HADRONS AND HADRONIC MATTER IN CHIRAL QUARK MODELS

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II. Order parameters, gap equations, phase diagram

III. Mesonic fluctuations, Bethe-Salpeter equation, Mott effect

IV. Hadron resonance gas, chemical freeze-out, QCD phase transition

V. Signals of the QGP, electromagnetic probes etc.

Hadrons and hadronic matter in chiral quark models

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Main motivations for effective chiral quark models of QCD$|_{T,\mu}$:

- Qualitative and quantitative interpretation and understanding of lattice QCD results
- Extension from finite-T/low-μ to low-T/high-μ region of the QCD phase diagram
- Application to HIC energy scan programs (RHIC, SPS, NICA, CBM) and compact stars
- Calculation of in-medium processes, prediction of QGP signals

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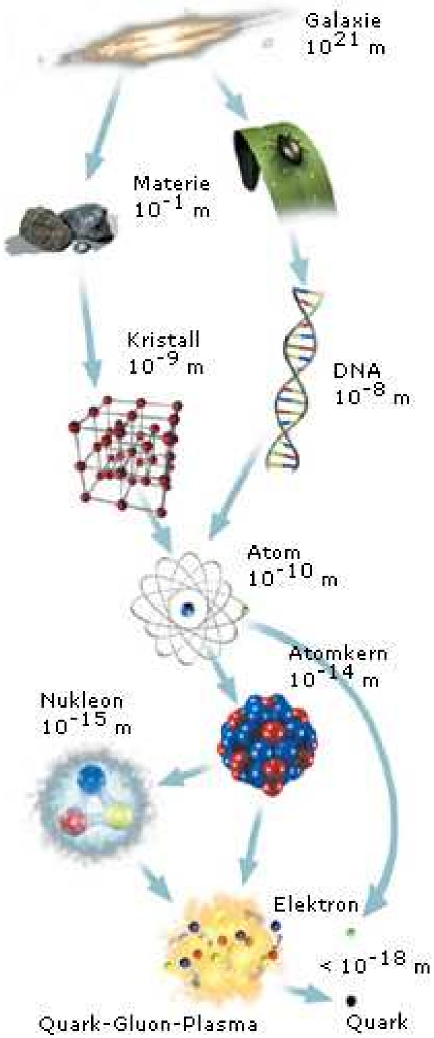
MANY PARTICLE SYSTEMS & QUANTUM FIELD THEORY



Elements	Bound states	System
humans, animals	couples, groups, parties	society
molecules, crystals	(bio)polymers	animals, plants
atoms	molecules, clusters, crystals	solids, liquids, ...
ions, electrons	atoms	plasmas
nucleons, mesons	nuclei	nuclear matter
quarks, anti-quarks	nucleons, mesons	quark matter

Highly Compressed Matter $\Leftrightarrow$ Pauli Principle

Partition function: $Z = \mathrm{Tr}\left\{\mathrm{e}^{-\beta(H-\mu_i Q_i)}\right\}$

PARTITION FUNCTION FOR QUANTUM CHROMODYNAMICS (QCD)

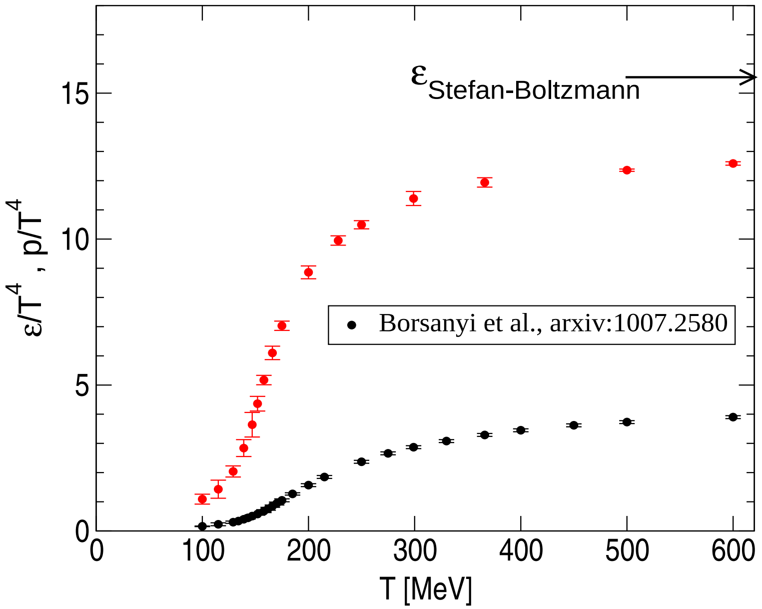
- Partition function as a Path Integral (imaginary time $\tau = i\,t$, $0 \le \tau \le \beta = 1/T$) $\Rightarrow$ PS I

$$Z[T,V,\mu] = \int \mathcal{D}\bar{\psi}\mathcal{D}\psi\mathcal{D}A \exp\left\{-\int_0^{\beta} d\tau \int_V d^3x\, \mathcal{L}_{QCD}(\psi,\bar{\psi},A)\right\}$$

- QCD Lagrangian, non-Abelian gluon field strength: $F^a_{\mu\nu}(A) = \partial_\mu A^a\nu - \partial_\nu A^a_\mu + g\, f^{abc}[A^b_\mu, A^c_\nu]$

$$\mathcal{L}_{QCD}(\psi,\bar{\psi},A) = \bar{\psi}[i\gamma^\mu(\partial_\mu - igA_\mu) - m - \gamma^0\mu]\psi - \frac{1}{4}F^a_{\mu\nu}(A)F^{a,\mu\nu}(A)$$

- Numerical evaluation: Lattice gauge theory simulations (hotQCD, Wuppertal-Budapest)



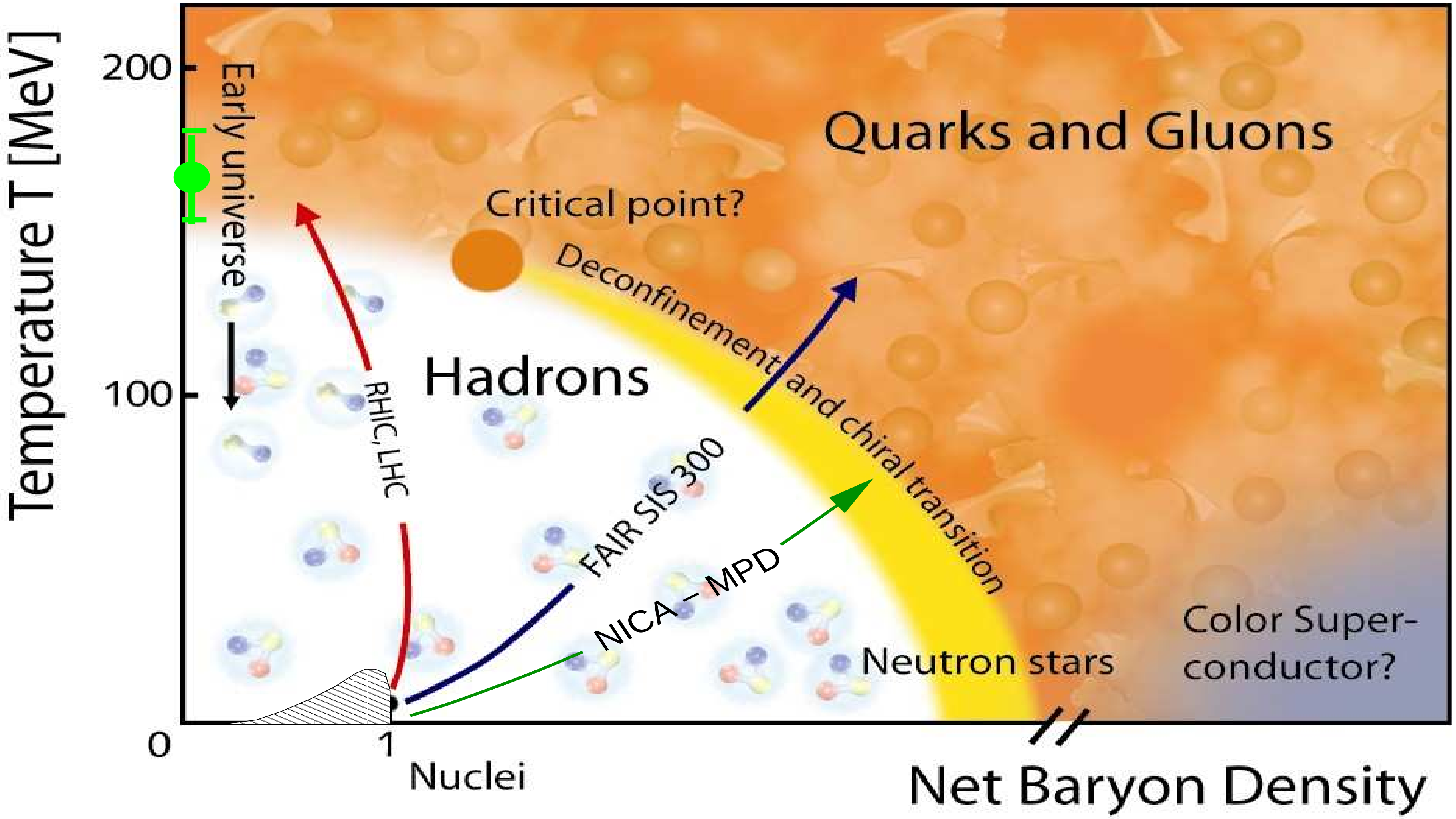
- Equation of state: $\varepsilon(T) = -\partial \ln Z[T,V,\mu]/\partial\beta$
- Phase transition at $T_c = 170$ MeV
- Problem: Interpretation ?

$\varepsilon/T^4 = \frac{\pi^2}{30}N_\pi \sim 1$ (ideal pion gas)

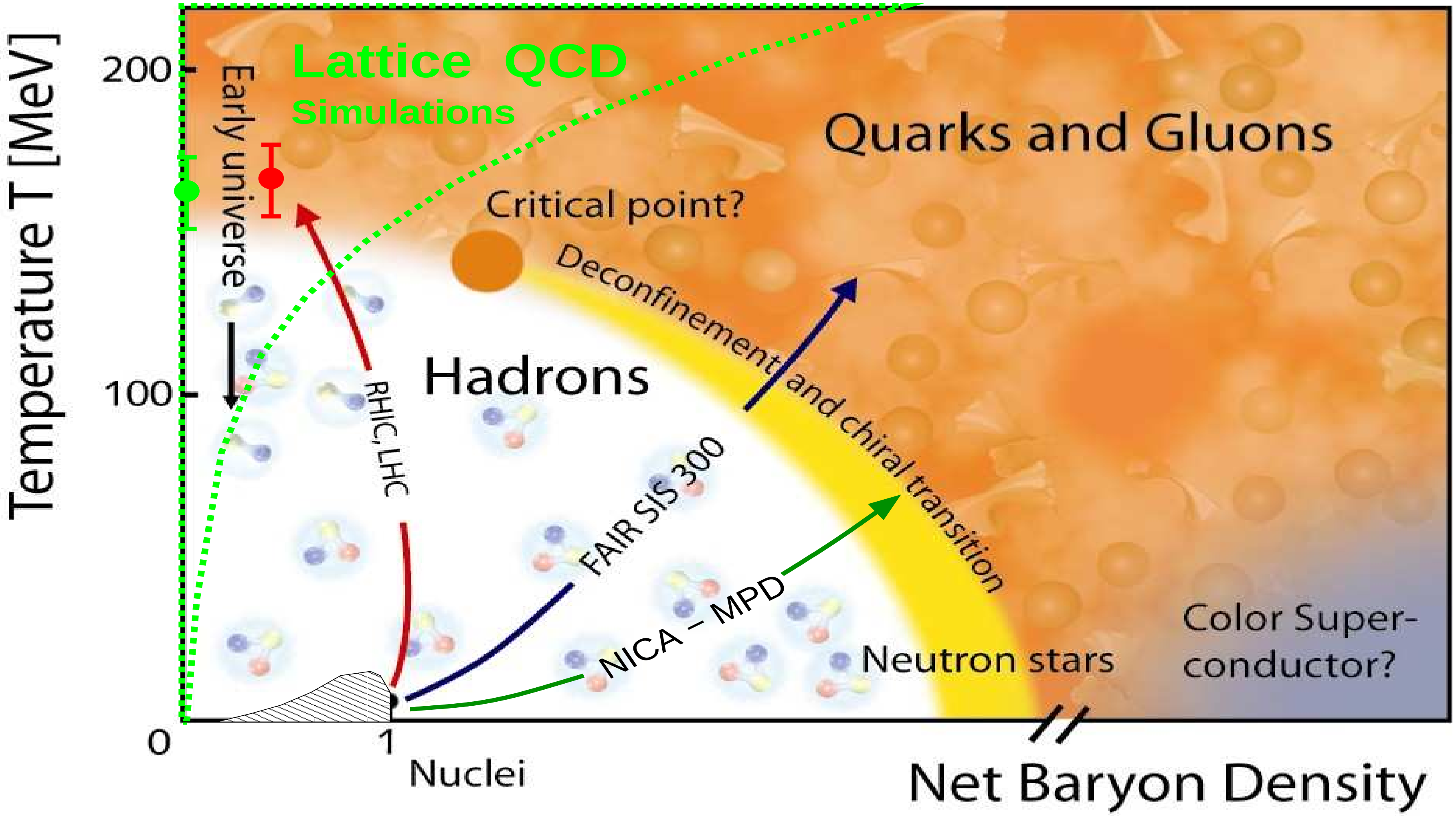
$\varepsilon/T^4 = \frac{\pi^2}{30}(N_G + \frac{7}{8}N_Q) \sim 15.6$ (quarks and gluons)

$\Longrightarrow$ Borsanyi et al., arxiv:1007.2580

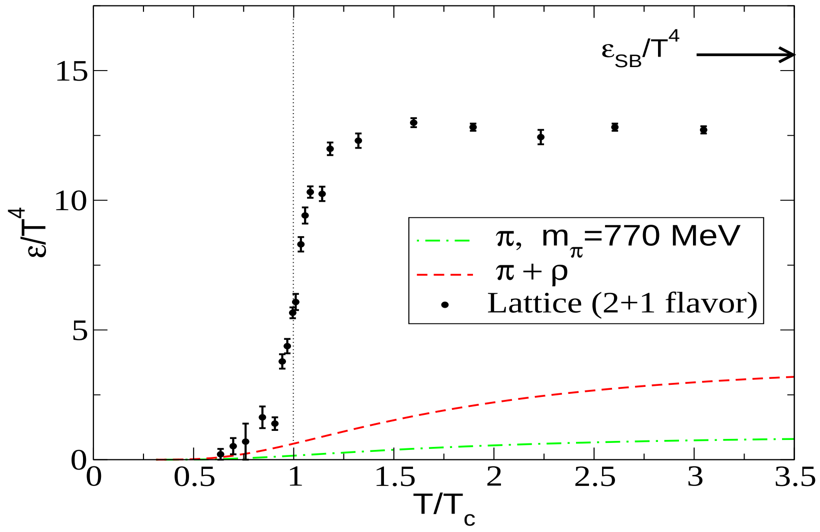
PHASEDIAGRAM OF QCD: LATTICE SIMULATIONS



PHASEDIAGRAM OF QCD: LATTICE SIMULATIONS



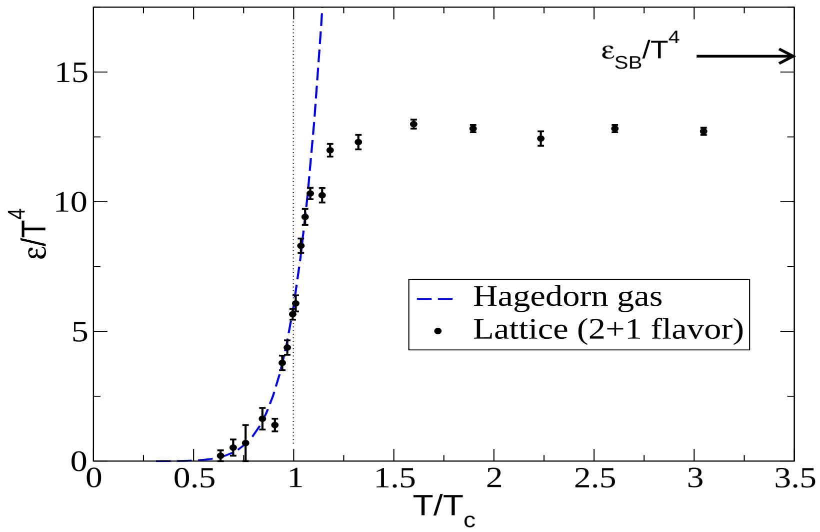
Lattice QCD EoS vs. resonance gas



Ideal hadron gas mixture ...

$$\varepsilon(T) = \sum_{i=\pi,\rho,\ldots} g_i \int \frac{d^3p}{(2\pi)^3} \frac{\sqrt{p^2+m_i^2}}{\exp(\sqrt{p^2+m_i^2}/T)+\delta_i}$$

missing degrees of freedom below and above T_c



Resonance gas ...

Karsch, Redlich, Tawfik, Eur.Phys.J. C29, 549 (2003)

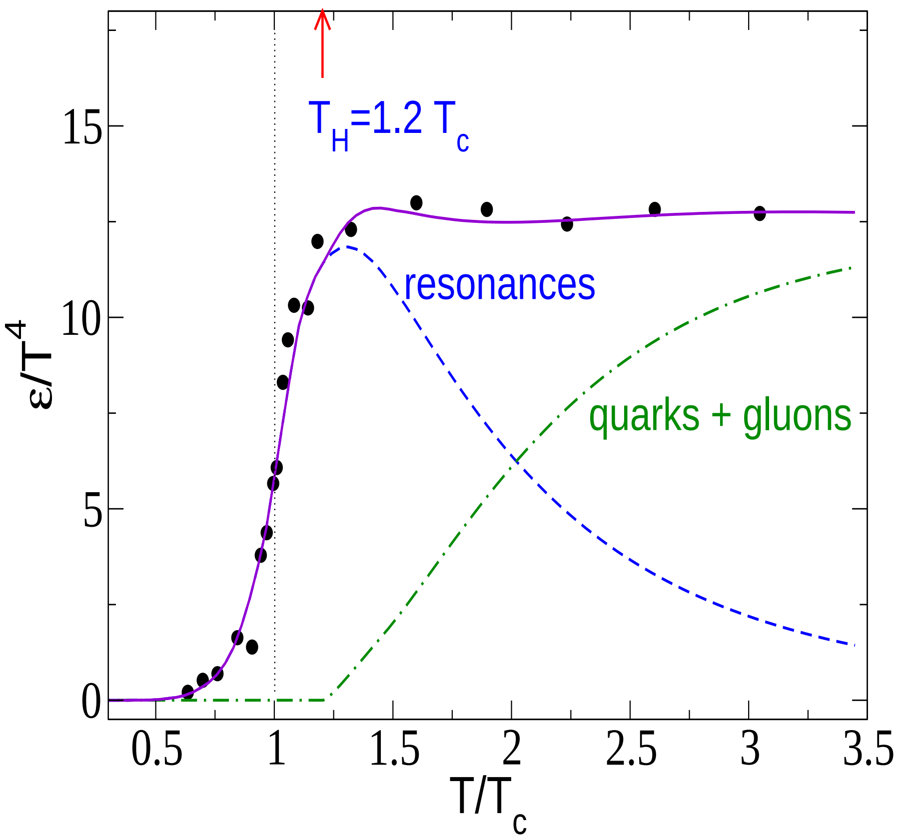
$$\begin{aligned} \varepsilon(T) &= \sum_{i=\pi,\rho,\ldots} \varepsilon_i(T) \\ &+ \sum_{r=M,B} g_r \int dm\, \rho(m) \int \frac{d^3p}{(2\pi)^3} \frac{\sqrt{p^2+m^2}}{\exp(\sqrt{p^2+m^2}/T)+\delta_r} \end{aligned}$$

$\rho(m) \sim m^\beta \exp(m/T_H)$ ... Hagedorn mass spektrum

too many degrees of freedom above T_c

Lattice QCD EoS and Mott-Hagedorn Gas

$$\varepsilon_{\rm R}(T,\{\mu_j\}) = \sum_{i=\pi,K,\dots} \varepsilon_i(T,\{\mu_i\}) + \sum_{r=M,B} g_r \int_{m_r} dm \int ds\, \rho(m) A(s,m;T) \int \frac{d^3p}{(2\pi)^3} \frac{\sqrt{p^2+s}}{\exp\left(\frac{\sqrt{p^2+s}-\mu_r}{T}\right)+\delta_r}$$



Hagedorn mass spectrum: $\rho(m)$

Spectral function for heavy resonances:

$$A(s,m;T) = N_s \frac{m\Gamma(T)}{(s-m^2)^2 + m^2\Gamma^2(T)}$$

Ansatz with Mott effect at $T = T_H = 192$ MeV:

$$\Gamma(T) = B\Theta(T-T_H)\left(\frac{m}{T_H}\right)^{2.5}\left(\frac{T}{T_H}\right)^{6}\exp\left(\frac{m}{T_H}\right)$$

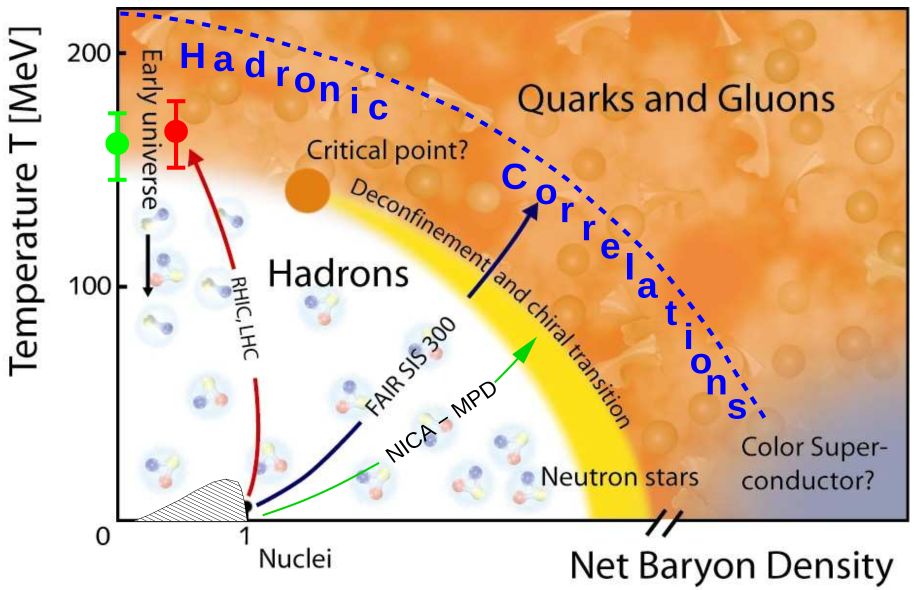
No width below T_H: Hagedorn resonance gas
Apparent phase transition at $T_c \sim 160$ MeV

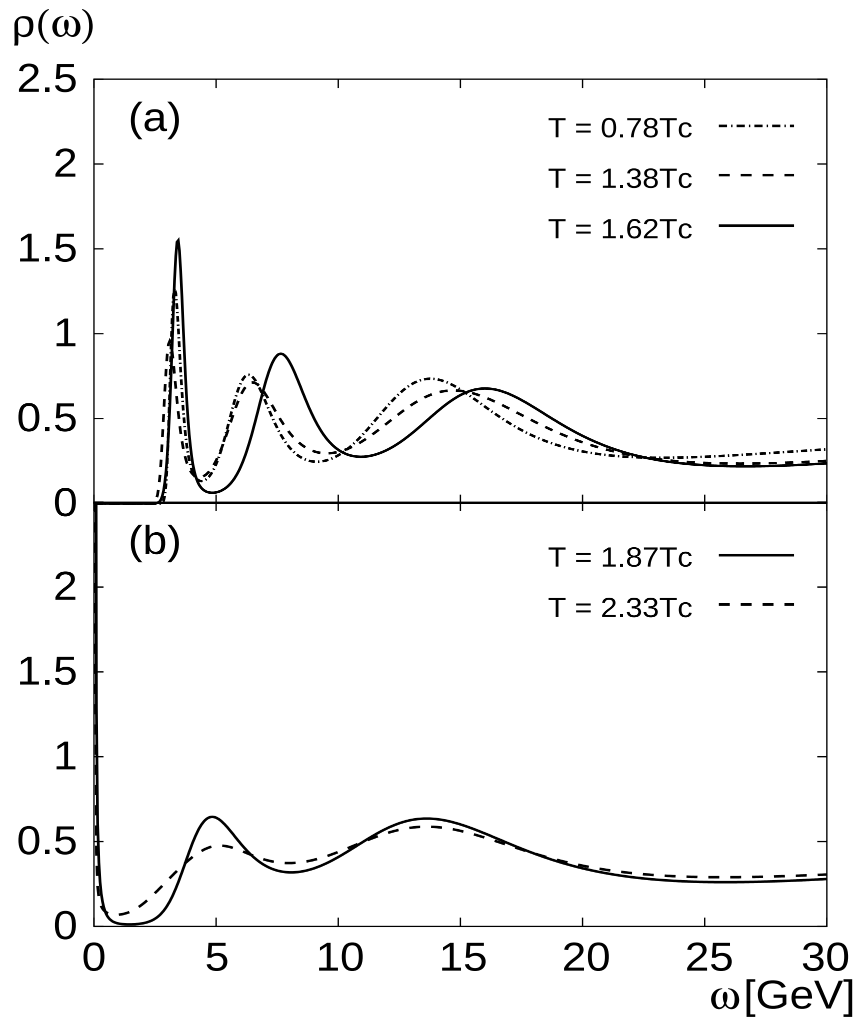
Blaschke & Bugaev, Fizika B13, 491 (2004)

Prog. Part. Nucl. Phys. 53, 197 (2004)

Blaschke & Yudichev (2006)

HADRONIC CORRELATIONS ABOVE T_c: LATTICE QCD





Hadron correlators $G_H \Longrightarrow$ spectral densities $\rho_H(\omega, T)$

$$G_H(\tau, T) = \int_0^\infty d\omega \rho_H(\omega, T) \frac{\cosh(\omega(\tau - T/2))}{\sinh(\omega/2T)}$$

Maximum entropy method

Karsch et al. PLB 530 (2002) 147

Result:

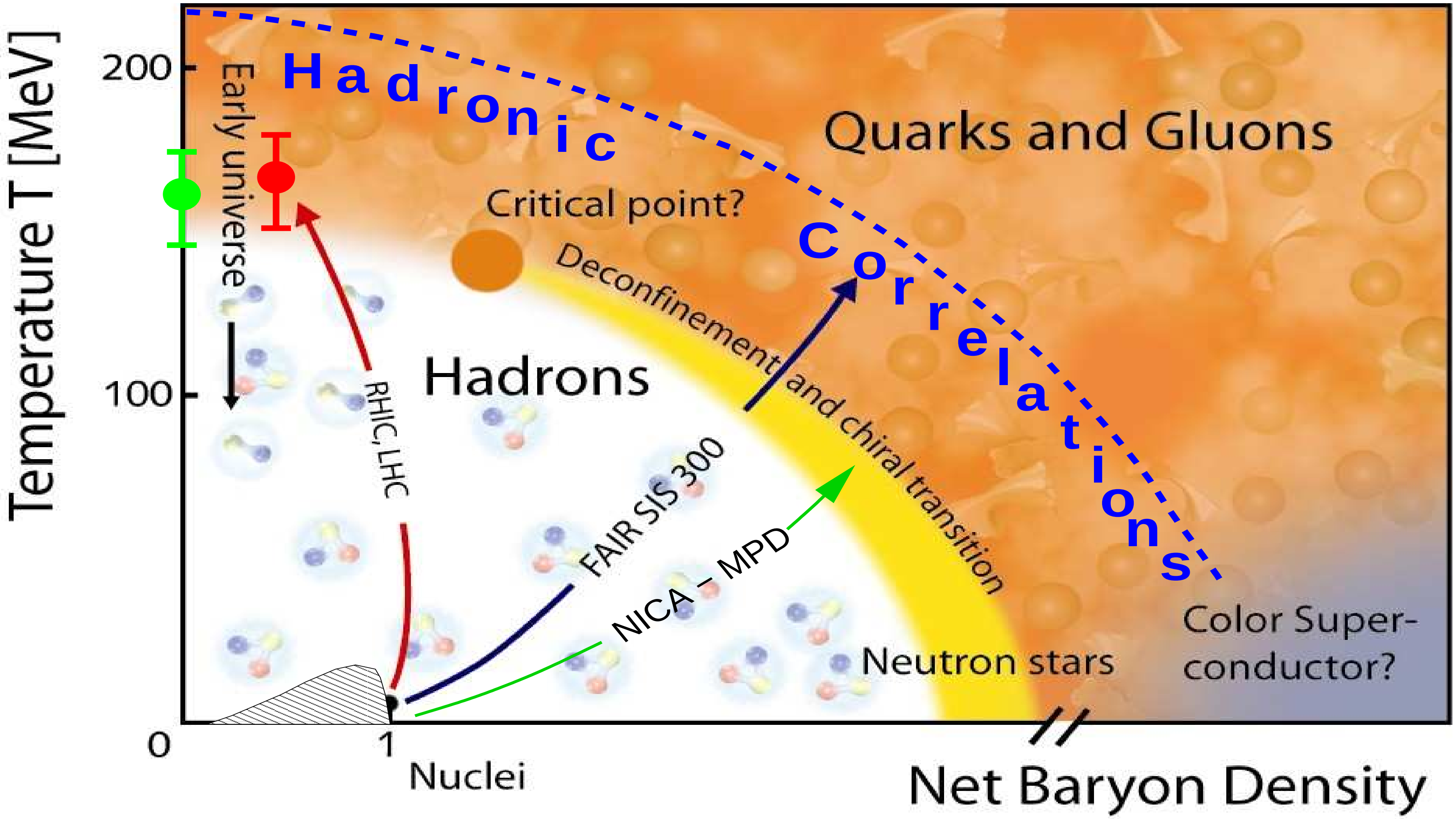
Correlations persist above T_c !

Karsch et al. NPA 715 (2003)

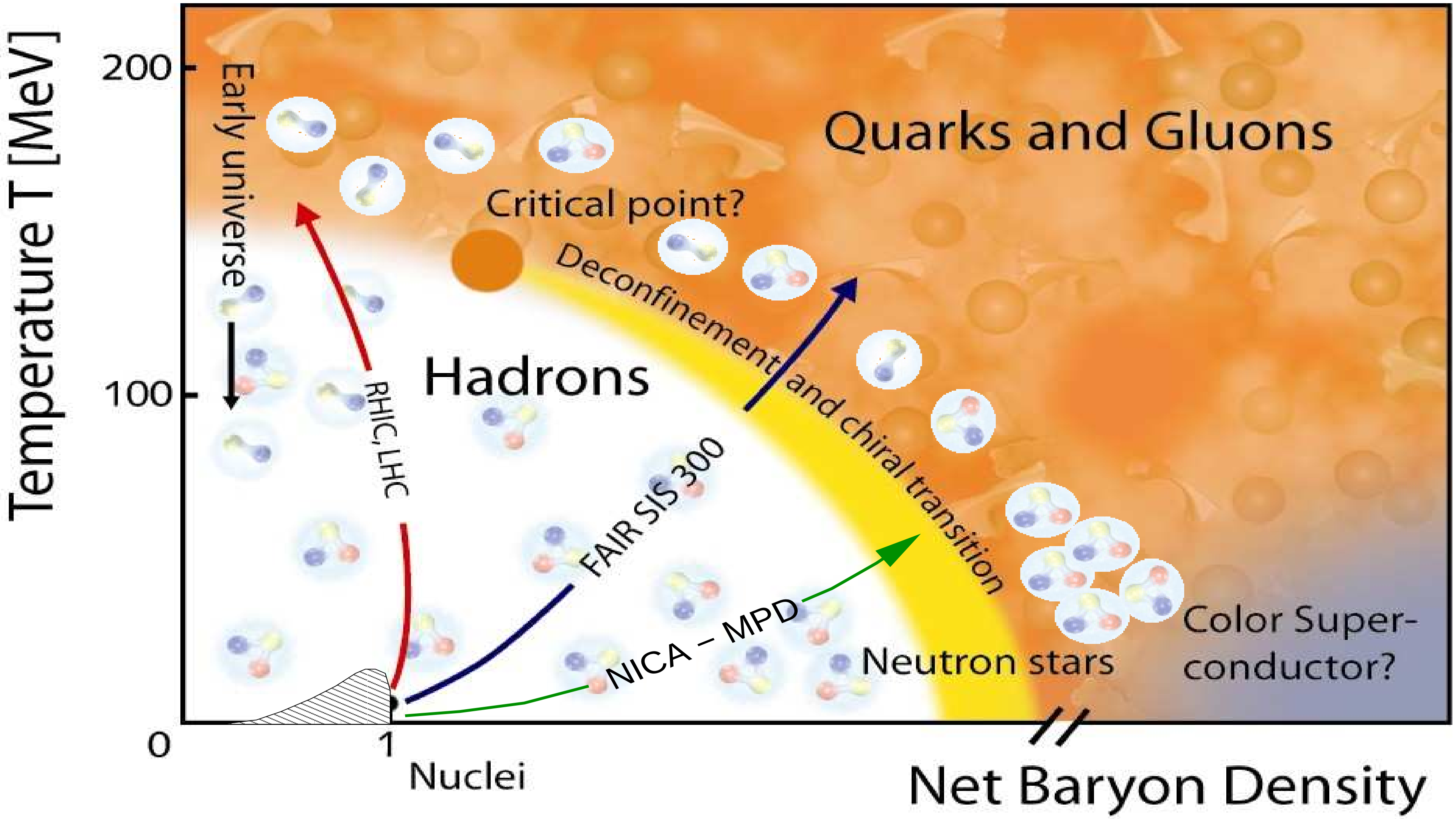
J/ψ and η_c survive up to $T \sim 1.6T_c$

Asakawa, Hatsuda; PRL 92 (2004) 012001

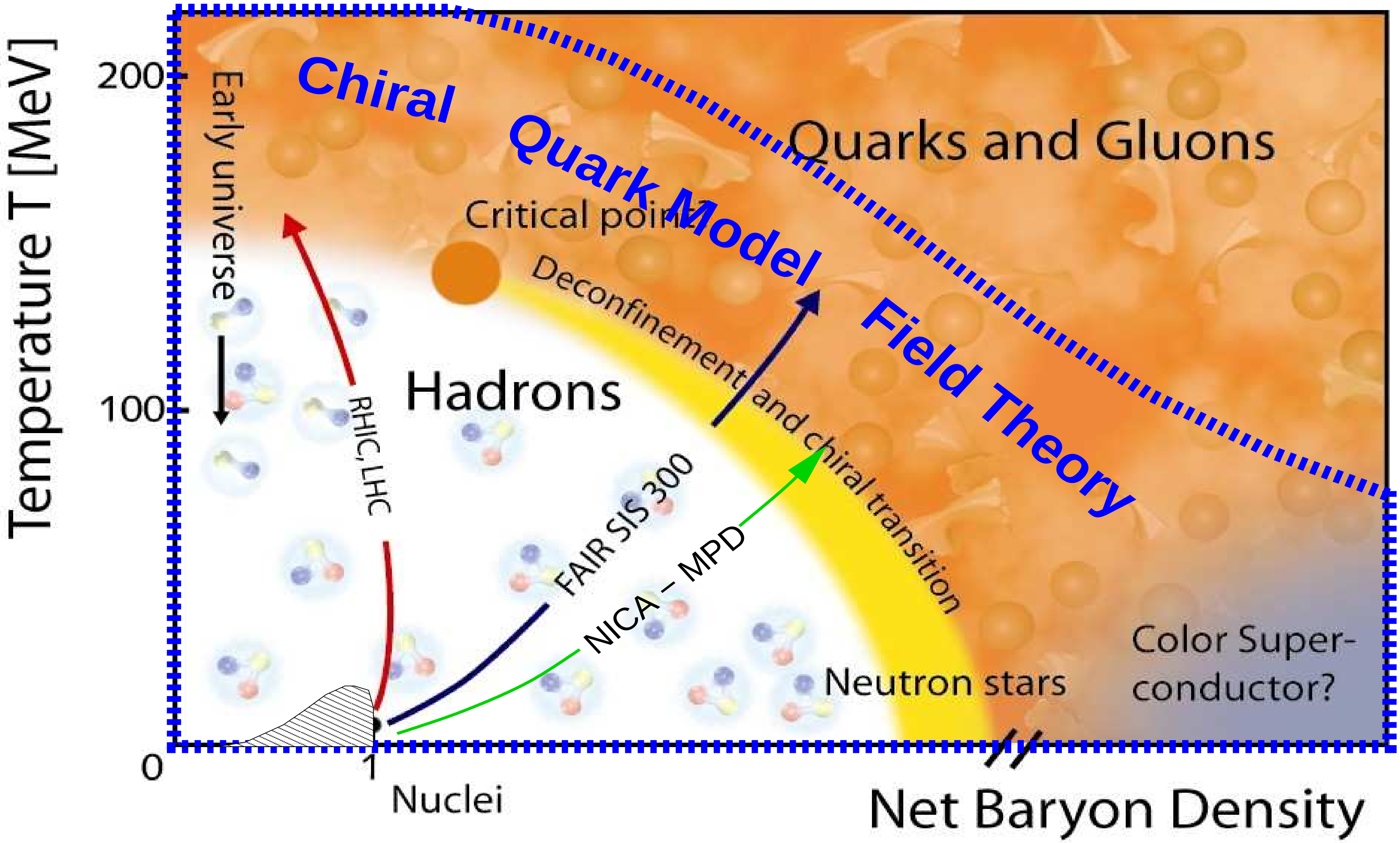
HADRONIC CORRELATIONS IN THE PHASEDIAGRAM OF QCD



HADRONIC CORRELATIONS IN THE PHASEDIAGRAM OF QCD



PHASEDIAGRAM OF QCD: CHIRAL MODEL FIELD THEORIES



Chiral Model Field Theory for Quark Matter

- Partition function as a Path Integral (imaginary time $\tau = i\, t$)

$$Z[T,V,\mu] = \int \mathcal{D}\bar{\psi}\mathcal{D}\psi \exp\left\{-\int^{\beta} d\tau \int_V d^3x [\bar{\psi}(i\gamma^\mu\partial_\mu - m - \gamma^0\mu)\psi - \mathcal{L}_{\rm int}]\right\}$$

- Current-current interaction (4-Fermion coupling)

$$\mathcal{L}_{\rm int} = \sum_{M=\pi,\sigma,\ldots} G_M(\bar{\psi}\Gamma_M\psi)^2 + \sum_D G_D(\bar{\psi}^C\Gamma_D\psi)^2$$

- Bosonization (Hubbard-Stratonovich Transformation)

$$Z[T,V,\mu] = \int \mathcal{D}M_M\mathcal{D}\Delta_D^\dagger\mathcal{D}\Delta_D \exp\left\{-\sum_M \frac{M_M^2}{4G_M} - \sum_D \frac{|\Delta_D|^2}{4G_D} + \frac{1}{2}\mathrm{Tr}\,\ln S^{-1}[\{M_M\},\{\Delta_D\}]\right\}$$

- Collective (stochastic) fields: Mesons (M_M) and Diquarks (Δ_D)
- Systematic evaluation: Mean fields + Fluctuations
  - Mean-field approximation: order parameters for phase transitions (gap equations)
  - Lowest order fluctuations: hadronic correlations (bound & scattering states)
  - Higher order fluctuations: hadron-hadron interactions

NJL MODEL FOR NEUTRAL 3-FLAVOR QUARK MATTER

Thermodynamic Potential $\Omega(T,\mu) = -T \ln Z[T,\mu]$

$$\Omega(T,\mu) \;=\; \frac{\phi_u^2+\phi_d^2+\phi_s^2}{8G_S} + \frac{|\Delta_{ud}|^2+|\Delta_{us}|^2+|\Delta_{ds}|^2}{4G_D} - T\sum_n \int \frac{d^3p}{(2\pi)^3}\frac{1}{2}\mathrm{Tr}\ln\left(\frac{1}{T}S^{-1}(i\omega_n,\vec{p})\right) + \Omega_e - \Omega_0.$$

$$\mathrm{InverseNambu-GorkovPropagator} \quad S^{-1}(i\omega_n,\vec{p}) = \begin{bmatrix} \gamma_\mu p^\mu - M(\vec{p}) + \mu\gamma^0 & \widehat{\Delta}(\vec{p}) \\ \widehat{\Delta}^\dagger(\vec{p}) & \gamma_\mu p^\mu - M(\vec{p}) - \mu\gamma^0 \end{bmatrix},$$

$$\widehat{\Delta}(\vec{p}) = i\gamma_5\epsilon_{\alpha\beta\gamma}\epsilon_{ijk}\Delta_{k\gamma}g(\vec{p}) \;\;;\;\; \Delta_{k\gamma} = 2G_D\langle\bar{q}_{i\alpha}i\gamma_5\epsilon_{\alpha\beta\gamma}\epsilon_{ijk}g(\vec{q})q^C_{j\beta}\rangle.$$

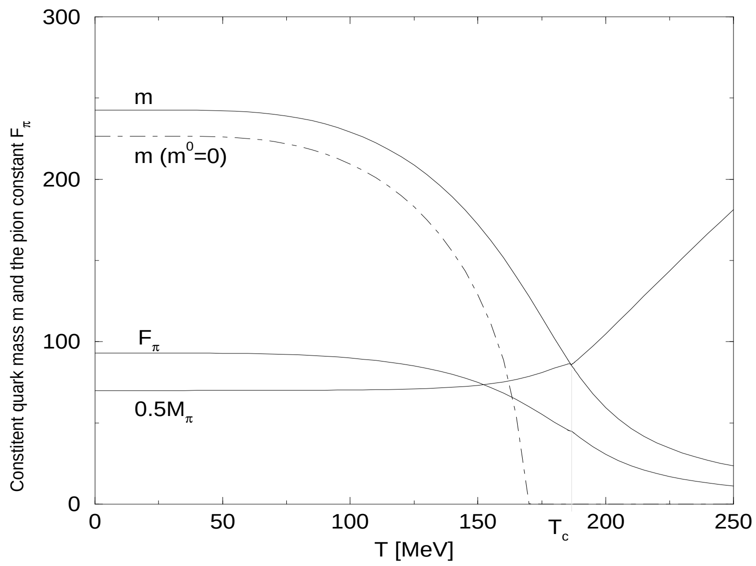
Fermion Determinant (Tr ln D = ln det D): $\ln\det[\beta\, S^{-1}(i\omega_n,\vec{p})] = 2\sum_{a=1}^{18}\ln\{\beta^2[\omega_n^2+\lambda_a(\vec{p})^2]\}$.

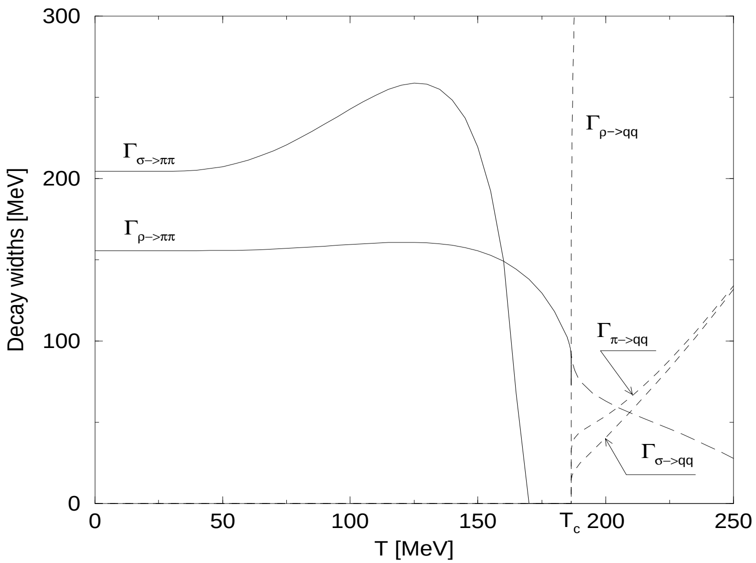
Result for the thermodynamic Potential (Meanfield approximation)

$$\Omega(T,\mu) \;=\; \frac{\phi_u^2+\phi_d^2+\phi_s^2}{8G_S} + \frac{|\Delta_{ud}|^2+|\Delta_{us}|^2+|\Delta_{ds}|^2}{4G_D} - \int\frac{d^3p}{(2\pi)^3}\sum_{a=1}^{18}\left[\lambda_a + 2T\ln\left(1+e^{-\lambda_a/T}\right)\right] + \Omega_e - \Omega_0.$$

Color and electric charge neutrality constraints: $n_Q = n_8 = n_3 = 0$, $n_i = -\partial\Omega/\partial\mu_i = 0$,
Equations of state: $P = -\Omega$, etc.

MOTT EFFECT: NJL MODEL PRIMER





Meson propagator: RPA-type resummation,

$$D_h(P) \sim [1 - G\Pi_h(P)]^{-1},$$

e.g. Pion Pseudoscalar polarization fuction ($m_q = m_{\bar{q}} = m$)

$$\Pi_\pi(\bar{M}_\pi, \vec{0}) = -\frac{N_c}{8\pi^2}\left\{2A(m) - (M_\pi - i\Gamma_\pi/2)^2\, B(M_\pi, \vec{0}; m, m)\right\}$$

Finite temperature (Matsubara)

$$A(m) = -4\int_\Lambda dp \frac{p^2}{\sqrt{E(p)}} \tanh(E(p)/2T) \quad \text{real}$$

$$B(P_0, \vec{0}; m, m) = 8\int_\Lambda dp \frac{p^2 \tanh(E(p)/2T)}{E(p)[4E^2(p) - P_0^2]} \quad \text{real for } T < T_c$$

Complex polarization function
$\Rightarrow$ Breit-Wigner type spectral function

$\Longleftarrow$ Blaschke, Burau, Volkov, Yudichev: EPJA **11** (2001) 319

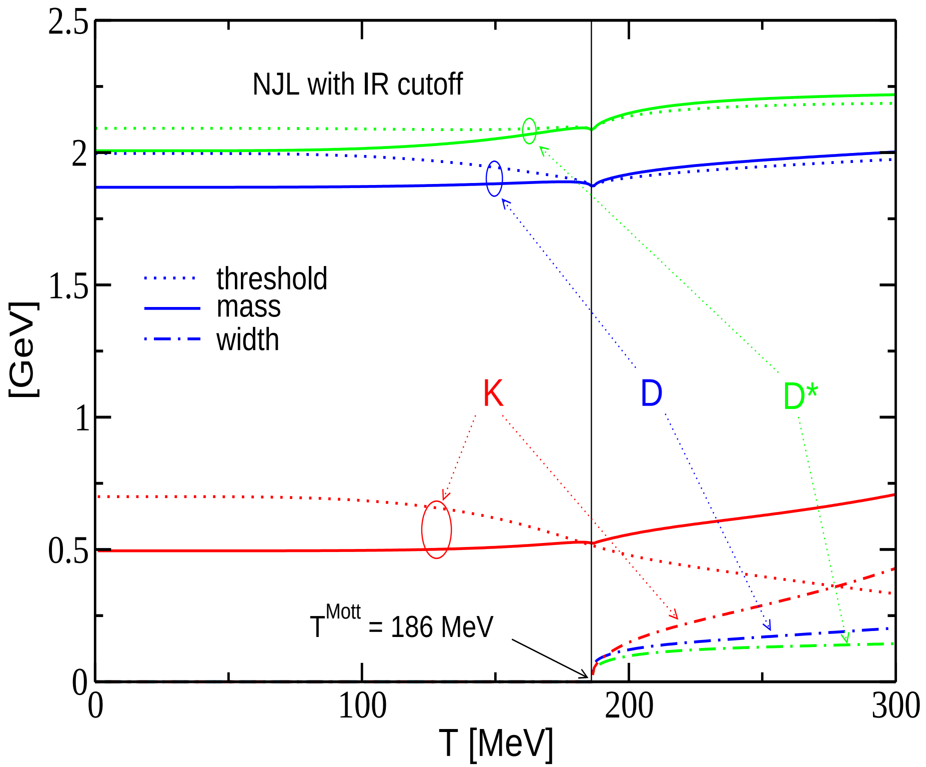
Charm meson sector, see
Gottfried, Klevansky, PLB **286** (1992) 221

Blaschke, Burau, Kalinovsky, Yudichev,
Prog. Theor. Phys. Suppl. **149** (2003) 182

Mott effect: Heavy meson generalization

$$\Pi_D(P^2;T) = 4I_1^{\Lambda}(m_u;T) + 4I_1^{\Lambda}(m_c;T) + 4\left(P^2 - (m_u - m_c)^2\right) I_2^{(\lambda_P,\Lambda)}(P^2, m_u, m_c;T),$$

$$I_2^{(\lambda_M,\Lambda)}(M, m_u, m_c;T) = \frac{N_c}{8\pi^2 M}\int_{\lambda_P}^{\Lambda} dp\, p^2 \left[\frac{\widetilde{E}_{uc}\tanh(E_u/2T)}{E_u(E_u^2 - \widetilde{E}_{uc}^2)} + \frac{\widetilde{E}_{cu}\tanh(E_c/2T)}{E_c(E_c^2 - \widetilde{E}_{cu}^2)}\right],$$



$$\widetilde{E}_{ij} = (m_i^2 - m_j^2 + M^2)/2M,$$

Infrared cutoff ($M_\pi(T_c) = 2m_u(T_c) = 2m_u^{\rm cr}$)

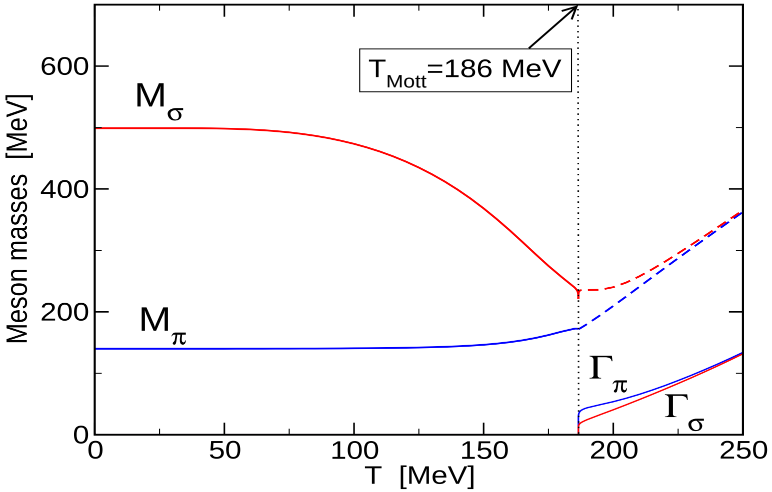
$$\lambda_P = [m_u^{\rm cr}\,\theta(m_u - m_u^{\rm cr}) + m_u\,\theta(m_u^{\rm cr} - m_u)] \times\theta(P^2 - 4(m_u^{\rm cr})^2)\sqrt{P^2/(2\,m_u^{\rm cr})^2 - 1},$$

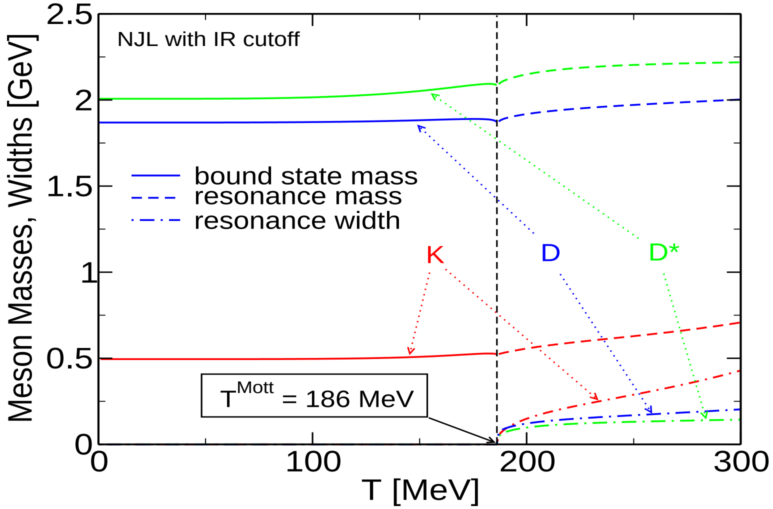
Meson spectral properties (mass M, width Γ)

$$G\,{\rm Re}\Pi(P^2 = M^2;T) = 1$$
$$\Gamma(T) = {\rm Im}\Pi(M^2;T)/[M(T)\,{\rm Re}\Pi'(M^2;T)]$$

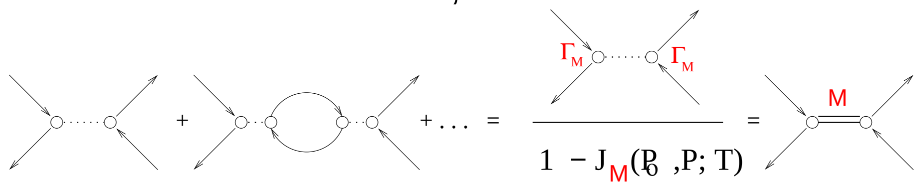
$\Longleftarrow$ Blaschke, Burau, Kalinovsky, Yudichev, Prog. Theor. Phys. Suppl. **149** (2003) 182.

Mott effect: NJL model primer





RPA-type resummation of quark-antiquark scattering in the mesonic channel M,



defines Meson propagator

$$D_M(P_0, P; T) \sim [1 - J_M(P_0, P; T)]^{-1},$$

by the complex polarization function J_M
$\rightarrow$ Breit-Wigner type spectral function

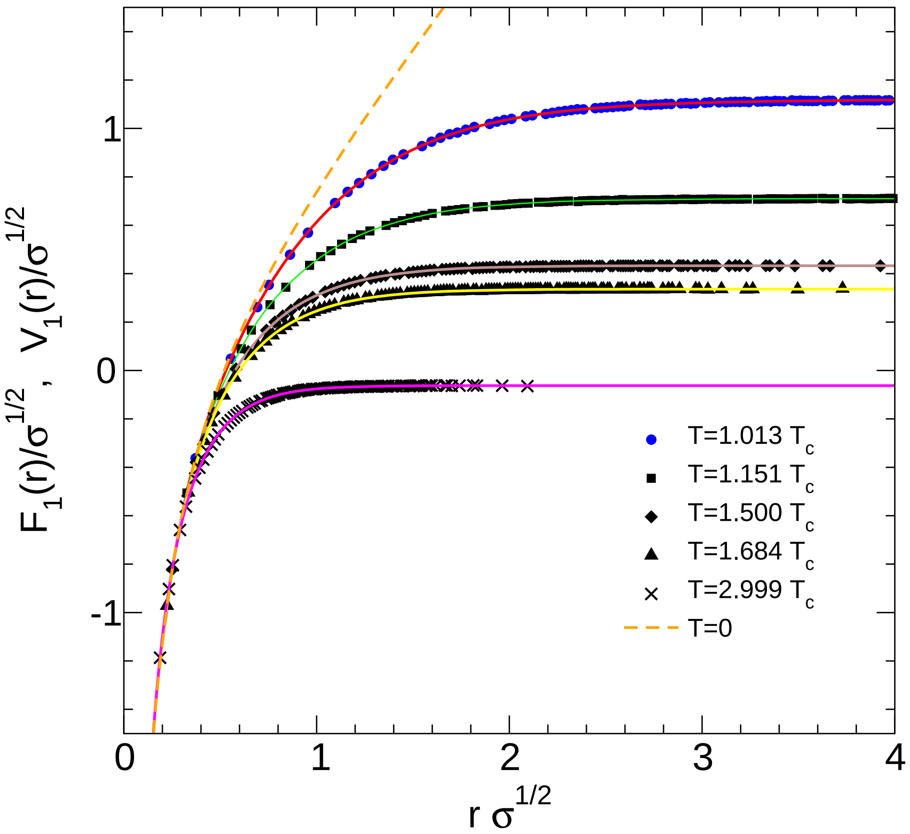
$$\mathcal{A}_M(P_0, P; T) = \frac{1}{\pi} \text{Im } D_M(P_0, P; T)$$
$$\sim \frac{1}{\pi} \frac{\Gamma_M(T) M_M(T)}{(s - M_M^2(T))^2 + \Gamma_M^2(T) M_M^2(T)}$$

For $T < T_{\rm Mott}$: $\Gamma \rightarrow 0$, i.e. bound state
$\mathcal{A}_M(P_0, P; T) = \delta(s - M_M^2(T))$

Light meson sector:
Blaschke, Burau, Volkov, Yudichev: EPJA **11** (2001) 319

Charm meson sector:
Blaschke, Burau, Kalinovsky, Yudichev,
Prog. Theor. Phys. Suppl. **149** (2003) 182

Heavy Quark Potential from Lattice QCD



Blaschke, Kaczmarek, Laermann, Yudichev, EPJC 43, 81 (2005); [hep-ph/0505053]

Color-singlet free energy F_1 in quenched QCD

$$\langle \mathrm{Tr}[L(0)L^\dagger(r)]\rangle = \exp[-F_1(r)/T]$$

Long- and short- range parts

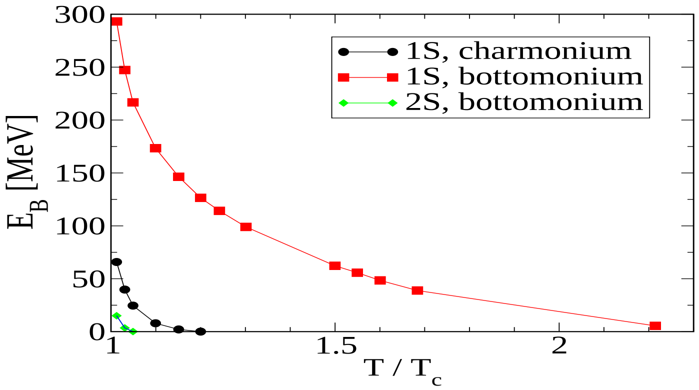
$$F_1(r,T) = F_{1,\mathrm{long}}(r,T) + V_{1,\mathrm{short}}(r)e^{-(\mu(T)r)^2}$$

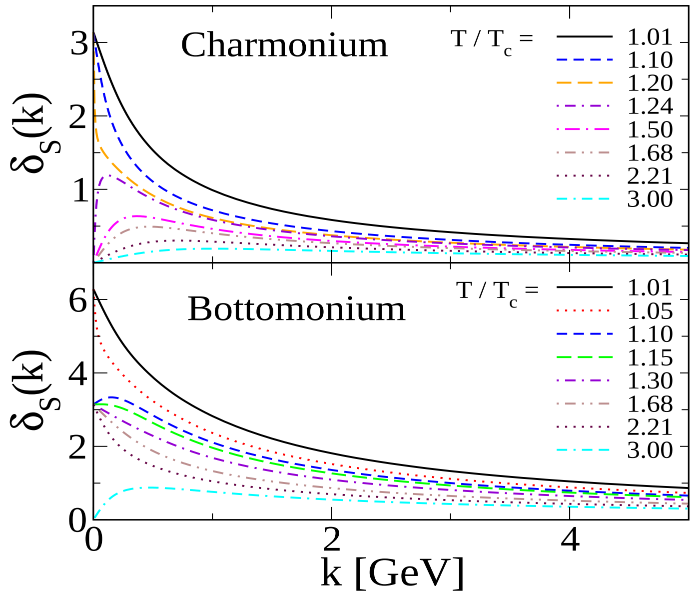
$$\begin{aligned} F_{1,\mathrm{long}}(r,T) &= \text{'screened' confinement pot.} \\ V_{1,\mathrm{short}}(r) &= -\frac{4}{3}\frac{\alpha(r)}{r},\ \alpha(r) = \text{running coupl.} \quad (1) \end{aligned}$$

Quarkonium ($Q\bar{Q}$)	1S	$1P_1$	2S
Charmonium ($c\bar{c}$)	J/ψ(3097)	χ_{c1}(3510)	ψ' (3686)
Bottomonium ($b\bar{b}$)	Υ (9460)	χ_{b1} (9892)	Υ' (10023)

$\Longrightarrow$ Lecture Petreczky

SCHROEDINGER EQN: BOUND & SCATTERING STATES





Quarkonia bound states at finite T:

$$[-\nabla^2/m_Q + V_{\text{eff}}(r,T)]\psi(r,T) = E_B(T)\psi(r,T)$$

Binding energy vanishes $E_B(T_{\text{Mott}}) = 0$: Mott effect

Scattering states:

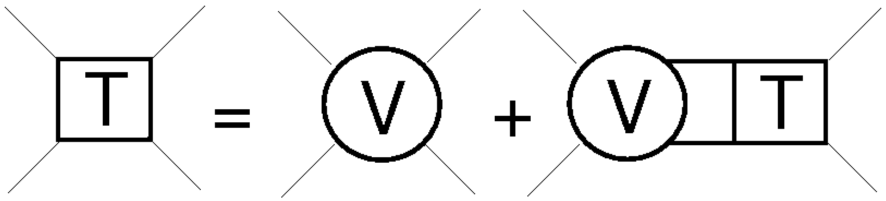
$$\frac{d\delta_S(k,r,T)}{dr} = -\frac{m_Q V_{\text{eff}}}{k}\sin(kr + \delta_S(k,r,T))$$

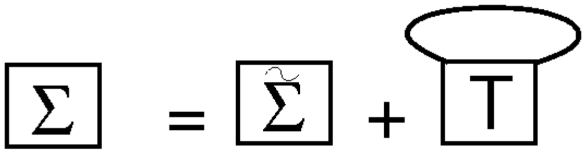
Levinson theorem:

Phase shift at threshold jumps by π when bound state $\rightarrow$ resonance at $T = T_{\text{Mott}}$

Blaschke, Kaczmarek, Laermann, Yudichev
EPJC 43, 81 (2005); [hep-ph/0505053]

T-MATRIX APPROACH TO QUARKONIA IN THE QGP



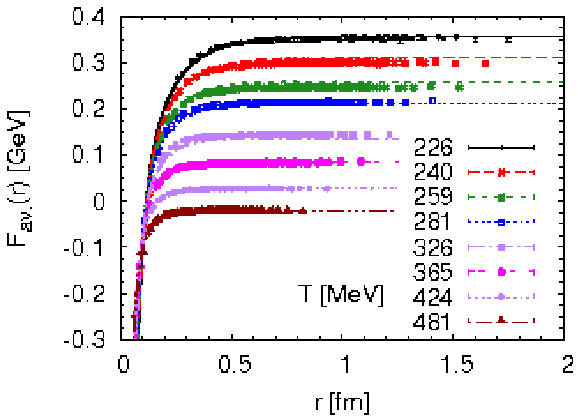


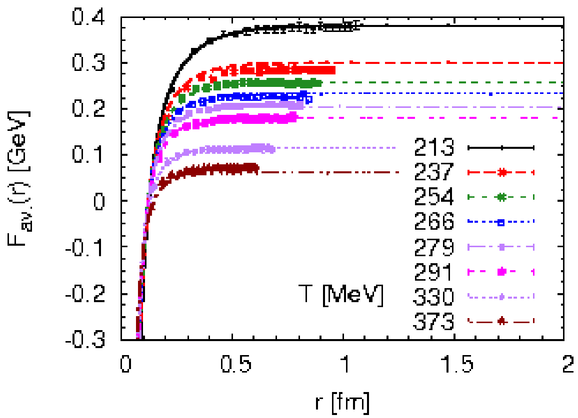
Riek & Rapp, PRC 82 (2010); arxiv:1005.0769

Open question: Wich potential to use?

$$
\begin{aligned}
U &= F - T\frac{dF}{dT} \\
V(r;T) &= F(r;T) - F(\infty,T) \text{ or } F \leftrightarrow U
\end{aligned}
$$

Result: J/ψ good resonance
below $1.5\,T_c$ for F, and $2.5\,T_c$ for U



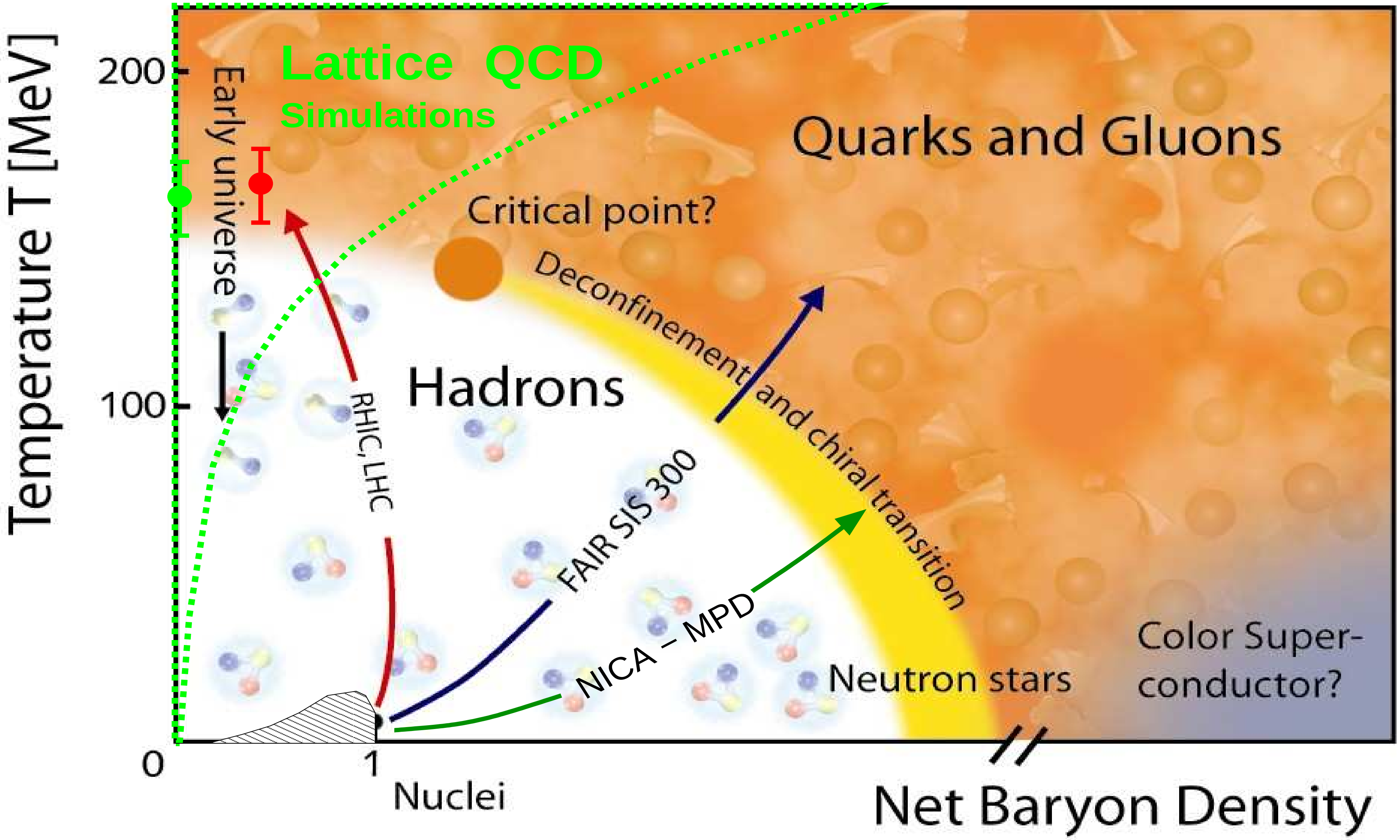


Field theoretic input:
Megias et al. JHEP (2006)

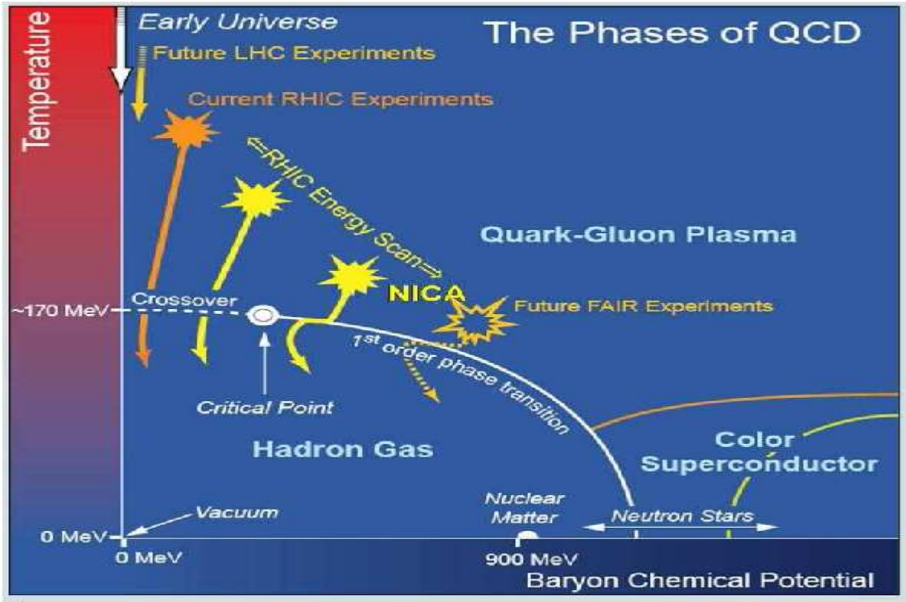
$$
\begin{aligned}
D_{00}(\vec{k}) &= D^{P}_{00}(\vec{k}) + D^{NP}_{00}(\vec{k}) \\
D^{P}_{00}(\vec{k}) &= \frac{1}{\vec{k}^2 + m_D^2} \\
D^{NP}_{00}(\vec{k}) &= \frac{m_G^2}{(\vec{k}^2 + \tilde{m}_D^2)^2}
\end{aligned}
$$

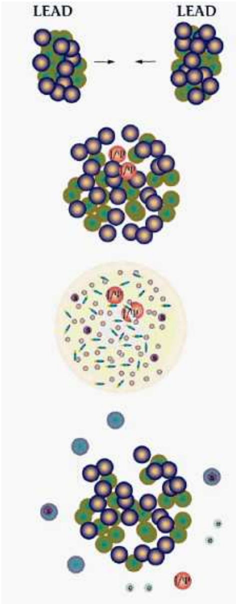
Lattice:Kaczmarek et al. (left), Petreczky et al. (right)

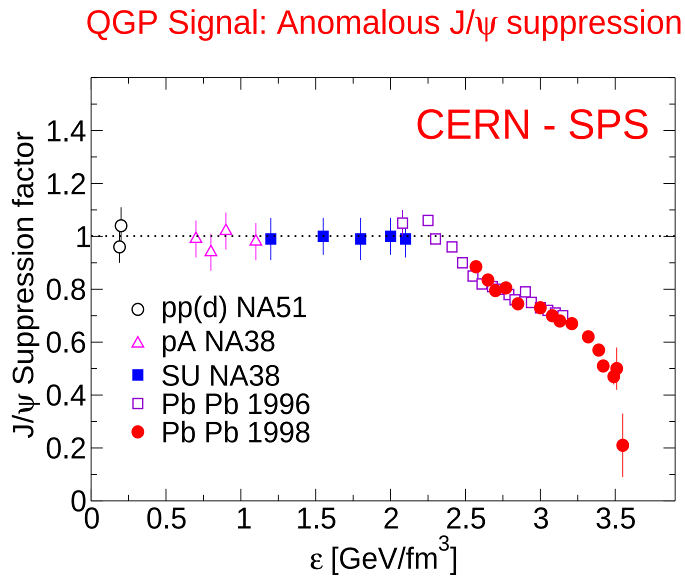
PHASEDIAGRAM OF QCD: HEAVY-ION COLLISIONS

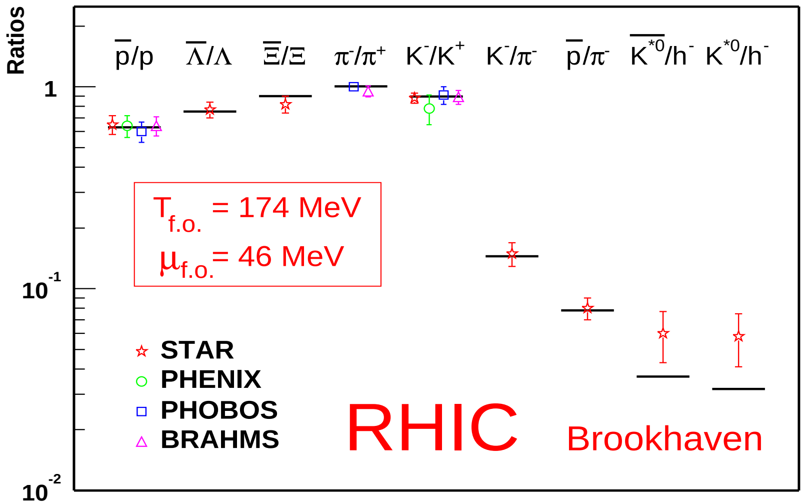


PHASEDIAGRAM: FREEZE-OUT IN HEAVY-ION COLLISIONS









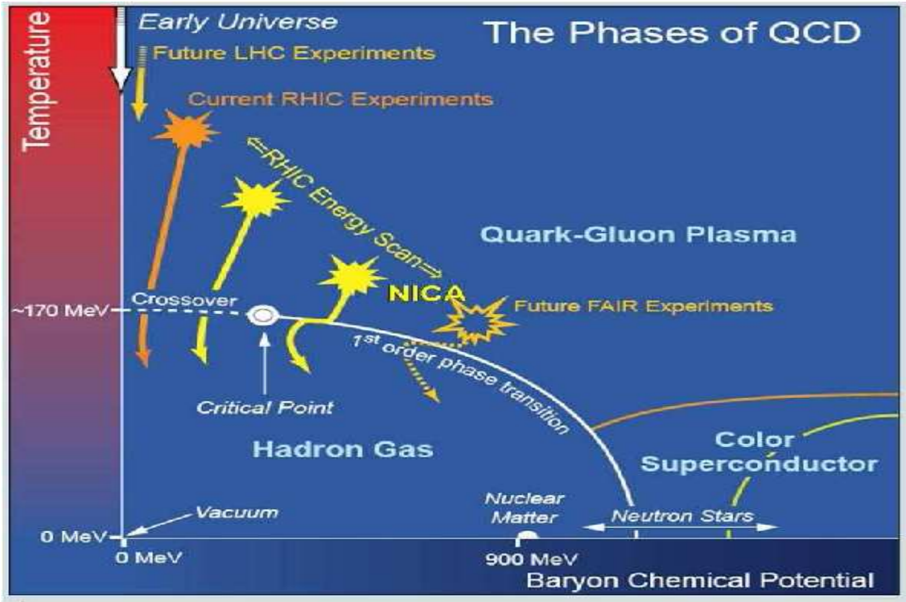
Statistical model describes composition of hadron yields in Heavy-Ion Collisions with few freeze-out parameters.

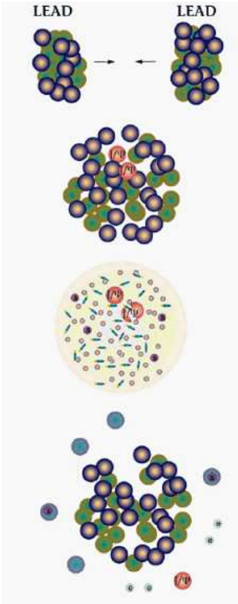
$$\ln Z[T,V,\{\mu\}] = \pm V \sum_i \frac{g_i}{2\pi^2} \int_0^\infty dp\, p^2 \ln[1 \pm \lambda_i \exp(-\beta\varepsilon_i(p))]$$

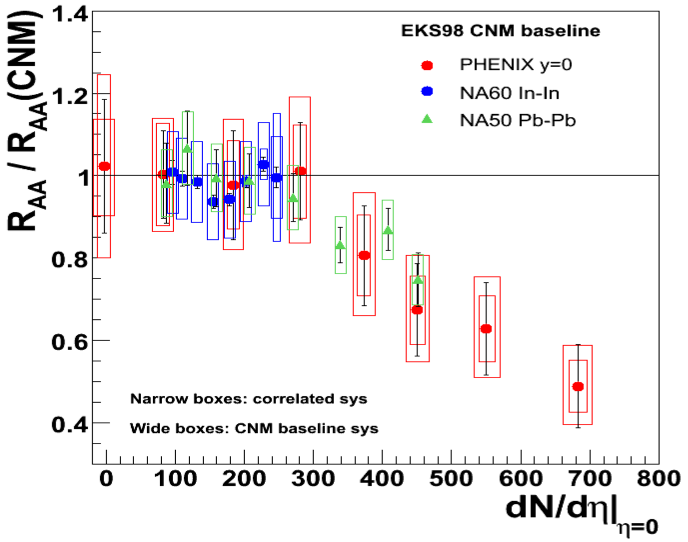
$$\lambda_i(T,\{\mu\}) = \exp[\beta(\mu_B B_i + \mu_S S_i + \mu_Q Q_i)]$$

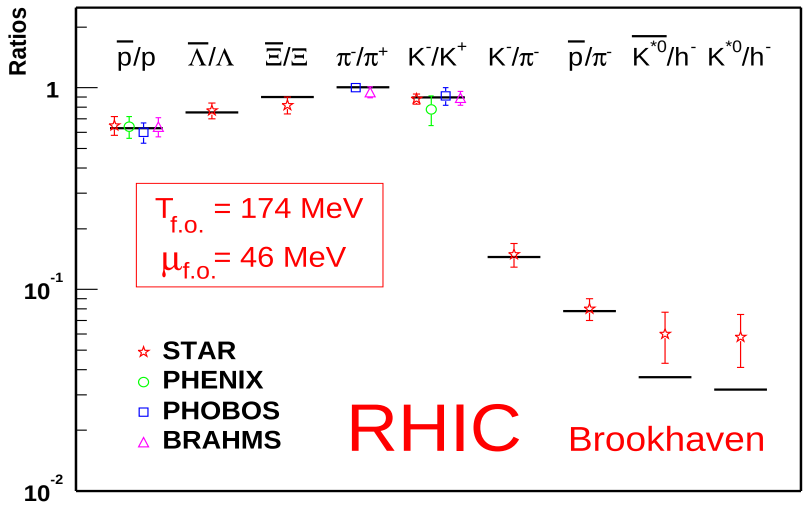
Braun-Munzinger, Redlich, Stachel, in *QGP III* (2003)

PHASEDIAGRAM: FREEZE-OUT IN HEAVY-ION COLLISIONS









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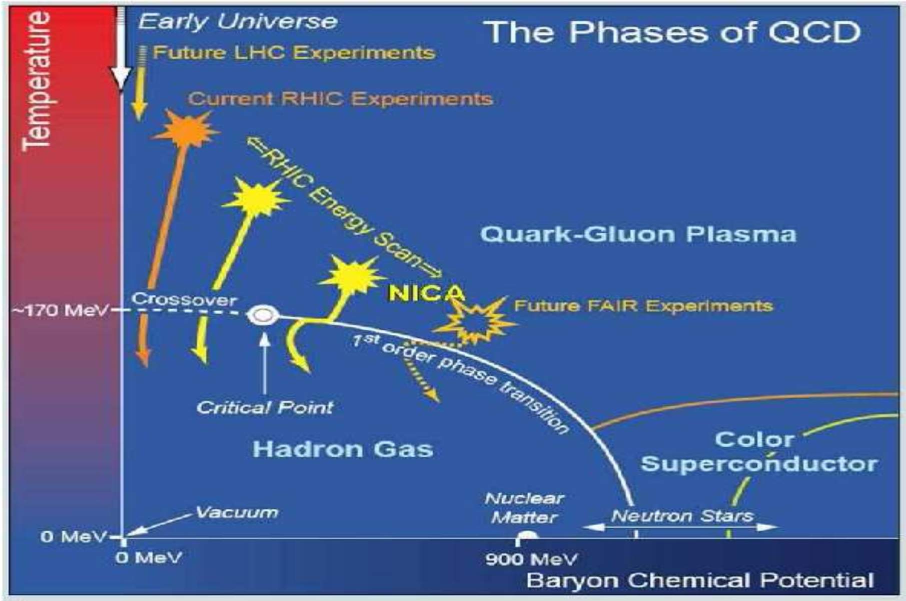
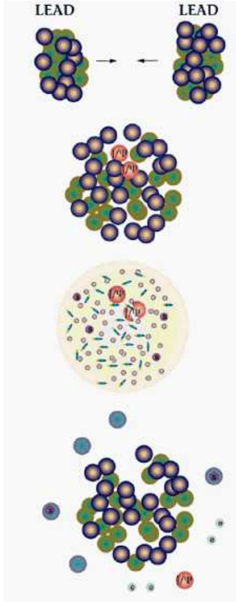
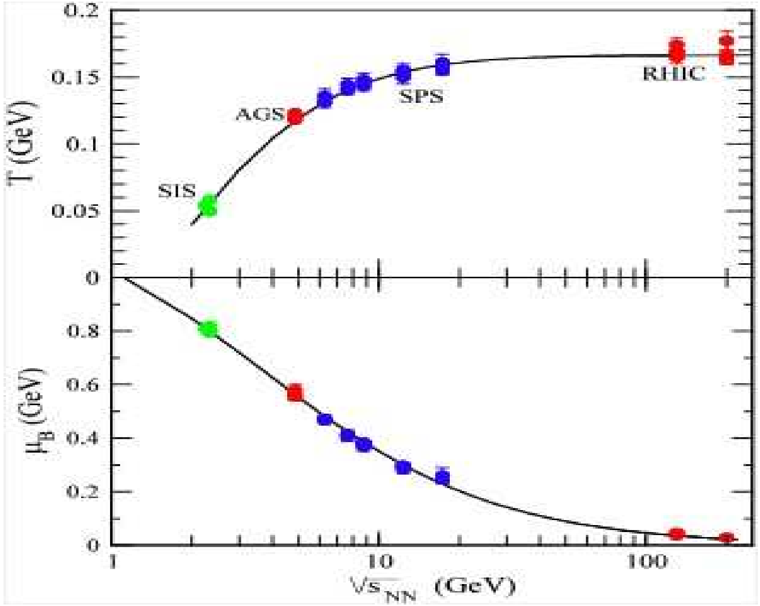
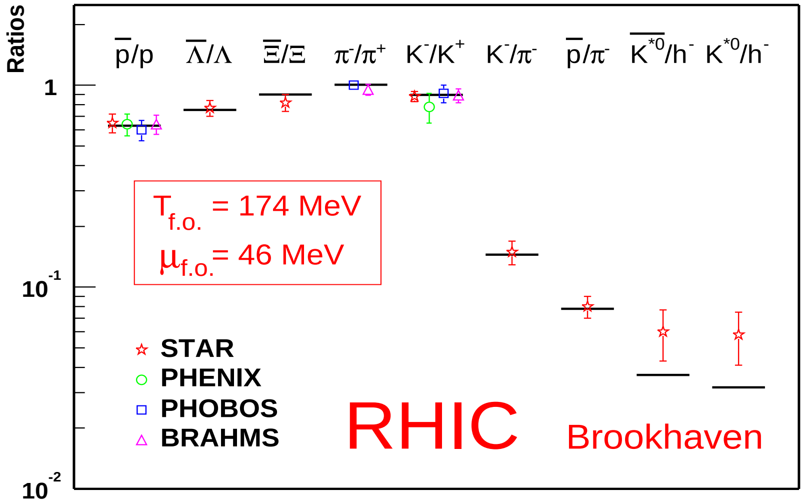
Braun-Munzinger, Redlich, Stachel, in *QGP III* (2003)

Phasediagram: Freeze-out in Heavy-Ion Collisions (II)









Statistical model describes composition of hadron yields in Heavy-Ion Collisions with few freeze-out parameters.

$$\ln Z[T,V,\{\mu\}] = \pm V \sum_i \frac{g_i}{2\pi^2}\int_0^\infty dp\, p^2 \ln[1 \pm \lambda_i \exp(-\beta\varepsilon_i(p))]$$

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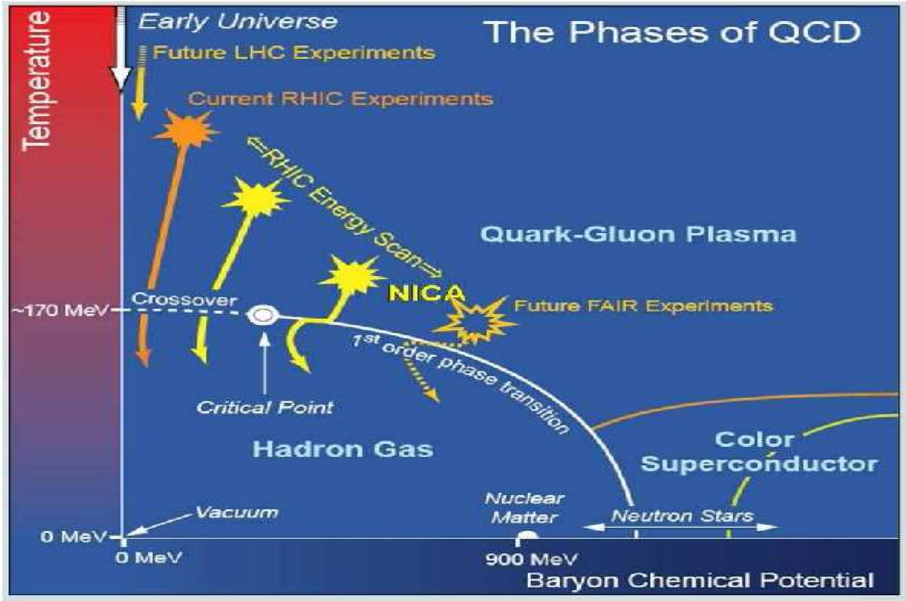
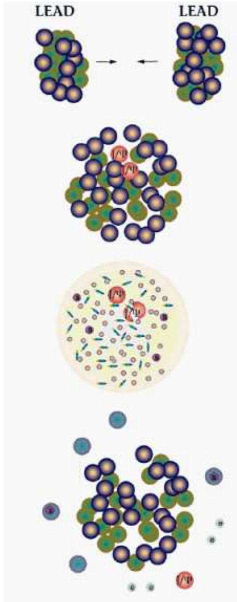
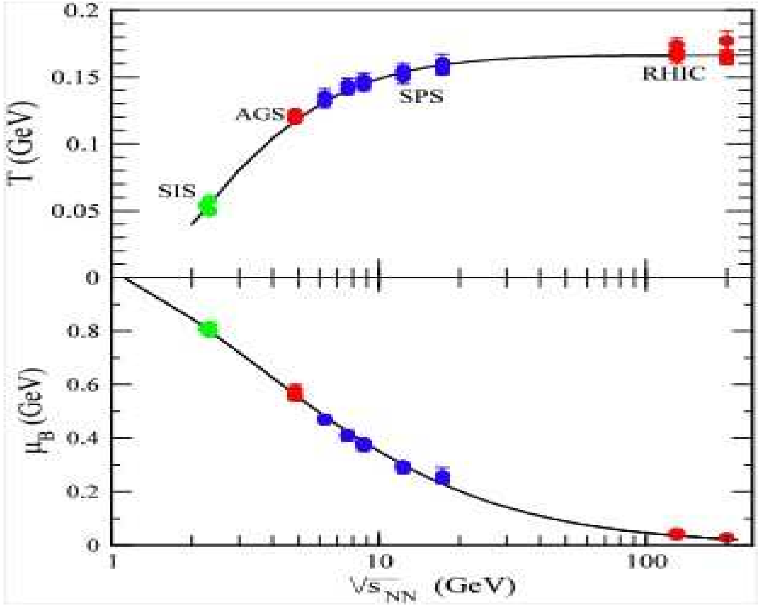
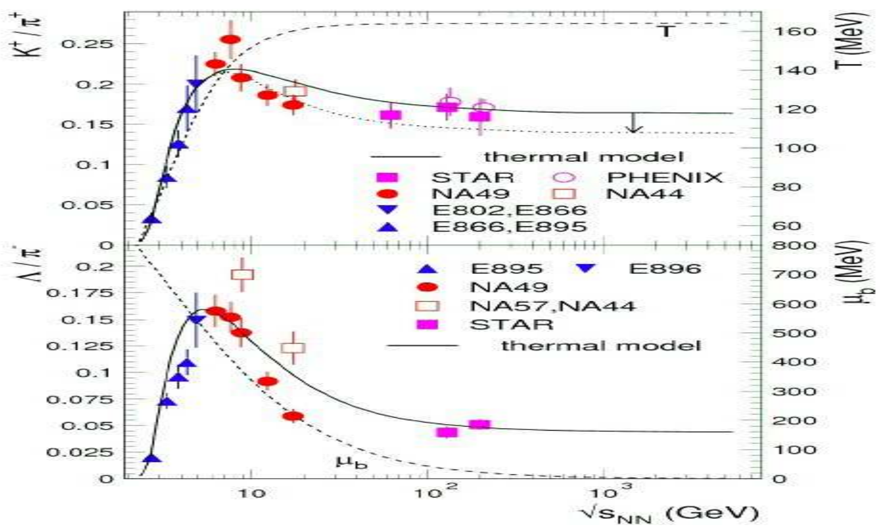
Braun-Munzinger, Redlich, Stachel, in *QGP III* (2003)

PHASEDIAGRAM: FREEZE-OUT IN HEAVY-ION COLLISIONS (III)



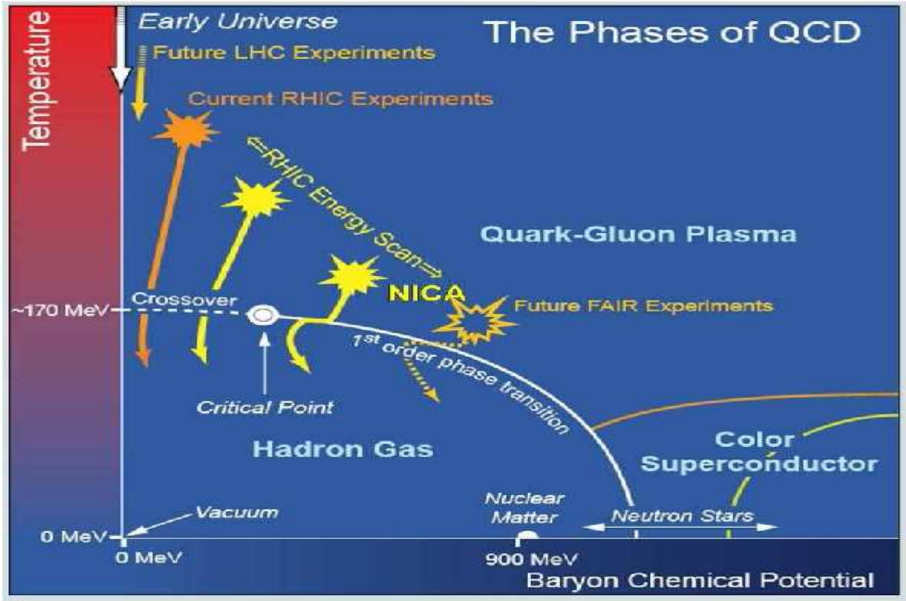


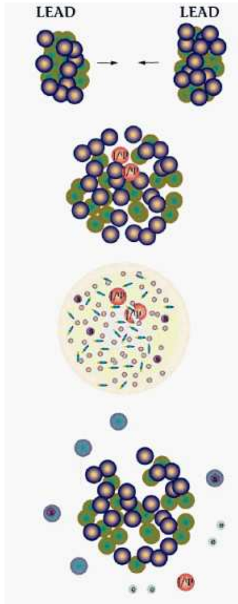


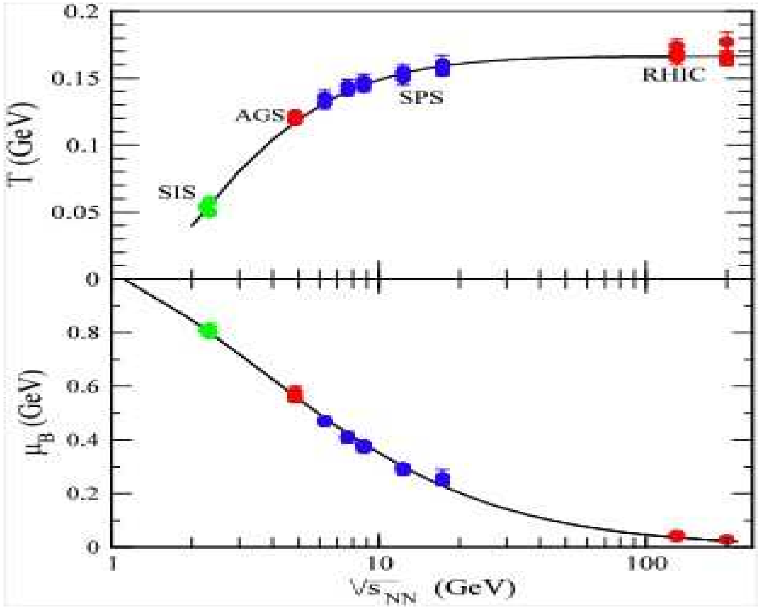


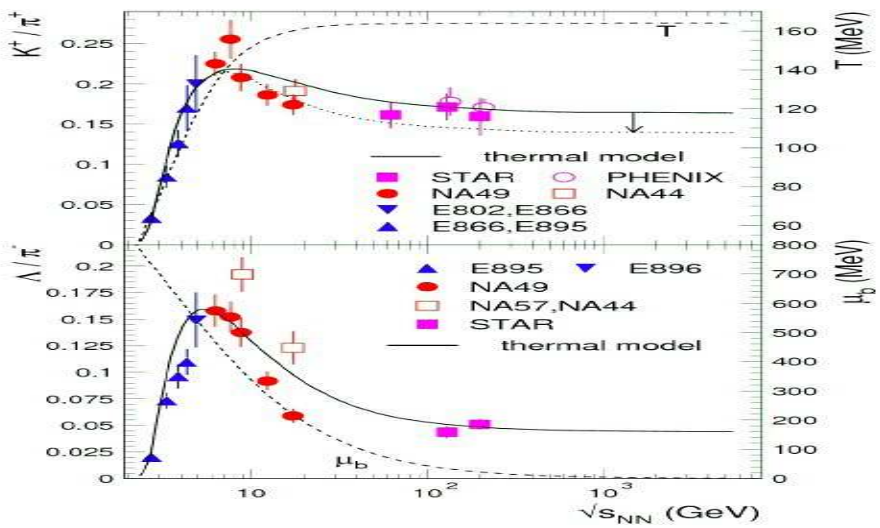
Strange MatterHorn (Pisarski)

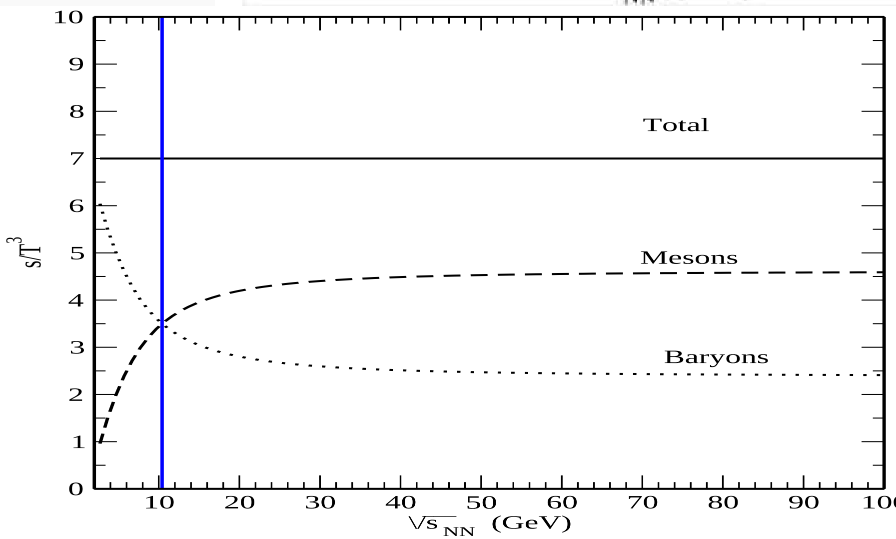
Phasediagram: Freeze-out in Heavy-Ion Collisions (III)





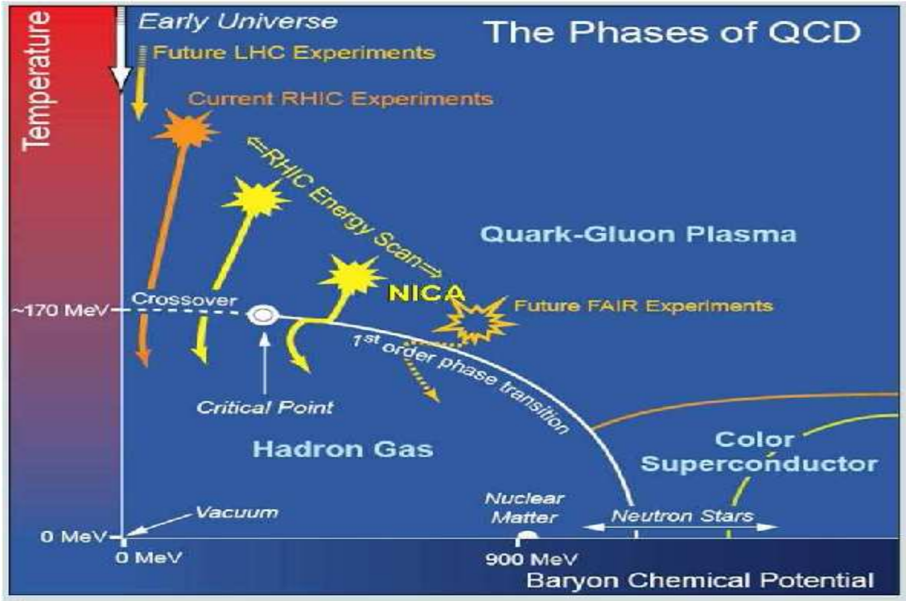


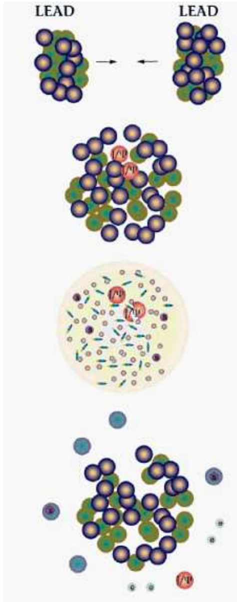


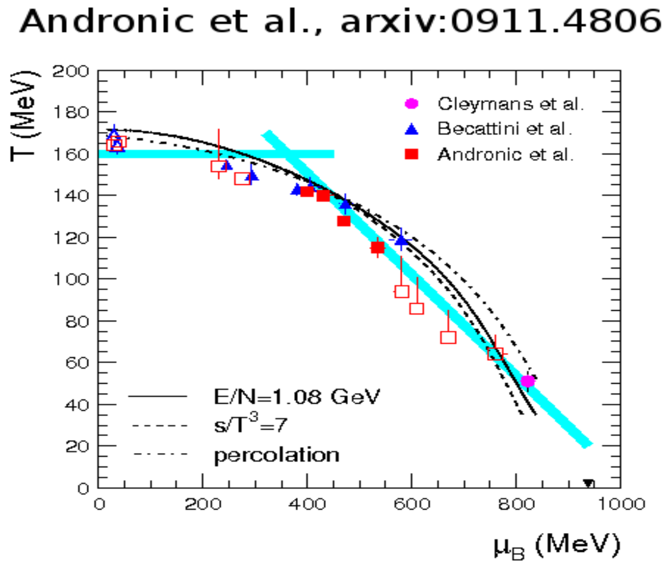


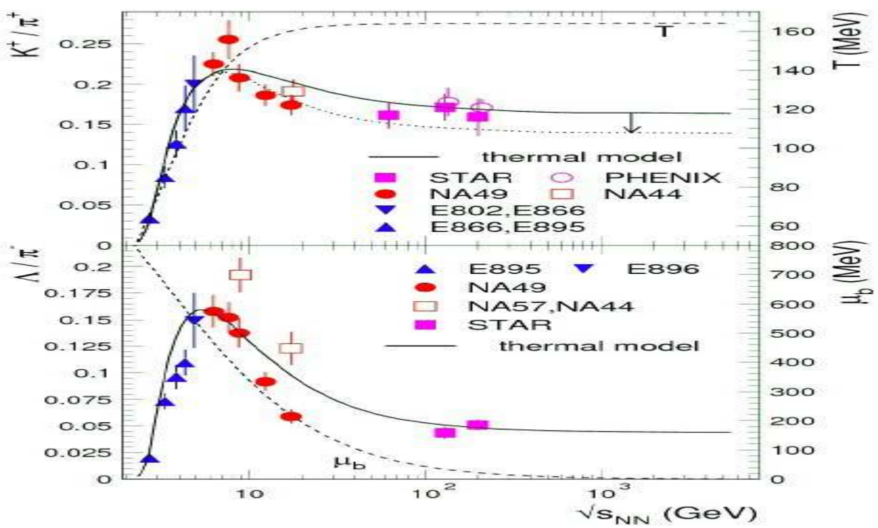
Baryon → Meson Dominance

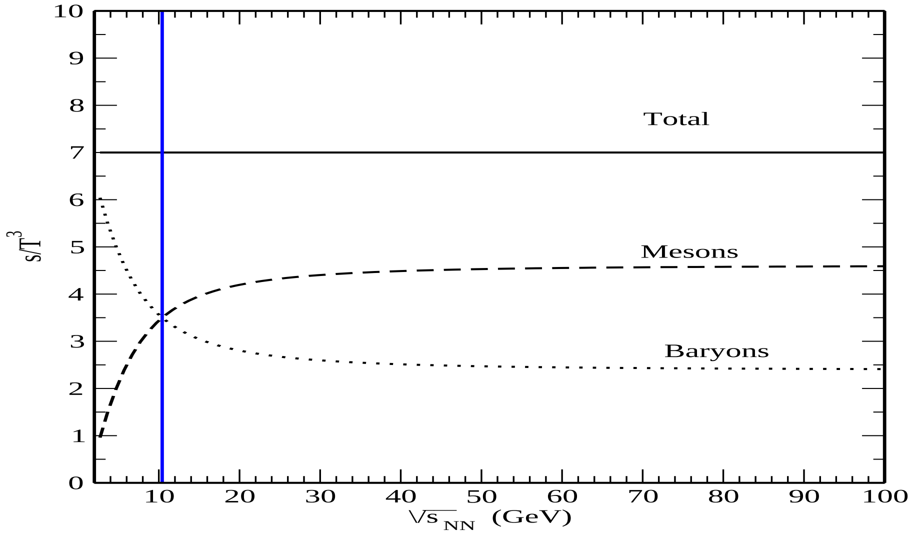
PHASEDIAGRAM: FREEZE-OUT IN HEAVY-ION COLLISIONS (IV)





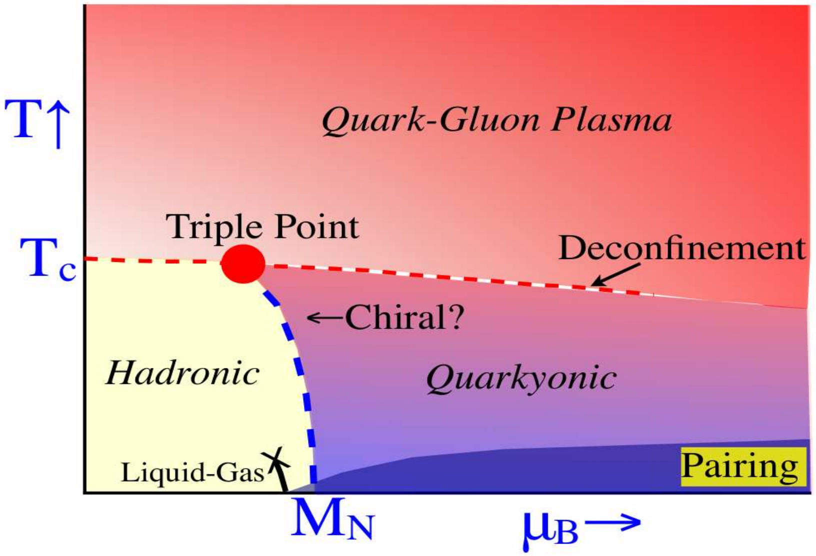




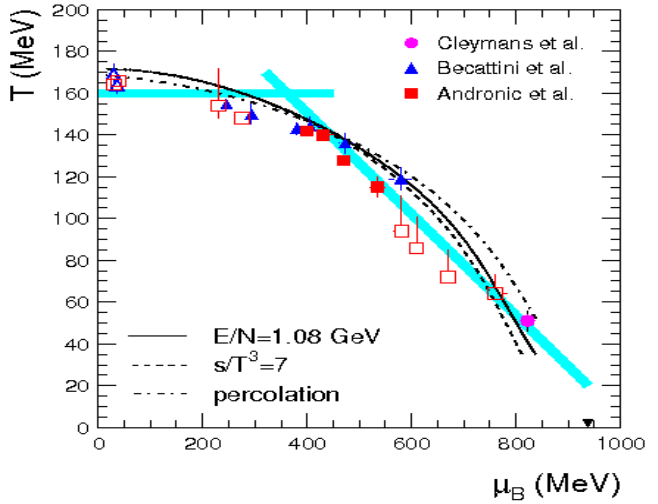


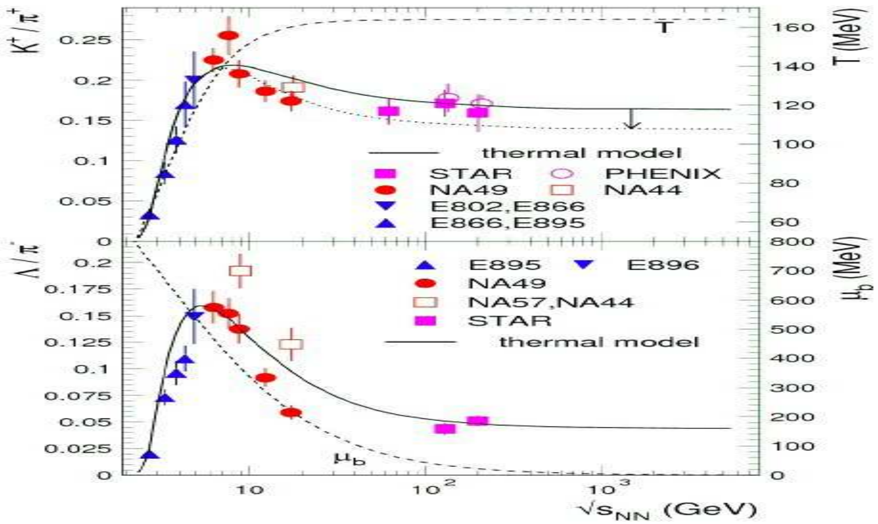
Baryon $\rightarrow$ Meson Dominance

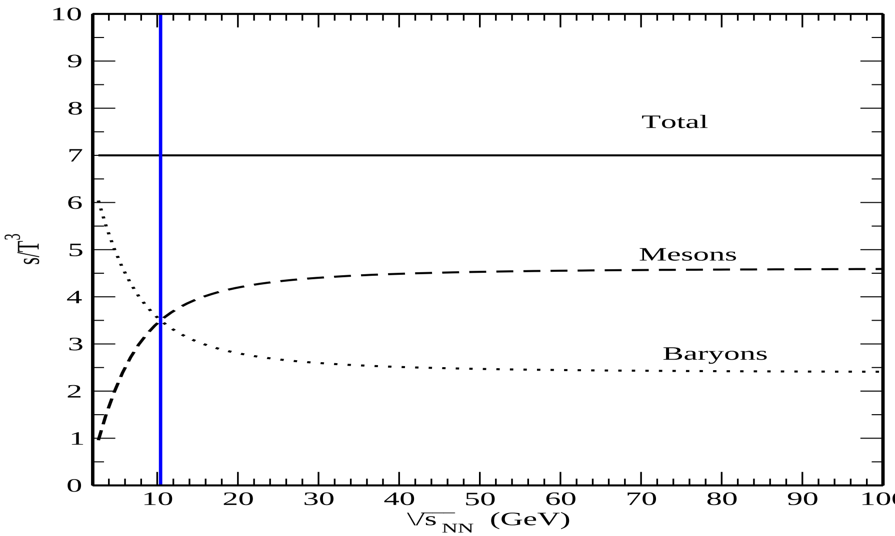
Phasediagram: Freeze-out in Heavy-Ion Collisions (V)



Andronic et al., arxiv:0911.4806



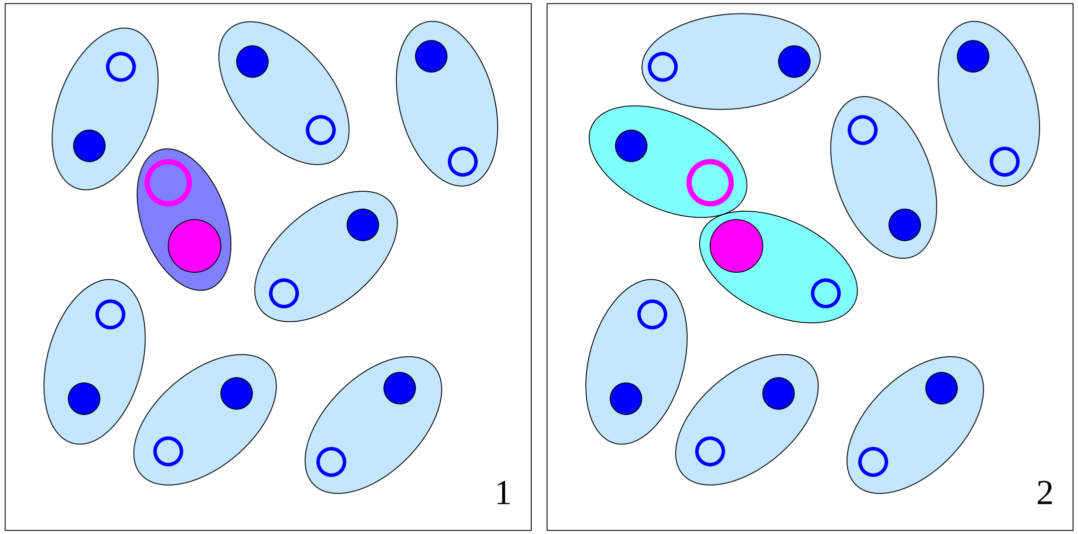


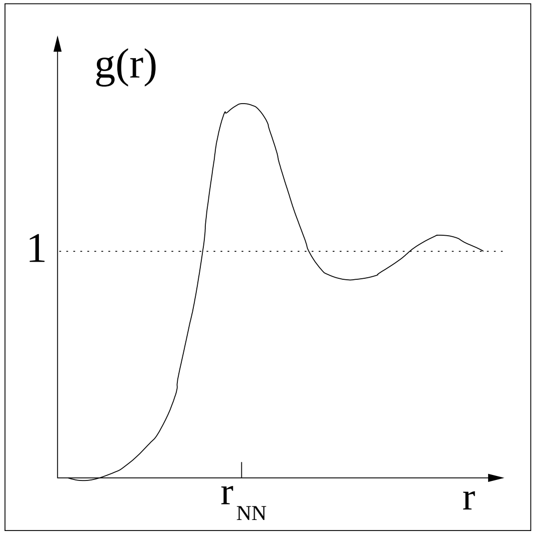


Andronic et al., arxiv:0911.4806; NPA (2010)

A SNAPSHOP OF THE sQGP

The Picture: String-flip (Rearrangement) $\Longleftrightarrow$ Pair correlation



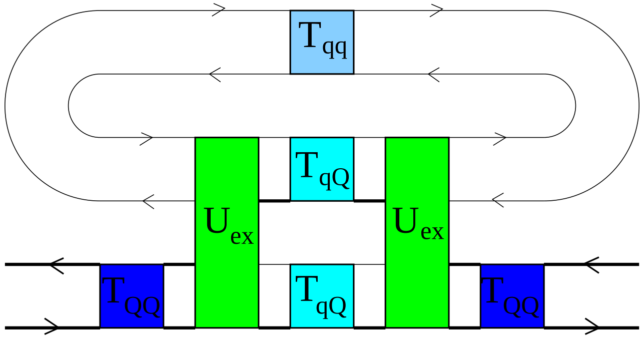


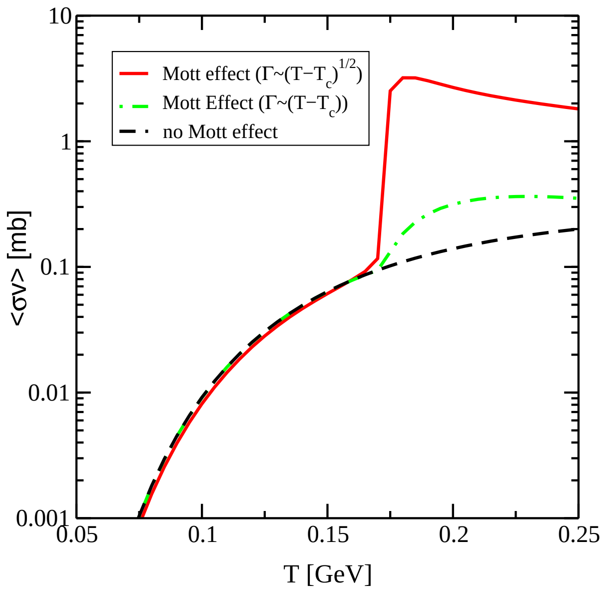
Horowitz et al. PRD (1985), D.B. et al. PLB (1985), Röpke, Blaschke, Schulz, PRD (1986)

Thoma,[hep-ph/0509154] Gelman et al., PRC 74 (2006)

- Strong correlations present: hadronic spectral functions above T_c (lattice QCD)
- Finite width due to rearrangement collisions (higher order correlations)
- Liquid-like pair correlation function (nearest neighbor peak)

Quantum kinetic approach to J/ψ breakup





Inverse lifetime for Charmonium states

$$\tau^{-1}(p) = \Gamma(p) = \Sigma^{>}(p) \mp \Sigma^{<}(p)$$

$$\Sigma^{\gtrless}(p,\omega) \;=\; \int_{p'}\int_{p_1}\int_{p_2} (2\pi)^4 \delta_{p,p';p_1,p_2}\, |\mathcal{M}|^2\, G_\pi^{\lessgtr}(p')\, G_{D_1}^{\gtrless}(p_1)\, G_{D_2}^{\gtrless}(p_2)$$

$G_h^{>}(p) = [1 \pm f_h(p)]A_h(p)$ and $G_h^{<}(p) = f_h(p)A_h(p)$

$$\tau^{-1}(p) = \int \frac{d^3\mathbf{p}'}{(2\pi)^3} \int ds' \quad f_\pi(\mathbf{p}', s')\; A_\pi(s') v_{\rm rel}\; \sigma^*(s)$$

In-medium breakup cross section

$$\sigma^*(s) = \int ds_1\, ds_2\; A_{D_1}(s_1)\; A_{D_2}(s_2)\; \sigma(s; s_1, s_2)$$

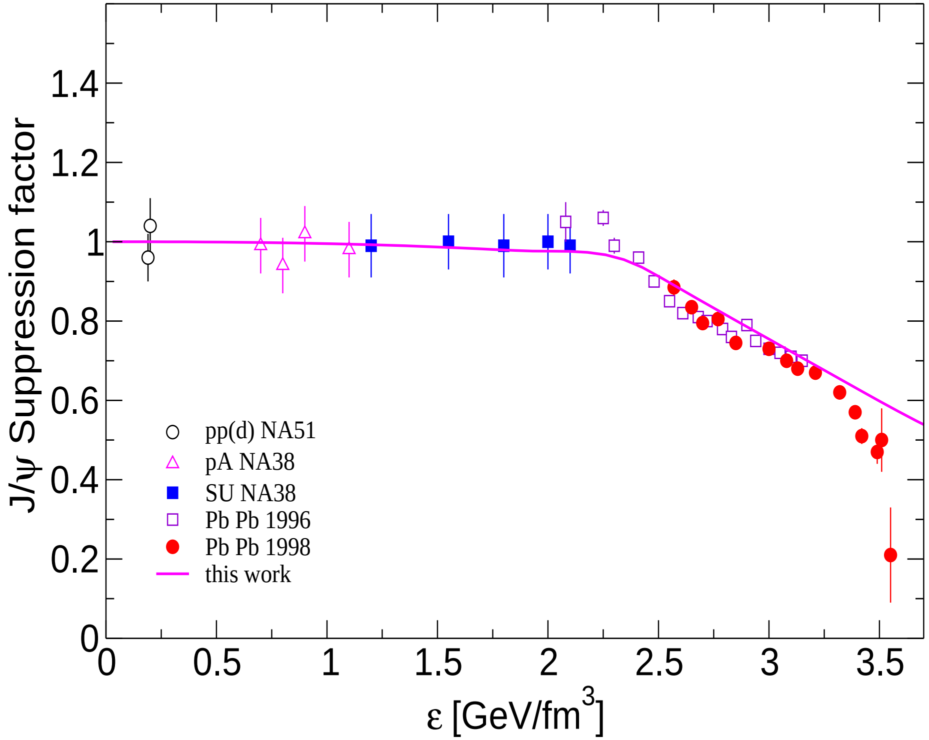
Medium effects in spectral functions A_h and $\sigma(s; s_1, s_2)$

$$A_h(s) \;=\; \frac{1}{\pi}\frac{\Gamma_h(T)\; M_h(T)}{(s - M_h^2(T))^2 + \Gamma_h^2(T) M_h^2(T)} \longrightarrow \delta(s - M_h^2)$$

resonance $\Leftarrow$ $\boxed{\text{Mott-effect}}$ $\Leftarrow$ bound state

Blaschke et al., Heavy Ion Phys. 18 (2003) 49

"ANOMALOUS" J/ψ SUPPRESSION IN MOTT-HAGEDORN GAS



Survival probability for J/ψ

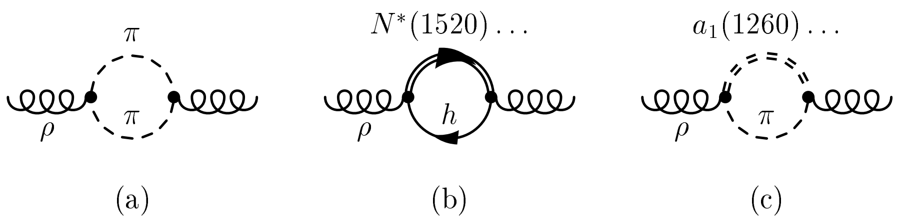
$$S(E_T)/S_N(E_T) \; = \; \exp\left[-\int_{t_0}^{t_f} dt\ \tau^{-1}(n(t))\right]$$

Threshold: Mott effect for hadrons

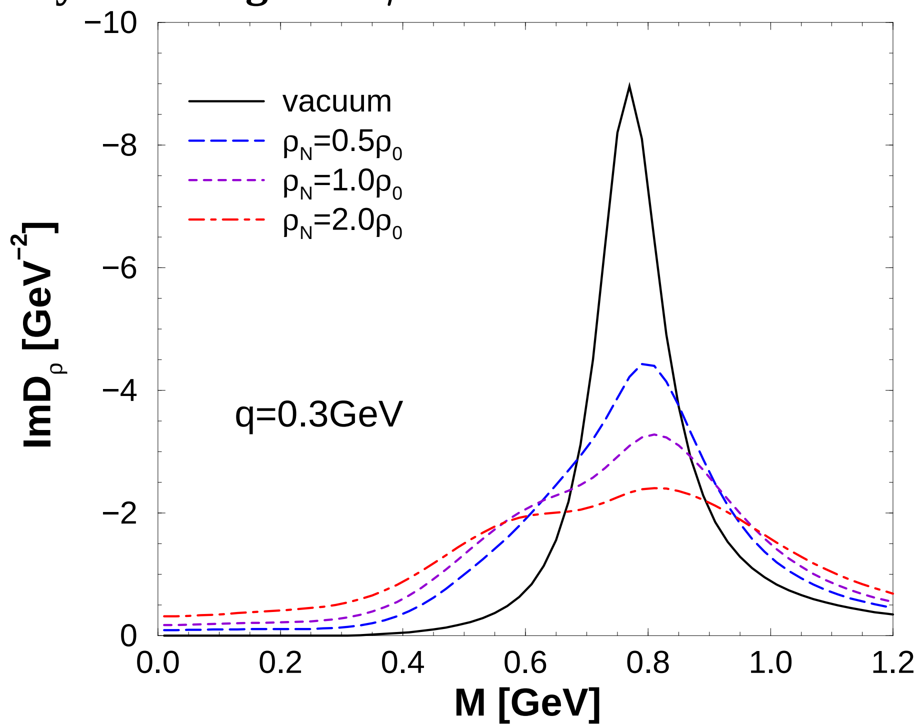
Blaschke and Bugaev, Prog. Part. Nucl. Phys. 53 (2004) 197

In progress: full kinetics with gain processes (D-fusion), HIC simulation

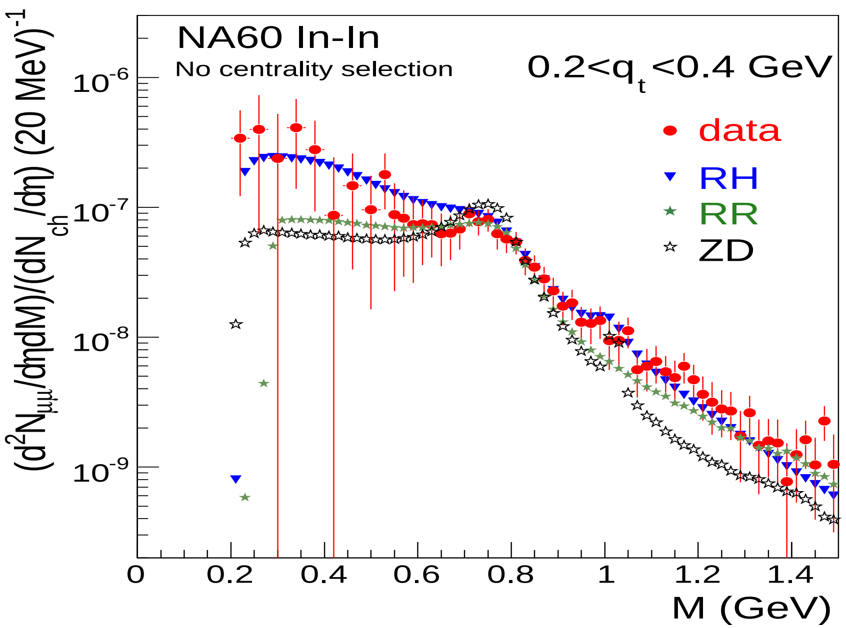
Low-mass dilepton production in heavy-ion collisions

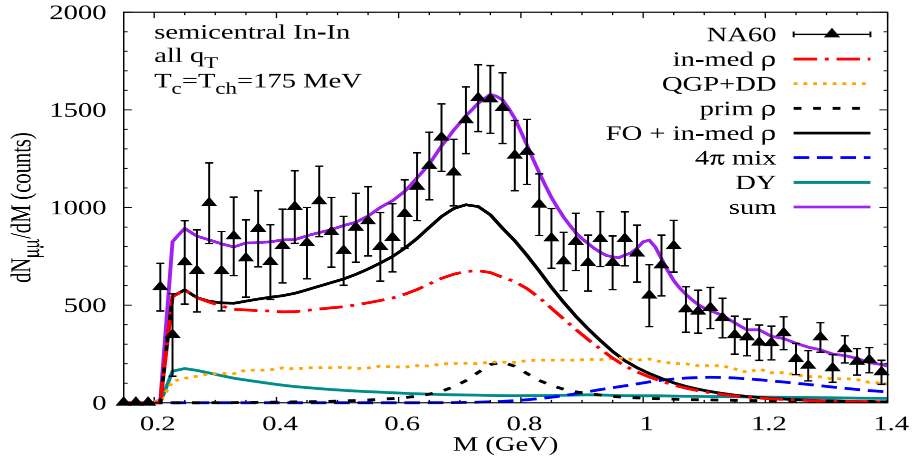


Feyman diagrams ρ-meson in hadronic matter



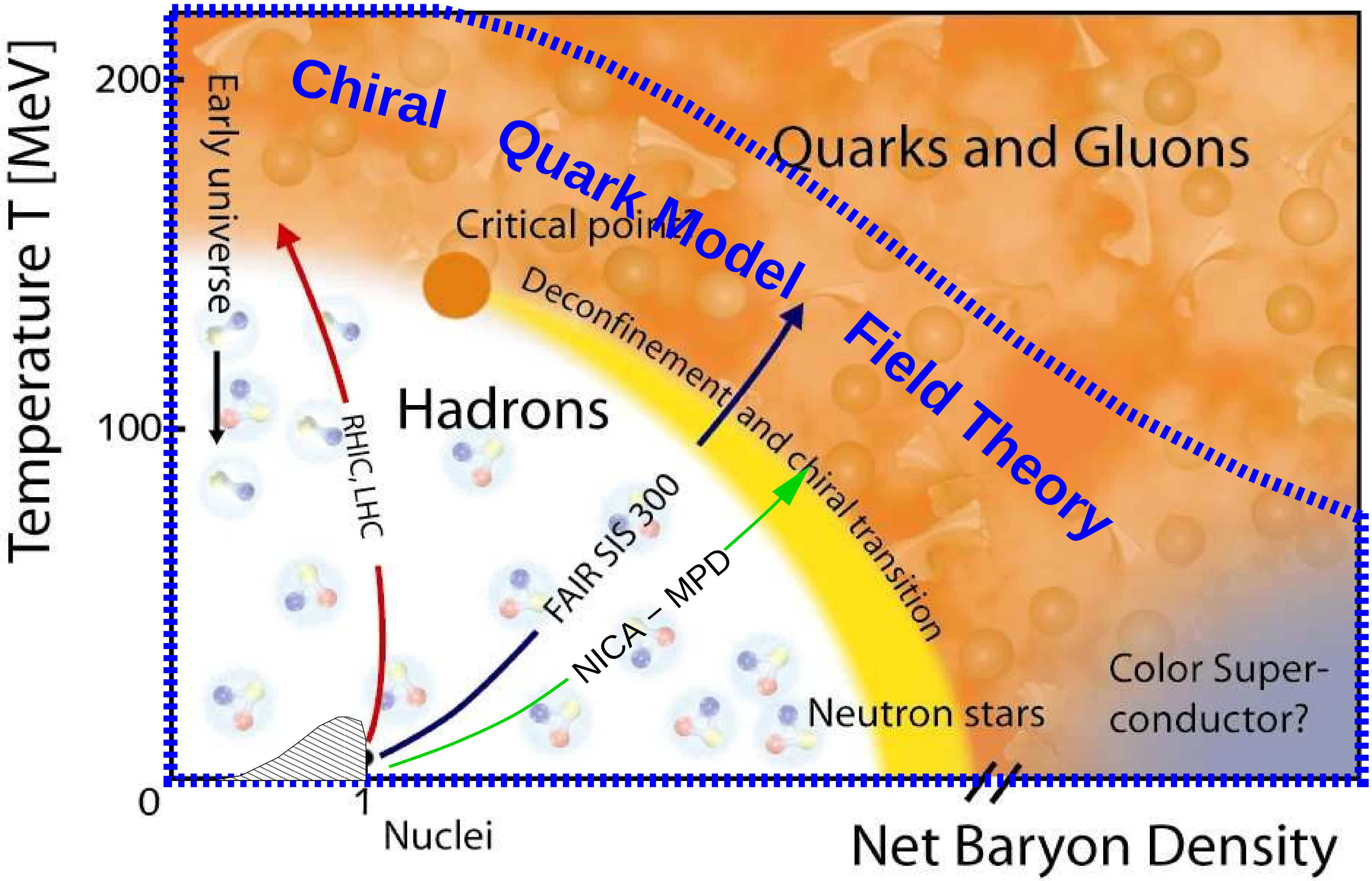
ρ-meson Spectral function in hadronic matter



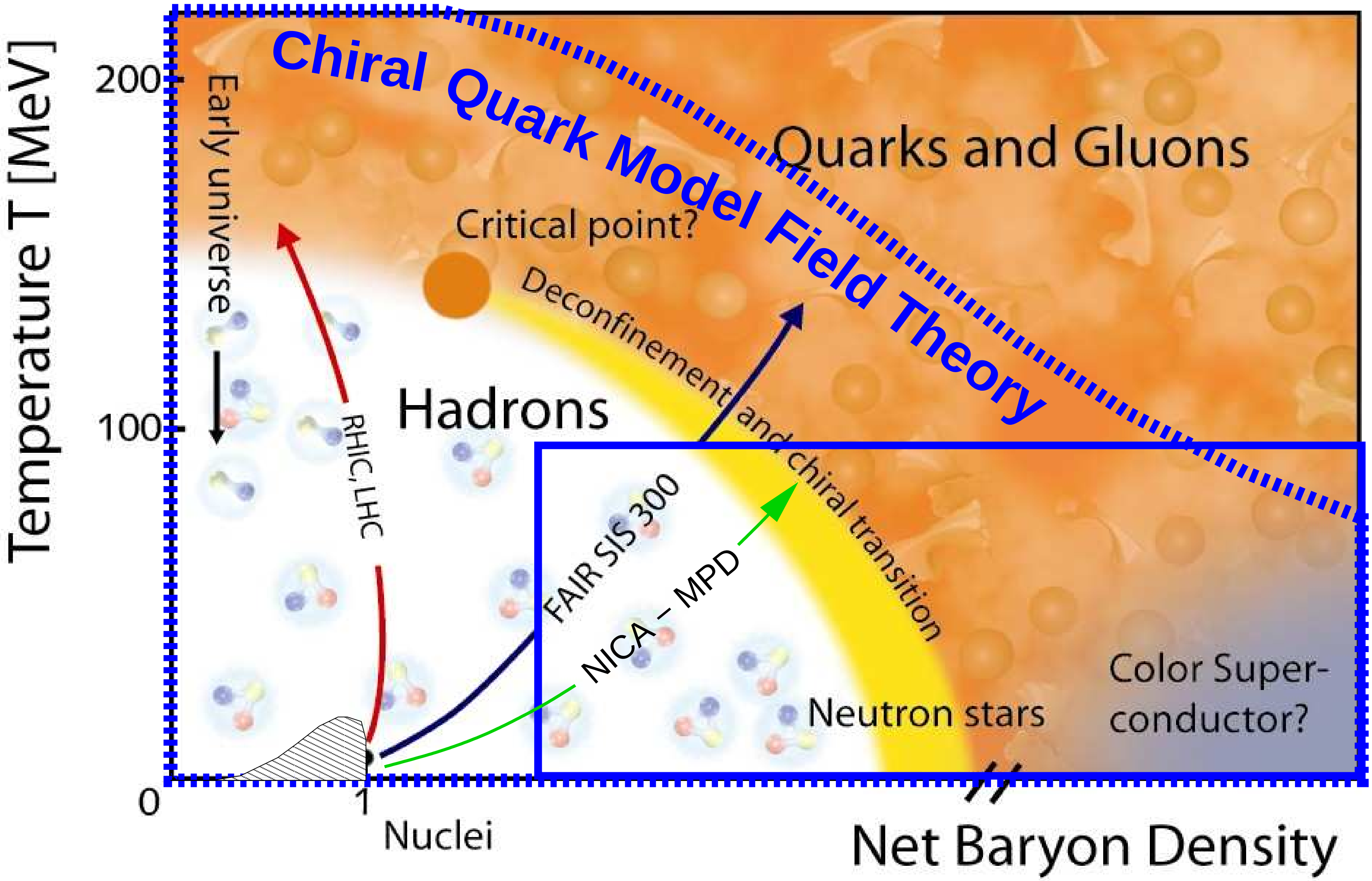


Dilepton mass spectrum: NA60 experiment

PHASEDIAGRAM OF DEGENERATE QUARK MATTER



PHASEDIAGRAM OF DEGENERATE QUARK MATTER



NJL MODEL FOR NEUTRAL 3-FLAVOR QUARK MATTER

Thermodynamic Potential $\Omega(T,\mu) = -T \ln Z[T,\mu]$

$$\Omega(T,\mu) \;=\; \frac{\phi_u^2+\phi_d^2+\phi_s^2}{8G_S} + \frac{|\Delta_{ud}|^2+|\Delta_{us}|^2+|\Delta_{ds}|^2}{4G_D} - T\sum_n \int \frac{d^3p}{(2\pi)^3}\frac{1}{2}\mathrm{Tr}\ln\left(\frac{1}{T}S^{-1}(i\omega_n,\vec{p})\right) + \Omega_e - \Omega_0.$$

$$\mathrm{Inverse Nambu-Gorkov Propagator} \quad S^{-1}(i\omega_n,\vec{p}) = \begin{bmatrix} \gamma_\mu p^\mu - M(\vec{p}) + \mu\gamma^0 & \widehat{\Delta}(\vec{p}) \\ \widehat{\Delta}^\dagger(\vec{p}) & \gamma_\mu p^\mu - M(\vec{p}) - \mu\gamma^0 \end{bmatrix},$$

$$\widehat{\Delta}(\vec{p}) = i\gamma_5\epsilon_{\alpha\beta\gamma}\epsilon_{ijk}\Delta_{k\gamma}g(\vec{p}) \;\; ; \;\; \Delta_{k\gamma} = 2G_D\langle\bar{q}_{i\alpha}i\gamma_5\epsilon_{\alpha\beta\gamma}\epsilon_{ijk}g(\vec{q})q^C_{j\beta}\rangle.$$

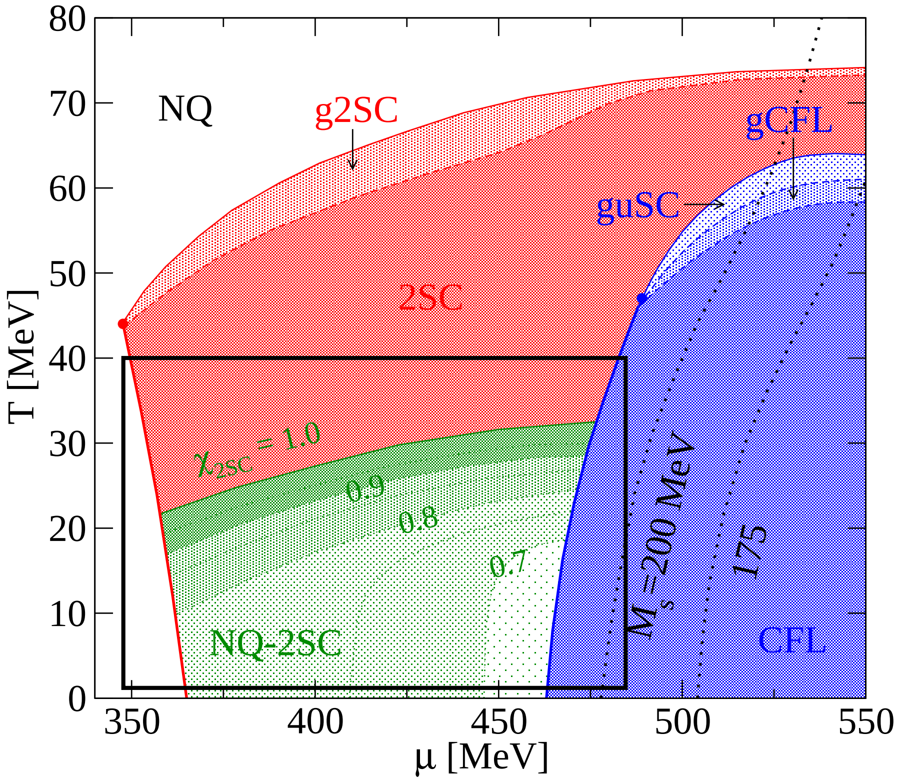
Fermion Determinant (Tr ln D = ln det D): $\ln\det[\beta\, S^{-1}(i\omega_n,\vec{p})] = 2\sum_{a=1}^{18}\ln\{\beta^2[\omega_n^2+\lambda_a(\vec{p})^2]\}$.

Result for the thermodynamic Potential (Meanfield approximation)

$$\Omega(T,\mu) \;=\; \frac{\phi_u^2+\phi_d^2+\phi_s^2}{8G_S} + \frac{|\Delta_{ud}|^2+|\Delta_{us}|^2+|\Delta_{ds}|^2}{4G_D} - \int \frac{d^3p}{(2\pi)^3}\sum_{a=1}^{18}\left[\lambda_a + 2T\ln\left(1+e^{-\lambda_a/T}\right)\right] + \Omega_e - \Omega_0.$$

Color and electric charge neutrality constraints: $n_Q = n_8 = n_3 = 0$, $n_i = -\partial\Omega/\partial\mu_i = 0$,
Equations of state: $P = -\Omega$, etc.

QUARK MATTER IN COMPACT STARS



Rüster et al: PRD 72 (2005) 034004
Blaschke et al: PRD 72 (2005) 065020
Abuki, Kunihiro: NPA 768 (2006) 118

The phases are characterized by 3 gaps:

- NQ: $\Delta_{ud} = \Delta_{us} = \Delta_{ds} = 0$;
- NQ-2SC: $\Delta_{ud} \neq 0$, $\Delta_{us} = \Delta_{ds} = 0$, $0 \leq \chi_{2SC} \leq 1$;
- 2SC: $\Delta_{ud} \neq 0$, $\Delta_{us} = \Delta_{ds} = 0$;
- uSC: $\Delta_{ud} \neq 0$, $\Delta_{us} \neq 0$, $\Delta_{ds} = 0$;
- CFL: $\Delta_{ud} \neq 0$, $\Delta_{ds} \neq 0$, $\Delta_{us} \neq 0$;

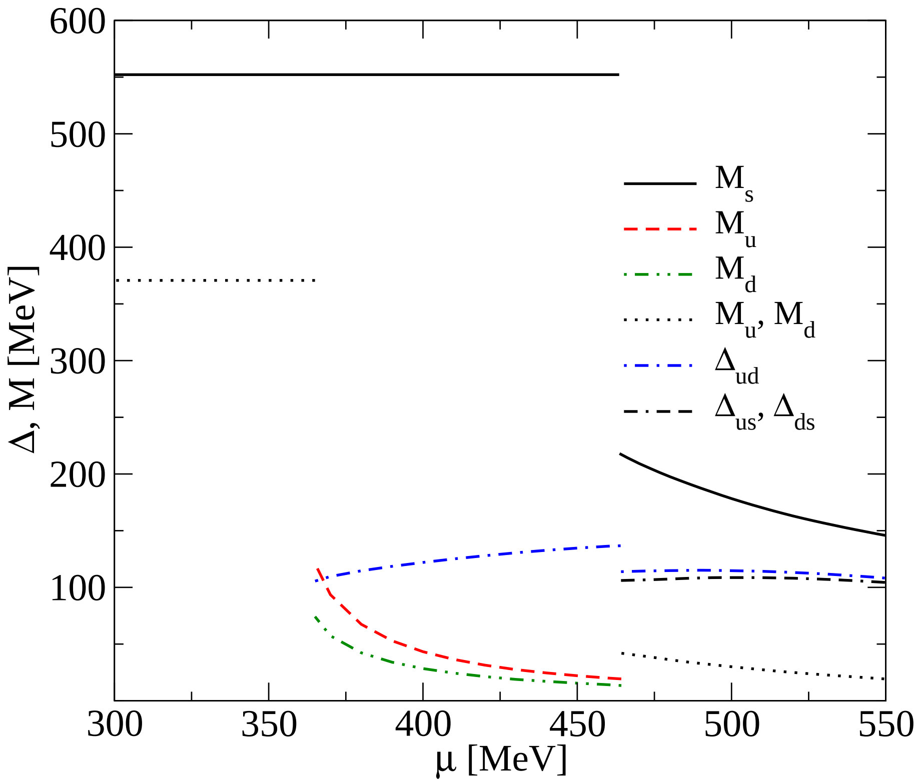
Result:

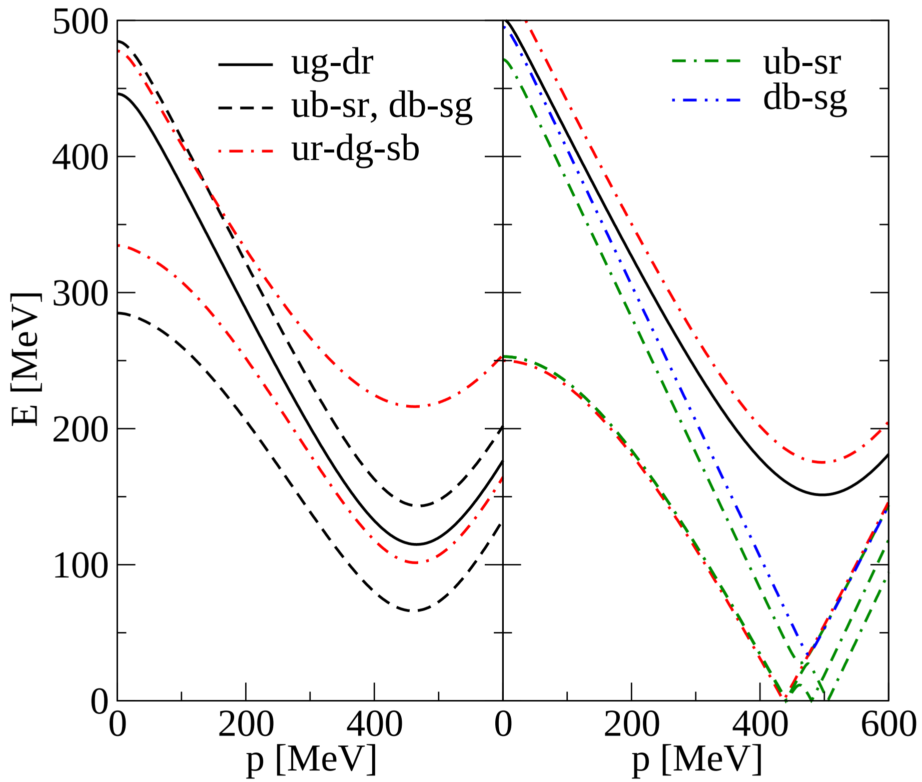
- Gapless phases only at high T,
- CFL only at high chemical potential,
- At T $\leq$25-30 MeV: mixed NQ-2SC phase,
- Critical point (T_c,μ_c)=(48 MeV, 353 MeV),
- Strong coupling, $\eta = 1$, changes?.

ORDER PARAMETERS: MASSES AND DIQUARK GAPS

Masses (M) and Diquark gaps (Δ) as a function of the chemical potential at $T = 0$

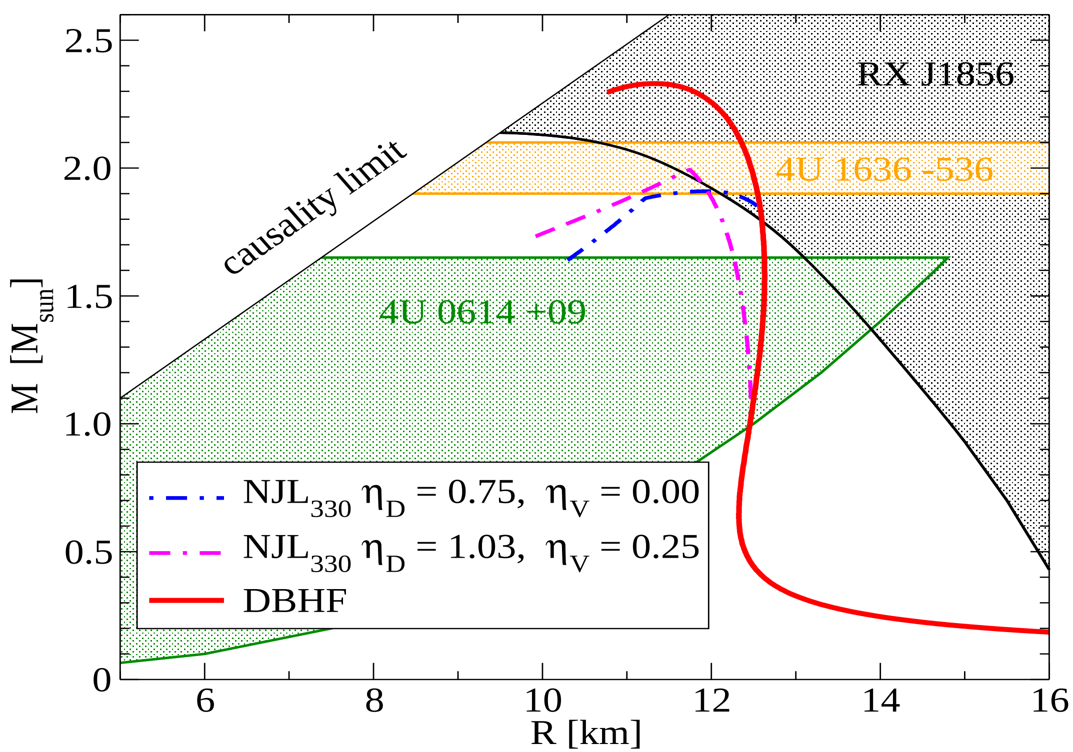
Left: Gap in excitation spectrum ($T = 0$)
Right: 'Gapless' excitations ($T = 60$ MeV)





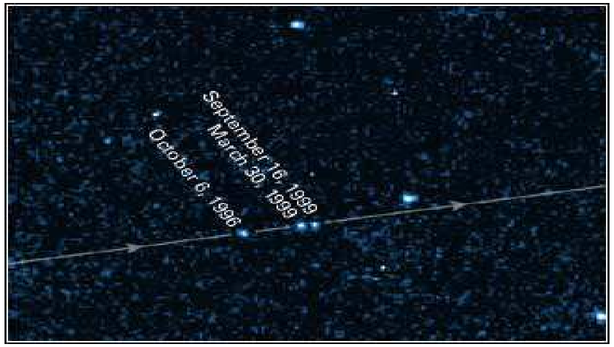
QUARK MATTER IN COMPACT STARS: MASS-RADIUS CONSTRAINT

Solve TOV Eqn. → Hybrid stars fulfill constraint!

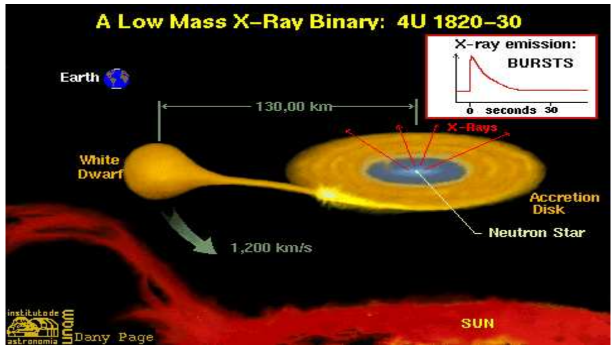


Klähn et al: Constraints on the high-density EoS ...
PRC 74 (2006); [nucl-th/0602038], [astro-ph/0606524]

- Isolated Neutron star RX J1856: M-R constraint from thermal emission

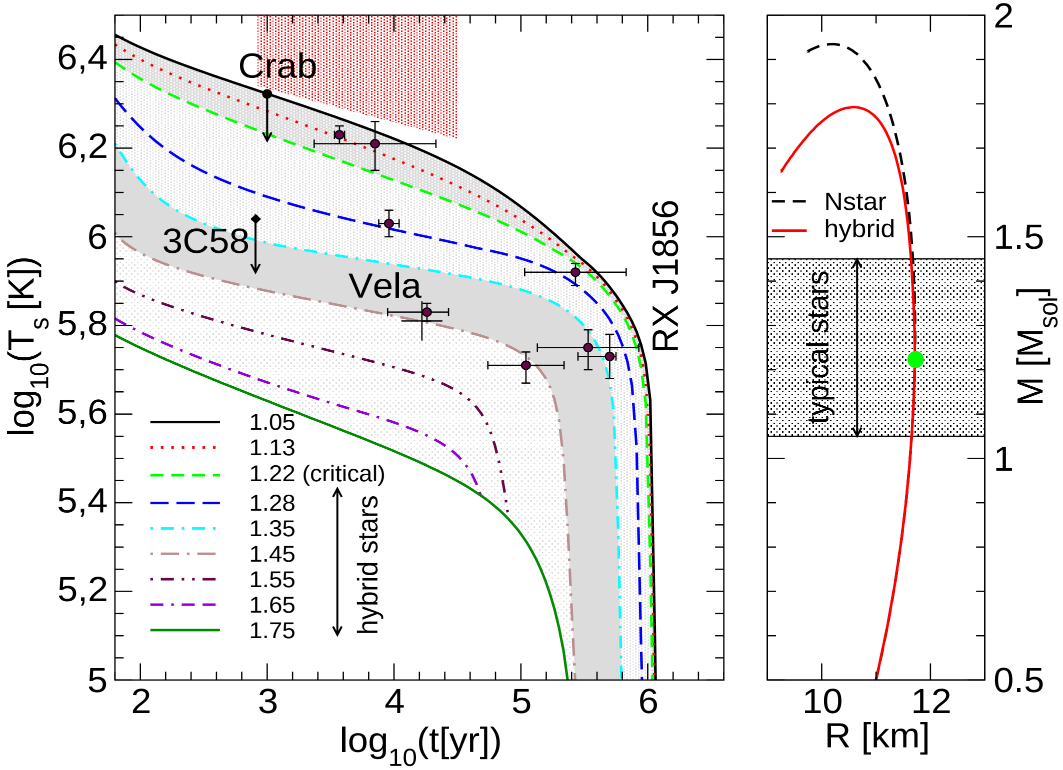


- Low-mass X-ray binary 4U 1636: Mass constraint from ISCO obs.



Quark matter in compact stars: Cooling constraint

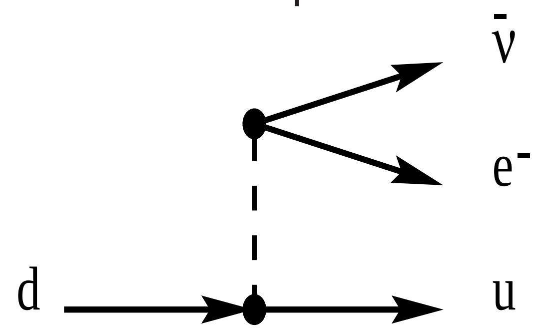
Quark matter in compact stars: color superconducting



- Neutrinos carry energy off the star, Cooling evolution (schematic) by

$$\frac{dT(t)}{dt} = -\frac{\epsilon_\gamma + \sum_{j=Urca,\dots} \epsilon_\nu^j}{\sum_{i=q,e,\gamma,\dots} c_V^i}$$

- Most efficient process: Urca



- Exponential suppression by pairing gaps! $\Delta \sim 10 \dots 100$ keV

Popov et al: Neutron star cooling constraints ...
PRC 74, 025803 (2006); [nucl-th/0512098]

Summary

- Mott-Hagedorn model as alternative interpretation of Lattice data
- Microscopic formulation of the hadronic Mott effect within a chiral quark model
- Mesonic (hadronic) correlations important for $T > T_c$
- Resonance gas with Mott effect and transition to PNJL plasma
- Step-like enhancement of threshold processes due to Mott effect
- Reaction kinetics for strong correlations in plasmas applicable @ SPS and RHIC
- Finite density: critical endpoint, color superconductivity, quark matter in compact stars

Lecture II: Order parameters and phase diagram for NJL-like models

- Polyakov-loop Nambu–Jona-Lasinio (PNJL) model
- Nonlocal (P)NJL models (PL-DSE approach)