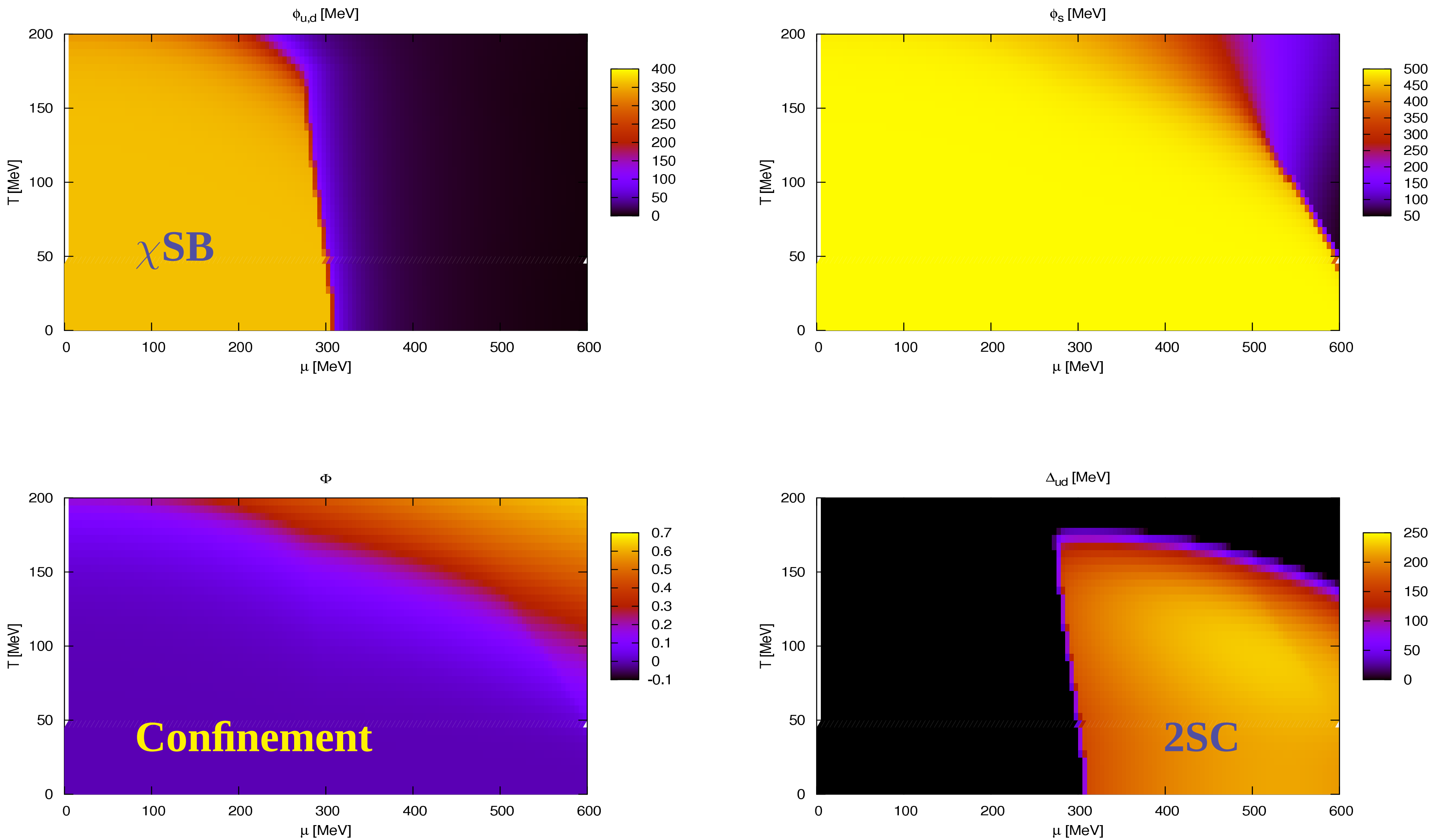
$$\begin{aligned}&\ln\det\left[1+L\,\mathrm{e}^{-(E-\mu)/T}\right]+\ln\det\left[1+L^\dagger\,\mathrm{e}^{-(E+\mu)/T}\right]\\ =\;&\ln\left[1+3\left(\Phi+\bar{\Phi}\mathrm{e}^{-(E-\mu)/T}\right)\mathrm{e}^{-(E-\mu)/T}+\mathrm{e}^{-3(E-\mu)/T}\right]\\ +\;&\ln\left[1+3\left(\bar{\Phi}+\Phi\mathrm{e}^{-(E+\mu)/T}\right)\mathrm{e}^{-(E+\mu)/T}+\mathrm{e}^{-3(E+\mu)/T}\right].\end{aligned}$$

Accordance with QCD lattice susceptibilities! Example:

$$\frac{n_q\,(T,\mu)}{T^3} = -\frac{1}{T^3}\frac{\partial\Omega\,(T,\mu)}{\partial\mu},$$

Ratti, Thaler, Weise, PRD 73 (2006) 014019.

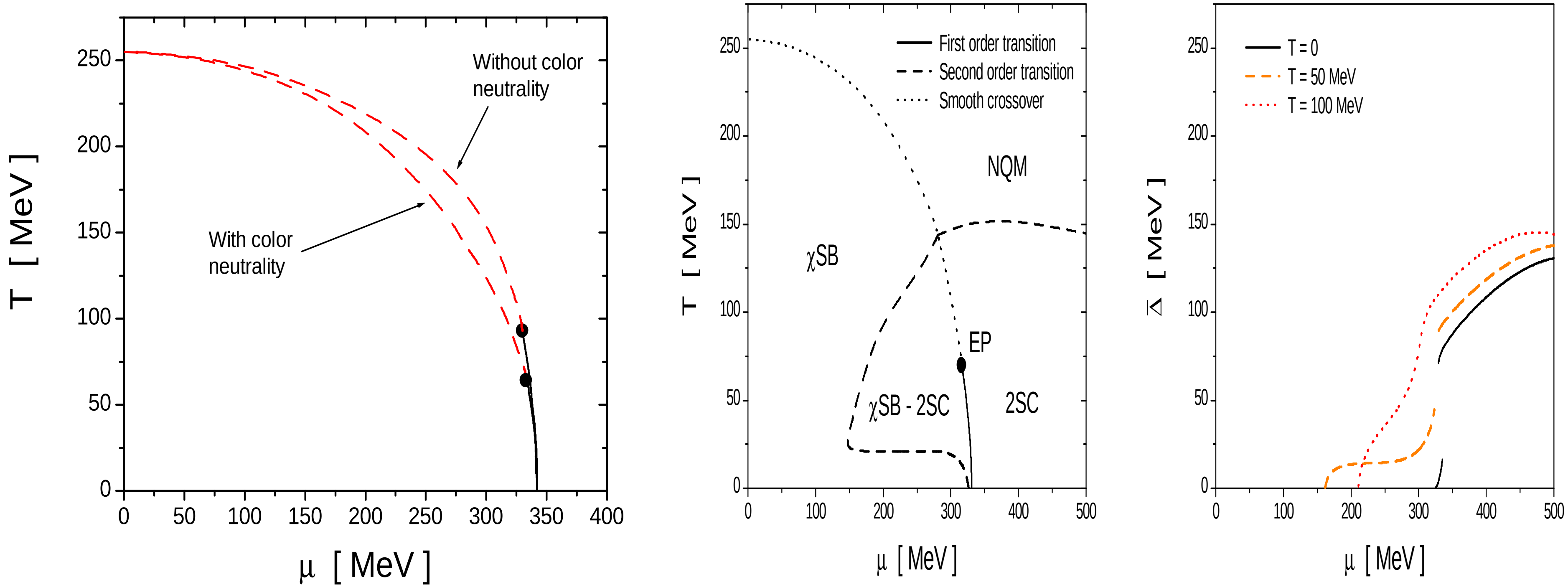
PHASES OF QCD @ EXTREMES: NO COLOR NEUTRALITY



Color Neutrality in the PNJL Phase Diagram

Color neutrality constraint: $\tilde{\mu} = \mu \mathbf{1} + \mu_8 \lambda_8 + i\phi_3 \lambda_3$; $\partial\Omega_{MF}/\partial\mu_8 = n_8 = n_r + n_g - 2n_b = 0$

Gap equations: $\partial\Omega_{MF}/(\partial\sigma, \partial\Delta, \partial\phi_3) = 0$



Gomez-Dumm, D.B., Grunfeld, Scoccola, PRD 78, 114021 (2008) [arXiv:0807.1660]

NONLOCAL POLYAKOV-LOOP CHIRAL QUARK MODEL

$$\Omega(T) \;=\; = \mathcal{U}(\Phi,\bar{\Phi}) - T\,\mathrm{Tr}_{\vec{p},n,\alpha,f,D}\left[\ln\{S_f^{-1}(p_n^\alpha,T)\} - \frac{1}{2}\Sigma_f(p_n^\alpha,T)\cdot S_f(p_n^\alpha,T)\right] ,$$

where the full quark propagator for the flavor $f = u, d, s$,

$$\begin{aligned} S_f^{-1}(p_n^\alpha,T) &= S_{f,0}^{-1}(p_n^\alpha,T) - \Sigma_f^{-1}(p_n^\alpha,T) \\ &= i\vec{\gamma}\cdot\vec{p}\,A_f((p_n^\alpha)^2,T) + i\gamma_4\omega_n\,C_f((p_n^\alpha)^2,T) + B_f((p_n^\alpha)^2,T) , \end{aligned}$$

is defined by the DSE for the quark selfenergy Σ, see below. The Polyakov-loop potential is:

$$\frac{\mathcal{U}(\Phi,\bar{\Phi})}{T^4} = -\frac{1}{2}a(T)\Phi^*\Phi + b(T)\ln\left[1 - 6\Phi^*\Phi + 4(\Phi^{*3}+\Phi^3) - 3(\Phi^*\Phi)^2\right] .$$

The Matsubara 4-momenta are defined as $(p_n^\alpha)^2 = (\omega_n^\alpha)^2 + \vec{p}^2, \quad \omega_n^\alpha = \omega_n + \alpha\phi_3, \quad \alpha = -1, 0, +1$, and are coupled to the Polyakov-loop variable $\Phi = \bar{\Phi} = \frac{1}{N_c}\left(1 + e^{i\frac{\phi_3}{T}} + e^{-i\frac{\phi_3}{T}}\right) = \frac{1}{N_c}\left(1 + 2\cos\left(\frac{\phi_3}{T}\right)\right)$. via the parameter ϕ_3.
Employing for the effective gluon propagator in a Feynman-like gauge, $g^2 D^{\mathrm{eff}}_{\mu\nu}(p-q) = \delta_{\mu\nu}D(p^2,q^2,p\cdot q)$, a rank-2 separable ansatz

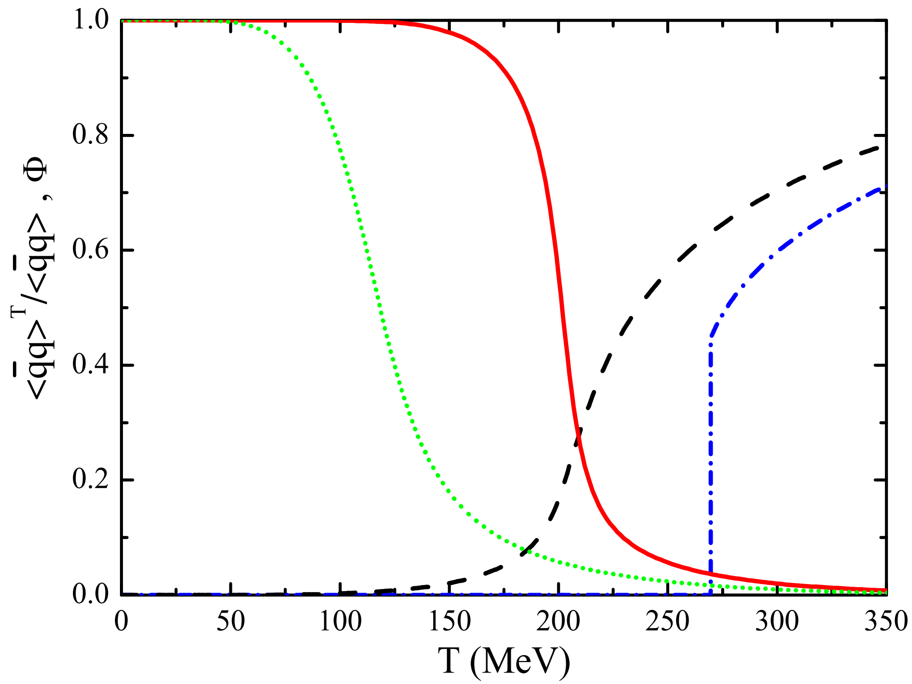
$$D(p^2,q^2,p\cdot q) = D_0\mathcal{F}_0(p^2)\mathcal{F}_0(q^2) + D_1\mathcal{F}_1(p^2)(p\cdot q)\mathcal{F}_1(q^2) ,$$

the propagator amplitudes are given by

$$\begin{aligned} B_f(p_n^2,T) &= \widetilde{m}_f + b_f(T)\mathcal{F}_0(p_n^2) , \\ A_f(p_n^2,T) &= 1 + a_f(T)\mathcal{F}_1(p_n^2) , \\ C_f(p_n^2,T) &= 1 + c_f(T)\mathcal{F}_1(p_n^2) , \end{aligned}$$

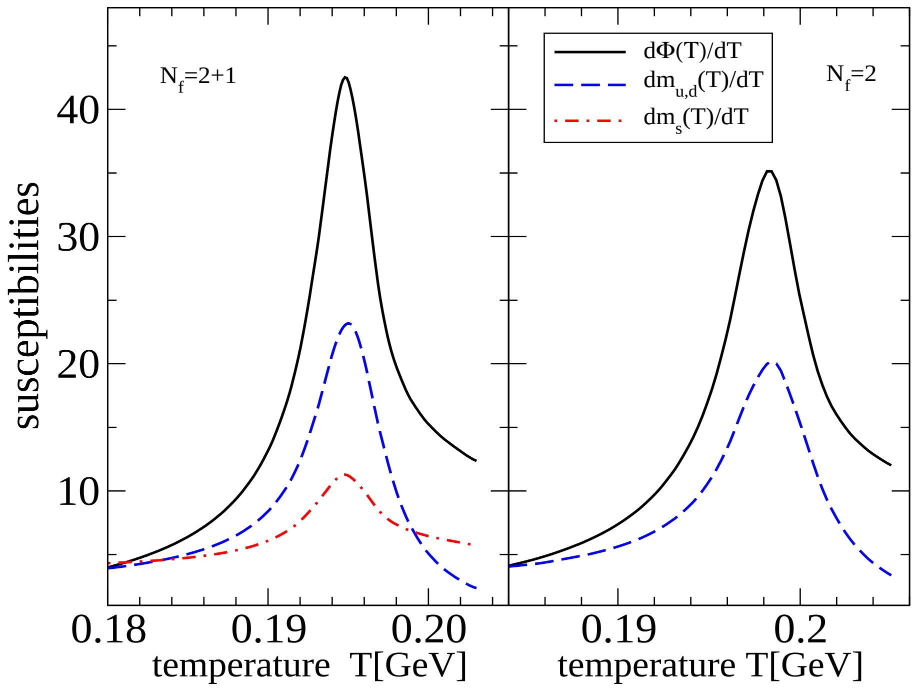
Nonlocal Polyakov Loop Chiral Quark Model

2-flavor, rank-1, 4D separable order parameters:



D.B., Buballa, Radzhabov, Volkov, Yad. Fiz. 71 (2008); arXiv:0705.0384

3-flavor, rank-2, 4D separable susceptibilities:



Horvatic, D.B., Klabucar, Kaczmarek, PRD 84 (2011)

Polyakov-loop Nambu–Jona-Lasinio Model (V)

Mesonic currents

$$J_P^a(x) = \bar{q}(x) i\gamma_5 \tau^a q(x) \quad \text{(pion)} \; ; \quad J_S(x) = \bar{q}(x)q(x) - \langle \bar{q}(x)q(x) \rangle \quad \text{(sigma)}$$

... and correlation functions

$$C_{ab}^{PP}(q^2) \equiv i \int d^4x e^{iq.x} \langle 0 | T \left(J_P^a(x) J_P^{b\dagger}(0) \right) | 0 \rangle = C^{PP}(q^2)\delta_{ab}$$

$$C_{ab}^{SS}(q^2) \equiv i \int d^4x e^{iq.x} \langle 0 | T \left(J_S(x) J_S^{\dagger}(0) \right) | 0 \rangle$$

Schwinger-Dyson Equations, $T = \mu = 0$

$$C^{MM}(q^2) = \Pi^{MM}(q^2) + \sum_{M'} \Pi^{MM'}(q^2)(2G_1)C^{M'M}(q^2)$$

One-loop polarization functions

$$\Pi^{MM'}(q^2) \equiv \int_\Lambda \frac{d^4p}{(2\pi)^4} \mathrm{Tr}\left(\Gamma_M S(p+q) \Gamma_{M'} S(q) \right)$$

Hartree quark propagator $S(p)$

Polyakov-loop Nambu–Jona-Lasinio Model (VI)

Example of the pion channel:

$$\Pi^{PP}(q^2) = -4iN_cN_f \int_\Lambda \frac{d^4p}{(2\pi)^4} \frac{m^2 - p^2 + q^2/4}{[(p+q/2)^2 - m^2][(p-q/2)^2 - m^2]} = 4iN_cN_f I_1 - 2iN_cN_f q^2 I_2(q^2)$$

Loop Integrals:

$$I_1 = \int_\Lambda \frac{d^4p}{(2\pi)^4} \frac{1}{p^2 - m^2} \quad ; \quad I_2(q^2) = \int_\Lambda \frac{d^4p}{(2\pi)^4} \frac{1}{[(p+q)^2 - m^2]\,[p^2 - m^2]}$$

With pseudoscalar decay constant (f_P) and gap equation for I_1

$$f_P^2(q^2) = -4iN_c m^2 I_2(q^2) \quad ; \quad I_1 = \frac{m - m_0}{8iG_1 m N_c N_f},$$

One obtains $\Pi^{PP}(q^2) = \frac{m-m_0}{2G_1 m} + f_P^2(q^2)\frac{q^2}{m^2}$; $\Pi^{SS}(q^2) = \frac{m-m_0}{2G_1 m} + f_P^2(q^2)\frac{q^2-4m^2}{m^2}$. In the chiral limit ($m_0 = 0$), the correlation functions

$$C^{MM}(q^2) = \Pi^{MM}(q^2) + \Pi^{MM}(q^2)(2G_1)C^{MM}(q^2) = \frac{\Pi^{MM}(q^2)}{1 - 2G_1\Pi^{MM}(q^2)} \quad , \quad M = P, S \; ,$$

have poles at $q^2 = M_P^2 = 0$ (Pion) and $q^2 = M_S^2 = (2m)^2$ (Sigma meson) $\Longrightarrow$ Check !

Polyakov-loop Nambu–Jona-Lasinio Model (VII)

Finite T, μ: $\quad p = (p_0, \vec{p}) \to (i\omega_n + \mu - iA_4, \vec{p}) \ ; \ i\int_\Lambda \frac{d^4p}{(2\pi)^4} \to -T\frac{1}{N_c}\mathrm{Tr}_c \sum_n \int_\Lambda \frac{d^3p}{(2\pi)^3}$

$$
\begin{aligned}
I_1 &= -i\int_\Lambda \frac{d^3p}{(2\pi)^3}\frac{1 - f(E_p-\mu) - f(E_p+\mu)}{2E_p} \\
I_2(\omega,\vec{q}) &= i\int_\Lambda \frac{d^3p}{(2\pi)^3}\frac{1}{2E_p 2E_{p+q}}\frac{f(E_p+\mu)+f(E_p-\mu)-f(E_{p+q}+\mu)-f(E_{p+q}-\mu)}{\omega - E_{p+q}+E_p} \\
&\quad +i\int_\Lambda \frac{d^3p}{(2\pi)^3}\frac{1-f(E_p-\mu)-f(E_{p+q}+\mu)}{2E_p 2E_{p+q}}\left(\frac{1}{\omega+E_{p+q}+E_p}-\frac{1}{\omega-E_{p+q}-E_p}\right)
\end{aligned}
$$

For a meson at rest in the medium ($\vec{q} = 0$)

$$
I_2\left(\omega,\vec{0}\right) = -i\int_\Lambda \frac{d^3p}{(2\pi)^3}\frac{1-f(E_p+\mu)-f(E_p-\mu)}{E_p\left(\omega^2-4E_p^2\right)}
$$

which develops an imaginary part

$$
\Im m \ (-iI_2(\omega,0)) = \frac{1}{16\pi}\left(1 - f\left(\frac{\omega}{2}-\mu\right) - f\left(\frac{\omega}{2}+\mu\right)\right)\sqrt{\frac{\omega^2-4m^2}{\omega^2}}\times\Theta(\omega^2-4m^2)\Theta(4(\Lambda^2+m^2)-\omega^2)
$$

with the Pauli-blocking factor: $N(\omega,\mu) = \left(1 - f\left(\frac{\omega}{2}-\mu\right) - f\left(\frac{\omega}{2}+\mu\right)\right)$

POLYAKOV-LOOP NAMBU–JONA-LASINIO MODEL (VIII)

Spectral function

$$F^{MM}(\omega,\vec{q}) \equiv \Im m\, C^{MM}(\omega+i\eta,\vec{q}) = \Im m\, \frac{\Pi^{MM}(\omega+i\eta,\vec{q})}{1-2G_1\Pi^{MM}(\omega+i\eta,\vec{q})}.$$

$$F^{MM}(\omega) = \frac{\pi}{2G_1}\frac{1}{\pi}\frac{2G_1\Im m\,\Pi^{MM}(\omega+i\eta)}{\left(1-2G_1\Re e\,\Pi^{MM}(\omega)\right)^2+\left(2G_1\Im m\,\Pi^{MM}(\omega+i\eta)\right)^2}.$$

For $\omega < 2m(T,\mu)$, $\Im m\,\Pi = 0$: decay channel closed $\to$ bound state!

$$F^{MM}(\omega) = \frac{\pi}{2G_1}\delta\left(1-2G_1\Re e\,\Pi^{MM}(\omega)\right) = \frac{\pi}{4G_1^2\left|\frac{\partial \Re e\,\Pi^{MM}}{\partial\omega}\right|_{\omega=m_M}}\delta(\omega-m_M)\ .$$

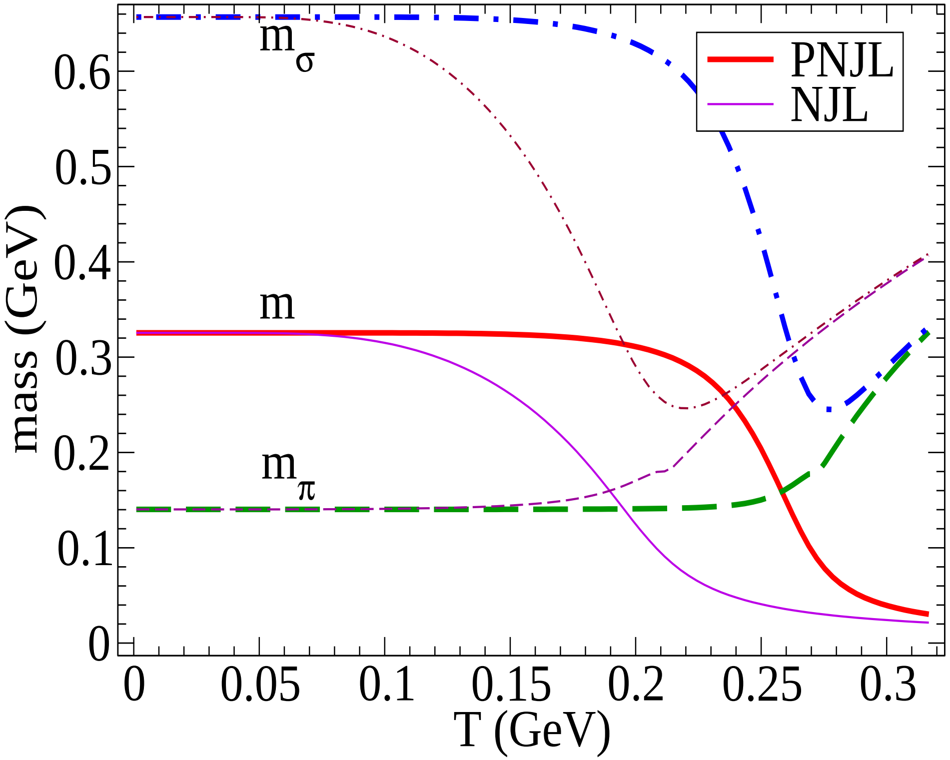
The meson mass m_M is the solution of

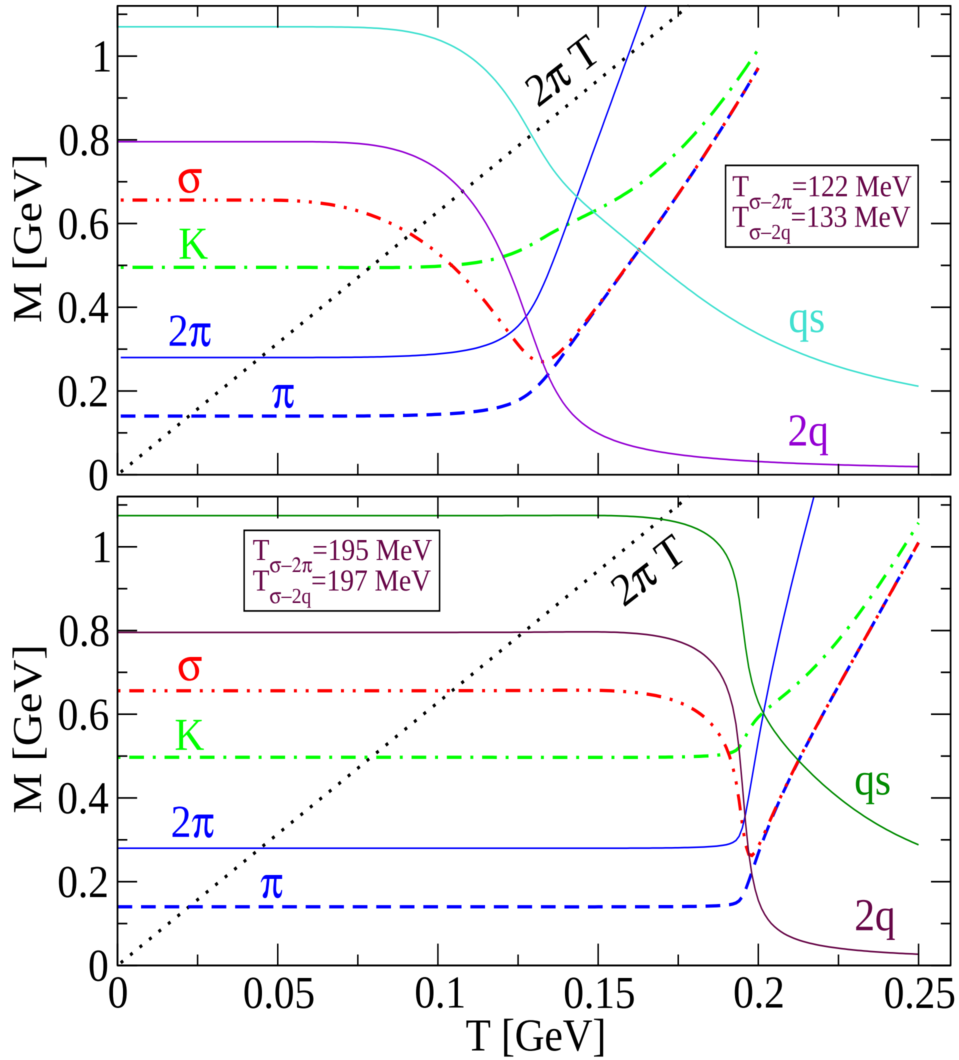
$$1-2G_1\Re e\,\Pi^{MM}(m_M) = 0$$

The decay width (inverse lifetime) is

$$\Gamma_M = 2G_1\Im m\,\Pi^{MM}(m_M)$$

PNJL vs. NJL model: mass spectrum



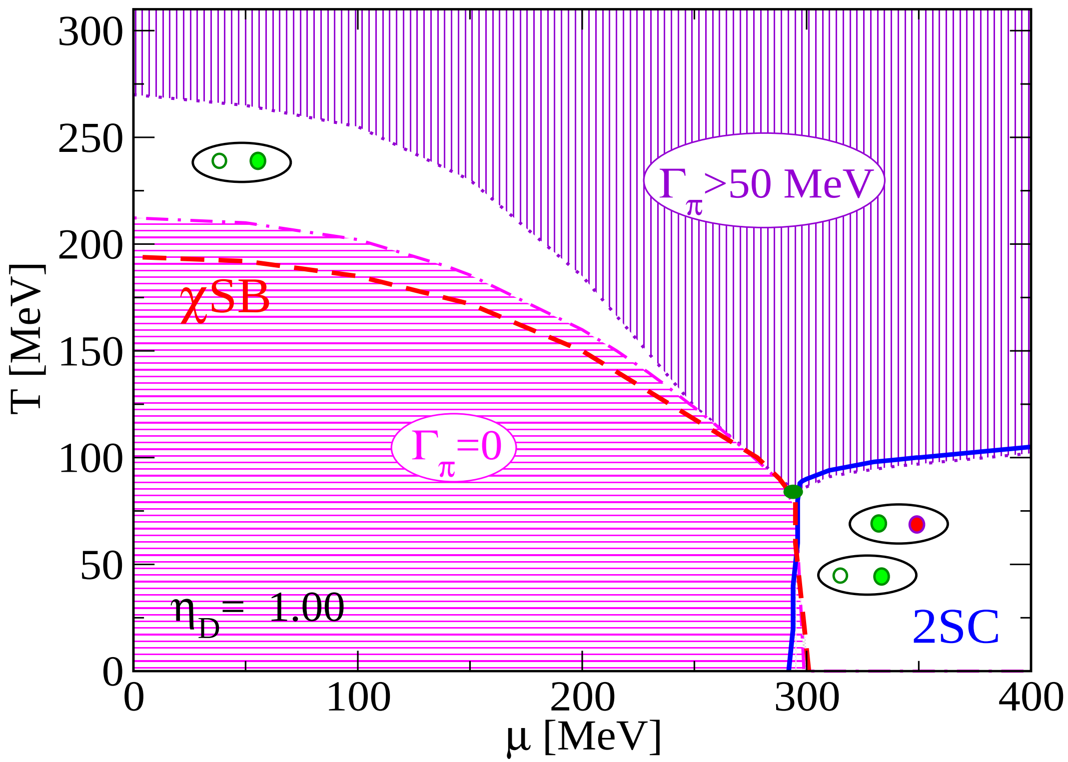


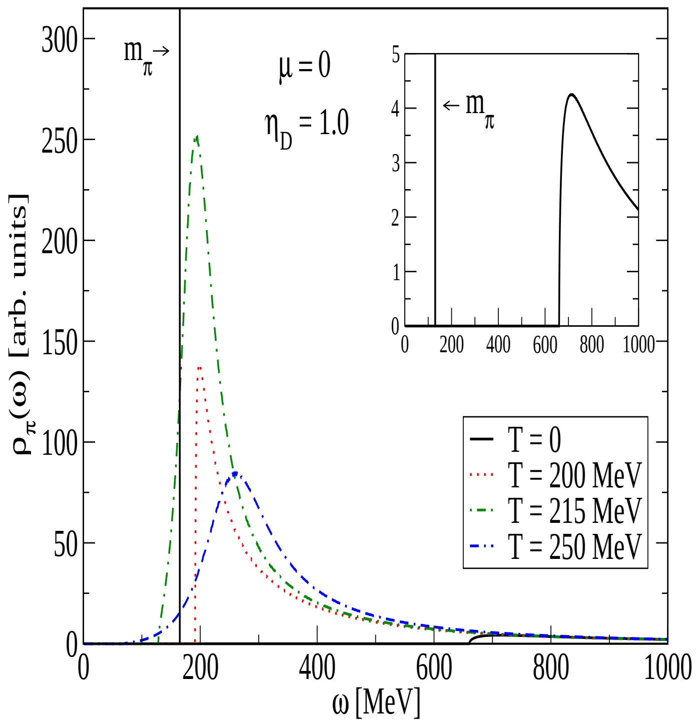
H. Hansen et al., PRD 75, 065004 (2007) ↑

D. Horvatic et al., PRD 84, 016005 (2011) →

PION CORRELATIONS IN THE PHASE DIAGRAM

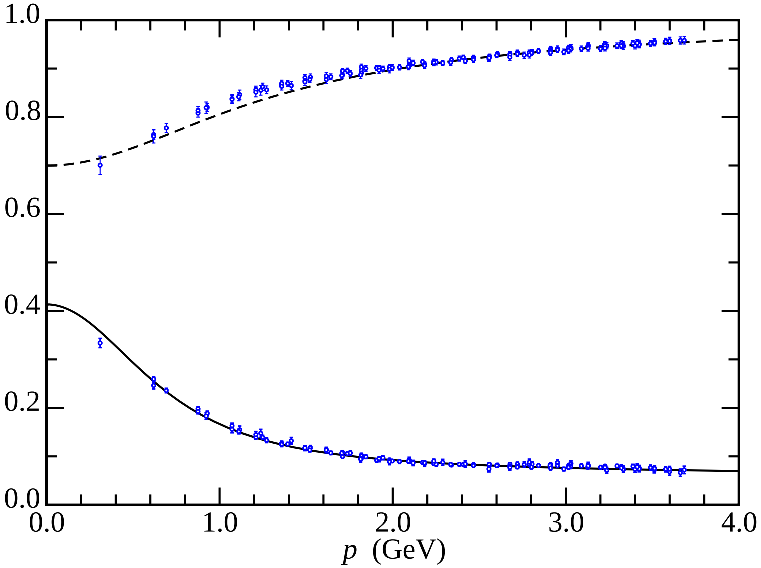
Stable pions for a HIC fireball lifetime $\tau_{\text{fireball}} < 4$ fm/c





Zablocki, D.B., Anglani, AIP Conf. Proc. 1038, 159 (2008); arXiv:0805.2687 [hep-ph]

Complex mass pole fit to Lattice propagator



Bhagwat, Pichowsky, Roberts, Tandy, Phys. Rev. **C68** (2003) 015203

$$S(p)^{-1} = i\not p A(p^2) + B(p^2)\ ,$$

$$M(p^2) = B(p^2)/A(p^2)$$

$$Z(p^2) = 1/A(p^2)$$

$S(p)$ sum of N pairs of complex conj. mass poles

$$S(p) = \sum_{i=1}^{N} \frac{1}{Z_2} \left\{ \frac{z_i}{i\not p + m_i} + \frac{z_i^*}{i\not p + m_i^*} \right\} = -i\not p \sigma_V(p^2) + \sigma_S(p^2)$$

Representation of the scalar amplitude

$$\sigma_S(p^2) = \sum_{i=1}^{N} Z_2^{-1} \left\{ \frac{z_i\, m_i}{p^2 + m_i^2} + \frac{z_i^*\, m_i^*}{p^2 + m_i^{*2}} \right\}$$

"Derivation" of the equivalent separable model (in Feynman-like gauge) $D_{\mu\nu}(p-q) = \delta_{\mu\nu}\, D(p,q)$ and

$$\begin{aligned} D(p,q) &= f_0(p^2)\, f_0(q^2) + f_1(p^2)\, p\cdot q\, f_1(q^2) \\ f_1(p^2) &= \frac{A(p^2)-1}{a} \quad ; \quad f_0(p^2) = \frac{B(p^2)-m_c}{b} \end{aligned}$$

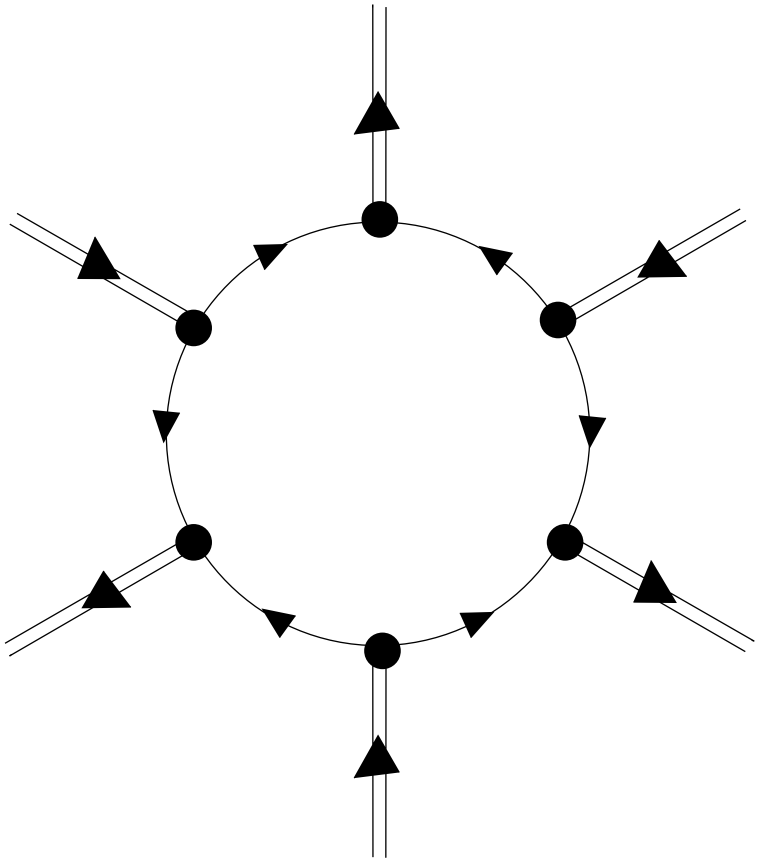
$$b^2 = \frac{16}{3}\int_q^{\Lambda} [B(q^2) - m_c]\sigma_s(q^2)$$

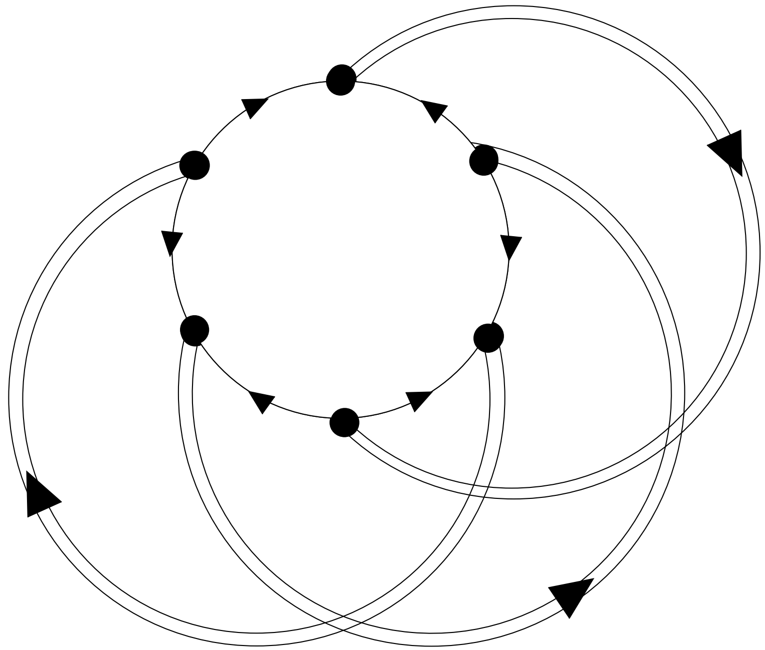
$$a^2 = \frac{8}{3}\int_q^{\Lambda} [A(q^2) - 1]\frac{q^2}{4}\sigma_v(q^2)$$

NUCLEONS IN THE NONLOCAL CHIRAL QUARK MODEL

$$Z_{\text{fluct}} = \int D\Delta^\dagger D\Delta \exp\{-\frac{|\Delta|^2}{4G_D} - Tr\ lnS^{-1}[\Delta,\Delta^\dagger]\}$$

Cahill, Roberts, Prashifka: Aust. J. Phys. 42 (1989) 129, 161
Cahill, ibid, 171; Reinhardt: PLB 244 (1990) 316; Buck, Alkofer, Reinhardt: PLB 286 (1992) 29





Quark sextett (diquark triplett): bound by exchange forces? sextett condensate?

Summary

- Compressed nuclear matter: **quarkyonic phase** (QP)! Coexisting chiral symm. + conf.
- Similarities: Mott-Hagedorn picture, string-flip model, confining DSE
- Here: PNJL model as microscopic formulation of the QP
- Color singlet quark triplets in chiral phase for $\mu > \mu_c$ (approx. massless baryons)
- Color neutrality by singlet projection = sum over color hexagon
- Prospects for CBM & NICA: dilepton enhancement (peak?) from diquark-antidiquark annih.
- Preparatory step to **compact stars**: single flavor CSL phase - OK with structure & cooling

Outlook: next steps ...

- Walecka model as limit of PNJL model: chiral transition effects in nuclear EoS
- Beyond meanfield: mesons and baryons in the PNJL, higher clusters: sextetting
- Astrophysics: Maximum mass & cooling of quarkyonic stars; quarkyonic supernovae